University of Cape Town

EEE3094S

Control

Lab Report 02

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10/10/2022

1 Introduction

Helicopters are a rotor craft that gains propulsion and lift from a set of horizontal, overhead rotary blades. These blades are affixed onto a set of hinges that allow for three angular degrees of freedom in their movement: vertical, longitudinal and lateral.

A helicopter is an example of aircraft that requires a great deal of control to ensure stable flight. It has four main inputs of control [1][2]: collective pitch control, throttle control, anti-torque control and cyclic pitch control.

This practical focuses on the vertical or altitude movement and control of the helicopter and thus throttle control as mentioned above. It tests the helicopter's ability to climb to a specified height and maintain it without oscillation. This operation requires the use of a controller which will ensure stability over the motion. The overall system can be shown in figure 1.0.1 below

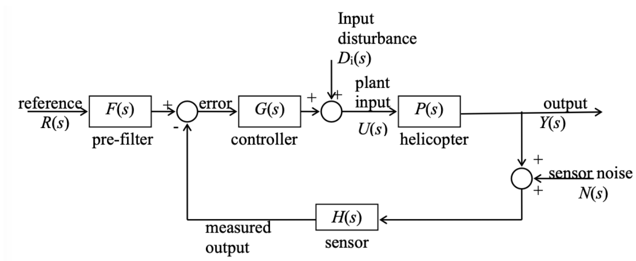


Figure 1.0.1: Helicopter System's Block Diagram

In the previous practical, the plant system P(s)in the system diagram in figure 1.0.1 above was identified using step and frequency response methods and the result was:

$$\frac{11,03}{s + 8,5s^2}$$

This practical is focused analysing the identified system in the time and frequency domains and designing a suitable controller or G(s) in the system diagram in figure 1.0.1 above.

2 Specifications

The specifications given to the controller project are:

1. A percentage overshoot of 20%
2. A 2% settling time ($\tau_{2\%}$) of 8s

Using the following equations, the damping ratio and resonant frequency can be calculated as:

$$\%overshoot = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \tag{1}$$

$$20\% = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\therefore \zeta = \sqrt{\frac{(\frac{ln(0,2)}{-\pi})^2}{1 + (\frac{ln(0,2)}{-\pi})^2}} = 0,456$$

$$\tau_{2\%} = \frac{ln(\frac{2}{100}\sqrt{1-\zeta^2})}{-\zeta\omega_n} \tag{2}$$

$$\therefore \omega_n = \frac{ln(\frac{2}{100}\sqrt{1-\zeta^2})}{-\zeta\tau_{2\%}} = \frac{ln(\frac{2}{100}\sqrt{1-0,456^2})}{-(0,456)(8)} = 1,104 rad/s$$

Therefore the required transfer function A(s) is:

$$A(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1,104^2}{s^2 + 2(0,456)(1,104)s + 1,104^2} = \frac{1,218816}{s^2 + 1,006848s + 1,218816}$$

The closed loop required sensitivity transfer function can be derived from the open loop required transfer function:

$$CL = 1 - A(s) = 1 - \frac{1,218816}{s^2 + 1,006848s + 1,218816}$$

The bode plot of the closed loop required transfer function is plotted below in figure 2.0.1.

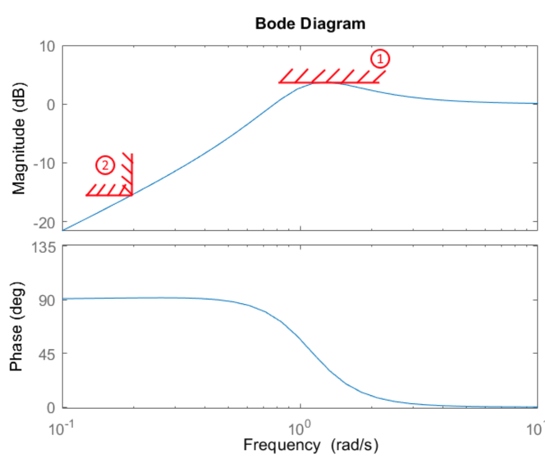


Figure 2.0.1: Specification Bode Plot with Boundaries

To derive the frequency specifications, two point were chosen from the bode plot and translated into specifications relating to the sensitivity function $\left|\frac{1}{1+L}\right|$. These correspond to the red grid boundaries and numbering on the bode plot and are as follows:

1. $\left|\frac{1}{1+L}\right| <$ 3dB $\forall\omega$
2. $\left|\frac{1}{1+L}\right| <$ -15dB, $\omega =$ 0,2rad/s
3. Zero steady state error

Note that point 3 is seen as $\left|\frac{1}{1+L}\right| \to -\infty$ as $\omega \to 0$

3 System Analysis

The system was modelled in MATLAB allowing for analysis of the system using time and frequency domain plots and experimentation with different controllers in a simulated environment. This was done using the model as shown by figure 3.0.1 below. Before the input is fed to the DAQ, an offset of 2.5V is added to account for the fact that 2.5V will make the vertical velocity of the helicopter zero. The saturation lines on the input and output model the maximum input and output of the DAQ and the -2.5V shows the previously mentioned fact of the vertical velocity being 0 if the input is 2.5V.

