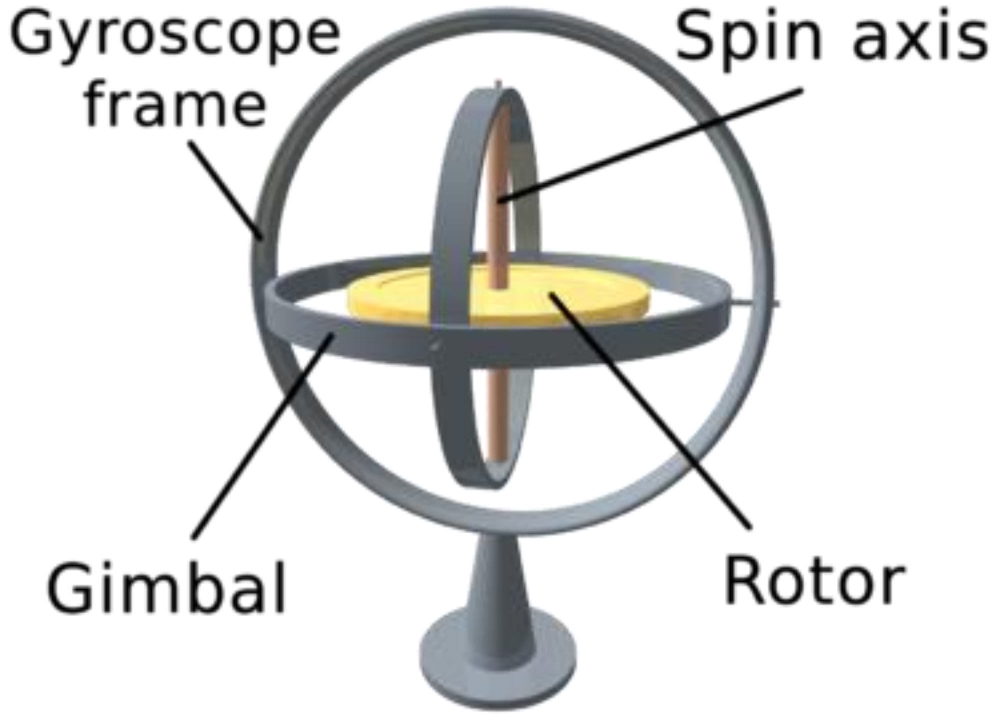
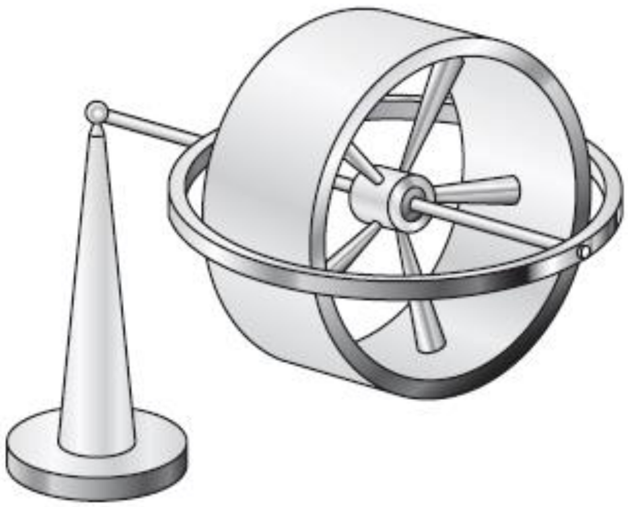
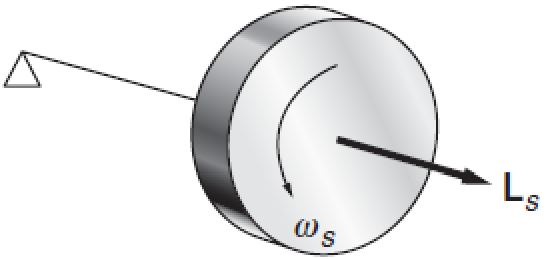
Gyroscope and its applications



Gyroscope

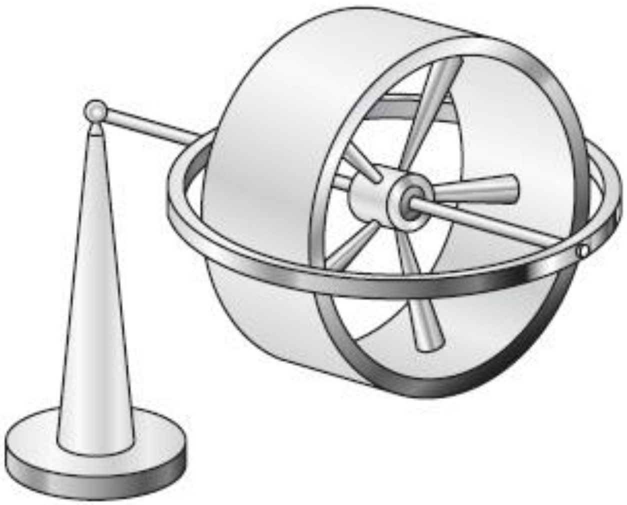
Gyroscope

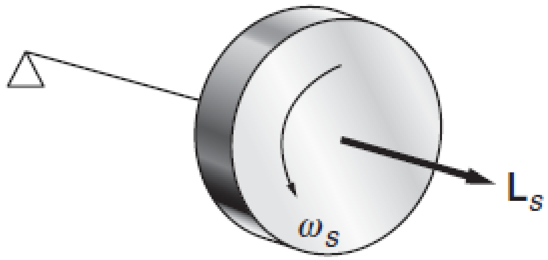




Uniform precession is consistent with $\boldsymbol{\tau} = \frac{d\boldsymbol{L}}{dt}$ and Newton's law.

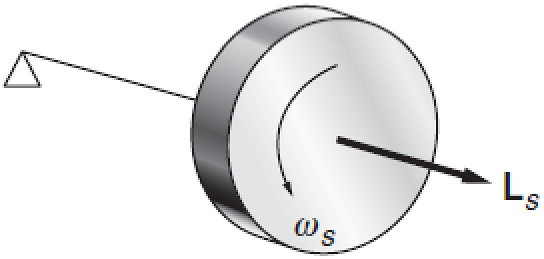
Gyroscope

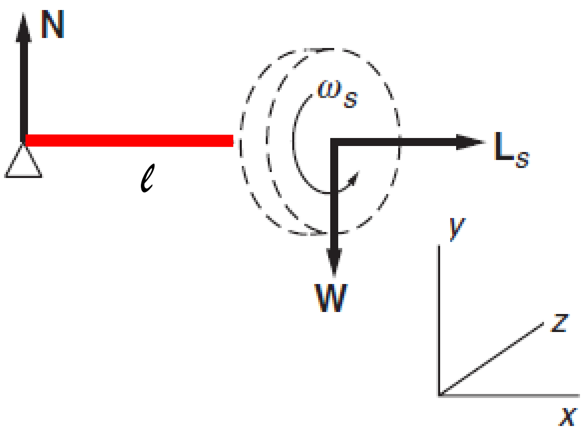


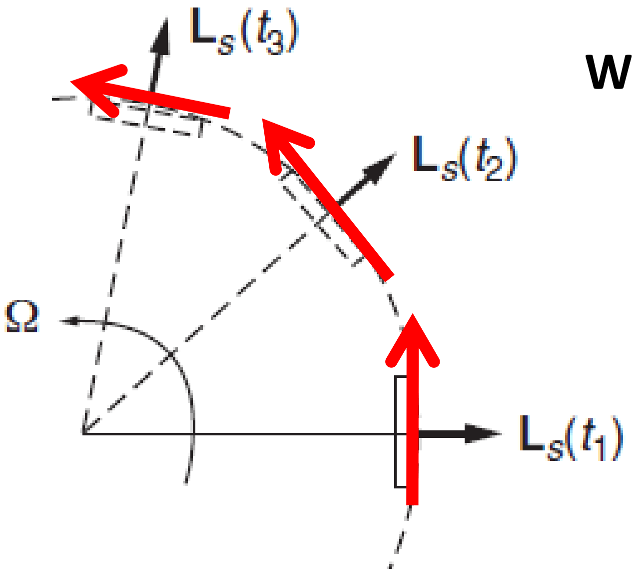


Why gyroscope does not fall?

Gyroscope Precision



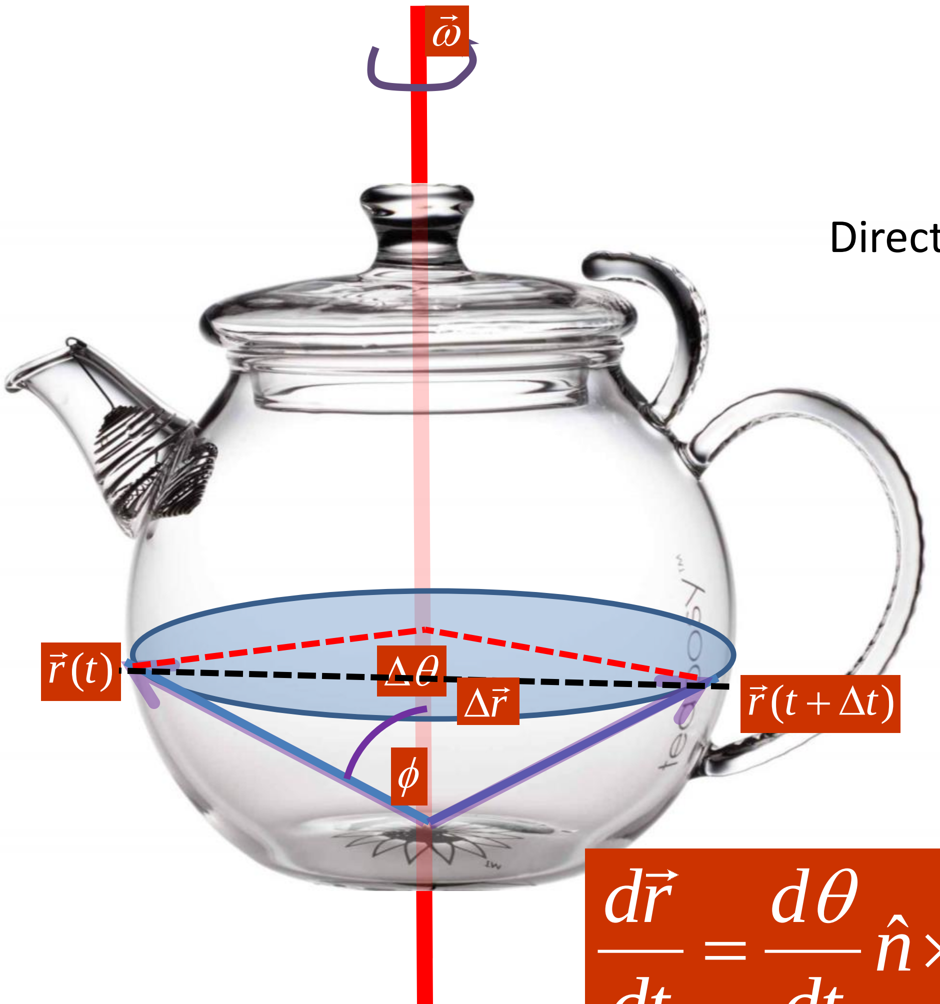




Why it does not swing about pivot like a pendulum?

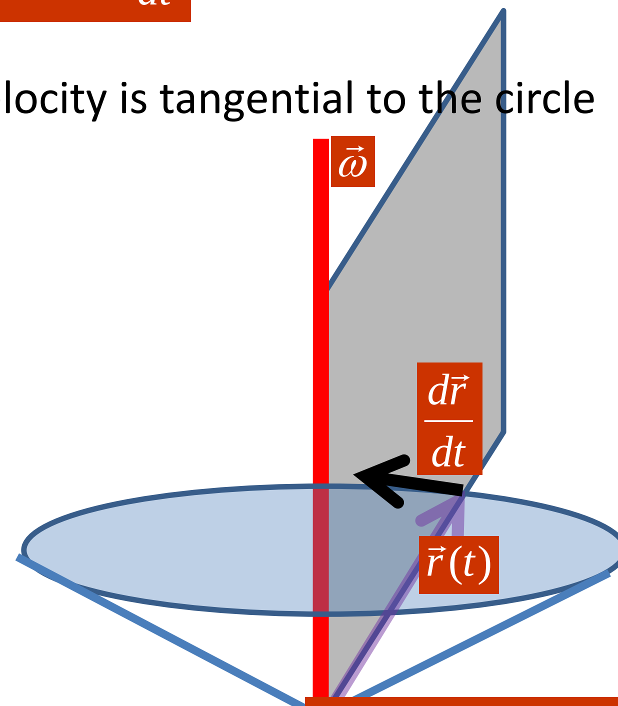
$$\left|\frac{dL_s}{dt}\right| = ?$$

Vector nature **of angular velocity** and **angular momentum**



$$\left|\frac{dr}{dt}\right| = r\sin\phi\,\frac{d\theta}{dt}$$

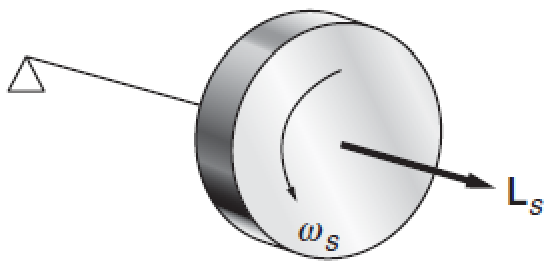
Direction of velocity is tangential to the circle

