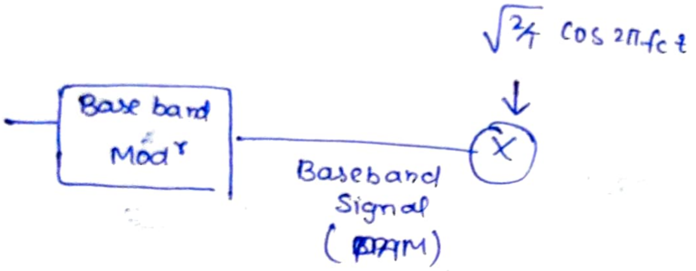
$$SER \approx 2\,Q\left(\sqrt{\frac{E_s}{N_0/2}}\right).$$

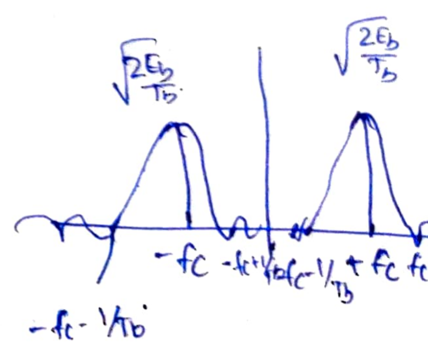
Passband Modulation Scheme



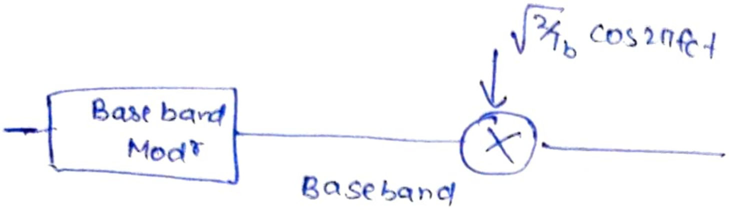
$$\int_0^{T_b} A^2\cos^2(2\pi f_c t)\,dt = \frac{A^2}{2}T_b = 1$$

$$A = \sqrt{2/T_b}$$

$$\left.\begin{aligned} 1 &\rightarrow \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \\ 0 &\rightarrow -\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \end{aligned}\right\} \quad 0 \le t \le T_b.$$



Passband Modulation Scheme



Baseband signal (PAM)

Pass band signal

(High frequency Translation).

Low frequency $\Leftrightarrow$ high freq:

↳ Antenna height reduction.

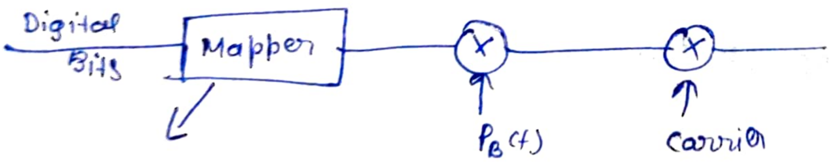
GPAM → Gaussian

$\times$ { Gaussian filter } $\Rightarrow$ Baseband → Baseband + Carrier

Carrier

Passband Schemes :-

BPSK, ASK, QPSK, OQPSK, QAM, MPSK, BFSK, MFSK, M-QAM, MSC, etc.



Passband.

map symbol to different amplitude level.

$\sqrt{\frac{2}{T}}\cos 2\pi f_c t$.

BPAM — $0 \to -A, -\sqrt{E}$

$1 \to A, +\sqrt{E}$

BPSK

Coherent BPSK

$$s_i(t) = \sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos\left(2\pi f_c t + (i-1)\pi\right) \quad ; \; i = 1, 2.$$

$$; \; 0 \le t \le T_b$$

$$0 \rightarrow s_i(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t\right).$$

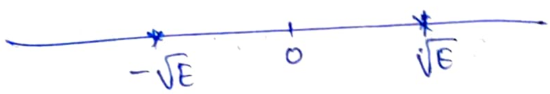
$$1 \rightarrow s_i(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t - \pi\right)$$

$$= -\sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t\right).$$

$$\phi(t) = \sqrt{\frac{2}{T_b}}\cos\left(2\pi f_c t\right) \quad ; \quad 0 \le t \le T_b.$$

$$\left.\begin{aligned} s_1(t) &= \sqrt{E}\,\phi(t) \\ s_2(t) &= -\sqrt{E}\,\phi(t). \end{aligned}\right\} \text{ — BPAM}$$

↳ In terms of vector representation.



$$BER_{BPSK} = Q\left(\frac{2\sqrt{E}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

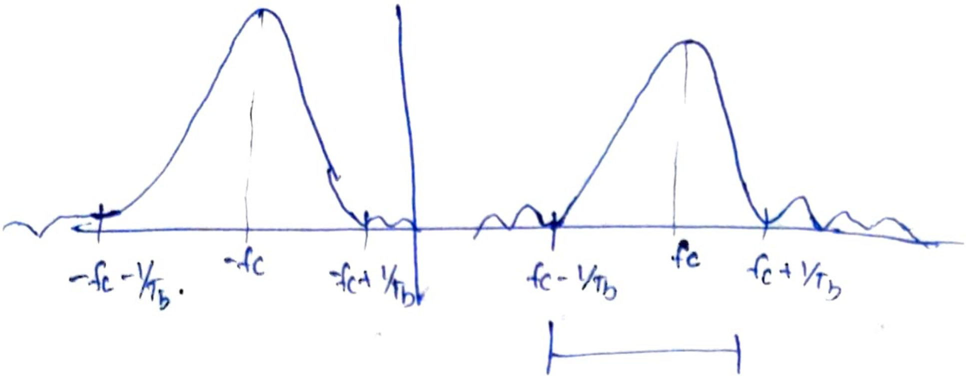
$$(E_{avg})_{BPSK} = \frac{E_b}{2} + E_b/2 = E_b.$$

$$B.W = f_c + 1/T_b.$$

$$PSD = \frac{1}{T_p}\left|S(f)\right|^2 \sum_{n=0}^{N} R_I(n)\, e^{-j2\pi f n T}$$

H.W

$$(PSD)_{BPSK} = \frac{E_b}{2}\left[\operatorname{sinc}^2\left(T_b(f-f_c)\right) + \operatorname{sinc}^2\left(T_b(f+f_c)\right)\right].$$



$B.W = \cancel{f_c} + 1/T_b \quad \cancel{-f_c} + 1/T_b$

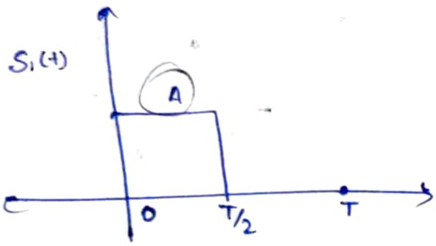
$= 2/T_b$.

