Fair Canonical Correlation Analysis

Zhuoping Zhou[1], **Davoud Atee Tarzanagh**[1], **Bojian Hou**[1]
Boning Tong, Jia Xu, Yanbo Feng, Qi Long[2], **Li Shen**[2]

University of Pennsylvania

DEPARTMENT of BIOSTATISTICS EPIDEMIOLOGY & INFORMATICS

Perelman SCHOOL OF MEDICINE UNIVERSITY of PENNSYLVANIA

Institute for Biomedical Informatics

NEURAL INFORMATION PROCESSING SYSTEMS

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[1] Equal contribution
[2] Corresponding authors

Canonical Correlation Analysis (CCA)

Given datasets $\mathbf{X} \in \mathbb{R}^{N \times D_x}$ and $\mathbf{Y} \in \mathbb{R}^{N \times D_y}$, CCA seeks the R–dimensional subspaces where the projections of $\mathbf{X}$ and $\mathbf{Y}$ are maximally correlated.

Optimization Problem: Find $\mathbf{U} \in \mathbb{R}^{D_x \times R}$ and $\mathbf{V} \in \mathbb{R}^{D_y \times R}$ such that

$$\begin{aligned} &\text{maximize} && \operatorname{trace}(\mathbf{U}^\top \mathbf{X}^\top \mathbf{Y} \mathbf{V}) \\ &\text{subject to} && \mathbf{U}^\top \mathbf{X}^\top \mathbf{X} \mathbf{U} = \mathbf{V}^\top \mathbf{Y}^\top \mathbf{Y} \mathbf{V} = \mathbf{I}_R. \end{aligned} \tag{CCA}$$

Applications:

- Economics, Psychology, Biology, Neuroscience, ...

Limitation:

- CCA can exhibit unfair/biased behavior analyzing data with protected attributes.

Disparity Error for Multiple Groups

Given $\left\{\left(\mathbf{X}^k, \mathbf{Y}^k\right)\right\}_{k=1}^K$, for each group $k \in \{1, \ldots, K\}$:

- **Group-Wise CCA:**

  Find canonical weights $\mathbf{U}^{k,\star} \in \mathbb{R}^{D_x \times R}$ and $\mathbf{V}^{k,\star} \in \mathbb{R}^{D_y \times R}$ as the solutions to (CCA) for the datasets $\mathbf{X}^k \in \mathbb{R}^{N_k \times D_x}$ and $\mathbf{Y}^k \in \mathbb{R}^{N_k \times D_y}$.

- **Correlation Disparity Error:**

$$\mathcal{E}^k\left(\mathbf{U}, \mathbf{V}\right) := \operatorname{trace}\left(\mathbf{U}^{k,\star\top}\mathbf{X}^{k\top}\mathbf{Y}^k\mathbf{V}^{k,\star}\right) - \operatorname{trace}\left(\mathbf{U}^\top\mathbf{X}^{k\top}\mathbf{Y}^k\mathbf{V}\right),$$

