(3.) Mobius transformation / bilinear transformation :

Defⁿ :- $w = \dfrac{az+b}{cz+d}$ , $ad - bc \neq 0$ , $a, b, c, d \in \mathbb{C}$ .

$w : \mathbb{C} \to \mathbb{C}$ if $c = 0$

$\mathbb{C} \setminus \{-\frac{d}{c}\} \to \mathbb{C} \setminus \{\frac{a}{c}\}$ if $c \neq 0$

$z \mapsto w$, given a point $w$,

we can retrieve $z, \mapsto w$.

$w = \dfrac{az+b}{cz+d} \Rightarrow cw.z + dw = az+b$

$\Rightarrow z = \dfrac{-dw+b}{cw-a}$ , provided $cw - a \neq 0$. $w \neq \frac{a}{c}$.

$= \dfrac{a'w+b'}{c'w+d'}$ , $a'd' - b'c' = ad - bc \neq 0$.

all points except $w = \frac{a}{c}$, has preimage, as above,

indeed unique

The case, $c = 0$ : $w \notin \mathbb{C}$.

$(ad \neq 0)$

Obs.

$\boxed{\text{one-one, onto correspondence from } \mathbb{C} \to \mathbb{C}}$

$\boxed{c \neq 0, \quad \mathbb{C} / \{-\frac{d}{c}\} \to \mathbb{C} \setminus \{\frac{a}{c}\}}$ .

one-one, onto

define $T(z) : \mathbb{C} \cup \{\text{"pt at infinity"}\} \longrightarrow \mathbb{C} \cup \{\text{"pt at infinity"}\}$

$T(z) = \dfrac{az+b}{cz+d}$ , $ad - bc \neq 0$, $\forall z \in \mathbb{C}$

$$T(\infty) = \begin{cases} \infty & c = 0 \\ a/c & c \neq 0 \end{cases}$$

and $T\left(\frac{-d}{c}\right) = \infty$ , $c \neq 0$.

- Now $T$ is one-one, onto correspondence.
- $T$ is contˢ on extended complex plane,

$c = 0$

$$T(z) = \frac{az+b}{d}$$

$$T(\infty) = \infty$$

$$\lim_{z\to\infty} T(z) = \infty$$

$$\left[\text{iff } \lim_{z\to 0} \frac{1}{T\left(\frac{1}{z}\right)} = 0.\right.$$

We have, $\lim_{z\to 0} \dfrac{1}{\dfrac{a\cdot\frac{1}{z}+b}{d}}$

$$\left. = \lim_{z\to 0} \frac{dz}{a+bz} = 0\right]$$

$c \neq 0$.

$$T(z) = \frac{az+b}{cz+d}.$$

$$T(\infty) = \frac{a}{c}$$

$$T\left(-\frac{d}{c}\right) = \infty$$

Check: $\lim_{z\to -\frac{d}{c}} T(z) = \infty$.

$$\lim_{z\to\infty} T(z) = \frac{a}{c}$$

iff $\lim_{z\to 0} T\left(\frac{1}{z}\right) = \frac{a}{c}$

$$\| $$

$$\frac{a+bz}{c+dz}$$

(2).

For $T$ :- there exists an inverse from extended plane to ext. pl.

$$T^{-1}(w) = z \quad \text{i.e. } T(z) = w = \frac{az+b}{cz+d}$$

We have: $T^{-1}(w) = \dfrac{-dw+b}{cw-a}$, $ad-bc\neq 0$.

which is again a bilinear transformat^n.

$c = 0$:

$$T^{-1}(\infty) = \infty.$$

$c \neq 0$

$$T^{-1}(\infty) = -\frac{d}{c},$$

$$T^{-1}\left(\frac{a}{c}\right) = \infty.$$

To $w = \dfrac{az+b}{cz+d}$ : $\mathbb{C} \to \mathbb{C}$ | inverse exists from $\mathbb{C} \to \mathbb{C}$ iff $c = 0$

" " $\mathbb{C}\setminus\{\frac{a}{c}\} \to \mathbb{C}\setminus\{-\frac{d}{c}\}$ iff $c \neq 0$.

