CS 225: Switching Theory

S. Tripathy
IIT Patna

Previous Class

Switching Algebra

This Class

- Switching Algebra
-
  - Switching circuit
  - Propositional calculus

Simpler Procedure for Canonical Sum-of-products

1. Examine each term: if it is a minterm, retain it; continue to next term
2. In each product which is not a minterm: check the variables that do not occur;
   for each xi that does not occur, multiply the product by (xi + xi')
3. Multiply out all products and eliminate redundant terms

Example: $T(x,y,z) = x'y + z' + xyz$

$$= x'y(z + z') + (x + x')(y + y')z' + xyz$$
$$= x'yz + x'yz' + xyz' + xy'z' + x'yz' + x'y'z' + xyz$$
$$= x'yz + x'yz' + xyz' + xy'z' + x'y'z' + xyz$$

Canonical product-of-sums obtained in a dual manner

Example:

$$T = x'(y' + z)$$
$$= (x' + yy' + zz')(y' + z + xx')$$
$$= (x' + y + z)(x' + y + z')(x' + y' + z)(x' + y' + z')(x + y' + z)(x' + y' + z)$$
$$= (x' + y + z)(x' + y + z')(x' + y' + z)(x' + y' + z')(x + y' + z)$$

Transforming One Form to Another

Example: Find the canonical product-of-sums for

$$T(x,y,z) = x'y'z' + x'y'z + x'yz + xyz + xy'z + xy'z'$$

$$T = (T')'$$

$$= [(x'y'z' + x'y'z + x'yz + xyz + xy'z + xy'z')']'$$

$$= [x'yz' + xyz']' \quad \text{//Complement } T' \text{ consists of minterms not contained in } T.$$

$$= (x + y' + z)(x' + y' + z)$$

Canonical forms are unique

Two switching functions are equivalent if and only if their corresponding canonical forms are identical

Functional Properties

Let binary constant a_i be the value of function f(x1, x2, …, xn) for the combination of variables whose decimal code is i. Thus,

$$f(x1, x2, \ldots, xn)= a_0 x_1' x_2' \ldots x_n' + a_1 x_1' x_2' \ldots x_n + \ldots + a_r x_1 x_2 \ldots x_n$$

The coefficient a_i is set to 1 (0) if the corresponding minterm is (is not) in the canonical form

Since there are 2^n coefficients, each of which can have two values, 0 and 1, there are 2^2^n possible switching functions of n variables

Example: Canonical sum-of-products form for two variables

$$f(x,y) = a_0 x'y' + a_1 x'y + a_2 xy' + a_3 xy$$

Thus 2^2^2 =16 functions corresponding to the 16 possible assignments of 0's and 1's to a_0, a_1, a_2, and a_3

List of Functions of Two Variables

a3	a2	a1	a0	f(x,y)	Name of Function	Symbol
0	0	0	0	0	Inconsistency	
0	0	0	1	$x'y'$	NOR	$x \downarrow y$
0	0	1	0	$x'y$		
0	0	1	1	x'	NOT	x'
0	1	0	0	xy'		
0	1	0	1	y'		
0	1	1	0	$x'y+xy'$	Exclusive OR	$x \oplus y$
0	1	1	1	$x'+y'$	NAND	$x \mid y$

a3	a2	a1	a0	f(x,y)	Name of Function	Symbol
1	0	0	0	xy	AND	$x.y$
1	0	0	1	$xy+x'y'$	Equivalence	$x \equiv y$
1	0	1	0	y		
1	0	1	1	$x'+y$	Implication	$x \longrightarrow y$
1	1	0	0	x		
1	1	0	1	$x+y'$	Implication	$y \longrightarrow x$
1	1	1	0	$x+y$	OR	$x + y$
1	1	1	1	1	Tautology	

The Exclusive-OR Operation

Exclusive-OR: modulo-2 addition, i.e., $A \oplus B = 1$ if either A or B is 1, but not both

Commutativity: $A \oplus B = B \oplus A$

Associativity: $(A \oplus B) \oplus C = A \oplus (B \oplus C) = A \oplus B \oplus C$

Distributivity: $(AB) \oplus (AC) = A(B \oplus C)$

If $A \oplus B = C$, then

$$A \oplus C = B$$
$$B \oplus C = A$$
$$A \oplus B \oplus C = 0$$

Exclusive-OR of an even number of elements, whose value is 1, is 0
Exclusive-OR of an odd number of elements, whose value is 1, is 1

Functionally Complete Operations

Every switching function can be expressed in canonical form consisting of a finite number of switching variables, constants and operations $+, ., '$

A set of operations is functionally complete (or universal) if and only if every switching function can be expressed by operations from this set

Example: Set $\{+, ., '\}$

Set $\{+, '\}$?

