CS225 Switching Theory

S. Tripathy
IIT Patna

Previous Class

Minimization/ Simplification **of Switching Functions**

K-map (SOP)

This Class

Minimization/ Simplification **of Switching Functions**

Quine-McCluskey (Tabular) Minimization

Some Definitions

- Implicant: A product term that has non-empty intersection with on-set F and does not intersect with off-set R .

- Prime Implicant: An implicant that is not covered by another implicant.

- Essential Prime Implicant: A prime implicant with at least one element that is not covered by one or more prime implicants

Procedure for Deriving Minimal Sum-of-products Expression

Procedure:

1. Obtain all essential prime implicants and include them in the minimal expression

2. Remove all prime implicants which are covered by the sum of some essential prime implicants

3. If the set of prime implicants derived so far covers all the minterms, it yields a unique minimal expression. Otherwise, select additional prime implicants so that the function is covered completely and the total number and size of the added prime implicants are minimal

Example: prime implicant xz is covered by the sum off our essential prime implicants, and

hence xz must not be included in any irredundant expression of the function

yz \ wx	00	01	11	10
00			1	
01	1	1	1	
11		1	1	1
10		1		

Tabulation Procedure for Obtaining the Set of All Prime implicants

Systematic Quine-McCluskey tabulation procedure: for functions with a large number of variables

- Fundamental idea: repeated application of the combining theorem $Aa + Aa' = A$ on all adjacent pairs of terms yields the set of all prime implicants

Example: minimize $f_1(w,x,y,z) = \sum(0,1,8,9) = w'x'y'z' + w'x'y'z + wx'y'z' + wx'y'z$

- Combine first two and last two terms to yield

$$f_1(w,x,y,z) = w'x'y'(z' + z) + wx'y'(z' + z) = w'x'y' + wx'y'$$

- Combine this expression in turn to yield

$$f_1(w,x,y,z) = x'y'(w' + w) = x'y'$$

- Similar result can be obtained by initially combining the first and third and the second and fourth terms

Tabulation Procedure (Contd.)

Two k-variable terms can be combined into a single (k-1)-variable term if and only if they have k-1 identical literals in common and differ in only one literal

- *Using the binary representation of minterms: two minterms can be combined if their binary representations differ in only one position*

Example: w'x'y'z (0001) and wx'y'z (1001) can be combined into -001,

indicating w has been absorbed and the combined term is x'y'z

Example of Different Notations

$F(A, B, C, D) = \sum m(4,5,6,8,10,13)$

	Full variable	Cellular	1,0,-
1	$\overline{A}B\overline{C}\overline{D}$	4	0100
	$A\overline{B}\overline{C}\overline{D}$	8	1000
2	$\overline{A}B\overline{C}D$	5	0101
	$\overline{A}BC\overline{D}$	6	0110
	$A\overline{B}C\overline{D}$	10	1010
3	$AB\overline{C}D$	13	1101

Notation Forms

- Full variable form - variables and complements in algebraic form
  - hard to identify when adjacency applies
  - very easy to make mistakes
- Cellular form - terms are identified by their decimal index value
  - Easy to tell when adjacency applies; indexes must differ by power of two (one bit)
- 1,0,- form - terms are identified by their binary index value
  - Easier to translate to/from full variable form
  - Easy to identify when adjacency applies, one bit is different
  - shows variable(s) dropped when adjacency is used
- Different forms may be mixed during the minimization

Tabulation Procedure (Contd.)

Procedure:

1. *Arrange all minterms in groups, with all terms in the same group having the same number of 1's. Start with the least number of 1's (called the index) and continue with groups of increasing numbers of 1's.*

2. *Compare every term of the lowest-index group with each term in the successive group. Whenever possible, combine them.*

   Repeat by comparing each term in a group of index i with every term in the group of index i + 1. Place a check mark next to every term which has been combined with at least one term.

3. *Compare the terms generated in step 2 in the same fashion: generate a new term by combining two terms that differ by only a single 1 and whose dashes are in the same position.*

   Continue until no further combinations are possible. The remaining unchecked terms constitute the set of prime implicants.