Obs (3) If $T$ and $S$ are two bilinear tra. then $S \cdot T$ is so.

Proof:- $T(z) = \dfrac{az+b}{cz+d}$, $ad-bc \neq 0$, $z \neq -\frac{d}{c}$ if $c \neq 0$

$S(z) = \dfrac{Az+B}{Cz+D}$. $AD - BC \neq 0$, $z \neq -\frac{D}{C}$ if $C \neq 0$

then $S \cdot T(z) = \dfrac{A\left(\frac{az+b}{cz+d}\right)+B}{C\left(\frac{az+b}{cz+d}\right)+D}$ for $z \neq -\frac{d}{c}$

also $\frac{az+b}{cz+d} \neq -\frac{D}{C}$

i.e. $(aC+Dc)z \neq -Dd - bC$

$$= \frac{(Aa+Bc)z + Ab + Bd}{(C \cdot a + Dc)z + Cb + Dd}$$

$z \neq \left(\dfrac{-Dd-bC}{aC+Dc}\right)$

$(Aa+Bc)(Cb+Dd) - (Ab+Bd)(C \cdot a + Dc) \neq 0$.

$\Rightarrow ad(AD - BC) + bc(BC - AD) + Aa\cancel{C}b + B\cancel{c}Dd$

$- Bd\cancel{D}c - Ab\cancel{C}\cancel{a}$

$\Rightarrow (ad-bc)(AD-BC) \neq 0$.

---

$$(S \cdot T)\left(-\frac{d}{c}\right) = \frac{(Aa+Bc)\cancel{c}(-d) + Abc + \cancel{Bd}c}{(C \cdot a + D\cancel{c})\cancel{c}(-d) + Cb \cdot c + \cancel{Dd}c}$$

$$= \frac{A(bc-ad)}{C(bc-ad)} = \frac{A}{C} \quad \text{if } C \neq 0.$$

$$= \infty \quad \text{if } C = 0.$$

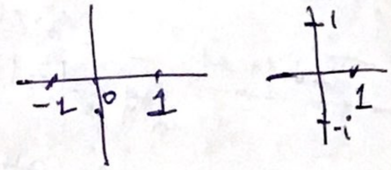
④ Given three distinct points $z_1, z_2, z_3$, and $w_1, w_2, w_3$ → distinct
there exists a bilinear transformation $T(z)$, s.t. → can be $\infty$
$T(z_i) = w_i \quad \forall\ i = 1, 2, 3$.

Example ①.

$z_1 = -1$, $\quad w_1 = -i$

$z_2 = 0$ $\quad w_2 = 1$

$z_3 = 1$ $\quad w_3 = i$




find $a, b, c, d$? let $T(z) = \dfrac{az+b}{cz+d}$, $ad - bc \neq 0$

$\cancel{T(1) = i = \dfrac{a+b}{c+d}}$

$T(0) = 1 = \dfrac{b}{d} \Rightarrow \boxed{d = b}$, $\quad ab - bc = b(a-c) \neq 0$.

$T(1) = i = \dfrac{a\cancel{z}+b}{c\cancel{z}+b} \Rightarrow ic\cancel{z} + ib = a\cancel{z} + b$.

$\Rightarrow b(1-i) = ic - a$ — ①

$T(-1) = -i = \dfrac{-a+b}{-c+b} \Rightarrow ic - ib = b - a$

$\Rightarrow ic + a = b(1+i)$ ②.

from ① & ②, $\cancel{2}ic = \cancel{2}b \Rightarrow \boxed{b = ic} = d$.

then $a = ib$.

so $T(z) = \dfrac{ibz + b}{-ibz + b} = \dfrac{b(iz+1)}{b(-iz+1)}$ with $b \neq 0$.

