PH-301.

MIDTERM.

Submitted by:-

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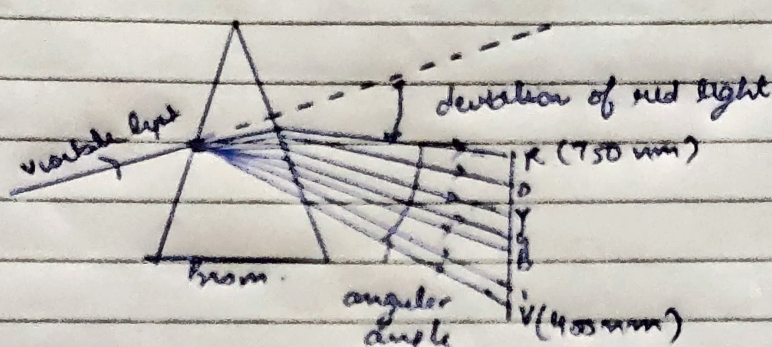
Question 1:-

PRISM

A prism works on the principle of dispersion of light which ~~...~~ means that different colours of light travel at different speeds, because of which, they get bent by different amounts and ~~so~~ come out all spread out instead of mixed up.

Voilet color has the lowest speed therefore it is on the bottom while red having the highest speed is at the top. This happens because index of refraction is increased for slower moving waves.

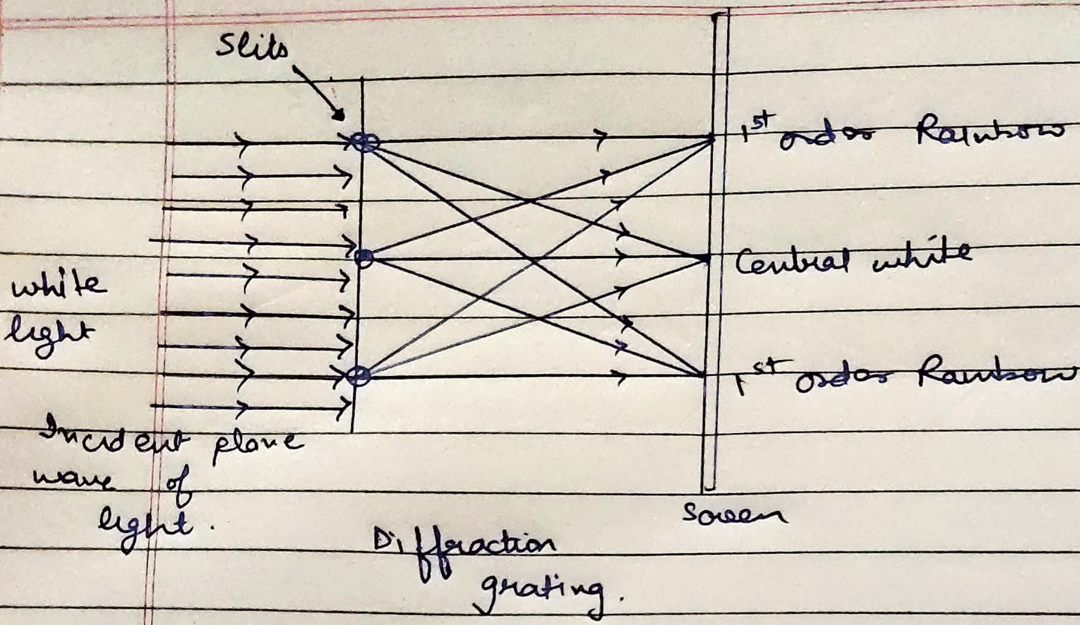
⇒ On passing white light through the prism it splits into 7 colors.



