CS 225: Switching Theory

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Previous Class

- Introduction to the course
  - Digital Design

This Class

- Number Systems and Codes

What is Number System?

- Number System?
  - Study of digital systems deals with discrete information
  - Numerical quantities are discrete information
  - Discrete information are represented by a finite set of symbols
- Symbols other than numeral like letters/characters are also included

Digital Number System

Positional Non-positional

- Can you find any number system with non-positional?
-
- What is the role of base?

Digital Number System

- Positional: Each position has a weight
  - Weight is unique for a particular number system
  - Ex: $346_{10} \neq 463_{10} \neq 346_8$
    - (changing the position changes the value)
- Decimal number system (base=10)

…..	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}	….
		3	4	6	.	2	7		

decimal point

$$\text{Ex.}346.27 = 3\times10^2 + 4\times10^1 + 6\times10^0 + 2\times10^{-1} + 7\times10^{-2}$$

Digital Number System

- Binary Positional System (base=2)

- Ex:$(101.11)_2 = 1x2^2 + 0x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$
- so on for octal and hexa-decimal

Representation of Integers

Base					
2	4	8	10	12	16
0000	0	0	0	0	0
0001	1	1	1	1	1
0010	2	2	2	2	2
0011	3	3	3	3	3
0100	10	4	4	4	4
0101	11	5	5	5	5
0110	12	6	6	6	6
0111	13	7	7	7	7

Base					
2	4	8	10	12	16
1000	20	10	8	8	8
1001	21	11	9	9	9
1010	22	12	10	α	A
1011	23	13	11	ß	B
1100	30	14	12	10	C
1101	31	15	13	11	D
1110	32	16	14	12	E
1111	33	17	15	13	F

Conversion of Bases

- Methods:
  - Polynomial
  - Iterative (Repeated division)
  - Special type
- Polynomial conversion

Binary to Decimal:

- $D = \sum 2^i C_i$, C_i is (coefficient) digit at i^{th} position
  - $11011_2 = ?_{10}$
  - $101.011_2 = ?_{10}$

Ans.: 5.375

Conversion of Bases

CONVERT Base 8 to base 10

$(432.2)_8 = (\ ?)_{10}$ $4 \cdot 8^2 + 3 \cdot 8^1 + 2 \cdot 8^0 + 2 \cdot 8^{-1} = (282.25)_{10}$

Base 2 to base 10

$(1101.01)_2 = (\ ?)_{10}$ $1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} = (13.25)_{10}$

Polynomial conversion

Decimal to Binary:

- $780_{10} = ?_2$

Complete….

0111 x 1100100 + 1000 x 1010 = 1010111100 + 1010000 = 1100001100

Iterative method (Repeated division)

$$(N)_{b1} = a_{q-1}b_2^{q-1} + a_{q-2}b_2^{q-2} + \cdots + a_1 b_2^1 + a_0 b_2^0$$

$$\frac{(N)_{b1}}{b_2} = \underbrace{a_{q-1}b_2^{q-2} + a_{q-2}b_2^{q-3} + \cdots + a_1}_{Q_0} + \frac{a_0}{b_2}$$

$$\left(\frac{Q_0}{b_2}\right)_{b_1} = \underbrace{a_{q-1}b_2^{q-3} + a_{q-2}b_2^{q-4} + \cdots}_{Q_1} + \frac{a_1}{b_2}$$

Algorithm: (Decimal to binary)

1. Start
2. Divide N by 2

   Q=N/2, R= N mod 2

3. if Q ≠ 0 Repeat step 2 with N=Q
4. Collect R as binary number being first R as LSB.
5. End

Ex.: $37_{10} = ?_2$

Conversion of Bases (Contd.)

Example: Convert $(548)_{10}$ to base 8

Q_i	r_i
68 8 1	$4=a_0$ $4=a_1$ $0=a_2$ $1=a_3$

Thus, $(548)_{10} = (1044)_8$

Example: Convert $(345)_{10}$ to base 6

Q_i	r_i
57 9 1	$3=a_0$ $3=a_1$ $3=a_2$ $1=a_3$

Thus, $(345)_{10} = (1333)_6$

Converting Fractional Numbers

Fractional number:

$$(N)_{b1} \rightarrow a_{-1}b_2^{-1}+a_{-2}b_2^{-2}+....+a_{-p}b_2^{-p}$$

$$b_2 \,.\, (N)_{b1} \rightarrow a_{-1}+a_{-2}b_2^{-1}+....+a_{-p}b_2^{-p+1}$$

Example: Convert $(0.3125)_{10}$ to base 8

$0.3125 * 8 = 2.5000$ hence $a_{-1} = 2$

$0.5000 * 8 = 4.0000$ hence $a_{-2} = 4$

Thus, $(0.3125)_{10} = (0.24)_8$

Decimal to Binary

Example: Convert $(432.354)_{10}$ to binary

Q_i	r_i
216	$0=a_0$
108	$0=a_1$
54	$0=a_2$
27	$0=a_3$
13	$1=a_4$
6	$1=a_5$
3	$0=a_6$
1	$1=a_7$
	$1=a_8$

$0.354 \cdot 2 = 0.708$ hence $a_{-1}=0$

$0.708 \cdot 2 = 1.416$ hence $a_{-2}=1$

$0.416 \cdot 2 = 0.832$ hence $a_{-3}=0$

$a_{-7}=1$

etc.

Thus, $(432.354)_{10} = (110110000.0101101\ldots)_2$

Octal/Binary Conversion

Example: Convert $(123.4)_8$ to binary

$(123.4)_8 = (?)_2$ $(001\ 010\ 011.100)_2$

Example: Convert $(1010110.0101)_2$ to octal

$$(1010110.0101)_2 = (001\ 010\ 110.010\ 100)_2 = (126.24)_8$$

Thanks