Analysis and Design of Beams for Bending
Lecture 15

Engineering Mechanics - ME102

Courtesy: TMH

Application

Forces that are *internal* to the structural members – beams – are the subject of this chapter

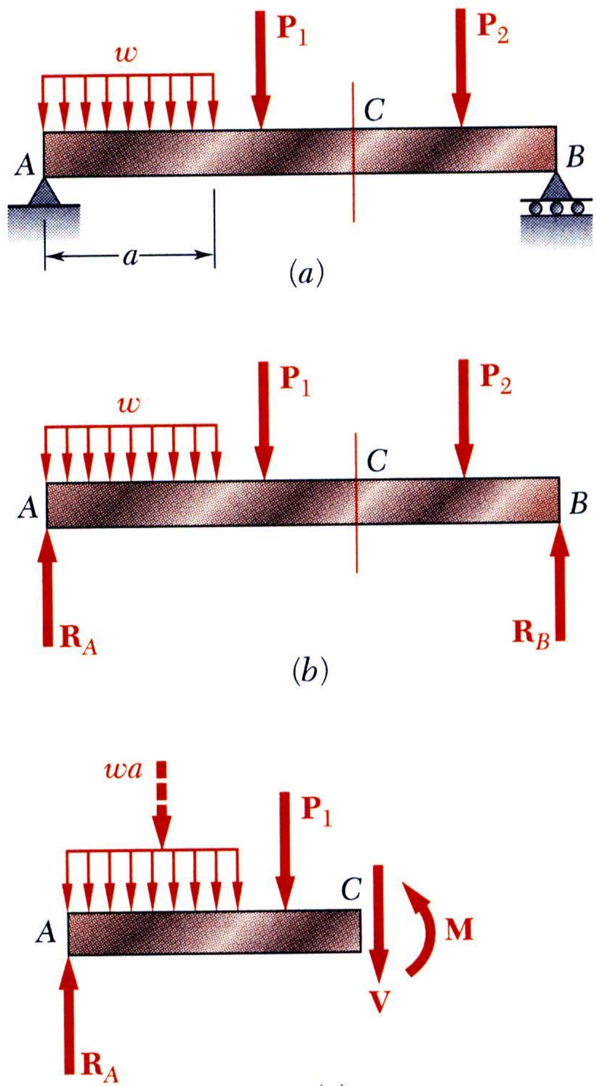
Courtesy: TMH

Introduction

- We will focus on beams:

  Beams **- usually long, straight, prismatic members designed to support loads applied at various points along the member.**

Introduction



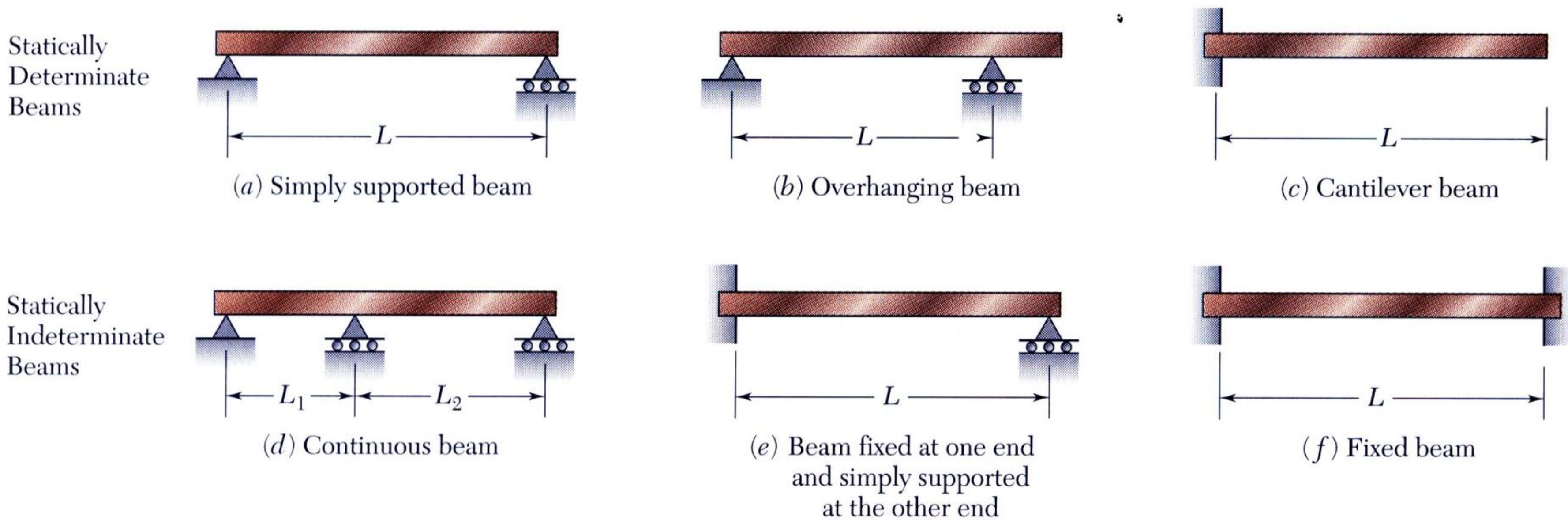
- Objective - Analysis and design of beams
- *Beams* - structural members supporting loads at various points along the member
- Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)
- Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

