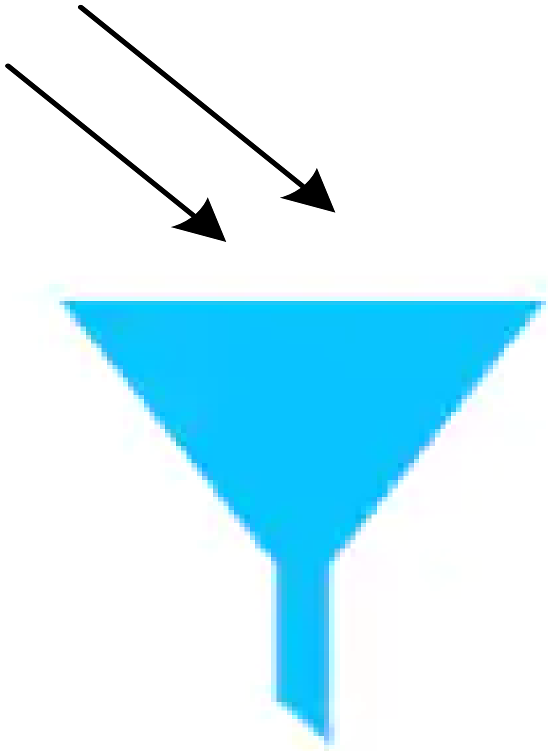
Introduction to Analogue Filter



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Separation of substances > Methods of separation II
Filtration

Filter

An electric filter is often a frequency selective circuit that passes a specified band frequencies and blocks or attenuates signals of frequencies outside this band.

Applications of Filter

The applications include:

- Filter Circuits are used to eliminate background Noise
- They are used in Radio tuning to a specific frequency
- Used in Pre-amplification, Equalization, Tone Control in Audio Systems
- They are also used in Signal Processing Circuits and Data Conversion
- Filter Circuits are extensively used in Medical Electronic Systems

Filter

An electric filter is often a frequency selective circuit that passes a specified band frequencies and blocks or attenuates signals of frequencies outside this band.

Classification of Filter

Filters may be classified in a number of ways:

- Analog or digital
- Passive or active
- Audio (AF) or radio frequency (RF)

Analog Filter:
Analog filters are designed to process analog signals.

Digital Filter:
Digital filters process analog signals using digital techniques.

Passive Filter:
Elements used in passive filters are resistors, capacitors, and inductors.

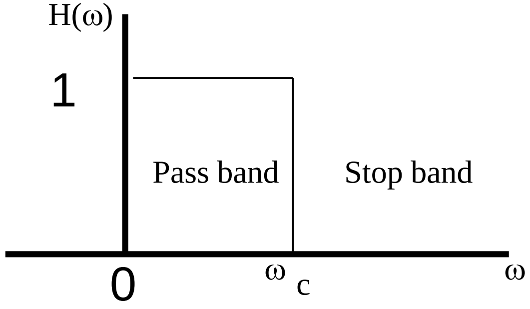
Active Filter:
This kind of filter employs transistors or OP-AMP in addition to resistors and capacitors. Inductors are not often used in active filters, because they are bulky and costly and may have large internal resistive components.

Types of filter

- Low pass filter
- High pass filter
- Band pass filter
- Band reject filter

Frequency Response of Ideal Filters

Low pass Filter



High pass Filter

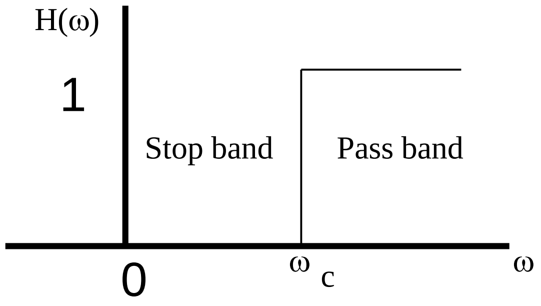
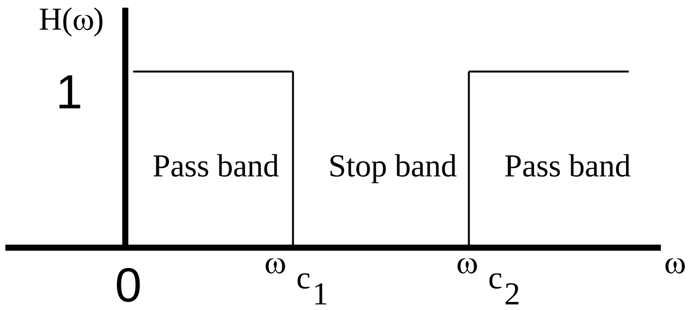


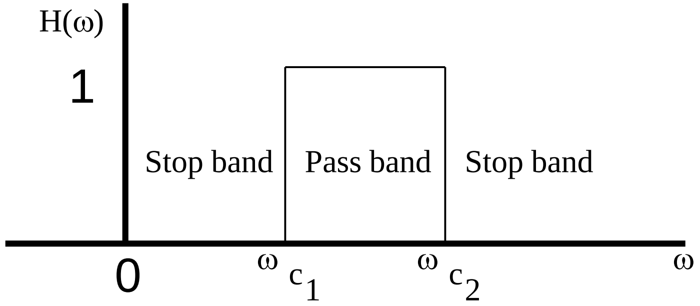
TABLE 14.5 Summary of the characteristics of filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

Band stop Filter



Band pass Filter



Ideal Frequency Response of four types of filter.

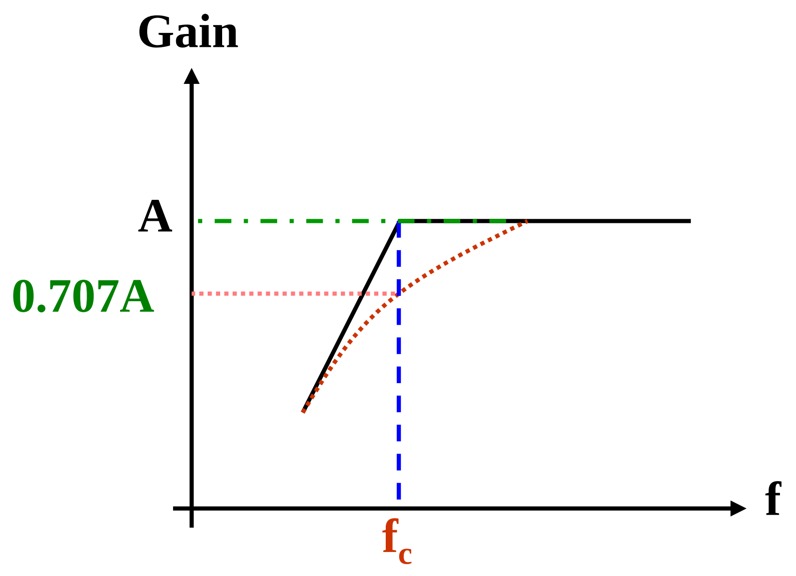
Cut off frequency

- Cutoff frequency is the frequency at which the output power is half of the input power.
- Or, at the frequency where the gain is down by 3 dB is called cut off frequency or 3dB frequency or 0.707 frequency or corner frequency or the break frequency. It is denoted by f_c

