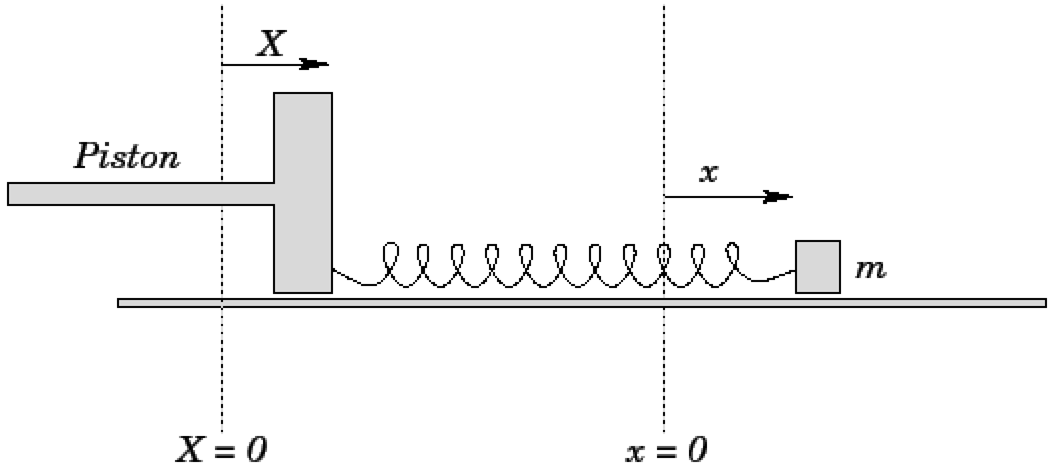
Oscillations and Waves
Recap

Transient Oscillator Response

Driven Damped Harmonic Oscillator



Piston executes simple harmonic oscillation of angular frequency, $\omega>0$, and amplitude $X_0>0$. This system is described by equation

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \omega_0^2 X_0 \cos\omega t \qquad \text{... (1)}$$

$$\gamma = \frac{b}{m}, \omega_0^2 = \frac{k}{m}$$

Transient Oscillator Response

Driven Damped Harmonic Oscillator equation

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \omega_0^2 X_0 \cos\omega t \quad \text{... (1)}$$

Solution of above equation is

$$x_{ta}(t) = x_0 \cos(\omega t - \varphi) \quad \text{... (2)}$$

$$x_0 = \frac{\omega_0^2 X_0}{\left[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2\right]^{1/2}}$$

$$\varphi = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)$$

Eq.(1) is second-ordered ordinary differential equation. The general solution of this equation should contain two arbitrary constants. However, Eq. (2) does not contain any arbitrary constants. Therefore, it can not be the general solution.

Transient Oscillator Response

Undriven Damped Harmonic Oscillator equation

$$\ddot{x} + \gamma \dot{x} + {\omega_0}^2 x = 0$$

If we add solution of this equation to Eq.(2), the resultant will still be solution of Eq. (1). The general solution of undriven damped harmonic oscillator equation is

$$x_{tr}(t) = Ae^{-\gamma t/2} \cos \omega_1 t + Be^{-\gamma t/2} \sin \omega_1 t$$

Where A and B are arbitrary constants and $\omega_1 = \left(\omega_0^2 - \frac{\gamma^2}{4}\right)^{1/2}$

The general solution is

$$x(t) = x_{ta}(t) + x_{tr}(t)$$

$$= x_0 \cos(\omega t - \varphi) + Ae^{-\gamma t/2} \cos \omega_1 t + Be^{-\gamma t/2} \sin \omega_1 t$$

… (3)

Transient Oscillator Response

$$x(t) = \underbrace{x_0 \cos(\omega t - \varphi)} + \underbrace{A e^{-\gamma t/2} \cos \omega_1 t + B e^{-\gamma t/2} \sin \omega_1 t}$$

Time asymptotic solution

- Oscillates at the driving frequency ω
- Constant amplitude
- Independent of initial conditions
- As time progresses the term becomes dominant

Transient solution

- Oscillates at the frequency ω_1
- Amplitude decays exponentially
- Depends on initial conditions
- As time progresses the term decays away

Transient Oscillator Response

$$\boldsymbol{x(t) = x_0 \cos(\omega t - \varphi) + Ae^{-\gamma t/2} \cos \omega_1 t + Be^{-\gamma t/2} \sin \omega_1 t}$$

... (3)

Lets take the initial conditions be $x(0) = \dot{x}(0) = 0$ to find A and B

$$\boldsymbol{x(0) = x_0 \cos(\varphi) + A = 0}$$

$$\boldsymbol{A = -x_0 cos\varphi}$$

$$\boldsymbol{\dot{x}(0) = x_0 \omega \sin \varphi - \frac{\gamma}{2} A + \omega_1 B = 0}$$

$$\boldsymbol{B = -x_0 \left[\frac{\omega \sin \varphi + \frac{\gamma}{2} \cos \varphi}{\omega_1} \right]}$$

For the driving frequencies close to the resonant frequency $|\omega - \omega_0| \sim \gamma$, we can write

$$\boldsymbol{x_0 \cong \frac{\omega_0 X_0}{[4(\omega_0 - \omega)^2 + \gamma^2]^{1/2}}}$$

$$\boldsymbol{\sin \varphi \cong \frac{\gamma}{[4(\omega_0 - \omega)^2 + \gamma^2]^{1/2}}}$$

$$\boldsymbol{\cos \varphi \cong \frac{2(\omega_0 - \omega)}{[4(\omega_0 - \omega)^2 + \gamma^2]^{1/2}}}$$

Transient Oscillator Response

$$x(t) = x_0 \cos(\omega t - \varphi) + Ae^{-\gamma t/2} \cos \omega_1 t + Be^{-\gamma t/2} \sin \omega_1 t$$