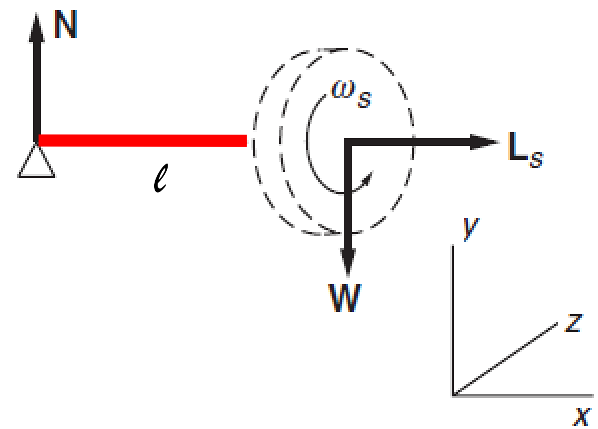


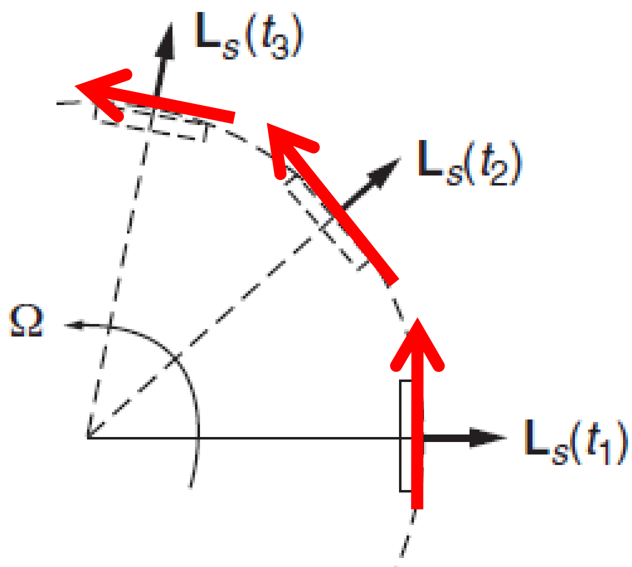
$$\frac{d\vec{r}}{dt} = \frac{d\theta}{dt}\hat{n}\times\vec{r} = \vec{\omega}\times\vec{r}$$

$$\frac{d\vec{A}}{dt} = \vec{\omega}\times\vec{A}$$

Gyroscope Precision







$$\frac{d\vec{A}}{dt} = \vec{\Omega} \times \vec{A}$$

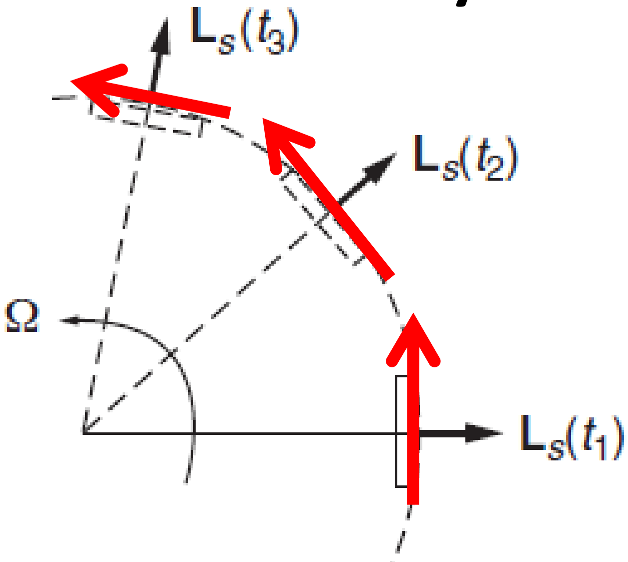
$$\left|\frac{dL_s}{dt}\right| = \Omega L_s$$

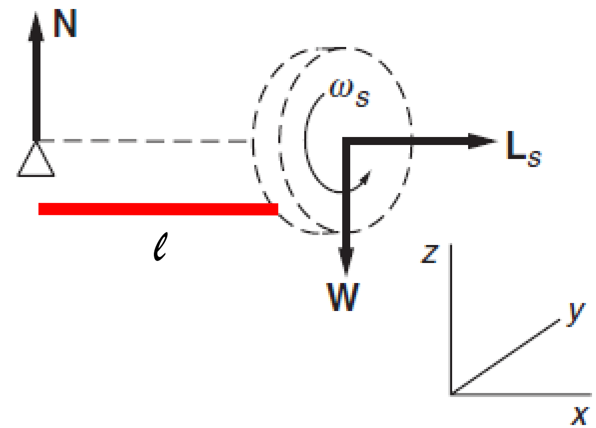
$$\tau = lW$$

Direction is tangential

There must be a torque on the gyroscope to account for the change in L_s

Gyroscope Precision





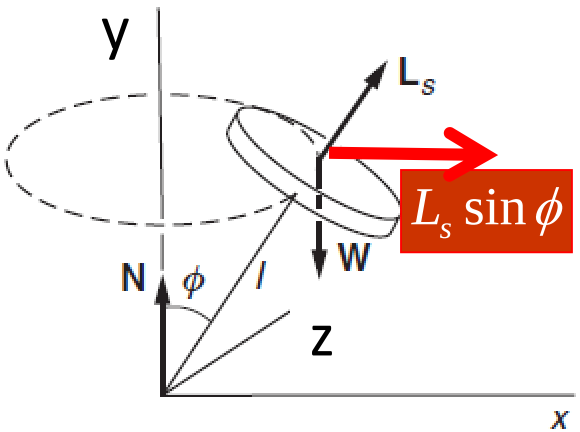
$$lW = \Omega L_s$$

$$lW = \Omega I_0 \omega_s$$

Rate of precession

$$\Omega = \frac{lW}{I_0 \omega_s}$$

Gyroscope Precision



Consider a gyroscope in uniform precession with its axle at angle ϕ with vertical

The horizontal component of angular momentum is $L_s \sin\phi$

$$\left|\frac{dL_s}{dt}\right| = \Omega L_s \sin\phi$$

$$lW \sin\phi = \Omega L_s \sin\phi$$

$$\Omega = \frac{lW}{I_0 \omega_s}$$

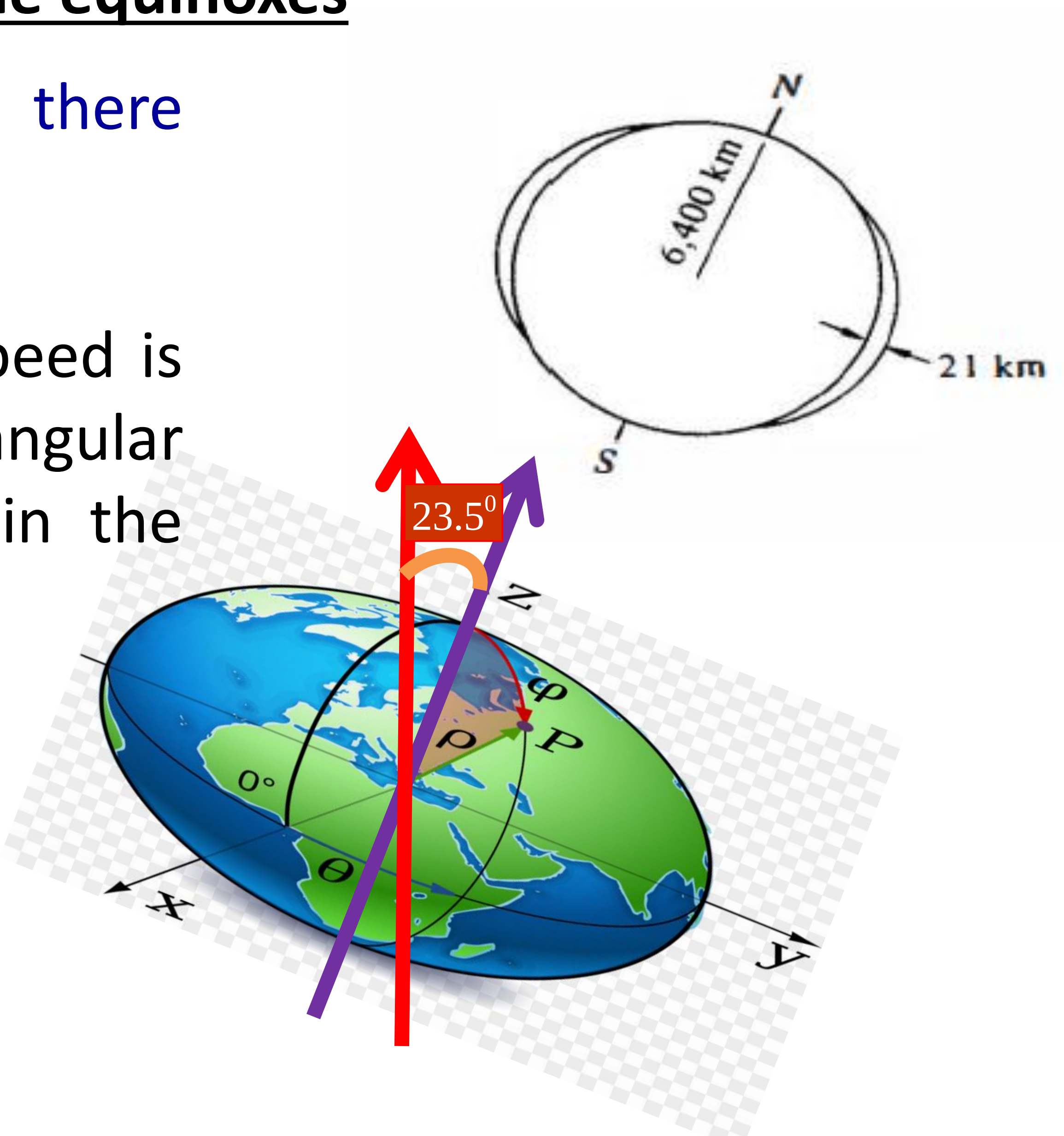
The precessional velocity is independent of ϕ

Some applications of Gyroscopic motion

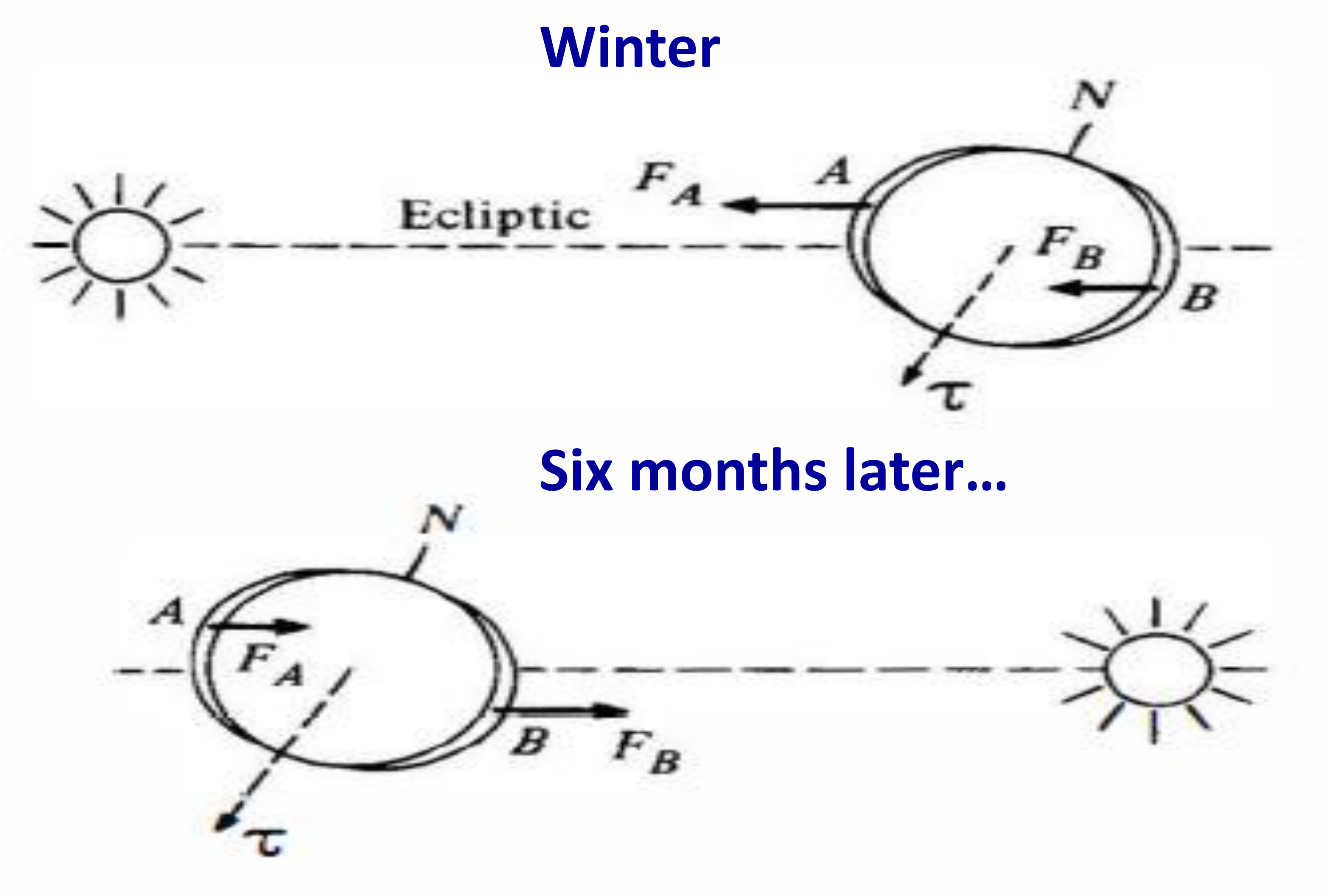
a. Precession of the equinoxes

First approximation: there are no torques

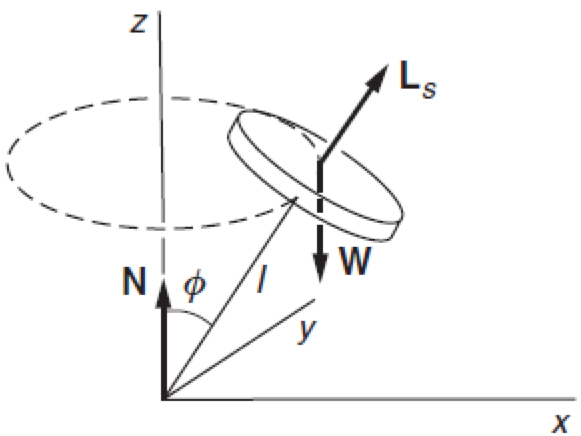
Earth's Rotational speed is constant and angular momentum points in the same direction.