Requires determination of the location and magnitude of largest bending moment

Introduction

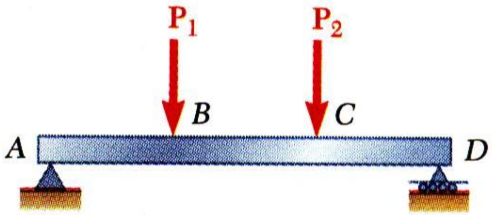
Classification of Beam Supports



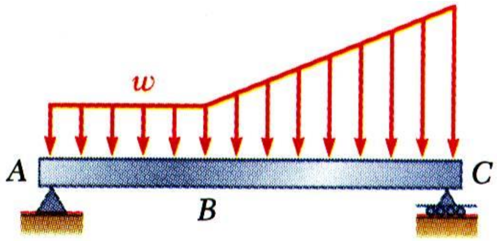
- Beams are classified according to the way in which they are supported.
- Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.

Courtesy: TMH

Various Types of Beam Loading and Support



(*a*) Concentrated loads

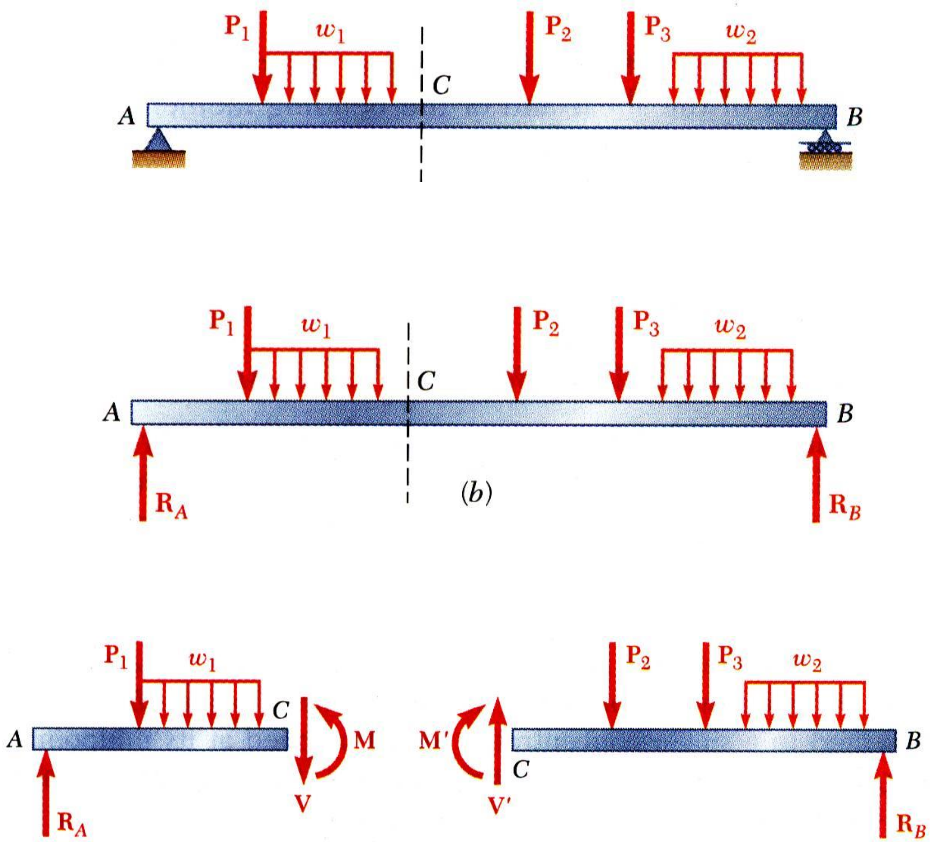


(*b*) Distributed load

- *Beam* - structural member designed to support loads applied at various points along its length.
- Beam can be subjected to *concentrated* loads or *distributed* loads or combination of both.
- *Beam design* is a two-step process:
  1) determine shearing forces and bending moments produced by applied loads
  2) select cross-section best suited to resist shearing forces and bending moments