Lets take the initial conditions be $x(0) = \dot{x}(0) = 0$ to find A and B **... (3)**

$$x_0 \cong \frac{\omega_0 X_0}{[4(\omega_0 - \omega)^2 + \gamma^2]^{1/2}} \qquad \sin\varphi \cong \frac{\gamma}{[4(\omega_0 - \omega)^2 + \gamma^2]^{1/2}}$$

$$\cos\varphi \cong \frac{2(\omega_0 - \omega)}{[4(\omega_0 - \omega)^2 + \gamma^2]^{1/2}}$$

$$x(t) \cong X_0 \left[\frac{2\omega_0(\omega_0 - \omega)}{[4(\omega_0 - \omega)^2 + \gamma^2]}\right] \left[\cos \omega_1 t - e^{-\gamma t/2} \cos \omega_0 t\right]$$

$$+X_0 \left[\frac{\omega_0 \gamma}{[4(\omega_0 - \omega)^2 + \gamma^2]}\right] \left[\sin \omega_1 t - e^{-\gamma t/2} \sin \omega_0 t\right] \qquad \text{... (4)}$$

Transient Oscillator Response

$$x(t) \cong X_0 \left[\frac{2\omega_0(\omega_0-\omega)}{[4(\omega_0-\omega)^2+\gamma^2]}\right]\left[\cos\omega_1 t - e^{-\gamma t/2}\cos\omega_0 t\right]$$
$$+X_0\left[\frac{\omega_0\gamma}{[4(\omega_0-\omega)^2+\gamma^2]}\right]\left[\sin\omega_1 t - e^{-\gamma t/2}\sin\omega_0 t\right] \qquad \text{... (4)}$$

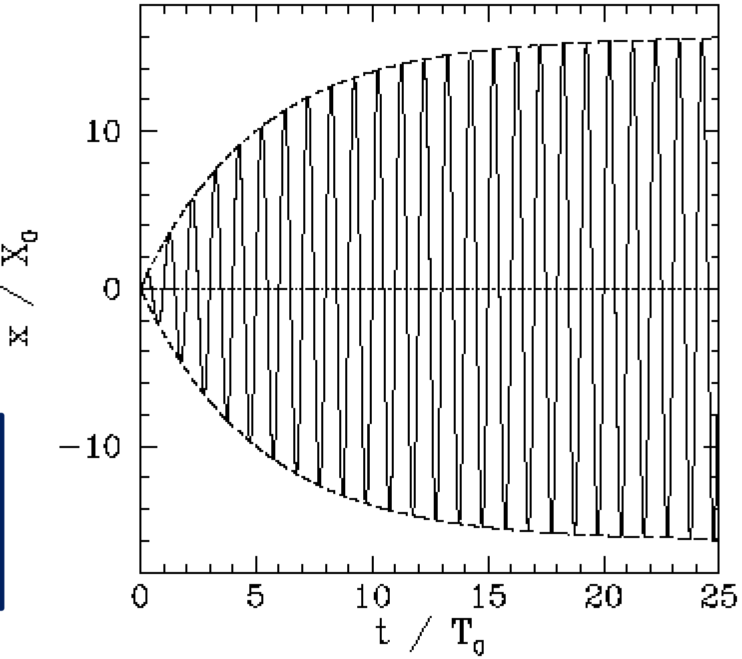
Case-I: Let the driving frequency equal to resonant frequency $\omega=\omega_0$

$$x(t) = X_0\frac{\omega_0}{\gamma}\left(1-e^{-\gamma t/2}\right)\sin\omega_0 t$$
$$= X_0 Q_f\left(1-e^{-\gamma t/2}\right) sin\,\omega_0 t$$

Where $Q_f = \frac{\omega_0}{\gamma}$

$T_0 = 2\pi/\omega_0$

$Q_f = \omega_0/\nu = 16$



Transient Oscillator Response

$$x(t) \cong X_0\left[\frac{2\omega_0(\omega_0-\omega)}{[4(\omega_0-\omega)^2+\gamma^2]}\right]\left[\cos\omega_1 t - e^{-\gamma t/2}\cos\omega_0 t\right]$$

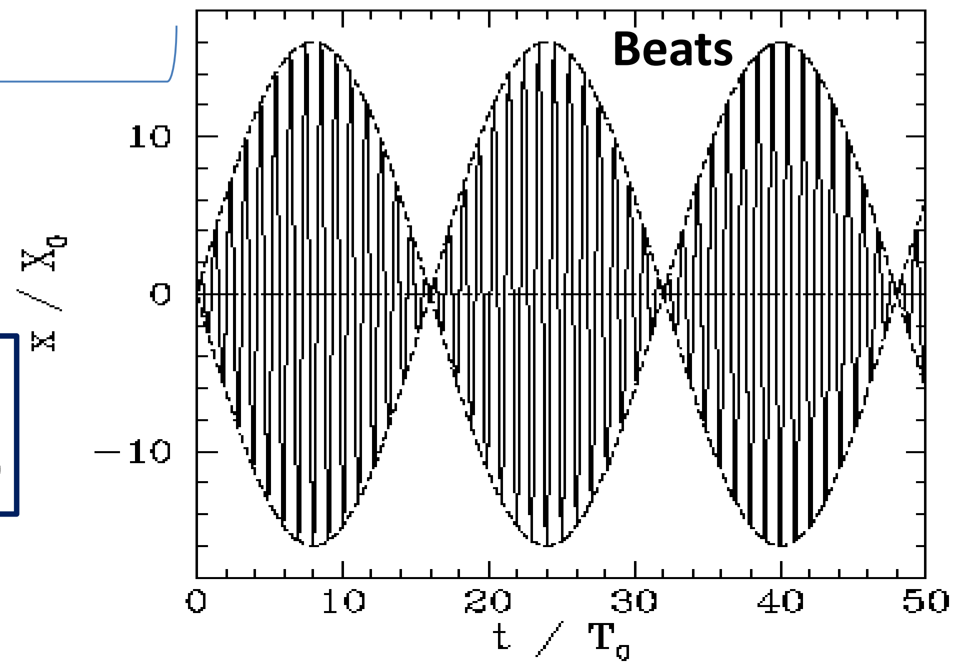
$$+X_0\left[\frac{\omega_0\gamma}{[4(\omega_0-\omega)^2+\gamma^2]}\right]\left[\sin\omega_1 t - e^{-\gamma t/2}\sin\omega_0 t\right] \qquad \ldots (4)$$

