The Strange Behavior of the Neutral K Meson

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Abstract

The K mesons are the lightest strange mesons, so their decays are dictated by the weak nuclear force. This makes them a uniquely powerful tool for examining the properties of weak interactions, and K mesons, or kaons, were instrumental in the discovery of both parity violation and charge-parity (CP) violation, which has dramatic consequences for our understanding of the fundamental laws of physics.

The neutral K mesons, K^0 and $\overline{K}^0$, also exhibit the fascinating phenomenon of strangeness oscillations, in which a beam consisting solely of K^0 mesons can evolve $\overline{K}^0$ components over time. This can be analyzed in terms of Feynman diagrams describing the transition and by examining the time evolution of the CP eigenstates of the system.

Strangeness: a quick review

We saw in chapter 10 of *Quantum Physics* that the discovery of the K mesons and Λ baryons was a puzzle for physicists. Here were particles that were produced in strong interactions but decayed with lifetimes more than a thousand times those typical of strong decays. Instead, the decays were by weak interactions. This, of course, begs the question: why can't the K and Λ decay strongly? To explain the phenomenon, physicists invented the concept of strangeness, and stated that strong and electromagnetic interactions had to conserve it. Once a particle was produced that had nonzero strangeness, the only way it could decay in the absence of other strange particles was by the weak force, which explained the extraordinarily long lifetimes (on the order of a nanosecond instead of a femtosecond) of the strange particles.[1] Like charge and lepton number, strangeness is an additive quantum number. That means, of course, that every strange particle has an antiparticle with the opposite strangeness.

Nowadays, with our knowledge of quarks, we say that a strange quark s has strangeness -1, and an anti-strange quark $\overline{s}$ has strangeness $+1$. These were arbitrary designations: the K^+ (quark content $u\overline{s}$) was defined to have strangeness $+1$ and other particles' strangeness values were determined by examination of processes conserving strangeness.

Neutral kaons

Neutral K mesons are produced in strong and electromagnetic interactions by pair production of strange quarks. Strangeness is conserved in this interaction, so the two possible neutral K mesons to be produced have quark contents of $d\overline{s}$ (K^0) and $\overline{d}s$ ($\overline{K}^0$).

[1] This is only necessarily true of the lightest strange baryons and mesons, as heavier strange hadrons can decay into Λ^0 and K states. The overall decay, however, necessarily includes the decay of the resulting low-mass strange particle, and thus still occurs on a weak timescale.

Unlike the neutral pion (π^0), the neutral K mesons are a particle-antiparticle pair. The π^0 is a superposition of two states $u\overline{u}$ and $d\bar{d}$. Since these states are indistinguishable by any quantum number, the neutral pion is its own antiparticle. On the other hand, the K^0 and $\overline{K}^0$ have different values of the strangeness quantum number, so they are distinct and are each others' antiparticles. Notice, however, that by the second-order weak interactions shown in the Feynman diagrams below, a K^0 can turn into a $\overline{K}^0$, and vice versa. We'll see more about these strangeness oscillations after a quick discussion of the CP eigenstates of the neutral kaon.

A couple of decay paths distinguish the K^0 and $\overline{K}^0$. Any strangeness-preserving decay can of course distinguish the two, since they have strangeness $+1$ and -1, respectively. Furthermore, the quark content of the two mesons means that the decay

$$K^0 \rightarrow \pi^- + \mu^+ + \nu_\mu$$

is far more likely to occur for the K^0 than the $\overline{K}^0$. Similarly, the $\overline{K}^0$ has a decay path involving a π^+ meson not usually seen for the K^0.

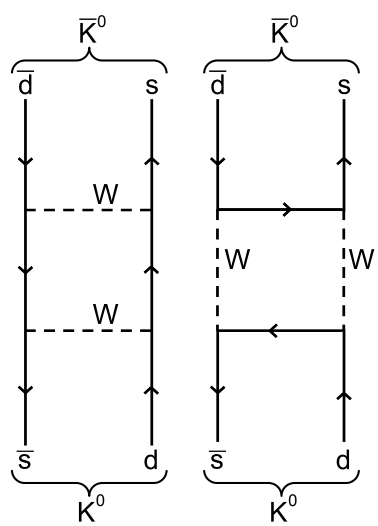

Figure 1: The two lowest-order Feynman diagrams contributing to the $K^0 \leftrightarrow \overline{K}^0$ transition. These give a physical basis for understanding neutral K meson oscillation.

