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Assignment - 1

PH-301

Que 1: In YDSE,

$n = 1.5$
$d = 0.1\,cm$ [Thin Mica].
$D = 50\,cm$
$y = 0.2\,cm$

$\Delta = 0$; for the central fringe.

$$\Delta = -\frac{yd}{D} + (n-1)t$$

$$t = \frac{yd}{(n-1)D} = \left(\frac{0.2 \times 0.1}{(0.5) \times 50}\right) cm$$

$$= 0.0008\,cm$$

$$= 0.008\,mm$$

$\Rightarrow$ The thickness of mica sheet $(t) = \boxed{0.0008\ cm}$

Que 2:- We know;

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$$, where $\delta \rightarrow$ phase difference

when 2 light waves ~~before~~ interfere.

In YDSE, I_1 and I_2 are equal, so we will take it as I_0

$$I = 2I_0 + 2I_0(\cos\delta)$$

$$= 2I_0(1 + \cos\delta)$$

$$= 4I_0\cos^2\left(\frac{\delta}{2}\right)$$

$$\boxed{I_{max} = 4I_0}$$

$$\Rightarrow \frac{I}{I_{max}} = \cos^2\left(\frac{\delta}{2}\right)$$

we~~know~~, $\delta = 2\pi\left(\frac{\Delta}{\lambda}\right)$

given $\Rightarrow$ $\Delta = \frac{\lambda}{5}$

$$\Rightarrow \delta = 2\pi \frac{\lambda}{5\lambda}$$

$$= \frac{2\pi}{5}$$

$$\Rightarrow \frac{I}{I_{max}} = \cos^2\left(\frac{\delta}{2}\right) = \cos^2\left(\frac{\pi}{5}\right) = 0.65$$

$$\boxed{\frac{I}{I_{max}}, \text{ when } \Delta = \frac{\lambda}{5} = 0.65}$$

Que 3:- for second order ; $n = 2$

$\lambda_0 = 650$ nm

As,

$$a(\sin\theta_n - \sin\theta_p) = n\lambda$$

$$a = \frac{1}{1000} \text{ cm}$$

$$\theta_i = 0$$

$$\frac{\sin(\theta_n)}{1000} = 2 \times 650 \text{ nm}.$$

$$\sin(\theta_n) = (2 \times 650 \times 10^{-9} \times 1000 \times 100) = 13 \times 10^{-2}$$

$$\sin(\theta_n) = 0.13$$

$$\theta_n = \sin^{-1}(0.13) = 7.469 \simeq 7.47$$

$$\boxed{\theta_n = 7.47^\circ}$$

Que 4:- $\nu = 4 \times 10^{14}$ Hz

$d = \frac{1}{10000}$ cm.

$\lambda = \frac{c}{\nu} = \frac{3}{4} \times \frac{10^{10}}{10^{14}} \frac{cm/s}{1s} = 0.75 \times 10^{-4}$ cm.

As by grating equation:

$d \sin\theta = n\lambda$

Now, if we want maximum value of n; the value of $\sin\theta$ has to be maximum.

$\Rightarrow \sin\theta = 1.$

$\Rightarrow d = n\lambda$

$\frac{d}{\lambda} = n$

$n = \frac{1}{10000 \times 0.75 \times 10^{-4}} = \frac{4}{3} = 1.33.$

~~$\Rightarrow$ sho~~

We know that n must be an integer $\Rightarrow$ the maximum n possible $\Rightarrow 1$

$\Rightarrow$ The 1^{st} order spectrum is the highest possible

Que 5:- $\lambda_1 = 5896$ A°

$\lambda_2 = 5890$ Å

$m = 3$ (as per for third order)

As, Resolving Power $R = \frac{\lambda}{\Delta\lambda}$

$\lambda = \frac{5890 + 5896}{2} = 5893$ Å

$\Delta\lambda = \lambda_1 - \lambda_2 = 6$ Å

$R = \frac{5893}{6} = 981.67.$

As;

$R = mN$ (where N = no of lines in grating)

$\Rightarrow \frac{R}{m} = N$

$N = \frac{981.67}{3} = 327.22$

$$\boxed{\text{No of lines present } (N) = 328}$$

Que 6:- According to Question:

width $(b) = 0.02$ cm

focal length $(f) = 20$ cm

$\lambda = 6000 Å = 6 \times 10^{-5}$ cm

In fraun Hofer Diffraction,

$$I = \frac{I_0 \sin^2(\beta)}{(\beta^2)} \quad \forall \quad \beta = \frac{b\pi \sin\theta}{\lambda}$$

for the minima $I = 0 \Rightarrow \beta = (n\pi)$ $n \neq 0$. (because of central maxima)