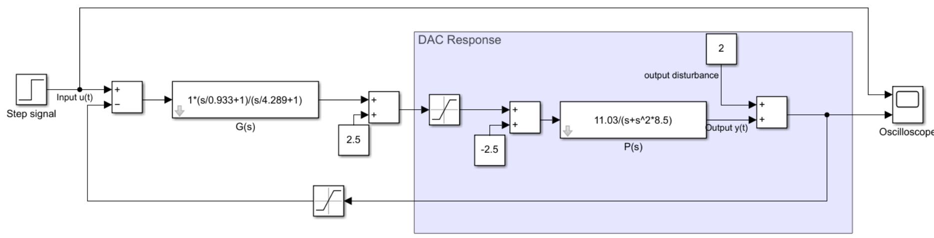


Figure 3.0.1: MATLAB Model

The step signal in figure 3.0.1 is 5 however in the actual system it needs to account for the ratio of voltage applied to height which was determined in System Identification as 0.41. This means that a voltage of 2.05 will result in a height of 5m ($V = 0.41h$).

The uncontrolled system in closed loop was analysed in several domains as shown by the following figures:

1. The Step Response

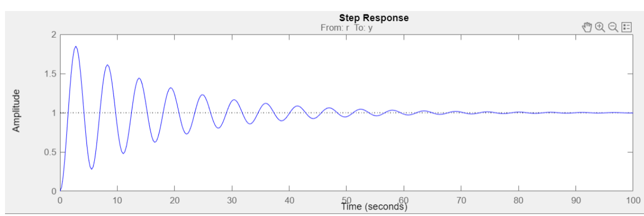


Figure 3.0.2: Step Response

The step response above shows that the uncontrolled response of the system is highly oscillatory. It also has a very slow time response of over 70 seconds and the overshoot is over 70%. All of these qualities are unwanted which is why a controller will be designed.

2. The Roots Locus Plot

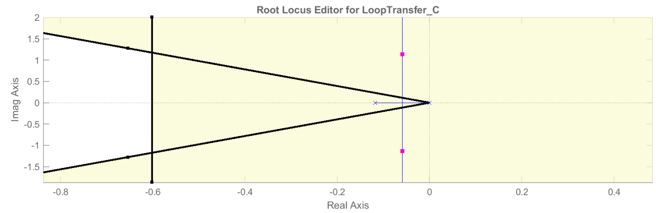


Figure 3.0.3: Root Locus Plot

The root locus shown above has plotted the specifications on it shown as yellow shaded areas. These areas represent the domains of the root locus that should not have poles in them. The uncontrolled system does not meet overshoot specifications or time specifications.

3. The Nyquist Plot

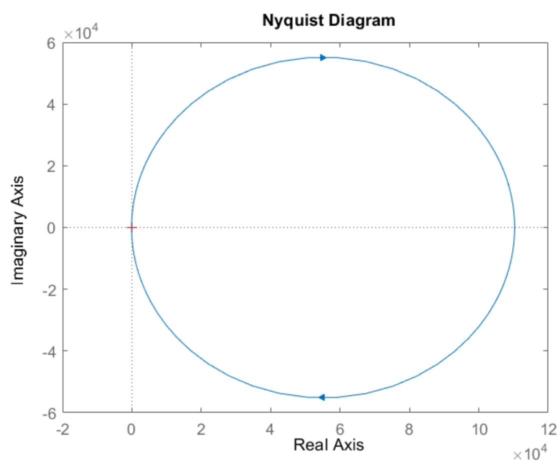


Figure 3.0.4: Nyquist Plot

The Nyquist plot shows that the number of times the -1 point is enclosed is zero and there are no open loop unstable poles which means that there will be no closed loop unstable poles. This means that no poles need to be shifted into the stable region.

4. The Inverse Nichols Chart

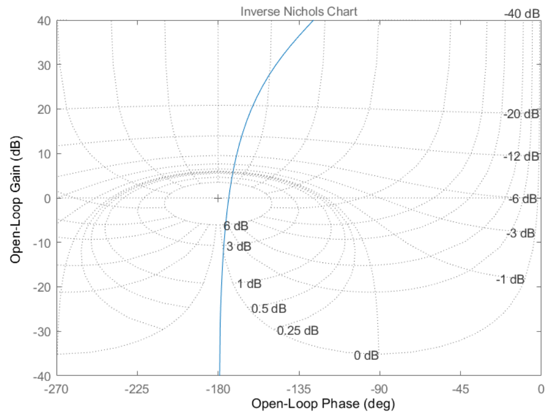


Figure 3.0.5: Inverse Nichols Chart

The Inverse Nichols chart shows that when the plant is put in closed loop, its sensitivity $\left|\frac{1}{1+L}\right|$ goes into the central region of extremely high gain on the chart. This property is undesired and so needs to be corrected by a controller. It does however show that the closed loop gain tends to $-\infty$ which is desired as it gives zero steady state error.

4 Controller Design

The controllers were initially assessed using MATLAB's sisotool to examine the root locus of the controlled system and determine if it meets the practical's specifications.