Kaons and symmetry

Symmetries and conservation laws lie at the heart of all physics. Newtonian mechanics can be summarized by conservation of energy, conservation of momentum, and $\vec{F} = m\vec{a}$, which describes how individual objects interact. On the subatomic level, K mesons are one of the best tools we have to investigate the inner workings of the weak force and reexamine our ideas of fundamental symmetries of the universe.

In physics, a symmetry can be viewed as a transformation that doesn't affect the way the world works. For instance, translational symmetry states that if you pick up the whole universe and move it one foot to the left, all the laws of physics remain the same. Another way of saying the same thing is that there is no such thing as absolute position; we can only measure the position of one object with respect to another. As a direct result of this symmetry, it can be (relatively) easily shown that linear momentum must be conserved. Symmetry, conservation, and indistinguishability of different states are all linked together by Noether's theorem, one of the most beautiful constructs in physics.

Symmetries can be divided into continuous and discrete symmetries. We can apply a continuous symmetry in whatever increments we like. For example, there are no limits on how much we can rotate a circle while preserving its image – $90°$ is as good as $153.2°$, which is just as good as $10^{-32}°$. Discrete symmetries, on the other hand, can only be applied in discrete packets. For instance, the rotational symmetry of a square is discrete; it's the same shape if we rotate it $90°$ or $180°$, but a different shape if we rotate it by $36°$. Discrete symmetries abound in particle physics, and they are often only partially true. As discussed at the end of Chapter 10 in Professor Townsend's *Quantum Physics*, these approximate symmetries undergo spontaneous symmetry breaking. Understanding the sources of this symmetry breaking is one of the most interesting problems in physics, and kaons are an excellent system to study the way in which symmetries are preserved and violated.

The three discrete symmetries most discussed in this paper are charge conjugation symmetry (C), parity transformation symmetry (P), and time reversal symmetry (T). Like their continuous counterparts, discrete symmetries in physics correspond to conservation laws, so P symmetry gives us parity conservation, and so on. A brief discussion of these symmetries will be followed by an examination of the K meson's implications for all three.

Parity transformation symmetry

We have seen the parity operator before, in the context of reflecting wavefunctions across the origin: $P\Psi(x) = \Psi(-x)$. The three-dimensional parity operator is simply an extension of this one-dimensional version

$$P\Psi(x, y, z) = \Psi(-x, -y, -z).$$

Just like before, applying the parity operator twice brings us right back to the state we started with. If the state is an eigenstate of parity, then we see

$$P^2\Psi(x, y, z) = p^2\Psi(x, y, z)$$

where p is the state's eigenvalue. Thus, just like in the one-dimensional version, the parity operator has eigenvalues of ± 1.

Observe that the inversion of all three spacial coordinates is equivalent to the inversion of one spacial coordinate and a subsequent rotation of $180°$ about that coordinate axis, so we can also view the three-dimensional parity transformation as looking at the universe in a mirror, which means that P symmetry is equivalent to the indistinguishability of right and left.

In elementary particle physics, particles are assigned parities according to their interactions with other particles. This process is somewhat arbitrary, but by convention we choose that quarks and leptons have positive parity. Parity is a multiplicative quantum number, so a multiparticle state with two odd-parity particles (such as a $\pi^0 + \pi^+$ state) has an even parity overall. Parity is conserved in strong and electromagnetic interactions, so we can assign every particle (including composite particles like hadrons) an intrinsic parity.[2]

Parity conservation is an intuitively obvious property – looking in a mirror and seeing a universe running differently seems very counterintuitive. We will see, however, that K mesons give a good hint at the violation of P symmetry in weak interactions.

