CS-206

ASSIGNMENT-6

- TARUSI MITTAL
- 1901CS65
- Tarusi Mittal

Tarush Mittal
1901CS65

Que 1:-

$a_n = a_n + a_{n-2} + 2n - 9$

For the next examples, we will prove $\{a_n\}$ is the solution of the given recurrence relation, by determining exact values of a_n, a_{n-1}, a_{n-2} and checking if equations holds or not for those values.

Sol) $a_n = a_{n-1} + 2a_{n-2} + 2n - 9.$

(a) $a_n = -n + 2$

$\Rightarrow$

$$a_{n-1} = -(n-1) + 2 = -n + 3$$

$$a_{n-2} = -(n-2) + 2 = -n + 4$$

$$2 \cdot a_{n-2} = 2(-n+4) = -2n + 8$$

$$R.H.S. = a_{n-1} + 2a_{n-1} + 2n - 9$$

$$= (-n+3) + (-2n+8) + 2n - 9$$

By distributive property:

$$RHS = -n + 3 - 2n + 8 + 2n - 9$$

$$= -n + 2$$

$$= a_n$$

$$= LHS.$$

Hence Proved.

(b) $a_n = 5(-1)^n - n + 2$

$$a_{n-1} = 5(-1)^{n-1} - (n-1) + 2 = -5(-1)^n - n + 3$$

$$a_{n-2} = 5(-1)^{n-2} - (n-2) + 2 = 5(-1)^n - n + 4$$

$RHS = a_{n-1} + 2a_{n-2} + 2n - 9 = (-5(-1)^n - n + 3) + 2(5(-1)^n - n + 4) + 2n - 9$

By distributive Property.

$$RHS = -5(-1)^n - n + 3 + 10(-1)^n - 2n + 8 + 2n - 9$$

$$= 5(-1)^n - n + 2$$

$$= a_n = LHS.$$

Hence Proved.

Tanusi Mittal
1901CS65

(c) $a_n = 3(-1)^n + 2^n - n + 2$

$a_{n-1} = 3(-1)^{n-1} + 2^{n-1} + (n-1) + 2 = 3(-1)^n + \frac{2^n}{2} - n + 3$

$a_{n-2} = 3(-1)^{n-2} + 2^{n-2} - (n-2) + 2 = 3(-1)^n + \frac{2^n}{4} - n + 4$

RHS: $= a_{n-1} + 2a_{n-2} + 2n - 9$

$= \left(-3(-1)^n + \frac{2^n}{2} - n + 3\right) + 2\left(3(-1)^n + \frac{2^n}{4} - n + 4\right) + 2n - 9$

By distributive property:

RHS: $= \left(-3(-1)^n + 6(-1)^n\right) + \left(\frac{2^n}{2} + \frac{2^n}{2}\right) + (-n - 2n + 2n) + (3 + 8 - 9)$

$= 3(-1)^n + 2^n - n + 2$

$= a_n$

$=$ LHS.

Hence, Proved.

(d) $a_n = 7.2^n - n + 2$

$a_{n-1} = 7.2^{n-1} - (n-1) + 2 = \frac{7.2^n}{2} - n + 3$

$a_{n-2} = 7.2^{n-2} - (n-2) + 2 = \frac{7.2^n}{4} - n + 4$

RHS $= a_{n-1} + 2a_{n-2} + 2n - 9$

$= \left(\frac{7.2^n}{2} - n + 3\right) + 2\left(\frac{7.2^n}{4} - n + 4\right) + 2n - 9$

By distributive property.

RHS $= \left(\frac{7.2^n}{2} + \frac{7.2^n}{2}\right) + (-n - 2n + 2n) + (3 + 8 - 9)$

$= 7.2^n - n + 2$

$= a_n$

$=$ LHS.

Hence Proved.

Tarusi Mittal
1901CS65
Tarusi Mittal

Que 2:-

(a) Let a_n be the number of bacteria after n hours have passed.

In every hour, bacteria triples.

$\Rightarrow$

$$\boxed{a_n = 3a_{n-1}}$$

(b)

$a_n = 3 \cdot a_{n-1}$
$a_0 = 100$ [given]

Applying recurrence relation

$$a_n = 3a_{n-1} = 3a_{n-1}$$
$$= 3(3a^{n-2}) = 3^2 a_{n-2}$$
$$= 3^2(3a^{n-3}) = 3^3 a_{n-3}$$
$$= 3^3(3a^{n-2}) = 3^4 a_{n-4}$$
$$\vdots$$
$$= 3^n a_{n-n}$$
$$= 3^n a_0$$
$$= 100 \cdot 3^n$$

Calculating at $n = 10$

$$= 100 \times 3^{10} = 5904900$$

$$= \boxed{5904900 \text{ bacteria}}$$

Tarusi Mittal
1901CS65

Que 3:-

Ans:

Big-O estimation

Big-O Notation: $f(n)$ is $O(g(n))$ if there exists constants C and k such that

$$|f(x)| \leq C|g(x)| \quad ; \quad x > k.$$

(a) $f(n) = (n \log n + n^2)(n^3 + 2)$

$\Rightarrow$ Simplifying $\rightarrow n^4 \log n + n^5 + 2n \log n + 2n^2$

Assume $g(n) = n^5$

Lets take $k = 4$. $n > 4$

$$|f(n)| = |n^4 \log n + n^5 + 2n \log n + 2n^2|$$

$$= n^4 \log n + n^5 + 2n \log n + 2n^2$$

$$\leq n^4 . n + n^5 + 2n . n + 2n^2 \qquad [\log n \leq n] \; (n \geq 0)$$

$$= n^5 + n^5 + 2n^2 + 2n^2$$

$$= 2n^5 + 4n^2$$

$$= 2n^5 + 4.1.1. n^2$$

$$< 2n^5 + n^3 . n^2$$

$$\leq 2n^5 + n^5$$

$$= 3n^5$$

$$= 3|n^5| \quad ; \quad c = 3$$

Big-O notation is $\boxed{O(n^5) \text{ with } k = 4, \; c = 3}$

Tarushi Mittal
1901CS65
Tarushi Mittal

(b) $f(n) = (n! + 2^n)(n^3 + \log(n^2+1))$

⇒ Simplifying

$= n^3 n! + n^3 2^n + n! \log(n^2+1) + 2^n \log(n^2+1)$

let $g(n) = n^3 n!$

We know;