4.1 Proportional Controller

A proportional controller of the form G(s) = k where k is a constant integer value. In this simulated case, the k value was set to 0,0033. This controlled system produced the root locus shown in figure 4.1.1 below and step response shown in figure 4.1.2:

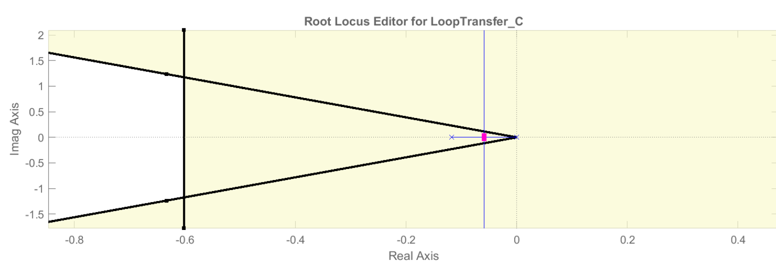


Figure 4.1.1: P Controller Root Locus

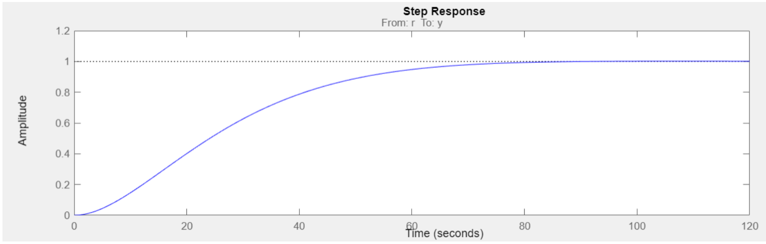


Figure 4.1.2: P Controller Step Response

From this we can conclude that although the proportional controller aids in reduction of overshoot, it cannot meet the time specifications that the practical requires. Therefore it is insufficient as a controller.

4.2 Proportional Integral Controller

A proportional integral controller of the form $G(s) = \frac{k(s/\alpha+1)}{s}$ where k and α are constant integer values and s is the Laplace transform variable. In this simulated case, the k and α values were set to 1. This controlled system produced the root locus shown in figure 4.2.1 below and step response shown in figure 4.2.2:

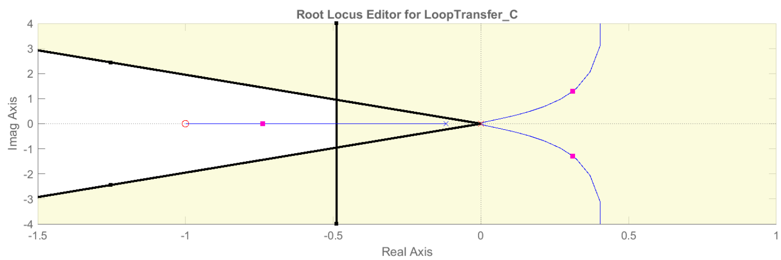


Figure 4.2.1: PI Controller Root Locus

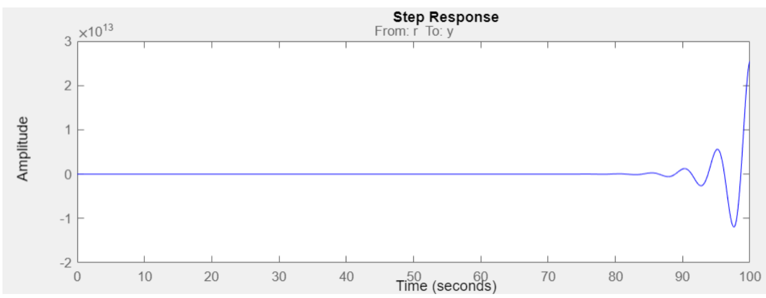


Figure 4.2.2: PI Controller Step Response

From this we can conclude that a proportional integral controller will cause the zeros to become unstable in closed loop configuration. Therefore it is insufficient as a controller.

4.3 Lead Controller

A proportional integral controller of the form $G(s) = \frac{k(s/\alpha+1)}{s/\beta+1}$ where k, α and β are constant integer values and s is the Laplace transform variable. For the lead controller, $\alpha < \beta$. In this simulated case, the k value was set to 1, the α values were set to 0,6 and the β value was set to 3. This controlled system produced the root locus shown in figure 4.3.1 below and step response shown in figure 4.3.2:

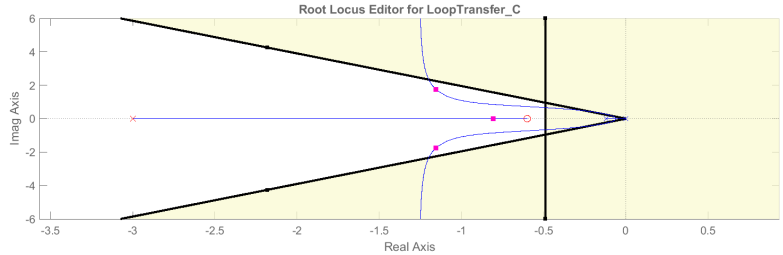


Figure 4.3.1: Example Lead Controller Root Locus

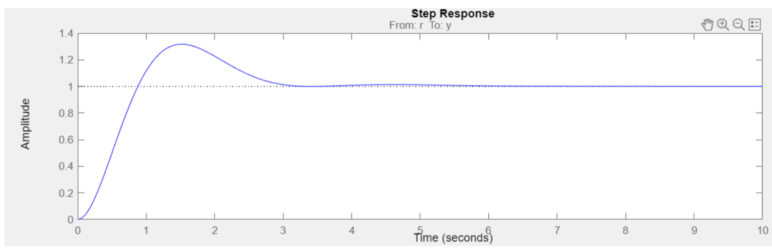


Figure 4.3.2: Example Lead Controller Step Response

This proposed controller seems most appropriate as it meets the project's specifications regarding settling time and overshoot. To implement this controller, accurate values for for k, α and β variables need to be calculated which can be done using frequency and root locus design methods.