Courtesy: TMH

Shear and Bending Moment in a Beam



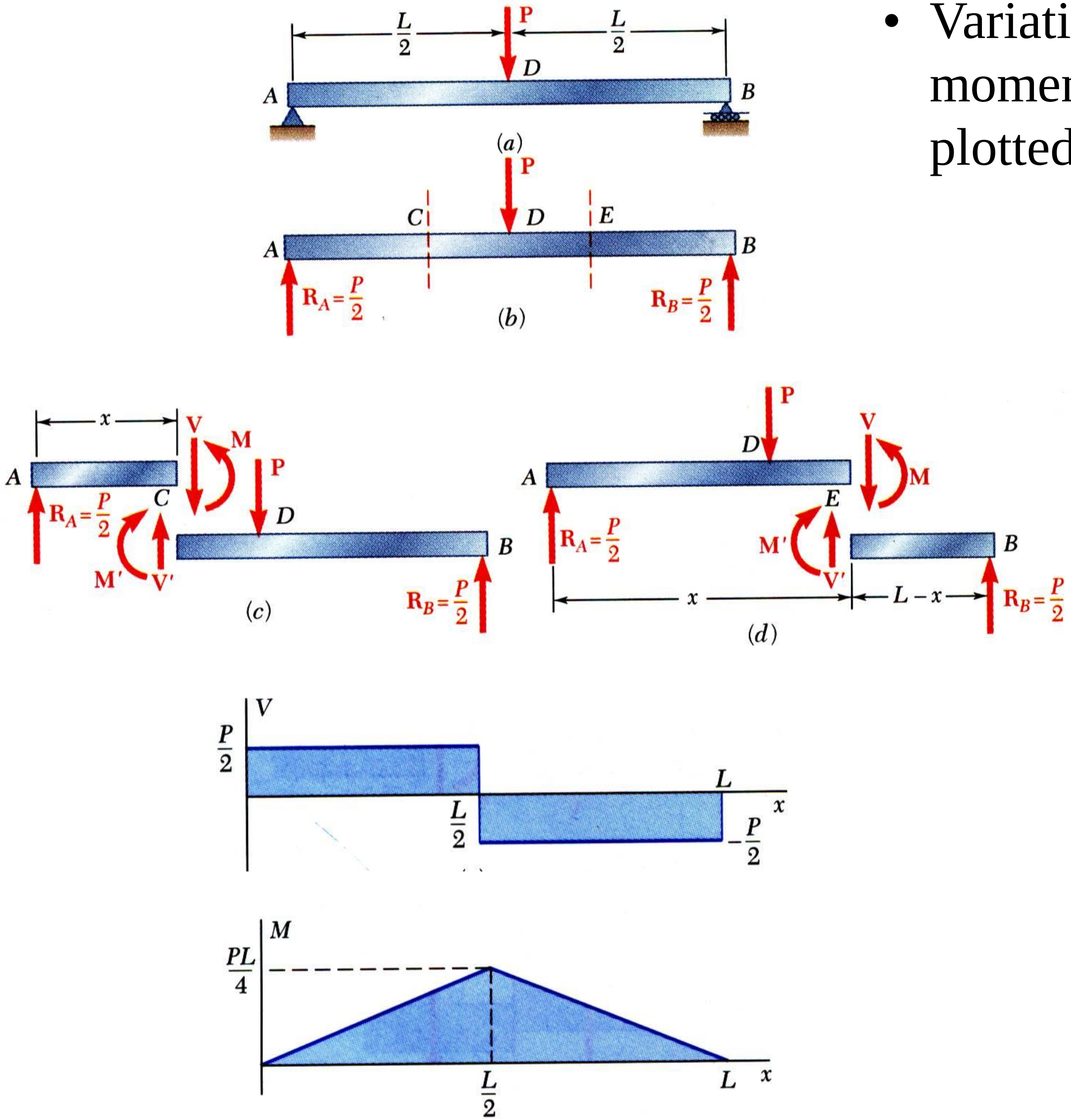
- Wish to determine bending moment and shearing force at any point (for example, point *C*) in a beam subjected to concentrated and distributed loads.

- Determine reactions at supports by treating whole beam as free-body.

- Cut beam at *C* and draw free-body diagrams for *AC* and *CB*. <u>By **definition, positive sense for internal force-couple systems are as shown for each beam section**</u>**.**

- From equilibrium considerations, determine **M** and **V** or **M'** and **V'**.