Yes, since using De Morgan's theorem, $x . y = (x' + y')'$.

Thus, $+$ and $'$ can replace the $.$ in any switching function

Set $\{., '\}$ Yes for similar reasons

NAND:?

NOR:?

Yes, $NAND(x,x) = x'$ and $NAND[NAND(x,y),NAND(x,y)] = xy$

Yes, $NOR(x,x) = x'$ and $NOR[NOR(x,y), NOR(x,y)] = x + y$

Isomorphic Systems

Isomorphism: Two algebraic systems are isomorphic if

- For every operation in one system, there exists a corresponding operation in the second system
- To each element xi in one system, there corresponds a unique element yi in the other system, and vice versa
- If each operation and element in every postulate of one system is replaced by the corresponding operation and element in the other system, then the resulting postulate is valid in the second system

Thus, two algebraic systems are isomorphic if and only if they are identical except the labels and symbols used to represent the operations and elements

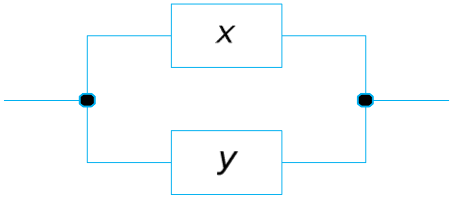
Series-parallel Switching Circuits

Gate: a two-state device capable of switching from one state, which permits flow of information, to another, which blocks it, and vice-versa

two-valued variable: denotes flowing (blocked) information

If a gate permits (blocks) the flow of information: literal associated with it takes value 1 (0)

Elementary series-parallel switching circuits



Parallel connection x + y

Series connection xy

Series-parallel circuits: any circuit constructed of either a series or parallel connection of two or more elementary series-parallel circuits

Transmission Function

Transmission function for a circuit: assumes value 1 (0) when there is (there is not) a path from one terminal of the circuit to the other through which information flows

Definition of transmission functions

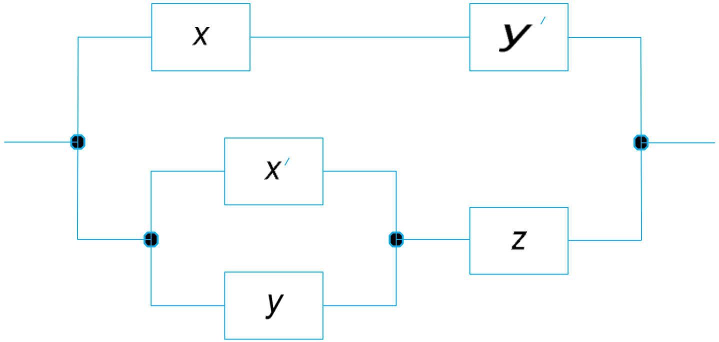
x y	x + y	xy
0 0	0	0
0 1	1	0
1 0	1	0
1 1	1	1

Analogy: OR <-> parallel; AND <-> series

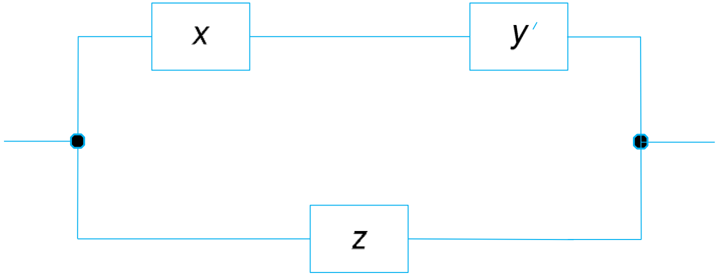
Complement of a given circuit: one that blocks all paths of information flow whenever the given circuit permits any

Thus, algebraic system for switching circuits isomorphic to switching algebra

Switching Circuit Simplification



Circuit realizing $T = xy' + (x' + y)z$



Simplified circuit realizing

$$\begin{aligned} T &= xy' + x'z + yz \\ &= xy' + x'z + y'z + yz \\ &= xy' + x'z + z \\ &= xy' + z \end{aligned}$$