Understanding volatilty in South Africa: a multivariate GARCH approach

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1. Introduction

Investors who manage their funds or are responsible for that of others seek stable and profitable returns on the portfolio decisions they make. This requires investors to optimise their portfolio of holdings, which may include a combination of equities, assets, commodities, bonds, etc, and assess whether the risk and reward metrics align with their portfolio return goals and/or benchmarks. Due to expectations from clients and investors on meeting certain performance standards, investors need to instil confidence in their ability to meet these requirements by using the correct measures of risk and return.

There is a plethora of investor risk management calculations (ROE, Sharpe ratio, diversification ratio, etc) that are used by investors to assess the current position of a portfolio and the exact returns gained or lost on individual returns. These calculations are based on price movements being directly affected by shifts in demand factors – which are affected by externalities. Externalities can be common or asset-specific, which may cause spillover to other assets due to correlation. The ability of the investor to capture externalities and incorporate them into the calculations for the portfolio is crucial for the investor to maintain an optimal portfolio. Therefore, utilising models that can sequentially deal with past information is of great importance for forecasting price movements and making better portfolio decisions.

The listed price of firms is affected by their reported performances, intended plans, overall well-being, and their ability to deal with exogenous shocks. As firms perform and react independently, to gain a good understanding of overall market movement, isolating key economic drivers in nations provides a clearer understanding of overall market contributors. This entails taking a broader approach by evaluating the key economic sectors and their price movement over time, instead of focusing on

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individual firms. Sectors may be unique in their production process, but common shocks pose a threat to all sectors (and their ability to deal with them). Key sectors may be more vulnerable than others, and it is important to understand if sectors that can deal with common shocks are exposed to the vulnerability of others. Furthermore, it is interesting to identify if unique shocks to a specific sector affect it in any way, and if it is transmitted to another. This valuable insight allows an investor to make better decisions with their portfolio composition, however, appropriate models have to be used to identify volatility and the spillover effects.

This paper investigates the effect of exogenous shocks over the past decade in South Africa for listed equities. As more than one sector is included, the multivariate generalised autoregressive conditional heteroskedasticity model (MV-GARCH) will be used to assess volatility and whether it is transmitted across sectors. The first section provides a breakdown of the South African economic climate and the importance of understanding why volatility modelling is helpful for portfolio management. Section two covers the theory behind the MV-GARCH model and the existing literature of findings from the models used in volatility assessment. Section three provides a summary of the data set used in the paper, along with a sector breakdown. Section four delivers a discussion about the empirical results from the model and lastly, the paper closes off with the conclusion.

2. South Africa Outline

South Africa is a nation that is endowed with an abundance of sought after commodities, however, it still remains its statues as a developing nation (Bakari (2017)). The World Bank has developed *The Global Investment Competitiveness Report* which highlights key factors that influence investor confidence (Bank (2020)). The report indicates that investor confidence is dependent on political stability and the macroeconomic environment, both of which are sensitive topics in South Africa. The frequent reports of corruption, recurrent cabinet shuffles and continuous political scuffles, create a hostile and unattractive political environment in South Africa. The macroeconomic environment consists of stagnant growth, socio-economic despair, extreme unemployment and many more issues.

The energy crisis, locally known as “load-shedding”, is currently the biggest challenge for the South African government to overcome, as this is a decisive input used in the productivity of firms and is also directly used by consumers. This current issue has caused severe issues for growth and development and has severely impaired several firms in their daily activities. Furthermore, another state-owned asset, Transnet, has suffered major logistical issues, creating bottlenecks for supply chains, firms and consumers. This, in addition to the energy crisis, has created an unfavourable outlook on productivity growth. Both issues have been raised by the International Monetary Fund (IMF) as key hurdles to overcome, due to key economic drivers being directly affected IMF. However, despite these tough economic conditions, certain sectors of the economy have been resilient.

Key drivers of the South African economy have managed to adapt to these conditions and provide positive returns in their respective sectors. This means that regardless of economic hardship, investors are still able to identify sectors that can deal with shocks and hold these equities, which aligns with the ideology of diversification. Therefore, it is important to investigate the sectors that are key economic drivers in South Africa and if exogenous shocks to one or more sector/s are transmitted to another. This is valuable information for investors who are holding local equities.

