CS-206

ASSIGNMENT - 9

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Que 1:- Find recursive defination of multiple of 5

Ans: Let S be the set of positive integer.

for any integer 'a' to be a multiple of 5, it must be of the form,

$$\boxed{a = 5k \quad k \in S}$$

Let a be i^{th} multiple of k and let $a_1 = 5$

$$\Rightarrow a_n = 5n$$
$$= 5(n-1+1)$$
$$= 5(n-1) + 5$$
$$= a_{n-1} + 5$$

$$\rightarrow \boxed{a_n = a_{n-1} + 5 \text{ and } a_1 = 5 \quad n \in S \ \& \ n \geq 2}$$

Que 2:- To find recursive defination of $\{a_n\}$, $n = 1, 2, 3, \ldots$

(a) $a_n = 6n$

$a_1 = 6 \times 1 = 6$

$a_n = 6n = 6(n-1+1)$

$= 6(n-1) + 6$

$\boxed{a_n = a_{n-1} + 6}$

$n \in$ Integers
$n > 1$
$a_1 = 6$

(b) $a_n = 2n + 1$

$a_1 = 2 \times 1 + 1 = 3$

Writing a_n in terms of a_{n-1}

$a_n = 2n + 1$

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$= 2(n-1+1)+1$

$= (2(n-1)+1)+2$

$a_n = a_{n-1} + 2$

$\boxed{a_n = a_{n-1} + 2}$ $(n \geq 2,\ a_1 = 3)$

(c) $a_n = 10^n$

$a_1 = 10^1$

writing in terms of a_{n-1}

$a_n = 10^n$

$= 10^{(n-1+1)} = 10^{n-1} \cdot 10$

$\boxed{a_n = 10\, a_{n-1}}$ $(a_1 = 10,\ n \geq 2)$

(d) $a_n = 5$

$a_1 = 5$

$\boxed{a_n = 5}$ $(n \geq 1)$

Que 3: How many bit strings of length 10 both begin and end with 1.

Ans: Let the string be $(a_1 a_2 \ldots a_{10})$

$a_1 = a_{10} = 1$.

All other values can be either 0 or 1.

As there are two possibilities for each position

No. of bit strings $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

$\boxed{\text{So, no of bit strings } = 2^8}$

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Que 4:-

Ans:

Password length must be between 8 to 12 (inclusive)

No. of lowercase english alphabets = 26

No. of uppercase english alphabets = 26.

No of special characters = 6

No of digits = 10

⇒ Total different characters possible = 26 + 26 + 6 + 10 = 68

(a) No. of password = 68^k for a k length string

⇒ Total no of passwords = $\sum_{k=8}^{12} 68^k$

= 9,920,671,339,261,325,541,376

(b) No. of possible passwords without any special character = $\sum_{k=8}^{12} 62^k$

= 3,279,156,377,874,257,103,616.

⇒ No. of passwords with at least one special character = a − b

= 6,641,514,961,387,068,437,760

Que 5:- To find no of positive integers less than 1,000,000, that are not divisible by 4 and 6.

→ Division rule:- If a finite set A is the union of pairwise disjoint subsets with d elements each then

$$n = \frac{|A|}{d} \text{ (Round down)}$$

Now, no. of integers not divisible by 4:

Let A contain integers less than 1,000,000

$|A| = 999999$

$d = 4.$

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$n_4 = \frac{|A|}{d} = \frac{999999}{4} = 249999.75 \approx 249999$

Thus, $999999 - 249999 = 750000$ — (n_4)

No. of integers not divisible by 6.

$|A| = 999999$

$d = 6$

$n_6 = \frac{999999}{6} = 166666.5 \approx 166666$

$n_6 = 999999 - 166666 = 833333$

No. of integers not divisible by 12

$n_{12} = 999999 - \frac{999999}{12} = 916666$

No. of integers not divisible by 4 and 6.

$= n_4 + n_6 - n_{12}$

$= 666667$

$\Rightarrow$ $\boxed{666667}$ integers are less than 1000000 are not divisible by 4 and 6

Que 6:-

Ans:-

No. of women $= 15$

No. of men $= 10$

Size of team $= 6$

The team has 3 men and 3 women (equal no)

No. of ways to select 3 men $= {}^{10}C_3$

No. of ways to select 3 women $= {}^{15}C_3$

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Since, the selection process of both men and women is independent, no. of ways to form a team =

$$^{10}C_3 \times {}^{15}C_3 = \frac{10!}{7!\,3!} \times \frac{15!}{12!\,3!}$$

$$= \frac{10\times9\times8}{3\times2\times1} \times \frac{15\times14\times13}{3\times2\times1} = 54600$$

→ $\boxed{\text{Ans} = 54600}$

Que 7:-

Ans:-

No. of horses = 12

No. of ways such that given 3 horses finish in top 3 = $^{12}C_3$

No. of ways 3 horses can finish in top 3 = $3!$

∴ No. of ways that different horses possibility ends up in top 3 $= {}^{12}C_3 \times 3! = \frac{12!}{9!\,3!} \times 3!$

$= 12\times11\times10$

$= 1320$

$\boxed{\text{Ans} = 1320}$

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