[CS502]-Graph Theory and Applications Fall 2022-23

Independent Study

Member-1:Vetsa Tarun RollNo:12041720 email:vetsatarun@iitbhilai.ac.in

Member-2:G.M.Imdadulla Khan RollNo:12040530 email:gmimdadulla@iitbhilai.ac.in

Graph Colouring Technique in Crime Science

INTRODUCTION:

Graph theory as a tool can be used for investigating the relationships between various entities.
A graph G(V, E) is a collection of vertex sets V(G) and edge sets E(G). Graph colouring refers to the coloring of vertices or edges of a graph.

Terms related to coloring:

Vertex colouring : It is the process of assigning distinct colours to the graph's vertices such that adjacent vertices have different colours.
Edge colouring : It is the process of assigning colours to the edges of a graph so that adjacent edges have different colours.
Chromatic number of a graph : It is the least number of colours required to colour the vertices so that no two adjacent vertices share the same colour.

APPLICATION:

1. Representation of Vertices and Edges:

This method examines three graph elements: the vertex set, the edge set, and the incidence function.

Vertices will be used to represent these elements. When any two elements are connected in any way, that connection is represented as an edge.

2. Method to Construct the Graph:

Initially, We have to provide algorithm for the formation of graph.
Consider the following crime scenario: A jewellery store has been robbed. Assume that the suspects are vertices. Give the vertices distinct colors (in this example,blue, red, and green respectively).
Formation of edges:
If a suspect says he is not the robber, then we draw directed edges (with the same colour as the suspect's vertex) to all the other vertices. i.e., this is similar to accusing all other suspects.
If a suspect accuses other suspect or suspects, then we draw directed edges from the accuser to all the suspects he/she accused.

Algorithm For basic Construction with example:

In this case, the statements of the suspects are recorded in the below table.
There are three suspects and their statements are:

Table I: Information of suspects for the graph G

SUSPECTS	INVESTIGATION
U	I am not the robber
V	U is the robber
W	I am not the robber

First, imagine that the suspects are the vertices U, V, and W.
Step 1: Give each of the vertices a unique colour by giving U, V, and W blue, red, and green respectively.
Step 2: Assign the appropriate colour to the edges and represent the connection between the vertices U, V, and W as edges.
Step 3:Draw lines from suspect U to V and W after he claims he is not the robber. Now give these edges the colour blue.
Step 4: Draw lines from V to U after suspect V claims that U is the robber. Now give these edges the colour red.

Step 5: Suspect W claims he is not the robber, after which lines are drawn from W to U and V. Now give these edges a green colour.

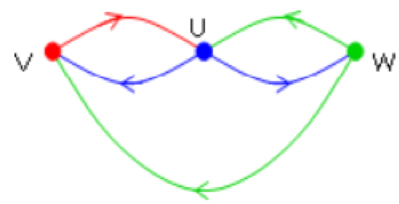

Fig-1.1: Graph for three suspects

3. Role of Colouring Technique:

Proof:

Considering the crime graph G, where the edges signify the connections between the vertices (U,V, and W), which are suspects.
Here We can develop three cases as there are three suspects.

Case 1: Police suspects U to be the robber:

Let's say that if U is the thief, then V or W aren't the thieves. When V claims that U is the robber, he is speaking the truth. Also, W's claim that he is not the robber also becomes a truth and is a contradiction that only one person is speaking facts. Hence U is not the thief.

Graph Colouring for Case 1:

- Let's assume that, U is the thief, neither V nor W are hence, V is telling the truth when he asserts that U committed the robbery.
- If U is the thief, then redraw the graph G by deleting all of the lines that branch off of U.
- Remove the line from W to V because V is speaking the truth when he claims that U is the robber.

- But W asserts that he is not the robber, which contradicts what one person claims to be true and is explicitly explained in graph G2.
- Two lines graphically pointing in the direction of U, which is contradictory. Thus, U is not the thief.

Fig-2: Graph Colouring if U is suspect

Case 2: Police suspects V to be the robber:

Let's say that if V is the thief, U or W are not. As he claims he is not the robber, U is here being truthful. V and W's statements must be lies. V now commits a lie when he claims that U is the robber. However, W asserts that he is not the robber, which is in conflict with what one person claims to be true. Therefore, V is not the thief.

Graph Colouring for Case 2:

- If V is the thief, then redraw the graph G by deleting all of the lines that branch off of V.
- Remove the line from W to U because U is speaking the truth when he claims he is not the robber.
- But W asserts that he is not the robber, which contradicts what one person claims to be true and is explicitly explained in graph G2.
- Two lines graphically pointing in the direction of V, which is contradictory. Thus, V is not the thief.

