Taruni Mittal
1901CS65

ASSIGNMENT - 3 CS206

Ans

(a) Suppose that n is even. Then $n = 2k$ for some integers k

Thus if $n^2 = (2k)^2 = 4k^2$

$$n^2 = 2(2k^2)$$

$$\therefore\ n^2 = 2(p) \text{ for some integer } p = 2k^2$$

Thus if n is even, n^2 is also even.

(b). Given: m and n and p are integers

$m+n$ and $n+p$ are even.

To prove: $m+p$ is even.

Let $m+n = 2k$ —— ①

$n+p = 2y$ —— ⑪

$$① + ⑪ \longrightarrow m + 2n + p = 2(k+y)$$

$$m+p = 2(k+y-n)$$

Thus $m+p = 2x$, for some $x = k+y-n$

Thus $m+p$ is even

Method of direct proof is required.

(c) Let there be a number $x = 2y+1$ (x is odd)

now $x = 2y+1$

$$= y^2+2y+1-y^2$$

$$x = (y+1)^2 - y^2$$

Hence, proved that every odd no. is subtraction of two square numbers.

Tanusi Mittal
1901CS65
[signature]

Ans 2:- (a) If we prove that i $\Rightarrow$ ii $\Rightarrow$ iii $\Rightarrow$ i it will be enough to say that they are equivalent.

If n is even $\longrightarrow P_1$

$n = 2k$ for some integer k

So, as integers are continuous series of odd, even, ... elements, $2k-1$ is odd.

Hence $n-1 = 2k-1$ is odd.

Therefore $\boxed{P_1 \longrightarrow P_2}$

$\Rightarrow$ If P_2 is true

$\therefore$ $n-1$ is odd

n must be even

$n = 2k$

$n^2 = (2k)^2 = 4k^2$

$n^2 = 2(2k^2)$

$n^2 = 2p$ for $p = 2k^2$

$\therefore$ $\boxed{n^2 \text{ is even}}$

$\therefore$ $P_1 \rightarrow P_2 \rightarrow P_3$

Now, if P_3 is true, n^2 is even

We know that every square has factors in multiple of 2.

$\Rightarrow$ If n^2 is even there must be at least.

Two 2s in prime factorisation of n^2

Taruni Mittal
1901CS65
Tarunimittal

$\therefore \quad n^2 = 2 \times 2 \times K$

$n^2 = 4K$

$n = 2Q$

$n = 2Q$ for some $Q = 2k$

$\boxed{n = 2Q}$

Thus $P_3 \rightarrow P_1$

$\therefore \quad P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow P_1$

hence, these statements are equivalent.

Ans 3:- suppose, there is a rational number r that satisfies the given equation.

Let $r = \frac{a}{b} \quad \forall (a,b) \in R \quad b \neq 0$

$$r^3 + r + 1 = 0$$

$$\left(\frac{a}{b}\right)^3 + \left(\frac{a}{b}\right) + 1 = 0$$

$$= \frac{a^3}{b^3} + \frac{a}{b} + 1 = \frac{a^3}{b^3} + \frac{ab^2}{b^3} + \frac{b^3}{b^3} = 0$$

$$= a^3 + ab^2 + b^3 = 0$$

With these we are left with 4 options.

1) a and b both are odd

$\therefore$ if a^3 is odd, b^3 is odd, ab^2 is odd.

'0' is even.

Thus sum of 3 odd no. cannot be even. So, this case is not possible.

Tarusi Mittal
1901CS65

2) a is odd, b is even.

a^3 is odd, b^3 is even, ab^2 is even.

Thus sum of 2 even and 1 odd no. cannot be even (0)

3) a is even, and b is odd.

a^3 is even, ab^2 is even and b^3 is odd

Thus sum of 2 even and 1 odd no. cannot be zero.

4) a and b both even

if a and b are both even, then $\frac{a}{b}$ is not in its lowest form. Thus it cannot be possible.

Since all the cases are not possible, this contradicts our assumption that a rational no. exist which satisfy the given equation.

<u>Hence Proved</u>.

3.(b) Let x be irrational, x cannot be written as ratio of two numbers. Let y be a rational no. $y = \frac{b}{a}$. Let us assume that $x+y$ is rational.

$$\therefore x + y = z \text{ (some rational no.)}$$

$$x + \frac{a}{b} = \frac{c}{d} \quad \left(y = \frac{a}{b}, \; z = \frac{c}{d} \right)$$

$$x = \frac{cb - ad}{bd}$$

Since a, b, c, d are all integers & $b \neq 0$ & $d \neq 0$

$\frac{cb - ad}{bd}$ is a rational no.

Tanvi Mittal
1901CS65
Tanvi Mittal

but x is not a rational no.

This contradicts our assumption that $x+y$ is rational. Hence $x+y$ is irrational if x is irrational and y is rational.

Ans 4:- (a) The contraposition of the statement is, "If n is odd, then n^3+5 is even." Hence to prove this.

Let n be odd. Thus $n = 2k+1$ (for some integer k)

$$n^3+5 = (2k+1)^3+5 = 8k^3+12k^2+6k+6$$
$$= 2(4k^3+6k^2+3k+3)$$

thus $n^3+5 = 2(p)$ for some integers

$$p = 4k^3+6k^2+3k+3$$

Thus n^3+5 is even

Since, its contraposition is true then the original statement is also true.

(b). Let n^3+5 be odd and n is not even. Thus

$$n = 2k+1$$
$$n^3+5 = (2k+1)^3+5 = 8k^3+12k^2+6k+6$$
$$= 2(4k^3+6k^2+3k+3)$$

Thus, n^3+5 must be even. Hence our assumption was wrong. Thus.

$$\boxed{n \text{ is an even number}}$$

Tanvi Mittal
1901CS65
Tanvi Mittal

Ans 5: (a) The proposition P(0) is vacuosly true because 0 is not a positive integer.

$\Rightarrow$ Vacuous proof has been used.

(b) $P(n) = (a+b)^n \geq a^n + b^n$; a & $b \in R^+$

Using the method of direct proof.

$P(1) = (a+b)^1 \geq a^1 + b^1$ —— (1)

$a+b = a+b$

$\Rightarrow$ (1) is true ;

$\Rightarrow$ $\boxed{P(1) \text{ is true}}$

-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-