$$dB = 10\log_{10}\left(\frac{P_2}{P_1}\right)$$

At the cut off frequency,

$$dB_{cut-off} = 10\log_{10}\left(\frac{1}{2}\right) = -3$$



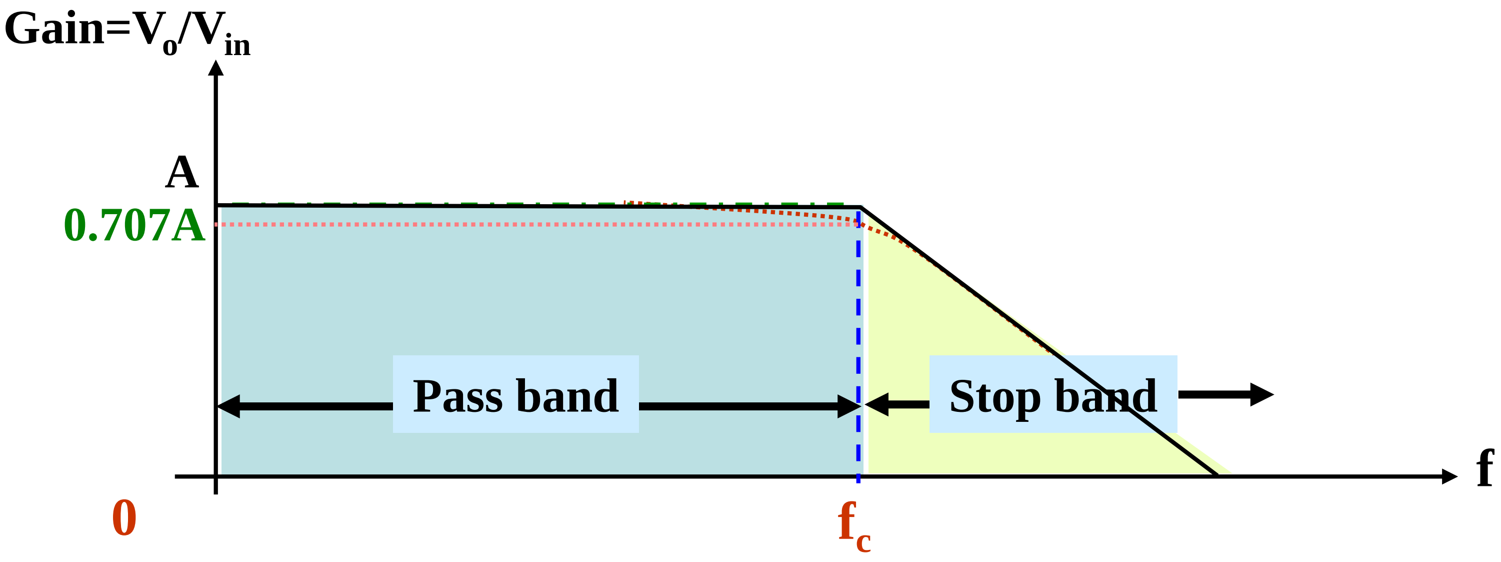
Low pass filter

- A low pass filter has a constant gain from 0Hz to a high cut off frequency (f_c). So that the band width also f_c.

- At f_c, gain is down by -3dB. After that ($f > f_H$) it decreases with the increase in input frequency.

  $0 \rightarrow f_c$ is pass band

  $f > f_c$ is stop band



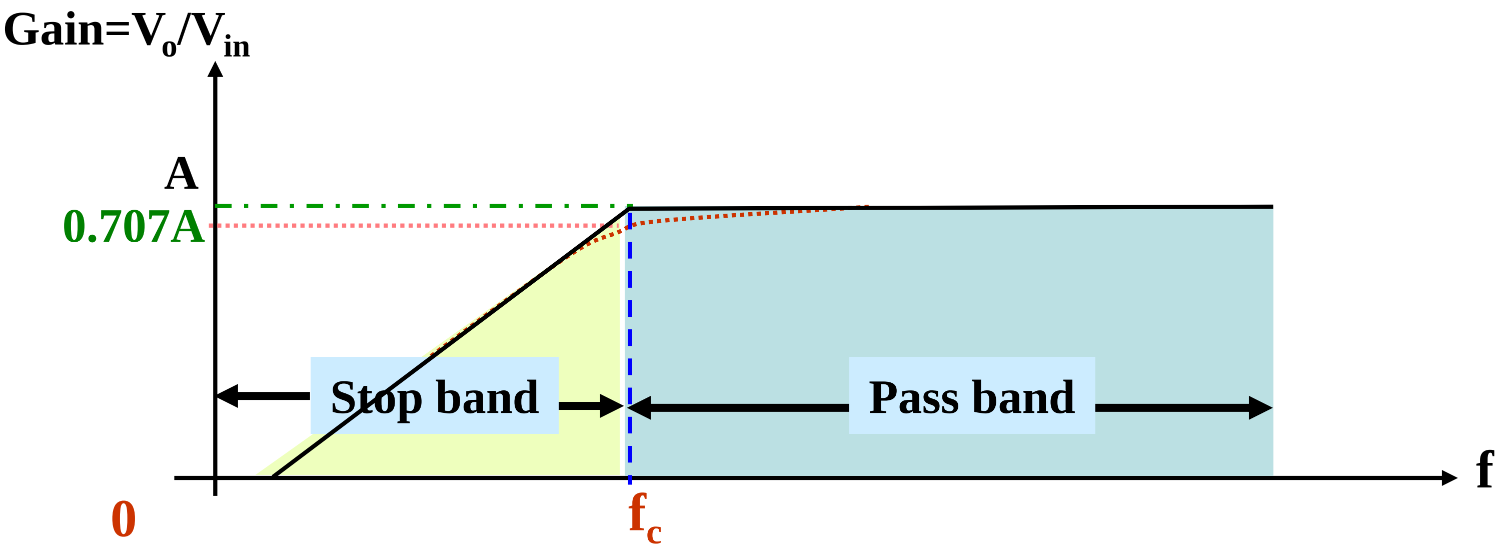
High pass filter

- This type of filter attenuates the output voltage for all frequency. Above f_c, the gain is constant.

- At f_c, gain is down by -3dB. After that ($f > f_H$) it seems to be constant with the increase of input frequency.

$0 \rightarrow f_c$ is stop band

$f > f_c$ is pass band



Band pass filter

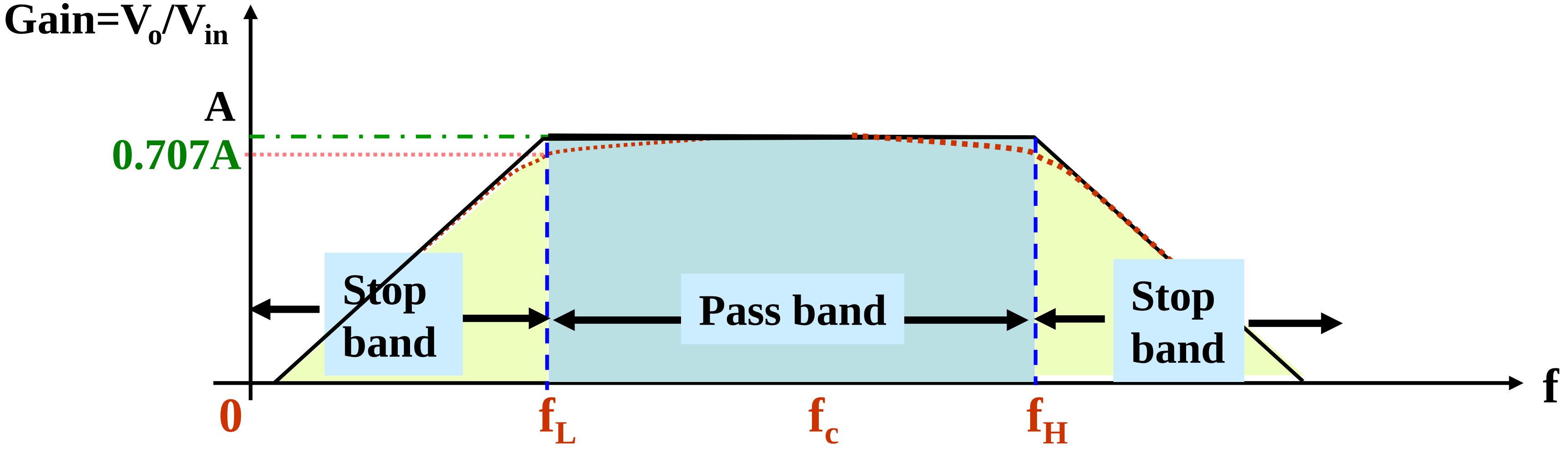
- Band pass filter passes only a band of frequencies while attenuating all frequencies outside the band.
- It has two cut off frequencies f_L and f_H.