$\Rightarrow$ $n\pi = \frac{b\pi \sin\theta}{\lambda}$

$\sin\theta = \frac{n\lambda}{b}$

$$\boxed{\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)}$$

for 1^{st} minima $n = \pm 1$

2^{nd} minima $n = \pm 2$

1^{st} minima

$$\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$$

$$= \sin^{-1}\left(\pm\frac{\lambda}{b}\right) = \sin^{-1}\left(\pm\frac{6\times10^{-5}}{0.02}\right) = \sin^{-1}(\pm 0.003)$$

$$\Rightarrow \boxed{\theta = \pm 0.17^\circ} \Rightarrow y = f\tan\theta = 20\tan(0.17)$$

$$\boxed{y = 0.06\,cm} \rightarrow \text{Position of } 1^{st} \text{ minima.}$$

2^{nd} minima

$$\theta = \sin^{-1}\left(\frac{n\lambda}{b}\right)$$

$$= \sin^{-1}\left(\pm\frac{2\lambda}{b}\right) = \sin^{-1}(0.006)$$

$$\Rightarrow \boxed{\theta = \pm 0.343^\circ} \Rightarrow y = f\tan\theta = 20\tan(0.343)$$

$$\boxed{y = 0.12\,cm} \rightarrow \text{Position of } 2^{nd} \text{ minima.}$$

Now to find maxima,

$$I = \frac{I_0 \sin^2\beta}{\beta^2}$$

differentiating $\frac{\sin^2\beta}{\beta^2}$ $\Rightarrow$ to find β for max I

$$\Rightarrow \frac{d}{dx}\frac{\sin^2\beta}{\beta^2} = 0$$

$$\frac{2\sin\beta\cos\beta\,\beta^2 - 2\beta\sin^2\beta}{\beta^4} = 0$$

$$\boxed{\tan\beta = \beta}$$

$\Rightarrow$ $\beta = \pm 1.43\pi$ for I^{st} max.

$\beta = \pm 2.46\pi$ for 2^{nd} max.

For 1^{st} maxima

$$\beta = \frac{\pi b \sin\theta}{\lambda} = \pm 1.43\pi$$

$$\sin\theta = \pm \frac{1.43 \times \lambda}{b}$$

$$\theta = \sin^{-1}\left(\frac{\pm 1.43 \times 6 \times 10^{-5}}{0.02}\right)$$

$$\theta = \pm 0.245$$

$\boxed{\theta_{1st\ max} = \pm 0.245}$ $\Rightarrow$ $y = f\tan\theta$

$= 20\tan(0.245)$

$\boxed{y = 0.085\ cm}$

Position of 1^{st} maxima.

For 2^{nd} maxima

$$\beta = \frac{\pi b \sin\theta}{\lambda} = \pm 2.46\pi$$

$$\sin\theta = \pm \frac{2.46 \times \lambda}{b}$$

$$\theta = \sin^{-1}\left(\frac{\pm 2.46 \times 6 \times 10^{-5}}{0.02}\right) = \pm 0.422$$

$\boxed{\theta_{2^{nd}\ max} = \pm 0.422}$ $\Rightarrow$ $y = f\tan\theta$

$= 20\tan(0.422)$

$\boxed{y = 0.147\ cm}$ → Position of 2^{nd} maxima.

	1^{st} θ	1^{st} y (cm)	2^{nd} θ	2^{nd} y (cm)
Minima	0.17°	0.06	0.343°	0.12
Maxima	0.245°	0.085	0.422°	0.147

Que 7:- According to Question :

There are 2 adjacent plane polarised waves A and B which are mutually perpendicular.

Also; In some orientation of analyzer intensity of wave B is 0. Also, when it is rotated by $60°$ intensity of A and B becomes equal.

By MALUS LAW

$$I = I_0 \cos^2(\phi)$$

$\Rightarrow$ Initially intensity of wave B is equal to 0, $\phi_B = 90$

and wave A is $\perp$ to B $\Rightarrow$ $\phi_A = 0$.

