ICS141: Discrete Mathematics for Computer Science I

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based on slides by Dr. Baek and Dr. Still

Originals by Dr. M. P. Frank and Dr. J.L. Gross

Provided by McGraw-Hill

Appendix to Lecture 7

Sample Proof for Example in Lecture 7

- This time a direct proof
  - Proves a more general Theorem first
  - Employs a Lemma
  - Uses Sets Notation

Direct Proof of $\forall n \in \mathbf{Z}: O(3n+2) \rightarrow O(n)$

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- **Definitions:**
  - Def_{Odd}: $O(x \in \mathbf{Z})$: $\exists k \in \mathbf{Z}$: $x=2k+1$
  - Def_{Even}: $E(x \in \mathbf{Z})$: $\exists k \in \mathbf{Z}$: $x=2k$
- **Theorem 1:**
  - $\forall n,m \in \mathbf{Z}: O(n) \wedge E(m) \rightarrow O(n+m)$.
- **Proof:**
  - $O(n) \wedge E(m)$
    $\rightarrow \exists x \in \mathbf{Z}: n=2x+1 \wedge \exists y \in \mathbf{Z}: m=2y$ $\quad \text{Def}_{\text{Odd}}, \text{Def}_{\text{Even}}$
    $\rightarrow \exists x,y \in \mathbf{Z}: n+m=2x+1+2y=2(x+y)+1$
    $\rightarrow \exists k=x+y \in \mathbf{Z}: n+m=2k+1$ $\quad \text{Def}_{\text{Odd}}$
    $\rightarrow O(n+m)$ ■

Direct Proof (cont.) of $\forall n \in \mathbf{Z}: O(3n+2) \rightarrow O(n)$

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- **Lemma:** $E(-2n-2)$
  - $-2n-2=2(-n-1)$
    $\rightarrow \exists k=-n-1 \in \mathbf{Z}: -2n-2=2k$ $\quad \text{Def}_{\text{Even}}$
    $\rightarrow E(-2n-2)$ ■
- **Theorem:** $\forall n \in Z: O(3n+2) \rightarrow O(n)$
  - Proof: $O(3n+2)$
    $\rightarrow O(3n+2) \wedge E(-2n-2)$ $\quad$ Conjunction Rule, Lemma
    $\rightarrow O(3n+2-2n-2)$ $\quad$ Theorem 1
    $\rightarrow O(3n-2n+2-2)$
    $\rightarrow O(n)$ ■