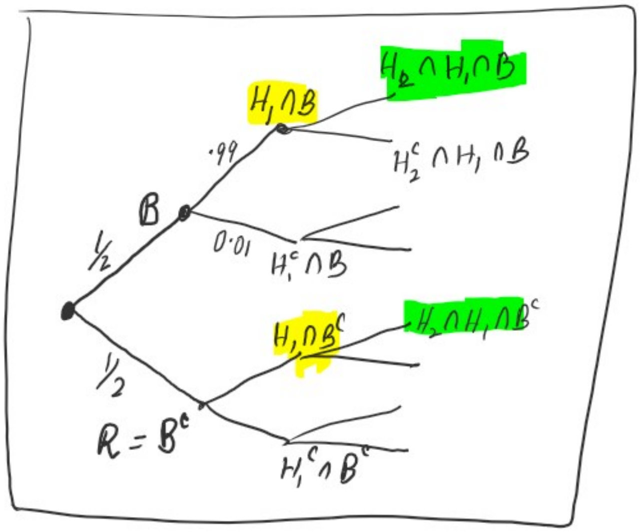
Independence of several events

Thursday, January 21, 2021 8:51 AM

Ex Two coins "Red" and "Blue"

if the coin is Red, $P(H) = 0.01$

if " " " Blue, $P(H) = .99$



B = coin is Blue

H_1 = first toss is H

H_2 = 2nd " " H

$$P(H_1 | B) = .99$$

$$P(H_2 | B) = 0.99$$

$$P(H_1 \cap H_2 | B) = 0.99 \times 0.99$$

$\Rightarrow H_1$ & H_2 are condi indep.

$$P(H_1) = P(B) \cdot P(H_1 | B) + P(B^c) \cdot P(H_1 | B^c)$$

$$= \frac{1}{2} \cdot 0.99 + \frac{1}{2} \cdot 0.01 = \frac{1}{2}$$

$$P(H_2) = \text{------} = \frac{1}{2}$$

$$P(H_1 \cap H_1) = P(B) \cdot P(H_1 \cap H_2 | B) + P(B^c) \cdot P(H_1 \cap H_2 | B^c)$$

$$= \frac{1}{2} \cdot (0.99)^2 + \frac{1}{2} (0.01)^2 \approx \frac{1}{2}$$

$\Rightarrow H_1$ & H_2 are not ind.

$$P(\text{11-th toss is head}) = \frac{1}{2}$$

$$P(\text{11-th toss is head} \mid \underline{\underline{H_1 \cap H_2 \cap \cdots \cap H_{10}}}) \approx P(\text{11-th toss is head} \mid B)$$

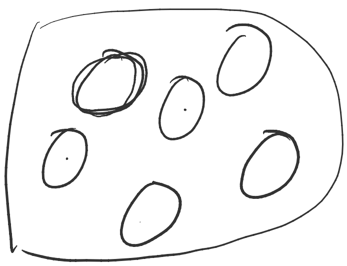
$$= .99$$

$$P\left(\bigcap_{i=1}^{10} H_i \mid B^c\right) = (0.01)^{10}$$

<u>Independence of several events</u>

We call $A_1, A_2, \cdots A_n$ are ind if

$$P\left(\bigcap_{i \in S} A_i\right) = \prod_{i \in S} P(A_i) \quad \text{for all subsets } S \text{ of } \{1, 2, \cdots n\}$$



A_1, A_2, A_3 are ind $\Rightarrow$

$$\left.\begin{aligned} P(A_1 \cap A_2) &= P(A_1) \cdot P(A_2) \\ P(A_1 \cap A_3) &= P(A_1) \cdot P(A_3) \\ P(A_2 \cap A_3) &= P(A_2) \cdot P(A_3) \end{aligned}\right\} \text{pairwise ind}$$

$$\not\Rightarrow P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3)$$

Pairwise ind does not imply ind

Toss a fair coin twice

$H_1 =$ first is head $\frac{1}{2}$

$H_2 =$ 2nd is head $\frac{1}{2}$

$D =$ two tosses have diff result $\frac{1}{2}$

$$P(H_1 \cap H_2) = P(H_1) . P(H_2)$$

$$P(H_1 \cap D) = \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = P(H_1) . P(D)$$ pairwise ind

$$P(H_1 \cap H_2 \cap D) = 0$$

⊗ $P(A_1 \cap A_2 \cap A_3) = \prod_{i=1}^{3} P(A_i)$ is not enough for ind

two ind rolls of a fair die.

$A_1 = \{$1st roll is 1, 2, or 3$\} \rightarrow \frac{1}{2}$

$A_2 = \{$1st roll is 3, 4, or 5$\} \rightarrow \frac{1}{2}$

$A_3 = \{$sum is 9$\}$ $\frac{4}{36} \rightarrow \frac{1}{9}$

$$P(A_1 \cap A_2 \cap A_3) = \frac{1}{36} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{9}$$

$$P(A_i \cap A_j) \neq P(A_i) . P(A_j)$$

<u>Ques:</u> Biased coin $P(H) = p$ $P(T) = 1 - p$

How can I make <u>fair dice</u> with this biased coin?

Hint: Already discussed in this class