After rotation by $60°$ $\Rightarrow$ $\phi_A = 60°$

$\phi_B = 30°$

Given that they have equal intensity after passing through it..

$\Rightarrow$ $I_A \cos^2(\phi_A) = I_B \cos^2(\phi_B)$ $\Rightarrow$ $\frac{I_A}{I_B} = \frac{\cos^2 30}{\cos^2 60} = \frac{3/4}{1/4}$

$$\boxed{I_A = 3 I_B}$$

$\Rightarrow$ Intensity of wave A is 3 times that of B.

Que 8:- Let the initial intensity of the linear polarized light be I_0

After 1st polarizer. Intensity $\rightarrow I_0 \cos^2(\theta) = I_1$

The second polarizer is at $(-\theta)$ angle $\Rightarrow$ from the 1st polarizer it is at an angle of 2θ.

$$I_2 = I_1 \cos^2(2\theta)$$

$$\boxed{I_2 = I_0 \cos^2\theta \cos^2(2\theta)}$$

For the final intensity to become 0.

$$I_2 = 0 = I_0 \cos^2\theta \cos^2(2\theta)$$

$$\Rightarrow \cos^2\theta \cos^2(2\theta) = 0$$

$$\cos^2\theta(2\cos^2\theta - 1) = 0$$

$\cos^2\theta = 0$ or $2\cos^2\theta - 1 = 0$

$\theta = (2n+1)\frac{\pi}{2}$ or $\cos\theta = \pm\frac{1}{\sqrt{2}}$

$$\theta = (2n+1)\frac{\pi}{4}$$

$$\boxed{\begin{array}{l}\text{For the final intensity to be } 0 \\ \theta = (2n+1)\frac{\pi}{2} \text{ or } \theta = (2n+1)\frac{\pi}{4}\end{array}}$$

Initially the light was linear polarized, the final state of emergent light will remain the same as linear polarized.

Que 9:- Amplitudes of 2 plane polarized light $\Rightarrow a_1, a_2$

Given, $a_1 = a_2$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

$E_x = a\cos\omega t$ $\quad E_y = a\cos(\omega t - \theta)$

Assuming that the major axis of ellipse is along x' or y' axes and that x axis makes an angle α with the x axis

$\Rightarrow E_x' = E_1\cos(\omega t - \phi)$ $\quad \Rightarrow E'_y = E_2(\sin(\omega t - \phi))$

$\frac{E'_x}{E_1 x} = \cos(\omega t - \phi) \rightarrow ①$ $\quad \frac{E'_y}{E_2} = \sin(\omega t - \phi) \rightarrow ②$

$(eq\ ①)^2 + (eq(2)^2)$

$\cos^2(\omega t - \phi) + \sin^2(\omega t - \phi) = 1$

$\Rightarrow \left(\frac{E'_x}{E_1}\right)^2 + \left(\frac{E'_y}{E_2}\right)^2 = 1$

The above equation represents the equation of an ellipse

$\Rightarrow$ for the rotated co-ordinates

$E_x = E_x' \cos\alpha - E'_y \sin\alpha$ —— ③

$E_y = E'_x \sin\alpha - E'_y \cos\alpha$ —— ④

eq ③ $\times (\cos\alpha)$ + eq ④ $\times (\sin\alpha)$

$E'_x = E_x \cos\alpha + E_y \sin\alpha$

|||ly $E'_y = -E_x \sin\alpha + E_y \cos\alpha$

By substitution solving.

$E_1^2 + E_2^2 = a_1^2 + a_2^2$

$\Rightarrow \frac{E_2}{E_1} = \frac{a_2 \sin\theta \cos\alpha}{a_1 \cos\alpha + a_2 \cos\theta \sin\alpha} = \frac{a_1 \sin\alpha - a_2 \cos\theta \cos\alpha}{a_2 \sin\theta \sin\alpha}$ —— ⑤

$\Rightarrow \tan(2\alpha) = \frac{2a_1 a_2 \cos\theta}{a_1^2 - a_2^2}$

$\Rightarrow a_1 = a_2$ (given)

$\rightarrow \tan(2\alpha) = \infty$

$\Rightarrow 2\alpha \rightarrow \frac{\pi}{2}$ $\left(\alpha = \frac{\pi}{4}\right) \Rightarrow$ The axis make $45°$ with x axis.

