Project 1: Martingale

American Roulette and Betting Strategy:

0-28-9-26-30-11-7-20-32-17-5-22-34-15-3-24-36-13-1-00-27-10-25-29-12-8-19-31-18-6-21-33-16-4-23-35-14-2

In an American Roulette there are 36 numbers with a single zero and a double zero.
The even numbers are Black, and the odd numbers are Red. The two zero numbers are Green.
The betting strategy is to always bet on black.
The probability of getting a Black is:

Number of black tiles = 18
Total number of tiles = 38

Probability of getting a Black P(B) = Number of black tiles / Total number of tiles = 18/38

Experiment 1: Unlimited Bankroll

1) **Probability of winning $80 within 1000 sequential bets**:

   a) **Method1:**
      We know that probability of winning a single bet is p = 18/38

      To get $80 we need to win at least 80 times within 1000 sequential bets. (Since each win will give us $1 extra than what we had previously. We need to win at least 80 times to get $80)

      $$\mathrm{P}(\mathrm{X} \geq 80) = \sum_{i=80}^{1000} \binom{1000}{i} (18/38)^{i} (20/38)^{1000-i}$$

      $\mathrm{P}(\mathrm{X} \geq 80)$ = 0.9999 ≅ 1

      Hence the probability of getting $80 within 1000 sequential bets is 0.9999 ≅ 1.

      With unlimited bankroll eventually we will get $80 with absolute certainty.

   b) **Method2:**
      From the experiment we know that $80 is reached in all 1000 episodes.
      The probability = Number episodes with $80 / total number of episodes
      Hence, the probability of winning $80 within 1000 sequential bets = 1000/1000 = 1

2) **Estimated expected value of winnings after 1000 sequential bets:**

   a) The estimated expected value of winnings after 1000 sequential bets is the mean of the winnings after 1000 bets in every simulated episode. The probability of getting $80 or more within 1000 bets is 1. The estimated expected value is approximately equal to $80*1 = $80

   b) The expected value

      $$\mathrm{E}(\mathrm{X}) = \sum_{i=0}^{n} i * \binom{n}{i} (p)^{i} (q)^{n-i}$$

      Since the value after getting $80 stays same. i.e., $80 for remaining bets

$$\mathrm{E}(\mathrm{X}) = \sum_{i=0}^{79} i * \binom{1000}{i} \left(\frac{18}{38}\right)^{i} \left(\frac{20}{38}\right)^{1000-i} + 80 * \sum_{i=80}^{1000} \binom{1000}{i} (18/38)^{i} (20/38)^{1000-i}$$

$$\mathrm{E}(\mathrm{X}) \cong \$80$$

3) **Does standard deviation reach a maximum value then stabilize or converge as the number of bets increases**

   The Standard Deviation converges as the number of bets increase. As the number of bets increase the chances of getting $80 increase. This will decrease the variations in the winnings and the standard deviation converges to 0 when the target winnings of $80 is reached.

Experiment 2: Limited Bankroll of $256

1) **Estimate the probability of winning $80 within 1000 sequential bets.**
   From the experiment we know that $80 was won in 646 episodes out of 1000 episodes of 1000 sequential bets each.
   Hence, the probability of winning $80 = 646/1000 = 0.646
2) **What is the estimated expected value of winnings after 1000 sequential bets?**
   The expected value is the $80 * 0.646 + (- $256)*0.354 = -$38.944
   Even though the win probability is more the amount you win is less than what you lose.
   So, the expected value after 1000 sequential bets is - $38.944
3) **Does standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increase?**
   The standard deviation reaches a maximum value and then stabilizes. The Standard deviation increases rapidly as loosing with a limited amount of bankroll is more often. The standard deviation reaches maximum and stabilizes as the chances of getting $80 increases with number of bets and stop betting.

