Probability Formulas

Poisson Distribution

For a random variable X representing the number of events occurring in a fixed interval, the probability mass function is:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

where λ is the average rate of occurrence.

Tchebysheff's Theorem

For any random variable X with mean μ and variance σ^2, the probability that X deviates from μ by at least k standard deviations is:

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}, \quad k > 1$$

Alternatively:

$$P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

Probability Distribution for a Continuous Random Variable

A continuous random variable X has a probability density function $f(x)$ such that:

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

where $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Uniform Probability Distribution

For a continuous random variable X uniformly distributed over $[a, b]$, the probability density function is:

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

The mean and variance are:

$$E(X) = \frac{a+b}{2}, \quad \mathrm{Var}(X) = \frac{(b-a)^2}{12}$$

Exponential Distribution

For a continuous random variable X with rate parameter $\lambda > 0$, the probability density function is:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The cumulative distribution function is:

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The mean and variance are:

$$E(X) = \frac{1}{\lambda}, \quad \mathrm{Var}(X) = \frac{1}{\lambda^2}$$

Bivariate Distribution

For two continuous random variables X and Y, the joint probability density function is $f(x, y)$, satisfying:

$$P((X, Y) \in A) = \iint_A f(x, y)\, dx\, dy$$

where $f(x, y) \geq 0$ and $\iint_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$.

Continuous Random Variables with Joint Distribution Function

The joint cumulative distribution function for continuous random variables X and Y is:

$$F(x, y) = P(X \leq x, Y \leq y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u, v)\, dv\, du$$

The joint density function is:

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$$

Marginal Probability and Density Functions

For a joint density function $f(x, y)$, the marginal density functions are:

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy, \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$$

The marginal cumulative distribution functions are:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(u)\, du, \quad F_Y(y) = P(Y \leq y) = \int_{-\infty}^{y} f_Y(v)\, dv$$

Independent Random Variables

Two random variables X and Y are independent if and only if:

$$f(x, y) = f_X(x) \cdot f_Y(y)$$

or equivalently:

$$F(x, y) = F_X(x) \cdot F_Y(y)$$

for all x, y.