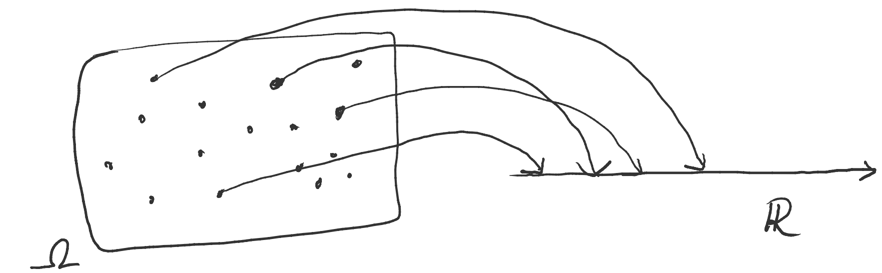
Definitions, PMF,Expectation

Friday, January 22, 2021 9:01 AM

Random Variable

X



toss a coin 3 times

$\Omega = \{HHH, (HTT, THT, TTH), TTT\}$

X = no of heads in the 3 tosses

$X \to \{0, 1, 2, 3\}$

A random variable is a real-valued fn of the exp. outcome

$$X : \Omega \longrightarrow \mathbb{R}$$

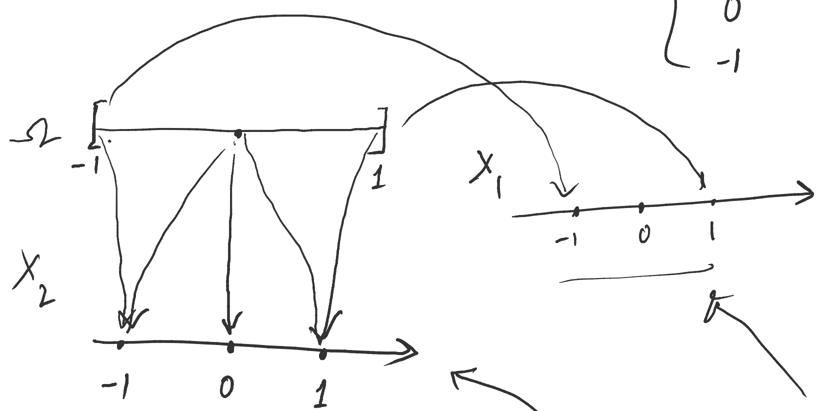
$\Omega = [-1, 1]$

$p \in$

$X : \Omega \to \mathbb{R}$

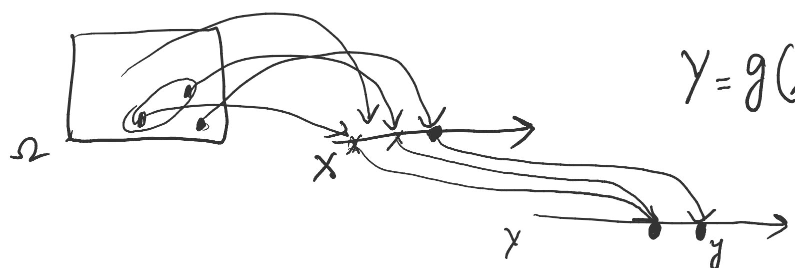
$X_1 = p$

$$X_2 = \begin{cases} 1 & \text{if } p > 0 \\ 0 & \text{if } p = 0 \\ -1 & \text{if } p < 0 \end{cases}$$

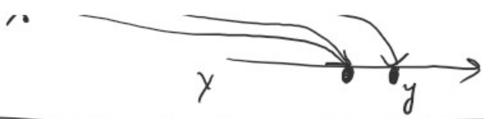


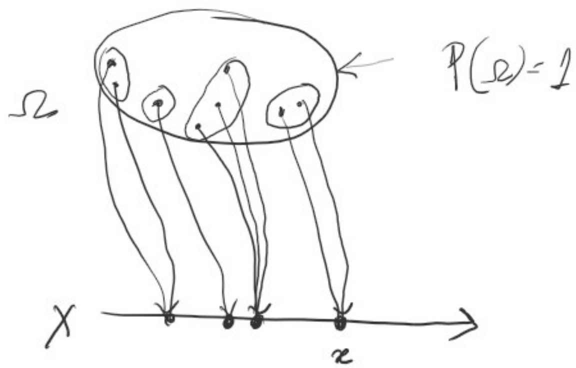
— Random variables a) discrete b) continuous

— $\Omega \longrightarrow$ many other random variables



$Y = g(X)$ is also a random variable.





$$P(X=x) = P\left(\{\omega \in \Omega \text{ s.t. } X(\omega)=x\}\right)$$

$X \longrightarrow$ random variable

$x \longrightarrow$ value

Probability Distribution of X

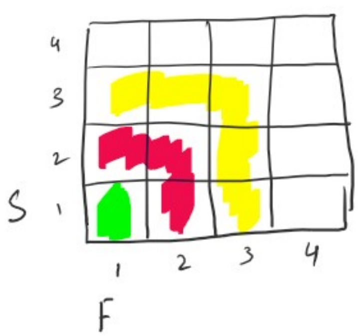
probability mass fn. (PMF) $\quad p_X(x) = P(X=x)$

$$= P\left(\underline{\{\omega \in \Omega \text{ s.t. } X(\omega)=x\}}\right)$$

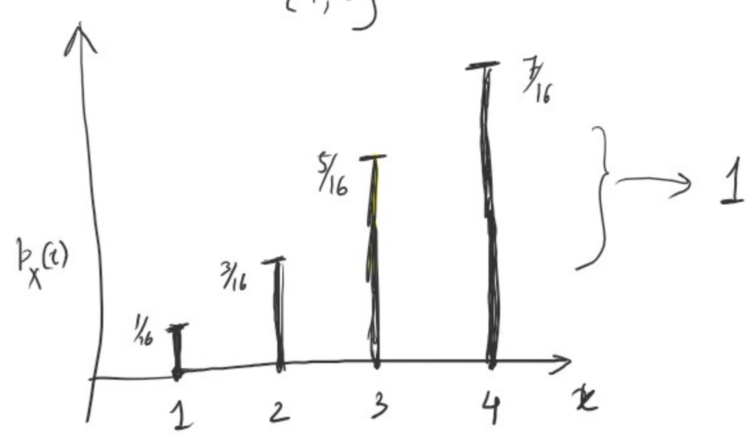
1. $p_X(x) \geq 0$

2. $\sum_x p_X(x) = 1$

Ex.



F = outcome of 1st roll
S = " " 2nd "
$X = \max\{F, S\}$

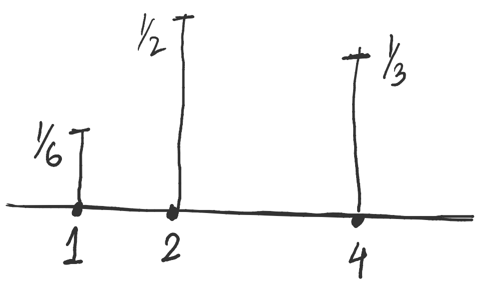


Function of X: $\quad Y = g(X) \;-\;$ r.v.

Function of X: $Y = g(X)$ – r.v.

$$\boxed{p_Y(y) = \sum_{\{x \mid g(x) = y\}} p_X(x)}$$

Expectation of X



$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{2} + 4 \cdot \frac{1}{3} = \boxed{2.5}$$

$$\sum_x x \cdot p_X(x) = E[X]$$