

CS 225: Switching Theory

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Previous Class

- Introduction to the course
 - Digital Design

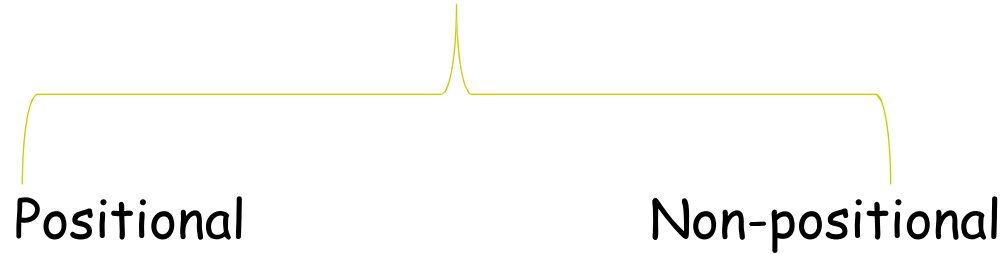
This Class

- Number Systems and Codes

What is Number System?

- Number System?
 - Study of digital systems deals with discrete information
 - Numerical quantities are discrete information
 - Discrete information are represented by a finite set of symbols
- Symbols other than numeral like letters/characters are also included

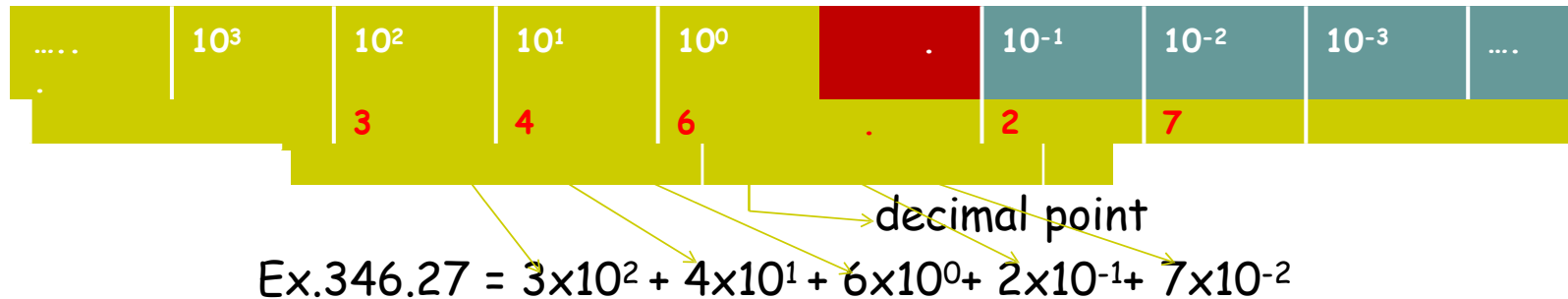
Digital Number System



- Can you find any number system with non-positional?
-
- What is the role of base?

Digital Number System

- Positional: Each position has a weight
 - Weight is unique for a particular number system
 - Ex: $346_{10} \neq 463_{10} \neq 346_8$
 - (changing the position changes the value)
- Decimal number system (base=10)



Digital Number System

- Binary Positional System (base=2)



- Ex: $(101.11)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$
- so on for octal and hexa-decimal

Representation of Integers

Base					
2	4	8	10	12	16
0000	0	0	0	0	0
0001	1	1	1	1	1
0010	2	2	2	2	2
0011	3	3	3	3	3
0100	10	4	4	4	4
0101	11	5	5	5	5
0110	12	6	6	6	6
0111	13	7	7	7	7

Base					
2	4	8	10	12	16
1000	20	10	8	8	8
1001	21	11	9	9	9
1010	22	12	10	α	A
1011	23	13	11	β	B
1100	30	14	12	10	C
1101	31	15	13	11	D
1110	32	16	14	12	E
1111	33	17	15	13	F

Conversion of Bases

- Methods:
 - Polynomial
 - Iterative (Repeated division)
 - Special type
- Polynomial conversion

Binary to Decimal:

- $D = \sum 2^i C_i$, C_i is (coefficient) digit at i^{th} position
 - $11011_2 = ?_{10}$
 - $101.011_2 = ?_{10}$

Ans.: 5.375

Conversion of Bases

CONVERT Base 8 to base 10

$$(432.2)_8 = (?)_{10} \quad 4 \cdot 8^2 + 3 \cdot 8^1 + 2 \cdot 8^0 + 2 \cdot 8^{-1} = (282.25)_{10}$$

Base 2 to base 10

$$(1101.01)_2 = (?)_{10} \quad 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} = (13.25)_{10}$$

Polynomial conversion

Decimal to Binary:

- $780_{10} = ?_2$

Complete....

$$0111 \times 1100100 + 1000 \times 1010 = 1010111100 + 1010000 = 1100001100$$

Iterative method (Repeated division)

$$(N)_{b_1} = a_{q-1}b_2^{q-1} + a_{q-2}b_2^{q-2} + \dots + a_1b_2^1 + a_0b_2^0$$

$$\frac{(N)_{b_1}}{b_2} = \underbrace{a_{q-1}b_2^{q-2} + a_{q-2}b_2^{q-3} + \dots + a_1}_{Q_0} + \frac{a_0}{b_2}$$

Algorithm: (Decimal to binary)

1. Start
2. Divide N by 2

$$\left(\frac{Q_0}{b_2}\right)_{b_1} = \underbrace{a_{q-1}b_2^{q-3} + a_{q-2}b_2^{q-4} + \dots + a_1}_{Q_1} + \frac{a_0}{b_2}$$

$$Q = N/2, \quad R = N \bmod 2$$

3. if $Q \neq 0$ Repeat step 2 with $N=Q$
4. Collect R as binary number being first R as LSB.
5. End

Ex.: $37_{10} = ?_2$

Conversion of Bases (Contd.)

Example: Convert $(548)_{10}$ to base 8

Q_i	r_i
68	$4=a_0$
8	$4=a_1$
1	$0=a_2$
	$1=a_3$

Thus, $(548)_{10} = (1044)_8$

Example: Convert $(345)_{10}$ to base 6

Q_i	r_i
57	$3=a_0$
9	$3=a_1$
1	$3=a_2$
	$1=a_3$

Thus, $(345)_{10} = (1333)_6$

Converting Fractional Numbers

Fractional number:

$$(N)_{b_1} \rightarrow a_{-1}b_2^{-1} + a_{-2}b_2^{-2} + \dots + a_{-p}b_2^{-p}$$

$$b_2 \cdot (N)_{b_1} \rightarrow a_{-1} + a_{-2}b_2^{-1} + \dots + a_{-p}b_2^{-p+1}$$

Example: Convert $(0.3125)_{10}$ to base 8

$$0.3125 * 8 = 2.5000 \text{ hence } a_{-1} = 2$$

$$0.5000 * 8 = 4.0000 \text{ hence } a_{-2} = 4$$

Thus, $(0.3125)_{10} = (0.24)_8$

Decimal to Binary

Example: Convert $(432.354)_{10}$ to binary

Q_i	r_i
216	$0=a_0$
108	$0=a_1$
54	$0=a_2$
27	$0=a_3$
13	$1=a_4$
6	$1=a_5$
3	$0=a_6$
1	$1=a_7$
	$1=a_8$

$$0.354 \cdot 2 = 0.708 \quad \text{hence } a_{-1} = 0$$

$$0.708 \cdot 2 = 1.416 \quad \text{hence } a_{-2} = 1$$

$$0.416 \cdot 2 = 0.832 \quad \text{hence } a_{-3} = 0$$

$$a_{-7} = 1$$

etc.

Thus, $(432.354)_{10} = (110110000.0101101\dots)_2$

Octal/Binary Conversion

Example: Convert $(123.4)_8$ to binary

$$(123.4)_8 = (?)_2 \quad (001\ 010\ 011.100)_2$$

Example: Convert $(1010110.0101)_2$ to octal

$$(1010110.0101)_2 = (001\ 010\ 110.010\ 100)_2 = (126.24)_8$$

Thanks