3. Literature Review

A great body of literature exists surrounding volatility modelling in the financial sector. Several models have been developed in the past and expanded upon by researchers who try to elaborate on the accuracy and applicability of certain models. Identifying the response within and between sectors has been studied by researchers, who have found valuable information in their methodologies and empirical results.

Classical econometrics makes use of ordinary least squares (OLS) regression to identify the impact of each variable on one another, by interpreting the coefficients, but its applicability to time series data is limited. Isolating sectors and using OLS can provide coefficients that represent sector impact on each other. The expected mean and variance are calculated using past information and serve as a long-term indicator of price movement, but it overlooks periods of high equity price movements that occur in both directions. The variance is of great interest to investors, as this is the expected deviation away from the price of an equity and this serves as a proxy for risk to the investor. However, a vast body of literature exists for the improvement of modelling risk/variance.

Whilst homoskedasticity is sought after in many models, Engle's 1982 *Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation* paper is one of the most influential papers in the world of finance, as it focuses on modelling heteroskedasticity with the use of autoregressive conditional heteroskedastic (ARCH) models. He argued that variance is heteroskedastic, as new information affects the price, which is taken into consideration by the ARCH model, as opposed to other models that ensure variance is homoskedastic (constant) throughout the estimation process. Engle argued that homoskedasticity should not be used as a proxy of current risk, as price movement is dependent on the current economic climate and/or news. Therefore, the current price movements may be beyond the long-term standard deviation due to the processing of information. Thus, Engle strongly suggested that the ARCH model be used, as the variance is conditional on the recent past and carries a bigger weight in the variance estimate (Engle (1982)).

Furthermore, the paper is expansive in the mathematics behind the model and how the ARCH model provides better coefficients compared to the OLS regression, as the constant variance in a normal regression is not able to utilise an exogenous variable to explain changes that occur in the variance.

This data-generating process of the ARCH model can provide more precise estimates of price. An important contribution from Engle is the use of the ARCH model to capture, predict and minimise periods of volatility. Engle mentions that periods of 'volatility clustering', defined by some exogenous factor causing variance measures to be outliers, are captured more precisely because the model is aimed at modelling variance. The ability of the ARCH model to incorporate Bayesian updating makes it highly valuable for investors to model risk and applies to understanding volatility dynamics (Engle (1982)).

Tim Bollerslev developed *Generalized Autoregressive Conditional Heteroskedasticity* in 1986 which expanded on Engle's work by incorporating a more flexible lag structure. He proves this by looking at inflation volatility and its correlation to certain macroeconomic factors – this is tested across the various models for comparative purposes. The GARCH model provides an adaptive learning mechanism with coefficients that are more accurate and robust, as indicated by his empirical results Bollerslev (2023). Engle has produced an updated paper about the application of ARCH and GARCH models in econometrics . This paper contains less literature on the mathematics behind the models and focuses on emphasising their superiority compared to normal regressions – especially for variance calculations. The variance interpretation is enhanced, as the ARCH approach makes use of more recent variance movements and their influence on price. This leap in improvement provides a variance indicator that is more representative of the current economic climate. Engle mentions how Bollerslev's GARCH approach is an improvement on his model, with it being more parsimonious, as it provides a better-weighted distribution for the past variance influence on the calculated variance estimate. The paper includes how these models are flexible to incorporate news and measure the source and extent of volatility, which is useful to investors for understanding the position of a portfolio (Engle (2001)). Burns et al. (1998) and Hassan & Malik (2007) expand upon this.

Burns et al. (1998) speak about asynchronous price data across different markets and time zones. This is a real-world issue with managers holding local and global equities with different price updating times. This affects the portfolio's hedging strategy, and value at risk estimates and could lead to more errors. Due to trading markets closing and opening at different times, this model approach aims to offer investors information to maintain optimal positions in the market targeted towards their portfolio goals by calculating an expected price at any time. The MV-GARCH approach is used across the different trading markets globally, and the authors emphasise the importance of correlations and how the model aids portfolio rebalancing based on predicted prices when markets are closed (Burns, Engle & Mezrich (1998)).

