MA 225 Probability Theory and Random Process.

Quiz - IV

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Ans:- $f_{x,y}(x,y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), & 0 < x \leq 1, \ 0 < y \leq 2. \\ 0, & \text{otherwise} \end{cases}$

To find $P(X > Y)$, $E(e^Y)$

$$\Rightarrow \quad P(X > Y) = \int_{x=0}^{1}\int_{y=0}^{x} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dy\,dx$$

$$= \int_{0}^{1} \frac{6}{7}\left[\left(x^2 y + \frac{xy^2}{4}\right)\right]_0^x dx$$

$$= \int_{x=0}^{1} \frac{6}{7}\left[x^3 + \frac{x^3}{4} - 0\right] dx$$

$$= \int \frac{6}{7}\left(\frac{5x^3}{4}\right) dx$$

$$= \frac{30}{28}\int_0^1 x^3\,dx$$

$$= \frac{30}{28} \times \left(\frac{x^4}{4}\right)_0^1 = \frac{15}{14}\left(\frac{1}{4}\right)$$

$$= \frac{15}{56}$$

$$\boxed{P(X > Y) = \frac{15}{56}}$$

$$\Rightarrow E(e^y) = \int_0^2 e^y\left(\frac{2}{7} + \frac{3y}{14}\right)dy$$

$$= \frac{2}{7}\int_0^2 e^y\,dy + \frac{3}{14}\int_0^2 y e^y\,dy$$

$$= \frac{2}{7}\left[e^y\right]_0^2 + \frac{3}{14}\left[\left[y\left[e^y\right]\right]_0^2 - \int_0^2 e^y\,dy\right]$$

$$= \frac{2}{7}(e^2-1) + \frac{3}{14}(2e^2 - (e^2-1))$$

$$= \frac{2}{7}(e^2-1) + \frac{3}{7}e^2 - \frac{3}{14}(e^2-1)$$

$$= \frac{e^2-1}{14} + \frac{3}{7}e^2$$

$$= \frac{e^2-1+6e^2}{14}$$

$$\boxed{E(e^y) = \frac{7e^2-1}{14}}$$

Answers:

$$P(X>Y) = \frac{15}{56}$$

$$E(e^y) = \frac{7e^2-1}{14}$$