4.3.1 Root Locus Design

The root locus in figure 3.0.3 above can be redrawn in figure 4.3.3 below to include the pole and zero of the controller. Using this diagram and assigned angle variables, the following calculations can be performed to determine the position of the pole and zero of the controller and thus the controller function G(s):

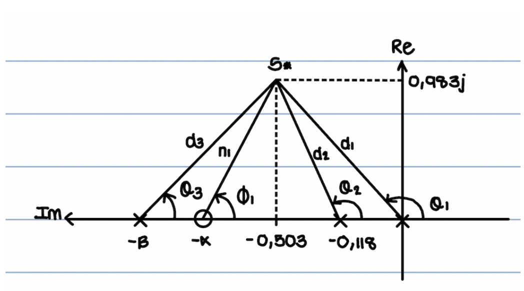


Figure 4.3.3: Root Locus Design

$$\theta_1 = 180^\circ - cos^{-1}(\zeta) = 180 - cos^{-1}(0,456) = 117,13^\circ$$

$$\theta_2 = 180° + cos^{-1}(\frac{0,983}{0,118 - 0,503}) = 111,39°$$

$$\phi_1 - \theta_3 = -180° + \theta_1 + \theta_2 = -180° + 117,13° + 111,39° = 48,52°$$

The final angles are determined using a trick where the line that bisects the horizontal line through the specification point and the line to the origin also bisects $\phi_1 - \theta_3$.

$$\phi_1 = \frac{\theta_1}{2} + \frac{\phi_1 - \theta_3}{2} = 58,57° + 24,26° = 82,83°$$

$$\theta 3 = \frac{\theta_1}{2} - \frac{\phi_1 - \theta_3}{2} = 58,57° - 24,26° = 34,31°$$

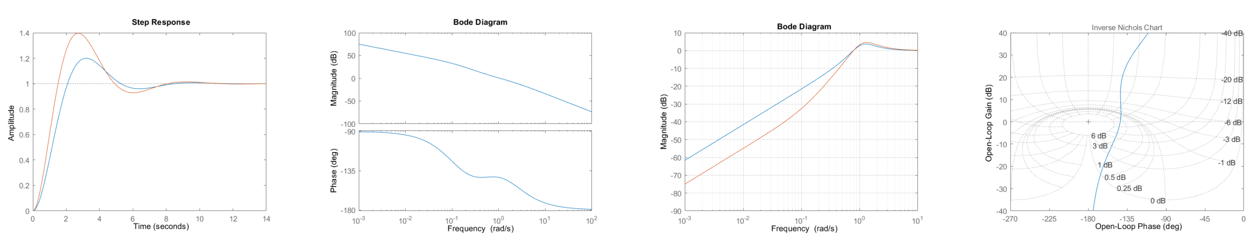
$$\phi_1 = tan^{-1}(\frac{0,983}{\alpha - 0,503}) \therefore \alpha = 0,627$$

$$\theta_1 = tan^{-1}(\frac{0,983}{\beta - 0,503}) \therefore \beta = 1,943$$

$$d_1 = \omega_n = 1,104$$
$$d_2 = |-0,503 + j0,983 + 0,118| = 1,056$$
$$d_3 = |-0,503 + j0,983 + 1,943| = 1,744$$
$$n_1 = |-0,503 + j0,983 + 0,627| = 0,991$$

$$\frac{11.03}{8,5} \cdot \frac{k\beta}{\alpha} = \frac{d_1 \cdot d_2 \cdot d_3}{n_1} = \frac{1,104 \cdot 1,744 \cdot 1,056}{0,991} = 2,052$$
$$\therefore k = 0,51$$

Therefore the final function of the controller G(s) is: $G(s) = \frac{0,51(\frac{s}{0,627}+1)}{(\frac{s}{1,943}+1}$
Simulating this proposed controller design in MATLAB produces the following graphs: figure 4.3.4a below shows the design's step response (red) against the specification step response (blue), figure 4.3.4b below shows the design's bode plot, figure 4.3.4c below shows the sensitivity function's bode plot of the design (red) against the specification (blue) and figure 4.3.4d shows the design's inverse Nichols Chart.



(a) Step Response (b) Bode Plot (c) Sensitivity Bode Plot (d) Inverse Nichols Chart

Figure 4.3.4: Root Locus Design

Figure 4.3.4a shows that the response this controller has acceptable time properties but the overshoot is 40% not the desired 20%. The sensitivity of the controller in figure 4.3.4c shows that it meets the gain specifications but the roll-off is different. The inverse Nichols chart in figure 4.3.4d shows how the controller has been moved out of the high closed loop gain area using a lead controller.