Band width = $f_H - f_L$

f_L to f_H is pass band

$f < f_L$ and $f > f_H$ is stop band

f_c is called center frequency



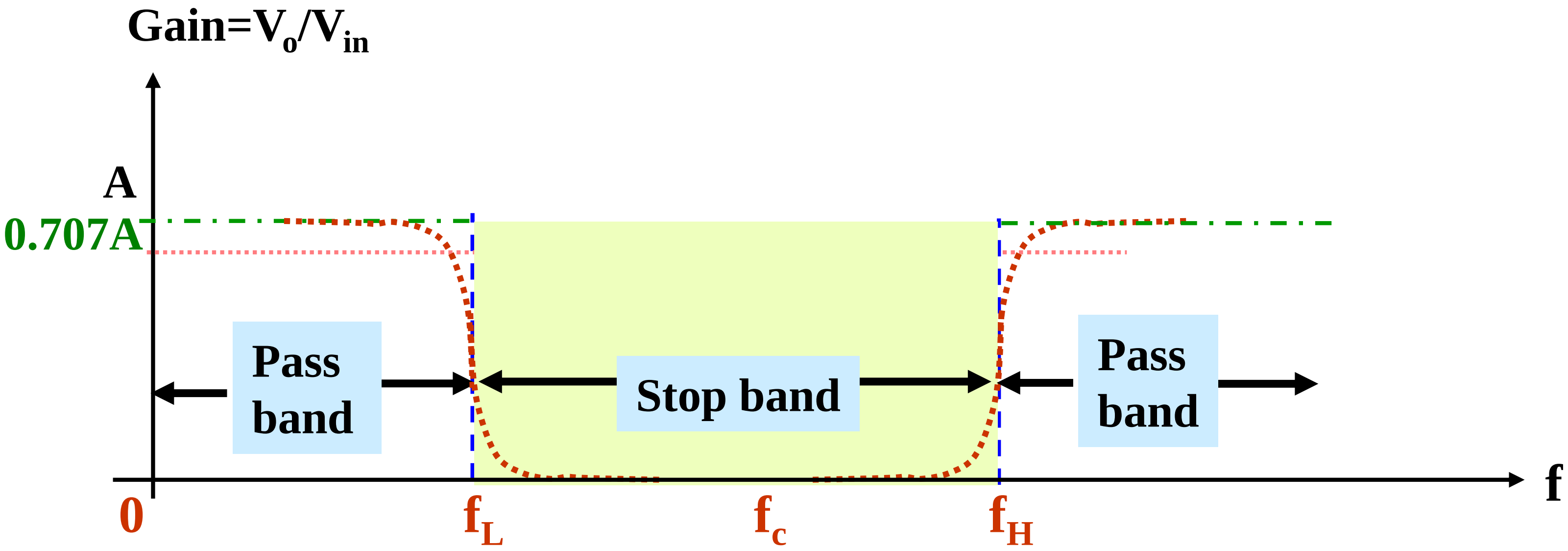
Band reject filter

- Band reject filters reject a specified band of frequencies while passing all the frequencies outside the band.
- It has two cut off frequencies f_L and $f_{H.}$

Band width = f_H-f_L f_L to f_H is stop band

$f<f_L$ and $f>f_H$ is pass band, and f_c is called center frequency

It is also called band elimination filter or Notch filter.



RC Low Pass Filter

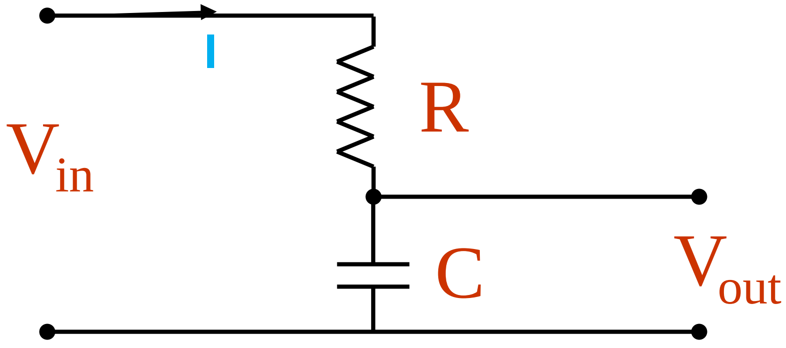


Fig. Circuit diagram.

- The voltage across the capacitor is $V_C = IX_C = I/\omega C$.
- Impedance, $Z_{RC} = (R^2 + (1/\omega C)^2)^{1/2}$
- The voltage across the series combination is $IZ_{RC} = I(R^2 + (1/\omega C)^2)^{1/2}$

So, the gain is

$$Gain = \frac{V_{out}}{V_{in}} = \frac{V_C}{V_{series}} = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$$

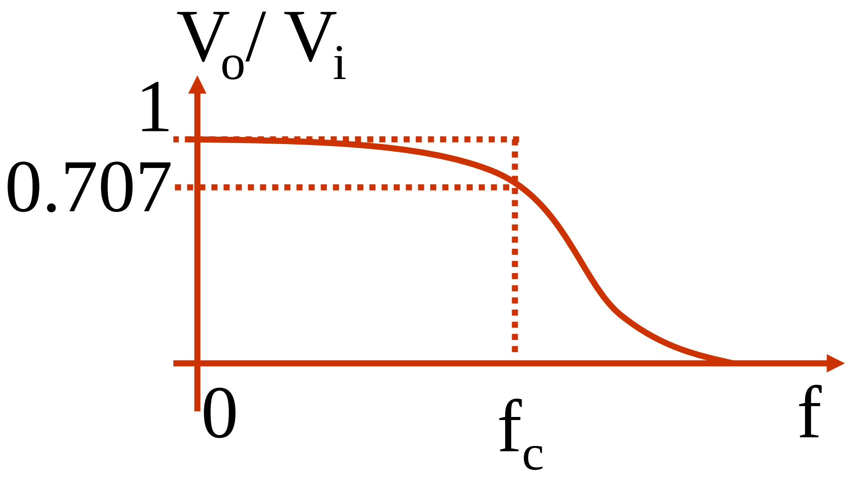


Fig. Frequency response.

