Course Name: Discrete Mathematics (CS206)

Assignment No.: 3 (4 September 2020) Submission Deadline: 10th September 2020 (6 PM)

Full Marks: 25

Submission
Link:https://docs.google.com/forms/d/e/1FAIpQLSdFALNskGDYjuMswUhygR5_BXR
xTa-gxpWePOEzECtvzNLtyA/viewform?usp=sf_link

1. (a) Show that the square of an even number is an even number using a direct proof.
   (b) Prove that if m + n and n + p are even integers, where m, n, and p are integers, then m + p is even. What kind of proof did you use?
   (c) Use a direct proof to show that every odd integer is the difference of two squares.
   (2 X 2 X 2 = 6 Marks)

2. Show that these statements about the integer n are equivalent:
   (a) p1: n is even.
   (b) p2: n − 1 is odd.
   (c) p3: n^2 is even.   (2 X 2 X 2 = 6 Marks)

3. (a) Use a proof by contradiction to show that there is no rational number r for which r^3 + r + 1 = 0.
   [Hint: Assume that r = a/b is a root, where a and b are integers and a/b is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]

   (b) Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

   (2+3= 5 Marks)

4. Show that if n is an integer and n^3 + 5 is odd, then n is even using

   (a) a proof by contraposition.
   (b) a proof by contradiction.   (2 X 2 = 4 Marks)

5. (a) Prove the proposition P (0), where P (n) is the proposition "If n is a positive integer greater than 1, then n^2 > n." What kind of proof did you use?

   (b) Let P (n) be the proposition "If a and b are positive real numbers, then $(a + b)^n \geq a^n + b^n$." Prove that P (1) is true. What kind of proof did you use?   (2 X 2 = 4 Marks)