CS225 Switching Theory

Minimization of Switching Funtion

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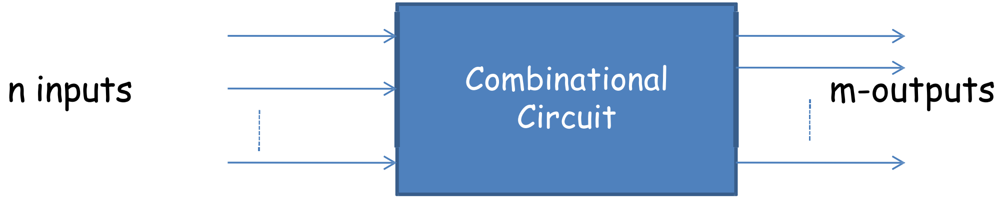
Previous Class

- Switching Algebra
  - Switching circuit
  - Propositional calculus

This Class

Minimization/ Simplification **of Switching Functions**

Combinational Circuit

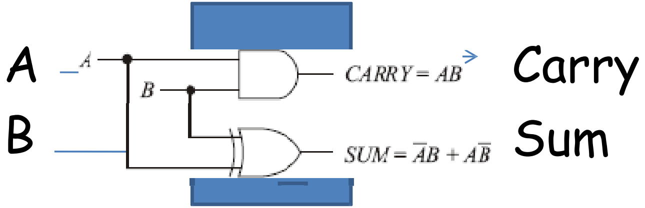


Design procedure: from the specification

1. Determine the required number of inputs and outputs
2. Derive the truth table
3. Find the Simplified Boolean expression for each output as a function of the input variable
4. Draw the logic diagram
5. Verify the correctness

EX.1:Binary Adder

- Half Adder:
- Four cases to remember (Two single-bit addition)
  - 0+0=0
  - 0+1=1
  - 1+0=1
  - 1+1=10 (Carry has been generated)



Combinational Circuit

- Analysis

1. Find the Boolean functions for each gate and obtain the output
2. Repeat step 1 until the output(s) of the circuit is obtained
3. Obtain the output Boolean function in terms of input variables

Find the simplified Boolean expression!

with minimum terms\ literals

Definitions

- Literals x_i or x_i'
- Product Term $x_2x_1'x_0$
- Sum Term $x_2 + x_1' + x_0$

- Minterm of n variables: A product of n literals in which every variable appears exactly once.

- Maxterm of n variables: A sum of n literals in which every variable appears exactly once.

- Adjacency of minterms (maxterms): Two minterms (maxterms) are adjacent if they differ by only one variable.

Implementation

Specification → Schematic Diagram
Net list, Switching expression

Obj min cost (max performance) → Search in solution space

Cost: wires, gates → Literals, product terms, sum terms

For two level logic (sum of products or product of sums), we want to minimize # of terms, and # of literals

Simplifying Switching Functions

Finding an equivalent switching expression that minimizes some cost criteria:

1. Minimize literal count

2. Minimize literal count in sum-of-products (or product-of-sums) expression

3. Minimize number of terms in a sum-of-products expression provided no other expression exists with the same number of terms and fewer literals

Example: $f(x,y,z) = x'yz' + x'y'z' + xy'z' + x'yz + xyz + xy'z$

$=$

Minimization

Boolean expressions can be minimized by combining terms

K-maps minimize equations graphically

Karnaugh Map: A 2-dimensional truth table

Implementation: Specification => Logic Diagram

Flow 1: Boolean Algebra

1. Specification
2. Truth table
3. Sum of products (SOP) or product of sums(POS) canonical form
4. Reduced expression using Boolean algebra
5. Schematic diagram of two level logic

Flow 2: K Map

1. Specification
2. Truth Table
3. Karnaugh Map (truth table in two dimensional space)
4. Reduce using K-Maps
5. Reduced expression (SOP or POS)
6. Schematic diagram of two level logic

K-Map: Truth Table in 2 Dimensions

2- Variables Truth Table

ID	A	B	f(A,B)
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

2- Variables K-Map

	B = 0	B = 1
A = 0	0 0	1 1
A = 1	2 1	3 1

Algebraic procedure to combine terms using the *Aa* + *Aa′* = *A* rule

$$f(A,B) = A + B$$

The Map Method

Karnaugh map: modified form of truth table

z \ xy	00	01	11	10
0	0	2	6	4
1	1	3	7	5

(a) Location of minterms in a three variable map.

z \ xy	00	01	11	10
0		1	1	
1			1	

(b) Map for function f(x, y, z) = $\sum(2,6,7) =$ $yz' + xy$

yz \ wx	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

(c) Location of minterms in a four-variable map.

yz \ wx	00	01	11	10
00		1	1	1
01		1	1	
11			1	
10			1	

(d) Map for function f(w, x, y, z) = $\sum(4,5,8,12,13,14,15) =$ $wx + xy' + wy'z'$

Simplification and Minimization of Functions

Cube: collection of 2^m cells, each adjacent to m cells of the collection

- Cube is said to **cover** these cells
- Cube expressed by a product of n-m literals for a function containing n variables
- m literals not in the product said to be **eliminated**

Example: $w'xy'z' + w'xy'z + wxy'z' + wxy'z$

$= xy'(w'z' + w'z + wz' + wz)$

$= xy'$

yz \ wx	00	01	11	10
00		1	1	1
01		1	1	
11			1	
10			1	

(d) Map for function f(w, x, y, z) = $\sum(4, 5, 8, 12, 13, 14, 15) =$

$wx + xy' + wy'z'$

Minimization (Contd.)

Example:

Use of cell 6 in forming both cubes justified by idempotent law

z \ xy	00	01	11	10
0	0	2	6	4
1	1	3	7	5

(a) Location of minterms in a three-variable map.

z \ xy	00	01	11	10
0		1	1	
1			1	

(b) Map for function $f(x, y, z) = \sum(2,6,7) = yz' + xy$

Corresponding algebraic manipulations:

$$\begin{aligned} f &= x'yz' + xyz' + xyz \\ &= x'yz' + xyz' + xyz' + xyz \\ &= yz'(x' + x) + xy(z' + z) \\ &= yz' + xy \end{aligned}$$

Minimization (Contd.)

Minimal expression: cover all the 1 cells with the smallest number of cubes such that each cube is as large as possible

- A cube contained in a larger cube must never be selected
- If there is more than one way of covering the map with a minimal number of cubes, select the cover with larger cubes
- A cube contained in any combination of other cubes already selected in the cover is redundant by virtue of the consensus theorem

Rules for minimization:

1. First, cover those 1 cells by cubes that cannot be combined with other 1 cells; continue to 1 cells that have a single adjacent 1 cell (thus can form cubes of only two cells)
2. Next, combine 1 cells that yield cubes of four cells, but are not part of any cube of eight cells, and so on
3. Minimal expression: collection of cubes that are as large and as few in number as possible, such that each 1 cell is covered by at least one cube

Thanks