- **Pairwise Correlation Disparity Error:**

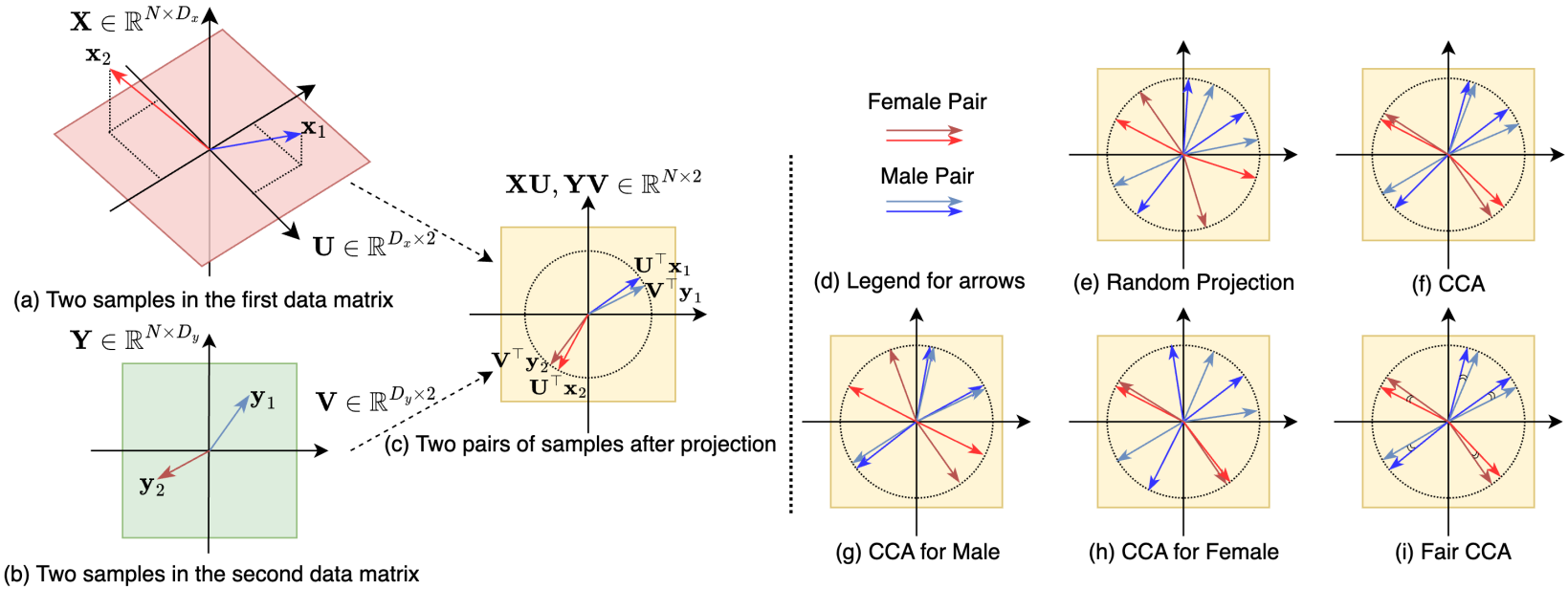
$$\Delta^{k,s}\left(\mathbf{U}, \mathbf{V}\right) := \phi\left(\mathcal{E}^k\left(\mathbf{U}, \mathbf{V}\right) - \mathcal{E}^s\left(\mathbf{U}, \mathbf{V}\right)\right), \quad \forall k \neq s, \quad s \in [K].$$

Here, $\phi : \mathbb{R} \to \mathbb{R}_+$ is a penalty function such as $\phi(x) = \exp(x)$ or $\phi(x) = x^2$.

Fair CCA

Fair CCA aims to discover linear transformations $\mathbf{U}$ and $\mathbf{V}$ which project $\mathbf{X}$ and $\mathbf{Y}$ to a *R*–dimensional subspace where

- $\mathbf{XU}$ and $\mathbf{YV}$ are maximally correlated,
- pairwise correlation disparity errors are minimized.



Fair CCA Optimization

- Goal 1: Maximize the correlation
- Goal 2: Minimize the pairwise correlation disparity error

Multi-Objective Fair CCA (MF-CCA):

$$\begin{aligned} \text{minimize } & \mathbf{F}(\mathbf{U},\mathbf{V}) := \left[-\text{trace}\left(\mathbf{U}^\top\mathbf{X}^\top\mathbf{Y}\mathbf{V}\right), \Delta^{1,2}(\mathbf{U},\mathbf{V}), \ldots, \Delta^{K-1,K}(\mathbf{U},\mathbf{V})\right] \\ \text{subj. to } & \mathbf{U}^\top\mathbf{X}^\top\mathbf{X}\mathbf{U} = \mathbf{V}^\top\mathbf{Y}^\top\mathbf{Y}\mathbf{V} = \mathbf{I}_R. \end{aligned}$$

Single-Objective Fair CCA (SF-CCA):

$$\begin{aligned} \text{minimize } & f(\mathbf{U},\mathbf{V}) := -\text{trace}\left(\mathbf{U}^\top\mathbf{X}^\top\mathbf{Y}\mathbf{V}\right) + \lambda \sum_{k\neq s,\ k,s\in[K]} \Delta^{k,s}(\mathbf{U},\mathbf{V}) \\ \text{subj. to } & \mathbf{U}^\top\mathbf{X}^\top\mathbf{X}\mathbf{U} = \mathbf{V}^\top\mathbf{Y}^\top\mathbf{Y}\mathbf{V} = \mathbf{I}_R. \end{aligned}$$

- Requires a tuning parameter $\lambda > 0$.

