CS 225: Switching Theory

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Previous Class

- Number Systems and Codes
  - Different Number systems (positional)
  - Conversion

This Class

- Number Systems and Codes
  - Binary Arithmetic
  - Codes
    - BCD, cyclic code etc.
    - Gray code
    - Parity and Error correcting code

Number Systems

Representing number N in base b: $(N)_b$

Decimal Number: $123.45=1\cdot10^2 + 2\cdot10^1 + 3\cdot10^0 + 4\cdot10^{-1} + 5\cdot10^{-2}$

Base b number: $N = a_{q-1}b^{q-1} + \cdots + a_0 b_0 + a_{-1}b^{-1} \cdots + a_{-p}b^{-p}$

$b > 1, 0 <= a_i <= b-1$

Integer part: $a_{q-1}a_{q-2} \cdots a_0$ Fractional part: $a_{-1}a_{-2} \cdots a_{-p}$

Most significant digit: a_{q-1} Least significant digit: a_{-p}

Binary number (b=2): $1101.01 = 1\cdot2^3 + 1\cdot2^2 + 0\cdot2^1 + 1\cdot2^0 + 0\cdot2^{-1} + 1\cdot2^{-2}$

Complement of digit a: $a' = (b-1)-a$

Decimal system: 9's complement of 3 = 9-3 = 6

Binary system: 1's complement of 1 = 1-1 = 0

Iterative method (Repeated division)

$$(N)_{b1} = a_{q-1}b_2^{q-1} + a_{q-2}b_2^{q-2} + \cdots + a_1 b_2^1 + a_0 b_2^0$$

$$\frac{(N)_{b1}}{b_2} = \underbrace{a_{q-1}b_2^{q-2} + a_{q-2}b_2^{q-3} + \cdots + a_1}_{Q_0} + \frac{a_0}{b_2}$$

$$\left(\frac{Q_0}{b_2}\right)_{b_1} = \underbrace{a_{q-1}b_2^{q-3} + a_{q-2}b_2^{q-4} + \cdots}_{Q_1} + \frac{a_1}{b_2}$$

Algorithm: (Decimal to binary)

1. Start
2. Divide N by 2

   Q=N/2, R= N mod 2
3. if Q ≠ 0 Repeat step 2 with N=Q
4. Collect R as binary number being first R as LSB.
5. End

Conversion (Octal)

- Octal Numbers conversions
- Decimal-to-Octal conversion
  - Repeated division by 8

- Binary-to-Octal conversion

1. Break the binary number into 3-bit groups
2. Replace each group with an octal equivalent

- Octal-to-decimal conversion

1. Convert the octal to groups of 3-bit binary
2. Convert the binary to decimal

Conversion (Hexadecimal)

- Hexadecimal Numbers conversions
  - Repeated division by 16

- Binary-to-hexadecimal conversion
  1. Break the binary number into 4-bit groups
  2. Replace each group with the hexadecimal equivalent
- Hexadecimal-to-decimal conversion
  1. Convert the hexadecimal to groups of 4-bit binary
  2. Convert the binary to decimal
- Decimal-to-hexadecimal conversion

Ex.:

- Conversion

Q1. $(41.6875)_{10} = (?)_2$

(101001.1011)2

Q2. $(153.513)_{10} = (?)_8$

$(231.406517)_8$

Binary Arithmetic

Bits		Sum	Carry	Difference	Borrow	Product
a	b	a+b		a-b		a · b
0	0	0	0	0	0	0
0	1	1	0	1	1	0
1	0	1	0	1	0	0
1	1	0	1	0	0	1

Binary Addition/Subtraction

Example: Binary addition

$$
\begin{array}{rl}
1111 & = \text{carries of } 1 \\
1111.01 & = (15.25)_{10} \\
+\ \underline{0111.10} & = (\ 7.50)_{10} \\
10110.11 & = (22.75)_{10}
\end{array}
$$

Example: Binary subtraction

$$
\begin{array}{rl}
1 & = \text{borrows of } 1 \\
10010.11 & = (18.75)_{10} \\
-\ \underline{01100.10} & = (12.50)_{10} \\
00110.01 & = (\ 6.25)_{10}
\end{array}
$$

Answer the following

Q3. $(1001.1)_2 + (010.1)_2$ =? Show Carries and Borrows

Q4. $(100.01)_2 - (010.1)_2$ =?

$9.5+2.5 = 12.0 = (1100.0)_2$

$4.25-2.5 = 1.75 = (01.11)_2$

Binary Multiplication/Division

Example: Binary Multiplication

$$
\begin{array}{r}
11001.1 = (25.5)_{10} \\
\underline{110.1} = (6.5)_{10} \\
110011 \\
000000 \\
110011 \\
\underline{110011} \\
10100101.11 = (165.75)_{10}
\end{array}
$$

Binary Multiplication/Division

Example: Binary Division

```
             10110 = quotient
       ___________
11001 | 1000100110
         11001
        ----------
        00100101
            11001
         ---------
           0011001
             11001
         ---------
             00000   remainder
```

Signed Numbers Representation

- Three main different ways
- Sign and magnitude
- r-1's complement
- r 's complement

Signed binary number

Positive numbers can be defined with Sign bit 0

- Ex. In 8-bit representation of +9 = 00001001

- Negative numbers can be represented in three different ways:
  - Signed magnitude: -9 = 10001001
  - Signed 1's complement: -9 = 11110110
  - Signed 2's complement: -9 = 11110111

- Undesired aspect in signed binary and 1 s complement method:
- representation of 0s:

  +0 has different code from -0

Radix Complements (r-1's complement)

- r-1's Complements:
  - Diminished Radix (r-1)'s Complement
    - (r-1)'s complement of a number N with n digits is $(r^n - 1) - N$.

Ex.:9's complement of 346 is 999-346=653 (10^3 -1= 999)

  - 1's complement of a binary number can be determined as just replacing 1's with 0's and vice- versa..

Radix Complements (r’s complement)

- Radix complement:
  - r’s complement of a number N with n digits is $r^n - N = (r^n - 1) - N + 1$.

Ex. 10’s complement of 346 is = 654 (653+1)

2’s complement of 1011= 0101

NB: complement of complement of N is N $r^n - (r^n - N) = N$

Thanks