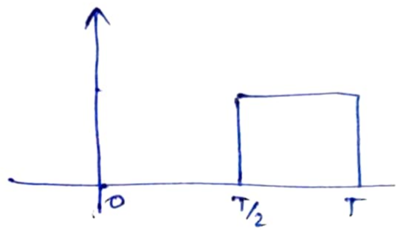
4/10/23

Tutorial

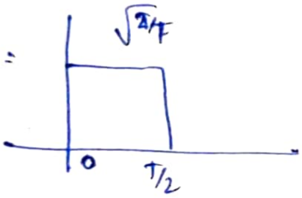
Mid-Sem

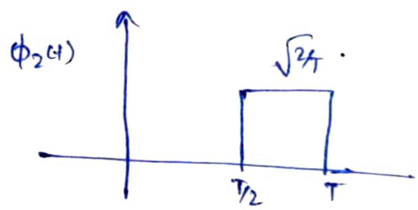
QC.





$\phi_1(t) =$





$s_1 = A \times \sqrt{T/2}\ \phi_1(t)$.

$A \times \sqrt{T/2} \times \quad \times \sqrt{8/3T} \times 3T/8$.

$\sqrt{\frac{3T}{8}}$

$A \times \frac{\sqrt{3} \times \sqrt{T}}{\sqrt{8} \times \sqrt{2}}$

$= \frac{\sqrt{3T}\,A}{4}$

$A \times \sqrt{T/2} \times \sqrt{\frac{8}{3T}} \times 3T/8$

$A \times \sqrt{T/2} \times \frac{\sqrt{3T}}{\sqrt{8}}$

$= \frac{\sqrt{3}\,AT}{4}$.

$A \times \sqrt{\frac{3T}{8}}$

Amplitude shift keying

b_i		$P_i(t)$	$S(t)$	
0	$\rightarrow$	0	0	
1	$\rightarrow$		$\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$	$0 \le t \le T_b$

$$E_{avg._{ASK}} = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot E_b = E_b/2$$

$$E_{avg.\ BPSK} = E_b.$$

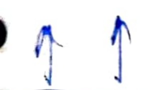
PSD of unipolar (ON-OFF)

$$= \frac{A^2 T_b}{4}\,\text{sinc}^2(f T_b) + A^2/4\ \delta(f).$$

(ON-OFF) baseband $\times \sqrt{2/T_b} \times \cos 2\pi f_c t$.

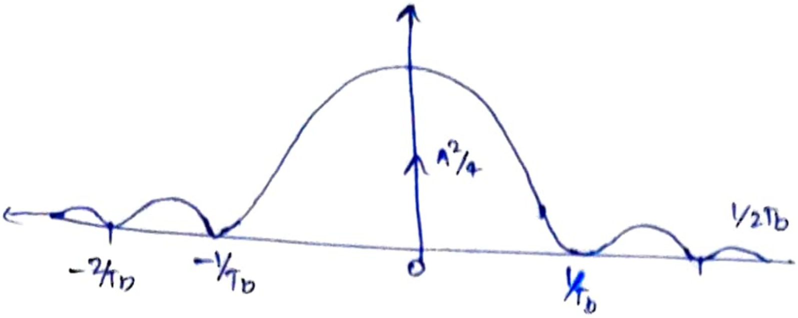
$$PSD(f) * \left[\frac{\delta(f+f_c)+\delta(f-f_c)}{2} \times \sqrt{2/T_b}\right]^2$$

$$PSD(f) * \left[\delta(f+f_c) + \delta(f-f_c) + 2\,\cancel{\delta(f-f_c)\,\delta(f+f_c)}^{\,0}\right] \times \frac{1}{2T_b}$$

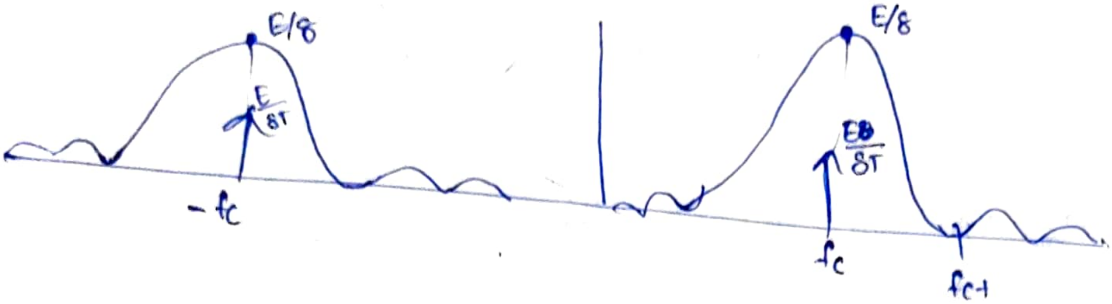


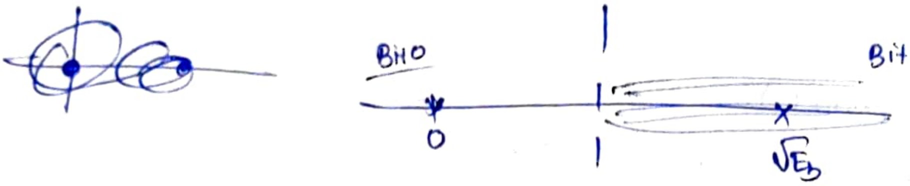
$$= \frac{1}{2T_b}\left[\frac{A^2 T_b}{4}\,\text{sinc}^2 (f+f_c)T_b + \frac{A^2}{4}\,\delta(f) * \delta(f+f_c)\right]$$

$$= \frac{A^2}{8}\left\{\text{sinc}^2 (f+f_c)T_b + \text{sinc}^2 (f-f_c)T_b\right].$$



$\frac{1}{2T_b}\left[\delta(f-f_c)+\delta(f+f_c)\right]$.