Charge conjugation symmetry

Charge conjugation is a transformation which, when applied to a system, causes all charges in the system to invert. The charges reversed include not just electric charge but also additive quantum numbers like strangeness, baryon number, and lepton number. Therefore under charge conjugation, every particle is transformed into its antiparticle. Just like for parity, applying a C transformation twice brings the system back to its original state, so the only real eigenvalues of the charge transformation are ± 1. The eigenstates of the C transformation are particles which are unchanged under a single charge conjugation. These particles are their own antiparticles, and are thus charge conjugation eigenstates. Other consequences of charge conjugation include the reversal of the directions of electric and magnetic fields, since these are caused by the presence and motion of electric charges, respectively.

[2] It turns out that apart from intrinsic parities of particles, the net angular momentum of a multi-particle state also contributes to the state's parity, but we can safely ignore that for now.

Time reversal symmetry

A time reversal transformation takes t to $-t$, much like the parity operator flips x to $-x$. More mathematically,

$$T\Psi(x,t) = \Psi(x,-t)$$

Like the parity operator, applying the T transformation twice brings us back to the starting system, so we see that the eigenvalues are ± 1.

Time reversal invariance states that the world should operate the same way if time ran backward. Since that's a tricky experiment to run, we can also view it as a statement about the reversibility of reactions on the subatomic scale. Of the big three discrete symmetries, it is the hardest to test directly.

Parity violation

In the mid-twentieth century, physicists studying cosmic rays found two seemingly similar particles, the θ^+ and the τ^+. They were produced in similar reactions, had nearly identical masses, spins, lifetimes, and so on, but their decays were different. The θ^+ decayed into two pions (a π^+ and a π^0), while the τ^+ decayed into three (either $\pi^+ + \pi^+ + \pi^-$ or $\pi^+ + \pi^0 + \pi^0$). Well, the pions all have negative parity, and parity is a multiplicative quantum number. By parity conservation, then, it seems that the θ^+ has a parity of $+1$, while the τ^+ has a parity of -1. Surely they couldn't be the same particle!

This baffled physicists for a while, but eventually Lee and Yang suggested that parity might not be conserved in weak interactions. Within a year, multiple experiments had confirmed the downfall of parity conservation. The two most famous of these experiments investigated the radioactive decay of cobalt-60 and the handedness of neutrinos.

- Beta decay of cobalt-60: This was the first experimental test of parity conservation in weak interactions. Wu et al. polarized the spins of cobalt-60 atoms in a magnetic field. Then they watched the cobalt and measured the direction in which electrons were emitted in beta decay. Astonishingly enough, they observed more electrons emitted in the opposite direction as the nuclear spin than in the same direction. The strange thing about this is that if you look at the experiment in a mirror universe, the nucleus is spinning the opposite way, but the electrons are emitted in the same direction. The mirror universe behaves differently!

- Handedness of neutrinos: The handedness of a particle can be defined by projecting its spin onto its momentum vector. If the spin and momentum vectors then point the same way, the particle is called right-handed. Experimentalists studying the decay of the negative pion observed that neutrinos are always left-handed. One decay mode for the π^- meson is

  $$\pi^- \to \mu^- + \overline{\nu}_\mu$$

  As we've seen, angular momentum is conserved in decays, so experiments starting with pions having no orbital angular momentum generate a muon and an anti-muon neutrino with the same handedness, which means both particles have spins either parallel or antiparallel to their momentum. Physicists observed that in the decay of a negative pion, the produced muon was *always* right-handed. Studies of other reactions involving neutrinos and anti-neutrinos demonstrated that all neutrinos are left-handed, and all anti-neutrinos are right-handed. This is a striking violation of parity symmetry: if you look in a mirror, you would see right-handed neutrinos, which don't exist on our side!

In light of the discovery of parity nonconservation, physicists concluded that the θ^+ and τ^+ were really the same particle. It is now known as the K^+ meson.

The conclusion that parity is not conserved in weak interactions led physicists to their next best guess, CP symmetry.

