Sample space, Events and Probability Laws

Friday, January 8, 2021 8:55 AM

Mathematical description of an uncertain situation

Probability Model

Every prob model involves an underlying process → Experiment.

Experiment produces one of the several possible outcomes

Set of all possible outcomes is called the Sample space, denoted by Ω

Ex 1 Tossing a coin

$\Omega = \{H, T\}$

Ex. Throwing a dice

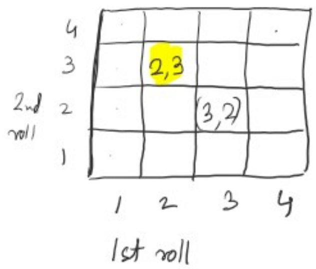
$\Omega = \{1, 2, 3, 4, 5, 6\}$ 10

(*) outcomes should be mutually exclusive

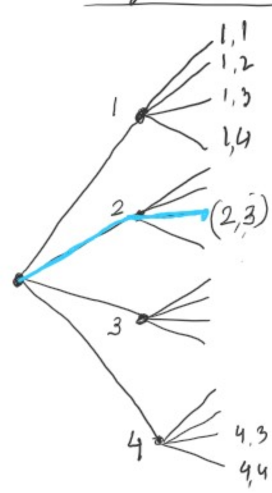
(*) " " " collectively exhaustive

$\Omega = \{H,$ T and its raining , T and its not raining$\}$

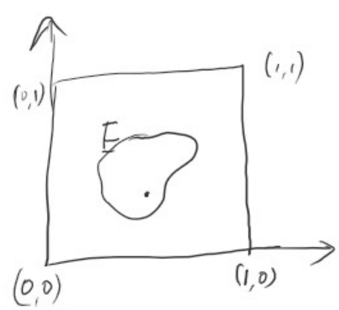
Ex 3 Rolling a 4-faced dice twice



Sequential Description



Ex 4



Throwing a dart on the unit sq

$\Omega = \{(x,y) \mid 0 \leq x, y \leq 1\}$

A subset of Ω is called an event E

$E_1 \cap E_2 \longrightarrow$ Both the events occured

$E_1 \cup E_2 \longrightarrow$ E_1, E_2 or both occured

(*) Not every subset of Ω is an event.

Probability Laws

$P(E)$ follows ~~few~~ the following axioms:

A1 $\quad 0 \leq P(E)$ for every event E [Nonnegativity]

A2 $\quad P(\Omega) = 1$ [Normalization]

A3 $\quad$ For any seq of $\underbrace{\text{mutually exclusive events}}_{\Downarrow}$ $E_1, E_2 \cdots E_n \cdots$

$(E_i \cap E_j = \emptyset$ for $i \neq j)$

$$P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$$

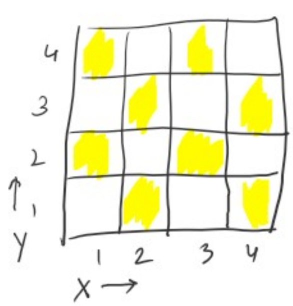
$A^c \rightarrow$ complement of event A.

$\hookrightarrow \Omega \setminus A$

$$1 \overset{A2}{=} P(\Omega) = P(A \cup A^c) \overset{A3}{=} P(A) + P(A^c)$$

$$P(A) = 1 - \underline{P(A^c)} \overset{A1}{\leq} 1$$

E3



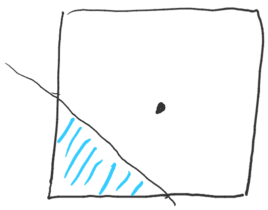
Prob law: Prob of every outcome is $1/16$

$$P\big((X,Y) \equiv (1,2) \text{ or } (3,4)\big) = P\big((X,Y) \equiv (1,2)\big) + P\big((X,Y) \equiv (3,4)\big)$$

$$= 1/16 + 1/16 = \frac{1}{8}$$

$$P(X+Y \text{ is odd}) = \frac{1}{16} \cdot 8 = \frac{1}{2}$$

E4



Prob law: $\text{Prob}(E) = \text{area of } E$

$$P((x,y) = (.5,.5)) = 0$$

$$P(x+y \le 1/2) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$