CS-206

MIDSEM - ASSIGNMENT

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Que 1:-

(a) Define syntax and semantics of a program.

Ans:- Syntax: Syntax refers to the rules that define the structure of a language. Syntax in computer programming means the rules that control the structure of the symbols, punctuation and words of a programming language. Syntax defines whether or not the sentence is properly constructed or not.

eg: in the programming language C++, if we are using the if-else or for loop we need to enclose them inside an curly brackets;

If the syntax of a code is not proper then the code will fail to compile.

Semantics: In programming language; semantics is the field concerned with the rigorous mathematical study of meaning of programming languages. Semantics describes the processes a computer follows when executing a program in that specific language. In short, it is about the meaning of sentence.

eg: in the programming language C++; temp-- is a semantically valid statement, where temp is a variable which means that the integer temp value is reduced by 1. It fails for any other type other that integer. Therefore semantics tell us whether the code will work or not.

(b) Answer the following with an example in each case:
How do you describe the semantics of statements in propositional and predicate logic?

Ans:- ⇒ In case of propositional logic, semantics means the truth valuation or assignment that assigns a truth value to every proposition (true/false) basically it is the truth table associated with it.

→ Negation

p	$\neg p$
T	F
F	T

eg: Negation of the statement 'Today is Monday' is 'Today is not Monday'.
Hence if P is true then $\neg P$ will be false and vice versa

→ Conjuction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

eg: If p = "Today is Monday"
q = "I will go to school today"
then $p \wedge q$ = Today is monday and I will go to school today.

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

eg: If p = "Today is monday"
q = "I will go to school today"
then $p \vee q$ = Today is monday or today I will go to school.

Exclusive OR

p	q	$p \oplus q$
T	T	T
T	F	T
F	T	T
F	F	F

eg: If p = "Today is monday"
q = " I will go to school today"
then $p \oplus q$ = either today is monday or I will go to school today but not both

Implication

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

eg:- If p = "Today is Monday"
q = "I will go to school today"
$p \rightarrow q$ = "If today is monday then I will go to school today".

ie. If p is true, then q is true, but if p is not true then q may or may not be true.

→ **Biconditional**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

eg If p = " Today is monday"
q = " I will go to school today"
$p \leftrightarrow q$ = " Today is monday if and only if I go to school today"

⇒ **Semantics of Predicate Logic**:
Predicates are proposition containing variables. The semantics of Predicate logic does two things. It assigns a meaning to the individuals, predicates and variables in the syntax. It also systematically determines the meaning of a propostion (order in which it combines)
eg let $X(a,b,c)$ denotes $a \times b \times c = 0$.
As predicate is an expression of more than one variables
A predicate with variable value assigned turns into proposition

→ **Quantifiers** (the variable of predicates)

1. **Universal quantifier**: Denoted by symbol '$\forall$' which means for all elements within its scope it is true.

$\forall x P(x)$ is read as i.e for all x $P(x)$ is equal to true.

eg: " Man is omnivorous " can be written as.

$P(x)$ = " x is omnivorous".

The propositional logic/form is $\forall x P(x)$ ⇒ for all man $P(x)$ is true

2. **Existential quantifiers**: Denoted by symbol '$\exists$' which means that statements within its scope are true for some value of the specific value.

$\exists x P(x)$ is read as there exist a x for which $P(x)$ is true.

eg: There exists some x for which $x-2 \geq 0$; domain: Real.

∴ $\exists x P(x)$ will be true if $x \geq 2$ otherwise it will be false.

Negation of quantifiers:

$$\sim \forall x P(x) = \exists x \sim P(x) \quad \text{—— (i)}$$
$$\sim \exists x P(x) = \forall x \sim P(x) \quad \text{—— (ii)}$$

eg (i) 'Not every human is indian'

$P(x)$ = x is indian $\forall x P(x)$ = Every one is indian

$\sim \forall x P(x) = \exists x \sim P(x)$

$\exists x \sim P(x)$ = " There exists some humans which are not indian.

⇒ $\sim \exists x P(x) \equiv \forall x \sim P(x)$

Hence they are equivalent.

Que 2:- To prove ; direct proof, proof by contradiction and proof by contraposition. All are equal.