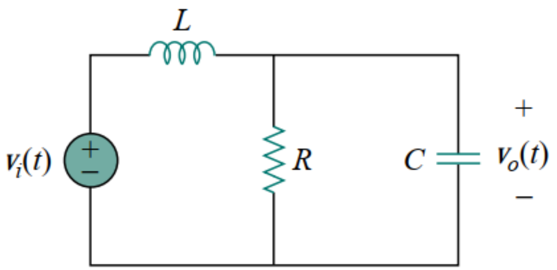
- At the angular frequency $\omega = \omega_c = 1/RC$, the capacitive reactance $1/\omega C$ equals the resistance R

$$R = X_C$$

$$f_c = \frac{1}{2\pi RC}$$

RC Low Pass Filter/Math Problem

- Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take R = 2 k&, L = 2 H, and C = 2 μF



The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \qquad s = j\omega \qquad (14.10.1)$$

But

$$R \left\| \frac{1}{sC} \right. = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}$$

Substituting this into Eq. (14.10.1) gives

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \qquad s = j\omega$$

or

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R} \qquad (14.10.2)$$

RC Low Pass Filter/Math Problem

Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take R = 2 k&, L = 2 H, and C = 2 μF

Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$, we conclude from Table 14.5 that the circuit in Fig. 14.39 is a second-order lowpass filter. The magnitude of $\mathbf{H}$ is

$$H = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}} \tag{14.10.3}$$

The corner frequency is the same as the half-power frequency, i.e., where $\mathbf{H}$ is reduced by a factor of $1\sqrt{2}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency, Eq. (14.10.3) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

or

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of R, L, and C, we obtain

$$2 = \left(1 - \omega_c^2\, 4 \times 10^{-6}\right)^2 + (\omega_c\, 10^{-3})^2$$

Assuming that ω_c is in krad/s,

$$2 = (1 - 4\omega_c)^2 + \omega_c^2 \qquad \text{or} \qquad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2, we get $\omega_c^2 = 0.5509$, or

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

RC High Pass Filter

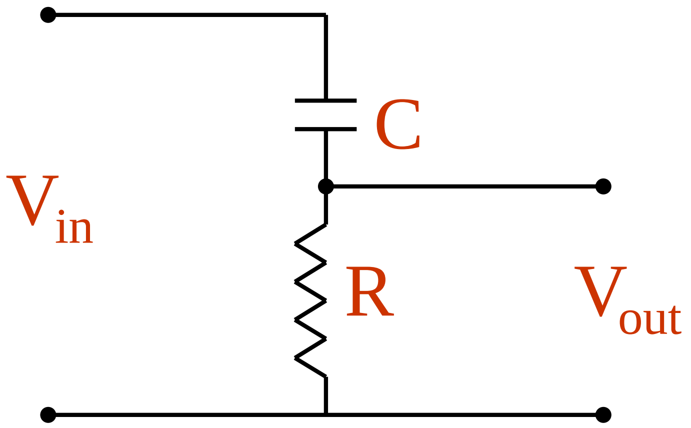


Fig. Circuit diagram.

$$V_{out} = \frac{2\pi RfC}{\sqrt{1+(2\pi RfC)^2}} V_{in}$$

At the angular frequency $\omega = \omega_0 = 1/RC$, the capacitive reactance $1/\omega C$ equals the resistance R

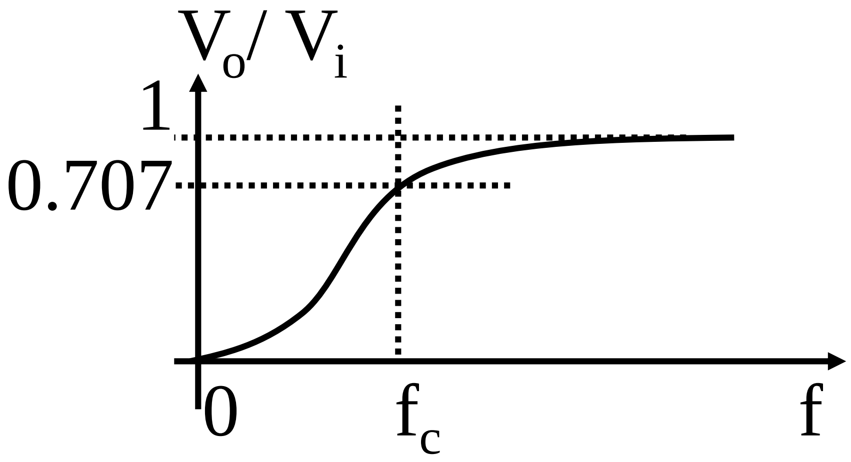


Fig. Frequency response.

