

# Assignment 1

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EE19BTECH11050

September 7, 2020

## Problem Statement

52. Many systems are piece-wise linear. That is, over a large range of variable values, the system can be described linearly. A system with amplifier saturation is one such example. Given the differential equation

$$\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x = f(x)$$

assume that  $f(x)$  is as shown in Figure P2.32.

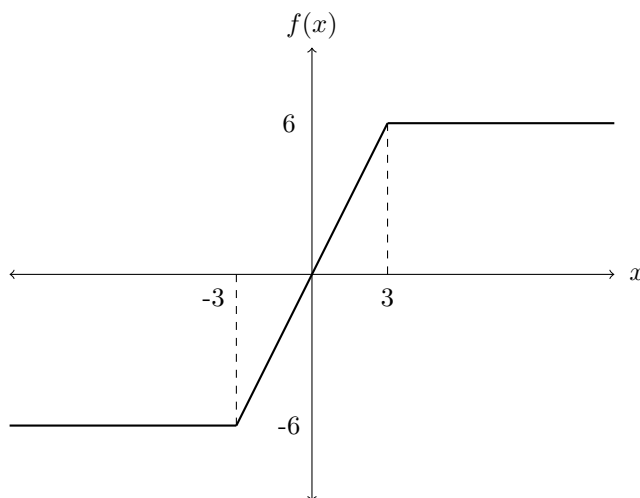


Figure P2.32

Write the differential equation for each of the following ranges of  $x$ :

- a.  $-\infty < x < -3$
- b.  $-3 < x < 3$
- c.  $3 < x < \infty$

## Solution

The function  $f(x)$  displayed in Figure P2.32 can be written as the following piece-wise function:

$$f(x) = \begin{cases} -6 & -\infty < x < -3 \\ 2x & -3 < x < 3 \\ 6 & 3 < x < \infty \end{cases}$$

Substitute the value of  $f(x)$  in the interval  $-\infty < x < -3$ . The resulting differential equation is

$$\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x = -6$$

$$\boxed{\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x + 6 = 0} \quad (1)$$

Substitute the value of  $f(x)$  in the interval  $-3 < x < 3$ . The resulting differential equation is

$$\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x = 2x$$

$$\boxed{\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 48x = 0} \quad (2)$$

Substitute the value of  $f(x)$  in the interval  $3 < x < \infty$ . The resulting differential equation is

$$\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x = 6$$

$$\boxed{\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x - 6 = 0} \quad (3)$$