Shear and Bending Moment Diagrams

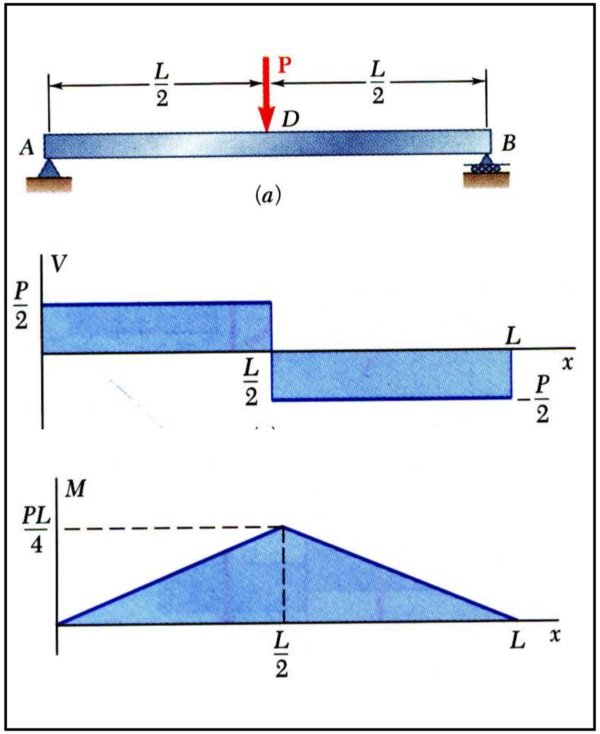


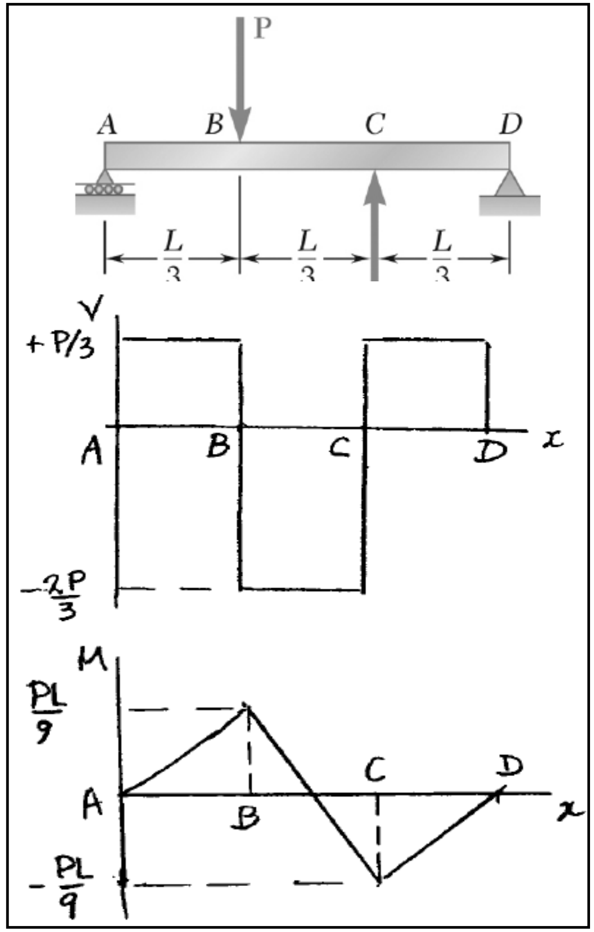
- Variation of shear and bending moment along beam may be plotted.
  - Determine reactions at supports.
  - Cut beam at C and consider member AC,

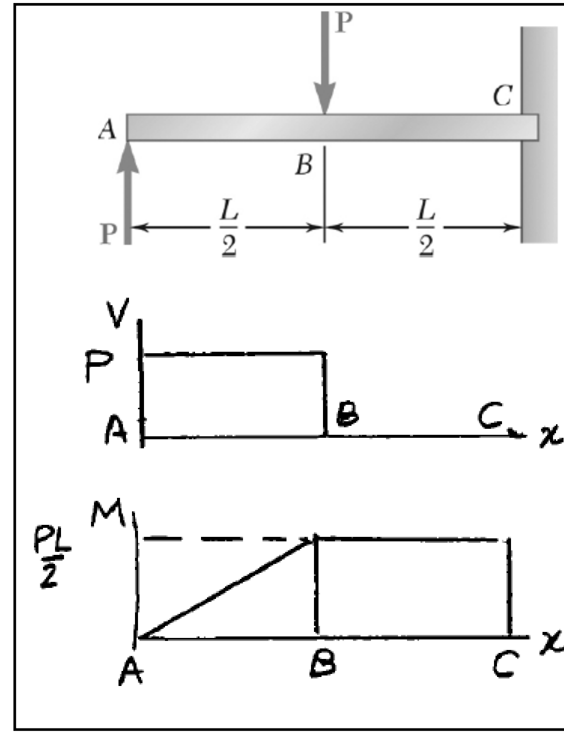
    $V = +P/2 \quad M = +Px/2$
  - Cut beam at E and consider member EB,