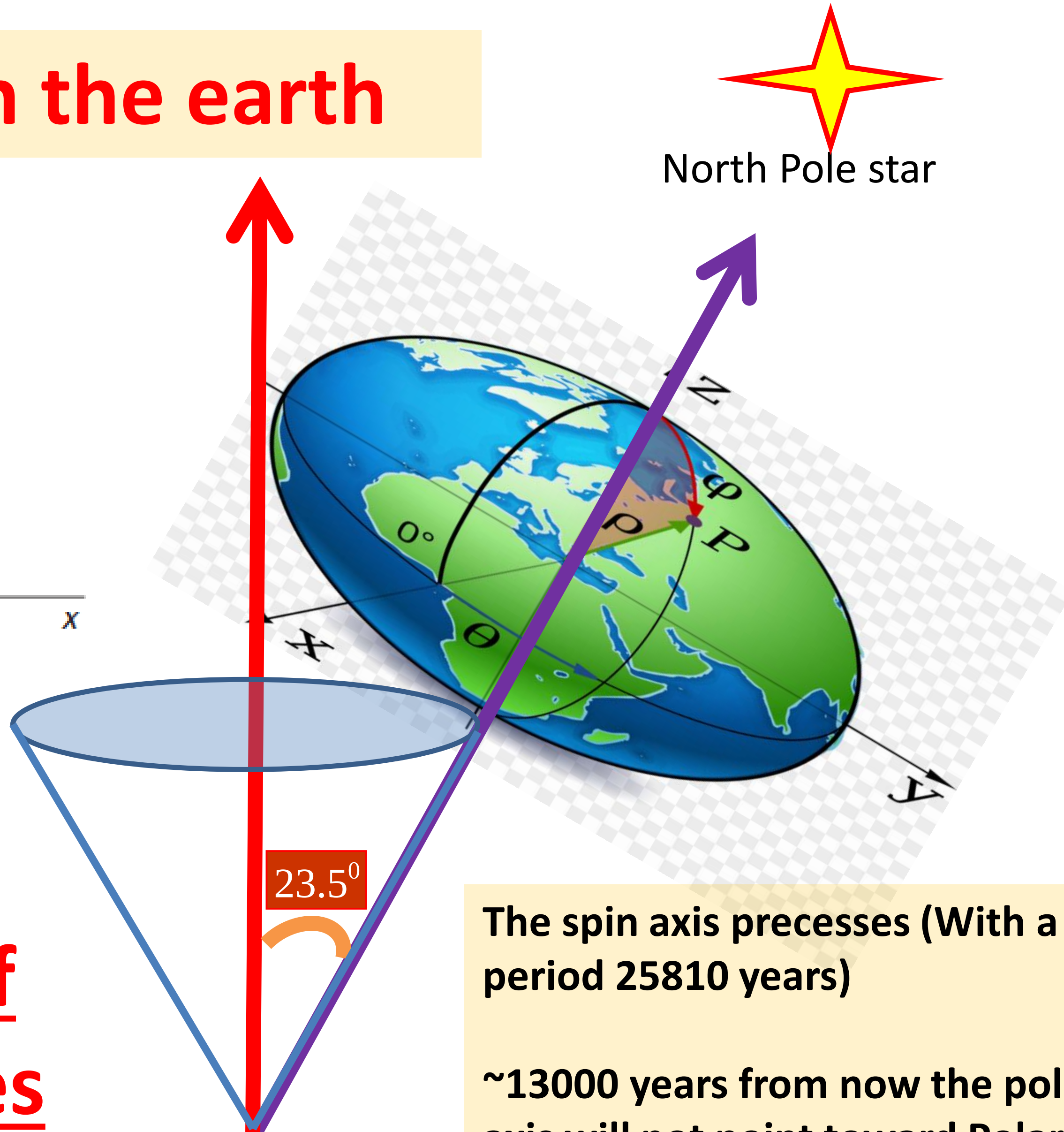


Torque on the earth



Torque on the earth





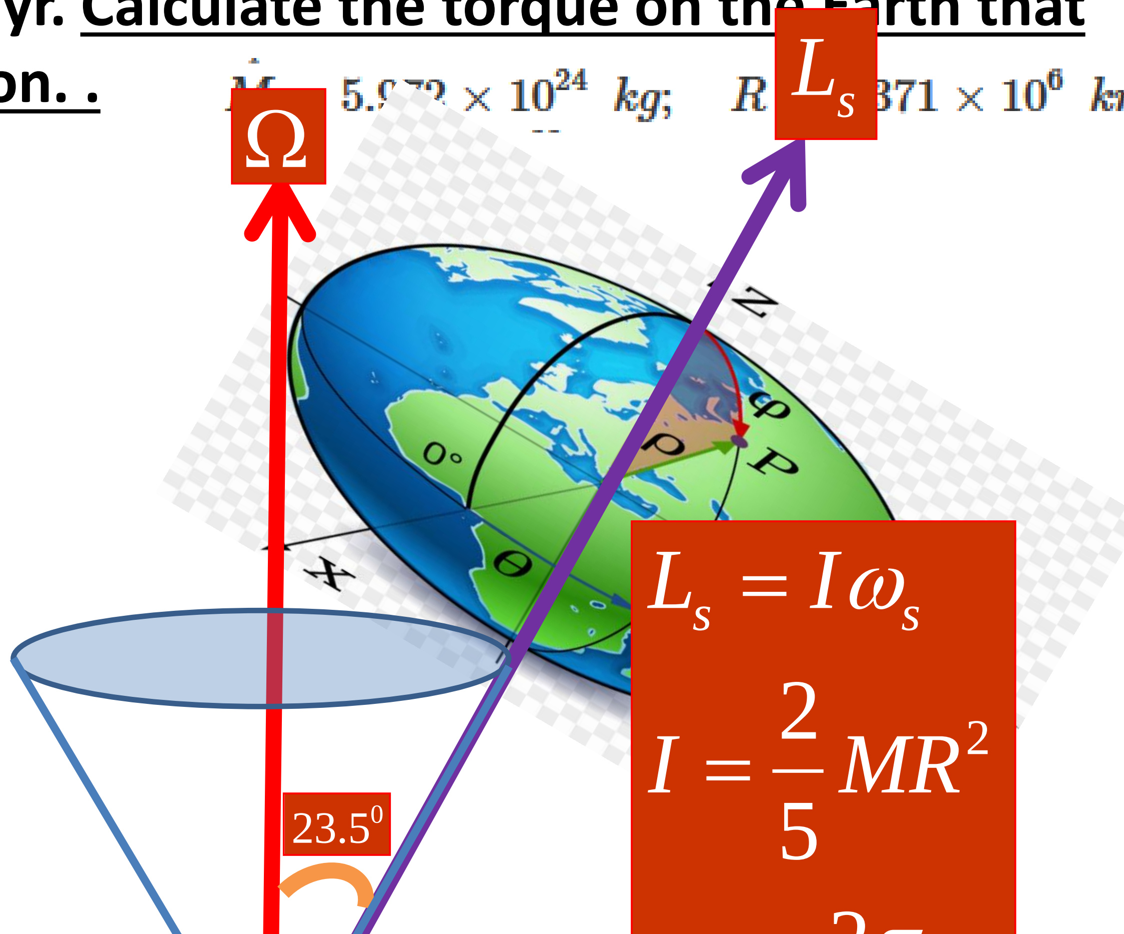
Precession of the equinoxes

The spin axis precesses (With a period 25810 years)

~13000 years from now the polar axis will not point toward Polaris, the north pole star. It will point 47° away.

In the motion called precession of the equinoxes, the Earth's axis of rotation precesses about the perpendicular to its orbital plane with a period of 25810 yr. <u>Calculate the torque on the Earth that is causing this precession. .</u>

$M = 5.972 \times 10^{24}\ kg; \quad R = 6.371 \times 10^{6}\ km$



$$\tau = \Omega \times L_s$$

$$\tau = \Omega L_s \sin\phi$$

$$L_s = I\omega_s$$

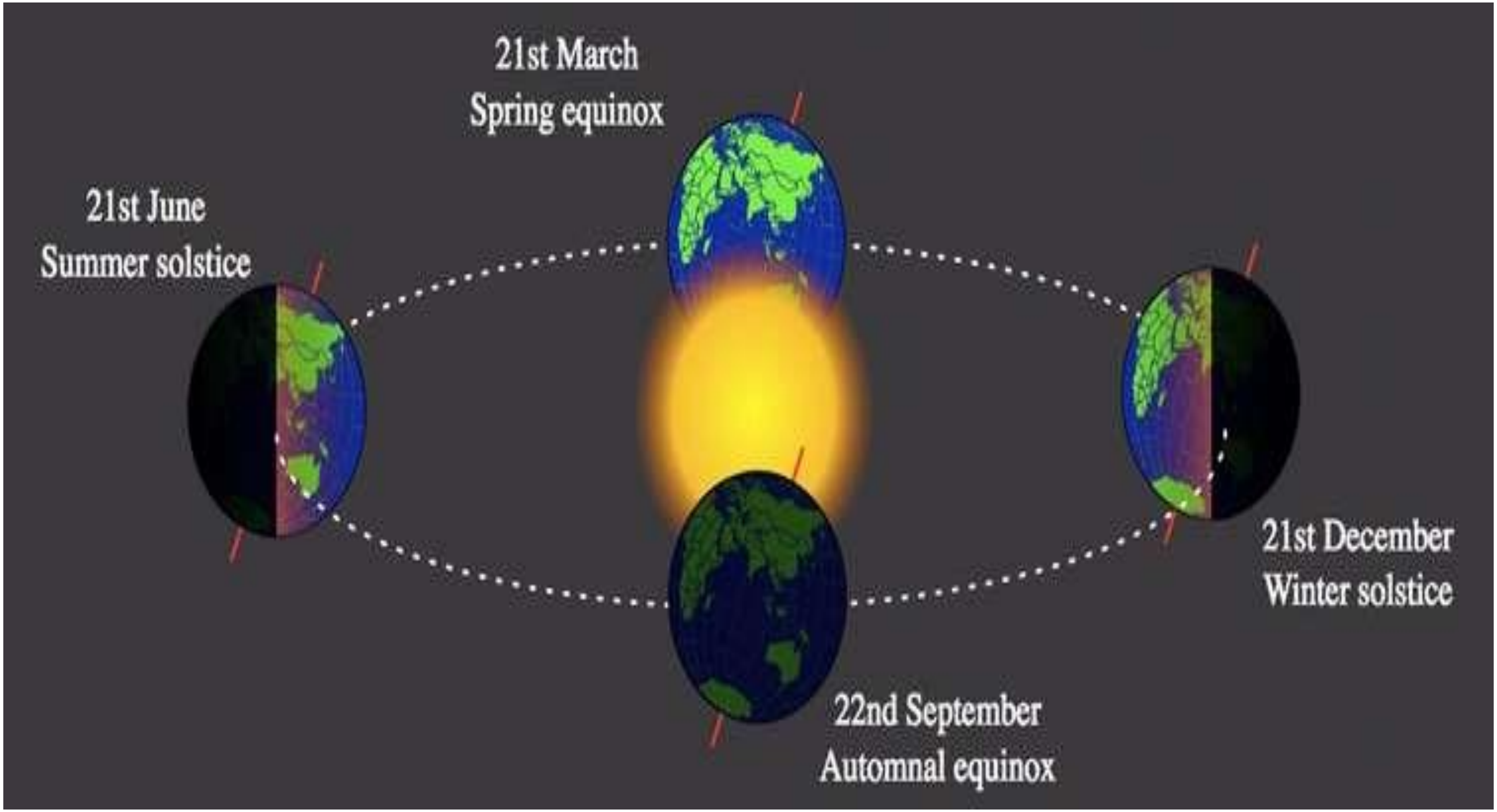
$$I = \frac{2}{5}MR^2$$

$$\Omega = \frac{2\pi}{T}$$

$$\tau = 2.166 \times 10^{22}\ N.m$$

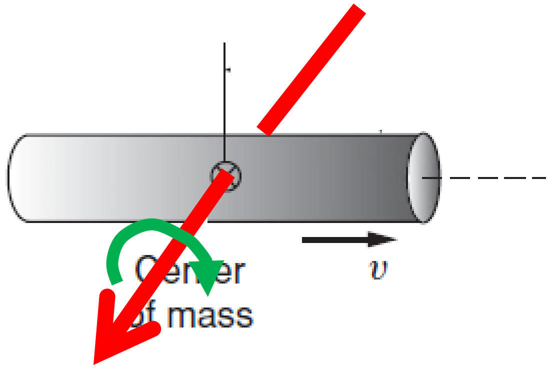
[redacted]