CP symmetry

A natural thought upon noticing the parity-violating interactions (Co-60 beta decay and neutrino handedness, for instance) is that if instead of applying only a parity transformation, we instead apply a parity transformation followed by a charge conjugation (switch particles for antiparticles), we might get the same laws of nature. For instance, reflecting a neutrino in a mirror results in a neutrino with the opposite handedness, which we never see in nature. But if we also change it to an antineutrino, then the handedness is as needed. Similarly, if we both look in a mirror and use anti-cobalt-60, then the positrons (anti-electrons) are emitted in the same direction as the nuclear spin, so it seems that CP symmetry should hold.

It turns out that like P-symmetry, CP-symmetry is not exact. This was first discovered in the neutral kaon system, which will be discussed extensively in the next section. It is, however, much closer to a true symmetry than parity, so in many situations it is safe to assume that CP is conserved.

CP eigenstates for kaon decay

The K meson decays in a weak interaction, so strangeness is not conserved. This means that the particle's strangeness eigenstate (K^0 or $\overline{K}^0$) is irrelevant to the decay. Instead, what matters is the particle's CP eigenstate, since CP is (mostly) conserved in weak interactions. Naturally, the strangeness eigenstates are not also CP eigenstates. In order to find the combination of strangeness eigenstates that gives CP eigenstates, we need to examine the results of the C and P operations on the K^0 and $\overline{K}^0$. Based on previous discussion of the C operator, we know that the charge conjugate of the K^0 is the $\overline{K}^0$, and vice versa. For reasons outside the scope of this paper, the charge conjugation operation also has a complex phase, which in the case of the K^0 and $\overline{K}^0$ introduces a negative sign in the result. Let's call the states describing the pure particles by the particle's names. Thus, K^0 denotes a state which, if measured, is guaranteed to give a K^0 particle. In general, the state K^0 will appear in equations, while the particle K^0 will be discussed in prose. The application of the charge conjugation operator yields

$$CK^0 = -\overline{K}^0 \qquad\qquad C\overline{K}^0 = -K^0$$

Experiments show that both strangeness eigenstates have odd parity, so we also have

$$PK^0 = -K^0 \qquad\qquad P\overline{K}^0 = -\overline{K}^0$$

From this, it is clear that applying both transformations yields

$$CP\ K^0 = \overline{K}^0 \qquad\qquad CP\ \overline{K}^0 = K^0$$

As expected, K^0 and $\overline{K}^0$ are *not* CP eigenstates. Instead, the relevant eigenstates are special superpositions of the two strangeness eigenstates. We creatively call them K_1 and K_2, and define them as

$$K_1 = \frac{1}{\sqrt{2}}\left(K^0 + \overline{K}^0\right) \qquad\qquad K_2 = \frac{1}{\sqrt{2}}\left(K^0 - \overline{K}^0\right)$$

Check for yourself that that K_1 and K_2 are eigenstates of CP with eigenvalues $+1$ and -1, respectively. Observe also that the strangeness eigenstates can be written in terms of the CP eigenstates:

$$K^0 = \frac{1}{\sqrt{2}}\left(K_1 + K_2\right) \qquad\qquad \overline{K}^0 = \frac{1}{\sqrt{2}}\left(K_1 - K_2\right)$$

Any state involving neutral kaons can be expressed as a superposition of the strangeness eigenstates or the CP eigenstates. If a bunch of K^0 mesons are produced in some interaction, we can view them as half K_1's and half K_2's. Or a K_2 beam can be viewed as half K^0's and half $\overline{K}^0$'s. It's a matter of which basis you choose to express the state of the system.

A key result of this change of basis is that since K_1 and K_2 are their own antiparticles (that is, they are charge conjugation eigenstates), they are permitted to have distinct masses and lifetimes. This is in contrast to the K^0 and $\overline{K}^0$, which are each others' antiparticles and are therefore required to have identical properties.

In weak decays, the CP eigenstate is (mostly) conserved, so K_1 has to decay to a CP-even final state. In particular, it can decay into two pions. By similar logic, K_2 can decay into three pions but not two. As a result of something called phase space that contributes to the probabilities of various decays, the two-pion decay is about 600 hundred times faster than the three-pion decay. As a result, the lifetime of the K_2 eigenstate is a couple of orders of magnitude longer than the K_1's. That means that if we start with a pure K^0 beam, the K_1 component decays away very quickly in a flurry of two-pion decays. A short while later, only K_2's are left. This allows for the production of an arbitrarily pure K_2 beam, which was instrumental in the discovery of CP violation in the weak interaction.