So $T(z) = \dfrac{(iz+1)}{(-iz+1)}$

(5). Given distinct pts $z_1, z_2, z_3$ & $w_1, w_2, w_3$, in $\mathbb{C}$.

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} \quad \text{—(1)}$$

for $z=z_1$, in above,

$$(w-w_1)(w_2-w_3)(z-z_3)(z_2-z_1) = (z-z_1)(z_2-z_3)(w-w_3)(w_2-w_1) \quad \text{—(2)}$$

$z=z_1$, $(w-w_1)=0$.

$z=z_3$, $w-w_3=0$.

$z=z_2$, $(w-w_1)(w_2-w_3) = (w-w_3)(w_2-w_1)$.

$$\Rightarrow \quad w(\cancel{w_2}-w_3-\cancel{w_2}+w_1) = -w_3w_2+\cancel{w_3w_1}+w_1w_2-\cancel{w_1w_3}$$

$$\Rightarrow \quad w = \frac{w_2(w_1-w_3)}{(w_1-w_3)} = w_2.$$

Q. Is above —(1)— a bilinear transform$^n$?

A bili. tr. $w=\dfrac{az+b}{cz+d}$, $ad-bc\neq 0$,

Can be written in the form $\boxed{Azw+Bz+Cw+D=0 \quad (AD-BC\neq 0)}$

$[\, c\cdot zw - a\cdot z + d\cdot w - b = 0.$ | $A=c$, $B=-a$, $C=d$, $D=-b$ ; $-bc+ad\neq 0$

For (1) or (2), can be written in this form,

Verify $AD-BC\neq 0$,

So, there is a bilinear transform$^n$, given by (1), which maps three distinct finite points $z_1, z_2, z_3$ onto a distinct finite, $w_1, w_2, w_3$

Q. Is it is a unique such map?

(8) The bilinear transf. $w = \frac{az+b}{cz+d}$, $ad-bc \neq 0$, transforms circles and lines into circles and lines

$c=0$, $w = \frac{a}{d}z + \frac{b}{d} = Az + B$, $\frac{a}{d} = A \neq 0$, $d \neq 0$ so $\frac{a}{d}, \frac{b}{d} \in \mathbb{C}$.

$c \neq 0$, $w = \frac{c \cdot (az+b)}{c \cdot (cz+d)} = \frac{ac \cdot z + bc \cancel{+ad} + ad}{c \cdot (cz+d)}$.

$$= \frac{a \cdot (cz+d) + (bc-ad)}{c \cdot (cz+d)}$$

$$\Rightarrow w = \frac{a}{c} + \frac{(bc-ad)}{c} \cdot \frac{1}{cz+d} . \quad ad-bc \neq 0$$

$\therefore$ (not a constant $f^n$)

We know that $Az+B$ : transforms circles & lines into circles & lines.

$\frac{1}{z}$ : "

---

$w = \frac{1}{z}$ : special case, a bilinear transformat$^n$ with $a=0, b=1, c=1, d=0$.

Recall :-

$A \neq 0$.

① $w = Az$ : transforms circles & lines to circles & lines.

② $w = z + B$ : 〃.

Justificatⁿ :- circle :- $|z| = r$, $\rightsquigarrow$ $|w| = |A||z| = |A| \cdot r$ again circle

eqⁿ of $\cancel{(x-a)^2 + (y-b)^2 = r^2}$.

① line :- $|z - z_1| = k \cdot |z - z_2|$ $\rightsquigarrow$ $\cancel{\left|\frac{w}{A} - \frac{z_1}{A}\right| = k}$

$k \in \mathbb{R}_{>0}$

$z_1 \neq z_2$,

$z \neq z_1$

a line passing thr. $z_1$ & $z_2$.