Case-II: No damping γ=0

$$x(t) = \underbrace{X_0\frac{\omega_0}{(\omega_0-\omega)}\sin[(\omega_0-\omega)t/2]}_{\text{A(t)}}\sin[(\omega_0+\omega)t/2]$$

$$T_0 = 2\pi/\omega_0$$

$$\omega_0 - \omega = \omega_0/16$$



Transient Oscillator Response

$$x(t) \cong X_0 \left[\frac{2\omega_0(\omega_0 - \omega)}{[4(\omega_0 - \omega)^2 + \gamma^2]}\right]\left[\cos\omega_1 t - e^{-\gamma t/2}\cos\omega_0 t\right]$$

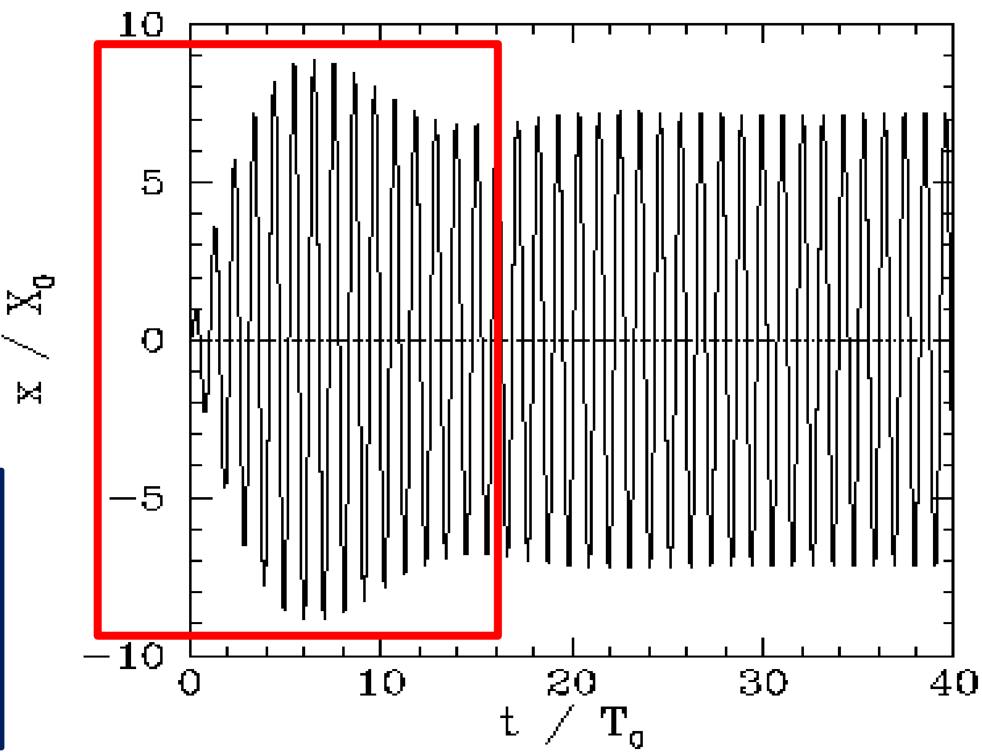
$$+X_0\left[\frac{\omega_0\gamma}{[4(\omega_0-\omega)^2+\gamma^2]}\right]\left[\sin\omega_1 t - e^{-\gamma t/2}\sin\omega_0 t\right] \quad \text{... (4)}$$

Transient solution, needed to produce beats, initially grows (red box), but then damps away leaving behind the constant amplitude time asymptotic solution

$$T_0 = 2\pi/\omega_0$$

$$\omega_0 - \omega = \omega_0/16$$

$$\gamma = \omega_0/16$$



Driven Damped Oscillator (Phase)

Driven Damped Harmonic Oscillator equation

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \omega_0^2 X_0 \cos\omega t$$ **... (1)**

Solution of above equation is

$$\boldsymbol{x_{ta}(t) = x_0 \cos(\omega t - \varphi)}$$ **... (2)**

$$\boldsymbol{x_0 = \frac{\omega_0^2 X_0}{\left[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2\right]^{1/2}}}$$

$$\boldsymbol{\varphi = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)}$$

$$\boldsymbol{\gamma\omega \ll (\omega_0^2 - \omega^2)}$$
$$\boldsymbol{\omega \approx 0}$$

$$\boldsymbol{\gamma\omega \ll (\omega^2 - \omega_0^2)}$$
$$\boldsymbol{\omega \approx \infty}$$

$$\boldsymbol{\omega \approx \omega_0}$$

$$\boldsymbol{\gamma \approx 0}$$

Case-I: $\omega \approx 0$

Driven Damped Harmonic Oscillator equation

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \omega_0^2 X_0 \cos\omega t \qquad \text{... (1)}$$

Solution of above equation is $x_{ta}(t) = x_0 \cos(\omega t - \varphi)$ **... (2)**

$$x_0 = \frac{\omega_0^2 X_0}{\left[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2\right]^{1/2}} \qquad \text{... (3)}$$

$$\varphi = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) \qquad \text{... (4)}$$

$$\left.\begin{aligned} &\gamma\omega \ll (\omega_0^2 - \omega^2) \\ &\omega \approx 0 \implies \varphi \approx 0 \end{aligned}\right\}$$

Motion is in phase with the force

Mathematical meaning:

$\ddot{x}$ *and* $\dot{x}$ in Eq. (1) are small, as they are proportional to ω^2 and ω, respectively. Therefore, first two terms in Eq.(1) are negligible. Therefore, we have $x \propto \cos\omega t \implies$ phase is zero

Physical meaning:

Since there is no acceleration, the net force is zero $\Rightarrow$ driving force balances the spring force. The negative sign in F=-kx, means that the spring force is 180° out of phase with motion. Therefore, the driving force balancing the spring force is in phase with motion.

Case-II: $\omega \approx \infty$

Driven Damped Harmonic Oscillator equation

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \omega_0^2 X_0 \cos\omega t \qquad \text{... (1)}$$

Solution of above equation is $x_{ta}(t) = x_0 \cos(\omega t - \varphi)$ **... (2)**

$$\varphi = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) \qquad \text{... (3)}$$

$$\left.\begin{array}{l} \gamma\omega \ll (\omega^2 - \omega_0^2) \\ \omega \approx \infty \implies \varphi \approx \pi \end{array}\right\}$$

Motion is 180° out of phase with the force

Mathematical meaning:

$\ddot{x}$ **term in Eq. (1) dominates, as it is proportional to ω^2,and we have $\ddot{x} \propto \cos\omega t$. Acceleration is in phase with the force but then x is 180° out of phase with acceleration (property of sinusoidal function). Therefore, x is 180° out of phase with the force.**

Physical meaning:

Since there is no acceleration, the net force is zero $\Rightarrow$ driving force balances the spring force. The negative sign in F=-kx, means that the spring force is 180° out of phase with motion. Therefore, the driving force balancing the spring force is in phase with motion.