Dispersion of white light

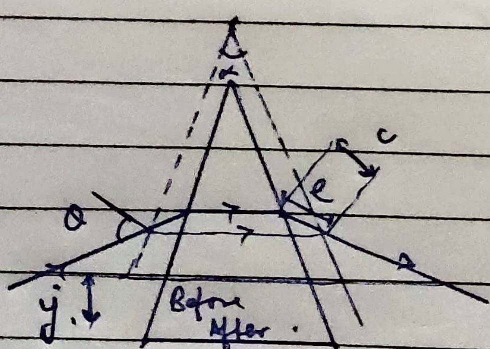
GRATING

Grating works on the principle of diffraction of light. A grating is an optical component with a periodic structure that splits and diffracts light into several beams travelling in different directions. Direction of beam depends on the spacing of grating and wavelength of the light. So it acts like a dispersive element. Grating separates incident polychromic light into its constituents (which are different wavelengths) at different angles. A grating diffracts diff wavelengths at diff angles due to interference of each wavelength, the diffracted beams may overlap.



The key difference between prism and grating in terms of spectrum creating is that in prism spectra, the spectrum is created due to dispersion of light whereas in grating, the spectrum is created due to diffraction of light.

(ii) Yes, a prism can be used for as phase shifting device. In that case it is known as prismatic phase shifter. It produces change of phase of beam, when it is mounted on a translation stage and placed in the beam oriented at the angle of minimum deviation. The translation causes the prism to move along its plane of symmetry.



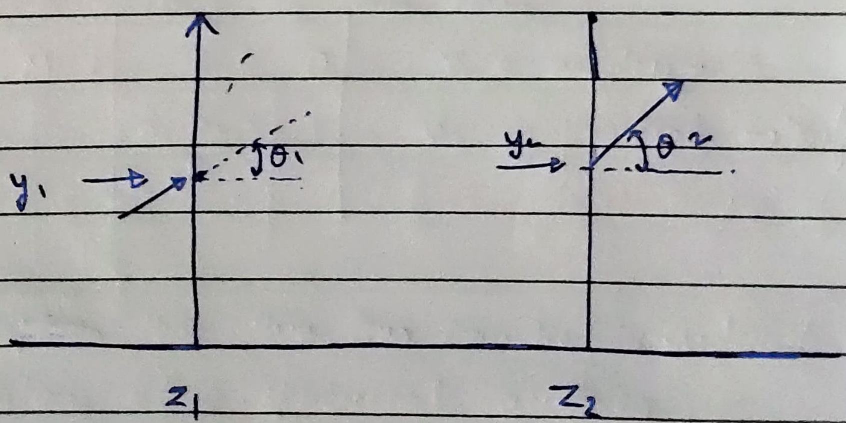
Question 2:-

⇒ Matrix Optics:- It is a technique for tracing paraxial rays. Under paraxial conditions, properties of rays in optical system can be treated with an elegant matrix formalism.

⇒ We ~~will~~ assume the rays to travel only in a single plane, (containing z axis), so that the formalism is applicable to system with planar geometry and only meridional rays are considered.

⇒ Let the traverse axis in this plane be y axis and thus plane of reference is (y, z) plane.

⇒ An optical system is a set of optical components placed between z_1 and z_2 planes, referred as input and output planes of system. The system is characterised completely by its effect on an incoming ray of position and direction (y_1, θ_1). It steers the ray so that it has new position and direction at the output plane (y_2, θ_2)



(Input and output of an optical system)

The relation between (y_1, θ_1) and (y_2, θ_2) is linear because by paraxial approximation we know that $\sin\theta \approx \theta$

$$\Rightarrow \quad y_2 = Ay_1 + B\theta_1$$
$$\theta_2 = Cy_1 + D\theta_1$$

here A, B, C, D are real no. These equations can be expressed more compactly in matrix notation as :-

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

Matrix "M"

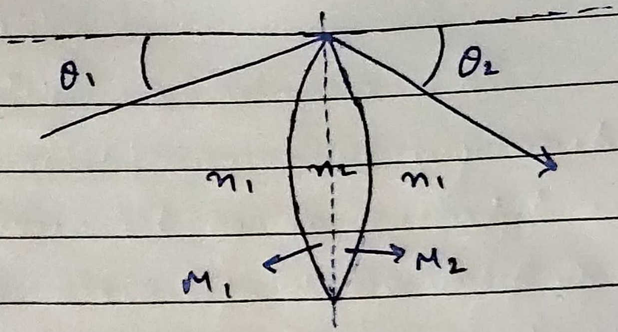
The matrix M, whose elements are A, B, C, D, characterizes the optical system completely since it permits (y_2, θ_2) to be determined for any (y_1, θ_1). Thus Matrix is called ray transfer matrix or ABCD matrix.
It is regarded as a transformation b/w spatial distribution of local spatial frequency at input and corresponding distribution at output.

