CS 225: Switching Theory

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Previous Class

- Number Systems
  - Different Number systems (positional)
  - Conversion
  - Representation (complement)
  - Binary Arithmetic
- Codes
  - BCD, cyclic code etc.
  - Gray code
  - Parity and Error correcting code

This Class

- Parity and Error correcting code
- Switching Algebra

Error-detecting Codes

p: parity bit;
Even parity used in codes.

Distance between codewords: no. of bits they differ in

Minimum distance of a code: smallest no. of bits in which any two code words differ

Minimum distance of above single error-detecting codes = 2

Decimal Digit	Even-parity BCD	2-out-of-5
	8 4 2 1 p	0 1 2 4 7
0	0 0 0 0 0	0 0 0 1 1
1	0 0 0 1 1	1 1 0 0 0
2	0 0 1 0 1	0 1 1 0 0
3	0 0 1 1 0	0 1 1 0 0
4	0 1 0 0 1	1 0 0 1 0
5	0 1 0 1 0	0 1 0 1 0
6	0 1 1 0 0	0 0 1 1 0
7	0 1 1 1 1	1 0 0 0 1
8	1 0 0 0 1	0 1 0 0 1
9	1 0 0 1 0	0 0 1 0 1

Hamming Codes: Single Error-correcting

Minimum distance for SEC or double-error detecting (DED) codes = 3
Example: {000,111}
Minimum distance for SEC and DED codes = 4

No. of information bits = m

No. of parity check bits, p1, p2, …, pk = k
No. of bits in the code word = m+k

Assign a decimal value to each of the m+k bits: from 1 to MSB to m+k to LSB

Perform k parity checks on selected bits of each code word: record results as 0 or 1

- Form a binary number (called position number), c1c2…ck, with the k parity checks

Hamming Codes (Contd.)

No. of parity check bits, k, must satisfy: $2^k >= m+k+1$

Example: if m = 4 then k =3

Place check bits at the following locations: 1, 2, 4, …, 2^{k-1}

Example code word: 1100110

- Check bits: p1= 1, p2 = 1, p3 = 0
- Information bits: 0, 1, 1, 0

Hamming Code Construction

Select p_1 to establish even parity in positions: 1, 3, 5, 7

Select p_2 to establish even parity in positions: 2, 3, 6, 7

Select p_3 to establish even parity in positions: 4, 5, 6, 7

Error position	Position number		
	c1	c2	c3
0 (no error)	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Hamming Code Construction (Contd.)

Position:	1 p_1	2 p_2	3 m_1	4 p_3	5 m_2	6 m_3	7 m_4
Original BCD message:			0		1	0	0
Parity Check in positions 1,3,5,7 requires p_1=1	1		0		1	0	0
Parity Check in positions 2,3,6,7 requires p_2=0	1	0	0		1	0	0
Parity Check in positions 4,5,6,7 requires p_3=1	1	0	0	1	1	0	0
Coded message	1	0	0	1	1	0	0

Hamming Code Construction

Ex: If the original message is to be send is 0010

Q1: The message to be send is ?

0 1 0 1 0 1 0

Q2. If the received message is 0 1 0 1 0 1 1

Error position is:

1 1 1 (7)

Do it your self

If the original message is to be send is 1001

Q3.: The message to be send is ?

Q4: If the received message is (flipppting the 3rd (from right) bit)

Error position is:

Hamming Code for BCD

Decimal digit	Position						
	p_1	p_2	m_1	p_3	m_2	m_3	m_4
0	0	1	0	0	0	0	0
1	1	1	0	1	0	0	1
2	0	1	0	1	0	1	0
3	1	0	0	0	0	1	1
4	1	0	0	1	1	0	0
5	0	1	0	0	1	0	1
6	1	1	0	0	1	1	0
7	0	0	0	1	1	1	1
8	1	1	1	0	0	0	0
9	0	0	1	1	0	0	1

	1	2	3	4	5	6	7
Position:	1	2	3	4	5	6	7
Intended message:	1	1	0	1	0	0	1
Message received:	1	1	0	1	1	0	1
4-5-6-7 parity check:				1	1	0	1
c_1=1 since parity is odd							
2-3-6-7 parity check:		1	0			0	1
c_2=0 since parity is even							
4-5-6-7 parity check:	1		0		1		1
c_3=1 since parity is odd							

SEC/DED Code

Add another parity bit such that all eight bits have even parity

- Two errors occur: overall parity check satisfied, but position number indicates error
  double error (cannot be corrected)

- Single error occurs: overall parity check not satisfied
  - Position no. is 0: error in last parity bit
  - Else, position no. indicates erroneous bit

- No error occurs: all parity checks indicate even parities

- Switching Algebra