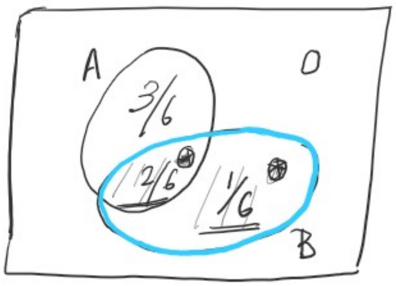
Definitions and Modeling

Wednesday, January 13, 2021 10:16 PM

$\Omega = \{ \qquad \}$ P



Ω

$P(A) = \frac{5}{6}$

$P(A|B)$ = Prob of A given B has happend

$\boxed{P(B|B) = 1}$

Original Model

$\frac{2}{3}$ $\frac{1}{3}$

$\Omega_{|B} = B$ $\quad P(\Omega_{|B}) = 1$ $\quad P(B|B) = 1$

Definitions

We define the conditional prob of A given B as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ we assume that } P(B) \neq 0$$

The conditional prob is undefined when the conditioning event has zero prob.

Ex Roll a dice twice

$\Omega = \{(1,1), (1,2) \cdots \quad (6,6)\}$

A = sum is even & B = first roll is 3.

$P(A) = 18/36$ $\quad P(B) = \frac{6}{36}$ $\quad P(A \cap B) = \frac{3}{36}$

Knowledge: B has occured

$P(A|B) = \frac{3}{6} = \frac{1}{2}$

$\Omega_{|B} = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$

$\Longrightarrow P(A|B) = \frac{P(A \cap B)}{} = \frac{3/36}{} = \frac{1}{2}$

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/36}{6/36} = \frac{1}{2}$$

Condition prob specify a Prob law

1. $P(A|B) \geq 0$ $\quad\quad$ $P(A) \geq 0$

2. $P(\underline{B}|B) = 1$ $\quad\quad$ $P(\Omega) = 1$

3. $A_1, A_2 \cdots$ s.t $A_i \cap A_j = \emptyset$

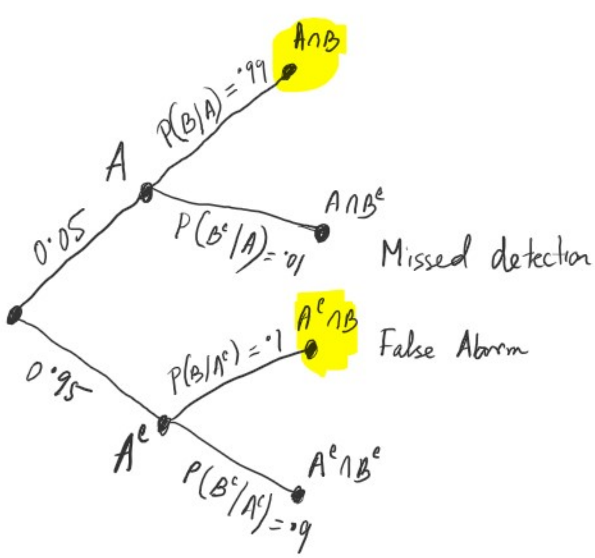
$$P(A_1 \cup A_2 | B) = P(A_1|B) + P(A_2|B)$$

$$= \frac{P((A_1 \cup A_2) \cap B)}{P(B)} = \frac{P((A_1 \cap B) \cup (A_2 \cap B))}{P(B)} = \frac{P(A_1 \cap B)}{P(B)} + \frac{P(A_2 \cap B)}{P(B)}$$

$$= P(A_1|B) + P(A_2|B)$$

Ex $\quad$ A = aircraft is present

B = the radar registers the presence of an aircraft



$$P(A \cap B) = .99 \times .05 = 0.0495$$

$$P(B) = P(A \cap B) + P(A^c \cap B)$$

$$= .0495 + .95 \times .1$$

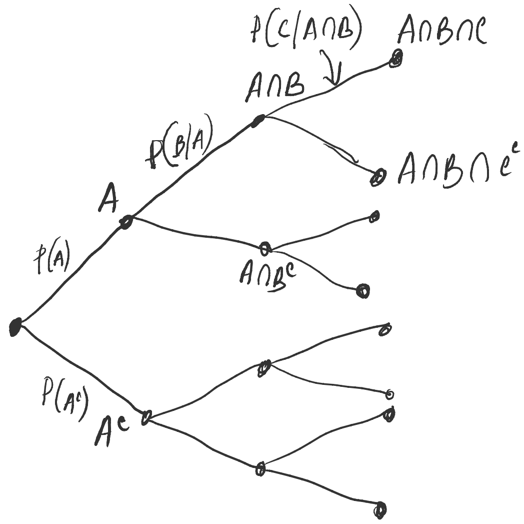
$$= 0.1445$$

$$\underline{P(A|B)} = \frac{P(A \cap B)}{P(B)} = \frac{.0495}{.1445}$$

$$= 0.34$$

$(A^c) = .9$ $\qquad$ $P(B)$

$= 0.34$



$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$$

<u>Multiplication Rule:</u> $\quad A_1, A_2 \cdots A_n$

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \cdots P\left(A_n \Big| \bigcap_{i=1}^{n-1} A_i\right)$$