Hassan & Malik (2007) use an MV-GARCH model to identify if there is a volatility transmission between sectors in the United States over time. This literature is important for understanding local exogenous shocks to sectors, and whether the volatility is transmitted to any other sector (Hassan & Malik (2007)). Their methodologies are used as a guideline for this paper, as the MV-GARCH model is extremely useful in capturing volatility, responsiveness and persistence of each sector in South Africa for the data set.

4. Data

The data set contains daily closed returns from 1 January 2013 to 12 December 2023 for the financial, industrial and resource sectors of South Africa. These results are important for investors to understand the movement of industries over time and the volatility present in each. The price movement for each sector can be viewed in Figure 4.1. Figure 4.1 has been formulated by taking the cumulative daily returns of all the equities in each sector throughout the specified period. There were a few extreme outliers in the data that have been restricted for graphical purposes – Figure 4.2 shows the unrestricted data. Figure 1 clearly shows periods of volatility clustering, as high price movements are grouped. This aligns with Engle's view on variance movement being conditional on current news - hence autocorrelation is present.

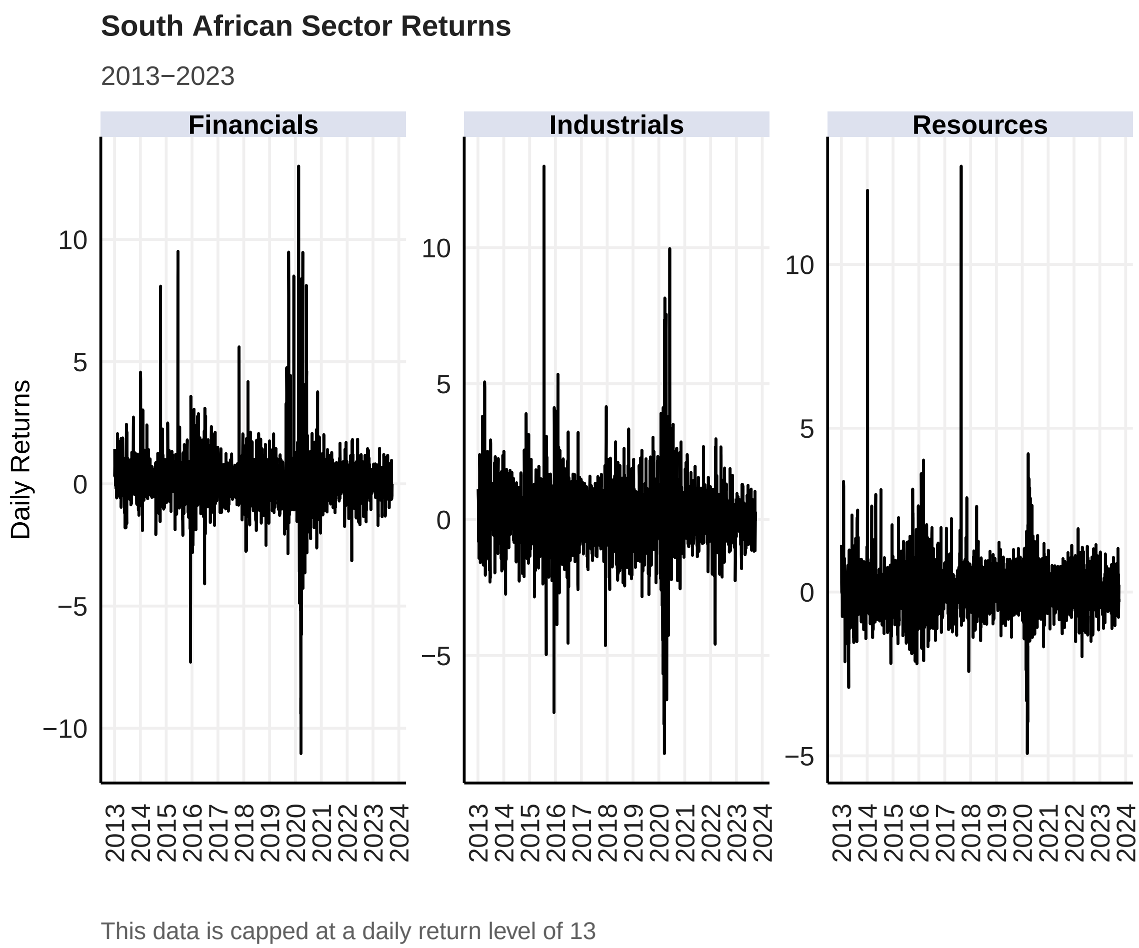

Figure 4.1: Cumulative Sector Returns (restricted data set)

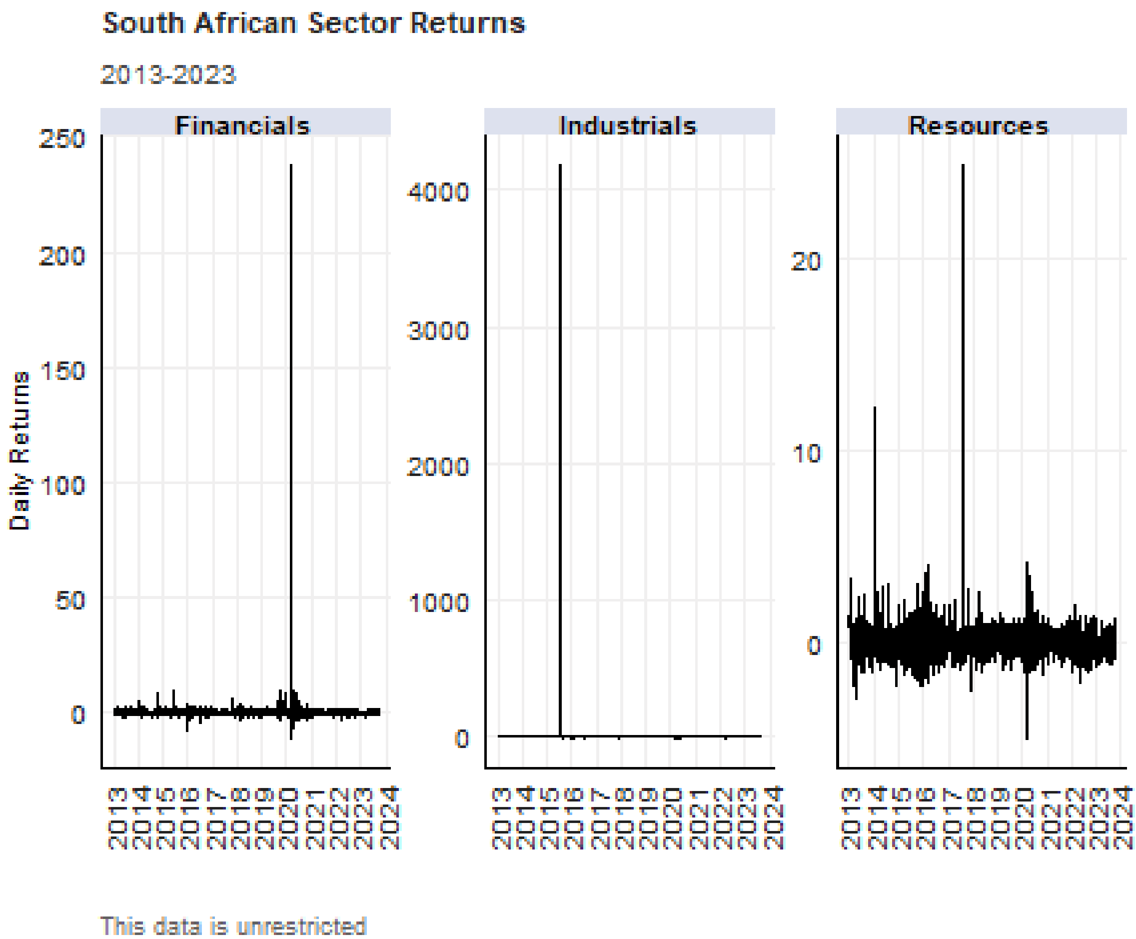

Figure 4.2: Cumulative Sector Returns (unrestricted data set)

Table 4.1 provides the descriptive statistics for each of the sector returns – this is done on the unrestricted data. The high kurtosis levels indicate that the returns are leptokurtic. However, looking at the descriptive data in Table 4.2, the results are more representative of the price movements.