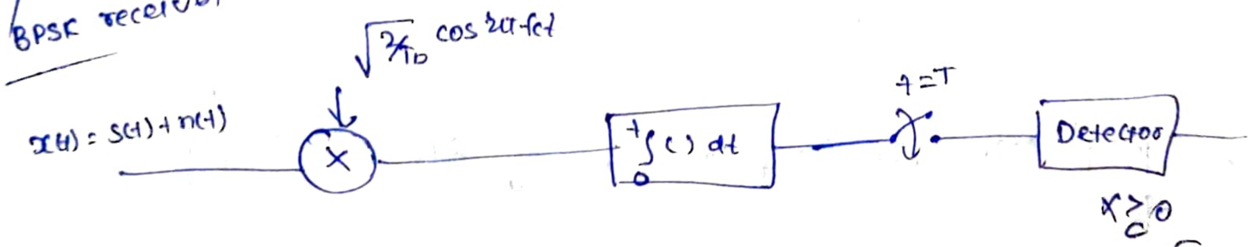


$\sqrt{E_b}/2$ — decision region.

$$P_e = Q\left(\frac{d}{\sqrt{2N_0}}\right) = Q\left(\frac{\sqrt{E_b}}{\sqrt{2N_0}}\right)$$

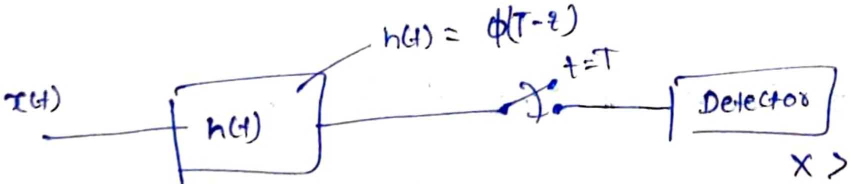
$$P_e = Q\left(\sqrt{\frac{E_b}{2N_0}}\right).$$

ASK/BPSK receiver



$x \gtrless 0$

$x \gtrless \sqrt{E_b}/2$

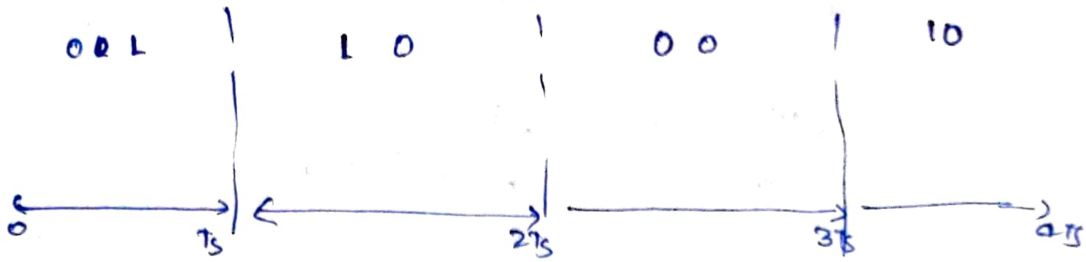


$x \gtrless \sqrt{E_b}/2$.

$x \gtrless 0$.

$\phi(t) = \sqrt{2/T_b}\cos 2\pi f_c t$

QPSK {Quadrature Phase Shift Keying}



$T_s = 2T_b$; $\boxed{R_s = 1/T_s = \frac{R_b}{2}}$

Symbol rate

$E_s = 2E_b$

$$S_i(t) = \begin{cases} \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + (2i-1)\pi/4\right) & 0 \le t \le T_s \\ 0 & \text{o.w} \end{cases}$$

$i = 1, 2, 3, 4.$

$$S_1(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \pi/4\right) \quad ; \quad 0 \le t \le T_s$$

$$= \sqrt{\frac{2E_s}{T_s}} \left\{ \cos 2\pi f_c t \; \underset{\substack{\downarrow \\ 1/\sqrt{2}}}{\cos \pi/4} - \sin 2\pi f_c t \cdot \underset{\substack{\downarrow \\ 1/\sqrt{2}}}{\sin \pi/4} \right\}$$

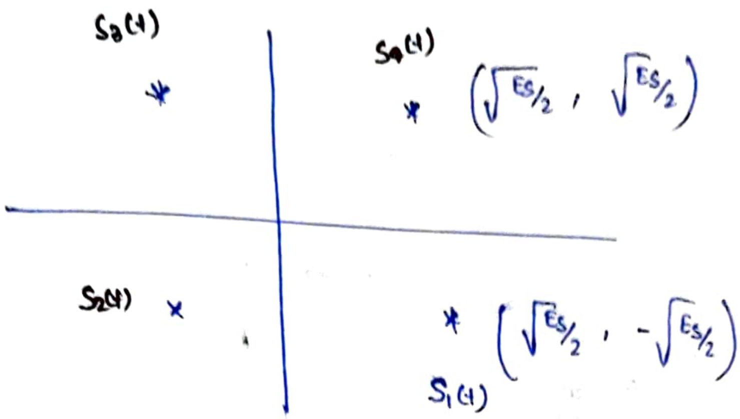
$$= \sqrt{\frac{2(E_s/2)}{T_s}} \left[\cos 2\pi f_c t - \sin 2\pi f_c t \right]$$

$$= \sqrt{\frac{2(E_s/2)}{T_s}} \cos 2\pi f_c t - \sqrt{\frac{2(E_s/2)}{T_s}} \sin 2\pi f_c t$$

$$= \sqrt{\frac{E_s}{2}} \cdot \underbrace{\sqrt{\frac{2}{T_s}} \cos 2\pi f_c t}_{\phi_1(t)} \cdot - \sqrt{\frac{E_s}{2}} \underbrace{\sqrt{\frac{2}{T_s}} \sin 2\pi f_c t}_{\phi_2(t)}$$

$$= \sqrt{\frac{E_s}{2}} \cdot \phi_1(t) - \sqrt{E_s/2}\; \phi_2(t).$$

$$S_i(t) = \pm\sqrt{\frac{E_s}{2}}\,\phi_1(t) \pm \sqrt{\frac{E_s}{2}}\,\phi_2(t)$$



