CS 225: Switching Theory

S. Tripathy
IIT Patna

Previous Class

Switching Algebra

This Class

- Switching Algebra
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  - Switching circuit
  - Propositional calculus

Simplification of Expressions

Example 1: Simplify $T(A,B,C,D) = A'C' + ABD + BC'D + AB'D' + ABCD'$

- Apply consensus theorem $A'C' + ABD + BC'D = A'C' + ABD$
- $T = A'C' + ABD + AB'D' + ABCD'$ [place as $x=A'$, $y=C'$, $z=BD$]
- Apply distributive law: $AD'(B' + BC) \rightarrow AD'(B' + C)$

- Thus, $T = A'C' + A[BD + D'(B' + C)]$

Example 2: Simplify $T(A,B,C,D) = A'B + ABD + AB'CD' + BC$

- $T = A'B + BD + ACD'$

Canonical Forms

Deriving an expression from a truth table:

- Find the sum of all terms that correspond to combinations for which function is 1
- Each term is a product of the variables on which the function depends
- Variable x_i appears in uncomplemented (complemented) form in the product if has value 1 (0) in the combination
- Truth table for $f = x'y'z' + x'yz' + x'yz + xyz' + xyz$

Decimal code	x	y	z	f
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1

Decimal code	x	y	z	f
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Canonical Sum-of-products

Minterm: a product term that contains each of the n variables as factors in either complemented or uncomplemented form

- It assumes value 1 for exactly one combination of variables

Canonical sum-of-products: sum of all minterms derived from combinations for which function is 1

- Also called disjunctive normal expression

Compact representation of switching functions: $\sum(0,2,3,6,7)$

Decimal code	x	y	z	f
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Canonical Product-of-sums

Maxterm: a sum term that contains each of the n variables in either complemented or uncomplemented form

- It assumes value 0 for exactly one combination of variables
- Variable xi appears in uncomplemented (complemented) form in the sum if it has value 0 (1) in the combination

Canonical product-of-sums: product of all maxterms derived from combinations for which function is 0

- Also called conjunctive normal expression

Compact representation of switching functions: $\prod (1,4,5)$

$$f = (x + y + z')(x' + y + z)(x' + y + z')$$

Decimal code	x	y	z	f
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Shannon's Expansion to Obtain Canonical Forms

Shannon's expansion theorem:

$f(x1, x2, \ldots, xn) = x1 \,.\, f(1, x2, \ldots, xn) + x1' \,.\, f(0, x2, \ldots, xn)$

$f(x1, x2, \ldots, xn) = [x1 + f(0, x2, \ldots, xn)] \,.\, [\, x1' + f(1, x2, \ldots, xn)]$

Shannon's expansion around two variables:

$$f(x1, x2, \ldots, xn) = x1x2f(1, 1, x3, \ldots, xn) + x1x2'f(1, 0, x3, \ldots, xn) + x1'x2f(0, 1, x3, \ldots, xn) + x1'x2'f(0, 0, x3, \ldots, xn)$$

Similar Shannon's expansion around all n variables yields the canonical sum-of-products

Repeated expansion of the dual form yields the canonical product-of-sums

Simpler Procedure for Canonical Sum-of-products

1. Examine each term: if it is a minterm, retain it; continue to next term
2. In each product which is not a minterm: check the variables that do not occur; for each xi that does not occur, multiply the product by $(x_i + x_i')$
3. Multiply out all products and eliminate redundant terms

Example: $T(x,y,z) = x'y + z' + xyz$

$$= x'y(z + z') + (x + x')(y + y')z' + xyz$$
$$= x'yz + x'yz' + xyz' + xy'z' + x'yz' + x'y'z' + xyz$$
$$= x'yz + x'yz' + xyz' + xy'z' + x'y'z' + xyz$$

Canonical product-of-sums obtained in a dual manner

Example:

$$T = x'(y' + z)$$
$$= (x' + yy' + zz')(y' + z + xx')$$
$$= (x' + y + z)(x' + y + z')(x' + y' + z)(x' + y' + z')(x + y' + z)(x' + y' + z)$$
$$= (x' + y + z)(x' + y + z')(x' + y' + z)(x' + y' + z')(x + y' + z)$$

Thanks