Case-II: $\omega \approx \infty$

Driven Damped Harmonic Oscillator equation

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \omega_0^2 X_0 \cos\omega t \qquad \dots (1)$$

Solution of above equation is $x_{ta}(t) = x_0 \cos(\omega t - \varphi)$ **... (2)**

$$\varphi = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) \qquad \dots (3)$$

$$\gamma\omega \ll (\omega^2 - \omega_0^2)$$

$$\omega \approx \infty \implies \varphi \approx \pi$$

Motion is 180° out of phase with the force

$$x_0 = \frac{\omega_0^2 X_0}{\left[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2\right]^{1/2}} \qquad \dots (4)$$

Physical meaning: The mass hardly moves

The amplitude x0 is proportional to $1/\omega^2$ and velocity is proportional to $1/\omega$. Therefore, x and v are always small and the spring and damping forces can be ignored. The mass then only feels the driving force. It can not tell if it's being driven by an oscillating driving force, or being pushed and pulled by oscillating spring force. They both feel same to the mass. Therefore, both phases must be the same. Spring force is 180° out of phase with motion. Therefore φ ≈ π.

Case-III: $\omega \approx \omega_0$

Driven Damped Harmonic Oscillator equation

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \omega_0^2 X_0 \cos\omega t \quad \text{... (1)}$$

Solution of above equation is $x_{ta}(t) = x_0 \cos(\omega t - \varphi)$ **... (2)**

$$\varphi = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) \quad \text{... (3)}$$

$$\omega \approx \omega_0 \implies \varphi \approx \frac{\pi}{2}$$

Motion lags driving force by a quarter of a cycle

Meaning:

When particle moves rightward past the origin, the force is already at its maximum. When the particle makes it out to the maximum value of x, the force is already back to zero.

From energy point of view this makes sense, the force is maximum when the particle is moving fastest. The velocity is fastest at the origin and we want our force to be in phase with velocity at resonance.

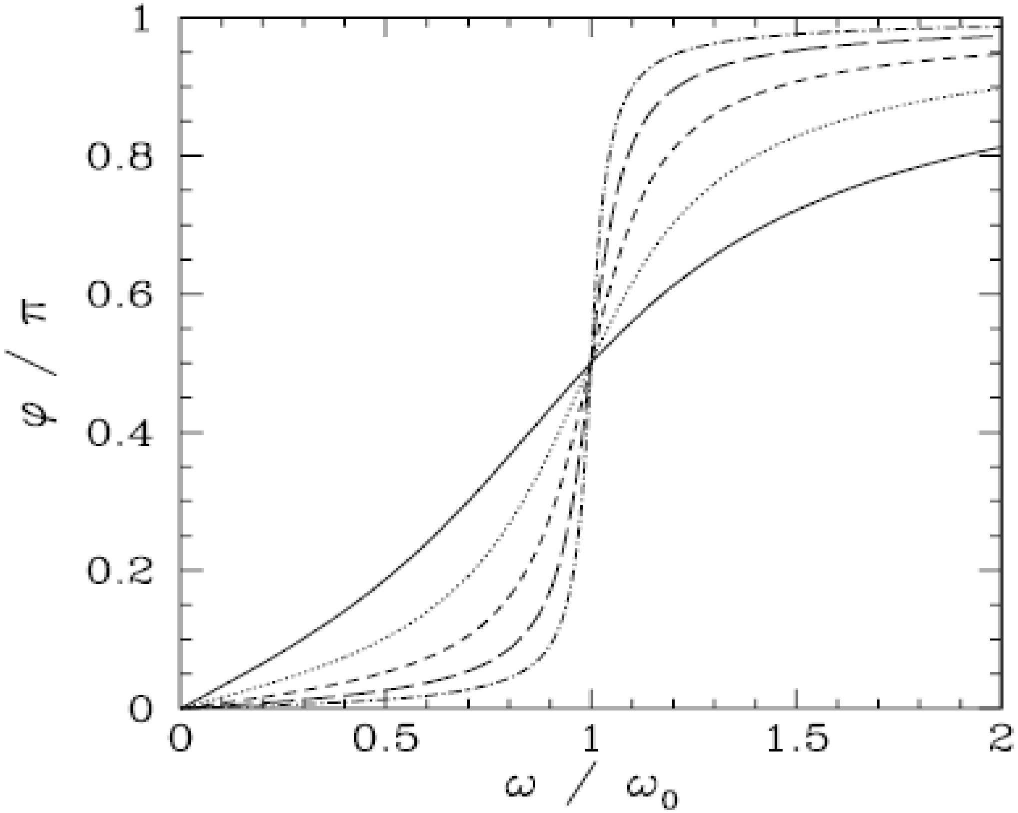
Using the property of sinusoidal function, we know that velocity is quarter cycle ahead of x.