$$\mathcal{E}/_{S_4(t)} = 1 - \int_0^\infty\int_0^\infty f_{x_1,x_2}(x_1, x_2)\, dx_1\, dx_2$$

x_1, & x_2 are Independent if $\phi_1(t)$ & $\phi_2(t)$ are orthogonal

$$= 1 - \int_0^\infty f_{x_1}(x_1)\, dx_1 \int_0^\infty f_{x_2}(x_2)\, dx_2$$

$$x_1|_{S_4(t)} \sim N\left(\sqrt{E_s/2}, \frac{N_0}{2}\right)$$

$$x_2|_{S_4(t)} \sim N\left(\sqrt{\frac{E_s}{2}}, N_0/2\right)$$

$$\mathcal{E}|_{S_4(t)} = 1 - \left(1 - Q\left(\sqrt{SNR}\right)\right)^2 \quad ; \; SNR = E_s/N_0$$

$$P_e|_{S_4(t)} = 2Q\left(\sqrt{E_s/N_0}\right) - Q^2\left(\sqrt{E_s/N_0}\right)$$

$$= P_e|_I + P_e|_Q - P_e|_I \cdot P_e|_Q.$$

we may ... any 00 0 to 1.

if $0 \to -\sqrt{E_s/2}$

$\phi \to \sqrt{E_s/2}$.

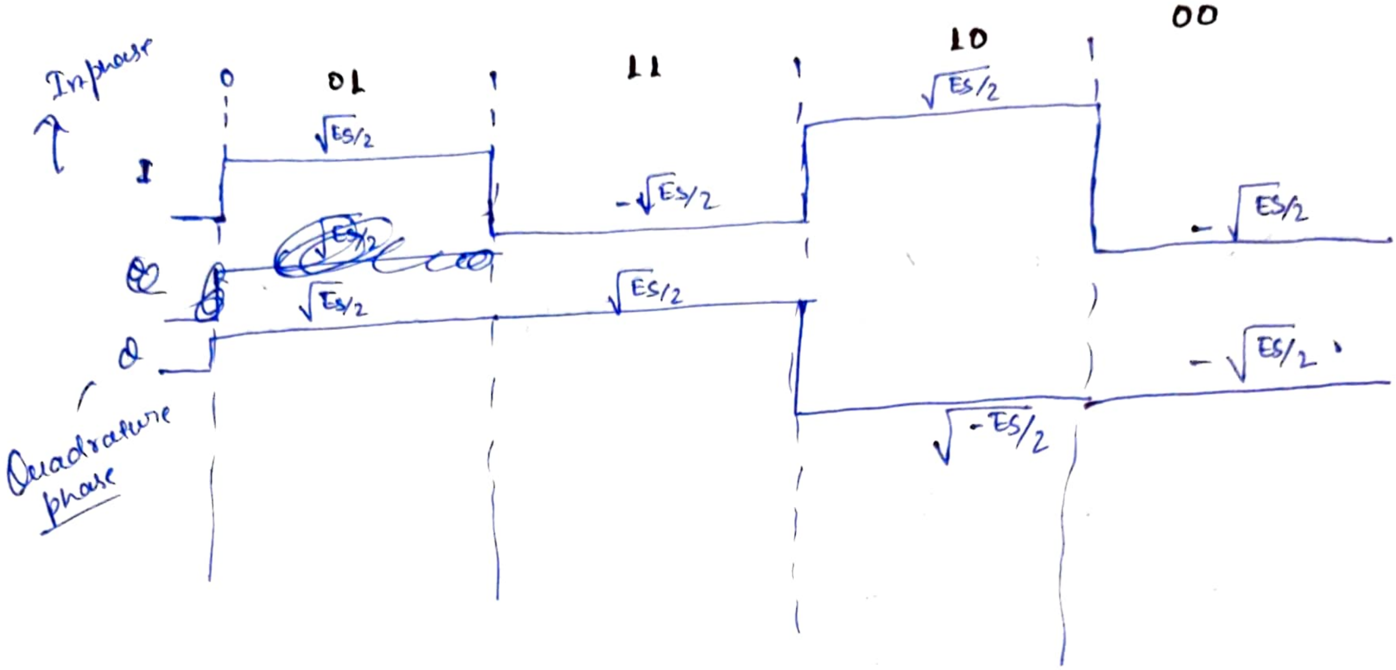
$01 \to s_4(t)$

$00 \to s_2(t)$

$10 \to s_1(t)$

$11 \to s_3(t)$

definitation



I_B, $Q_B \to$ NRZ

Base band

$$(PSD)_{I\text{-}pass} = PSD\left(\pm\sqrt{\frac{E_s}{T_b}}\cos 2\pi f_c t\right)$$

$$= PSD\left[\pm\sqrt{E_s/2}\ \sqcap\ *\ \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t\right].$$

$$(PSD)_{Q\text{-}pass} = PSD\left(\pm\sqrt{\frac{E_s}{2}}\sqrt{\frac{2}{T_b}}\sin 2\pi f_c t\right)$$

$$= PSD\left(\pm\sqrt{E_s/2}\ \sqcap\ \times\ \sqrt{\frac{2}{T_b}}\sin 2\pi f_c t\right).$$

$$= PSD\left(\pm\sqrt{E_s/2}\ \sqcap\right) * PSD\left(\sqrt{\frac{2}{T_b}}\sin 2\pi f_c t\right).$$

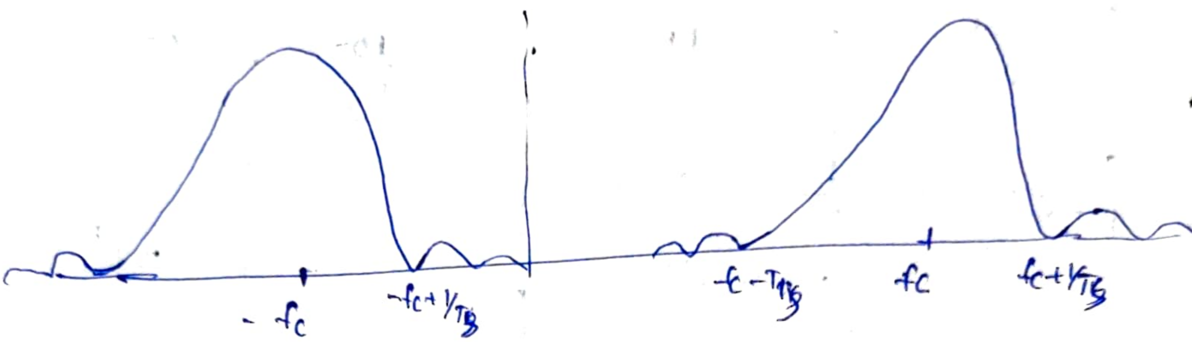
Convolution

$\delta(f-f_c)+\delta(f+f_c)$

due to square.