This controller was then implemented using the MATLAB model as shown in figure 3.0.1 above. This circuit takes the 2,5V offset and the saturation of the DAQ into account. Figure 4.3.5 below shows the resulting step response:

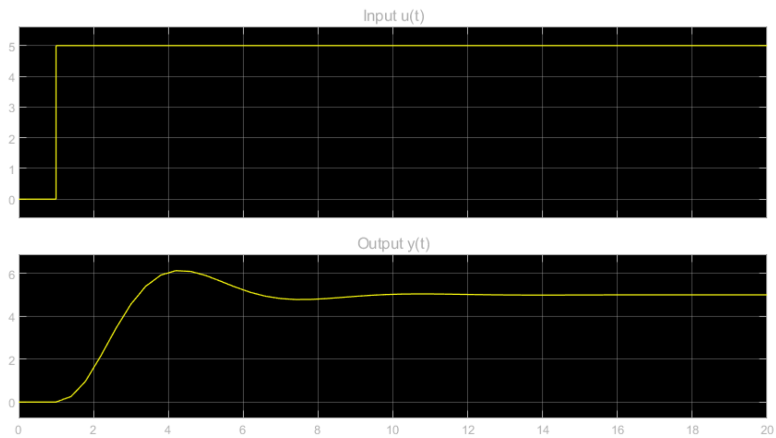


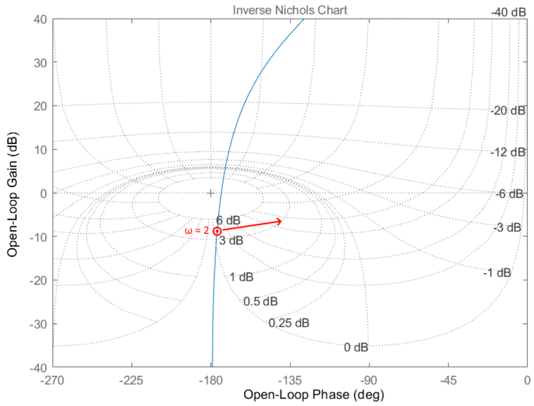
Figure 4.3.5: Root Locus Design Controller Step Response

Again, figure 4.3.5 shows the undesired overshoot that comes with this controller design.

4.3.2 Frequency Design

From the specification (2) as outlined in section 2 above. We can let $\omega = 0,2$ rad/s and determine the loop transfer function to be $|L|$ = 28,9 dB. On the inverse Nichols chart in figure 3.0.5 above this falls between -20dB & -40dB in the closed loop response which satisfies the first specification. Therefore no gain is required to be added and the gain of the controller k can be set to 1.

A frequency point ω = 2rad/s was chosen to design the controller around. At this point the magnitude of the loop function $|L|$ = -9,79dB and the phase of the loop function $\measuredangle$ = -176°. This point can be seen on the inverse Nichols chart in figure 4.3.6 below:



It is evident from the inverse Nichols chart that the system requires a phase lead and the maximum phase lead $\phi = 40°$. From this, the variables of the controller function can be calculated as follows:

Figure 4.3.6: Inverse Nichols Chart with Maximum Phase Lead

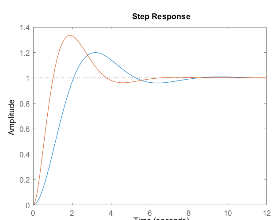
$$\alpha = \frac{\omega}{tan(\frac{\phi_{max}+90°}{2})} = \frac{2}{tan(\frac{40°+90°}{2})} = 0,933$$

$$\beta = \frac{\omega^2}{\alpha} = \frac{2^2}{0,933} = 4,289$$

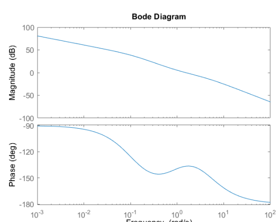
Therefore the final function of the controller G(s) is:

$$G(s) = \frac{k(\frac{s}{\alpha}+1)}{(\frac{s}{\beta}+1)} = \frac{(\frac{s}{0,933}+1)}{(\frac{s}{4,289}+1)}$$

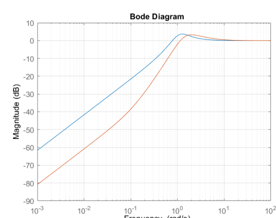
Simulating this proposed controller design in MATLAB produces the following graphs: figure 4.3.7a below shows the design's step response (red) against the specification step response (blue), figure 4.3.7b below shows the design's bode plot, figure 4.3.7c below shows the sensitivity function's bode plot of the design (red) against the specification (blue) and figure 4.3.7d shows the design's inverse Nichols Chart.



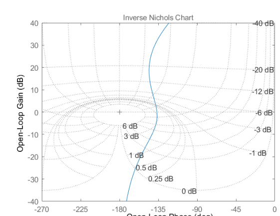
(a) Step Response



(b) Bode Plot



(c) Sensitivity Bode Plot



(d) Inverse Nichols Chart

Figure 4.3.7: Frequency Design

Figure 4.3.7a shows that the response this controller has acceptable time properties but the overshoot less than 40% but still not the desired 20%. The sensitivity of the controller in figure 4.3.7c shows that it meets the gain specifications but the roll-off is different. The inverse Nichols chart in figure 4.3.7d shows how the controller has been moved out of the high closed loop gain area using a lead controller.