$$R = X_C$$

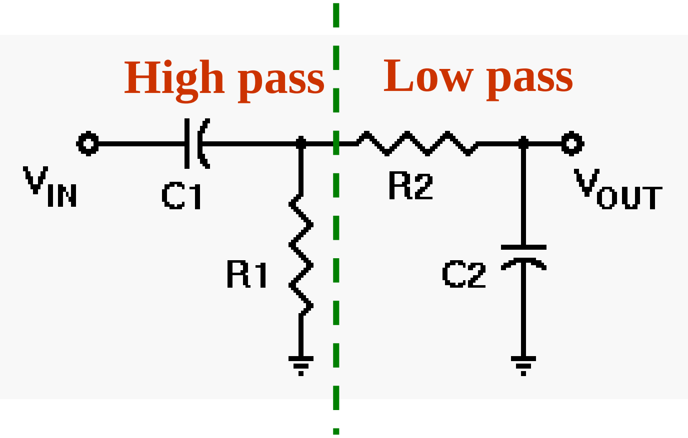
$$f_c = \frac{1}{2\pi RC}$$

Assignment

- **Problem 1: Design a low pass filter using RL circuit.**

- **Problem 2: Design a High pass filter using RL circuit.**

Band pass filter

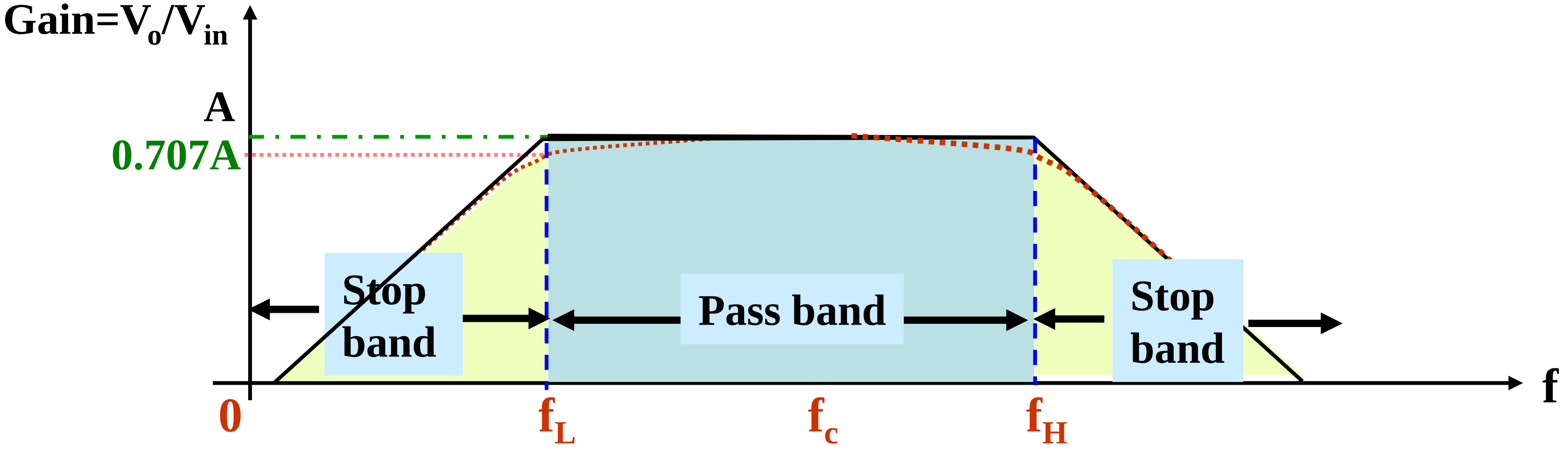


$$V_{R1} = \frac{2\pi R_1 f_1 C_1}{\sqrt{1+(2\pi R_1 f_1 C_1)^2}} V_{in}$$

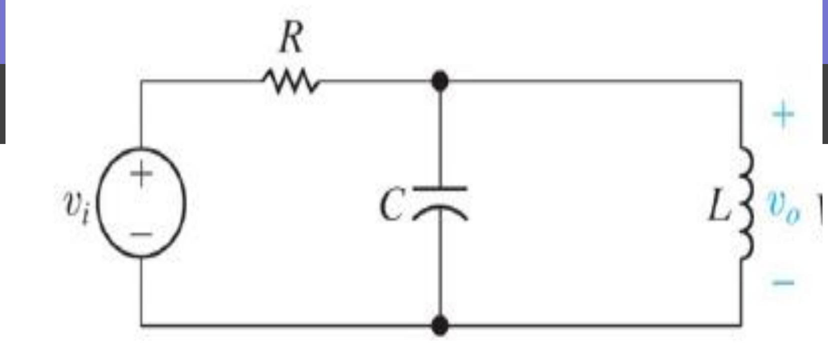
$$f_H = \frac{1}{2\pi R_1 C_1}$$

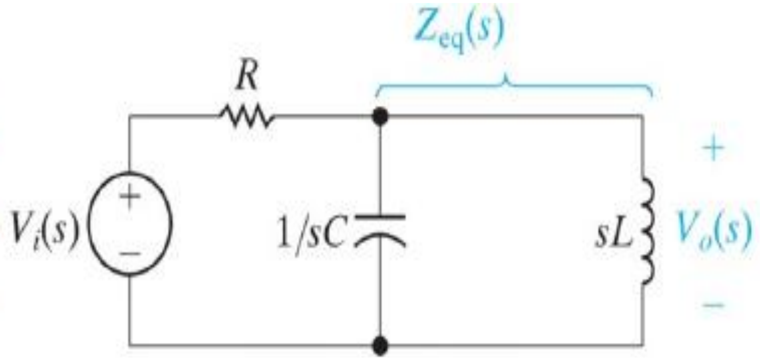
$$V_{out} = \frac{1}{\sqrt{1+(2\pi f_2 R_2 C_2)^2}} V_{R1}$$

$$f_L = \frac{1}{2\pi R_2 C_2}$$



Band pass filter

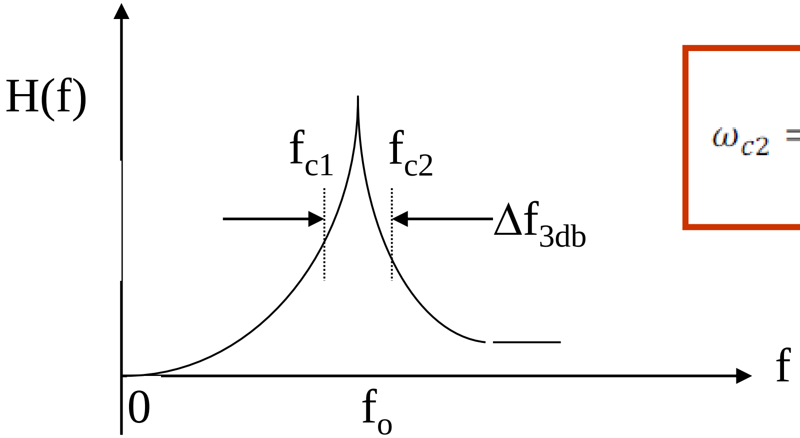




$$Z_{eq}(s) = \frac{\frac{L}{C}}{sL + \frac{1}{sC}}$$

$$therefore \quad H(s) = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$\boxed{\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}};}$$



$$\boxed{\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}}$$

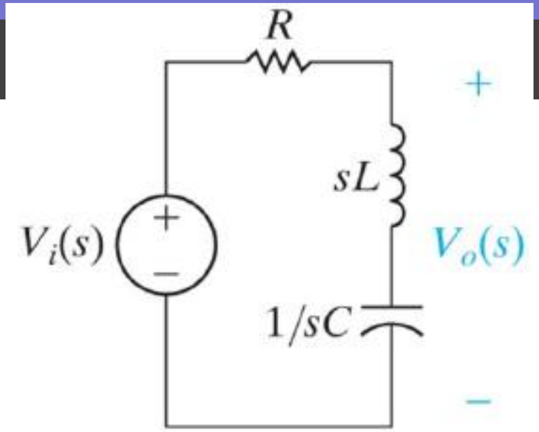
$$\boxed{\beta = \omega_{c2} - \omega_{c1} = \frac{1}{RC}}$$

$$Q = \frac{\omega_o}{\beta} = \sqrt{\frac{CR^2}{L}}$$

$$\boxed{f_o = \frac{1}{2\pi\sqrt{LC}}}$$

It is also known as TUNED circuit, signal of a particular frequency can be selected by adjusting the capacitor or inductor.

Band Reject or Notch Filter



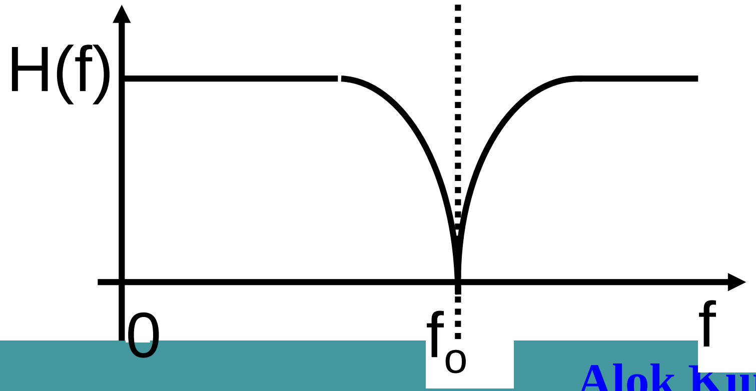
$$H(s) = \frac{sL + \frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\boxed{\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

$$\boxed{\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

$$\boxed{\beta = \omega_{c2} - \omega_{c1} = \frac{R}{L}}$$

$$Q = \sqrt{\frac{L}{CR^2}} \quad \text{and} \quad \omega_o = \sqrt{1/LC}$$



Advantages of Active Filters over Passive Filters

1. Gain and frequency adjustment flexibility

An OP-APM is capable of providing a gain, the input signal is not attenuated as it is in a passive filter. In addition, the active filter is easier to tune or adjust.