Case-IV: $\gamma = 0$

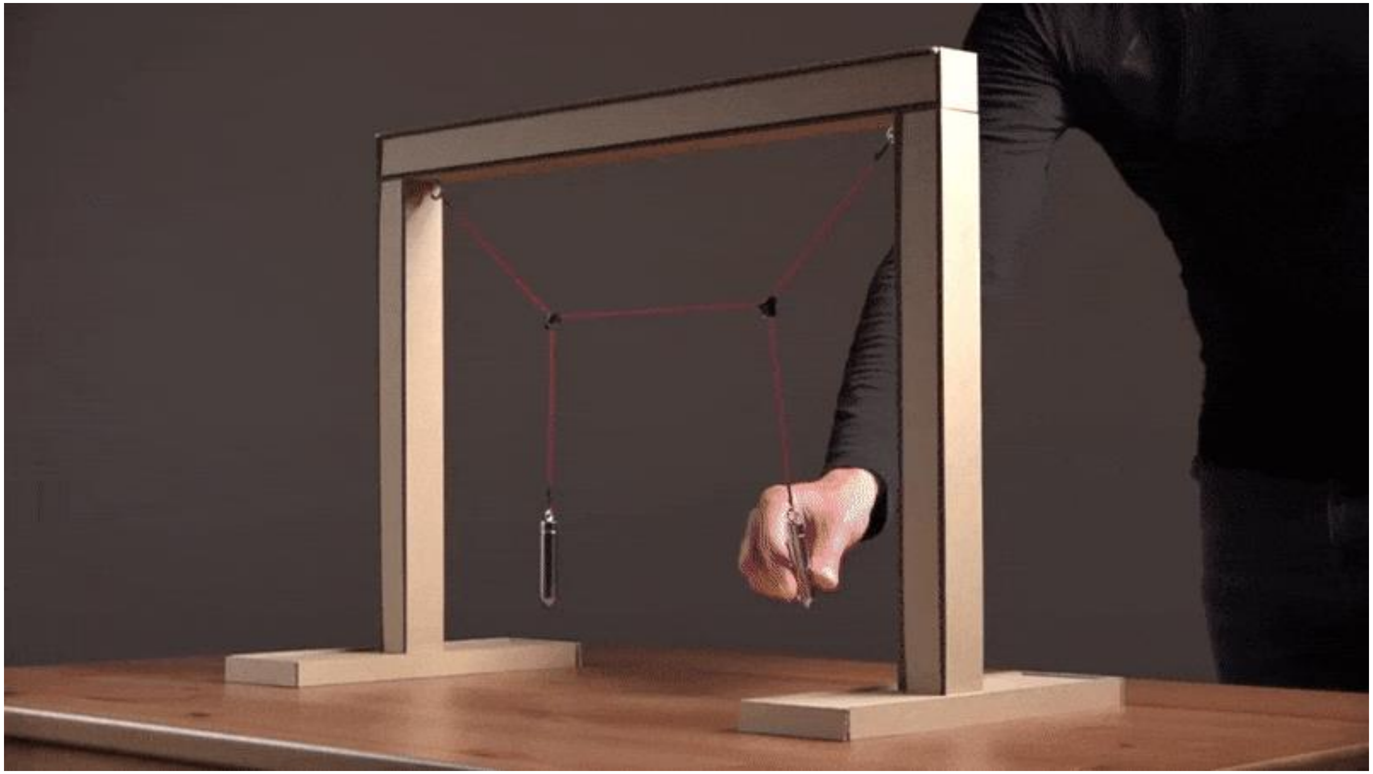
$$\varphi = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right) \qquad \tan\varphi \approx 0 \Longrightarrow \varphi = 0 \; or \; \pi$$

Motion is either in phase or 180° out of phase with the force, depending on which of ω_0 or ω is higher.

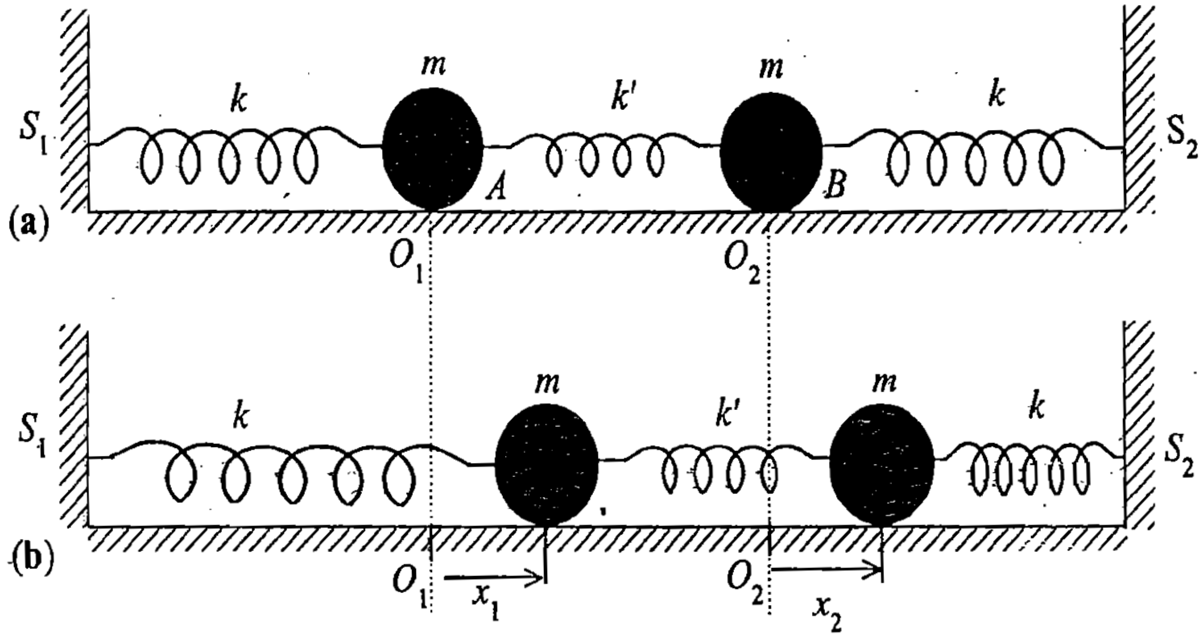
φ $vs.$ ω for Driven Damped Oscillator:

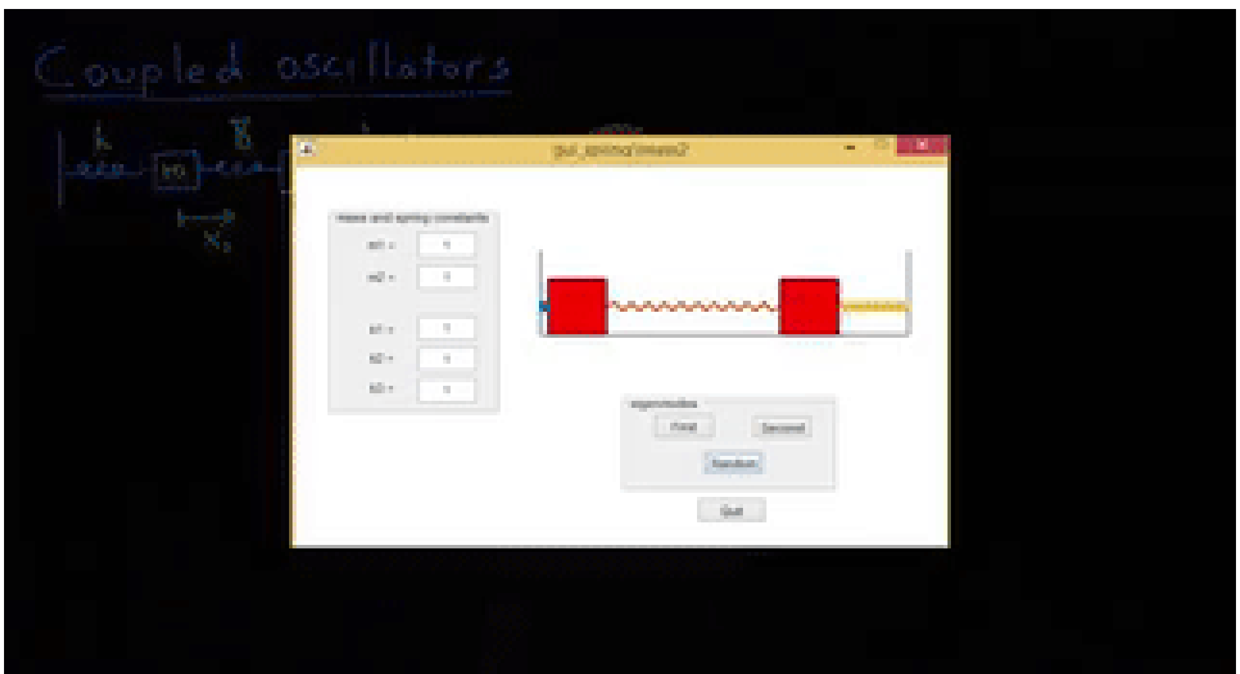


Coupled Oscillations

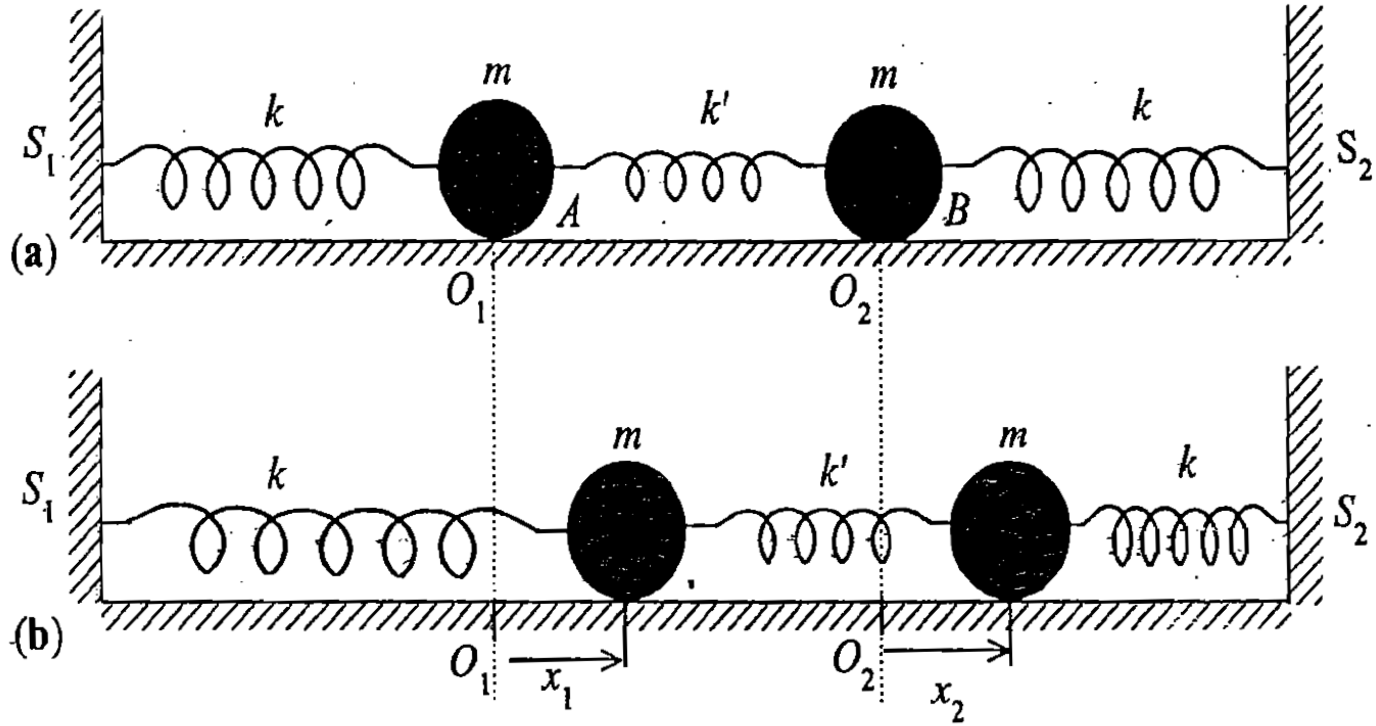


Coupled Oscillators





Coupled Oscillators



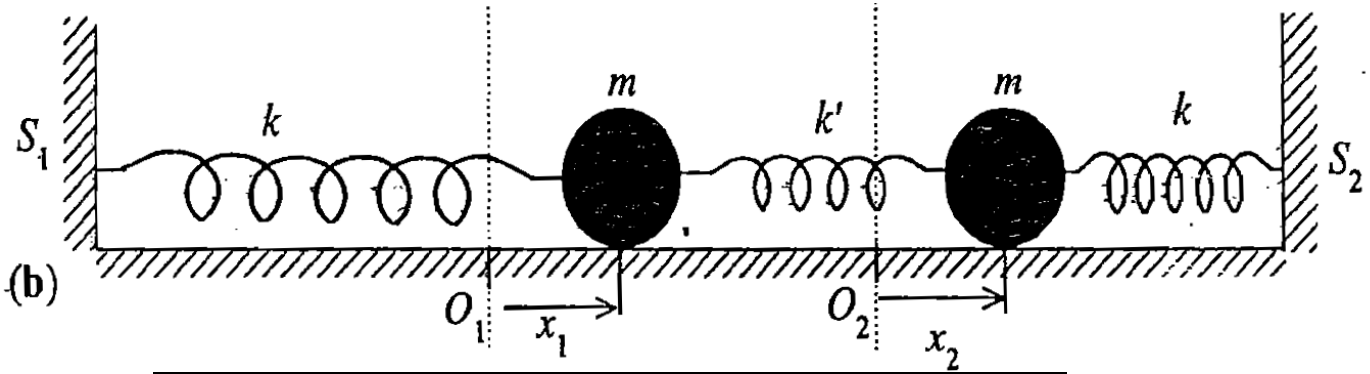
Equations of motion are

$$m\ddot{x}_1 = -kx_1 + k'(x_2 - x_1)$$

$$m\ddot{x}_2 = -kx_2 - k'(x_2 - x_1)$$

Simultaneous coupled differential equation

Matrix equation of coupled oscillator



$$m\ddot{x}_1 = -kx_1 + k'(x_2 - x_1)$$

$$m\ddot{x}_2 = -kx_2 - k'(x_2 - x_1)$$

$$m\frac{d^2}{dt^2}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} -k-k' & k' \\ k' & -k-k' \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$m\frac{d^2[X]}{dt^2} = K[X]$$

$$m\frac{d^2}{dt^2}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} -k-k' & k' \\ k' & -k-k' \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\frac{d^2[X]}{dt^2} = \begin{bmatrix} -\left(\omega_0{}^2+\omega_c{}^2\right) & \omega_c{}^2 \\ \omega_c{}^2 & -\left(\omega_0{}^2+\omega_c{}^2\right) \end{bmatrix}[X]$$

Let trail solution be $[X]=[V]e^{\alpha t}$

$$\omega_0{}^2 = \frac{k}{m},\ \omega_c{}^2 = \frac{k'}{m}$$

$$\frac{d^2[X]}{dt^2} = \begin{bmatrix} -\left(\omega_0^{\ 2} + \omega_c^{\ 2}\right) & \omega_c^{\ 2} \\ \omega_c^{\ 2} & -\left(\omega_0^{\ 2} + \omega_c^{\ 2}\right) \end{bmatrix} [X] \longleftarrow [X] = [V] e^{\alpha t}$$

$$[V]\alpha^2 = \begin{bmatrix} -\left(\omega_0^{\ 2} + \omega_c^{\ 2}\right) & \omega_c^{\ 2} \\ \omega_c^{\ 2} & -\left(\omega_0^{\ 2} + \omega_c^{\ 2}\right) \end{bmatrix} [V]$$

$$\begin{bmatrix} -\left(\omega_0^{\ 2} + \omega_c^{\ 2}\right) & \omega_c^{\ 2} \\ \omega_c^{\ 2} & -\left(\omega_0^{\ 2} + \omega_c^{\ 2}\right) \end{bmatrix} [V] - \alpha^2 [I][V] = 0$$

$$\left\{[A] - \lambda [I]\right\}[V] = 0$$

Find eigen values and eigen vectors of the above eigen equation

Finding the energy eigen values λ of [A]

$$\left\{\left[A\right]-\lambda\left[I\right]\right\}\left[V\right]=0$$

$$\text{determinant: }\left|A-\lambda I\right|=0$$

$$\left|[A]-\lambda[I]\right|=0$$