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	Statistics	Financials	Industrials	Resources
1	Observations	2684.00	2684.00	2684.00
2	NAs	0.00	0.00	0.00
3	Minimum	-11.04	-8.61	-4.93
4	Quartile 1	-0.33	-0.49	-0.33
5	Median	0.12	0.13	0.05
6	Arithmetic Mean	0.32	3.25	0.10
7	Geometric Mean			
8	Quartile 3	0.59	0.76	0.47
9	Maximum	238.16	4186.93	24.97
10	SE Mean	0.13	2.21	0.02
11	LCL Mean (0.95)	0.08	-1.07	0.06
12	UCL Mean (0.95)	0.57	7.58	0.14
13	Variance	43.44	13058.35	1.11
14	Stdev	6.59	114.27	1.05
15	Skewness	34.98	36.59	10.87
16	Kurtosis	1258.74	1336.70	239.48

Table 4.1: Descriptive Statistics Table

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	Statistics	Financials	Industrials	Resources
1	Observations	2684.00	2684.00	2684.00
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4	Quartile 1	-0.33	-0.49	-0.33
5	Median	0.12	0.13	0.05
6	Arithmetic Mean	0.16	0.14	0.09
7	Geometric Mean			
8	Quartile 3	0.59	0.76	0.47
9	Maximum	13.00	13.00	13.00
10	SE Mean	0.02	0.02	0.02
11	LCL Mean (0.95)	0.11	0.10	0.06
12	UCL Mean (0.95)	0.20	0.19	0.12
13	Variance	1.41	1.58	0.78
14	Stdev	1.19	1.26	0.88
15	Skewness	1.56	0.80	4.37
16	Kurtosis	27.58	14.87	60.77

Table 4.2: Descriptive Statistics Table (restricted)

The MarchTest is used to test if there is a need for the MV-GARCH model. The p-values of all the Marchtests indicate that the GARCH model is appropriate to use for the data. There is significant autocorrelation present, which further supports the need for a GARCH model. The MV-GARCH model is most appropriate for identifying how each sector responds to shocks and if other sectors are exposed to this shock. This model will provide valuable information to investors who want to identify the risk of holding stocks that display co-movement during common and/or unique sector shocks, as this information helps with hedging purposes and portfolio diversification. The BEKK (Baba, Engle, Kraft and Kroner) (Hassan & Malik (2007)) test is a type of GARCH model that is used for estimating the conditional variance between each sector. Furthermore, the BEKK model is limited to handling a maximum of three variables at a time, as this means that 24 parameters need to be estimated. This model is a type of GARCH model that Hassan and Malik (2007) use in their paper to identify volatility transmission (Hassan & Malik (2007)).

5. Empirical Results

Three sectors in South Africa are investigated using the MV-GARCH model with BEKK parameters. These results are presented in Table 5.1 and Table 5.2, with each representing the unrestricted and restricted data sets. The BEKK parametrisation identifies the conditional variance of each sector and the conditional covariance between each sector and can provide some insight into the effect of news through the error/residual term.