$k > 4$; $2^n < n!$

If; $k = 2$, $x > 2$

$|f(n)| = n^3 n! + n^3 2^n + n! \log(n^2+1) + 2^n \log(n^2+1)$

$\leq n^3 n! + n^3 2^n + n!(n^2+1) + 2^n(n^2+1)$ $(\log n \leq n)$

$< n^3 n! + n! n^3 + n!(n^2+1) + n!(n^2+1)$

$= 2n^3 n! + 2n!(n^2+1)$

$< 2n^3 n! + 2n!(n^2+n^2)$

$= 2n^3 n! + 2n!(2n^2)$

$= 2n^3 n! + 4n^2 n!$

$= 2n^3 n! + 2.2.n^2 n!$

$< 2n^3 n! + 2.n.n^2 n!$

$= 2n^3 n! + 2n^3 n!$

$= 4n^3 n!$

$= 4|n^3 n!|$

$\Rightarrow C = 4$

The Big-O notation is $\boxed{O(n^3 n!) \text{ with } k=2 \text{ and } C=4}$

Taruni Mittal
1901CS65
Taruni Mittal

Que 4:-

Given:

$m := 0$
for $i := 1$ to n
  for $j := i+1$ to n
    $m := \max(a_i a_j, m)$

⇒ The only operation is that of multiplication

$\# = \max(a_i a_j, m)$

This line contains 2 operations (i) and (j)

i can take values from 1 to n → n values

j can take value from i+1 to n ~~therefore~~ j can take n-i values.

So total operations (maximum)

$n \times (n-1) \times 2 = 2n(n-1)$

$= 2n^2 - 2n \longrightarrow O(n^2)$

So the function is $\boxed{O(n^2)}$

Que 5:-

(a) Linear Search:

Here, we first check for the element in the first element, then second element and so on:

If the list contains n elements, then we will need to compare the element with n elements hence n comparisons

If the list is on 2n elements, then we will have to make 2n comparisons

⇒ Number of comparisons double.

Tarusi Mittal
1901CS65
Tarusi Mittal

(b) Binary search:

Here we divide the set into two halves and check in which half the element is present. Then it divides that part of the set into two halves again and check again in which half the element is present..

In the set contains n elements. The set is divided into $\log_2 n$ and hence comparisons made are $(\log_2 n)$

If now there are 2n elements.

$$\lceil \log_2(2n) \rceil = \lceil \log_2 2 + \log_2 n \rceil$$
$$= \lceil 1 + \log_2 n \rceil$$
$$= \log_2 n + 1$$

$$\lceil \log_2(2n) \rceil = \log_2 n + 1$$

$\Rightarrow$ Number of comparisons increases by 1

Que 6:- Big O Notation

$f(n)$ is $O(g(n))$ if $\exists c, k$, such that $|f(n)| \le c|g(n)|$ $n > k$

(a) $f(n) = n \log(n^2+1) + n^2 \log n$

For convienience, let $k = 3$; for $n > 3$ $\log(n^2+1) \le n$

$$|f(n)| = n \log(n^2+1) + n^2 \log n \le n \cdot n + n^2 \log n$$
$$\le n^2(1 + \log n) \le n^2(\log n + \log n)$$
$$\le 2n^2 \log n$$
$$\le 2|n^2 \log n|$$

$\therefore f(n)$ is $\boxed{O(n^2 \log n)}$ with $k = 3$, $c = 2$

Tanusi Mittal
1901CS65
Tanusi Mittal

(b) $f(n) = (n\log n + 1)^2 + (\log n + 1)(n^2 + 1)$

$= n^2(\log n)^2 + 2n\log n + 1 + n^2\log n + \log n + n^2 + 1$

Let $k = 3$.

$\therefore$ for $n > 3$ $((\log n)^2 > 1$, $n > 1$, $n^2 > 1$, $\log n > 1$

$|f(n)| \leq n^2(\log n)^2 + 2n\log n + n^2 + n^2\log n + \log n + n^2 + n^2$

$\leq n^2(\log n)^2 + 2n^2(\log n)^2 + n^2(\log n)^2 + n^2(\log n)^2 + n^2(\log n)^2$
$+ n^2(\log n)^2 + n^2[\log(n)]^2$

$\leq 8|n^2[\log(n)]^2|$

$\therefore$ $f(n)$ is $\boxed{O(n^2(\log n)^2)}$ with $k = 3$, $c = 8$.

(c) $f(n) = n^{2^n} + n^{n^2}$

for $n > 4$; $n^2 \leq 2^n$

$\therefore$ Let $k = 4$

$|f(n)| = |n^{2^n} + n^{n^2}|$

$= n^{2^n} + n^{n^2}$

$\leq n^{2^n} + n^{2^n}$

$\leq |2n^{2^n}|$

$\therefore$ $f(n)$ is $\boxed{O(n^{2^n})}$, with $k = 4$, $c = 2$

—x——x——x——x——x——x——x——x——x—