Different seasons on the earth



Consider a cylinder moving parallel to its axis with velocity v in free space. A perturbing force F acts on the cylinder for time Δt. Find the angular frequency of rotation

First consider cylinder spin is 0

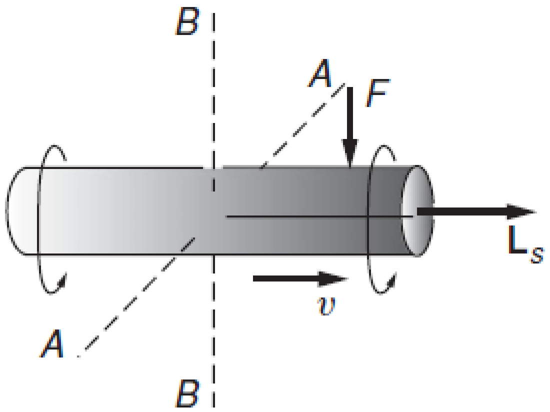


__Angular impulse__ $\tau\Delta t = Fl\Delta t$

__Change in angular momentum__ $\Delta L_A = I_A\omega$

$$\omega = \frac{Fl\Delta t}{I_A}$$

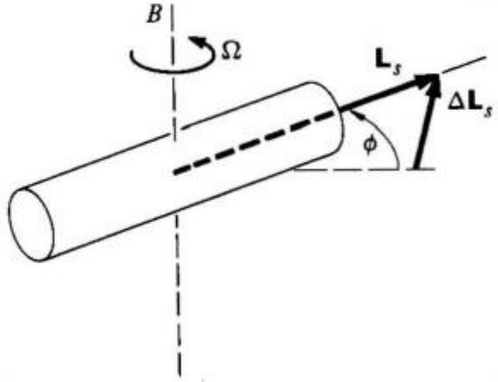
Consider a cylinder spinning rapidly with angular momentum L_s moving parallel to its axis with velocity v in free space. A perturbing force F acts on the cylinder for time Δt. Find the angle through which the cylinder processes.



The cylinder is spinning with angular momentum L_s

The situation is similar to that of **gyroscope**: Torque along the AA axis causes precession around the BB axis

$$\boxed{\Omega = \frac{Fl}{L_s}}$$

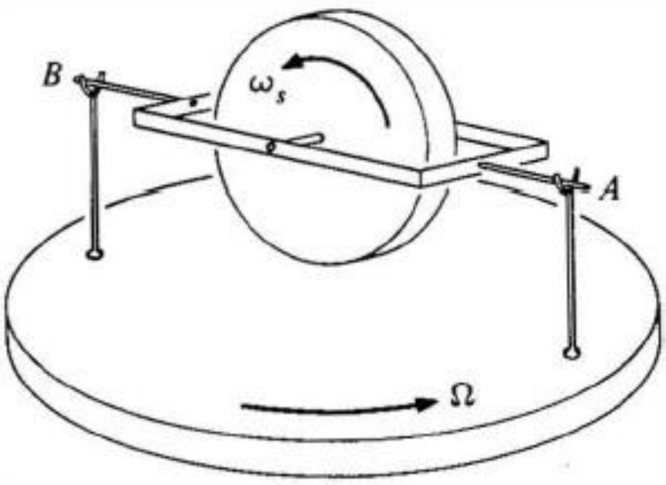


$$\boxed{\phi = \Omega \Delta t = \frac{FL}{L_s} \Delta t}$$

The cylinder slightly changes its orientation while the force is applied. No tumbling

Gyrocompass motion

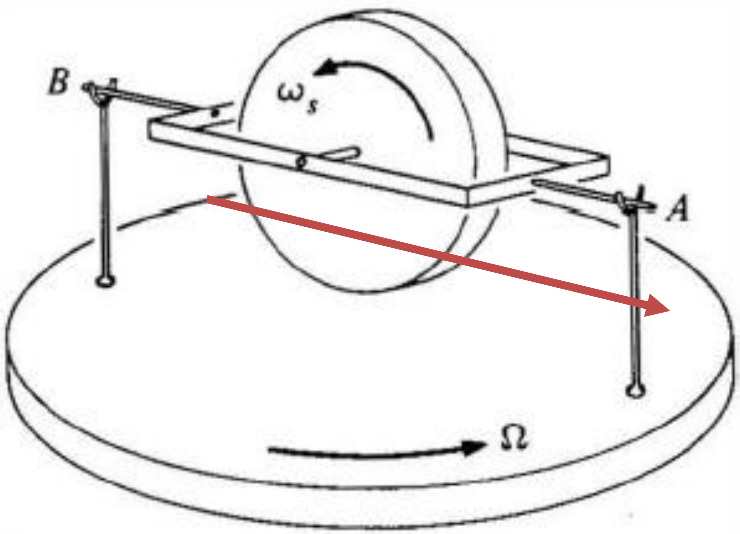
Consider a gyrocompass consisting of a balanced spinning disk held in a light frame supported by a horizontal axle. The assembly is on a turntable rotating at steady angular velocity Ω. Find the equation of motion. When the spin axis is near the vertical, show that gyro executes simple harmonic motion with $\theta = \theta_0 \sin \beta t$, where $\beta = \sqrt{\frac{\omega_s \Omega I_s}{I_\perp}}$.



Flywheel free to rotate about two perpendicular axes tends to orient its spin axis parallel to the axis of rotation of the system.

Compass comes to rest with its axis parallel to the polar axis.

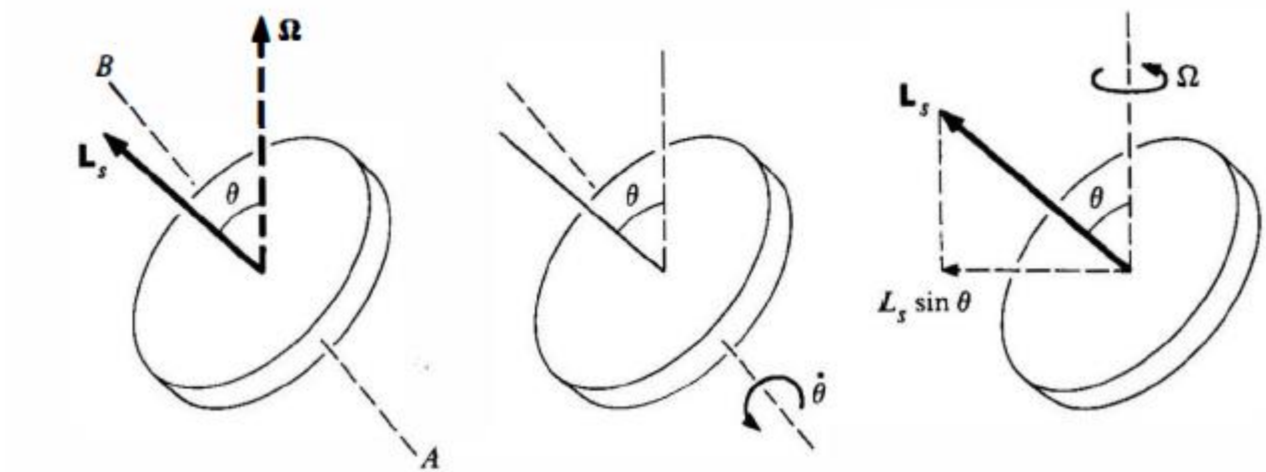
Gyrocompass motion



No torque along the horizontal AB axis because the axle is pivoted. Therefore, the angular momentum (L_h) along the AB direction is constant.

$$\frac{dL_h}{dt} = 0$$

Gyrocompass motion

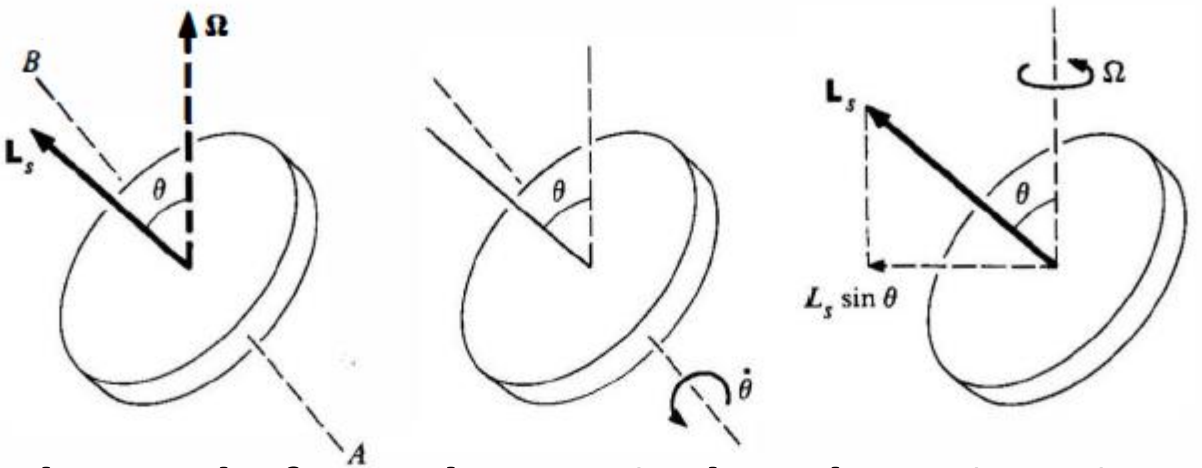


θ: the angle from the vertical to the spin axis

***$I_\perp$*: The moment of inertia about AB axis.**

$$L_h = I_\perp \dot{\theta} \quad \Rightarrow \quad \frac{dL_h}{dt} = I_\perp \ddot{\theta}$$

Gyrocompass motion



θ: the angle from the vertical to the spin axis

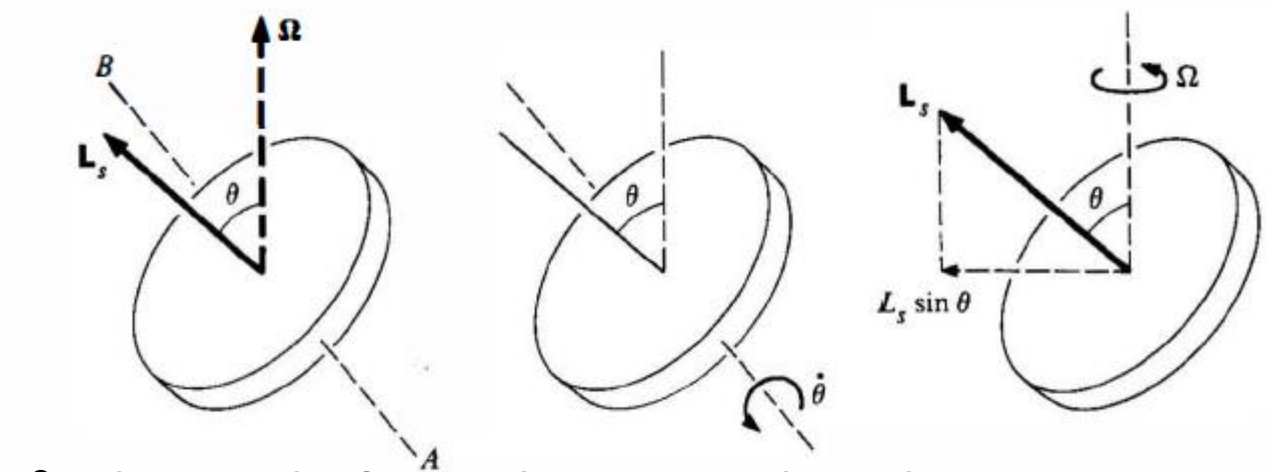
$I_\perp$: The moment of inertia about AB axis.

$$L_h = I_\perp \dot{\theta} \quad \Rightarrow \quad \frac{dL_h}{dt} = I_\perp \ddot{\theta}$$

L_h can change because of a change in direction of L_s

$$\frac{dL_h}{dt} = \Omega L_s \sin\theta$$

Gyrocompass motion



θ: the angle from the vertical to the spin axis

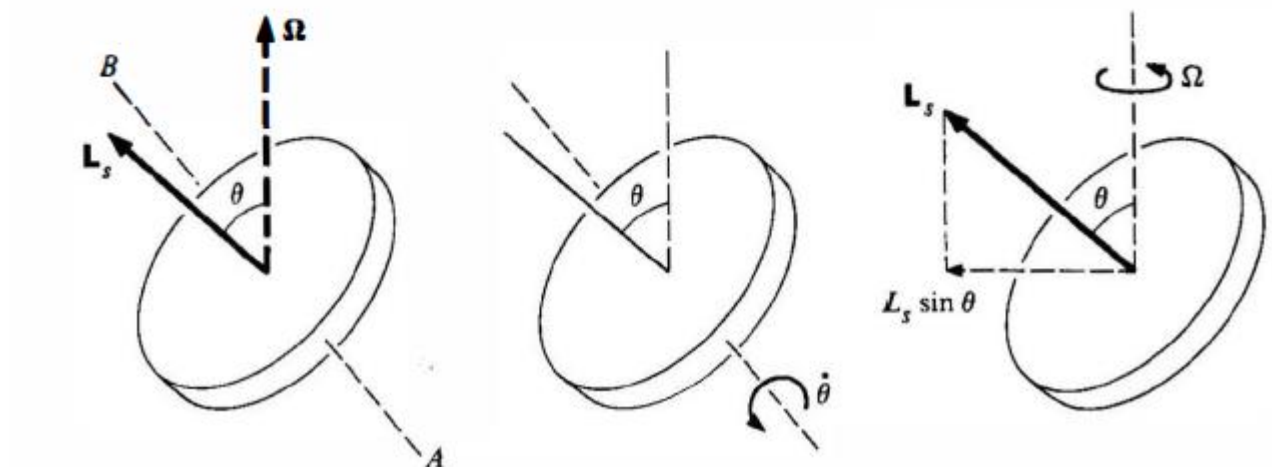
$I_\perp$: The moment of inertia about AB axis.