$$\begin{vmatrix} -\left(\omega_0{}^2+\omega_c{}^2\right)-\alpha^2 & \omega_c{}^2 \\ \omega_c{}^2 & -\left(\omega_0{}^2+\omega_c{}^2\right)-\alpha^2 \end{vmatrix}=0$$

Characteristic equation:

$$\left(\alpha^2\right)^2+2\left(\omega_0^2+\omega_c^2\right)\alpha^2+\omega_0^4+2\left(\omega_0^2\times\omega_c^2\right)=0$$

Eigen values of [A]:

$$\alpha_1=i\omega_0,\alpha_2=-i\omega_0,\alpha_3=i\sqrt{\omega_0^2+2\omega_c^2},\alpha_4=-i\sqrt{\omega_0^2+2\omega_c^2}$$

Find the eigen vectors of [A]

Finding Eigen vectors of [A]

Substitute each values of α^2 in the Eigen value equation

$$\left\{[A]-\lambda[I]\right\}[V]=0$$

Putting α_1^2 in the Eigen value equation to get first set of Eigen vectors

$$\begin{bmatrix} -\left({\omega_0}^2+{\omega_c}^2\right)-\alpha_1^2 & {\omega_c}^2 \\ {\omega_c}^2 & -\left({\omega_0}^2+{\omega_c}^2\right)-\alpha_1^2 \end{bmatrix}\begin{bmatrix} V_1^{\alpha 1} \\ V_2^{\alpha 1} \end{bmatrix}=0$$

Substituting $\alpha_1{}^2 = -\omega_0$

$$\begin{bmatrix} -\left(\omega_0{}^2 + \omega_c{}^2\right) + \omega_0{}^2 & \omega_c{}^2 \\ \omega_c{}^2 & -\left(\omega_0{}^2 + \omega_c{}^2\right) + \omega_0{}^2 \end{bmatrix} \begin{bmatrix} V_1^{\alpha 1} \\ V_2^{\alpha 1} \end{bmatrix} = 0$$

$$V_1^{\alpha 1} = V_2^{\alpha 1}$$

Eigen vector corresponding the eigen value $\lambda = \alpha_1^2$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Similarly substituting $\alpha_2{}^2 = -\omega_0$

Eigen vector corresponding the eigen value $\lambda = \alpha_2^2$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Putting $\alpha_3^2 = -(\omega_0^2 + 2\omega_c^2)$ in the Eigen value equation

$$\begin{bmatrix} -\left(\omega_0{}^2 + \omega_c{}^2\right) + \left(\omega_0^2 + 2\omega_c^2\right) & \omega_c{}^2 \\ \omega_c{}^2 & -\left(\omega_0{}^2 + \omega_c{}^2\right) + \left(\omega_0^2 + 2\omega_c^2\right) \end{bmatrix} \begin{bmatrix} V_1^{\alpha_3} \\ V_2^{\alpha_3} \end{bmatrix} = 0$$

$$V_1^{\alpha_3} = -V_2^{\alpha_3}$$

Eigen vector corresponding the eigen value $\lambda = \alpha_3^2$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Substitution of $\alpha_4{}^2 = -\left(\omega_0^2 + 2\omega_c^2\right)$