Example

Example: apply procedure to $f_2 \sum(w,x,y,z)$ = (0,1,2,5,7,8,9,10,13,15)

Step (i)

	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

Step (ii)

	w	x	y	z	
0,1	0	0	0	--	✓
0,2	0	0	--	0	✓
0,8	--	0	0	0	✓
1,5	0	--	0	1	✓
1,9	--	0	0	1	✓
2,10	--	0	1	0	✓
8,9	1	0	0	--	✓
8,10	1	0	--	0	✓
5,7	0	1	--	1	✓
5,13	--	1	0	1	✓
9,13	1	--	0	1	✓
7,15	--	1	1	1	✓
13,15	1	1	--	1	✓

Step (iii)

	w	x	y	z	
0,1,8,9	--	0	0	--	A
0,2,8,10	--	0	--	0	B
1,5,9,13	--	--	0	1	C
5,7,13,15	--	1	--	1	D

P= {x′y′, x′z′, y′z, xz}

Find the K-map and match with this result

Prime Implicant Chart

Prime implicant chart: pictorially displays covering relationships between prime implicants and minterms

Example: prime implicant chart for $f_2(w,x,y,z) = \sum(0,1,2,5,7,8,9,10,13,15)$

	✓0	1	✓2	✓5	✓7	✓8	9	✓10	✓13	✓15
$A = x'y'$	×	×				×	×			
✓ $B = x'z'$	×		⊗			×		⊗		
$C = y'z$		×		×			×		×	
✓ $D = xz$				×	⊗				×	⊗

Cover: a row is said to cover the columns in which it has x's

Problem: select a minimal subset of prime implicants such that each column contains at least one x in the rows corresponding to the selected subset and the total number of literals in the prime implicants selected is as small as possible

Essential rows: if a column contains a single x, the prime implicant corresponding to the row in which the x appears is essential, e.g., B, D

Cover remaining minterms 1 and 9 using A or C: thus, two minimal expressions:

$$f_2 = x'z' + xz + x'y' \text{ or } f_2 = x'z' + xz + y'z$$

Tabulation Procedure using Decimal Notation in the Presence of Don't-cares

Example: apply procedure to $f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25,27,29,31)$

$+ \sum_{\phi}(1,2,12,24)$

(a)

Index	Step 1	
1	1	00001 ✓
	2	00010 ✓
	12	01100 ✓
2	17	10001 ✓
	18	10010 ✓
	20	10100 ✓
	24	11000 ✓
	13	01101 ✓
3	19	10011 ✓
	21	10101 ✓
	25	11001 ✓
	15	01111 ✓
4	23	10111 ✓
	27	11011 ✓
	29	11101 ✓
5	31	11111 ✓

(b)

Step 2

(1,17)	-0001	H
(2,18)	-0010	G
(12,13) :	0110-	F
(17,19) :	100-1	✓
(17,21) :	10-01	✓
(17,25) :	1-001	✓
(18,19) :	1001-	E
(20,21) :	1010-	D
(24,25) :	1100-	C
(13,15) :	011-1	✓
(13,29) :	-1101	✓
(19,23) :	10-11	✓
(19,27) :	1-011	✓
(21,23) :	101-1	✓
(21,29) :	1-101	✓
(25,27) :	110-1	✓
(25,29) :	11-01	✓
(15,31) :	-1111	✓
(23,31) :	1-111	✓
(27,31) :	11-11	✓
(29,31) :	111-1	✓

(c)

Step 3

(17,19,21,23)	10--1	✓
(17,19,25,27)	1-0-1	✓
(17,21,25,29)	1--01	✓
(13,15,29,31)	-11-1	B
(19,23,27,31)	1--11	✓
(21,23,29,31)	1-1-1	✓
(25,27,29,31)	11--1	✓

(d)

Step 4

(17,19,21,23,25,27,29,31) A
1---1

(17,19,25,27, 21,23,29,31)
1---1

(17,21,25,29,19,23,27,31)
duplicate 1---1

P= {vz, wxz, vwx'y', vw'xy', vw'x'y, v'wxy', w'x'yz', w'x'y'z'}

Don’t-care Combinations

Don’t-cares: not listed as column headings in the prime implicant chart

Example: $f_3(v,w,x,y,z) = \sum(13,15,17,18,19,20,21,23,25,27,29,31) + \sum_{\phi}(1,2,12,24)$

	✓13	✓15	✓17	18	✓19	✓20	✓21	✓23	✓25	✓27	✓29	✓31
✓ $A = vz$			×		×		×	⊗	×	⊗	×	×
✓ $B = wxz$	×	⊗									×	×
$C = vwx'y'$									×			
✓ $D = vw'xy'$						⊗	×					
$E = vw'x'y$				×	×							
$F = v'wxy'$	×											
$G = w'x'yz'$				×								
$H = w'x'y'z$			×									

Selection of nonessential prime implicants facilitated by listing prime implicants in decreasing order of the number of minterms they cover

Essential prime implicants: A, B, and D. They cover all minterms except 18, which can be covered by E or G, giving rise to two minimal expressions

Obtain the result using K-map and match