$z = z_1 + (z_2 - z_1)\,t$ $\rightsquigarrow$

$\frac{w}{A} = z_1 + (z_2 - z_1)\,t$

$= Az_1 + (z_2 - z_1)At$

$\cancel{\frac{w}{A} = \frac{w_1}{A} + \frac{(w_2 - w_1)}{A}t}$

$\Rightarrow \cancel{w = w_1 + (w_2 - w_2)t}$

line passing through $Az_1$ & $Az_2$.

② $w = z + B$, line: $Ax + By = C$ $\rightsquigarrow$ $A(u - b_1) + B(v - b_2) = C$

$u = x + b_1$

$v = y + b_2$

circle: $z = re^{i\theta}$ $\rightsquigarrow$ $w - B = re^{i\theta}$

$\Rightarrow w = B + re^{i\theta}$.

So $Az + B$ transforms circles to circles.
& lines to lines.

§ $\boxed{w = \frac{1}{z}, \ z \neq 0.}$ $\mathbb{C}\setminus\{0\} \longrightarrow \mathbb{C}\setminus\{0\}$.

$T(z) = \frac{1}{|z|^2} \cdot z$, $\quad w = \overline{T(z)}$

I.T.

~~Observat~~
We discuss the ~~ma~~ images of curves under $w = \frac{1}{z}$.

Obs- 1) $\mathbb{C}\setminus\{0\} \to \mathbb{C}\setminus\{0\}$.

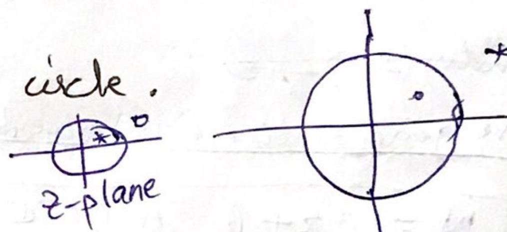
$w_0 \neq 0$, then $w_0 = \frac{1}{(w_0)^{-1}}$

and if $w_0 = \frac{1}{(w_1)}$

then $\frac{1}{w_1} = \frac{1}{(w_0)^{-1}}$ iff $w_0^{-1} = w_1$.

one-one, onto correspondence from $\mathbb{C}\setminus\{0\} \longrightarrow \mathbb{C}\setminus\{0\}$

2) ~~Inversion~~ Inversion w.r.t unit circle.




[$|z| < 1$ iff $|w| > 1$]

3) Let $z = x + iy \neq 0$, $w = u + iv$, then

$$u = \frac{x}{x^2+y^2}, \quad v = \frac{-y}{x^2+y^2}$$

This gives us, $u^2+v^2 = \frac{1}{x^2+y^2}$,

$$x = \frac{u}{u^2+v^2}, \quad y = \frac{-v}{u^2+v^2},$$

4) circles and lines, under $w = \frac{1}{z}$, are mapped to circles and lines.

eqn representing arbitrary circle or a line:

$$A(x^2+y^2) + Bx + Cy + D = 0, \quad B^2 + C^2 > 4AD.$$

$A \neq 0$, circle, $\left(x^2 + \frac{Bx}{A}\right) + \left(y^2 + \frac{Cy}{A}\right) = -\frac{D}{A}$.

$$\Rightarrow \left(x + \frac{B}{2A}\right)^2 + \left(y + \frac{C}{2A}\right)^2 = -\frac{D}{A} + \frac{B^2}{4A^2} + \frac{C^2}{4A^2}$$

$$= \frac{B^2 + C^2 - 4AD}{4A^2}$$

$$= \left(\frac{\sqrt{B^2+C^2-4AD}}{2A}\right)^2$$

circle with, centre : $\left(\frac{-B}{2A}, \frac{-C}{2A}\right)$

radius : $\frac{\sqrt{B^2+C^2-4AD}}{2A}$ provided $B^2+C^2-4AD > 0$.

arbitrary radius

~~$B \neq 0$~~, $A = 0$, ~~both~~ $\underbrace{B \text{ \& } C \text{ are not both zero}}$, $Bx + Cy + D = 0$

$\Uparrow$

Same cond$^n$ $B^2 + C^2 > 0$

Four cases :, Image of $A(x^2+y^2) + Bx + Cy + D = 0$ is

$D(u^2+v^2) + Bu - Cv + A = 0$.

