Multiplication rule, Total probability law

Friday, January 15, 2021 8:46 AM

$$P(A\cap B\cap C) = P(A)\cdot P(B|A)\cdot P(C|A\cap B)$$

$$= \underline{P(A\cap B)}\cdot P(C|A\cap B)$$

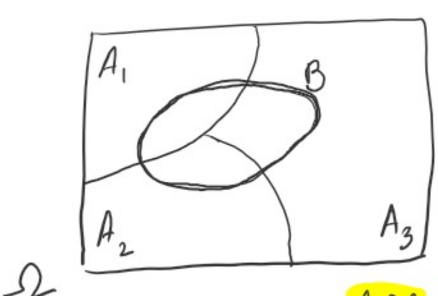
$$= P(A)\cdot P(B|A)\cdot P(C|A\cap B)$$

Ex Three cards are drawn from a deck of 52 cards without replacement. What is the prob. that none of these three cards is a heart?

Sol. A_i = event that i-th card is not a heart. $1\le i\le 3$

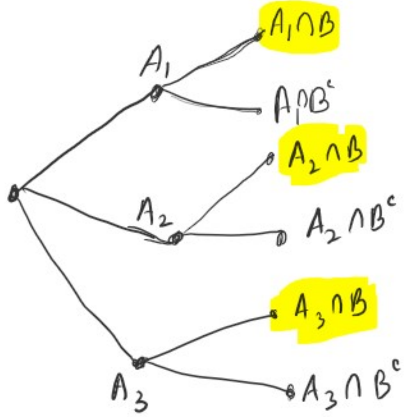
$$P(A_1\cap A_2\cap A_3) = P(A_1)\cdot P(A_2|A_1)\cdot P(A_3|A_2\cap A_1)$$

$$= \frac{39}{52}\cdot\frac{38}{51}\cdot\frac{37}{50}$$



Given $P(A_1)$ $P(A_2)$ $P(A_3)$

$P(B|A_1)$ $P(B|A_2)$ $P(B|A_3)$



$$P(B) = ??$$

$$= P\left((A_1\cap B)\cup(A_2\cap B)\cup(A_3\cap B)\right)$$

$$= P(A_1\cap B) + P(A_2\cap B) + P(A_3\cap B)$$

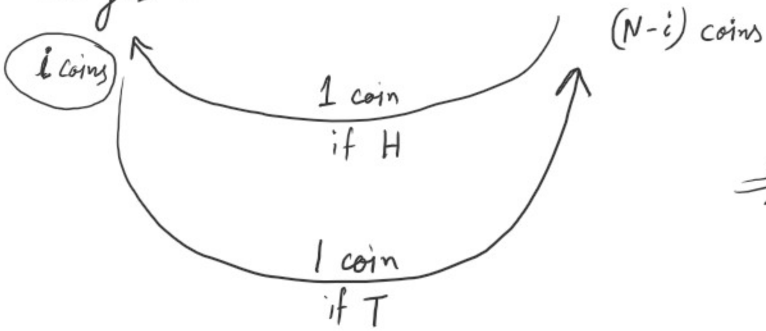
$$= P(A_1)\cdot P(B|A_1) + P(A_2)\cdot P(B|A_2) + P(A_3)\cdot P(B|A_3)$$

Total Prob law

Let $A_1, A_2 \cdots A_n$ be disjt events that forms a partition of Ω and assume that $P(A_i)>0$ for $i=1,2,\cdots n$. Then for any event B, we have

$$P(B) = \sum_{i=1}^{n} P(A_i) \cdot P(B|A_i)$$

Ex Players A and Player B. What is the prob that A wins ??



E = A wins all N coins

$\Rightarrow P_i$ = prob of E when A starts with i coins

H = first toss is head

$$P_i = P(E) = P(H) \cdot P(E|H) + P(H^c) \cdot P(E|H^c)$$

	A	B
	i	$N-i$
$H \Rightarrow$	$i+1$	$N-i-1$

$$\boxed{\left(\tfrac{1}{2}+\tfrac{1}{2}\right) P_i = \frac{1}{2} \cdot P_{i+1} + \frac{1}{2} \cdot P_{i-1}} \Rightarrow P_{i+1} = f(P_i, P_{i-1})$$

Given, $P_0 = 0$ $\quad P_N = 1$

$$\frac{1}{2} P_i + \frac{1}{2} P_i = \frac{1}{2} P_{i+1} + \frac{1}{2} P_{i-1}$$

$$\Rightarrow P_{i+1} - P_i = P_i - P_{i-1}$$

Ex

$$P_i = i \cdot P_1$$

$$1 = P_N = N \cdot P_1 \Rightarrow P_1 = \frac{1}{N}$$

$$P_i = \frac{i}{N}$$

$$Q_i = \frac{N-i}{N}$$ Q_i = Prob of B winning if B starts with (N-i) coins

$$\boxed{P_i + Q_i = 1}$$

Game will not go on forever.!!

$\boxed{\text{Gambler's Ruin Prob}}$