Few Common Distributions

Friday, January 29, 2021 8:38 AM

Bernoulli Random Variable

$$X = \begin{cases} 1 & \text{if } A \text{ happens} \\ 0 & \text{o/w} \end{cases}$$

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

$X_i \longrightarrow A_i$ $\qquad X_i = \begin{cases} 1 & \text{if } A_i \text{ happens} \\ 0 & \text{o/w} \end{cases}$

$A_i^c \longrightarrow (1 - X_i)$

$\boxed{E[X] = p}$

$A_i \cap A_j \longrightarrow X_i X_j$

$A_i^c \cap A_j^c \longrightarrow (1-X_i)(1-X_j)$

$A_i \cup A_j \longrightarrow 1 - (1-X_i)(1-X_j)$

$\bigcup_{i=1}^{n} A_i \longrightarrow \left[1 - \prod_{i=1}^{n} (1-X_i)\right]$

$$P(A_1 \cup A_2 \cup A_3) = E\left[1 - (1-X_1)(1-X_2)(1-X_3)\right]$$

$$= E\left[\cancel{1} - \cancel{1} + X_1 + X_2 + X_3 - X_1X_2 - X_2X_3 - X_1X_3 + X_1X_2X_3\right]$$

$$= E[X_1] + E[X_2] + E[X_3] - E[X_1X_2] - E[X_2X_3] - E[X_1X_3] + E[X_1X_2X_3]$$

$$= P(A_1) + \text{- - - - - - - -}$$

$\boxed{E[X+Y] = E[X] + E[Y]}$ Proof

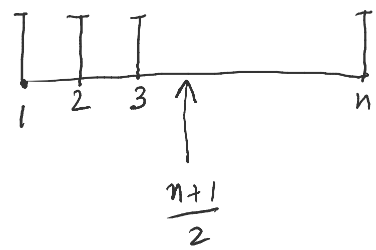
Discrete Uniform Random variable

Discrete Uniform R...

PMF:

$$p_X(x) = \begin{cases} \frac{1}{n} & x = 1, 2, \dots n \\ 0 & o/w \end{cases}$$

$$E[X] = \sum_{x=1}^{n} x \cdot \frac{1}{n} = \frac{n+1}{2}$$



$$Var(X) = E[X^2] - (E[X])^2 = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}$$

$$E[X^2] = \frac{1}{n}\sum_{x=1}^{n} x^2 = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$$

<u>General case</u>

$$p_X(x) = \begin{cases} \frac{1}{b-a+1} & \text{if } x = a, a+1, \dots, b \\ 0 & o/w \end{cases}$$

$1 = a$
$n = b - a + 1$

$$E[X] = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)(b-a+2)}{12}$$

Geometric Random Variables

Suppose we repeatedly and <u>indly</u> toss a baised coin which has $P(H) = p$. Let X = number of tosses needed for the first H to appear. This X

is called geometric random variable.

$$p_X(x) = \underline{(1-p)^{x-1}} \cdot \underline{p} \qquad x = 1, 2, 3, \ldots$$

$$\sum_{x=1}^{\infty} p_X(x) = \sum_{x=1}^{\infty} (1-p)^{x-1} p = p \sum_{k=0}^{\infty} (1-p)^k = p \frac{1}{1-1+p} = 1$$

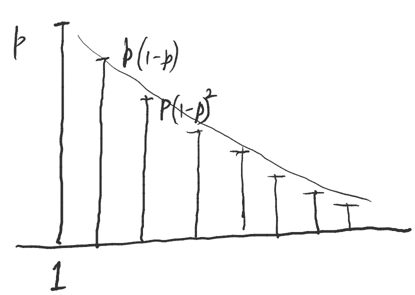
$$E[X] = \sum_{x=1}^{\infty} x \cdot (1-p)^{x-1} p = p \sum_{x=0}^{\infty} \frac{d}{dq}\left(q^x\right) \qquad q = 1-p$$

$$= p \frac{d}{dq}\left(\sum_{x=0}^{\infty} q^x\right)$$

$$= p \frac{d}{dq}\left(\frac{1}{1-q}\right) = \frac{p}{(1-q)^2} = \frac{1}{p}$$

$$\boxed{E[X] = \frac{1}{p}}$$

$$\mathrm{Var}(X) = \underline{\underline{E[X^2]}} - \left(E[X]\right)^2 = \frac{1-p}{p^2}$$



Binomial Random Variable

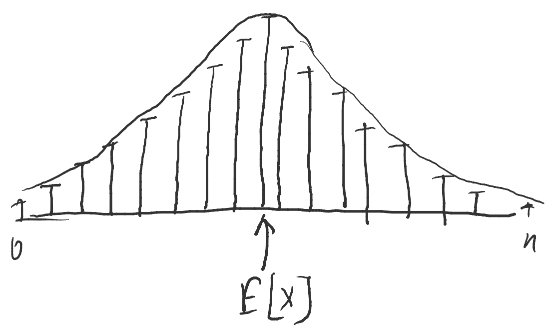
n ind coin tossed, $P(H) = p$, $P(T) = (1-p)$

X = no of heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad \text{if } x = 0, 1, 2, \ldots, n$$

$$\sum_{x=0}^{n} p_X(x) = 1 \quad \text{(Applying Binomial Th)}$$

$$E[X] = \sum_{x=0}^{n} x \cdot \binom{n}{x} p^x (1-p)^{n-x} = \underline{np}$$



$$Var(X) = \underline{np(1-p)}$$