- lines : $A=0$, passing through origin (i.e. $D=0$) $\rightsquigarrow$ line passing through origin.
  does not pass through origin (i.e. $D \neq 0$) $\rightsquigarrow$ circle pass. through origin.

- circles, : $A \neq 0$ passing through origin ($D=0$) $\rightsquigarrow$ line } not passing through origin
  not passing through origin ($D \neq 0$) $\rightsquigarrow$ circle. } not passing through origin

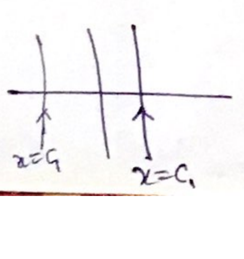
5) line-~~segments~~ $x = c_1 > 0 \rightsquigarrow u - c_1(u^2+v^2) = 0$.

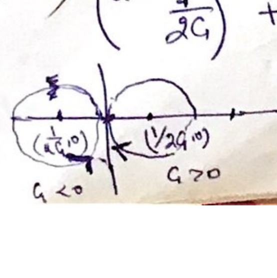
$$\parallel$$

line, $(u^2+v^2) - \frac{u}{c_1} = 0$.

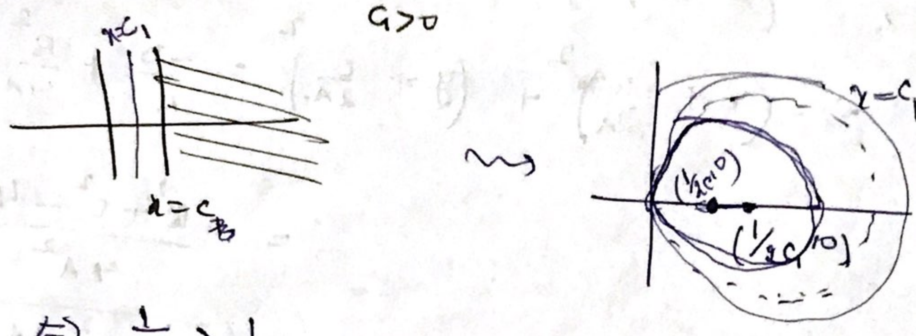
also know $(c_1, y) \rightsquigarrow \left(\frac{c_1}{c_1^2+y^2}, \frac{-y}{c_1^2+y^2}\right)$

$$\left(u - \frac{1}{2c_1}\right)^2 + v^2 = \left(\frac{1}{2c_1}\right)^2$$



$C_1 > 0$




$C > C_1 \iff \dfrac{1}{2C_1} > \dfrac{1}{2C}$

The half plane $x \geq C_1$, $(C_1 > 0)$ is mapped onto the disk

$$\left(u - \frac{1}{2C_1}\right)^2 + v^2 \leq \left(\frac{1}{2C_1}\right)^2$$

---

as $C_1 \to 0$, $\dfrac{1}{2C_1} \to \infty$. $\therefore$ radius of circle increases as $C_1 \to 0$.

So the half plane $x > 0 \rightsquigarrow u > 0$.

Similarly $x < 0 \rightsquigarrow u < 0$

$x = 0$, $\left(0, \dfrac{-1}{y}\right) \rightsquigarrow u = 0$ all $y$.

