



# ICS141: Discrete Mathematics for Computer Science I

Dept. Information & Computer Sci., University of Hawaii

Originals slides by Dr. Baek and Dr. Still, adapted by J. Stelovsky

Based on slides Dr. M. P. Frank and Dr. J.L. Gross

Provided by McGraw-Hill



# Lecture 1

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## Course Overview

## Chapter 1. The Foundations

### 1.1 Propositional Logic



# Class Information

- Instructor: **Jan Stelovsky** (POST 305C)
  - Email: [janst@hawaii.edu](mailto:janst@hawaii.edu)
  - Skype: **havajsky**
  - Office hours: M/W 3:00 p.m. – 4:00 p.m. or by appointment
- TA: **Christopher Mullins** (POST 303–5)
  - Email: [cmmullin@hawaii.edu](mailto:cmmullin@hawaii.edu)
  - Office hours: TBA
- Textbook: *Discrete Mathematics and Its Applications* (6<sup>th</sup> Edition), by Kenneth H. Rosen, McGraw-Hill
- Web site:
  - Lecture: [www2.hawaii.edu/~janst/141](http://www2.hawaii.edu/~janst/141)
  - Recitation: [www2.hawaii.edu/~cmmullin/141](http://www2.hawaii.edu/~cmmullin/141)
- **Grading**
  - Final exam: **40%**
  - Midterm Exam: **25%**
  - Homework assignments: **25%**
  - Quizzes: **5%**
  - Recitation attendance: **5%**

# What is Mathematics, really?

- It's *not* just about numbers!
- Mathematics is *much* more than that:

Mathematics is, most generally, the study of  
any and all *absolutely certain* truths about  
any and all *perfectly well-defined* concepts.

- These concepts can be *about* numbers, symbols, objects, images, sounds, *anything*!
- It is a way to interpret the world around you.

# So, what's *this* class about?

What are “discrete structures” anyway?

- “**Discrete**” - Composed of distinct, separable parts. (Opposite of *continuous*.)

*discrete:continuous :: digital:analog*

- “**Structures**” - Objects built up from simpler objects according to some definite pattern.
- “**Discrete Mathematics**” - The study of discrete, mathematical (i.e. well-defined conceptual) objects and structures.

# Discrete Objects/Concepts and Structures We Study



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## ■ DM PART I

- Propositions
- Predicates
- Proofs
- Sets
- Functions
- Orders of Growth
- Algorithms
- Integers
- Summations
- Sequences
- Strings
- Permutations
- Combinations
- Probability

## ■ DM PART II

- Relations
- Graphs
- Trees
- Boolean Functions /  
Logic Circuits
- Automata



# Why Study Discrete Math?

- The basis of all of digital information processing is: Discrete manipulations of discrete structures represented in memory.
- It's the basic language and conceptual foundation for all of computer science.
- Discrete math concepts are also widely used throughout math, science, engineering, economics, biology, *etc.*, ...
- A generally useful tool for rational thought!



# Uses for Discrete Math in Computer Science

- Advanced algorithms & data structures
- Programming language compilers & interpreters
- Computer networks
- Operating systems
- Computer architecture
- Database management systems
- Cryptography
- Error correction codes
- Graphics & animation algorithms, game engines, *etc....*
- *i.e., the whole field!*





# Course Topics

- Logic and Proofs (Chap. 1)
- Basic Structures (Chap. 2)
  - Sets, Functions, Sequences and Summations
- Algorithms, Integers, and Matrices (Chap. 3)
- Induction and Recursion (Chap. 4)
- Counting (Chap. 5)
- Discrete Probability (Chap. 6)



# 1.1 Propositional Logic

- Logic
  - Study of reasoning.
  - Specifically concerned with whether reasoning is correct.
  - Focuses on the relationship among statements, not on the content of any particular statement.
  - Gives precise meaning to mathematical statements.
- ***Propositional Logic*** is the logic that deals with statements (propositions) and compound statements built from simpler statements using so-called *Boolean connectives*.
- Some applications in computer science:
  - Design of digital electronic circuits.
  - Expressing conditions in programs.
  - Queries to databases & search engines.



# Definition of a *Proposition*

**Definition:** A *proposition* (denoted  $p, q, r, \dots$ ) is simply:

- a *statement* (i.e., a declarative sentence)
  - with some definite meaning,  
(not vague or ambiguous)
- having a *truth value* that's either *true* (**T**) or *false* (**F**)
  - it is **never** both, neither, or somewhere “in between!”
    - However, you might not *know* the actual truth value,
    - and, the truth value might *depend* on the situation or context.
- Later, we will study *probability theory*, in which we assign *degrees of certainty* (“between” **T** and **F**) to propositions.
  - But for now: think True/False only! (or in terms of **1** and **0**)



# Examples of Propositions

- It is raining. (In a given situation)
- Beijing is the capital of China. (T)
- $2 + 2 = 5$ . (F)
- $1 + 2 = 3$ . (T)
- A fact-based declaration is a proposition, even if no one knows whether it is true
  - 11213 is prime.
  - There exists an odd perfect number.