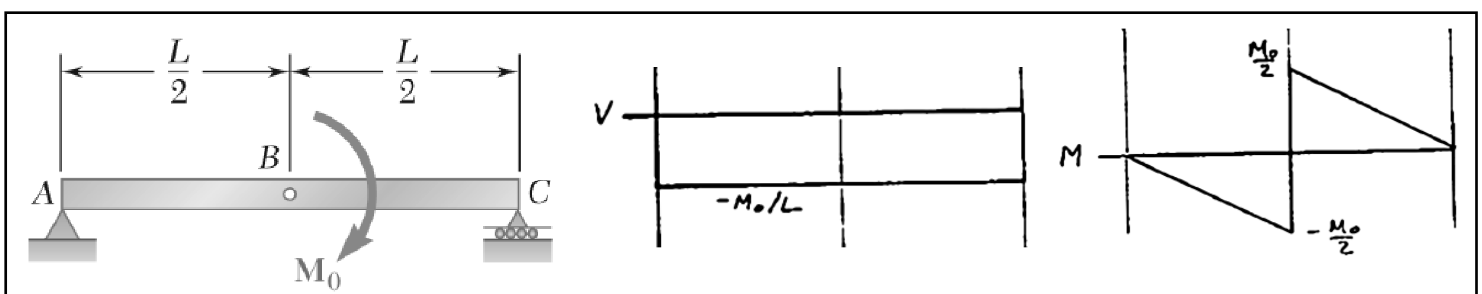
    $V = -P/2 \quad M = +P(L-x)/2$
  - For a beam subjected to <u>concentrated loads</u>, shear is constant between loading points and moment varies linearly.

Shear Force and Bending Moment Diagrams –
Examples for practice

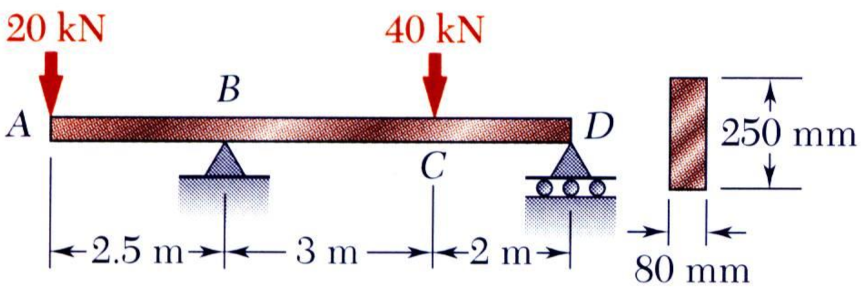






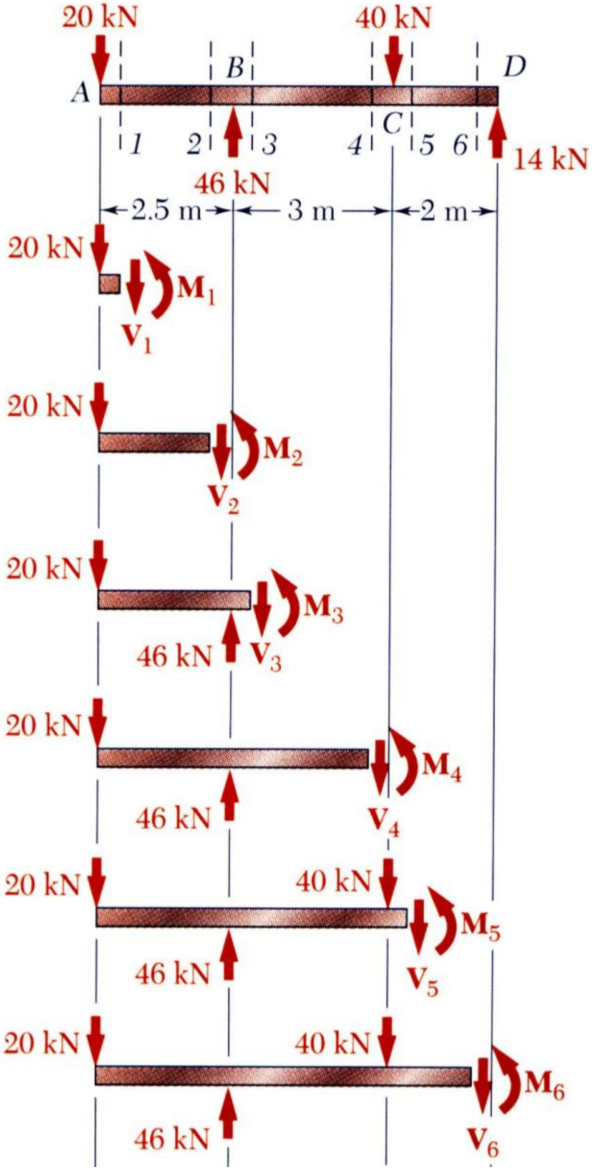


Sample Problem 5.1



For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

Sample Problem 5.1



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces

  from $\sum F_y = 0 = \sum M_B: \quad R_B = 40\,\text{kN} \quad R_D = 14\,\text{kN}$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\text{ kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$

$$\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\text{ kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$

