Power Gain

$$P_{in} = P_l + P_{rad}$$
$$= \tfrac{1}{2} |I_{in}|^2 (R_l + R_{rad})$$

$r_{min} = \frac{2d^2}{\lambda}$.

$$\boxed{G_p(\theta,\phi) = \frac{4\pi\, U(\theta,\phi)}{P_{in}}}$$

Radiation Efficiency $\boxed{\eta_r = \frac{G_p}{G_d} = \frac{P_{rad}}{P_{in}}}$; G_p and G_d expressed in dB.

$$\eta_r = \frac{R_{rad}}{R_{rad} + R_l}$$

Directive Gain proof.

For Hertzian

$$G_d(\theta,\phi) = \frac{4\pi f^2(\theta)}{\int f^2(\theta)\, d\Omega} = \frac{4\pi \sin^2\theta}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin^3\theta\, d\theta\, d\phi} = \frac{4\pi \sin^2\theta}{2\pi \times 4/3}$$
$$= 1.5 \sin^2\theta.$$

For Half-wave

$$G_d(\theta,\phi) = \frac{4\pi \dfrac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \dfrac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta\, d\phi} \quad 1.21\text{ck}$$

Power received $\boxed{P_r = \int P_{avg} \cdot ds = P_{avg}\, S}$

Effective Area $\boxed{A_e = \frac{P_r}{P_{ave}}}$

$$P_r = \frac{E^2 \lambda^2}{640\pi^2}$$
$$P_{avg} = \frac{E^2}{2\eta_0} = \frac{E^2}{240\pi}.$$

$$\boxed{A_e = \frac{\lambda^2}{4\pi} D}$$

$$\boxed{A_e = \frac{\lambda^2}{4\pi} G_d(\theta,\phi)}$$

Hold for any Antenna

Half-wave Dipole Antenna

$0 \le \theta \le \pi$

$0 \le \phi \le 2\pi$

$$A_{zs} = \frac{\mu I_0 e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \beta \sin^2\theta}$$

$$H_{\phi s} = \frac{j I_0 e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta} \quad ; \quad E_{\theta s} = \eta H_{\phi s}$$

$$P_{avg} = \frac{1}{2}\eta |H_{\phi s}|^2 a_r = \frac{\eta I_0^2 \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2 \sin^2\theta}\hat{a}_r$$

$$P_{rad} = \int P_{avg} \cdot ds$$

$$P_{rad} = 36.56\, I_0^2$$

$$\boxed{R_{ad} = \frac{2P_{rad}}{I_0^2} = 73\,\Omega}$$

$P_{rad} = \frac{1}{2} I_0^2 R_{rad}$, $R_{rad} = 73\,\Omega$

$$\boxed{|H_{\phi s}| = \frac{I_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}}$$

$$\boxed{|E_{\theta s}| = \frac{\eta_0 I_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}}$$

$$Z_{in} = R_{in} + jX_{in}$$

$$\boxed{Z_{in} = 73 + j\,42.5}$$

Quarter-wave Monopole

$$P_{rad} \simeq 18.28\, I_0^2$$

$$R_{rad} = \frac{2P_{rad}}{I_0^2} = 36.5\,\Omega$$

$$Z_{in} = 36.5 + j21.25\,\Omega$$

Small loop Antenna

$$E_{\phi s} = \frac{\eta \pi I_0 S}{r\lambda^2}\sin\theta\, e^{-j\beta r} = \frac{120\pi^2 I_0}{r}\frac{S}{\lambda^2}\sin\theta\, e^{-j\beta r}$$

$$H_{\theta s} = -\frac{E_{\phi s}}{\eta} \qquad E_{rs} = E_{\theta s} = H_{rs} = H_{\phi s} = 0$$

$$\boxed{|E_{\phi s}| = \eta \frac{\pi I_0}{r}\frac{S}{\lambda^2}\sin\theta} \qquad \boxed{|H_{\phi s}| = \frac{\pi I_0}{r}\frac{S}{\lambda^2}\sin\theta}$$

For a single turn, $S = \pi \rho_0^2$; For N-turn, $S = N\pi\rho_0^2$.

↓ for square

$S = a^2$

$S = Na^2$

$$\boxed{R_{rad} = \frac{320\pi^4 S^2}{\lambda^4}}$$

Antenna

gamma — $\boxed{\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}}$

$Z_{in} = 73 + j\,42.5$ — Half wave

$Z_{in} = 36.5 + j\,21.25$ — Quarter wave

$\boxed{S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}}$

Standing wave ratio

Radiation Intensity

$$U(\theta, \phi) = r^2 P_{avg.} \quad (W/sr)$$

$$P_{rad} = \oint_S P_{avg.}\, ds = \boxed{\oint_S P_{avg}\, r^2 \sin\theta\, d\theta\, d\phi}$$

$$\boxed{P_{rad} = \int_S U(\theta,\phi) \sin\theta\, d\theta . d\phi}$$

$d\Omega = \sin\theta\, d\theta\, d\phi$

unit - sr (steradian)

$$\boxed{U_{avg.} = \frac{P_{rad}}{4\pi}}$$

Directive Gain — measure of concentration of the radiated power in a particular dir^n (θ, ϕ).

$$\boxed{G_d(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}} = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}}$$

or,

$$\boxed{P_{avg} = \frac{G_d\, P_{rad}}{4\pi r^2}}$$

The directive gain $G_d(\theta,\phi)$ depends on antenna pattern.