<u>Argue</u> for $y > 0$
$y < 0$

Recale, $w = f(z) = \frac{az+b}{cz+d}$, $ad-bc \neq 0$. 13.10.20

[if $c=0$, $w = \frac{a}{d}z + b$  $ad \neq 0$.

if $c \neq 0$, $w = \frac{c(az+b)}{c(cz+d)} = \frac{ac\cdot z + bc - ad + ad}{c\cdot(cz+d)}$

$= \frac{a\cdot(cz+d) + (bc-ad)}{c\cdot(cz+d)}$

$= \frac{a}{c} + \frac{(bc-ad)}{cz+d}$. ]

**Note**: If $ad-bc=0$ then $f(z)$ is constant.

[in case of $c=0$]

- So $f(z)$ is composition of $(f_1 \circ f_2)$ where. $f_2(z) = \frac{a}{d}z$, $f_1(z) = z+b$

and if $c\neq0$, then $f(z)$ is composition of $(f_1 \circ f_2 \circ f_3 \circ f_4)$ where

$f_4(z) = cz+d$,

$f_3(z) = \frac{1}{z}$

$f_2(z) = (bc-ad)\cdot z$.

$f_1(z) = \frac{a}{c} + z$.

(a) Def (fixed point): A fixed point of a mapping $w=f(z)$ is a point $z_0$ s.t. $f(z_0) = z_0$.

- Fixed points of $w = f(z) = \frac{az+b}{cz+d}$

$f(z) = z$ iff $z = \frac{az+b}{\ }$

$\dot{\imath}$

→ The quadratic polynomial is zero poly iff $c = 0$, $b = 0$ and $a = d$.

iff $w = f(z) = \frac{az}{d} = z$ [Remember $ad - bc \neq 0$ so $ad \neq 0$.]

→ So, the quadratic equation $cz^2 - (a-d)z - b = 0$, (if not zeropoly) has at most two solutions.

Theorem :- $w = f(z)$ : a billinear transformation, which is not identity, can have at most two fixed points.

Corollary :- If $S$ and $T$ are two billinear transform$^n$ which agree at three distinct points of extended complex plane then $S = T$.

Proof :- Ex :- $(S \circ T^{-1})$ is a billinear transformat$^n$.

and $(S \circ T^{-1})$ has three fixed points.

$\Rightarrow S \circ T^{-1} = id$

$\Rightarrow S = T$.

Ex (Tutorial) Find all billinear transformation which has fixed points $i$ & $-i$.

[A general method to find a $w = \frac{az+b}{cz+d}$, $ad-bc \neq 0$. which maps three distinct pts. $z_1, z_2, z_3$ to distinct $w_1, w_2, w_3$ respectively]

**Definition :-**

(a) The cross ratio of the complex nos. $z, z_1, z_2, z_3$ is the number

$$\frac{z-z_1}{z-z_3} \cdot \frac{z_2-z_3}{z_2-z_1}$$

(b) The cross ratio of $\infty, z_1, z_2, z_3$ is

$$\lim_{z\to\infty} \frac{(z-z_1)}{(z-z_3)} \frac{(z_2-z_3)}{(z_2-z_1)} = \lim_{z\to\infty} \left(\frac{1-\frac{z_1}{z}}{1-\frac{z_3}{z}}\right) \cdot \frac{z_2-z_3}{z_2-z_1}$$

$$= \frac{z_2-z_3}{z_2-z_1}$$

**Theorem** (Invariance of cross ratio under billinear transf).

Suppose $w = f(z)$ is a billinear transformation. Suppose $z_1, z_2, z_3$ are three distinct points and $w_i = f(z_i)$. with $w_1, w_2, w_3$ are distinct.

Then $$\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} = \frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}$$

Proof :- Let $R(z) := \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ : a billinear trans.

$R(z_1) = 0$
$R(z_2) = 1$
$R(z_3) = \infty$.

~~Proof~~

**Proof:-** Consider $T(z) = \dfrac{z - w_1}{z - w_3} \cdot \dfrac{w_2 - w_3}{w_2 - w_1}$

: a billinear trans.

$T(w_1) = 0$,
$T(w_2) = 1$
$T(w_3) = \infty$.

