

# HOME ASSIGNMENT

B. Tech. II yr (III Sem) 2020-2021.

MA201 (Mathematics - III)  
Time: 2 days

Total Marks: 10

1. Let  $|z + \frac{1}{z}| = a$ , where  $z$  is a complex number. What are greatest and least possible values of  $|z|^2$ . [1]
2. Show that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin, although the Cauchy-Riemann equations are satisfied at that point. [1.5]
3. Prove that, if  $u = x^2 - y^2$ ,  $v = -y/x^2 + y^2$ , both  $u$  and  $v$  satisfy Laplace's equation, but  $u + iv$  is not analytic function of  $z$ . [1.5]
4. State Cauchy-Goursat's Theorem and prove it for a rectangular domain (i.e.  $C$ , a closed contour, has a rectangular shape). Also, evaluate 
$$\int_{(0,3)}^{(2,4)} (2y + x^2) dx + (3x - y) dy$$
 [2+1]  
along the parabola  $x = 2t$ ,  $y = t^3 + 3$ .
5. Define radius of convergence of a power series. Find the radius of convergence of the series  $\sum_{n=0}^{\infty} \frac{n\sqrt{2} + i}{1 + 2in} z^n$ . [1.5]
6. Prove that, if  $w = g(\xi)$  is analytic function of  $\xi$  which is itself an analytic function of  $z$ , then  $w$  is an analytic function of  $z$  and 
$$\frac{dw}{dz} = \left( \frac{dw}{d\xi} \right) \cdot \left( \frac{d\xi}{dz} \right)$$
. [1.5]