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	Coefficient	Estimate	Std.Error	t.value	Pr
1	mu1.Financials	0.32	0.33	0.98	0.32536315
2	mu2.Industrials	3.25	3.02	1.08	0.28063270
3	mu3.Resources	0.10	0.26	0.38	0.70138429
4	A011	6.59			NaN
5	A021	-0.05	2.22	-0.02	0.98156858
6	A031	0.07	0.12	0.57	0.57078451
7	A022	114.27			NaN
8	A032	-0.03	0.07	-0.37	0.71046912
9	A033	1.05	0.10	10.70	2.22e-16***
10	A11	0.10	0.01	9.75	2.22e-16***
11	A21	0.02	0.08	0.24	0.81099501
12	A31	0.02	0.01	2.86	0.00422241**
13	A12	0.02	0.01	3.82	0.00013261***
14	A22	0.10	0.01	11.22	2.22e-16***
15	A32	0.02	0.00	5.12	3.0281e-07***
16	A13	0.02	0.18	0.11	0.91128638
17	A23	0.02	1.62	0.01	0.99015553
18	A33	0.10	0.19	0.51	0.60787898
19	B11	0.80	0.01	68.89	2.22e-16***
20	B21	0.02	0.13	0.15	0.87879699
21	B31	0.02	0.01	1.92	0.05512446
22	B12	0.02	0.00	51.53	2.22e-16***
23	B22	0.80	0.01	121.59	2.22e-16***
24	B32	0.02			NaN
25	B13	0.02	0.02	1.24	0.21510030
26	B23	0.02	0.15	0.13	0.89686714
27	B33	0.80	0.01	83.42	2.22e-16***

Table 5.1: BEKK11 Model

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	Coefficient	Estimate	Std.Error	t.value	Pr
1	mu1.Financials	0.17	0.02	10.67	2.22e-16***
2	mu2.Industrials	0.22	0.02	11.65	2.22e-16***
3	mu3.Resources	0.05	0.02	3.11	0.00188300**
4	A011	0.24	0.03	8.01	1.1102e-15***
5	A021	0.35	0.10	3.56	0.00037423***
6	A031	0.28	0.07	4.09	4.3256e-05***
7	A022	0.29	0.10	2.97	0.00294364**
8	A032	0.17	0.12	1.42	0.15656910
9	A033	0.60	0.02	23.85	2.22e-16***
10	A11	0.55	0.03	16.27	2.22e-16***
11	A21	0.01	0.02	0.83	0.40656331
12	A31	0.06	0.01	4.41	1.0297e-05***
13	A12	0.05	0.02	3.06	0.00221959**
14	A22	0.61	0.03	22.75	2.22e-16***
15	A32	0.01	0.01	0.97	0.33015808
16	A13	0.01	0.02	0.71	0.47599918
17	A23	0.05	0.02	2.52	0.01185218*
18	A33	0.52	0.03	18.93	2.22e-16***
19	B11	0.52	0.02	22.52	2.22e-16***
20	B21	-0.50	0.03	-15.80	2.22e-16***
21	B31	0.09	0.04	2.38	0.01712790*
22	B12	0.41	0.02	20.30	2.22e-16***
23	B22	0.85	0.03	24.52	2.22e-16***
24	B32	-0.03			NaN
25	B13	-0.26	0.03	-9.00	2.22e-16***
26	B23	-0.09	0.10	-0.93	0.35273150
27	B33	0.00			NaN

Table 5.2: BEKK11 Model (restricted data)

The table can be interpreted as follows:
Significance codes: 0 = *** & 0.001 = ** & 0.05 = *
Financials = 1
Industrials = 2
Resources = 3
eg: A21 refers to the spillover effects of the Industrial sector on the Financial Sector

The mean equations represent the expected return from each sector, with the null hypothesis representing a mean return of zero – these estimates are *mu.financials, mu.industrials, mu.resources* in both tables. Table 5.1 provides statistically insignificant mean returns for each sector, which means that there is not enough evidence to reject the null hypothesis. The restricted data set mean returns, as seen in Table 5.2, are statistically significant for all sectors (p-value < 0.05).

Understanding volatility persistence within each sector provides the investor with an understanding of how long to expect periods of irregular movement. The volatility persistence is estimated using the lagged squared standardised residuals – these estimates are from *A011* to *A033* in both tables. This analyses how much of the current volatility is conditional on past movements. The *A011*, *A022* and *A033* estimates (Table 5.1) indicate the volatility persistence for each sector, with the industrial sector displaying the highest levels of persistence. These same estimates from the restricted data set (Table 5.2) are all statistically significant (p-value < 0.05) and positive. Volatility persistence from the industrial and resource sectors remains persistent in the financial sector, as *A021* and *A031* are statistically significant (p-value < 0.05). These coefficients provide the investor with an idea of how long volatility will remain in each sector and how much from other sectors will persist.