This controller was then implemented using the MATLAB model as shown in figure 3.0.1 above. This circuit takes the 2,5V offset and the saturation of the DAQ into account. Figure 4.3.8 below shows the resulting step response:

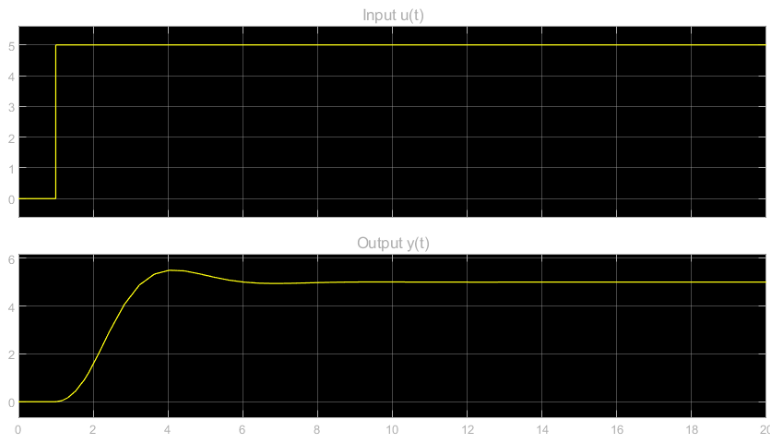


Figure 4.3.8: Root Locus Design Controller Step Response

Comparing the two controller's designed in the Root Locus Design and Frequency Design sections above. Specifically the actual step responses in figures 4.3.5 and 4.3.8 above. The controller designed in the Frequency Design section above is favoured as the step response better meets the practical's specifications.

5 Implementation

From the design sections above it is clear that the lead controller is best suited for this system and this section outlines its component calculation and implementation. The circuit diagram showing the controller implementation is in figure 5.0.1 below:

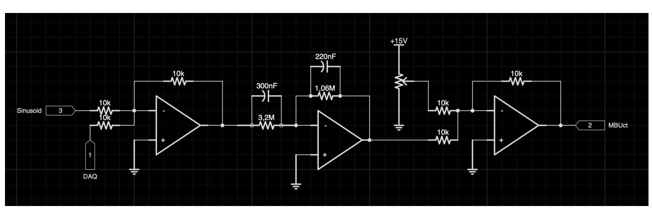


Figure 5.0.1: Circuit Diagram

Using the frequency domain transfer function the resistor and capacitor values can be calculated as follows:

$$\alpha = R_1C_1 = \frac{1}{0,933} = 1,07181$$

$$\beta = R_2 C_2 = \frac{1}{4,289} = 0,233154$$

Setting the capacitor values to be $C_1 = 330\text{nF}$ and $C_2 = 220\text{nF}$ the resistor values can be calculated as $R_1 = 3,24\text{M}\Omega$ and $R_2 = 1,059790\text{M}\Omega$. The closest available standard resistor values are $R_1 = 3,2\text{M}\Omega$ (combination of 1MΩ and 1,2MΩ) and $R_2 = 1,056\text{M}\Omega$ (combination of 1MΩ and 56kΩ).

When this circuit was tested it did not track the input reference as initially expected. This could be due to varying circuit limiting or external environment factors that hinders the circuit from operating as hypothesised or issues with the accuracy of the MBUct system used to simulate the helicopter. The circuit was therefore changed to apply a gain to the signal. This can be seen by the new circuit in figure 5.0.2 below.

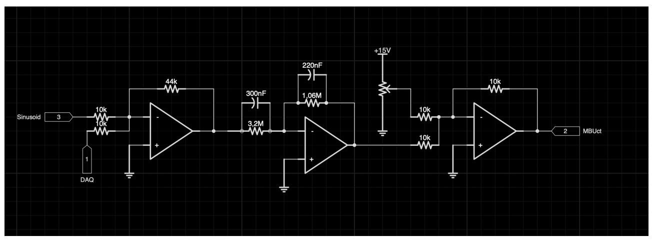
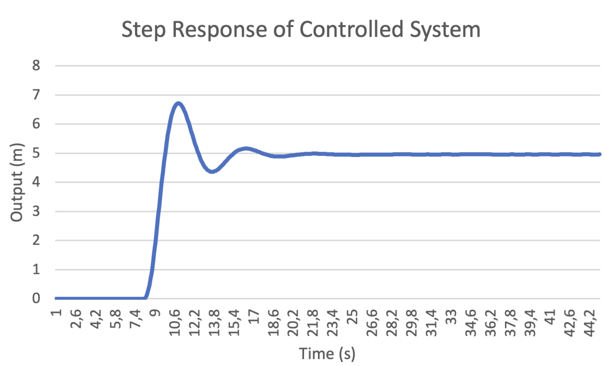


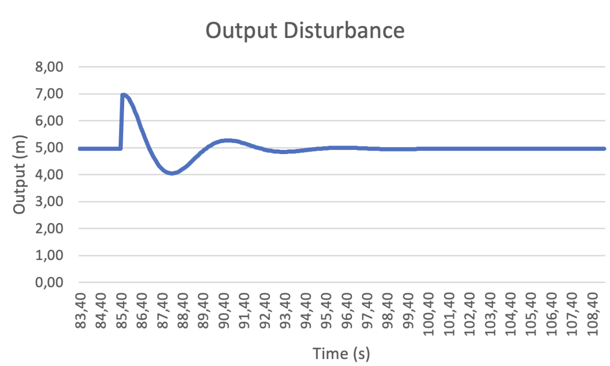
Figure 5.0.2: Revised Circuit Diagram

6 Testing and Results

The proposed circuit in figure 5.0.2 above was built as shown by figure 9.1.1 in Appendix 9.1 below. The circuit was connected to the MBUct Helicopter simulation software and tested. The following results were noted: Figure 6.0.1a below shows the step response of the controlled system. Although the overshoot is too large which contradicts specification 1, the system stabilizes well and within the settling time constraint and thus meets specification 2. And figure 6.0.1b below shows the system's response when dealing with an output disturbance. The system is initially stable when the output disturbance is introduced, the system then rejects the disturbance and re-stabilises to its previous position.



