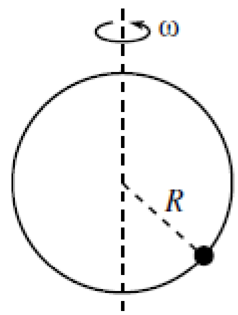
PH103 Physics-I: Assignment-1

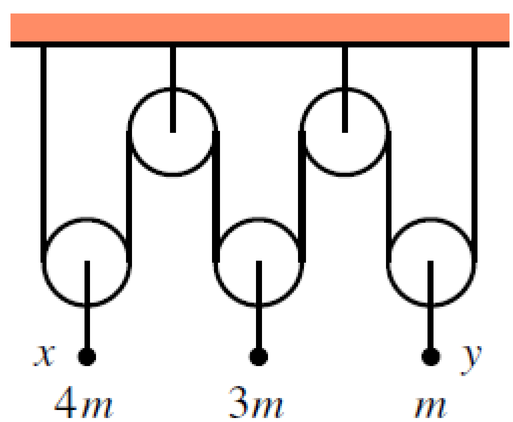
Spring semester 2020

Submission Date: 12/02/2020

1. A bead is free to slide along a frictionless hoop of radius R. The hoop rotates with constant angular speed ω around a vertical diameter (see figure). Find the equation of motion for the angle θ made by R with the rotation axis (use Lagrange's equation). What are the equilibrium positions? What is the frequency of small oscillations about the stable equilibrium? There is one value of ω that is rather special; what is it and why is it special?

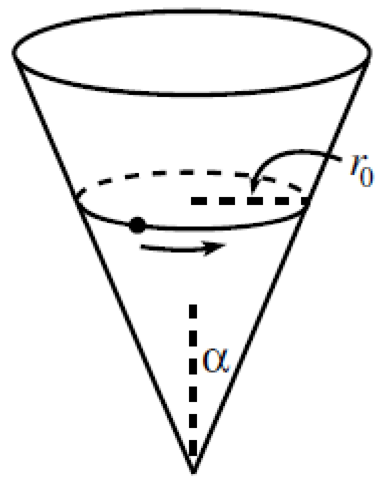


2. Consider Atwood's machine shown in Figure below. The masses are 4m, 3m and m. Let x and y be the heights of the left and right masses, relative to their initial positions. Write equation of motion using Lagrange's equation. Find the conserved momentum.



3. A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical. The half-angle at the tip is α (see figure on next page). Let r be the distance from the particle to the axis, and let θ be the angle around the cone. Find the equations of motion using Lagrange's equation. If particle moves in a circle of

radius r_0, what is the frequency ω of this motion? If the particle is then perturbed slightly from this circular motion, what is the frequency Ω of the oscillation about the radius r_0? Under what conditions does $\Omega=\omega$?

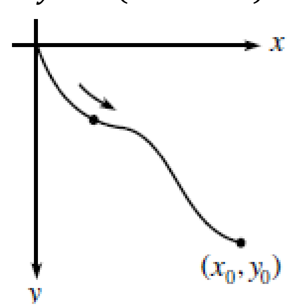


4. A bead is released from rest at the origin and slides down a frictionless wire that connects the origin to a given point, as shown in figure below. You wish to shape the view so that bead riches the endpoint in the shortest possible time. Let the desired curve be described by the function y(x), with downward taken to be positive. Show that y(x) satisfies

$$\mathbf{1+\dot{y}^2=\frac{B}{y}},$$

where B is a constant. Then show that x and y can be written as

$$x=a(\theta-\sin\theta),$$
$$y=a(1-\cos\theta).$$



5. A nucleus, originally at rest, decays radioactively by emitting an electron of momentum 1.73 MeV/c, and at right angles to the direction of the electron a neutrino with momentum 1.0 MeV/c. (The MeV, million electron volts, is unit of energy used in modern physics, equals to 1.60×10^{-13} J. MeV/c is a unit of linear momentum equal to 5.34×10^{-22} kg. m/s.) In what direction does the nucleus recoil? What is its momentum in MeV/c? If the mass of the residual nucleus is 3.90×10^{-25} kg what is its kinetic energy, in electron volts?

6. A spring of rest length La (no tension) is connected to a support at one end and has a mass M attached at the other. Neglect the mass of the string, the dimension of the mass M, and assume that the motion is confined to a vertical plane. Also assume that spring only stretches without bending but it can swing in the plane. (a) Using the angular displacement of the mass from the vertical and the length that the spring has stretched from its rest length, find Lagrange's equations. (b) Solve these equations for small stretching and angular displacements.