$$\frac{dL_h}{dt} = I_\perp \ddot{\theta} + \Omega L_s \sin\theta$$

$$\frac{dL_h}{dt} = 0 \Longrightarrow I_\perp \ddot{\theta} + \Omega L_s \sin\theta = 0$$

$$\ddot{\theta} + \frac{L_s \Omega}{I_\perp} sin\theta = 0$$ **Equation of motion**

Gyrocompass motion



θ: the angle from the vertical to the spin axis

$I_{\perp}$: The moment of inertia about AB axis.

$$\ddot{\theta} + \frac{L_s \Omega}{I_{\perp}} \sin\theta = 0$$

When spin axis is near the vertical, sinθ≈θ

$$\ddot{\theta} + \frac{L_s \Omega}{I_{\perp}} \theta = 0$$

$$\theta = \theta_0 \sin \beta t$$

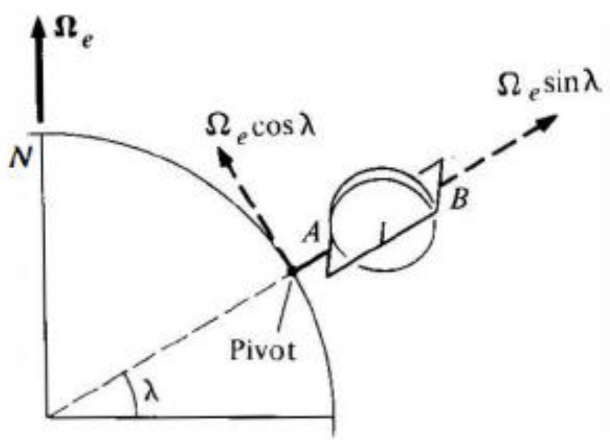
$$\beta = \sqrt{\frac{L_s \Omega}{I_{\perp}}} = \sqrt{\frac{\omega_s \Omega I_s}{I_{\perp}}}$$

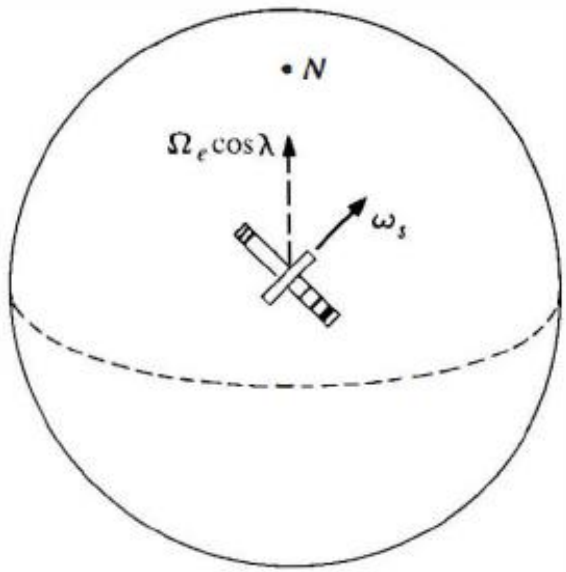
Gyrocompass motion

$$\ddot{\theta} + \frac{L_s\Omega}{I_\perp}\theta = 0$$

$$\theta = \theta_0 \sin \beta t$$

$$\beta = \sqrt{\frac{L_s\Omega}{I_\perp}} = \sqrt{\frac{\omega_s \Omega I_s}{I_\perp}}$$





For a thin disk

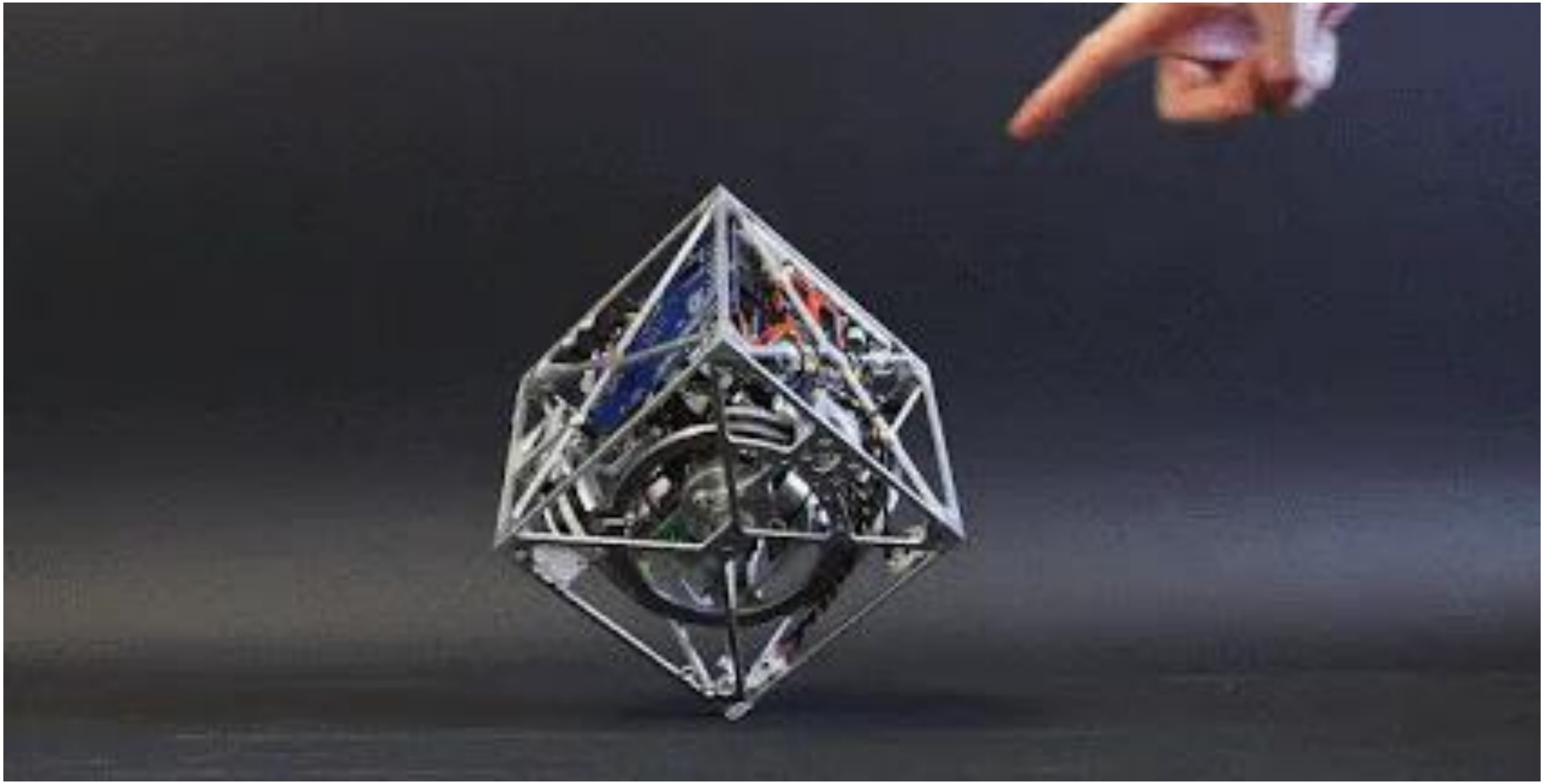
$$\frac{I_s}{I_\perp} = 2$$

$$\Omega_e = 2\pi/day$$

For a gyro rotating at 20000 rpm, the period at the equator is 11s.

Near north pole the period becomes too long that the gyrocompass is not effective.

Cubli



Fictitious Forces

Rotating or Accelerating Frame of Reference

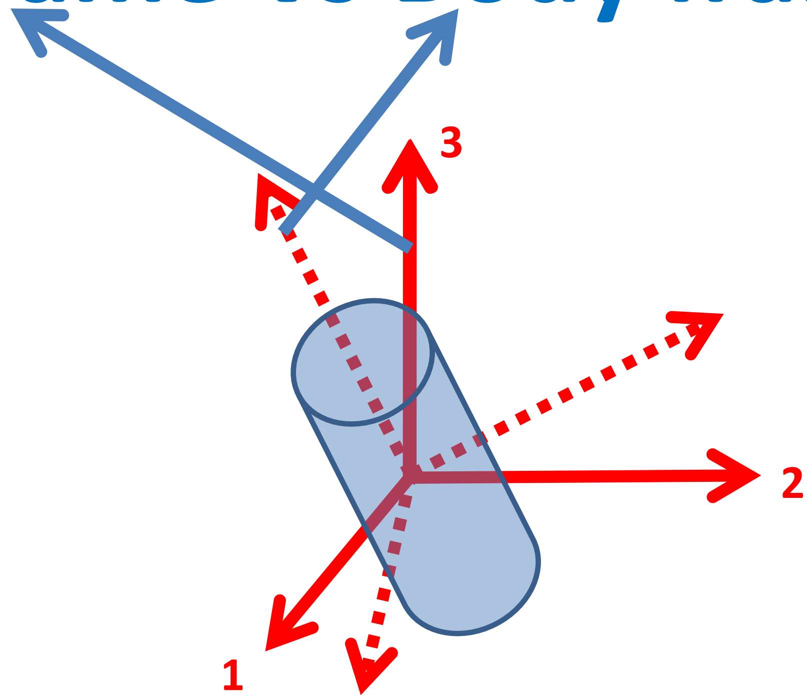
Newton's laws hold only in <u>inertial frames of reference</u>.

Many <u>non-inertial</u> (that is, accelerated or rotating) frames of reference that we might reasonably want to study, such as elevators, merry-go-rounds

Is there any possible way to modify Newton's laws so that they hold in non-inertial frames, or do we have to give up entirely on F = *m*a?

- ✓ **Inertial frame: Non-Rotating frame**
- ➢ **Non-Inertial frame: Rotating frame**

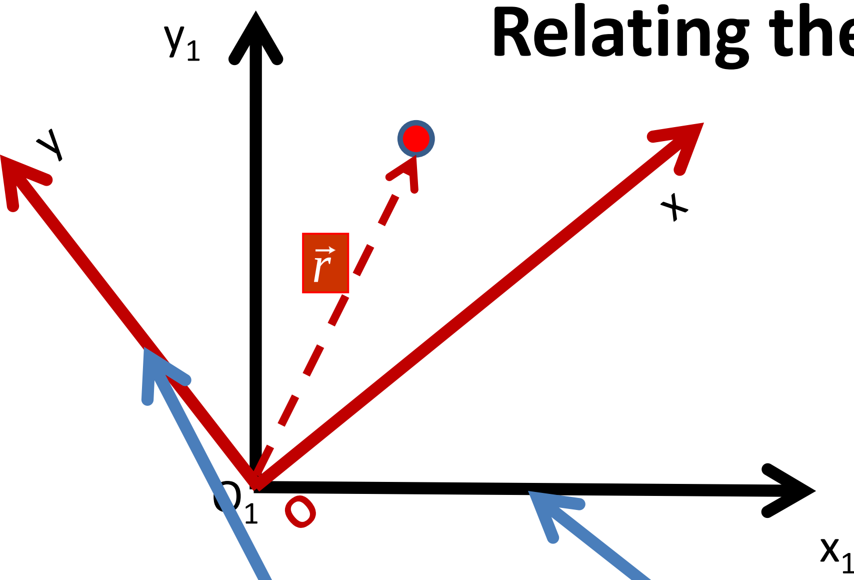
Lab frame Vs Body frame



Lab frame: Non-Rotating frame or Inertial Frame

Body frame: Rotating frame or Non-Inertial frame

Relating the Co-ordinates



$$\text{Angular velocity} = \vec{\omega}$$

Lab frame: Non-Rotating frame

Body frame: Rotating frame

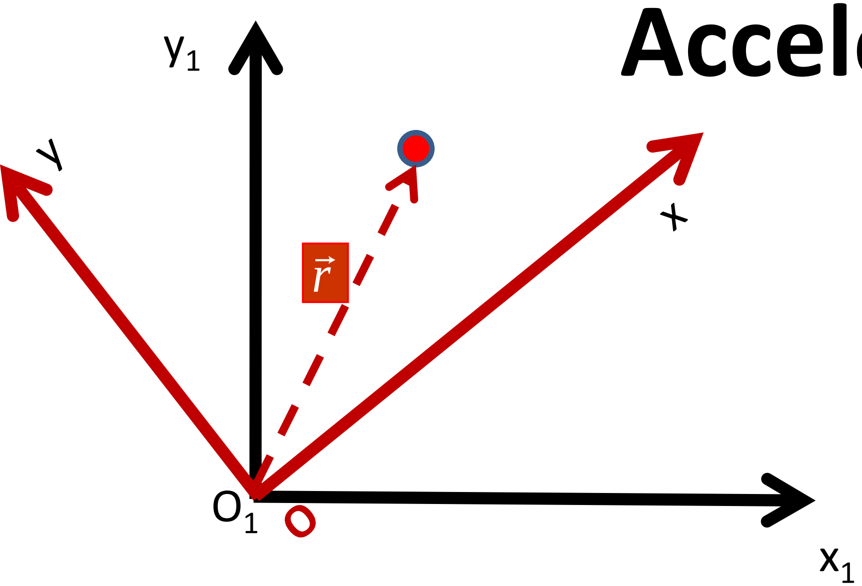
Velocity as observed in Non-Rotating frame/Lab frame

$$\left.\frac{d\vec{r}}{dt}\right|_{NR} = \left(\frac{d\vec{r}}{dt}\right)_R + \left(\vec{\omega}\times\vec{r}\right)$$

(Derivation: Introduction to mechanics, Kleppner & Kolenkow, Page 356)

What will be the acceleration in Non-Rotating frame?