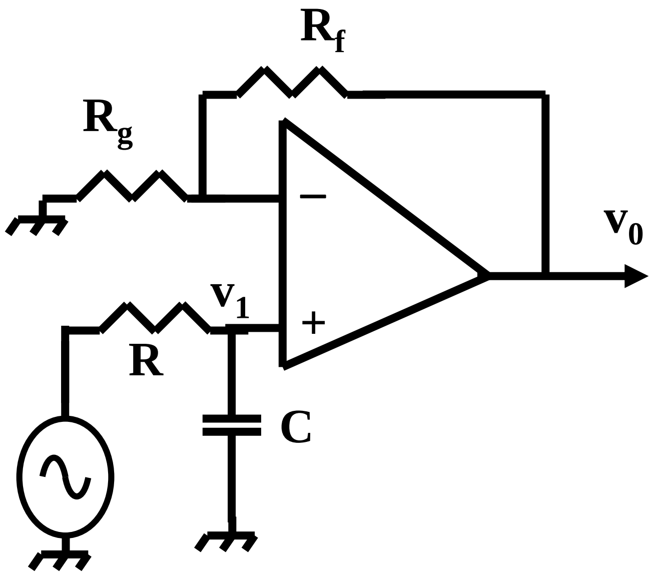
2. No loading problem

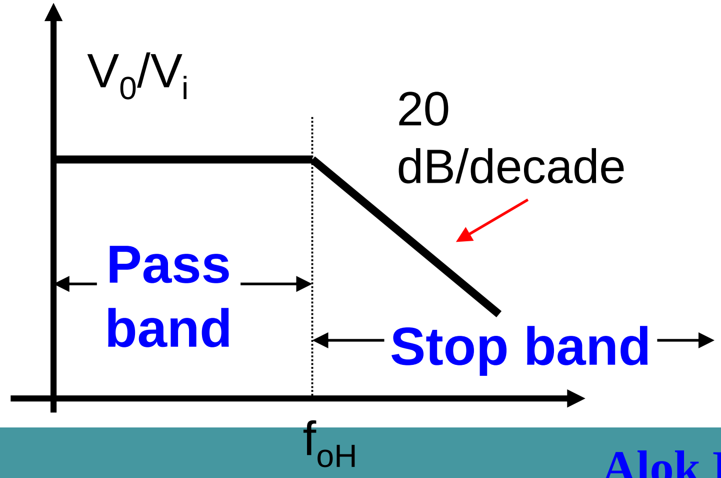
The active filter does not cause loading of the source or load, because of the high input resistance and low output resistance

3. Cost

The active filters are more economical than passive filters.

Active Low pass filter





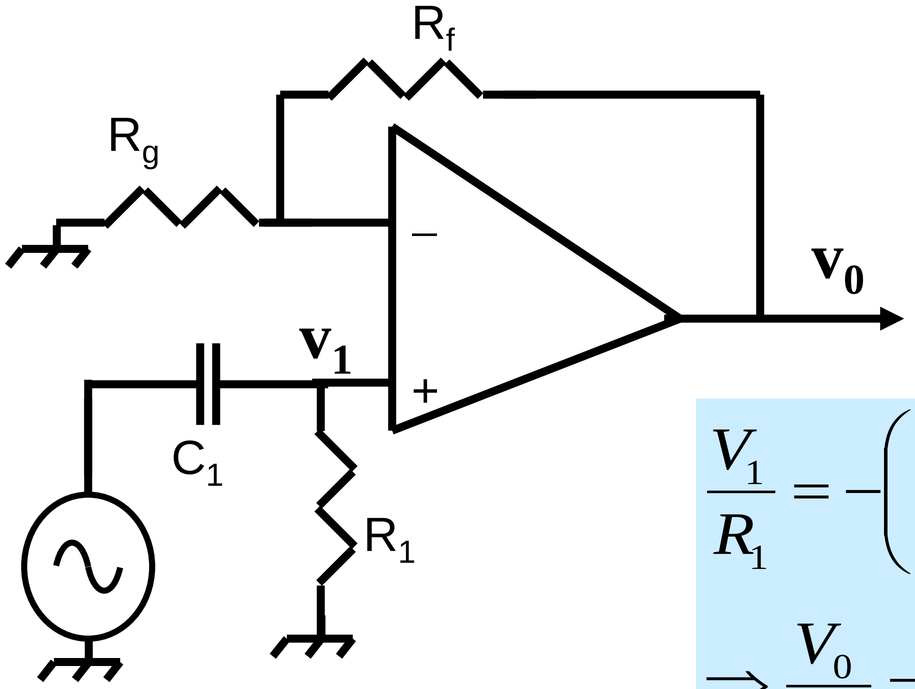
$$V_1 = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_i = \frac{1}{1 + j\omega RC} V_i$$

$$\frac{V_1}{R_1} = -\left(\frac{V_1 - V_0}{R_F}\right)$$

$$\Rightarrow \frac{V_0}{R_F} = \left(1 + \frac{R_F}{R_1}\right) V_1 = \left(1 + \frac{R_F}{R_1}\right) \frac{V_i}{1 + j\omega RC}$$

$$\frac{V_0}{V_i} = \frac{A_F}{1 + j\frac{\omega}{\omega_H}} \qquad f_H = \frac{1}{2\pi RC}$$

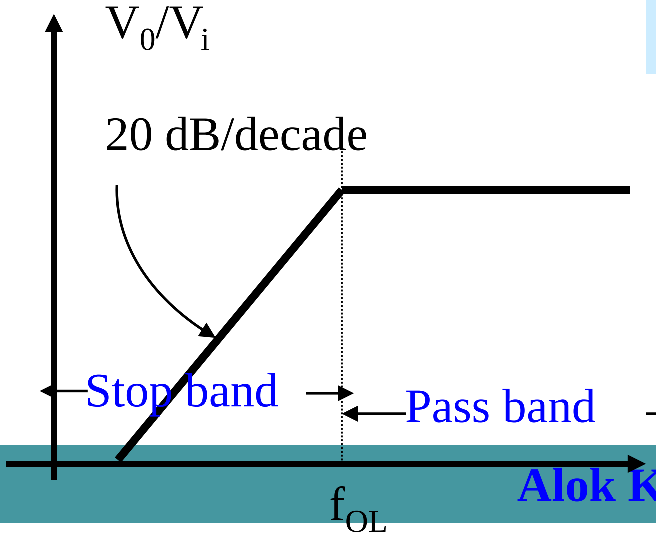
Active High pass filter



$$V_1 = \frac{R}{R + \frac{1}{j\omega C}} V_i = \frac{j\omega RC}{1 + j\omega RC} V_i$$

