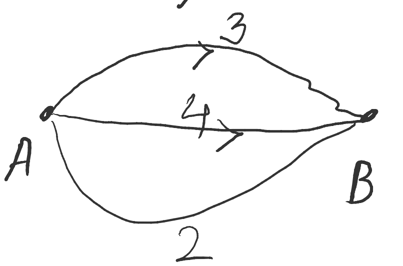
Basic Counting

Wednesday, January 6, 2021 4:51 PM

Rule of Sum

If ~~X~~ Event X can happen in x ways
and " Y can " " y ways

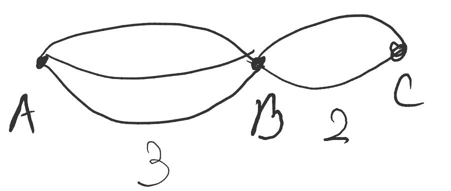
Then X or Y can happen in $x+y$ ways

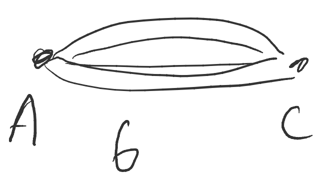


9 ways

Rule of Product

$(X \wedge Y)$ can happen in $x \cdot y$ ways





# of subsets of $|A| = n$

$A = \{a_1, a_2 \ldots a_n\}$

$\underline{\underline{2^n}}$

$(\underline{1}\ \underline{0}\ \underline{1}\ \ldots\ 0\ 0\ 0\ 0)_n$

$$\{a_1, a_3\}$$

Permutation

How many way can n distinct objects be arranged in a line?

Ans $n!$

$$n \cdot (n-1)\,(n-2) \cdots 1.$$

In circular arrangement — $(n-1)!$

Sampling / Selection

n distinct objects

we want to pick r objects

	Ordered Selection	Unordered Selection
Without repetition	$\frac{n!}{(n-r)!}$	$\binom{n}{r}$
With repetition	n^r	$\binom{n+r-1}{r}$

$\{x_1 \quad x_2 \qquad x_n\}$ — n ~~ob~~ dist obj

$\{x_1, x_2, x_2, x_{\cancel{3}}, \; x_4, x_4\}$

$$\left(\bullet \mid 0\,0\,0 \mid \underline{\quad} \mid 0\,0 \mid \quad \mid \right)$$

1st 2nd 3rd n-th

$r \longrightarrow 0\text{'s}$

$n-1 \longrightarrow 1\text{'s}$

$$\binom{n+r-1}{r}$$

Partition

A partition of a positive int n is a collection of positive ints whose sum is n.

$4 = \underline{4}$

$2+2$

$3+1$

$2+1+1$

$1+1+1+1$

No of partitions of n is denoted ~~by~~ $\pi(n)$

$$(3+1 \equiv 1+3)$$

$\pi(n,r)$ = no of partitions of n into r ~~parts~~ parts

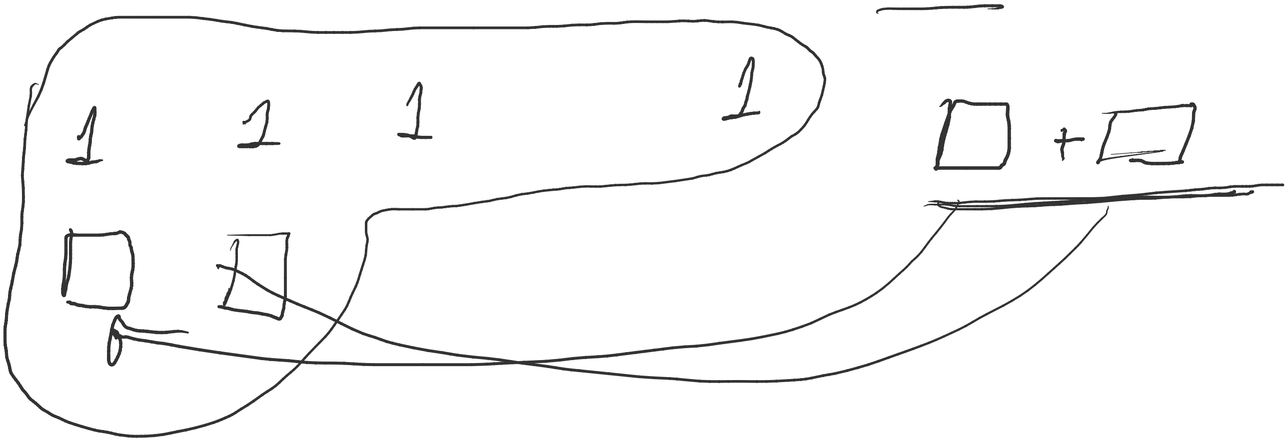
$$\pi(n) = \sum_{r=1}^{n} \pi(n,r)$$

$$\left.\begin{array}{l} x_1 + x_2 + \cdots + x_r = n \\ \text{s.t. } x_1 \geq x_2 \geq \cdots \geq x_r \geq 1 \end{array}\right\} - (1)$$

$$\boxed{\text{No of sols of } (1) = \pi(n,r)}$$

1. $$\boxed{\Pi(n,r) = \sum_{t=1}^{r} \Pi(n-r,t)}$$

$$x_1 + x_2 + x_3 \quad + x_r = (n-r)$$



2. $$\boxed{\Pi(n,r) = \Pi(n-1,r-1) + \Pi(n-r,r)}$$

Case I $\quad x_r = 1$

Case II $\quad x_r > 1 \Rightarrow x_r \geq 2$

$$\Pi\left(\underline{(n-r)}, r\right)$$

$$\left\{\begin{array}{l} x_1 + x_2 + \cdots + x_r = n \\ \underline{x_1 \geq x_2 \quad \geq x_r \geq 2} \end{array}\right\}$$

$$y_i = x_i - 1$$

$$\left\{\begin{array}{l} y_1 + y_2 + \cdots + y_r = \underline{n-r} \\ y_1 \geq y_2 \geq \cdots \geq y_r \geq 1 \end{array}\right\}$$

$$\Pi(n-r, r)$$

Refer:

Chapter 1 & 2

Counting: The Art of Enumerative

Counting: The Art of Enumerative Combinatorics

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