Acceleration



$$\left.\frac{d\vec{r}}{dt}\right|_{NR} = \left(\frac{d\vec{r}}{dt}\right)_R + (\vec{\omega}\times\vec{r})$$

$$\left.\frac{d\vec{r}}{dt}\right|_{NR} = \vec{\alpha} + \vec{\beta}$$

$$\left.\frac{d^2\vec{r}}{dt^2}\right|_{NR} = \left.\frac{d\vec{\alpha}}{dt}\right|_{NR} + \left.\frac{d\vec{\beta}}{dt}\right|_{NR}$$

$$\left.\frac{d\vec{\alpha}}{dt}\right|_{NR} = \left.\frac{d\vec{\alpha}}{dt}\right|_{R} + (\vec{\omega}\times\vec{\alpha})$$

$$\left.\frac{d\vec{\beta}}{dt}\right|_{NR} = \left.\frac{d\vec{\beta}}{dt}\right|_{R} + (\vec{\omega}\times\vec{\beta})$$

Acceleration

$$\left.\frac{d^2\vec{r}}{dt^2}\right|_{NR} = \left.\frac{d\vec{\alpha}}{dt}\right|_{R} + \left(\vec{\omega}\times\vec{\alpha}\right) + \left.\frac{d\vec{\beta}}{dt}\right|_{R} + \left(\vec{\omega}\times\vec{\beta}\right)$$

$$\vec{\alpha} = \left(\frac{d\vec{r}}{dt}\right)_{R}$$

$$\vec{\beta} = \left(\vec{\omega}\times\vec{r}\right)$$

$$\left.\frac{d^2\vec{r}}{dt^2}\right|_{NR} = \left(\frac{d^2\vec{r}}{dt^2}\right)_{R} + \left(\vec{\omega}\times\left(\frac{d\vec{r}}{dt}\right)_{R}\right) + \left.\frac{d\left(\vec{\omega}\times\vec{r}\right)}{dt}\right|_{R} + \left(\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right)$$

$$\left(\frac{d^2\vec{r}}{dt^2}\right)_{NR} = \left(\frac{d^2\vec{r}}{dt^2}\right)_{R} + \vec{\omega}\times\left(\frac{d\vec{r}}{dt}\right)_{R} + \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\left(\frac{d\vec{r}}{dt}\right)_{R} + \left(\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right)$$

Acceleration and Force in rotating frame

$$\left(\frac{d^2\vec{r}}{dt^2}\right)_{NR} = \left(\frac{d^2\vec{r}}{dt^2}\right)_R + 2\vec{\omega}\times\left(\frac{d\vec{r}}{dt}\right)_R + \left(\frac{d\vec{\omega}}{dt}\times\vec{r}\right) + \left(\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right)$$

$$[\vec{a}]_{NR} = \left([\vec{a}]_R + 2\vec{\omega}\times[\vec{v}]_R + \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right)$$

$$\left[\vec{F}\right]_R = \left(\left[\vec{F}\right]_{NR} - m\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right] - 2m\vec{\omega}\times[\vec{v}]_R - m\left[\frac{d\vec{\omega}}{dt}\times\vec{r}\right]\right)$$

Non-Rotating frame (NR): Inertial frame (i)

Rotational frame (R): Non-Inertial frame (ni)

Force

$$\left[\vec{F}\right]_{ni} = \left(\left[\vec{F}\right]_{i} - m\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right] - 2m\vec{\omega}\times\left[\vec{v}\right]_{ni} - m\left[\frac{d\vec{\omega}}{dt}\times\vec{r}\right]\right)$$

Centrifugal force

Coriolis force

Azimuthal force

All these forces are non-physical.

The forces arise from kinematics and are not due to physical interactions.

Centrifugal force increases with r, whereas real forces always decrease with distance.

Accelerated Frame of Reference

Is there any possible way to modify Newton's laws so that they hold in non-inertial frames, or do we have to give up entirely on F = *m*a?

$$\left[\vec{F}\right]_i = ma_i$$

$$\left[\vec{F}\right]_{ni} = ma_{ni}$$

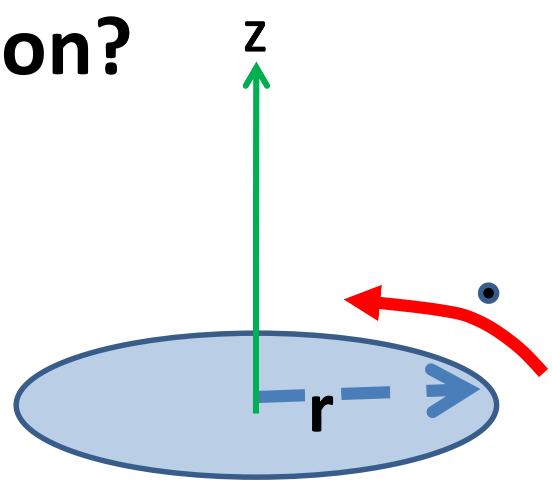
$$\left[\vec{a}\right]_{ni} = \left(\left[\vec{a}\right]_i - \left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right] - 2\vec{\omega}\times\left[\vec{v}\right]_{ni} - \left[\frac{d\vec{\omega}}{dt}\times\vec{r}\right]\right)$$

1. Consider a person standing motionless with respect to a carousel, a distance r from the center. Let the carousel rotate in the X-Y plane with angular velocity $\boldsymbol{\omega} = \omega \mathbf{e}_z$.

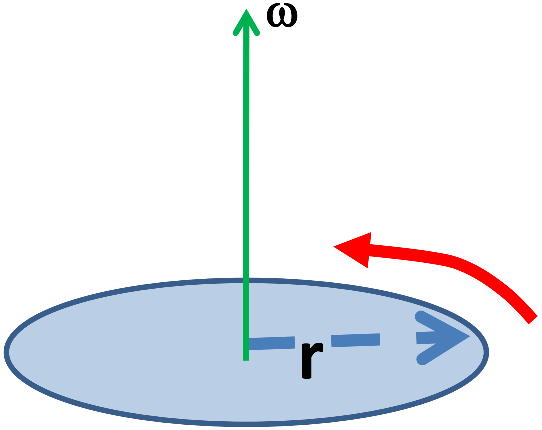
(1)What are the fictitious forces present?

(2)What is the direction of centrifugal force?

(3)What is the magnitude of the centrifugal force felt by the person?



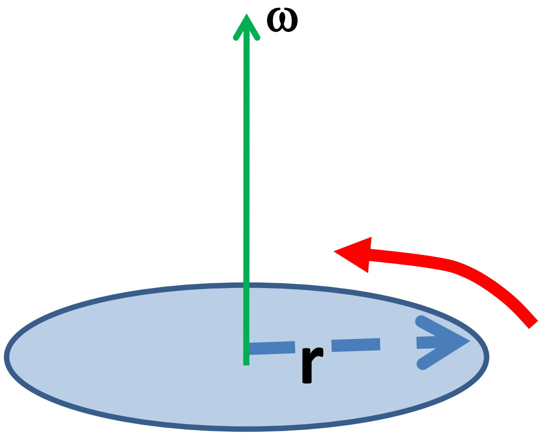
- On a Carousel



Person is standing motionless, V=0

$$\left[\vec{F}\right]_{ni} = \left(\left[\vec{F}\right]_{i} - m\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right] - 2m\vec{\omega}\times\left[\vec{v}\right]_{ni} - m\left[\frac{d\vec{\omega}}{dt}\times\vec{r}\right]\right)$$

- On a Carousel



$$\left[\vec{F}\right]_{centrifugal} = -m\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right]$$

Points radially outwards

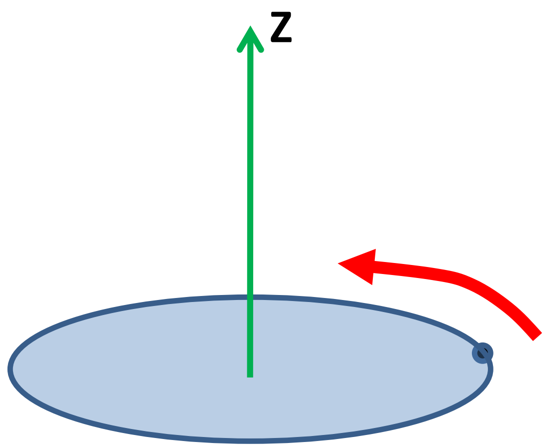
$$\left[\vec{F}\right]_{centrifugal} = mr\omega^2\hat{e}_r$$

If person is not moving with respect to carousel, and if ω is constant, then Centrifugal force is the only non-zero fictitious force.

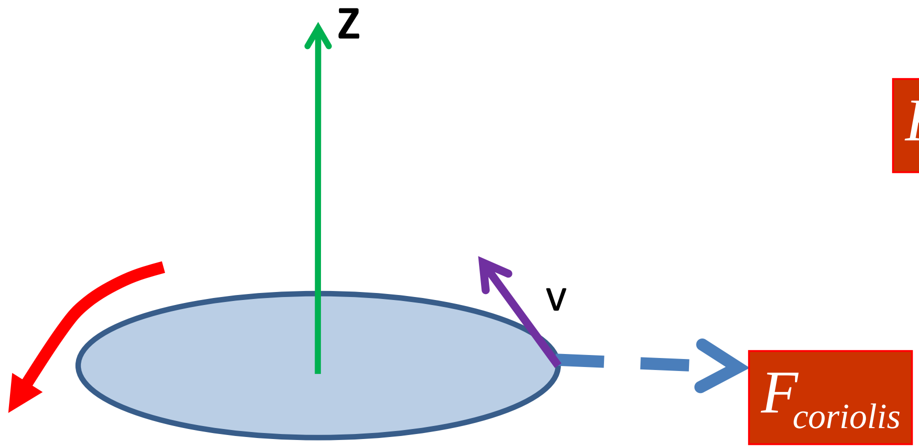
2. Consider a person moving tangentially inward on a carousel with a velocity **v.** Let the carousel rotate in the X-Y plane with angular velocity $\boldsymbol{\omega} = \omega \mathbf{e}_z$.

(1)What are the fictitious forces present?

(2)What is the magnitude and direction of coriolis force?



- Moving tangentially inward on a carousel



$$F_{coriolis} = 2m\omega v$$

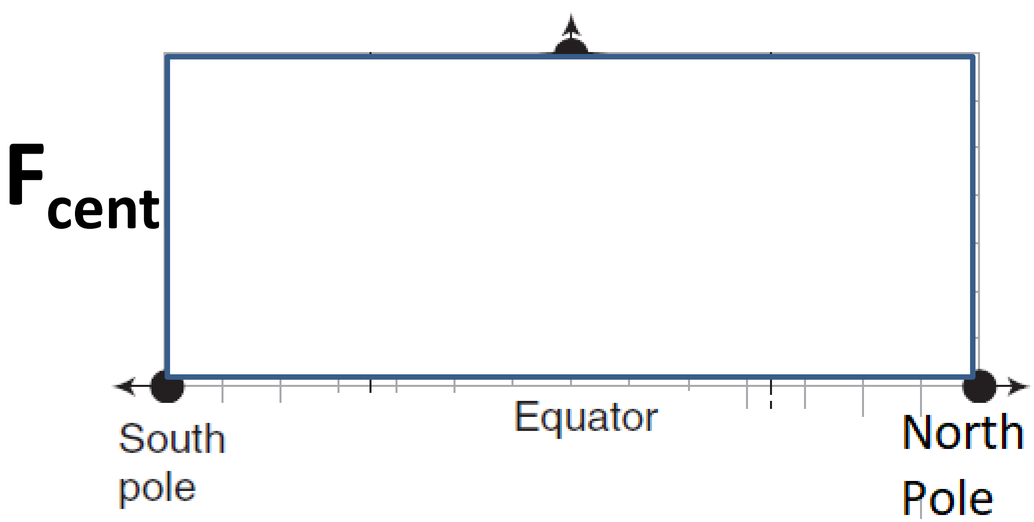
- Coriolis force points radially outward

Quantitative analysis of fictitious forces in earth's frame

3. A person is standing still on a location P as shown in figure 1 on Earth.

a. Plot the nature of $\mathbf{F}_{\mathbf{cent.}}$

b. What is the effective gravity felt by the person due to centrifugal force?