Directivity

$$D = \frac{U_{max}}{U_{avg}} \qquad \boxed{D = \frac{4\pi\, U_{max}}{P_{rad}}}$$

For Hertzian dipole,

$$G_d(\theta,\phi) = 1.5 \sin^2\theta \qquad D = 1.5$$

For $\lambda/2$ dipole,

$$G_d(\theta,\phi) = \frac{\eta}{\pi R_{rad}} f^2(\theta) \qquad D = 1.64$$

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

Antenna

Hertzian Dipole

$$A = \frac{\mu [I] dl}{4\pi r} \hat{a}_z \qquad [I] = I_0 \cos \omega (t - r/u)$$
$$= I_0 \cos(\omega t - \beta r)$$
$$\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda}$$

$$A_{zs} = \frac{\mu I_0 dl}{4\pi r} e^{-j\beta r}$$

$$A_s = (A_{rs}, A_{\theta s}, A_{\phi s}) \; ; \; A_{rs} = A_{zs}\cos\theta, \; A_{\theta s} = -A_{zs}\sin\theta$$
$$A_{\phi s} = 0.$$

$$B_s = \mu H_s = \nabla \times A_s$$

$$H_{\phi s} = \frac{I_0 dl}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2}\right] e^{-j\beta r}$$

$$H_{rs} = 0 = H_{\theta s}$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} \; ; \; \nabla \times H_s = j\omega\epsilon E_s$$

$$E_{rs} = \eta \frac{I_0 dl}{2\pi} \cos\theta \left[\frac{1}{r^2} - \frac{j}{\beta r^3}\right] e^{-j\beta r}$$

$$E_{\theta s} = \eta \frac{I_0 dl}{4\pi} \sin\theta \left[\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3}\right] e^{-j\beta r}$$

$$E_{\phi s} = 0$$

$$\boxed{P_{avg.} = \tfrac{1}{2} \eta \, |H_{\phi s}|^2 \hat{a}_r} \qquad P_{rad} = \int P_{avg} \cdot dS$$

imp

$$\boxed{P_{rad} = \frac{I_0^2 \pi \eta}{3} \left[\frac{dl}{\lambda}\right]^2 = 40\pi^2 \left[\frac{dl}{\lambda}\right]^2 I_0^2} \qquad P_{rad} = I_{rms}^2 R_{rad}$$

imp

$$\boxed{R_{rad} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2.}$$

imp

$$\boxed{|H_{\phi s}| = \frac{I_0 \beta dl \sin\theta}{4\pi r}}$$

$$|H_{\phi s}| = \frac{I_0 \beta \, dl \sin\theta}{4\pi r}$$

For Hertzian dipole,

normalize $|E_s|$

$$f(\theta) = |\sin\theta|$$

Normalized power pattern

$$f^2(\theta) = \sin^2\theta$$

$$\underline{U(\theta) = f^2(\theta)}$$

Antenna

total Received power

$$\boxed{P_r = G_{dr} G_{dt} \left[\frac{\lambda}{4\pi r}\right]^2 P_t}$$ — friis equation

$$\boxed{P_t = P_r \left[\frac{4\pi r}{\lambda}\right]^2 \frac{1}{G_{dr} G_{dt}}}$$

total transmitted power.

G_{dr} - Directive gain (Received)

G_{dt} = Directive gain (transmitted)

$$G_{dt} = \frac{4\pi U}{P_t} = \frac{4\pi r^2 P_{avg.}}{P_t.}$$

$$P_{avg.} = G_{dt} \frac{P_t}{4\pi r^2}$$

$$P_r = P_{avg.} A_{er} = \frac{\lambda^2}{4\pi} G_{Dr} P_{avg.}$$

$$P_r = \frac{\lambda^2}{(4\pi r)^2} G_{Dr} G_{Dt} P_t.$$

$$\boxed{P_r = G_{dr} G_{dt} \left(\frac{\lambda}{4\pi r}\right)^2 P_t.}$$ — friis transmission formula.

Q. Based on

1. Maximum effective aperature
2. P_{rad}, Maximum directivity.
3. HPBW.

$$\sigma_{copper} = 5.7 \times 10^7 \text{ S/m.}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

$$P_r = P_t G_d G_t \left(\frac{\lambda}{4\pi R}\right)^2.$$