Figures:

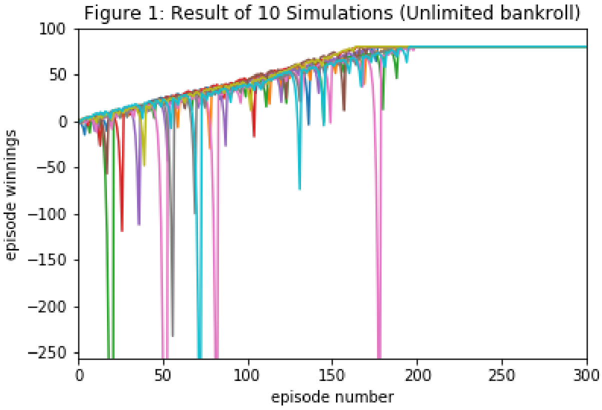

Figure 1: Result of 10 Simulations (Unlimited bankroll)

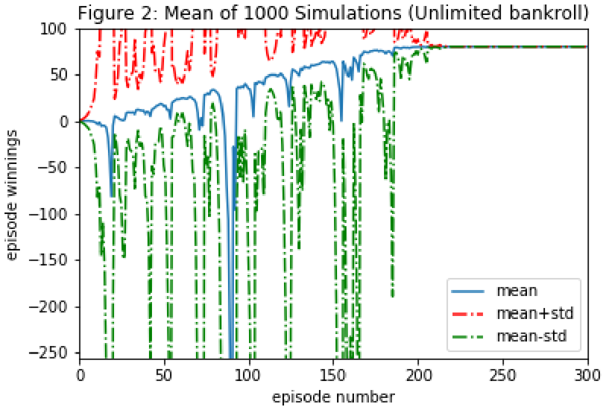

Figure 2: Mean of 1000 Simulations (Unlimited bankroll)

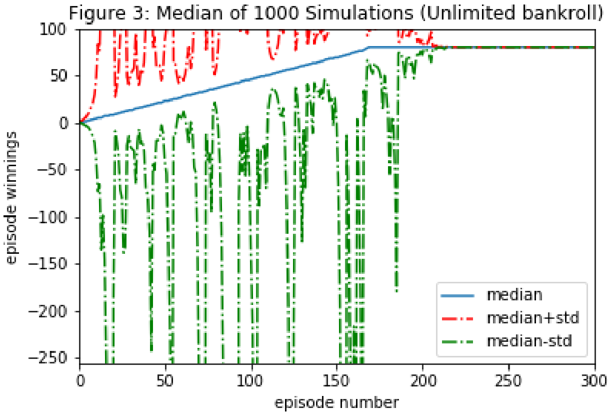

Figure 3: Median of 1000 Simulations (Unlimited bankroll)

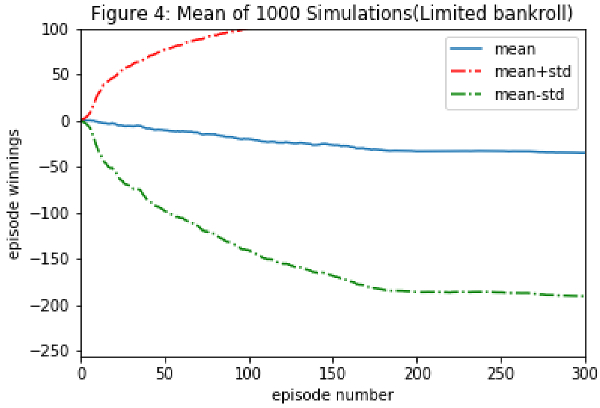

Figure 4: Mean of 1000 Simulations(Limited bankroll)

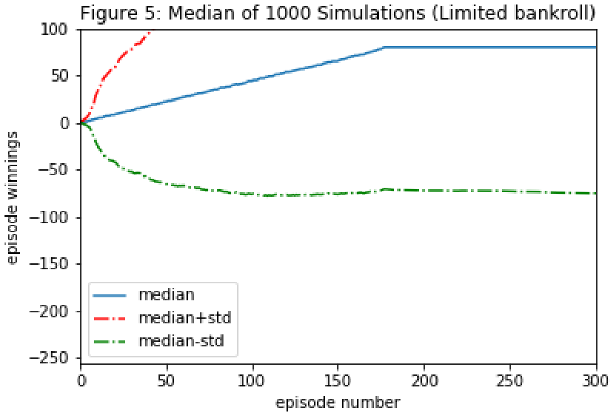

Figure 5: Median of 1000 Simulations (Limited bankroll)