↳ By substituting in eq ⑤

$\Rightarrow \frac{E_2}{E_1} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$

$\Rightarrow \dfrac{E_2}{E_1} = \tan\dfrac{\theta}{2}$ $\quad$ ($\theta = \dfrac{2\pi}{3}$ given)

$\Rightarrow \dfrac{E_2}{E_1} = \sqrt{3}$ $\quad \Rightarrow$ It is Right elliptically Polarized since ratio is +ve

$\Rightarrow$ The major axis is along y' axis

Putting $E_2 = \sqrt{3}E_1$ in eq ③ and eq ④

$E_x' = E_1 \cos(\omega t - \phi)$

$E_y' = \sqrt{3}\, E_1 \sin(\omega t - \phi)$

To determine the state of polarization, we can take $t = 0$, and at that instant so that ϕ can be $\phi = 0$,

$\Rightarrow E_x' = E_1 \cos(\omega t) = E_1$

$E_y' = \sqrt{3}\, E_1 \sin(\omega t) = 0$

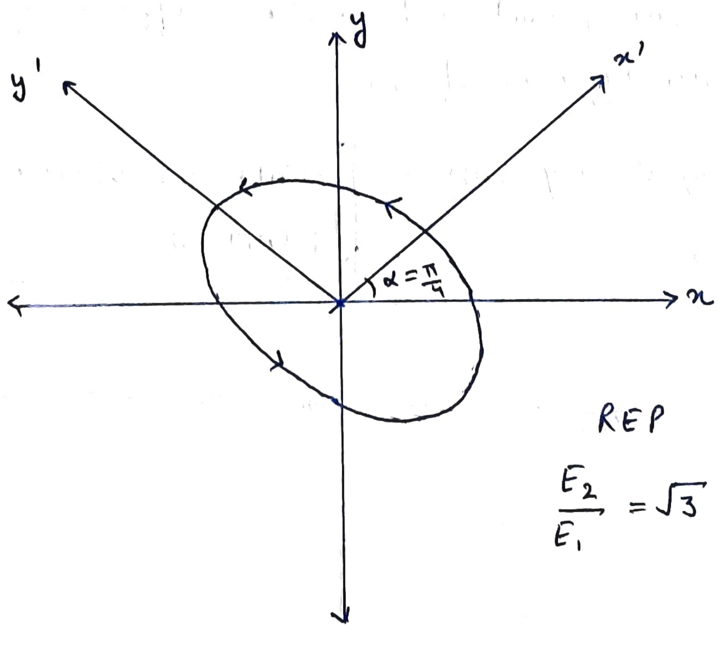
$\Rightarrow \therefore$ At $t = \dfrac{\pi}{2\omega}$

$$\boxed{E_x' = 0 \qquad E_y' = \sqrt{3}\, E_1}$$

$\therefore$ at $t = \dfrac{\pi}{\omega}$

$$\boxed{E_x' = -E_1 \qquad E_y' = 0}$$

we know, the points to plot and also know that it is a REP with electric vector in anti clockwise direction.



Ques 6:- Given - left circularly polarised beam, $\lambda_0 = 589.3$ nm

→ Calcite crystal with its optic axis cut parallel to surfaces.

→ Thickness = 0.005141 nm , $\lambda_0 = 1.65836$, $n_e = 1.48641$

Since the beam is normally incident as given in the question, on the calcite crystal, let plane $x=0$, represent surface of crystal on which beam is incident :-

Let y and z components be,

$$E_y(x=0) = E_0 \sin\phi \cos\omega t$$

$$E_z(x=0) = E_0 \cos\phi \cos\omega t.$$

m, $\theta = \theta_0 - \theta_e = n_0 k d - n_e k d = kd(n_0 - n_e)$

$\theta = \frac{\omega d}{c}(n_0 - n_e)$ ⇒ Represents phase difference b/w o & e beams.

d = thickness

$$\theta = \frac{2\pi d}{\lambda_0}(n_0 - n_e)$$

$$= \frac{2 \times \pi \times 0.005141 \times 10^{-9} \times 0.17195}{589.3 \times 10^{-9}}$$

$$\boxed{\theta = 3\pi}$$

If the thickness of crystal d is such that θ is an odd multiple of π, then crystal is said to be a HALF WAVE PLATE (HWP) and - a phase difference of π implies a path difference of λ/2.

⇒ The emergent ray will be $\boxed{\text{RIGHT CIRCULARLY POLARISED}}$

—x —x —x —x —x —x —x —x —x —x —x —