CS225 Switching Theory

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Previous Class

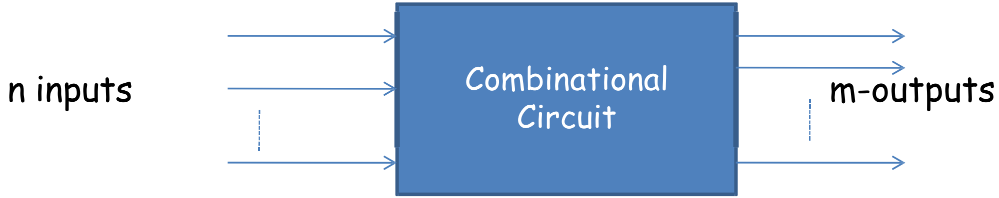
Minimization/ Simplification **of Switching Functions**

K-map (SOP)

This Class

Minimization/ Simplification **of Switching Functions**

Combinational Circuit



Design procedure:

from the specification

1. Determine the required number of inputs and outputs
2. Derive the truth table
3. Find the Simplified Boolean expression for each output as a function of the input variable
4. Draw the logic diagram
5. Verify the correctness

Simplification and Minimization

Example:

Use of cell 6 in forming both cubes justified by idempotent law

z \ xy	00	01	11	10
0	0	2	6	4
1	1	3	7	5

(a) Location of minterms in a three-variable map.

z \ xy	00	01	11	10
0		1	1	
1			1	

(b) Map for function
f(x, y, z) = $\sum(2, 6, 7) = \quad yz' + xy$

Corresponding algebraic manipulations:

$$
\begin{aligned}
f &= x'yz' + xyz' + xyz \\
&= x'yz' + xyz' + xyz' + xyz \\
&= yz'(x' + x) + xy(z' + z) \\
&= yz' + xy
\end{aligned}
$$

Simplification and Minimization of Functions Contd..

Cube: collection of 2^m cells, each adjacent to *m* cells of the collection

- Cube is said to **cover** these cells
- Cube expressed by a product of *n-m* literals for a function containing *n* variables
- *m* literals not in the product said to be **eliminated**

Example: $wxy'z' + wxy'z + wxyz' + wxyz$

$= wx\,(y'z' + y'z + yz' + yz)$

$= wx$

yz \ wx	00	01	11	10
00		1	1	1
01		1	1	
11			1	
10			1	

(d) Map for function f(w, x, y, z) = $\sum(4, 5, 8, 12, 13, 14, 15) =$

$wx + xy' + wy'z'$

Minimization (Contd.)

Minimal expression: covers all the 1 cells with the smallest number of cubes such that each cube is as large as possible

- A cube contained in a larger cube must never be selected
- If there is more than one way of covering the map with a minimal number of cubes, select the cover with larger cubes
- A cube contained in any combination of other cubes already selected in the cover is redundant by virtue of the consensus theorem

Rules for minimization:

1. First, cover those 1 cells by cubes that cannot be combined with other 1 cells; continue to 1 cells that have a single adjacent 1 cell (thus can form cubes of only two cells)
2. Next, combine 1 cells that yield cubes of four cells, but are not part of any cube of eight cells, and so on
3. Minimal expression: collection of cubes that are as large and as few in number as possible, such that each 1 cell is covered by at least one cube

Minimization (Contd.)

Example: $f(w, x, y, z) = \sum(1, 5, 6, 7, 11, 12, 13, 15)$

Only one irredundant form: $f = wxy' + wyz + w'xy + w'y'z$

- Dotted cube xz is redundant

yz \ wx	00	01	11	10
00			1	
01	1	1	1	
11		1	1	1
10		1		

Map for function

$f = wxy' + wxy + w'xy + w'y'z$

Minimal Product-of-sums

Dual procedure: product of a minimum number of sum factors, provided there is no other such product with the same number of factors and fewer literals

- Variable corresponding to a 1 (0) is complemented (uncomplemented)
- Cubes are formed of 0 cells

Example: either one of minimal sum-of-products or minimal product-of- sums can be better than the other in literal count

yz \ wx	00	01	11	10
00				
01		1		1
11				
10		1		1

(a) Map of $f(x,y,z) = \sum(5,6,9,10)$
$= w'xy'z + wx'y'z + w'xyz' + wx'yz'$

yz \ wx	00	01	11	10
00	0	0	0	0
01	0	1	0	1
11	0	0	0	0
10	0	1	0	1

(b) Map of $f(x,y,z) = \prod(0,1,2,3,4,7,8,11,12,13,14,15)$
$= (y+z)(y'+z')(w+x)(w'+x')$

Example:

Simplification of Two bit adder

Simplification of Full Adder