So,
$$\begin{matrix} z_1 \\ z_2 \\ z_3 \end{matrix} \xrightarrow{f(z)} \begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix} \xrightarrow{T(z)} \begin{matrix} 0 \\ 1 \\ \infty \end{matrix}.$$

So $f^{-1}(T^{-1}(R(z)))$ :
$$\begin{matrix} z_1 & \xrightarrow{R} & 0 & \xrightarrow{T^{-1}} & w_1 & \xrightarrow{f^{-1}} & z_1 \\ z_2 & \longrightarrow & 1 & \longrightarrow & w_2 & \longrightarrow & z_2 \\ z_3 & \longrightarrow & \infty & \longrightarrow & w_3 & \longrightarrow & z_3 \end{matrix}$$

That is $f^{-1}(T^{-1}(R(z)))$ has three distinct fixed points.

$\Rightarrow$ $f^{-1}(T^{-1}(R(z))) = Id$ (by earlier result)

$\Rightarrow$ $R(z) = T(f(z))$

$\Rightarrow$ $\dfrac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)} = T(w) = \dfrac{w - w_1}{w - w_3} \cdot \dfrac{w_2 - w_3}{w_2 - w_1}$

[by def^n of R & T]

Eg :- Find a billinear transf^n that maps $1, i, -i$ to $-1, 0, 1$.

Ans:- $\left[ w = \dfrac{z - i}{iz - 1} \right.$ - check ?? $\left. \right]$

⑧ (analyticity)

$$w = f(z) = \frac{az+b}{cz+d}, \quad ad-bc \neq 0.$$

$$= \frac{p(z)}{q(z)} \quad \& \quad q(z)=0 \text{ iff } z = -\frac{d}{c}$$

so

* $f(z)$ is analytic everywhere except at $z = -\frac{d}{c}$. (in case of $c \neq 0$)

* $z = -\frac{d}{c}$ : isolated singular point

: pole of order 1.

* $$\operatorname{Res}_{z=-\frac{d}{c}} f(z) = \lim_{z \to -\frac{d}{c}} \left(z + \frac{d}{c}\right) \cdot f(z).$$

$$= \frac{a\left(-\frac{d}{c}\right) + b}{c} = \frac{(bc-ad)}{c^2} \ (\neq 0).$$

[in case $c=0$, $f(z)$ is analytic everywhere].

Now $\underset{\left[z \neq -\frac{d}{c}\right]}{f'(z)} = \begin{cases} a & \text{if } c=0. \\ \dfrac{a.(cz+d) - c(az+b)}{(cz+d)^2} & \text{if } c \neq 0. \\ \quad \parallel & \\ \dfrac{ad-bc}{(cz+d)^2} & \end{cases}$

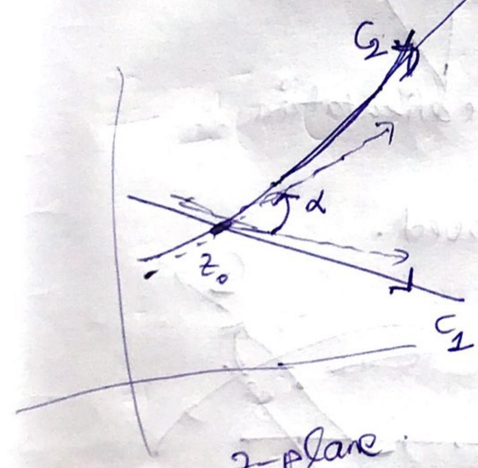
[So, $f'(z) \neq 0$ for all $z$ since $ad-bc \neq 0$.]