$$\frac{E_s}{2} \times T_b \; sinc^2(f T_b) + \frac{2}{T_b} \cdot \left(\delta(f-f_c) + \delta(f+f_c)\right)$$

$$\frac{E_s}{2} \, sinc^2\left(T_b (f-f_c)\right) + E_s \, sinc^2\left(T_b (f+f_c)\right)$$



$f_c + 1/T_b - f_c + 1/T_s$

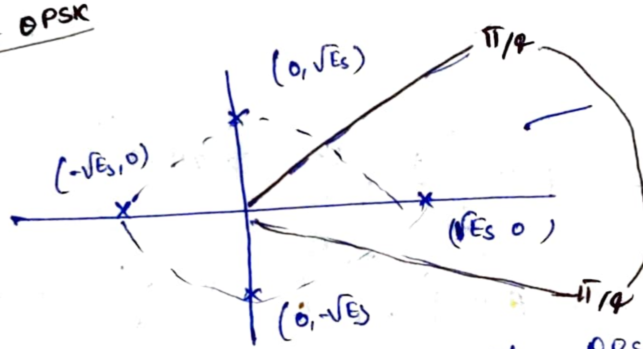
Bandwidth $= 2/T_s$.

$B.W = 2R_s$ QPSK.

$R_s = 1/T_s$

$= 1/2T_b$.

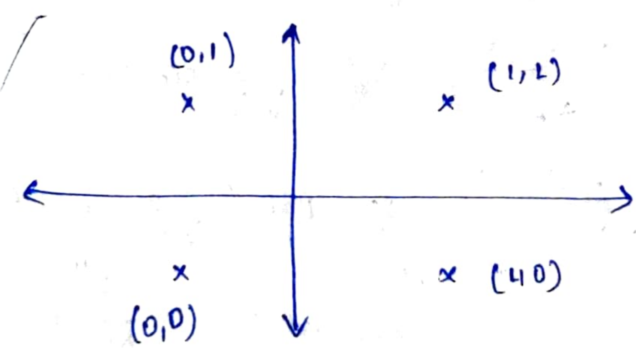
other form of QPSK



decision region.

Another version of QPSK.

Constellation of QPSK



discontinuity

10 01

clockwise

Band-limited channel

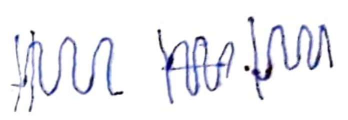
ISI

phase-traverse

00	180°	11	90°	01	180°	10	90°	11
$S_3(t)$		$S_1(t)$		$S_2(t)$		$S_4(t)$		$S_1(t)$
(3rd)		1st		(2nd)		(4th)		1st

→ The QPSK has maximum phase shift of 180° and minm phase shift 90° (other than 0°).

00 01 10 11



At the point where phase shift occur

⇓

lead to spread in the signal

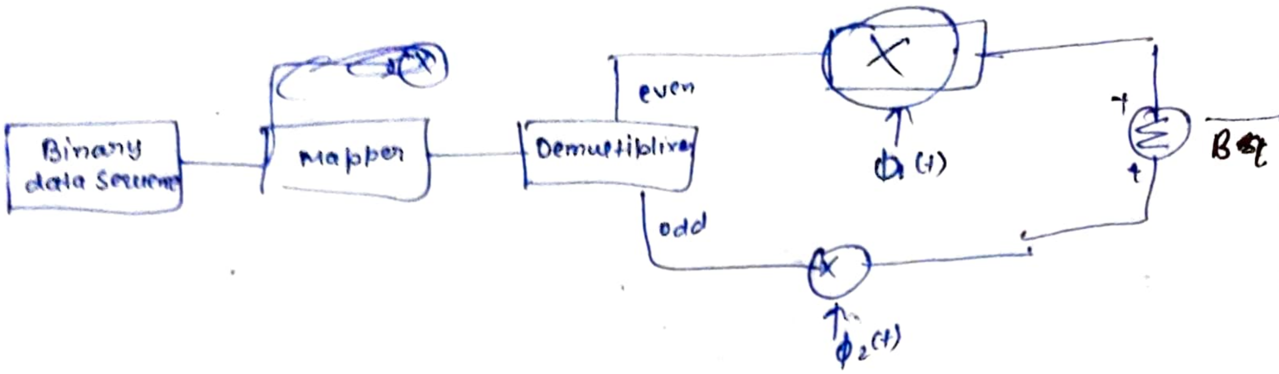
⇓

Bandwidth Increases.

discontinuity

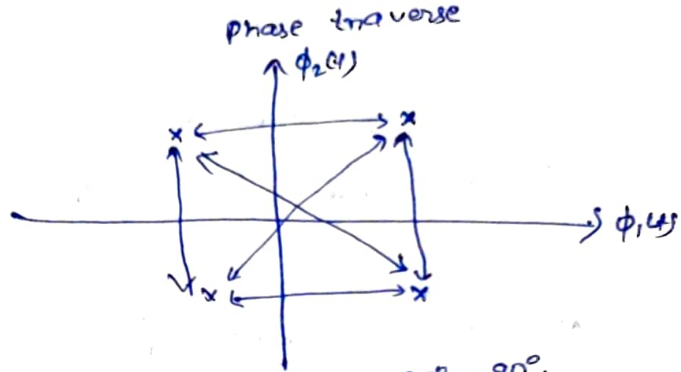
Offset QPSK

BOOK

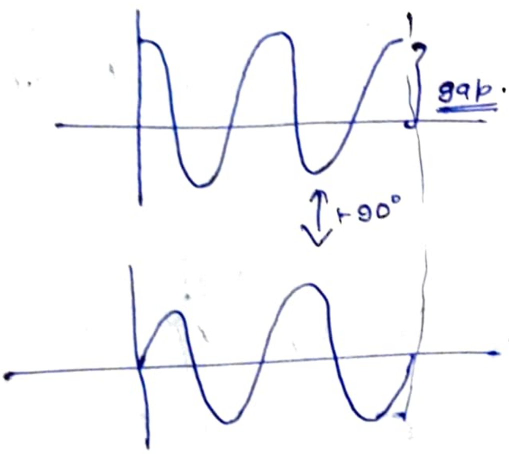


OQPSK (off-set QPSK)

