# ASSIGNMENT - 3 CS206

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Q1

a) Suppose that $n$ is even. Then $n = 2k$ for some integers $k$. Thus if $n^2 = (2k)^2 = 4k^2$

$$n^2 = 2(2k^2)$$

$\therefore$ $n^2 = 2(P)$ for some integer $P = 2k^2$

Thus if $n$ is even $n^2$ is also even

b) given: $m$ & $n$ & $p$ are integers
$m+n$ & $n+p$ are even

To proof: $m+p$ is even

Let $m+n = 2k$ —(i)

$n+p = 2y$ —(ii)

$i + ii \Rightarrow$ $m + 2n + p = 2(k+y)$

$m + p = 2(k+y-n)$

Thus $m+P = 2x$ for some $x = k+y-n$

Thus $m+p$ is even.

Method of direct proof is required.

c) Let there be a number $\boxed{x = 2y+1}$ ($x$ is odd)

now $x = 2y+1$

$= y^2 + 2y + 1 - y^2$

$x = (y+1)^2 - (y)^2$

Hence proved that every odd no. is subtraction of 2 squares

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Q2

a) If we prove that $i \Rightarrow ii \Rightarrow iii \Rightarrow i$ it will be enough to say that they are equivalent

If $n$ is even $\longrightarrow P1$

$n = 2k$ for some integer $k$

now we know that integers are continuous series of odd, even, odd... elements

thus $2k-1$ is odd

Hence $n-1 = 2k-1$ is odd

Thus $P_1 \longrightarrow P_2$

Now if $P_2$ is true

$\therefore$ $n-1$ is odd

$n$ must be even

$n = 2k$

$n^2 = (2k)^2 = 4k^2$

$n^2 = 2(2k^2)$

$n^2 = 2p$ for $p = 2k^2$

$\therefore$ $n^2$ is even

$\therefore$ $P_1 \to P_2 \to P_3$

Now if $P_3$ is true, $n^2$ is even

we know that every square has factors in multiple of 2.

Thus if $n^2$ is even there must be at least Two 2s in prime factorisation of $n^2$

$\therefore \quad n^2 = 2 \times 2 \times K$

$n^2 = 4K$

$n^2 = 2(2K)$

$n^2 = 2Q \qquad$ for some $Q = 2K$

$n = 2Q$

Thus $\quad P_3 \rightarrow P_1$

$\therefore \quad P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow P_1$

hence these statements are equivalent

Q3

Suppose that there is a rational number $r$ that satisfies the given equation.

Let $r = \frac{a}{b} \quad \forall (a,b) \in R \quad b \neq 0$

$r^3 + r + 1 = 0$

$\left(\frac{a}{b}\right)^3 + \left(\frac{a}{b}\right) + 1 = 0$

$= \frac{a^3}{b^3} + \frac{a}{b} + 1 = \frac{a^3}{b^3} + \frac{ab^2}{b^3} + \frac{b^3}{b^3} = 0$

$= a^3 + ab^2 + b^3 = 0$

Now 4 possible option arise.

1.) a and b both are odd

$\therefore$ If $a^3$ is odd, $b^3$ is odd, $ab^2$ is odd

'0' is even

Thus sum of 3 odd no. can not be even. So this case is not possible

2 a is odd b is even

$a^3$ is odd, $b^3$ is even, $ab^2$ is even

Thus sum of 2 even and 1 odd no. cannot be even (0).

3. a is even and b is odd

$a^3$ is even, $ab^2$ is even and $b^3$ is odd

Thus sum of 2 even and 1 odd no. cannot be zero

4. a and b both even

if a and b are both even, then $\frac{a}{b}$ is not in its lowest form. Thus it cannot be possible.

Since all the cases are not possible, this contradicts our assumption that a rational no. exist which satisfy the given equation. Hence proved

Q3

(b.) (a.) Let $x$ be irrational, $x$ cannot be written as ratio of two numbers. Let $y$ be a rational no. $y = \frac{b}{a}$
Let us assume that $x+y$ is rational.

$$\therefore \quad x + y = z \text{ (some rational no.)}$$

$$x + \frac{a}{b} = \frac{c}{d} \quad \left( y = \frac{a}{b} \quad z = \frac{c}{d} \right)$$

$$x = \frac{cb - ad}{bd}$$

Since $a, b, c, d$ are all integers & $b \neq 0$ & $d \neq 0$

$\frac{cb-ad}{bd}$ is a rational no.

but $x$ is not a rational no.

This contradicts our assumption that $x+y$ is rational
Hence $x+y$ is irrational if $x$ is irrational & $y$ is rational.

Q4

a) The contraposition of the statement is, "If $n$ is odd then $n^3+5$ is even." Hence to prove this let $n$ be odd. Thus $n = 2K+1$ (for some integer $K$)

$$n^3 + 5 = (2K+1)^3 + 5 = 8K^3 + 12K^2 + 6K + 6$$
$$= 2\left(4K^3 + 6K^2 + 3K + 3\right)$$

thus $n^3 + 5 = 2(P)$ for some integer $P = 4k^3 + 6k^2 + 3k + 3$

Thus $n^3 + 5$ is even

Since its contraposition is true then the original statement is also true.

(b.) Let $n^3 + 5$ be odd and $n$ is not even. Thus

$n = 2K + 1$

$n^3 + 5 = (2K+1)^3 + 5 = 8K^3 + 12K^2 + 6K + 6$

$= 2(4K^3 + 6K^2 + 3K + 3)$

Thus $n^3 + 5$ must be even. Hence our assumption was wrong. Thus $n$ must be an even number.

Q5

(a) The proposition $P(0)$ is vacuously true because 0 is not a positive integer.

Vacuous proof has been used.

(b) $P(n) = (a+b)^n \geq a^n + b^n$ $\quad a$ & $b \in R^+$

Using direct proof method

$P(1) = (a+b)^1 \geq a^1 + b^1$ $\quad$ -(i)

$a + b = a + b$

thus equation (1) is true. Hence $P(1)$ is true.