ASSIGNMENT
5
CS-206

Tarusi Mittal

Roll No. : 1901 CS 65
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Sign : Tarusi Mittal

ASSIGNMENT – 5.
CS-206.

Tanish Mittal
1901CS65
Tanish Mittal

Ques:- (a) $R = \{(a,b) \mid a \text{ divides } b\}$.

(1) R^{-1}

Using the inverse relation defination

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$

$= \{(b,a) \mid a \text{ divides } b\}$

$= \{(a,b) \mid b \text{ divides } a\}$

2) $\overline{R}$

Using the complementary relation defination.

$\overline{R} = \{(a,b) \mid (a,b) \notin R\}$

$= \{(a,b) \mid a \text{ does not divide } b\}$

(b) 1. Everyone who has visited Web Page a has also visited web Page b.

→ A = set of All Webpages

$R = \{(a,b) \mid$ everyone who has visited web page a has also visited webpage b$\}$

. It is reflexive as because if you visit webpage ~~b~~ a then you have also visited webpage ~~b~~ a

It is transitive as if someone has visited webpage a he has visited webpage b and if someone has visited webpage b he has visited webpage c. So if someone has visited webpage a he has visited webpage c.

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Tarun Mittal

2. There are no common links found on both web page a and web page b.

→ R = {(a,b) | a and b have no common links}.

The relation is symmetric, because if webpage a and webpage b have no common links, then webpage b and webpage a have no common links.

3. There is at least one common link on web page a and web page b.

→ R = {(a,b) | there is at least one common link on web page a and web page b}

The relation is symmetric, because if webpage a and webpage b have a common link, then webpage b and webpage a also have a common link.

4. There is a web page that includes link to both web page a and b.

→ R = {(a,b) | there is a webpage that include links to both webpage a and webpage b}

The relation is symmetric, because if there is a webpage that includes links to webpage a and b then the same webpage include links to webpage b and webpage a.

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Que2:

A7. $(a,b) \in S \circ R$, which means there is c such that $(a,c) \in R$ and $(c,b) \in S$. $\Rightarrow$ a is a parent of c as well as c and b are siblings

$\Rightarrow$ a is the parent of b

$$\boxed{S \circ R = \{(a,b) \mid a \text{ is a parent of } b\}}$$

||ly if $(a,b) \in R \circ S$, then there is a c such that $(a,c) \in S$ ie a and c are siblings and $(c,b) \in R$ or c is a parent of b.

$\Rightarrow$ a is either uncle or aunt of b

$$\boxed{R \circ S = \{(a,b) \mid a \text{ is an uncle or aunt of } b\}}$$

Que 3:

A7(a) $f(x) = e^x$ from R to R

This function is not onto because its domain is the set of positive real numbers. Hence it is not invertible.

Suppose the codomain is the set of positive numbers (real).

one-one : Let $f(x) = f(y)$, then $e^x = e^y$ or $e^{(x-y)} = 1$.

$\Rightarrow x = y$, hence it is one to one

Onto: Let be be a +ve real number. We have to show that for any b, there exists an x in R such that $e^x = b$

we have $x = \log(b)$, which is defined for all +ve real numbers

hence it is onto.

This makes the function invertible.

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(b) $f(x) = |x|$

The function is not one to one (for example, $f(2) = 2 = f(-2)$)
So, it is not invertible. On the restricted domain, the function is the identity function from the set of non-negative real numbers to itself, $f(x) = x$, so it is one-to-one and onto and therefore invertible.
In fact, it is its own inverse.

Que 4:

A→ (a) $((a,b),(c,d)) \in R$: R is an equivalence relation.

A = Set of ordered pairs of positive integers.

$$R = \{((a,b),(c,d)) \mid ad = bc\}$$

To prove: R is an equivalence relation.

Proof:

Reflexive: Let $(a,b) \in A$

Since $ab = ba$ (commutative property of multiplication)

$$((a,b),(a,b)) \in R$$

Thus R is reflexive.

Symmetry: Let $((a,b),(c,d)) \in R$

$ad = bc$

$da = cb$ (commutative property)

$cb = da$

$\Rightarrow ((c,d),(a,b)) \in R$

Thus R is symmetric.

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Transitive: $((a,b),(c,d)) \in R$ and $((c,d),(e,f)) \in R$

$$ad = bc$$
$$cf = dc$$

Since a,b,c,d,e,f are all +ve integers, they are all nonzero:

$$a = \frac{bc}{d} \qquad f = \frac{de}{c}$$

Multiplying $\Rightarrow$ $af = \frac{bc}{d} \times \frac{de}{c} = be$

$af = be$, then implies

$$((a,b),(e,f)) \in R.$$

Thus R is transitive.