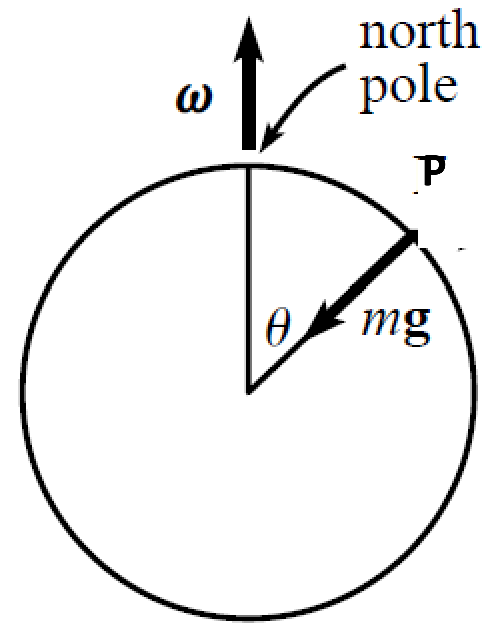
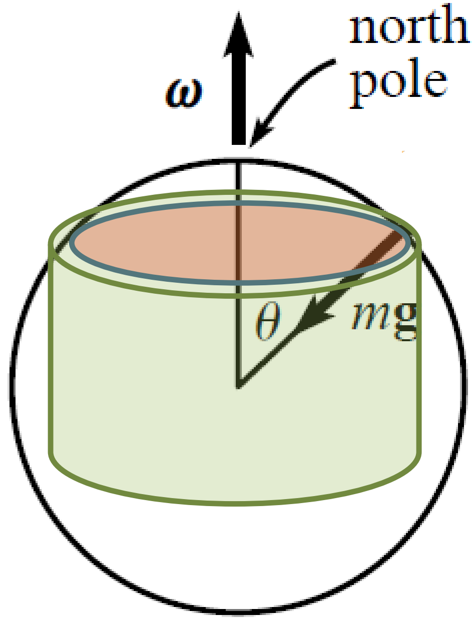


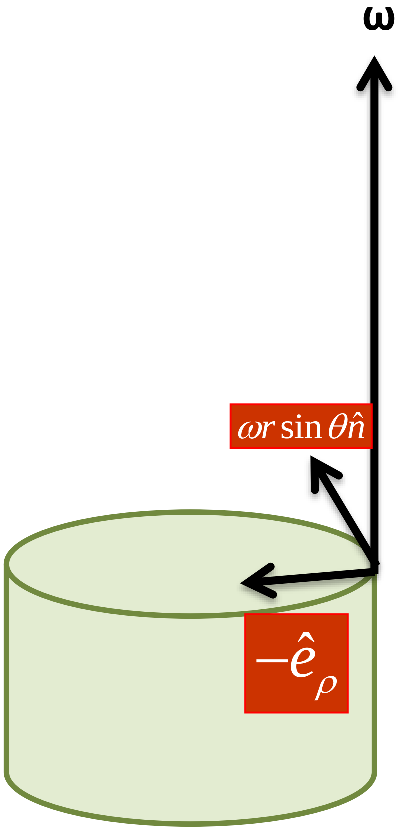
Fig 1

$$\boxed{\left[\vec{F}\right]_{centrifugal} = -m\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right]}$$

What is the Direction of $\boxed{\left[\left(\vec{\omega}\times\vec{r}\right)\right]}$ **?**

Direction of $\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right]$?





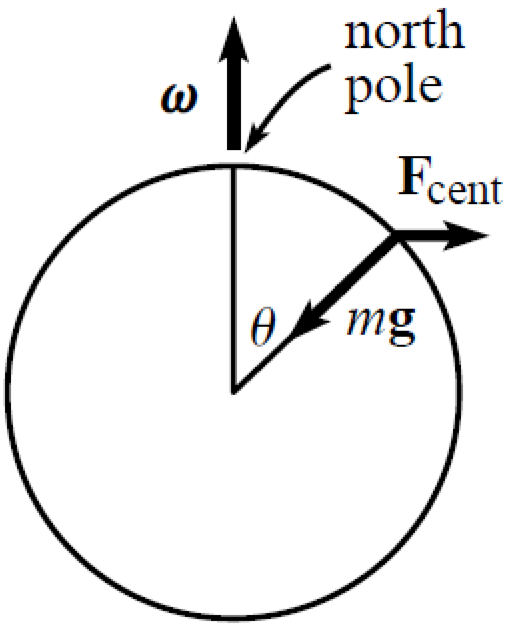
$$\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right]=\left[\omega\hat{e}_z\times\omega r\sin\theta\hat{n}\right]$$

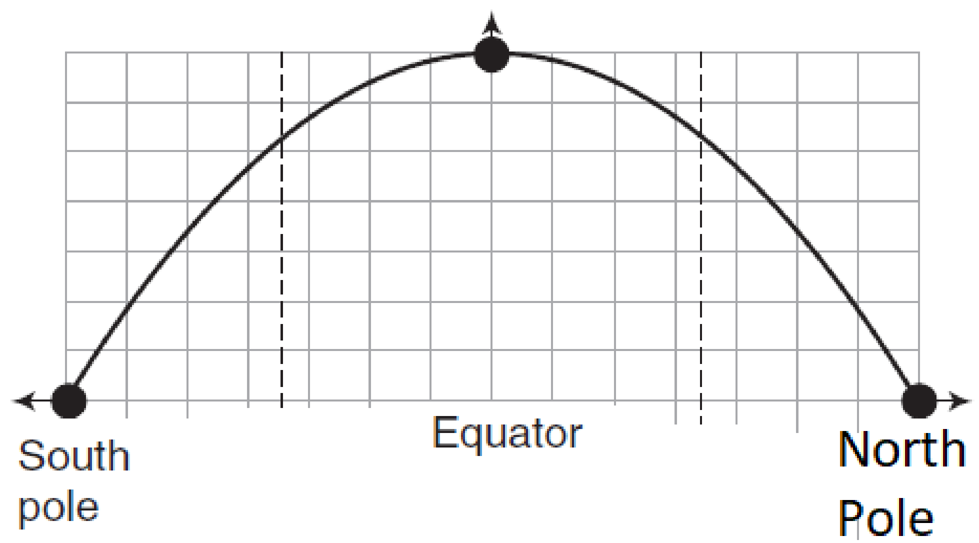
$$=\omega^2 r\sin\theta\left(-\hat{e}_\rho\right)$$

Centrifugal force

$$\left[\vec{F}\right]_{centrifugal} = -m\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right]$$

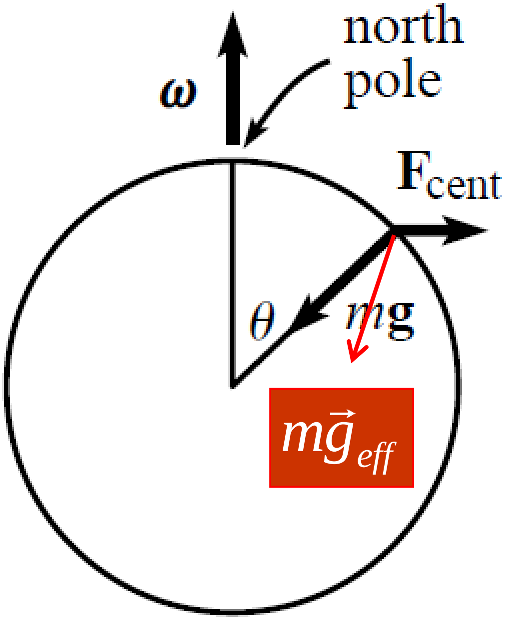
$$\left[\vec{F}\right]_{centrifugal} = mr\omega^2 \sin\theta(+\hat{e}_\rho)$$





Effective gravity: Centrifugal force

$$\left[\vec{F}\right]_{centrifugal} = -m\left[\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)\right]$$



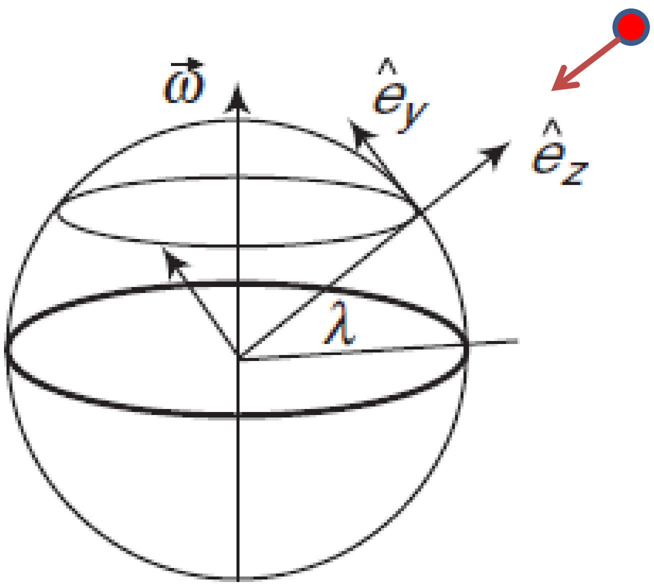
$$m\vec{g}_{eff}$$

$$m\vec{g}_{eff} = m(g\left(-\hat{e}_r\right) + mr\omega^2 \sin\theta\hat{e}_\rho)$$

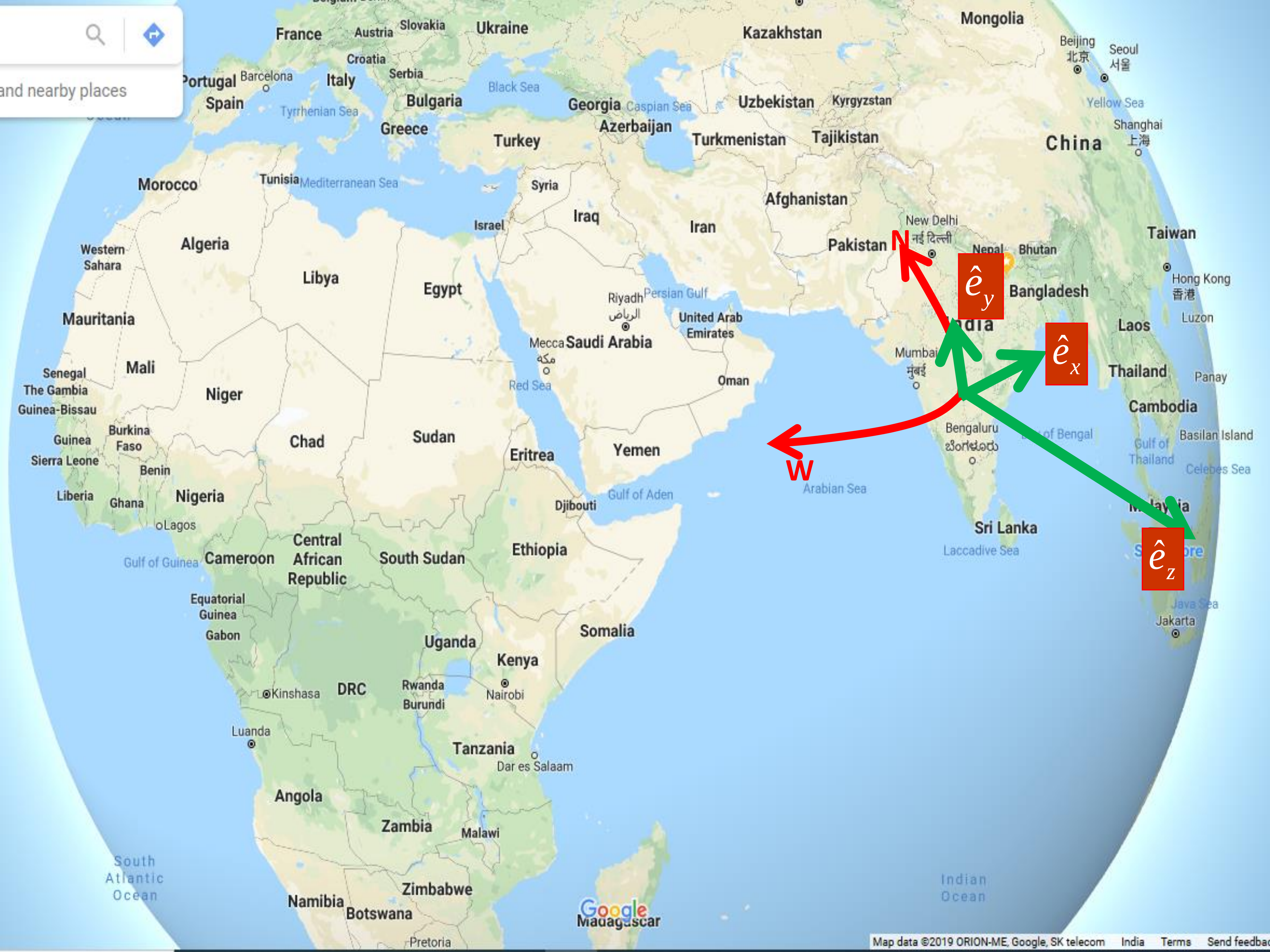
4. Consider an object is dropped under gravity in the $-\mathbf{e}_z$ direction as shown in figure. In the problem, consider λ as the latitude and ω as the angular velocity of the earth.

a. What is the nature of the Coriolis force?

b. Find the coriolis speed and deflection of the object due to the force.



c. What is the nature of Coriolis force if the object is thrown upward.