modified version of QPSK

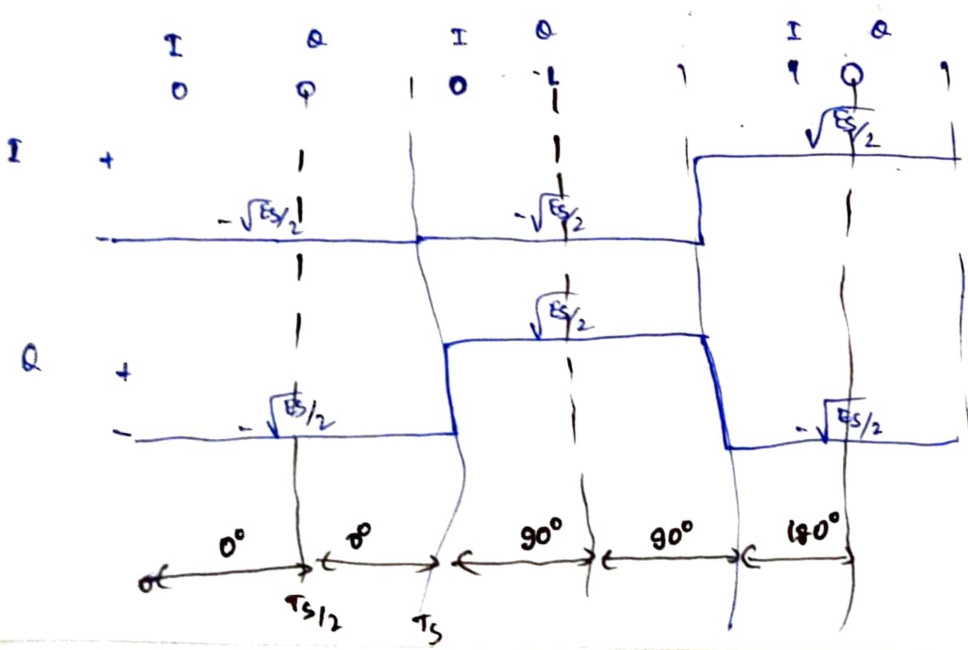


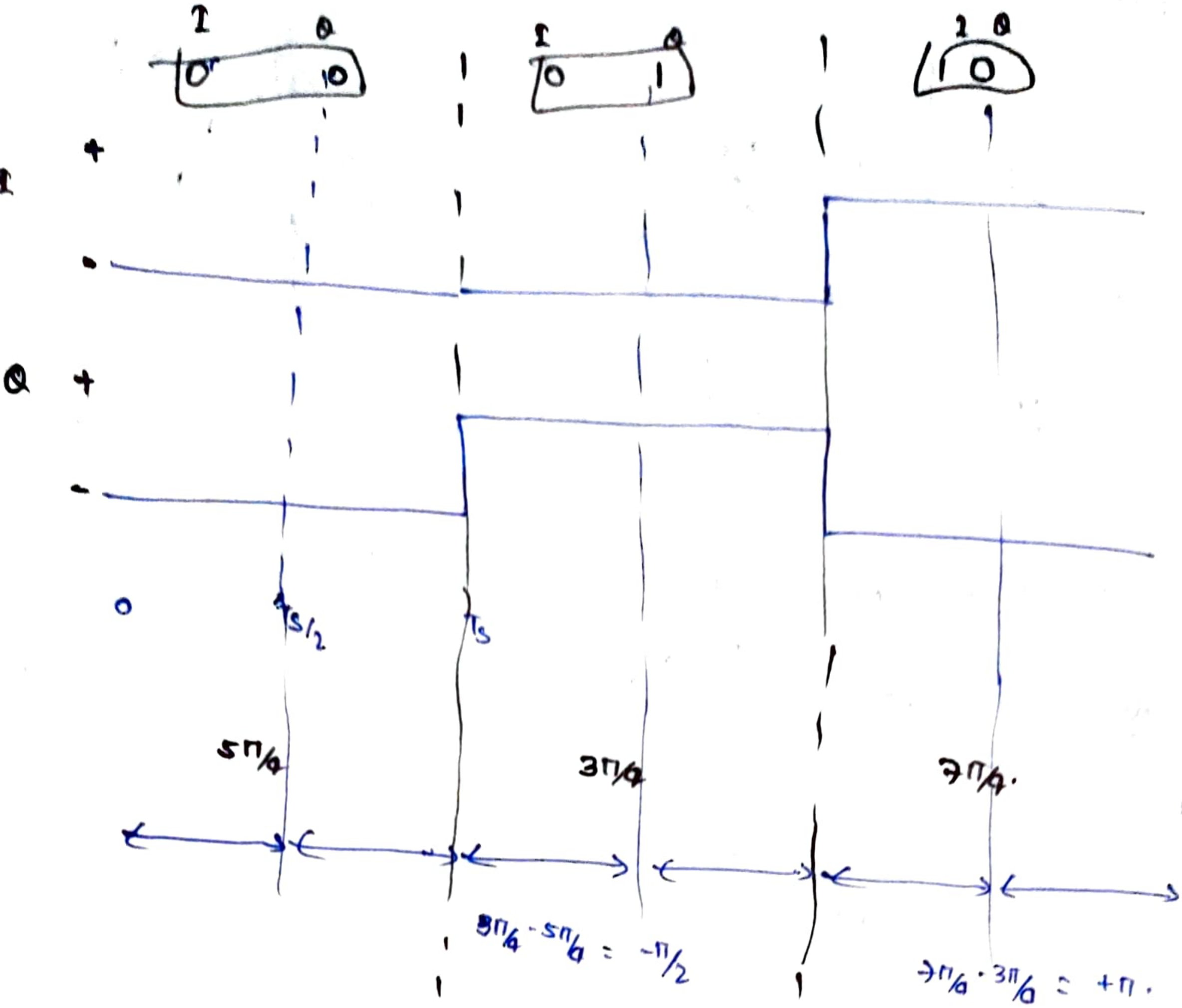
90°, -90°, 180°, -180°

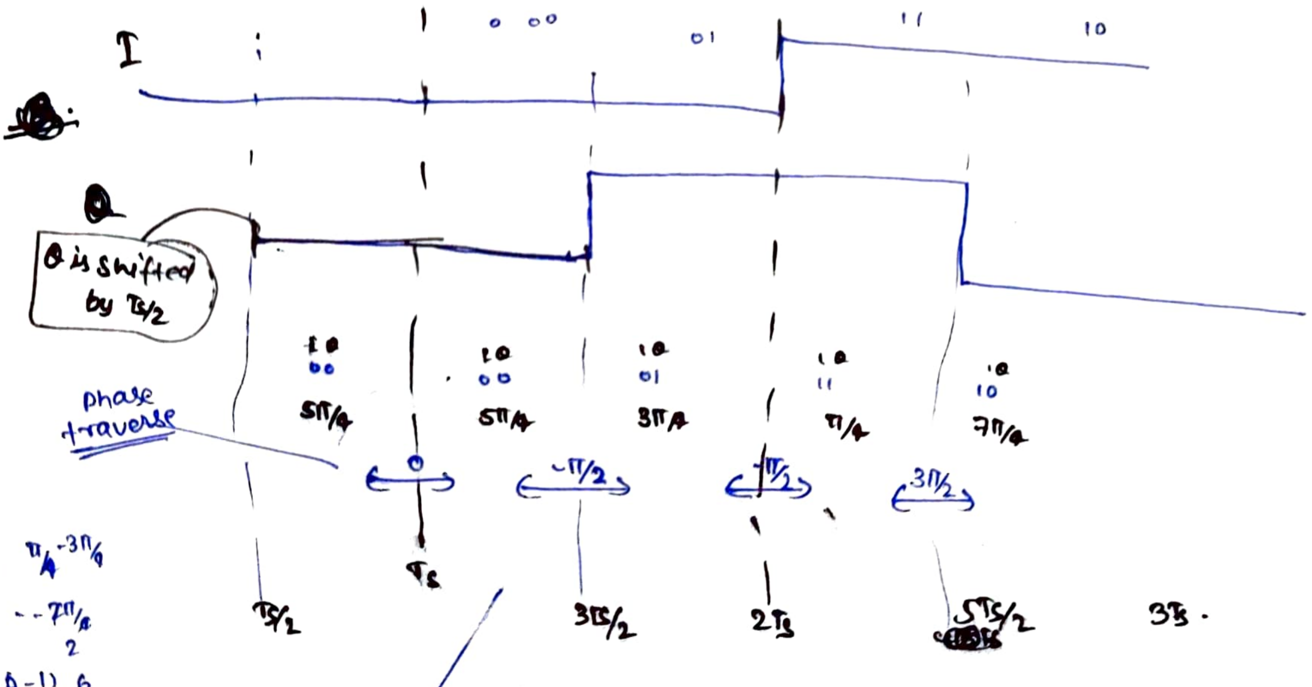


3π/4

OQPSK ↳ we delay one of the phase.