Datasets

The datasets used in experiments include synthetic data and real data which is obtained from the fields of health and education.

Database	Modalities	Sensitive Attribute
Synthetic Data	$\mathbf{X}$ and $\mathbf{Y}$ are generated using a Gaussian distribution	5 groups
National Health and Nutrition Examination Survey (NHANES)	$\mathbf{X}$: Phenotypic Measure $\mathbf{Y}$: Environmental Measure	Education (3 groups)
Mental Health and Academic Performance Survey (MHAAPS)	$\mathbf{X}$: Psychological Performance $\mathbf{Y}$: Academical Performance	Sex (2 groups)
Alzheimer's Disease Neuroimaging Initiative (ADNI)	$\mathbf{X}$: Amyloid PET Scan $\mathbf{Y}$: Tau PET Scan	Sex (2 groups)

Table 1: Dataset Descriptions

Evaluation Criteria

For each projection dimension $r \in \{1, \ldots, R\}$:

- **Correlation:**

$$\rho_r = \frac{\mathbf{u}_r^\top \mathbf{X}^\top \mathbf{Y} \mathbf{v}_r}{\sqrt{\mathbf{u}_r^\top \mathbf{X}^\top \mathbf{X} \mathbf{u}_r \mathbf{v}_r^\top \mathbf{Y}^\top \mathbf{Y} \mathbf{v}_r}},$$

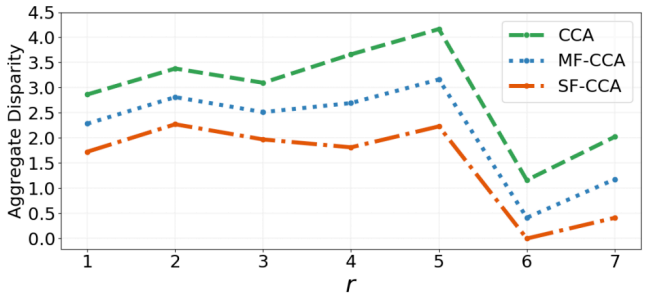
- **Maximum Gap of Correlation Disparity Error:**

$$\Delta_{\text{max},r} = \max_{i,j \in [K]} |\mathcal{E}^i(\mathbf{u}_r, \mathbf{v}_r) - \mathcal{E}^j(\mathbf{u}_r, \mathbf{v}_r)|,$$

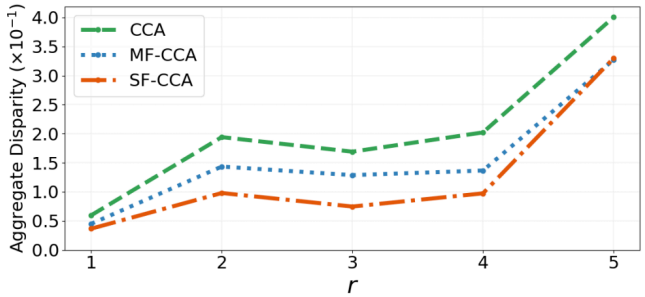
- **Aggregate Correlation Disparity Error:**

$$\Delta_{\text{sum},r} = \sum_{i,j \in [K]} |\mathcal{E}^i(\mathbf{u}_r, \mathbf{v}_r) - \mathcal{E}^j(\mathbf{u}_r, \mathbf{v}_r)|.$$

