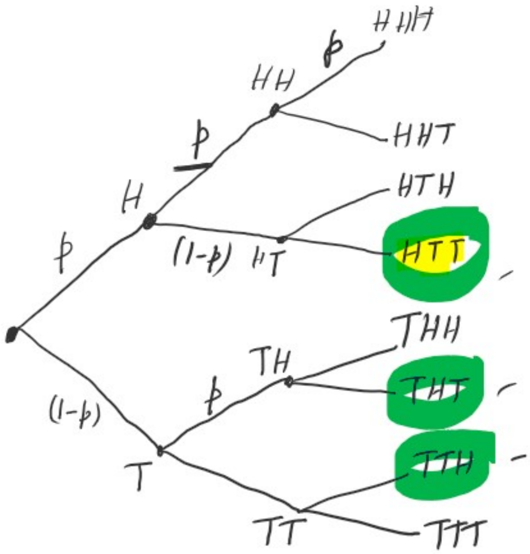
Definitions and examples

Tuesday, January 19, 2021 8:44 AM

Ex A biased Coin $P(H) = p$

Toss the coin 3 times



$$P(HTT) = p(1-p)(1-p)$$

$$P(1 \text{ head}) = 3p(1-p)^2$$

$$P(\underline{\text{first toss is H}} \mid \underline{1\text{-head}})$$

$$= 1/3 \qquad = \frac{P(\overset{(HTT)}{\text{first H and 1-head}})}{P(1\text{-head})}$$

Notion of Indep. Two events are indep iff

$$\boxed{P(A|B) = P(A)}$$

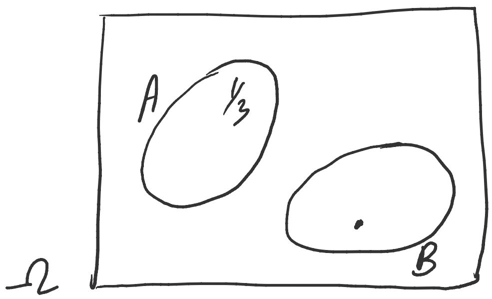
$$\frac{P(A|B)}{P(A)} \cdot P(B) = P(A \cap B)$$

Defi

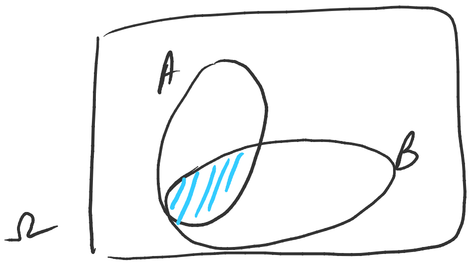
$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

If $P(B) = 0$

$$P(A \cap B) = 0 = P(A) \cdot 0 = P(A) \cdot P(B)$$



$\Rightarrow$ A & B are dept.



(*) If A & B are ind, A and B^c are also ind

$$A = (A\cap B) \cup (A\cap B^c)$$

$$\Rightarrow P(A) = P(A\cap B) + P(A\cap B^c) \Rightarrow \underline{P(A\cap B^c) = P(A)\cdot P(B^c)}$$

<u>Conditional Indep</u>

$$P(A\cap B / C) = P(A/C)\cdot P(B/C)$$

If A & B are condi ind, given C,

$$\underline{P(A|B\cap C)} = \underline{P(A|C)}$$

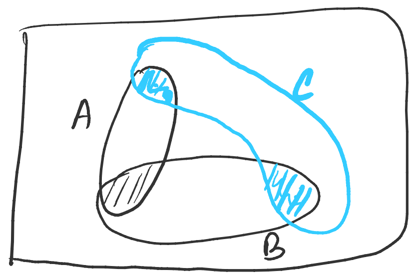
$$\boxed{P(B|A\cap C) = P(B|C)}$$

$$P(A|C)\,P(B|C) = P(A\cap B|C) = \frac{P(A\cap B\cap C)}{P(C)} = \frac{\cancel{P(C)}\cdot P(B|C)\cdot P(A|B\cap C)}{\cancel{P(C)}}$$

$$\Rightarrow P(A|C) = P(A|B\cap C)$$

Ques: If A & B are ind, ~~does~~ can A & B are condi ind?

C always



ind $\not\Rightarrow$ cond ind