A11 is a matrix coefficient that represents the volatility spillovers between sectors – these estimates are from *A11* to *A33* in both tables. It indicates the extent to which the past historical data influences the current covariance between and within each sector. The restricted data (Table 5.1) indicates that only five of the variables are significant (p-value < 0.05), with all estimates being positive. The financial sector is affected by volatility spillover from the resource sector (*A13* p-value). The industrial sector is vulnerable to volatility spillover from the financial and resource sectors, with each having an equal impact (equal estimates from *A21* & *A23*) and the resource sector being more statistically significant. The resource sector is unperturbed by any volatility from the other sectors (p-value > 0.05). The restricted data results (Table 5.2) have six significant variables present and all estimates are positive. The financial sector is vulnerable to volatility spillover from the resource sector, as the p-value is significant (*A31*). Volatility in the financial sector is transmitted to the industrial sector, indicated by the significant p-value (*A12*), but the converse is not present. Lastly, the resource sector remains unaffected by volatility movement in the other sectors. The volatility spill-over indicator provides invaluable information to an investor, as holding any financial or industrial sector equity requires constant vigilance of the other sectors, whereas the resource sector is unaffected. This information remains constant for both data sets.

The B matrix coefficients provide the shock impact of each sector. This is interpreted as the ability of the sector to deal with news from its own and other sectors – these estimates are from *B11* to *B33* in both tables. Table 5.1} only contains positive estimates, for the B coefficients, but only four of these estimates are significant at the five per cent level. The financial sector is significantly affected by news from its sector. Furthermore, although the p-value from the resource sector is not significant (p-value $= 0.055$), it can be substantiated that the resource sector news impacts the financial sector.

The industrial sector is influenced by shocks from its sector and the financial sector (p-value < 0.05). Lastly, the shock impact on the resource sector is only affected by news from its sector. Table 5.2 comprises a mix of positive and negative estimates of which six of them are significant at the five percent level. The financial sector is highly influenced by news in its sector and from the other two sectors, as all p-values are significant at the five per cent level. The industrial and resource sector news each respectfully has a positive and negative impact on the financial sector, with the former having a greater effect. The industrial sector is positively impacted by its shocks and shocks from the financial sector, with an unfortunate NaN for the resource impact. The resource sector's news impact cannot be interpreted, as a NaN is present, but any shock to the financial sector is significant and negatively impacts this sector. These findings are important to investors, as they can better understand the impact that shocks (endogenous and exogenous) have on sectors.

6. Conclusion

Since the introduction of the GARCH model, there have been several iterations of it, each slightly different, to provide the user with more sensible and robust estimates. The world of finance research and understanding of variance has been greatly improved using heteroskedastic conditional variance modelling. Logically, during economic swings or exogenous and/or endogenous shocks, it makes sense to expect local equity prices to remain in a volatile state (i.e. volatility clustering), this is taken into consideration for the risk/variance estimates. Volatility modelling has been sought after by many researchers, as identifying the closest estimate of deviation compared to the actual change in listed price allows for better portfolio optimisation. The BEKK parametrisation is a version of the MV-GARCH model that breaks down the volatility between three variables of interest and provides a much deeper understanding of volatility persistence, spillover and reaction to shocks.

This paper's findings indicate that although the South African economy is faced with a multitude of economic issues, the volatility created by this is identified with the use of the MV-GARCH model. The empirical findings are of great use to investors who want to hold local equities from the financial, industrial and resource sectors. The results can illustrate the source of volatility in their sector and how long it will remain (persistence), whether the volatility is carried over to another sector (spillover), and how well each sector can deal with shocks. These findings are invaluable for portfolio optimisation and deliver a better understanding of the resilience of these sectors in the South African climate.

This research can be expanded upon by including more key economic sectors of South Africa, as this will further help investors with portfolio optimisation, hedging and many more applications. However, this paper can be improved by dealing with the NaNs present in the data, as it will help identify which independent variable estimates are statistically significant (p-value < 0.05).

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