Assignment 0
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1.1

(a)

We can define X as

$$X = CDF^{-1}(Y)$$

(assuming CDF is invertible)

Proof : let $Z = CDF^{-1}(Y)$ we will prove that Z and X have the same probability distribution.

Now, since cdf is always non decreasing :

$$\begin{aligned} P(Z < Z_0) &= P(CDF^{-1}(Y) < Z_0) \\ &= P(Y < CDF(Z_0)) \\ &= CDF(Z_0) \end{aligned}$$

thus CDF (Z) = CDF (X)

and are proposition is correct.

(b)

X is an exponential random variable

$$PDF(X) = \lambda . \exp(-\lambda x)$$

thus, $CDF(X) = 1 - \exp(-\lambda x)$

thus, $CDF^{-1}(Y) = -\log(1 - Y) / \lambda$

thus, $X = -\log(1 - Y) / \lambda$

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(a)

let F(n) = expected number of tosses to get n heads
let p be probability of head

then $F(n) = p . (F(n-1) + 1) + (1 - p) (F(n-1) + 1 + F(n))$

thus, $p. F(n) = F(n-1) + 1$
$F(n) = F(n-1)/p + 1/p$

here n =2 and p = 0.75
F(0) = 0
So, F(1) = 1/p = 4/3
So, $F(2) = 1/p + 1/p^2 = 4/3 + 16/9 = 28/9$