CS-206

ASSIGNMENT-7

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Que 1:

```
Void function (int n){
    int i=1, s=1;
    while (s<=n){
        i++;
        s+=i
        printf("*");
    }
}
```

When $s > n$; the loop will break.

The value of i and s increases by 1 & i resp. in each step.

After k iterations the value of s will be greater than n.

But value of s will be $1+2+3+ \dots +k$

$\Rightarrow \quad \frac{k(k+1)}{2} > n$

$\frac{k^2+k}{2} > n$

$k = \sqrt{n}$

$\therefore$ The $\boxed{\text{Time Complexity will be } O(\sqrt{n})}$

Que 2:-

```
Void Read (int n){
    Int k=1;
    while (k<=n)
    k = 3*k;
    }
```

Until $k <= n$, the loop will run.

If we assume that after i^{th} iteration the value of $k = 3^{i}$

$3^i > n$

Log 3 both side .

$\log_3 3^i > \log_3 n$

$i > \log_3 n$

$\therefore$ The $\boxed{\text{Time Complexity of the function is } \Omega(\log_3 n)}$

Hence Proved.

Que 3:- Big-O notation for any function $f(n)$ suggests that if

$f(n) = O(g(n))$

there exists constants $c, n_0 > 0$.

Such that $0 \leq f(n) \leq c\,g(n)$

Let $f(n) = 2(\lfloor \frac{n}{2} \rfloor + 1)!$ $\quad$ $\lfloor\ \rfloor \rightarrow$ floor function.

$g(n) = 2(\lceil \frac{n}{2} \rceil)!$ $\quad$ $\lceil\ \rceil \rightarrow$ ceil function

n	1	2	3	4	5
$f(n)$	1!	3!	3!	5!	5!
$g(n)$	2!	2!	4!	4!	6!

If n is even $\frac{f(2m)}{f(2m)} = \frac{(2m+1)!}{(2m)!} = 2m+1 \xrightarrow[m\to\infty]{} \infty$

n is odd $\frac{f(2m+1)}{f(2m+1)} = \frac{(2m+1)!}{(2m+2)!} = \frac{1}{2m+2} \xrightarrow[m\to\infty]{} 0$

Thus $f(n)$ and $g(n)$ continuously overtake each other

Thus neither $f(n) \rightarrow O(g(n))$ nor $g(n) = O(f(n))$

Hence Proved.

Que 4:-

```
int fun(int n){
    int count=0;
    for (int i=n ;i>0;i/=2)   ——— ①
    for (int j=0; j<i; j++)   ——→ ②.
        count += 1
    return count;
}
```

For every value of i in 1st loop; the 2nd loop runs exactly i times.

Change of values of n are from:

$n \to \frac{n}{2} \to \frac{n}{4} \to \dots \to 1 \to 0.$

⇒ Total steps $= n + \frac{n}{2} + \frac{n}{4} + \dots + 1$

$= n(1 + \frac{1}{2} + \frac{1}{4} + \dots +)$

$= n\left(1 + \frac{\frac{1}{2}(1-(\frac{1}{2})^k)}{1-\frac{1}{2}}\right)$

$= n(1+1)$ k → k is a large no.

$= 2n.$

⇒ $\boxed{\text{Time Complexity} = O(n)}$

Qus 5:

```
void function (int n)
    int i, j, k, count = 0;
    for (i = n/2 ; i <= n ; i++)          —— ①
        for (j = 1 ; j + n/2 <= n ; j = j + 1) —— ②
            for (k = 1 ; k <= n ; k * 2)   —— ③
                count++;
}
```

Every loop is independent of the other loop.

3rd loop.

Let No of steps = x.

∴ 3rd loop will break when $k > n$.

At the end of x^{th} iteration the value of $k = 2^x$

$$2^x > n$$
$$x \geq \log_2 n$$

3rd loop runs $\log_2 n$ times

2nd loop / 1st loop

It will break when $j > \frac{n}{2}$ $(j + n/2 > n)$ $(n - \frac{n}{2} = \frac{n}{2})$

Both loops runs $n/2$ times.

Total steps $= \frac{n}{2}\left(\frac{n}{2}\left(\log_2 n\right)\right)$

$$= \frac{n^2}{4}(\log_2 n)$$

$$\boxed{\text{Time Complexity} = O(n^2 \log n)}$$

—x—x—x—x—x—x—x—x—x—x—