$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$

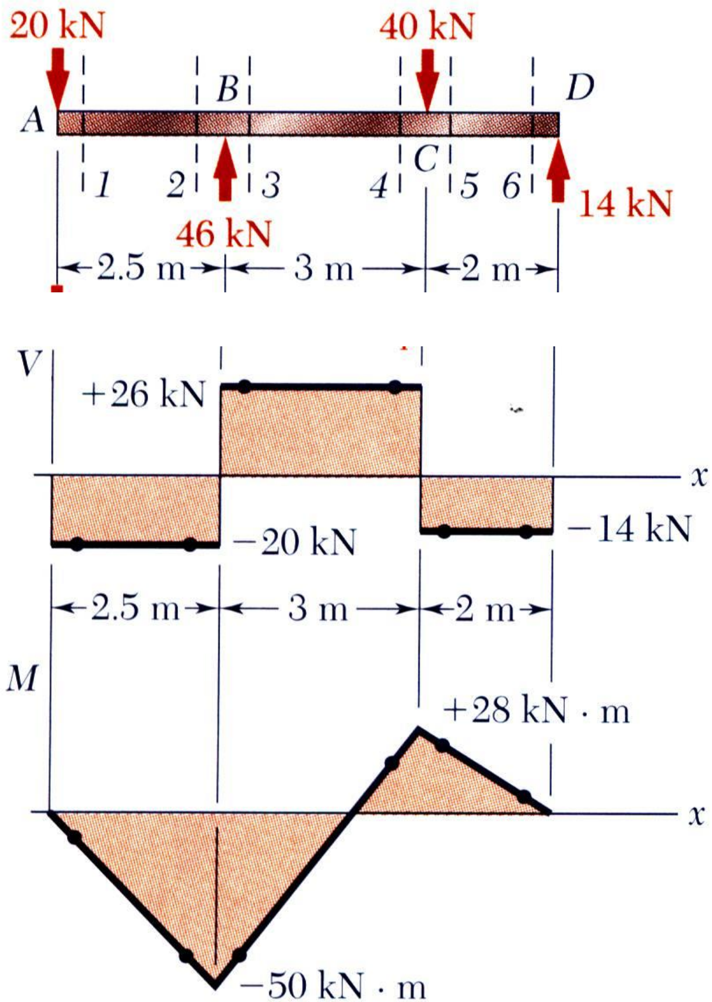
$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$

$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$

$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

Courtesy: TMH

Sample Problem 5.1



- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\mathrm{kN} \quad M_m = |M_B| = 50\,\mathrm{kN\cdot m}$$

- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

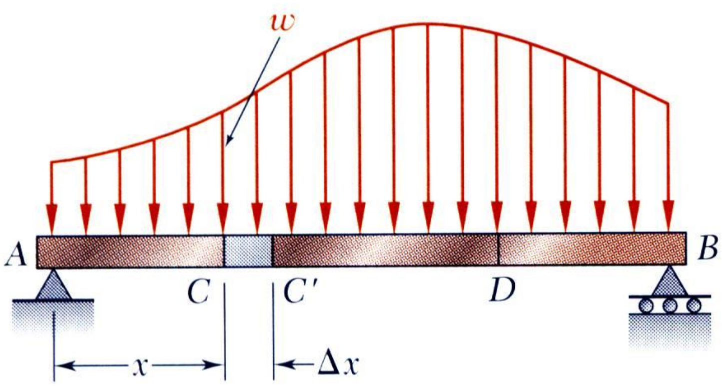
$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6}(0.080\,\mathrm{m})(0.250\,\mathrm{m})^2$$
$$= 833.33\times10^{-6}\,\mathrm{m}^3$$

$$\sigma_m = \frac{|M_B|}{S} = \frac{50\times10^3\,\mathrm{N\cdot m}}{833.33\times10^{-6}\,\mathrm{m}^3}$$

$$\boxed{\sigma_m = 60.0\times10^6\,\mathrm{Pa}}$$

Courtesy: TMH

Relations Among Load, Shear, and Bending Moment

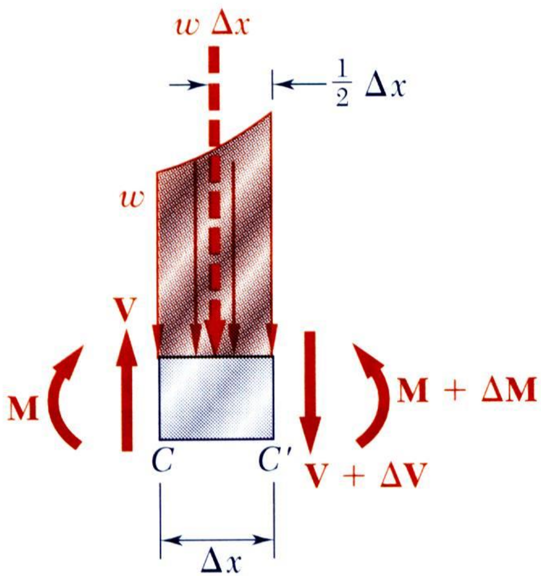


- Relations between load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) - w\Delta x = 0$$

$$\frac{dV}{dx} = \lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\, dx = -(\text{area under load curve between C and D})$$