$$\frac{V_1}{R_1} = -\left(\frac{V_1 - V_0}{R_F}\right)$$

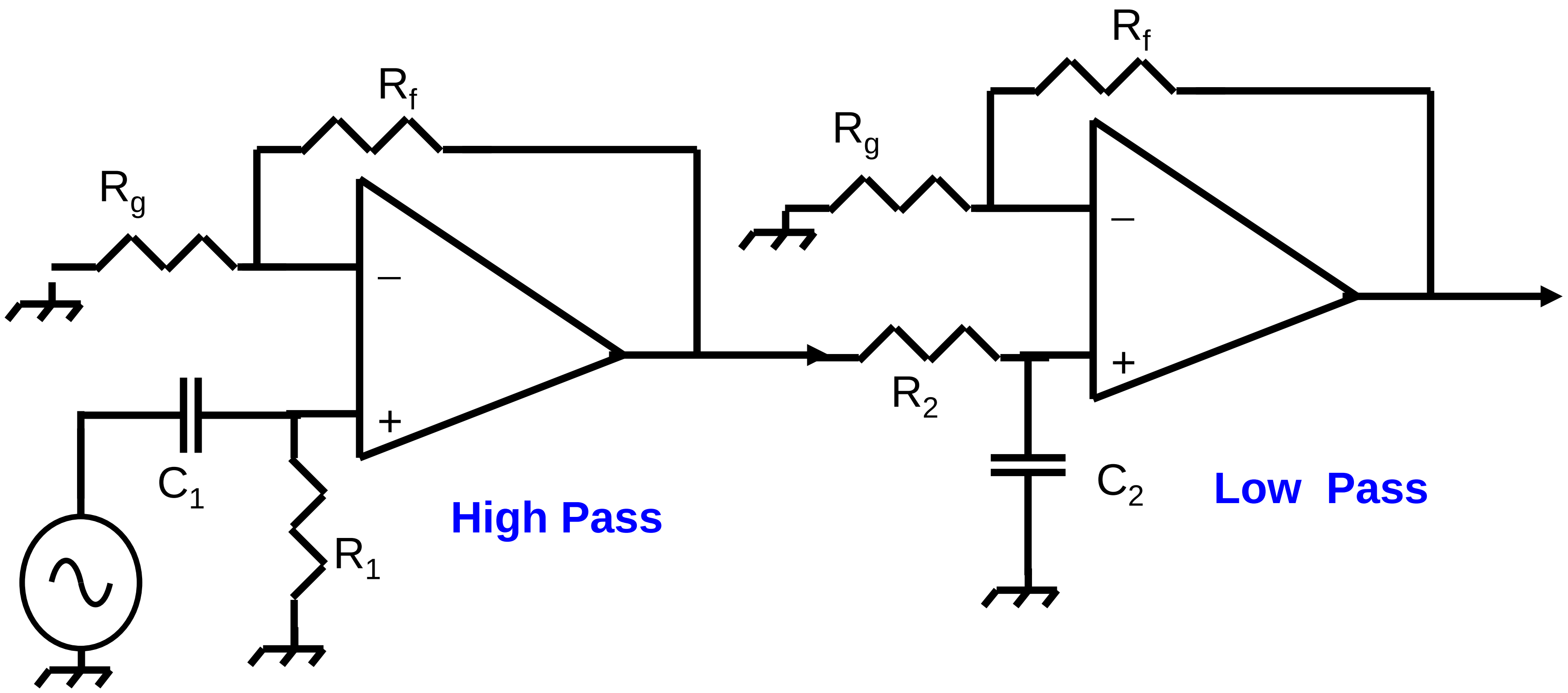
$$\Rightarrow \frac{V_0}{R_F} = \left(1 + \frac{R_F}{R_1}\right) V_1 = \left(1 + \frac{R_F}{R_1}\right) \frac{j\omega RCV_i}{1 + j\omega RC}$$

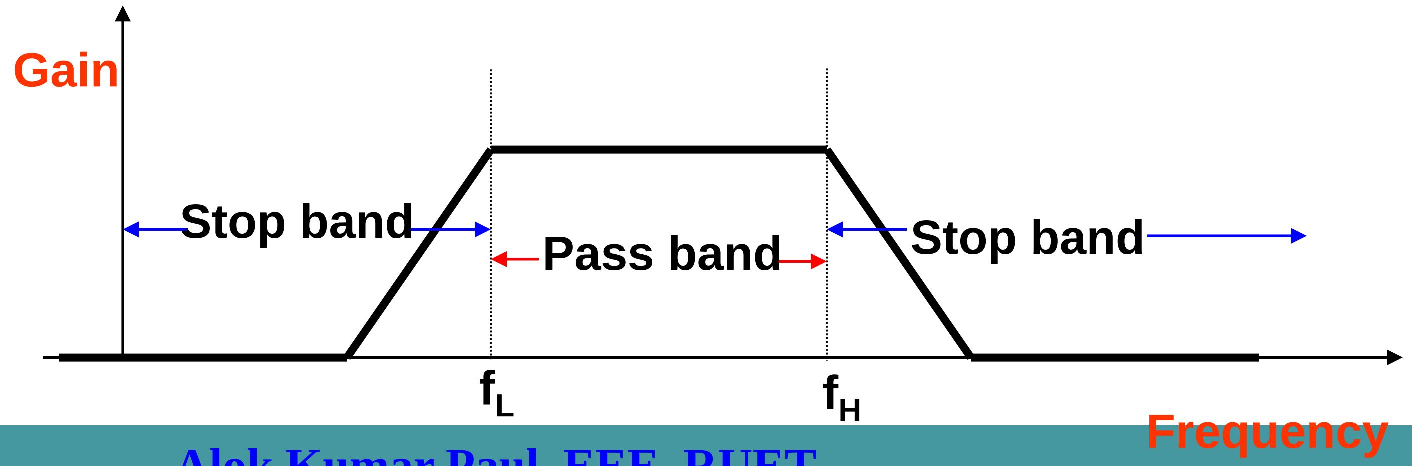


$$\frac{V_0}{V_i} = \frac{A_F\, j\left(\frac{\omega}{\omega_H}\right)}{1 + j\frac{\omega}{\omega_H}}$$

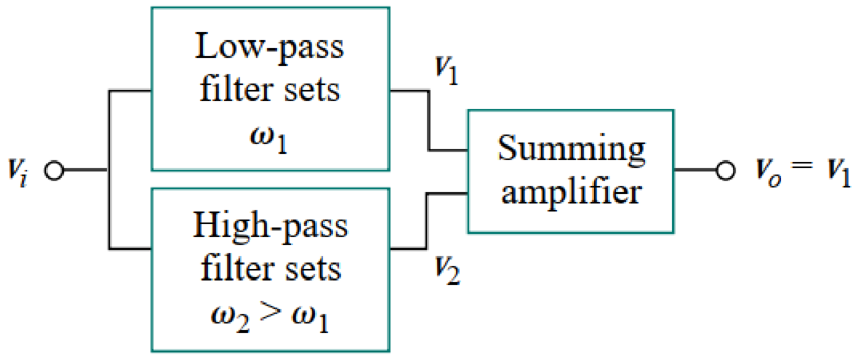
$$f_H = \frac{1}{2\pi RC}$$

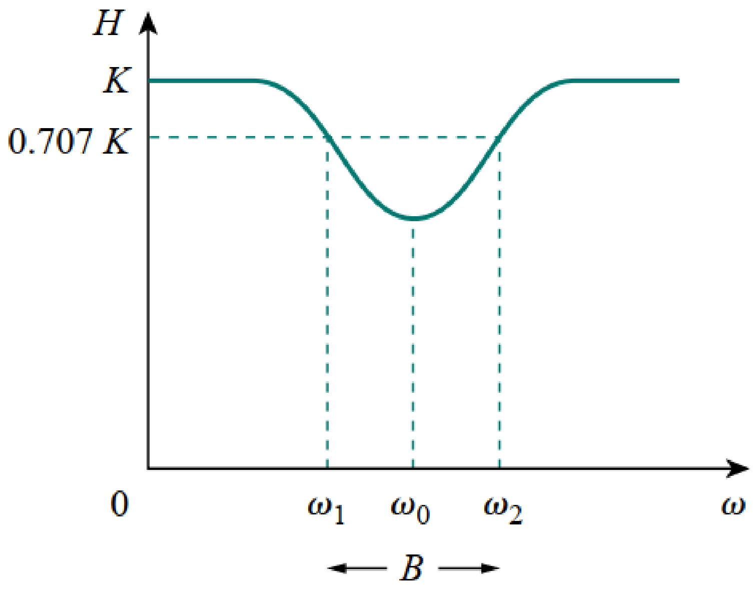
Active Band pass filter (Wide)