There are some philosophical implications to the distinction between CP and strangeness eigenstates. In particular, what is a particle? Is it the thing produced in a strong interaction, or the thing that decays away in a weak one? Is it the thing with a definite lifetime, or the thing with a definite production cross-section? In the end, though, these are primarily questions of philosophy rather than physics, and it makes little difference which states you define as particles.

Strangeness oscillations

Because kaons decay by weak interactions, it is the CP, rather than strangeness, eigenstates that have the time evolution that we have come to expect of systems described by the Schrödinger equation. Here, we will examine the time evolution of the strangeness states in the rest frame of the particle. All the results translate directly into the lab frame by the application of special relativity. We'll start, as many experiments do, with a pure K^0 beam. These can be produced by strong interactions of K^+ with matter, which to preserve strangeness must generate K^0 rather than $\overline{K}^0$. In terms of the CP eigenstates K_1 and K_2, the initial beam is therefore

$$\Psi(0) = \frac{1}{\sqrt{2}}\,(K_1 + K_2)$$

You can substitute the definitions for K_1 and K_2 into the equation above to verify this decomposition into the CP eigenstates.

We have seen that it is possible for a K^0 to turn into a $\overline{K}^0$ via second-order weak interactions. What we want to know now is how the fraction of the beam that is K^0 mesons changes over time, in terms of the probability that an individual particle decays as a K^0. Denote this probability by $P[K^0(0) \to K^0(t)]$.

To find the time evolution of the CP eigenstates, recall that when we used separation of variables on the time-dependent Schrödinger equation, we found that the time evolution of a state with energy E was a complex phase

$$f(t) = e^{-iEt/\hbar}$$

Since we're working in the rest frame of the particle, the energy of the K_1 state is just $E_1 = m_1c^2$, where m_1 is the mass of the K_1. In addition to this, the fact that we're working with two states that decay with definite lifetimes τ_1 and τ_2 means that we need to add a real exponential decay to this phase evolution to account for the fact that there's a definite probability that after a time t, the particle no longer exists. Putting this all together, we find that the time evolution of the K_1 state is

$$K_1(t) = e^{-t/2\tau_1} e^{-im_1 c^2 t/\hbar} K_1(0)$$

The real exponential term has a factor of two in the exponent's denominator in order to produce the expected decay rate.

Observe that the probability that an initial K_1 state is still a K_1 after a time t is the square of the complex coefficient above

$$P[K_1(0) \to K_1(t)] = \left(e^{-t/2\tau_1} e^{-im_1 c^2 t/\hbar}\right)^* \left(e^{-t/2\tau_1} e^{-im_1 c^2 t/\hbar}\right) = e^{-t/\tau_1}$$

This is exactly what we would expect for a decaying particle, and is why the factor of two is introduced in the exponent.

Based on this modified picture of the time evolution of a decaying particle, the time evolution of the beam's state $\Psi(t)$ is

$$\Psi(t) = \frac{1}{\sqrt{2}} \left(e^{-t/2\tau_1} e^{-im_1 c^2 t/\hbar} K_1 + e^{-t/2\tau_2} e^{-im_2 c^2 t/\hbar} K_2\right)$$

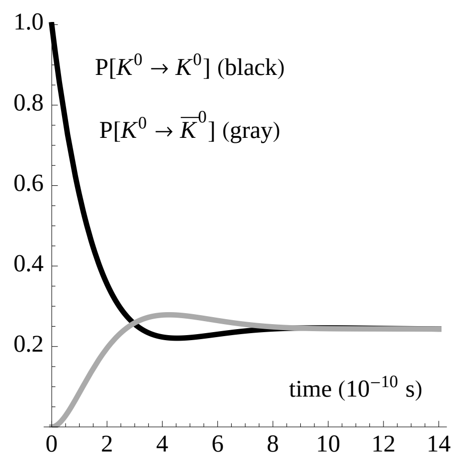

Figure 2: Probabilities of detecting K^0 (black) and $\overline{K}^0$ (gray) in a beam starting off as pure K^0. The times are in the rest frame of the beam.