<u>ABCD matrix for thin lens</u>
We know that ray transfer matrix for a spherical interface can be written as.

$$M = \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R} & 1 \end{bmatrix}$$

n_1 = medium
n_2 = spherical interface

We know that thin lens can be considered as a combination of 2 spherical interfaces



$$\begin{bmatrix} 1 & 0 \\ \frac{n_1-n_2}{R_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n_2-n_1}{R_2} & 1 \end{bmatrix}$$

Required Matrix $M = M_1 M_2$

$$= \begin{bmatrix} 1 & 0 \\ \frac{n_1-n_2}{R_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n_2-n_1}{R_2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (n_1-n_2)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) & 1 \end{bmatrix}$$

Also, we know

$$\frac{1}{f} = \left(\frac{n_2-n_1}{n_1}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow M = \begin{bmatrix} 1 & 0 \\ \frac{-n_1}{f} & 1 \end{bmatrix}$$

$\Rightarrow$ ABCD matrix of this lens

$$= \begin{bmatrix} 1 & 0 \\ \left(\frac{-n_1}{f}\right) & 1 \end{bmatrix}$$

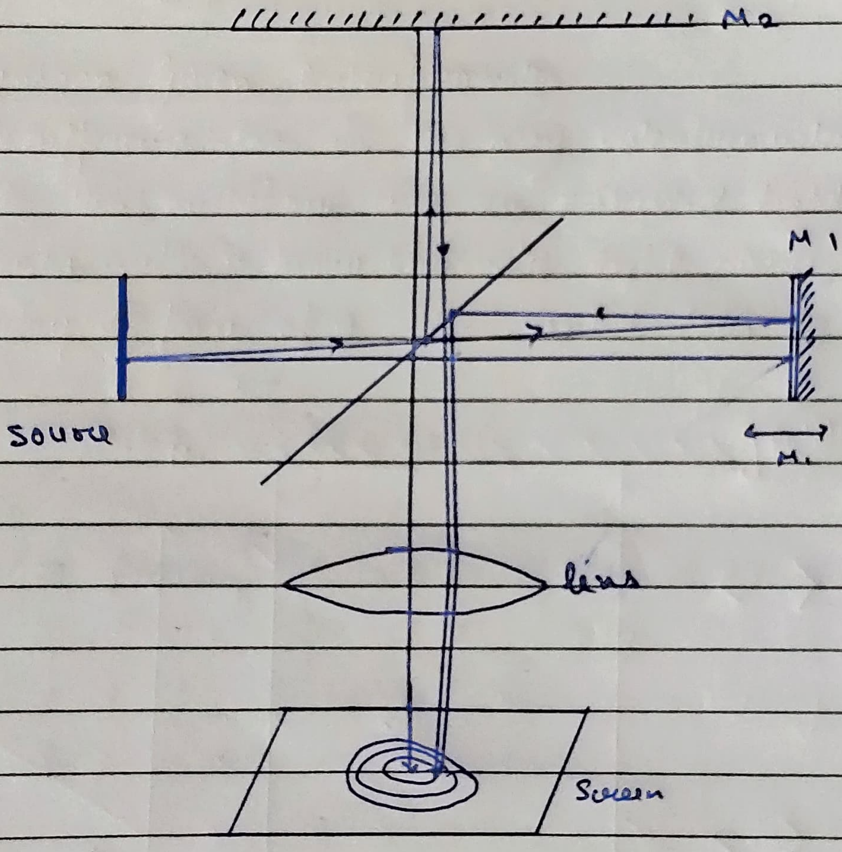
n_1 = refractive index of medium

f = focal length of lens.