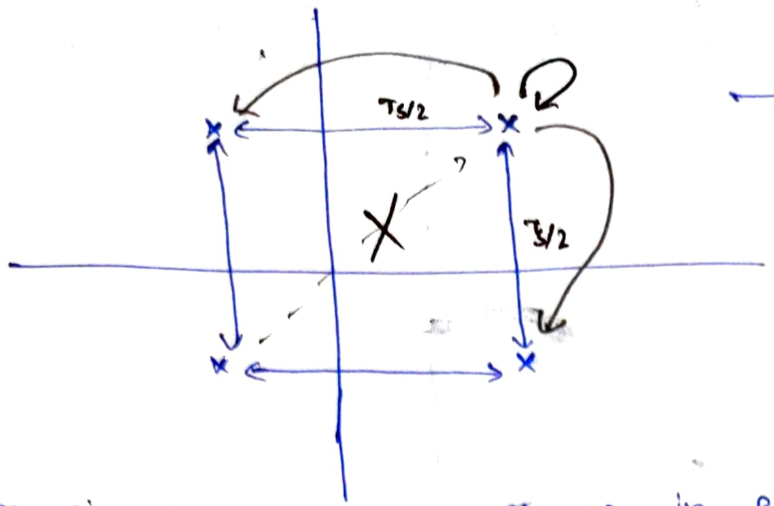


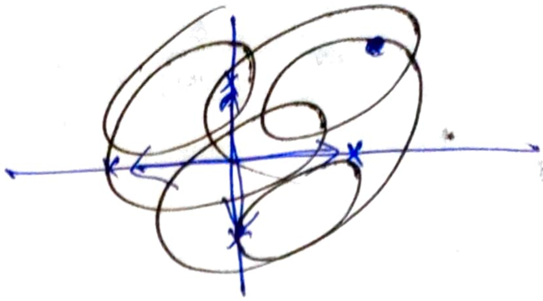




Phase is changing more frequently by $(-\pi/2)$.

Constellation of OQPSK





The signal point will change in every $T_s/2$ point.