From the definitions of K_1 and K_2, this can be rewritten in terms of the strangeness eigenstates K^0 and $\overline{K}^0$

$$\Psi(t) = \frac{1}{2}\left[\left(e^{-t/2\tau_1} e^{-im_1 c^2 t/\hbar} + e^{-t/2\tau_2} e^{-im_2 c^2 t/\hbar}\right) K^0 + \left(e^{-t/2\tau_1} e^{-im_1 c^2 t/\hbar} - e^{-t/2\tau_2} e^{-im_2 c^2 t/\hbar}\right) \overline{K}^0\right]$$

Well, that's a bit of a mouthful. It makes it clear, however, that the coefficient c_+ in front of the K^0 state is

$$c_+ = e^{-t/2\tau_1} e^{-im_1c^2t/\hbar} + e^{-t/2\tau_2} e^{-im_2c^2t/\hbar}$$

The magnitude squared of this coefficient gives the probability of observing a K^0 in the beam after time t:

$$P[K^0(0) \to K^0(t)] = \frac{1}{4}\left(e^{-t/\tau_1} + e^{-t/\tau_2} + 2e^{-\frac{1}{2}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)t}\cos\left(\frac{(m_1-m_2)c^2}{\hbar}\,t\right)\right)$$

The probability of observing a $\overline{K}^0$ in the beam after time t is exactly the same, but with a negative cosine term. The resulting probabilities over time are plotted in Figure 2. These strangeness oscillations are a bizarre phenomenon. Starting with a pure K^0 beam, there are times at which the beam actually contains more $\overline{K}^0$ than K^0.

CP violation

Not content with introducing strangeness and violating parity conservation, the kaons were also the first evidence of CP violation. If CP were conserved, the strangeness and CP eigenstates of the neutral kaon would be all we needed to analyze the (rather strange) system. In that case, after all the K_1's had decayed away, we would expect to see only three-pion decays of K_2's. Instead, Cronin and Fitch observed that a beam of pure K^0, after traveling over fifty feet (plenty of time for all the K_1 components to decay away), around one in five hundred decays were in fact two-pion decays. That means that the long-lived neutral kaon is actually a mixture of both K_2 and K_1, with substantially more K_2. And that means that like parity, CP is not a perfect symmetry in weak interactions. Another score for kaons in that fascinating game of Confuse the Physicists.

Consequences of CP violation

CP violation has several fascinating consequences. For one thing, it is able to favor matter over antimatter, which can at least partially explain why the universe we live in appears to be made entirely of matter.[3] In the Big Bang, since the forces present were primarily electromagnetic and strong forces, CP should have been an exact symmetry, which means that for a while, at least, the universe had exactly as much matter as antimatter. The question then becomes what allowed the universe to evolve into predominantly matter, and CP violation says that through weak interactions, matter and antimatter can be distinguished and matter preferentially generated.

CP violation also means that we can finally absolutely define the positive charge, whereas previously we simply had to stick with a convention based on rubbing glass and silk together. We can say that positive is the charge of the lepton preferentially emitted in the decay of the neutral K meson.

Furthermore, a beautiful consequence of quantum field theory is the CPT theorem, which predicts that the combined operation of time reversal, parity transformation, and charge conjugation produces a universe in which the laws of physics are identical to our own. One of the most remarkable things about the CPT theorem is that it's based only on the basics; things like special relativity and the idea that only local properties can affect an interaction, which means that if we discover CPT-violating reactions, we'll have to seriously rethink the foundations of modern physics.

If the CPT theorem holds (which most physicists believe it does), then CP violation means that weak interactions also violate time reversal symmetry. Direct time reversal symmetry breaking is a lot harder to detect than the other symmetries we've looked at so far, but physicists are currently diligently looking for things like a non-zero electric dipole moment of the neutron, which would imply T violation and allow a confirmation of the CPT theorem.

[3] If the universe had matter and antimatter sections, we would expect that the interface would have an abundance of annihilation, which would appear as a bright layer without necessarily many stars around.

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