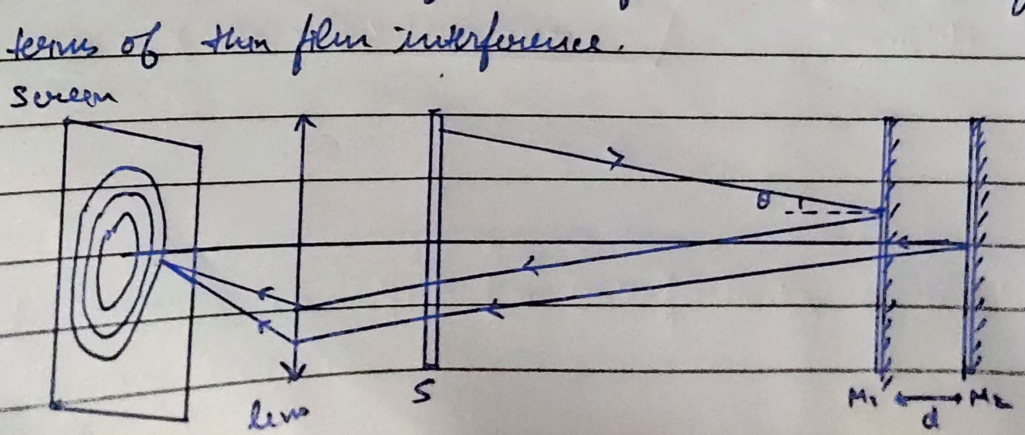
Question 3:-

SETUP

For the most common arrangement, a Michelson interferometer is illuminated by an extended source s and consists of a 50% beam spitter BS and 2 mirrors named M_1 & M_2.
The interference pattern is observed on a screen that is either way very far from the beam splitter, or a lens is placed one focal length in front of the screen.
⇒ The rotation or translation of one or both mirror change the interferance pattern.



Here shown is the interference of michelson interferometer in terms of thin film interference.



WORKING

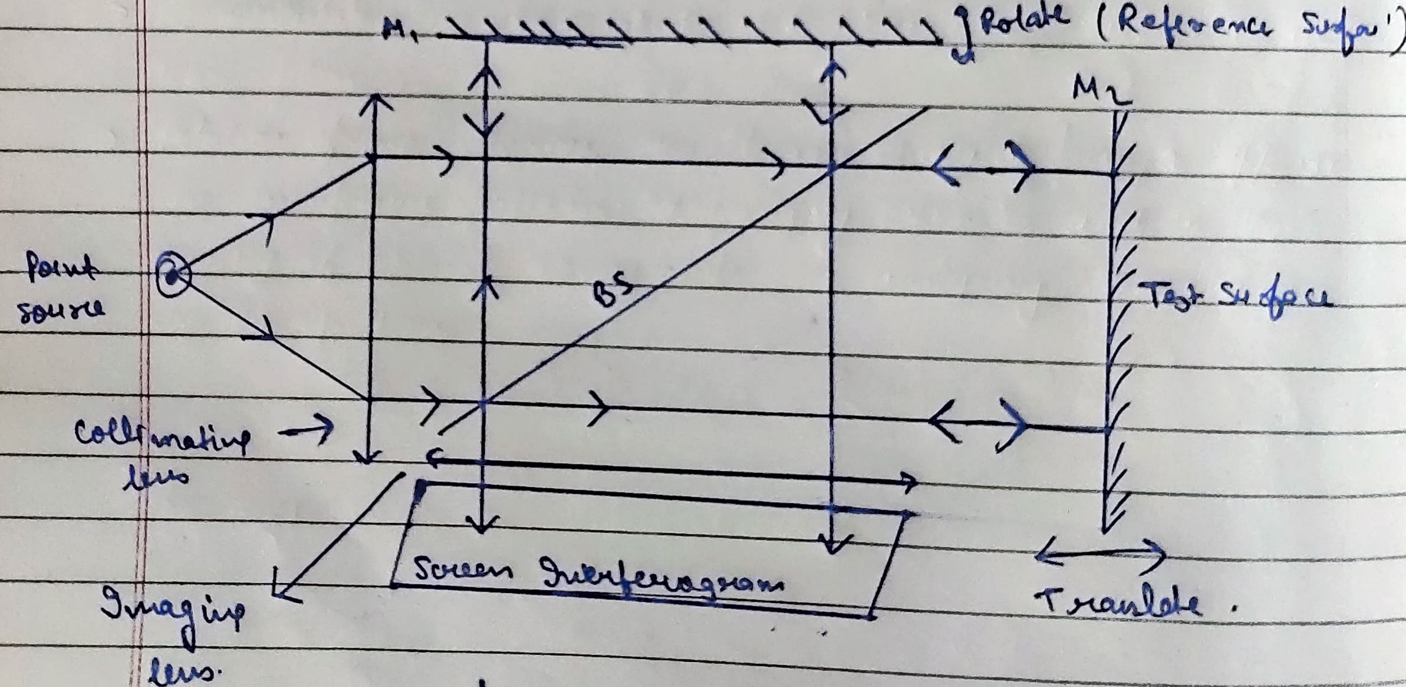
⇒ Michelson Interferometer.