(a) Step Response of the Controlled System



(b) Output Disturbance

7 Digital Controller

7.1 Design

A digital controller was also designed for the helicopter. A digital system, which is used to implement the controller (eg. a microcontroller), is a discrete system and this sampling introduces inaccuracy. For controllers, digital sampling introduces a phase lag to a system which should be incorporated when designing the lead controller.

The digital controller designed for the helicopter will have fast sampling and so the phase lag can be ignored. The discrete transfer equation can be calculated using the bilinear transform. A sampling speed of T = 0,05s is chosen to be fast enough.

The bilinear transform is: $s \approx \frac{2}{T}\frac{z-1}{z+1}$
This is used in the continuous time function of the controller to get an approximate in the discrete time domain:

$$G(z) = \frac{4.4(\frac{2}{T}\frac{z-1}{z+1}/0,933 + 1)}{(\frac{2}{T}\frac{z-1}{z+1}/4,289 + 1)}$$

$$G(z) = \frac{18,69z - 17,84}{z - 0,8063}$$

7.2 Simulation

The discrete model is added to the continuous time MATLAB model in figure 3.0.1 above to produce figure 7.2.1 shown below:

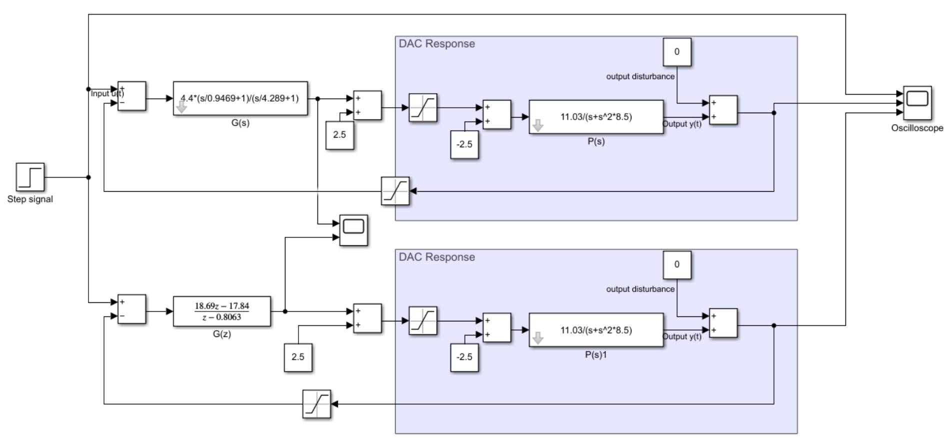
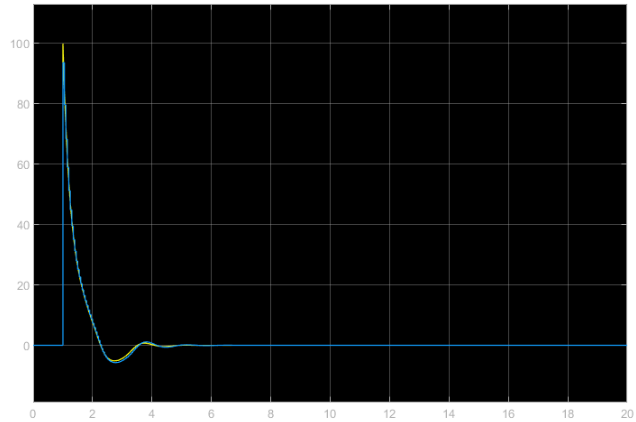
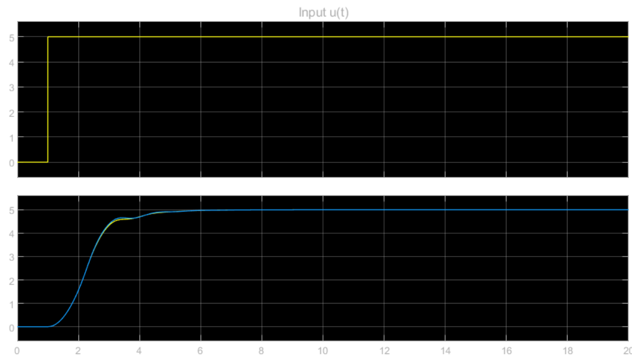


Figure 7.2.1: MATLAB Model including the discrete time controller

The two oscilloscopes in figure 7.2.1 are used to compare the signal at the input to the plant shown in figure 7.2.2a and the overall movement of the helicopter 7.2.2b. In both the continuous time response is in yellow and the discrete time response in blue.



(a) Input to plant



(b) Step Response

Figure 7.2.2: Discrete Controller system response

Figure 7.2.2a shows the sampling in that the input to the plant is changing in steps rather than continous and it would change every 0.05s. This gives an output in figure 7.2.2b that is almost identical to the continuous time controller.

8 Conclusion

In order to better control the response of a plant, it can be placed in closed loop and have a controller added before the input to the plant. In this case the plant was the height of the helicopter and the final designed controller was:

$$G(s) = \frac{(\frac{s}{0{,}933} + 1)}{(\frac{s}{4{,}289} + 1)}$$

This analogue controller could also be implemented as a digital controller (with some error) with the sampling time of T = 0.05s as:

$$G(z) = \frac{18,69z - 17,84}{z - 0,8063}$$

The actual response of the helicopter did not completely match the simulated response but this can be attributed to the saturation effects as well as the uncertainty surrounding using actual equipment. The above controllers did, however, allow the helicopter to be better controlled and so could be implemented.

