MA225
Probability Theory and Random Processes
Midsem Assignment
Full Marks: 15

Instructions:

a) Submission Deadline: **11:59pm on 26th February, 2021**

b) You have to send only one pdf file of your answer sheet via **email (subhabrata@iitp.ac.in)**. File name should be **"roll number.pdf"**; e.g., if roll no is 2021CS89, then the file name should be "2021CS89.pdf"

Q1. Let $G = (V, E)$ be a simple undirected graph with vertex set V and edge set E. A vertex u is called a neighbor of v if $(u, v) \in E$. A dominating set of G is a set $D \subseteq V$ such that every vertex $v \in V \setminus D$ has at least one neighbor in D. The domination number of G is the minimum cardinality of a dominating set of G.

Let X be the random set of vertices formed by picking each vertex with probability p independently. Let Y be the random set of vertices in $V \setminus X$ that do not have any neighbor in X.

a) Find the expected value of $|X| + |Y|$.

b) For what value of p, this expected value is minimized?

c) How can you get an upper bound on domination number using expectation argument?

[5 + 1 + 2]

Q2. Let $R(k, k)$ be the smallest integer n such that in any two coloring of the edges of a complete graph K_n on n vertices by red and blue, either there is a red K_k (i.e., a complete subgraph on k vertices all of whose edges are colored red) or there is a blue K_k. Show that for any integer n,

$$R(k, k) > n - \binom{n}{k} 2^{1-\binom{k}{2}}.$$

[7]