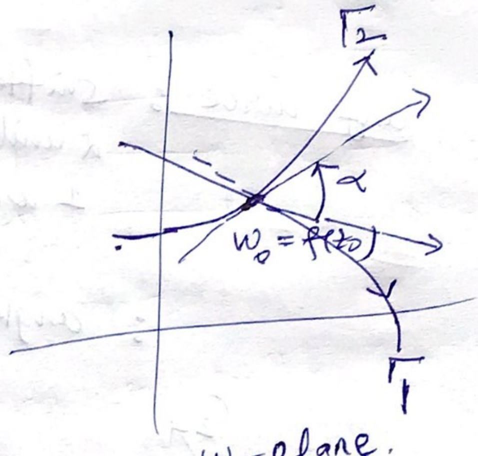
# Conformal Mapping

**Def$^n$:-** suppose that $w = f(z)$ is a complex mapping defined in a domain D.

We say that $f(z)$ is conformal at $z_0$ in D if it preserves the angle between oriented curves in magnitude and sense.

that is if $C_1$ and $C_2$ are two smooth oriented curves inside D and they intersect at a point $z_0$,



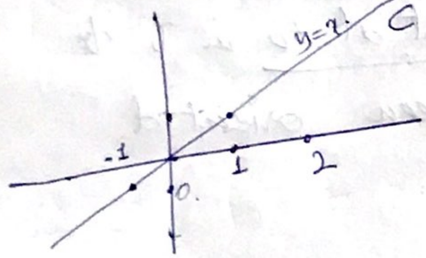




then, the images of $C_1$ and $C_2$, say $\Gamma_1$ and $\Gamma_2$ respecti-vely intersect at $f(z_0) =: w_0$.

the angle between $C_1$ and $C_2$ at $z_0$ (which is the angle between the tangents) and the angle between $\Gamma_1$ and $\Gamma_2$ at $w_0$ are same (both in magnitude and in sense).

$\Gamma_1$ & $\Gamma_2$ : are also oriented smooth curves

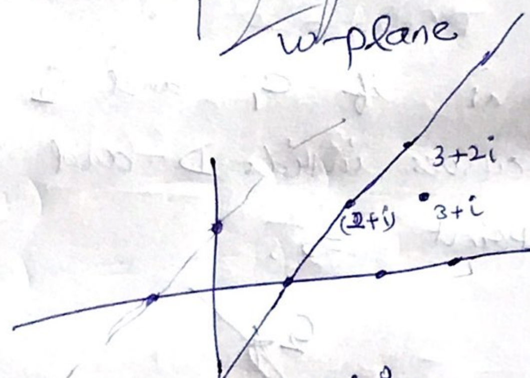
Eg! - $f(z) = z + b$, $b \in \mathbb{C}$ [translation]





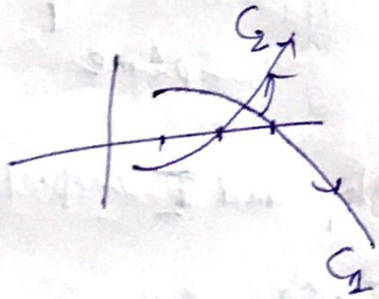
Say, $b = 2 + i$

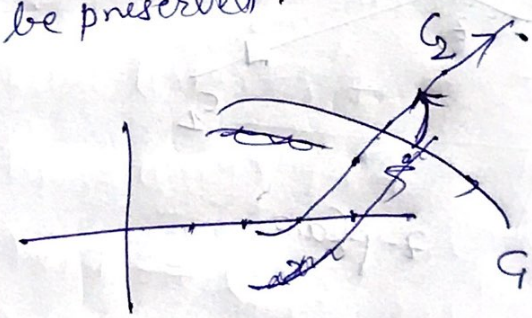
z-plane




any curve: shifted ~~by~~
2 units right
1 unit up. with same orientation.

: angles will be preserved.



Theorem :- Suppose $f(z)$ is an analytic function.

Then $f(z)$ is conformal at a point $z_0$ iff.

① $f'(z_0) \neq 0$.

Eg: (a) $w = z^2$, is not conformal at $z=0$.
conformal everywhere else.

~~Fact~~ (b) A billinear transformati' is conformal at all points.