CS 225: Switching Theory

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Previous Class

- Number Systems
  - Different Number systems (positional)
  - Conversion
  - Representation (complement)
  - Binary Arithmetic
- Codes
  - BCD, cyclic code etc.
  - Gray code
  - Parity and Error correcting code

This Class

- Switching Algebra

Switching Algebra

Basic postulate:
Existence of two-valued switching variable that takes two distinct values 0 and 1

Switching Algebra:
Algebraic system of set **{0, 1},** binary operations **OR** and **AND**, and unary operation **NOT**

OR operation	AND operation	NOT operation
$0 + 0 = 0$	$0 . 0 = 0$	$0' = 1$
$0 + 1 = 1$	$0 . 1 = 0$	
$1 + 0 = 1$	$1 . 0 = 0$	$1' = 0$
$1 + 1 = 1$	$1 . 1 = 1$	

OR: called logical sum

AND: called logical product

NOT called complementation:

Basic Properties

Perfect induction: Proving a theorem by verifying every combination of values that the variables may assume

If x is a switching variable, then: $x + 1 = 1$

$x \, . \, 0 = 0$

$x + 0 = x$

$x \, . \, 1 = x$

Idempotency: $x + x = x$ $x \, . \, x = x$

Proof of $x + x = x$: $1 + 1 = 1$ and $0 + 0 = 0$

Commutativity: $x + y = y + x$
$x \, . \, y = y \, . \, x$

Basic Properties (Contd.)

Associativity: $(x + y) + z = x + (y + z)$

$(x \,.\, y) \,.\, z = x \,.\, (y \,.\, z)$

Complementation: $x + x' = 1$

$x \,.\, x' = 0$

Distributive: $x \,.\, (y + z) = x \,.\, y + x \,.\, z$

$x + y \,.\, z = (x + y) \,.\, (x + z)$

Proof by perfect induction using a **truth table**:

x	**y**	**z**	**xy**	**xz**	**y+z**	**x(y+z)**	**xy+xz**
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	0	1	0	0
0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	1	1	1
1	1	1	1	1	1	1	1

Basic Properties (Contd.)

Principle of Duality:

- Preceding properties grouped in pairs
    - One statement can be obtained from the other by interchanging operations OR and AND and constants 0 and 1
    - The two statements are said to be **dual** of each other
- This principle stems from the symmetry of the postulates and definitions of switching algebra w.r.t. the two operations and constants

- **Implication**: necessary to prove only one of each pair of statements

Switching Expressions and Their Manipulation

Switching expression:

- combination of finite number of switching variables and constants via switching operations (AND, OR, NOT)
  - Any constant or switching variable is a switching expression
  - If T1 and T2 are switching expressions, so are T1', T2', T1+T2 and T1T2
  - No other combination of constants and variables is a switching expression

Absorption law: $x + xy = x$

$x(x + y) = x$

Proof 1:

Laws of Switching Algebra

Another important law 2: $x + x'y = x + y$

$x(x' + y) = xy$

Consensus theorem : $xy + x'z + yz = xy + x'z$

$(x + y)(x' + z)(y + z) = (x + y)(x' + z)$

Proof 3:

Switching Expression Simplification

Literal: variable or its complement
Redundant literal: if value of switching expression is independent of literal x_i then x_i is said to be redundant

Example: Simplify $T(x,y,z) = x'y'z + yz + xz$

$$\begin{aligned} x'y'z + yz + xz &= z(x'y' + y + x) \\ &= z(x' + y + x) \\ &= z(y + 1) \\ &= z1 = z \end{aligned}$$

Thus, literals x and y are redundant in $T(x,y,z)$

Important note: No inverse operations are defined in Switching Algebra, So cancellations are not allowed

- $A + B = A + C$ does not imply $B = C$

  Counterexample: $A = B = 1$ and $C = 0$
- Similarly, $AB = AC$ does not imply $B = C$

De Morgan's Theorems

Involution: $(x')' = x$

De Morgan's theorem for two variables:

$$(x + y)' = x' \, . \, y'$$
$$(x \, . \, y)' = x' + y'$$

Proof by perfect induction:

x	y	x'	y'	x+y	(x+y)'	x'y'
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

De Morgan's theorems for n variables:

$$[f(x_1, x_2, \ldots, x_n, 0, 1, +, .)]' = f(x_1', x_2', \ldots, x_n', 1, 0, ., +)$$

Simplification Examples

Example 4: Simplify $T(x,y,z) = (x + y)[x'(y' + z')]' + x'y' + x'z'$

Thus, $T(x,y,z) = 1$, independently of the values of the variables

Example 5: Prove $xy + x'y' + yz = xy + x'y' + x'z$

Switching Functions

Let $T(x_1, x_2, \ldots, x_n)$ be a switching expression:

- Since each variable can assume 0 or 1, → 2^n combinations are possible

Determining the value of an expression for an input combination:

Example: $T(x,y,z) = x'z + xz' + x'y'$

$$T(0,0,1) = 0'1 + 01' + 0'0' = 1$$

Truth table for T

x	y	z	T
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1

x	y	z	T
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Switching Functions (Contd.)

Switching function $f(x_1, x_2, \ldots, x_n)$: values assumed by an expression for all combinations of variables $x_1, x_2, \ldots, x_n$

Complement function: $f'(x_1, x_2, \ldots, x_n)$ assumes value 0 (1) whenever $f(x_1, x_2, \ldots, x_n)$ assumes value 1 (0)

Logical sum of two functions:

$f(x_1, x_2, \ldots, x_n) + g(x_1, x_2, \ldots, x_n) = 1$ for every combination in which either f or g or both equal 1

Logical product of two functions:

$f(x_1, x_2, \ldots, x_n) \,.\, g(x_1, x_2, \ldots, x_n) = 1$ for every combination for which both f and g equal 1

Switching Functions (Contd.)

sum, product and complementation of functions:

x	y	z	f	g	f'	f+g	fg
0	0	0	1	0	0	1	0
0	0	1	0	1	1	1	0
0	1	0	1	0	0	1	0
0	1	1	1	1	0	1	1
1	0	0	0	1	1	1	0
1	0	1	0	0	1	0	0
1	1	0	1	1	0	1	1
1	1	1	1	0	0	1	0

Simplification of Expressions

Example 1: Simplify $T(A,B,C) = A'C' + ABD + BC'D + AB'D' + ABCD'$

- Apply consensus theorem to first three terms -> $BC'D$ is redundant
- Apply distributive law to last two terms -> $AD'(B' + BC)$ -> $AD'(B' + C)$

- Thus, $T = A'C' + A[BD + D'(B' + C)]$

Example 2: Simplify $T(A,B,C,D) = A'B + ABD + AB'CD' + BC$

- $T = A'B + BD + ACD'$

Thanks