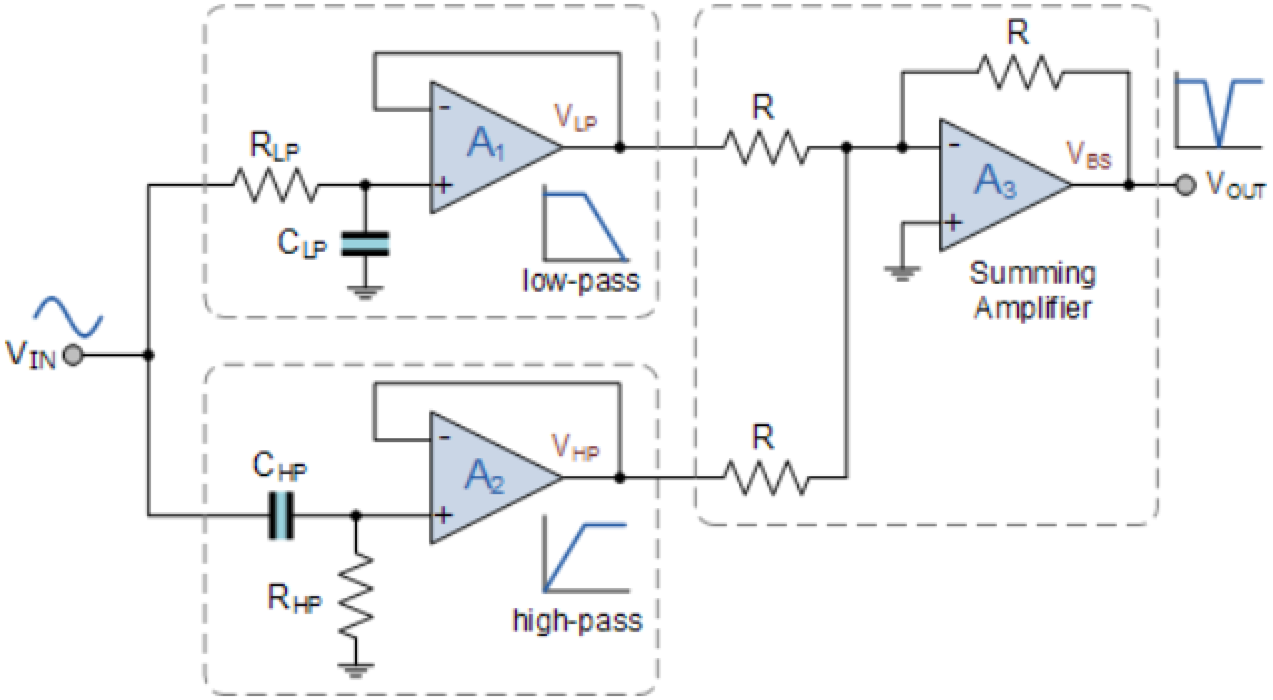




Active Band Reject filter







Frequency

Design a low-pass active filter with a dc gain of 4 and a corner frequency of 500 Hz.

$$\omega_c = 2\pi f_c = 2\pi(500) = \frac{1}{R_f C_f} \tag{14.12.1}$$

The dc gain is

$$H(0) = -\frac{R_f}{R_i} = -4 \tag{14.12.2}$$

We have two equations and three unknowns. If we select $C_f = 0.2\ \mu$F, then

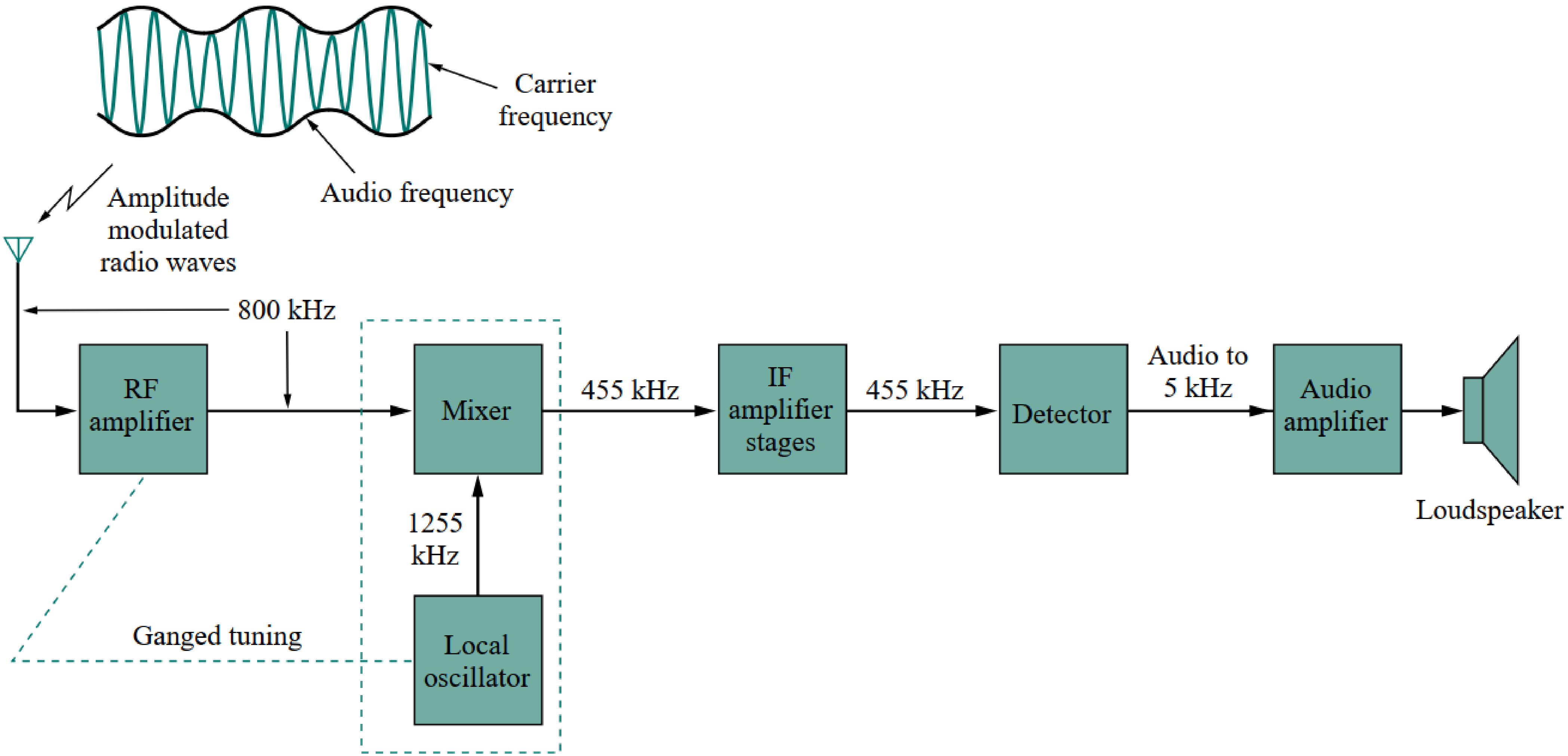
$$R_f = \frac{1}{2\pi(500)0.2 \times 10^{-6}} = 1.59\ \text{k}\Omega$$

and

$$R_i = \frac{R_f}{4} = 397.5\ \Omega$$

We use a 1.6-kΩ resistor for R_f and a 400-Ω resistor for R_i. Figure 14.42 shows the filter.

Design a highpass filter with a high-frequency gain of 5 and a corner frequency of 2 kHz. Use a 0.1-μF capacitor in your design.
Answer: R_i = 800 & and R_f = 4 k&.


Carrier
frequency
Audio frequency
Amplitude
modulated
radio waves
800 kHz
RF
amplifier
Mixer
455 kHz
IF
amplifier
stages
455 kHz
Detector
Audio to
5 kHz
Audio
amplifier
Loudspeaker
1255
kHz
Local
oscillator
Ganged tuning

Questions??