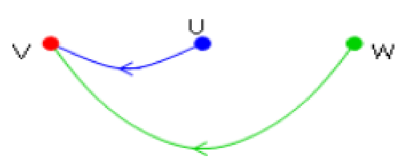

Fig-3: Graph Colouring if V is suspect

Case 3: Police suspects W to be the robber:

Let's say that if W is the thief, V or U are not. As he claims he is not the robber, U is here being truthful. From the statements of V and W must be lies and their statements are in accordance with this. This situation, where one person is telling the truth, is logically consistent. Therefore, W is the thief.

Graph Colouring for Case 3:

- If W is the thief, then redraw the graph by erasing all of the lines that branch off of W.
- Remove all lines pointing at U because, as he claims, he is not the robber in this instance in consideration that U is speaking the truth.
- Remove the line from V to U because, according to V, U is the thief and a liar, as shown in the graph G3.
- From W to U, there is a single line that is logically consistent and depicts one person telling the truth. W is therefore the thief.

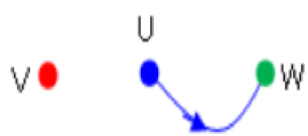

Fig-4: Graph Colouring if W is suspect

Observations:
We can create a tuple which shows the number of lines leading to the suspects, In the given example, graphical sequence (2,2,1), which shows the number of lines leading to (U,V,W). This process accurately determines whether a person is a robber based on whether they are telling the truth.

Basic Algorithm to construct Graphical Sequence:

Step 1: Consider a vertex in the graph G.

Step 2: The number of edges incident on to the vertex indicates the no.of people who accuse him as the suspect. This count can be used for varying conditions of the no. of people telling the truth.

Step 3: Repeat the iteration process.

- In this investigation, we came to the conclusion that W is the robber based on the condition that only one person telling the truth.
- If the condition involves that two people are telling the truth, then the thief would be either V or W. As a result, we can only say with certainty that W is not the person in question in this situation.

4. General Case : In case of more than three suspects :

Here we discuss about the case of more suspects X1,X2,X3... for a complicated set of acquisitions. For example, we extend the robbery case to four suspects in the crime scene. The information of the suspects are given in the following table.

Table II: Information of Suspects for the graph G*

SUSPECTS	INVESTIGATION
P	I am not the robber
Q	P is the robber
R	I am not the robber
S	Q is the robber

Proof:

Consider the crime graph G*, where the edges signify the connections between the suspects at the vertex points P, Q, R, and S.
Give the vertices the colours blue, green, red, and yellow.
If Suspect P claims he is not a thief, then lines should be drawn from P to Q, R, and S, and the corresponding vertex and edges should be given the colour blue.
When Q, the suspect, claims that P is the thief, a line connecting Q and P is drawn, and the colour green is assigned.
When suspect R claims he is not the thief, lines should be drawn from R to P, Q,

and S, and the colour green should also be assigned.
When suspect S claims that suspect Q is the thief, yellow is assigned to the line that connects S and Q.This is a representation of the four-vertex graph that shows how the robber can be located.

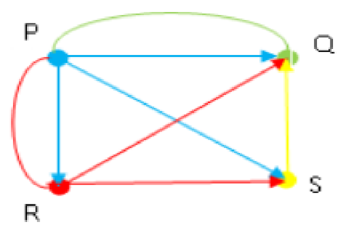

Fig-5: Graph Colouring for four Suspects.

As a result of the investigation into the crime problem, we arrive at the following sequence (2,3,1,2), which shows the number of lines leading (P,Q,R,S). We can determine potential solutions by counting the number of edges that that are incident on to the vertices, which detects the number of people telling the truth. If a person is a robber, this sequence can clearly identify how many of them are telling the truth.

Observations:
1. In the cases of this extension of proposition 1, which is proven above, it is evident from the solution set that R is the robber for the condition that one person is telling the truth.
2. We arrive at the options P or S for two people telling the truth. We are unable to identify the precise individual, but we can say that Q and R are definitely not the right choices.
3. Our accurate conclusion that Q is the robber is based on three people telling the truth. With fewer suspects, these solutions assist the crime scientist in directly identifying the offender. This method reduces confusion over suspects in higher orders.

Conclusion:

In this paper, we offer the information needed by law enforcement to detect crimes. This study combines graph colouring methods with crime scene analysis to create a mathematical model of crime.

References:

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