Conclusion: Since R is reflexive, symmetrical and transitive, R is an equivalence relation.

(b) Give a description of each of the congruence classes modulo 6.

A→ A = set of all integers

$R = \{(x,y) \mid x \bmod 6 = y \bmod 6\}$

$[0]_R = \{y \mid y \bmod 6 = 0 \bmod 6\} = \{\ldots, -6, 0, 6, 12, \ldots\}$

$[1]_R = \{y \mid y \bmod 6 = 1\} = \{\ldots, -5, 1, 7, 13, \ldots\}$

$[2]_R = \{y \mid y \bmod 6 = 2\} = \{\ldots, -4, 2, 8, 14, \ldots\}$

$[3]_R = \{y \mid y \bmod 6 = 3\} = \{\ldots, -3, 3, 9, 15, \ldots\}$

$[4]_R = \{y \mid y \bmod 6 = 4\} = \{\ldots, -2, 4, 10, 16, \ldots\}$

$[5]_R = \{y \mid y \bmod 6 = 5\} = \{\ldots, -1, 5, 11, 17, \ldots\}$

$\boxed{y = c + 6k \text{ with } c \in (0,1,2,3,4,5) \text{ and } k \text{ is an integer}}$

Taruni Mittal
1901CS65
Tarunimittal

Ques:-

(a) Which of these are posets?

1) $(Z, =)$ 2) $(Z, =)$ 3) $(Z, \geq)$ 4) $(Z, 1)$

1) It is a poset

As $(Z, =)$ is,

Reflexive: $a \in Z$ & $a = a$; so it is reflexive.

Antisymmetric: when $a = b$; $b = a$ and $a, b \in Z$ then $a = b$ & thus it is antisymmetric.

Transitive: $a = b$, $b = c$ with $a, b, c \in Z$ then $a = c$; so it is transitive.

Since it is reflexive, antisymmetric and transitive.
It is a poset.

2) Same as 1st part; It is a poset.

3) $(Z, \geq)$

It is a poset.

As. 1) Reflexive: $a \geq a$; it is reflexive.

2) Antisymmetric: $a \geq b$; $b \geq a$ with $a, b \in Z$, then $a = b$ and thus the relation is antisymmetric.

3) Transitive: $a \geq b$ and $b \geq c$, $a, b, c \in Z$, then $a \geq c$ thus it is transitive.

Since it is both three, it is a poset

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Tanvi Mittal

a) $(Z, |)$

It is a poset.

As

1) Reflexive : As $a|a$ when $a \in Z$; it is reflexive.

2) Antisymmetric: when $a|b$ and $b|a$ and $a, b \in Z$ then $a|b$ (a must be equal to b), so it is antisymmetric.

3) Transitive: when $a|b$ and $b|c$ and $a, b, c \in Z$ then $a|c$, so it is transitive

Since it is both three, it is a poset.

b) Comparable in the poset $(Z^+, |)$

a) $(5|15)$ = true → Comparable
b) $(6|9)$ = false → Not comparable
c) $(8|16)$ = true → Comparable
d) $(7|7)$ = true → Not Comparable

Tanusri Mithal
1901CS65
Tanusri Mithal

c) Two incomparable elements in the posets.

Let us define a relation R on set S such that (S, R) is a poset.

Two elements are incomparable if $(a,b) \notin R$ and $(b,a) \notin R$

a) $(P(\{0,1,2\}), \subseteq)$

$$S = P(\{0,1,2\})$$
$$R = \{(a,b) \mid a \subseteq b\}$$

eg $\{0,1\} \in P(\{0,1,2\})$ and $\{1,2\} \in P(\{0,1,2\})$ while

$$\{0,1\} \nsubseteq \{1,2\}$$
$$\{1,2\} \nsubseteq \{0,1\}$$

Thus $\{0,1\}$ and $\{1,2\}$ are incomparable elements.

b) $(\{1,2,4,6,8\}, |)$

$$S = \{1,2,4,6,8\}$$
$$R = \{(a,b) \mid a \text{ divides } b\}$$

eg $6 \in \{1,2,4,6,8\}$ and $8 \in \{1,2,4,6,8\}$

6 doesnot divide 8
8 doesnot divide 6

Thus 6 and 8 are two incomparable elements.