Ans:- To prone that all these three are equal, we need to show that the truth table values corresponding to all these are equal.

1. Direct Proof: This is the most basic form of proof Here we assume p and prone that q follows from p. ie. $\boxed{p \rightarrow q}$

2. Proof by Contraposition: In this, we assume q is false therefore p is false ie we prove the contraposition of the statement $p \rightarrow q$

ie. $\boxed{\sim q \rightarrow \sim p}$

3. Proof by contradiction: In this we start by saying that p is false ie $-p$ is true; and then when it leads to the contradiction of $q \wedge q$. we say assumption was wrong and show $-p \rightarrow (q \wedge -q)$ and hence says this is always false.

So to prove by contradiction we show

$(p \wedge \sim q) \rightarrow$ false.

		Direct proof			Contraposition			Contradiction
p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$\sim q \rightarrow -p$	$p \wedge \sim q$	false	$(p \wedge \sim q) \rightarrow$ false
T	T	T	F	F	T	F	F	T
T	F	F	F	T	F	T	F	F
F	T	T	T	F	T	F	F	T
F	F	T	T	T	T	F	F	T

As all are same: So these are logically equivalent.

Que 3:- To prove equivalence classes of R form a partition in A. non empty set A where R is an equivalence relation.

Ans:- **Method:** Proof By Contradiction

Solution:

Let us assume that distinct set of equivalence classes of R doesnot form a partition in A.

ie. if $A_1, A_2, A_3, \ldots, A_n$ denotes the equivalence classes of R on A, then.

(a) $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n \neq A$

(b) $A_i \cap A_j \neq 0$ where $i \neq j$.

(a) By the defination of union, we can say that if.

$x \in A_1 \cup A_2 \cup \ldots \cup A_n$ then x belongs to at least one equivalence class A_i

Also by the defination of equivalence classes.

$x \in A \longrightarrow ①$

Thus $A_1 \cup A_2 \cup \ldots \cup A_n \subseteq A \longrightarrow ②$.

Now, $x \in A$, then $x R x$ since R is reflexive.

$\Rightarrow x = [x]$

$[x] = A_i$, since $[x]$ is an equivalence class of R.

$\Rightarrow A \subseteq A_1 \cup A_2 \cup \ldots \cup A_n$ —— ③

By point ② and ③.

$A = A_1 \cup A_2 \cup \ldots \cup A_n$.

$\Rightarrow$ The assumption (a) is wrong and hence it implies (a) is not true for the statement (R is any equivalence relation on any non empty set A.

(b) First proving one result for further use.

$$a R b \rightarrow [a] = [b]$$

Given: R is an equivalence class on A.

Let $x \in [a]$, then $x R a$ by defination of equivalence class.

Now, we have $a R b$ and $x R a$ thus, by transitivity $x R b$;

Since $x R b$, $x \in [b]$ and now if $x \in [a]$, then. $x \in [b]$ thus $[a] \subseteq [b]$ —— ①

Next, let $x \in [b]$, then $x R b$ by defination of equivalence class.

Since $a R b$, we also have $b R a$ (symmetry)

Hence $x R b$ and $b R a$ thus $x R a$ (transistivity)

Since $x R a$; $x \in [a]$ by defination of equivalence class.

and now if $x \in [b]$, then $x \in [a]$;

thus $[b] \subseteq [a]$ —— ②.

From ① and ②

$$[a] = [b].$$

Hence Proved

⇒ <u>Now</u>;

Since $A_i \cap A_j \neq \phi$ $(i \neq j)$

let us take an element which is in both A_i and A_j

By defination of equivalence class:

$$A_i = [a]$$
$$A_j = [b]$$

then mRa and mRb (defination of equivalence class)

By symmetry: aRm and mRa

By transitivity: aRm and mRb

aRb

$\therefore [a] = [b]$

$\Rightarrow \quad A_i \cap A_j = \phi$

This implies our assumption (b) was wrong.

$\Rightarrow$ Both (a) and (b) contradicts with our assumption made at first.

So by contradiction we proved that if R is an equivalence relation of A, then the equivalence classes R forms a partition of A.

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