Michelson Interferometer works on the principle of division of amplitude. According to this; the incident beam of light falls on the BS, which roughly reflects half of the intensity of the wave front in one direction and transmits the other half of it on another direction.

These 2 beams travel different optical path, are subsequently recombined to produce interference patterns.

⇒ Twymann - Green Interferometer

The difference lies in the way interferometers are illuminated while the michelson uses an extended light source, this is used with a monochromatic light source; located at principal focus of a well corrected lens.



Fixed mirror in Michelson Interferometer is not tiltable in Tyman Green.

APPLICATIONS

Michelson Interferometer

→ This interferometer is used in cases that involve small changes in optical path length.

→ It has optical instrument of high precision and versatality.

→ It can be used to produce circular and straight line fringes of both monochromatic and white light.

→ It can resolve stars which have angular size of the order of 0.01''

Twyman - Green Interferometer.

→ It is very useful instrument for measuring defects in optical components.

→ It is used for testing prisms and small objectives; invention of lases increases its utility.

→ It evalutats in vivo breakup characteristic of the human tear film.

→ Rotation of one mirror results in straight fringes appearing in interferance pattern, a fringing, which is used to test quality of optical components by analysing the changes in the pattern

Question 4:-

Wavelength of the given radiation $= 632.8\ nm$

Reflectivity of the plates (R) $= 0.99$

Inter-mode separation $= 300\ MHz = 3\times10^8\ Hz$.

(i) The Minimum Plate separation:

As;

$$m = \frac{2P}{\pi\sqrt{F}} \quad \text{where} \quad F = \frac{4R}{(1-R)^2}$$

$$d = \frac{m\lambda}{2}$$

$$P = \frac{\lambda}{\Delta\lambda}$$

Coefficient of finesse (F) $= \dfrac{4\times(0.99)}{(1-0.99)^2}$

$$\boxed{F = 39600}$$

wavelength separation between laser modes =

$$\Delta\lambda = \frac{\lambda^2}{c}|(\Delta\nu)|$$

$$= \left(\frac{(632.8\times10^{-9})^2\times(3\times10^8)}{3\times10^8}\right)$$

$$\boxed{\Delta\lambda = 4.00435\times10^{-13}\ m}$$

Resolving Power of Interferometer

$$P = \frac{\Delta\lambda}{\lambda} = \frac{632.8\times10^{-9}}{4.00435\times10^{-13}}$$

$$\boxed{P = 1580281.44}$$

Mode number m =

$$m = \frac{2P}{\pi\sqrt{F}} = \frac{2 \times 1580\,281.44}{\pi\sqrt{39600}}$$

$$\boxed{m = 5055.53}$$

Minimum Plate Separation:

$$d = \frac{m\lambda}{2}$$

$$= \frac{5055.53 \times 632.8 \times 10^{-9}}{2}$$

$$= 0.00159\,m$$
$$= 1.59 \times 10^{-3}\,m.$$

$$\boxed{d = 1.59 \times 10^{-3}\,m}$$

(ii) Free spectral range of interferometer.