9 Appendix

9.1 Appendix 01: Circuit

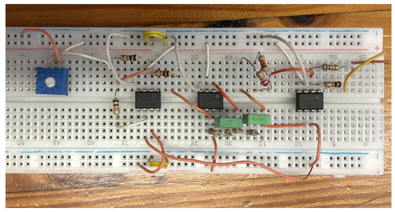
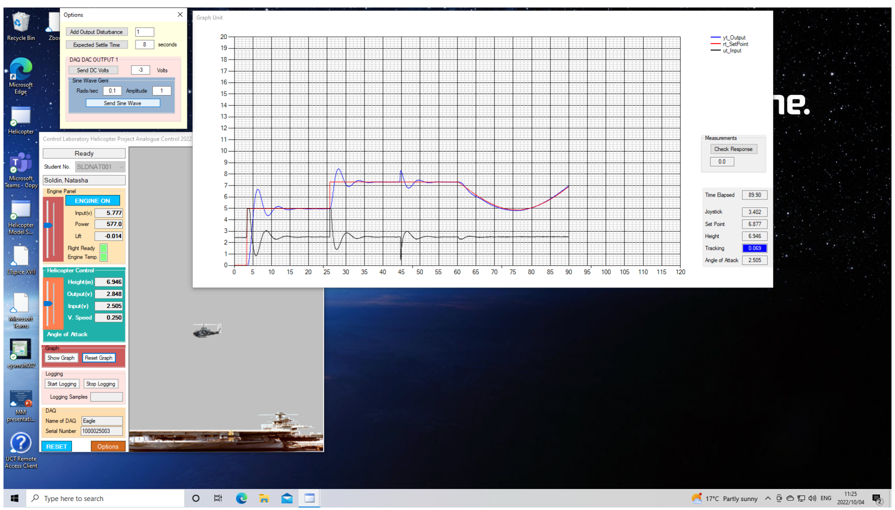
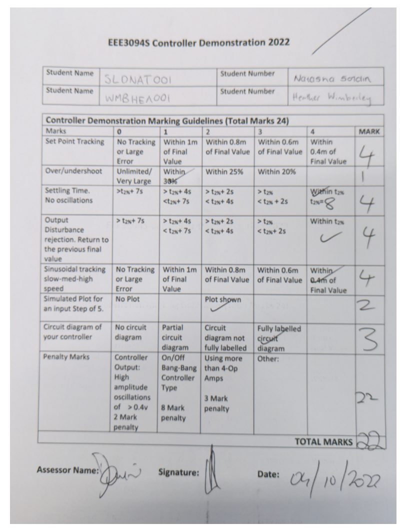


Figure 9.1.1: Circuit

9.2 Appendix 02: Demonstration



(a) Demonstration Graph



EEE3094S Controller Demonstration 2022

Student Name	SLDNAT001	Student Number	Natasha Soldin
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Controller Demonstration Marking Guidelines (Total Marks 24)						
Marks	0	1	2	3	4	MARK
Set Point Tracking	No Tracking or Large Error	Within 1m of Final Value	Within 0.8m of Final Value	Within 0.6m of Final Value	Within 0.4m of Final Value	4
Over/undershoot	Unlimited/ Very Large	Within 30%	Within 25%	Within 20%		1
Settling Time. No oscillations	$>t_{2\%}+7s$	$>t_{2\%}+4s$ $<t_{2\%}+7s$	$>t_{2\%}+2s$ $<t_{2\%}+4s$	$>t_{2\%}$ $<t_{2\%}+2s$	Within $t_{2\%}$ $t_{2\%}=8$	4
Output Disturbance rejection. Return to the previous final value	$>t_{2\%}+7s$	$>t_{2\%}+4s$ $<t_{2\%}+7s$	$>t_{2\%}+2s$ $<t_{2\%}+4s$	$>t_{2\%}$ $<t_{2\%}+2s$	Within $t_{2\%}$	4
Sinusoidal tracking slow-med-high speed	No Tracking or Large Error	Within 1m of Final Value	Within 0.8m of Final Value	Within 0.6m of Final Value	Within 0.4m of Final Value	4
Simulated Plot for an input Step of 5.	No Plot		Plot shown			2
Circuit diagram of your controller	No circuit diagram	Partial circuit diagram	Circuit diagram not fully labelled	Fully labelled circuit diagram		3
Penalty Marks	Controller Output: High amplitude oscillations of >0.4v 2 Mark penalty	On/Off Bang-Bang Controller Type 8 Mark penalty	Using more than 4-Op Amps 3 Mark penalty	Other:		22
					TOTAL MARKS	22

Assessor Name: Signature: Date: 04/10/2022

(b) Demonstration Mark

Figure 9.2.1a above shows the simulated set-up on a PC in the control lab. The above graph shows the system's output for four instances: (1) The graph initially tracks the input step or input reference. (2) Then the reference step is increased and the graph tracks it again. (3) An output disturbance is applied to the system and the graph indicates that the system rejects the disturbance and tracks back to its previous state. (4) The reference is changed to a sinusoid and the graph indicates that the system follows and tracks the sinusoid.

References

[1] Control functions. https://www.britannica.com/technology/helicopter/Control-functions.

[2] Helicopter flight controls, Jul 2022. https://en.wikipedia.org/wiki/Helicopter_flight_controls.