Eigen vector corresponding the eigen value $\lambda = \alpha_4^2$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\frac{d^2}{dt^2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\left(\omega_0{}^2 + \omega_c{}^2\right) & \omega_c{}^2 \\ \omega_c{}^2 & -\left(\omega_0{}^2 + \omega_c{}^2\right) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} e^{\alpha t}$$

$$\alpha_1 = i\omega_0, \alpha_2 = -i\omega_0, \alpha_3 = i\sqrt{\omega_0^2 + 2\omega_c^2}, \alpha_4 = -i\sqrt{\omega_0^2 + 2\omega_c^2}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

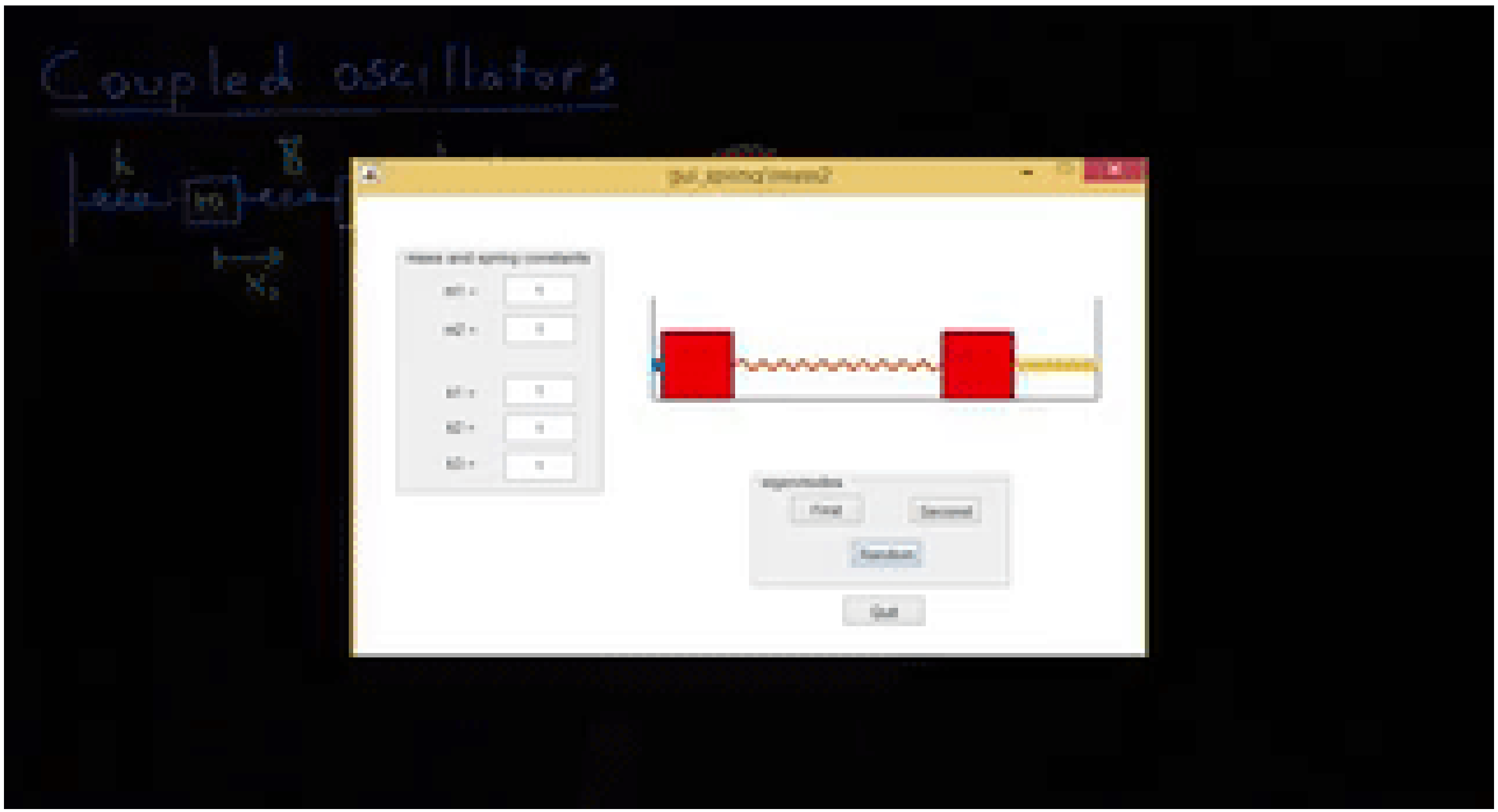
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{i\omega_0 t} + a_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-i\omega_0 t} + a_3 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{i\sqrt{\omega_0^2 + 2\omega_c^2}\, t} + a_4 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-i\sqrt{\omega_0^2 + 2\omega_c^2}\, t}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{i\omega_0 t} + a_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-i\omega_0 t} + a_3 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{i\sqrt{\omega_0^2 + 2\omega_c^2}\, t} + a_4 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-i\sqrt{\omega_0^2 + 2\omega_c^2}\, t}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \boxed{a_1 e^{i\omega_0 t} + a_2 e^{-i\omega_0 t}} + \boxed{a_3 e^{i\sqrt{\omega_0^2 + 2\omega_c^2}\, t} + a_4 e^{-i\sqrt{\omega_0^2 + 2\omega_c^2}\, t}} \\ \boxed{a_1 e^{i\omega_0 t} + a_2 e^{-i\omega_0 t}} - \boxed{a_3 e^{i\sqrt{\omega_0^2 + 2\omega_c^2}\, t} - a_4 e^{-i\sqrt{\omega_0^2 + 2\omega_c^2}\, t}} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(\sqrt{\omega_0^2 + 2\omega_c^2}\, t + \phi_2) \\ A_1 \cos(\omega_0 t + \phi_1) - A_2 \cos(\sqrt{\omega_0^2 + 2\omega_c^2}\, t + \phi_2) \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(\sqrt{\omega_0^2 + 2\omega_c^2}\, t + \phi_2) \\ A_1 \cos(\omega_0 t + \phi_1) - A_2 \cos(\sqrt{\omega_0^2 + 2\omega_c^2}\, t + \phi_2) \end{bmatrix}$$



Exercise: Write coupled equations for 3-masses coupled oscillator shown in figure below. Find the eigen values and eigen vectors