- Relations between shear and bending moment:

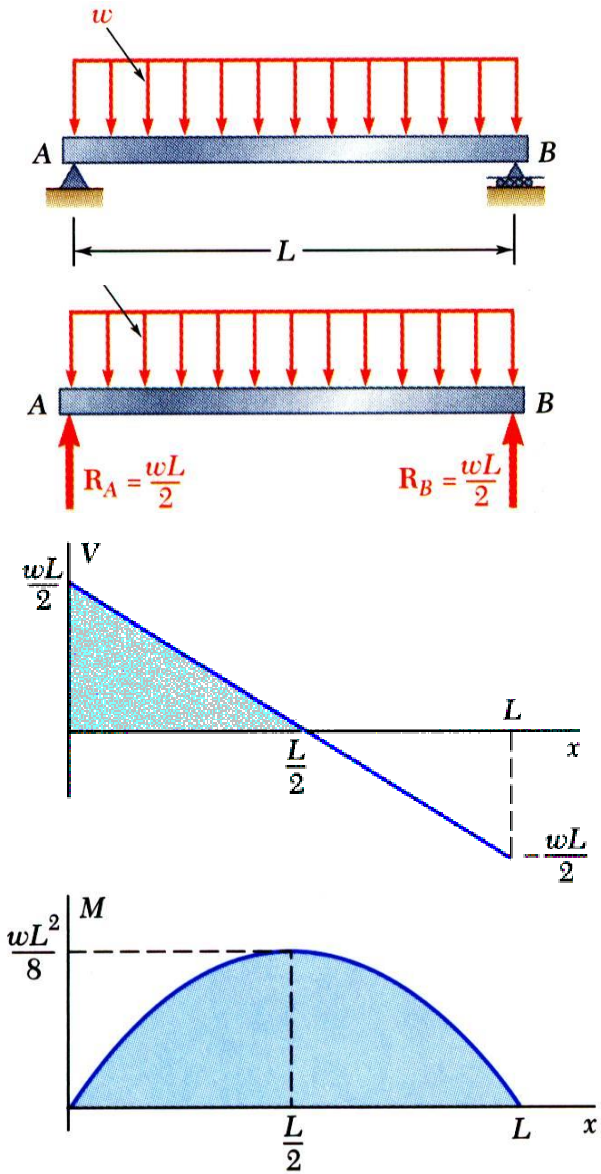
$$\sum M_{C'} = 0: \qquad (M + \Delta M) - M - V\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\frac{dM}{dx} = \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \lim_{\Delta x \to 0} \left(V - \tfrac{1}{2} w \Delta x\right) = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\, dx = (\text{area under shear curve})$$

Courtesy: TMH

Relations Among Load, Shear, and Bending Moment



- Reactions at supports, $R_A = R_B = \dfrac{wL}{2}$
- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