⇒ In frequency.

$$\nu_{free} = \frac{c}{2d}.$$

$$= \frac{3 \times 10^{8}}{2 \times 1.59 \times 10^{-3}} = 9.43 \times 10^{10}\,Hz.$$

⇒ In wavelength

$$\lambda_{free} = \frac{\lambda^2}{c}\,\nu_{free} = \frac{(632.8 \times 10^{-9}) \times (9.43 \times 10^{10})}{3 \times 10^{8}}$$

$$\lambda_{free} = 1.26 \times 10^{-10}\,m$$

(xi) Higest Order of fringes.

$$\delta = \frac{4\pi}{\lambda} nd\cos\theta$$

$$= 2m\pi$$

$$\Rightarrow \cos\theta = \frac{m\lambda}{2nd} \quad ; \quad n = 1$$

$$\cos\theta = \frac{m \times 632.8 \times 10^{-9}}{2 \times 1.39 \times 10^{-3}}$$

$$= \frac{m}{5056.89}$$

$$\Rightarrow \boxed{\text{Highest Order of fringe} = 5056}$$

P.T.O. $\longrightarrow$

Question 5:- MTF ⇒ Modulation Transfer Function:

MTF is a type of high quality optical system used to compare the performance of different optical systems. It is a function of Spatial Resolution (ξ) which refers to the system resolvable smallest line-pair.

Resolution Association

Resolution is the ability of the imaging system to distinguish the object detail.
It is expressed in terms of line-pair per milimeter.
Usually, high resolution images exhibit a large amount of details with minimum blurring.

Contrast Association

Contrast is how accurately the minimum and maximum intensity values are transferred from object to image
It is the degree of difference between elements that forms images

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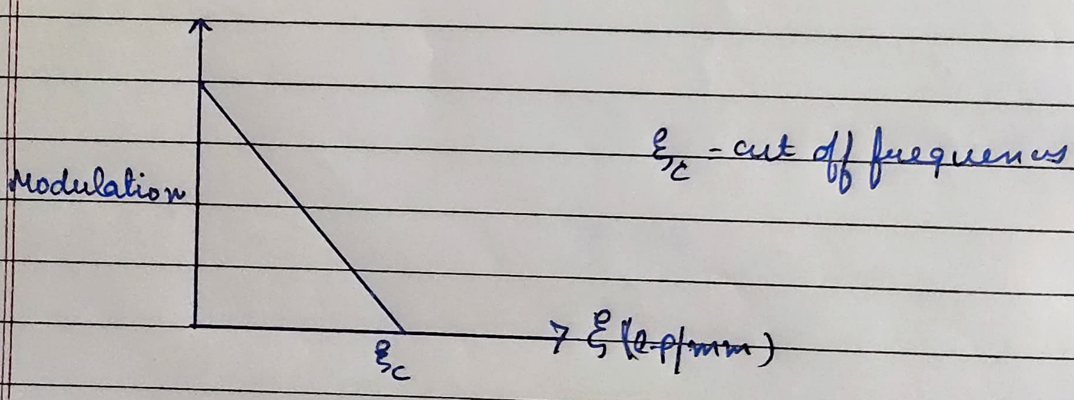

MTF for an abberration free lens with rectangular aperture.

⇒ Via the graph we can observe that:
→ as line spaces decrease (ie the frequency increases) on test target, it becomes increasingly difficult for lens to efficiently transfer this decrease in contrast as a result MTF decreases.

For an Aberration free image with a circular pupil :

$$MTF(\xi) = \frac{2}{\pi}(\theta - \cos\theta \cdot \sin\theta) \quad ; \theta = \cos^{-1}\left(\frac{\xi}{\xi_c}\right)$$

$\xi_c \rightarrow$ cut off frequency

⇒ MTF is a way to incorporate resolution and contrast into a single specification. Transfer functions, in general relate an output to its input. Ily MTF describes the ability of the lens to transfer contrast at a particular resolution from object to image.

⇒ Every component within a system has an associated MTF which as result contributes to overall MTF.

⇒ We can also determine the overall performance of a system by analyzing system MTF curve.

—x — x —x —x — x —x —x —x — x —x —x