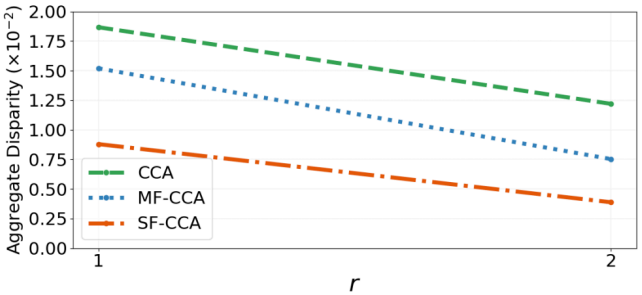
Aggregate Disparity Performance



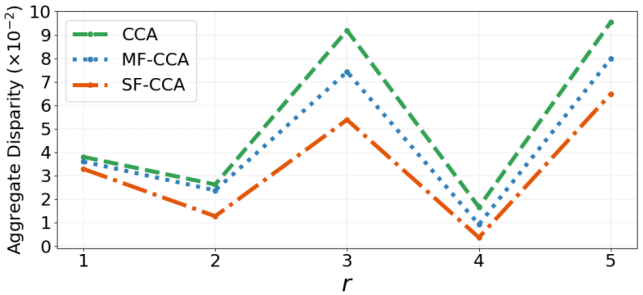
(a) Synthetic Data



(b) NHANES



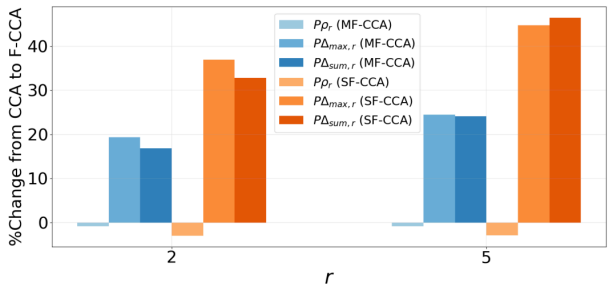
(c) MHAAPS



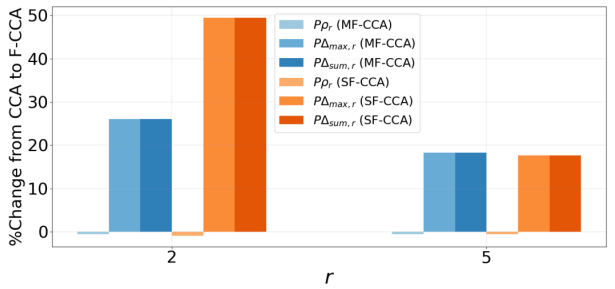
(d) ADNI

- SF-CCA outperforms MF-CCA in terms of aggregate disparity error $\Delta_{\text{sum},r}$.
- Fair CCA outperforms CCA across all datasets on each projection dimension.

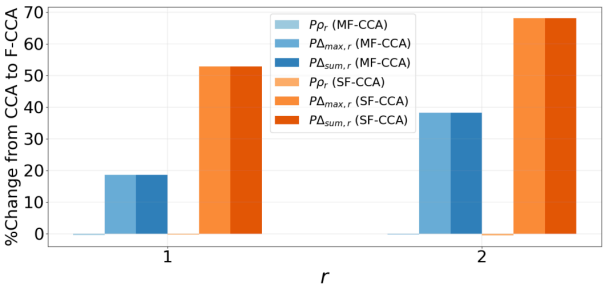
Percentage Changes Performance



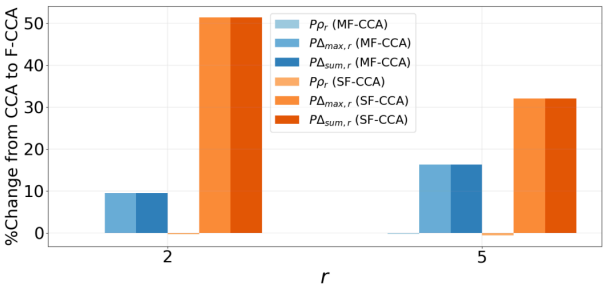
(a) Synthetic Data



(b) NHANES



(c) MHAAPS



(d) ADNI

- Percentage changes of ρ_r (P_{ρ_r}) are slight.
- Percentage changes of $\Delta_{\max,r}$ and $\Delta_{\text{sum},r}$ ($P_{\Delta_{\max,r}}$ and $P_{\Delta_{\sum,r}}$) are substantial.
- Fairness improvement is signified without significant correlation sacrifice.

Thank You!

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- ADNI data were obtained from `https://adni.loni.usc.edu`, funded by NIH U01 AG024904.

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