and nearby places
France
Austria
Slovakia
Ukraine
Kazakhstan
Mongolia
Beijing
北京
Seoul
서울
Croatia
Serbia
Portugal
Barcelona
Italy
Spain
Tyrrhenian Sea
Bulgaria
Black Sea
Georgia
Caspian Sea
Uzbekistan
Kyrgyzstan
Yellow Sea
Azerbaijan
Shanghai
上海
Greece
Turkey
Turkmenistan
Tajikistan
China
Morocco
Tunisia
Mediterranean Sea
Syria
Afghanistan
Israel
Iraq
Iran
New Delhi
नई दिल्ली
Taiwan
Western Sahara
Algeria
Pakistan
Nepal
Bhutan
Hong Kong
香港
Libya
Egypt
Riyadh
الرياض
Persian Gulf
Bangladesh
United Arab Emirates
Luzon
Mauritania
Mecca
مكة
Saudi Arabia
Laos
Mumbai
मुंबई
Senegal
Mali
Oman
Thailand
Panay
The Gambia
Niger
Red Sea
Guinea-Bissau
Cambodia
Burkina Faso
Guinea
Chad
Sudan
Bengaluru
ಬೆಂಗಳೂರು
Basilan Island
Sierra Leone
Benin
Eritrea
Yemen
Gulf of Thailand
Celebes Sea
Liberia
Ghana
Nigeria
Arabian Sea
Djibouti
Gulf of Aden
Lagos
Central African Republic
Sri Lanka
Gulf of Guinea
Cameroon
South Sudan
Ethiopia
Laccadive Sea
Equatorial Guinea
Java Sea
Gabon
Jakarta
Somalia
Uganda
Kenya
Kinshasa
DRC
Rwanda
Burundi
Nairobi
Luanda
Tanzania
Dar es Salaam
Angola
Zambia
Malawi
South Atlantic Ocean
Indian Ocean
Zimbabwe
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Botswana
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India
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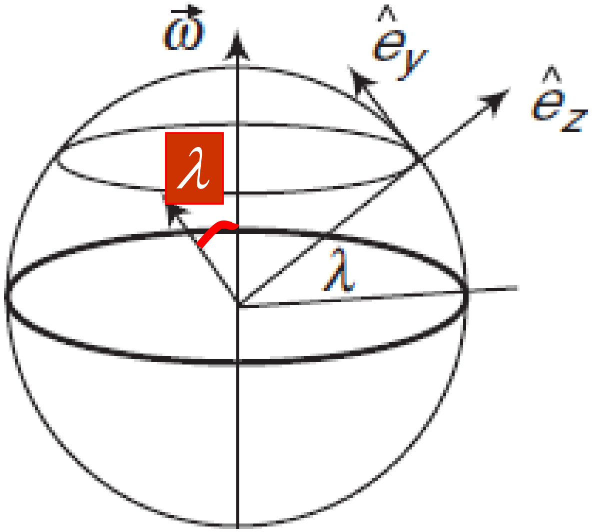
N
W
$\hat{e}_y$
$\hat{e}_x$
$\hat{e}_z$

$$\left[\vec{F}\right]_{corr} = \left(-2m\vec{\omega}\times[\vec{v}]_{ni}\right)$$



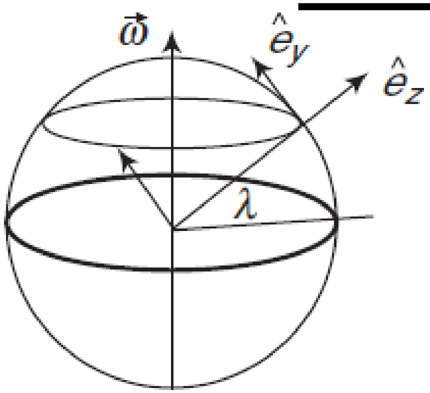
$$\vec{\omega} = \omega_y\hat{e}_y + \omega_z\hat{e}_z$$

$$\vec{\omega} = \omega\cos\lambda\hat{e}_y + \omega\sin\lambda\hat{e}_z$$

$$\left[\vec{F}\right]_{corr} = -2m\omega\left(\cos\lambda\hat{e}_y + \sin\lambda\hat{e}_z\right)\times\left(v_x\hat{e}_x + v_y\hat{e}_y + v_z\hat{e}_z\right)$$

$$\vec{F}_{\text{Coriolis}} = -2m\omega\,[(\cos\lambda v_z - \sin\lambda v_y)\,\hat{e}_x + \sin\lambda v_x\hat{e}_y + (-\cos\lambda v_x)\hat{e}_z].$$

Coriolis Force (A quantitative analysis in earth frame)



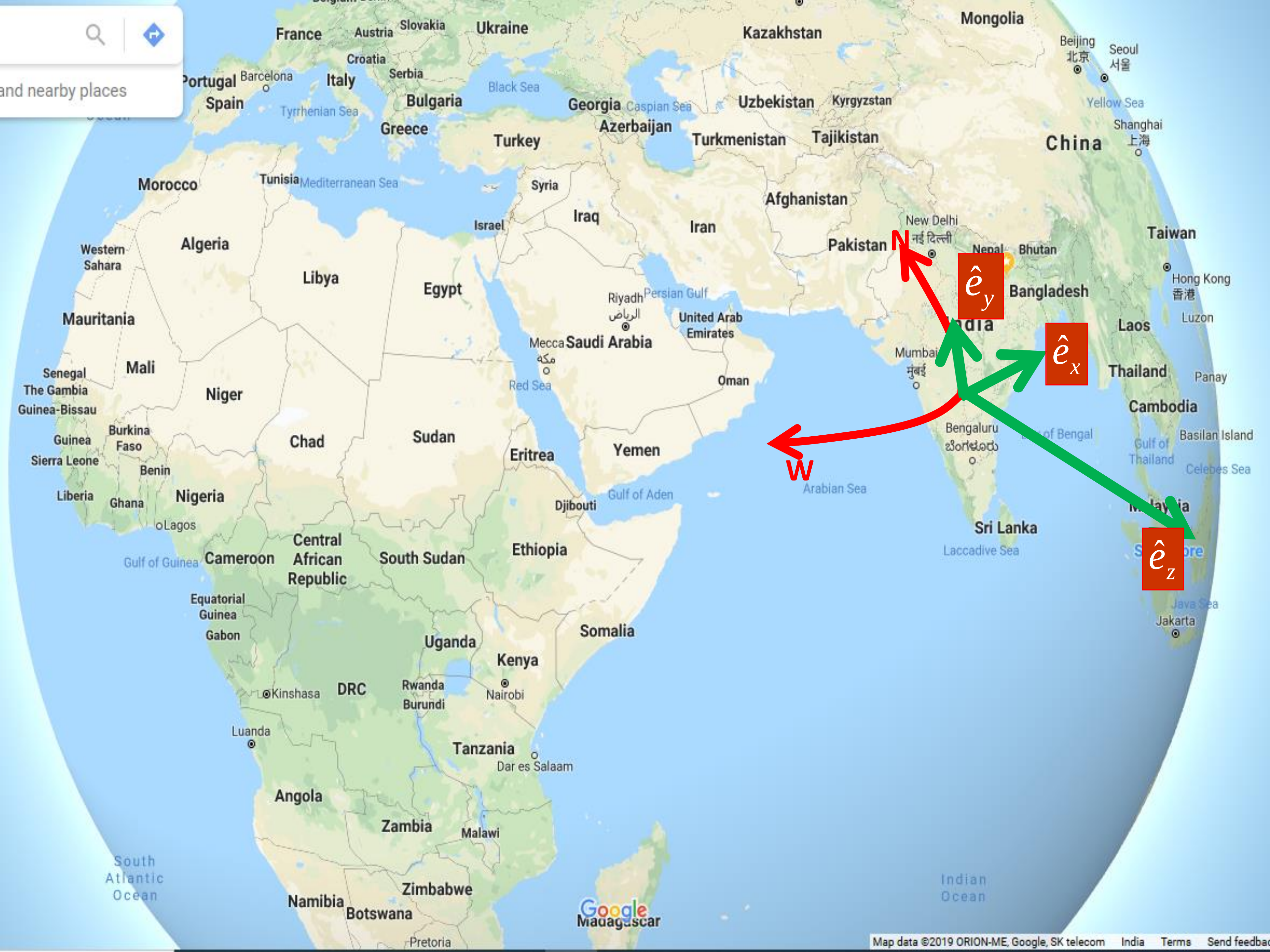
$$\vec{\omega} = \omega_y \hat{e}_y + \omega_z \hat{e}_z$$

$$\vec{F}_{\text{Coriolis}} = -2m\omega\left[(\cos\lambda v_z - \sin\lambda v_y)\,\hat{e}_x + \sin\lambda v_x \hat{e}_y + (-\cos\lambda v_x)\hat{e}_z\right].$$

If an object is dropped under gravity

$$\vec{v} = -gt\hat{e}_z$$

$$\vec{F}_{\text{Coriolis}} = -2m\omega\left[(\cos\lambda v_z)\hat{e}_x\right] = 2m\omega gt\cos\lambda\hat{e}_x,$$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Black Sea
Georgia
Caspian Sea
Uzbekistan
Kyrgyzstan
Yellow Sea
Greece
Turkey
Azerbaijan
Turkmenistan
Tajikistan
China
Shanghai
上海
Morocco
Tunisia
Mediterranean Sea
Syria
Afghanistan
Israel
Iraq
Iran
New Delhi
नई दिल्ली
Pakistan
Taiwan
Western Sahara
Algeria
Libya
Egypt
Nepal
Bhutan
Hong Kong
香港
Riyadh
الرياض
Persian Gulf
United Arab Emirates
Bangladesh
Luzon
Mauritania
Mecca
مكة
Saudi Arabia
Laos
Mumbai
मुंबई
Senegal
Mali
The Gambia
Guinea-Bissau
Niger
Red Sea
Oman
Thailand
Panay
Cambodia
Burkina Faso
Guinea
Chad
Sudan
Eritrea
Yemen
Bengaluru
ಬೆಂಗಳೂರು
Basilan Island
Sierra Leone
Benin
Gulf of Thailand
Celebes Sea
Liberia
Ghana
Nigeria
Lagos
Djibouti
Gulf of Aden
Arabian Sea
Sri Lanka
Central African Republic
Gulf of Guinea
Cameroon
South Sudan
Ethiopia
Laccadive Sea
Equatorial Guinea
Gabon
Somalia
Java Sea
Jakarta
Uganda
Kenya
Nairobi
Kinshasa
DRC
Rwanda
Burundi
Luanda
Tanzania
Dar es Salaam
Angola
Zambia
Malawi
South Atlantic Ocean
Indian Ocean
Zimbabwe
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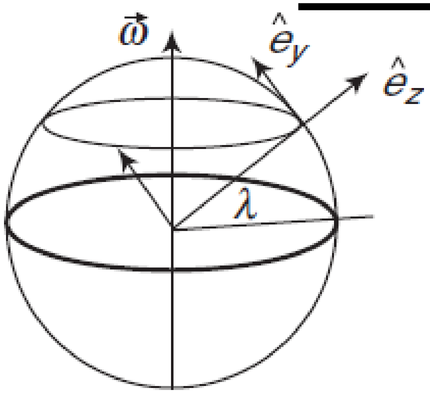
$\hat{e}_y$
$\hat{e}_x$
$\hat{e}_z$

Coriolis Force (A quantitative analysis in earth frame)



$$\vec{\omega} = \omega_y \hat{e}_y + \omega_z \hat{e}_z$$

$$\vec{F}_{\text{Coriolis}} = -2m\omega\,[(\cos\lambda v_z - \sin\lambda v_y)\,\hat{e}_x + \sin\lambda v_x \hat{e}_y + (-\cos\lambda v_x)\hat{e}_z].$$

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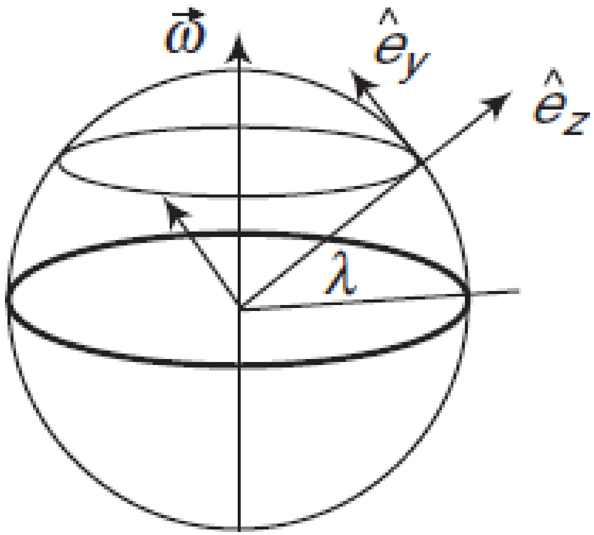
$$\vec{F}_{\text{Coriolis}} = -2m\omega\,[(\cos\lambda v_z)\hat{e}_x] = 2m\omega gt\cos\lambda \hat{e}_x,$$

$$v_x^{\text{East}} = \int_0^\tau (2\omega gt\cos\lambda)dt = 2\omega g\cos\lambda\int_0^\tau t\,dt = (\omega g\cos\lambda)\tau^2.$$

$$\delta x^{\text{East}} = \int_0^\tau v_x dt = (\omega g\cos\lambda)\int_0^\tau t'^2 dt' = \omega g\cos\lambda\frac{\tau^3}{3}.$$

Coriolis Force (A quantitative analysis in earth frame)



If an object is thrown upward under gravity

$$\vec{v} = v_z \hat{e}_z$$

$$\vec{F}_{corr} = -2m\omega\cos\lambda v_z \hat{e}_x \text{ (Towards west)}$$

Coriolis effect on mass dropped from IIT Patna admin building

A mass m is released with zero initial velocity from top of IIT Patna admin building. IIT Patna is located at latitude of 25.5^0. Assume that the admin building is 50 m. tall. Determine the amount of sideways coriolis deflection

$$\vec{F}_{corr} = 2m\omega\cos\lambda gt\hat{e}_x$$

$$x_{deff} = \frac{\omega\cos\lambda gt^3}{3}$$

Time of fall to cover 50 m=

$$t = \sqrt{\frac{2h}{g}} = 3.1943\sec$$

$$x_{deff} = 0.7cm$$

- Self-study (Introduction to mechanics, Kleppner & Kolenkow)
  - Example 8.10: Weather systems (page 364)
  - Example 8.11:Foucault pendulum (page 367)