# Examples of Non-Propositions

The following are **NOT** propositions:

- Who's there? (interrogative, question)
- Just do it! (imperative, command)
- La la la la la. (meaningless interjection)
- Yeah, I sorta dunno, whatever... (vague)
- $1 + 2$  (expression with a non-true/false value)
- $x + 2 = 5$  (declaration about semantic tokens of non-constant value)



# Truth Tables

- An *operator* or *connective* combines one or more *operand* expressions into a larger expression. (e.g., “+” in numeric expressions.)
- **Unary** operators take *one* operand (e.g.,  $-3$ );  
**Binary** operators take *two* operands (e.g.  $3 \times 4$ ).
- **Propositional** or **Boolean operators** operate on propositions (or their truth values) instead of on numbers.
- The **Boolean domain** is the set  $\{T, F\}$ . Either of its elements is called a **Boolean value**.  
An  $n$ -tuple  $(p_1, \dots, p_n)$  of Boolean values is called a **Boolean  $n$ -tuple**.
- An  $n$ -operand truth table is a table that assigns a Boolean value to the set of all Boolean  $n$ -tuples.



# Some Popular Boolean Operators

<u>Formal Name</u>	<u>Nickname</u>	<u>Arity</u>	<u>Symbol</u>
Negation operator	NOT	Unary	$\neg$
Conjunction operator	AND	Binary	$\wedge$
Disjunction operator	OR	Binary	$\vee$
Exclusive-OR operator	XOR	Binary	$\oplus$
Implication operator	IMPLIES	Binary	$\rightarrow$
Biconditional operator	IFF	Binary	$\leftrightarrow$

# The Negation Operator

- The unary ***negation operator*** “ $\neg$ ” (*NOT*) transforms a proposition into its logical *negation*.
- *E.g.* If  $p =$  “I have brown hair.”  
then  $\neg p =$  “It is not the case that I have brown hair” or “I do **not** have brown hair.”
- The *truth table* for NOT:

$p$	$\neg p$
T	F
F	T

Operand  
column

Result  
column





# The Conjunction Operator

- The binary ***conjunction operator*** “ $\wedge$ ” (*AND*) combines two propositions to form their logical *conjunction*.

- *E.g.* If  $p$  = “I will have salad for lunch.” and  $q$  = “I will have steak for dinner.”

then,  $p \wedge q$  = “I will have salad for lunch **and** I will have steak for dinner.”

# Conjunction Truth Table

Operand columns

$p$	$q$	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- Note that a conjunction  $p_1 \wedge p_2 \wedge \dots \wedge p_n$  of  $n$  propositions will have  $2^n$  rows in its truth table

# The Disjunction Operator

- The binary ***disjunction operator*** “ $\vee$ ” (*OR*) combines two propositions to form their logical *disjunction*.

- *E.g.* If  $p$  = “My car has a bad engine.” and  $q$  = “My car has a bad carburetor.”

then,  $p \vee q$  = “My car has a bad engine, **or** my car has a bad carburetor.”

Meaning is like “and/or” in informal English.

# Disjunction Truth Table

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note difference  
from AND

- Note that  $p \vee q$  means that  $p$  is true, or  $q$  is true, **or both** are true!
- So, this operation is also called ***inclusive or***, because it **includes** the possibility that both  $p$  and  $q$  are true.



# The Exclusive-Or Operator

- The binary ***exclusive-or operator*** “ $\oplus$ ” (*XOR*) combines two propositions to form their logical “exclusive or”
- *E.g.* If  $p =$  “I will earn an A in this course.” and  $q =$  “I will drop this course.”, then
$$p \oplus q = \text{“I will **either** earn an A in this course, **or** I will drop it (**but not both!**)”}$$

# Exclusive-Or Truth Table

$p$	$q$	$p \oplus q$
T	T	<b>F</b>
T	F	T
F	T	T
F	F	F

Note difference  
from OR.

- Note that  $p \oplus q$  means that  $p$  is true, or  $q$  is true, but **not both**!
- This operation is called **exclusive or**, because it **excludes** the possibility that both  $p$  and  $q$  are true.

# Natural Language is Ambiguous

- Note that the English “or” can be ambiguous regarding the “both” case!

- “Pat is a singer or Pat is a writer.” - ✓

- “Pat is a man or Pat is a woman.” - ⊕

$p$	$q$	$p$ "or" $q$
T	T	?
T	F	T
F	T	T
F	F	F

- Need context to disambiguate the meaning!
- For this class, assume “or” means inclusive ( $\vee$ ).



# The Implication Operator

- The conditional statement (aka ***implication***)  
 $p \rightarrow q$  states that  $p$  implies  $q$ .
- */e.e., If  $p$  is true, then  $q$  is true; but if  $p$  is not true, then  $q$  could be either true or false.*
- *E.g., let  $p$  = “You study hard.”  
 $q$  = “You will get a good grade.”  
 $p \rightarrow q$  = “If you study hard, then you will get a good grade.” (else, it could go either way)*
  - *$p$ : hypothesis or antecedent or premise*
  - *$q$ : conclusion or consequence*



# Implication Truth Table

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	<b>F</b>
F	T	T
F	F	T

The only False case!

- $p \rightarrow q$  is **false** only when  $p$  is true but  $q$  is **not** true.
- $p \rightarrow q$  does **not** require that  $p$  or  $q$  are ever true!
- E.g. “ $(1=0) \rightarrow$  pigs can fly” is TRUE!