- Moment curve,

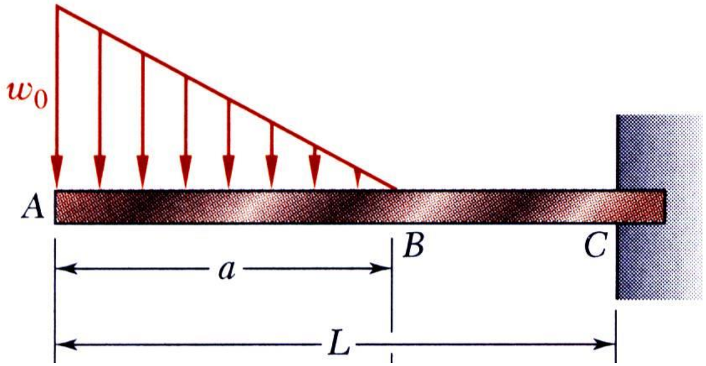
$$M - M_A = \int_0^x V dx$$

$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

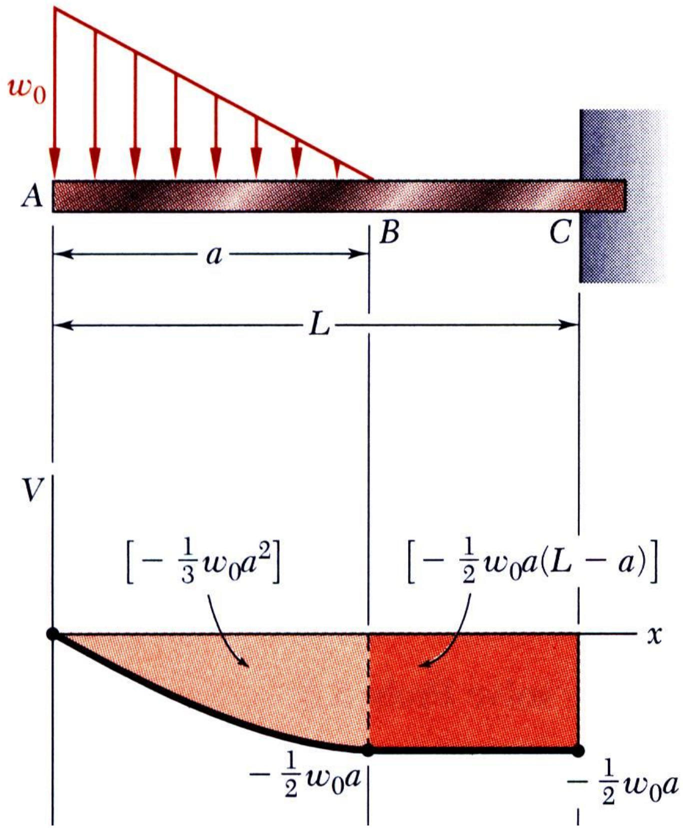
Courtesy: TMH

Sample Problem 5.5



Draw the shear and bending moment diagrams for the beam and loading shown.

Sample Problem 5.5



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.

$$\sum F_y = 0 = -\tfrac{1}{2}w_0a + R_C \qquad R_C = \tfrac{1}{2}w_0a$$

$$\sum M_C = 0 = \tfrac{1}{2}w_0a\left(L - \frac{a}{3}\right) + M_C \qquad M_C = -\tfrac{1}{2}w_0a\left(L - \frac{a}{3}\right)$$

Results from integration of the load and shear distributions should be equivalent.

- Apply the relationship between shear and load to develop the shear diagram.

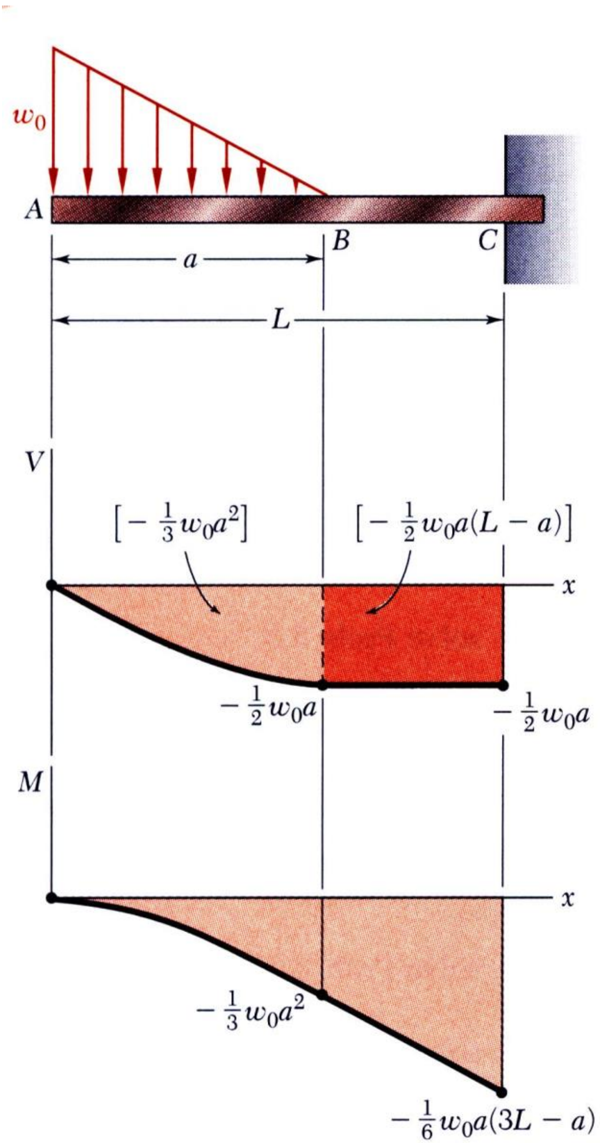
$$V_B - V_A = -\int_0^a w_0\left(1 - \frac{x}{a}\right)dx = -\left[w_0\left(x - \frac{x^2}{2a}\right)\right]_0^a$$

$$V_B = -\tfrac{1}{2}w_0a = -(\,area\ under\ load\ curve)$$

  - No change in shear between *B* and *C*.
  - Compatible with free body analysis

Courtesy: TMH

Sample Problem 5.5



- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$M_B - M_A = \int_0^a \left(-w_0 \left(x - \frac{x^2}{2a} \right) \right) dx = \left[-w_0 \left(\frac{x^2}{2} - \frac{x^3}{6a} \right) \right]_0^a$$

$$M_B = -\tfrac{1}{3} w_0 a^2$$

$$M_B - M_C = \int_a^L \left(-\tfrac{1}{2} w_0 a \right) dx = -\tfrac{1}{2} w_0 a (L - a)$$

$$M_C = -\tfrac{1}{6} w_0 a (3L - a) = \frac{a\,w_0}{2} \left(L - \frac{a}{3} \right)$$

Results at *C* are compatible with free-body analysis