To change OQPSK to QPSK,

we shift $m\left(\frac{T_s}{2}\right)$ to $\frac{T_s}{2} + T_s$.

or $m\left(\frac{T_s}{2}\right)$ to $(m+1)\left(\frac{T_s}{2} + T_s\right)$

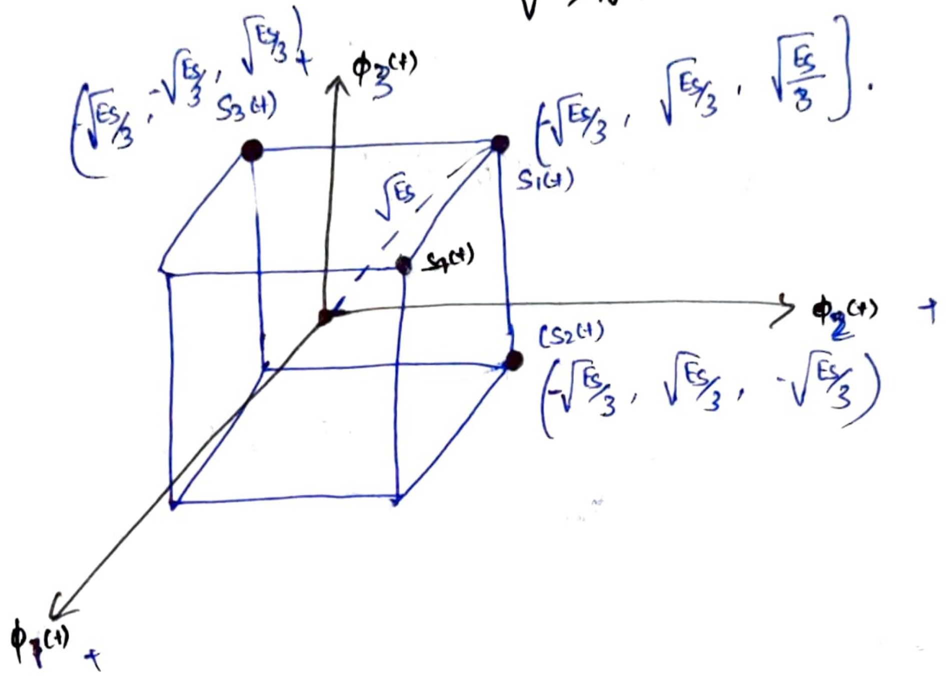
Tutorial

Q. 8.25. Show that the correlation coefficient of two adjacent signal points corresponding to the vertices of N-dimensional hypercube with its centre at the origin is given by

$$\gamma = \frac{N-2}{2}$$

and their Euclidean distance is

$$d = 2\sqrt{E_s/N}.$$



correlation b/w $S_1(t)$ & $S_3(t)$

$$\langle S_1(t), S_3(t)\rangle = |S_1(t)|\,|S_3(t)|\cos\theta.$$

$$\frac{E_s}{3} - \frac{E_s}{3} + \frac{E_s}{3}$$

$$= \frac{E_s}{3}.$$

$$\frac{E_s/3}{E_s} = 1/3.$$

Observation :- There will only one sign change in N-dimension similar to 3-Dimension case.

$$\langle s_1(1) \ldots s_1(N)\rangle = E_s/N + \ldots E_s/N - E_s/N$$

$$= (N-1)\, E_s/N - E_s/N$$

$$= \frac{(N-2)\, E_s/N}{E_s}$$

$$= \frac{N-2}{N}.$$

$\boxed{\cos\theta = \rho}$ — correlation coefficient

$S_1 =$

$S_2 =$

$$\cos\theta = \rho = \frac{\langle s_1 \cdot s_2\rangle}{|s_1|\,|s_2|} = \frac{-\frac{E_s}{N} + E_s/N + \ldots + \frac{E_s}{N}}{\sqrt{E_s}\ \sqrt{E_s}}$$

$$= \frac{\frac{(N-1)}{N} E_s - E_s/N}{E_s}$$

$$= \frac{N-2}{N}.$$

$$d = \sqrt{(S_{11} - S_{21})^2 + (\underbrace{\cancel{S_{12} - S_{22}}}_{0})^2 + \ldots + (\underbrace{\cancel{S_{1N} - S_{2N}}}_{0})^2}$$

$$= \sqrt{\left(\sqrt{\frac{E_s}{N}} - \left(-\sqrt{\frac{E_s}{N}}\right)\right)^2} = 2\sqrt{E_s/N}.$$

Eculidian distance

Q. 8.12

$\langle s_1(t) \cdot s_2(t) \rangle = 0.$

$\int_{-\infty}^{\infty} s_1(t)\, s_2(t)\, dt = 0$

$n(t) \sim N\left(0, \frac{N_0}{2}\right).$

$n_1 = \int_0^T s_1(t)\, n(t)\, dt$

$n_2 = \int_0^T s_2(t)\, n(t)\, dt$

$E[n_1 n_2] = \int_0^T \int_0^T E[n(t)\, n(u)]\, s_1(t)\, s_2(u)\, dt\, du$

$= \int_0^T \int_0^T \frac{N_0}{2} \delta(t-u)\, s_1(t)\, s_2(u)\, dt\, du$

$= \frac{N_0}{2} \int_0^T s_1(t)\, s_2(t)\, dt.$

$= 0.$

$\delta(t-u)$

$\delta(t_0)$

M-ary PSK (M-PSK)

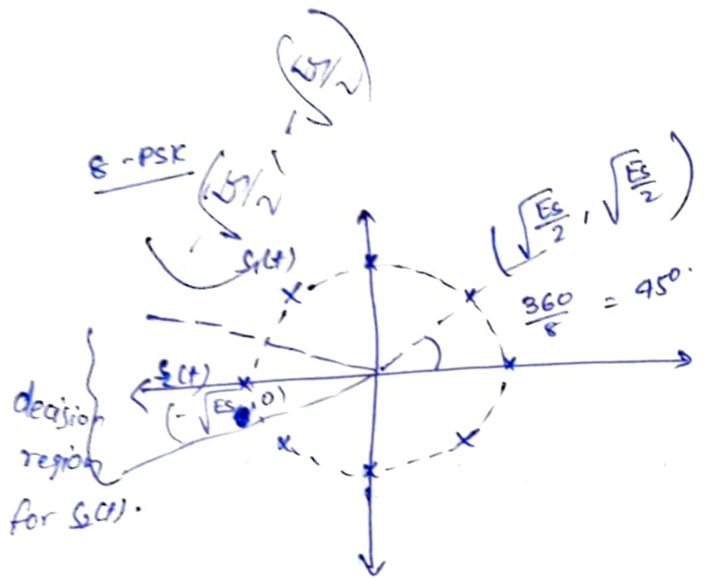
$\frac{360}{4}$

QPSK → 4 symbols → 2 bits — Angle (90°)

M-PSK → M symbols — $\log_2 M$ bits symbol.

phase shift b/w $\left\{ \frac{360°}{M} \right\}$

8-PSK



$\frac{360}{8}$

$T_s = \log_2 M \; T_b = 3T_b$

$E_s = 3E_b$.

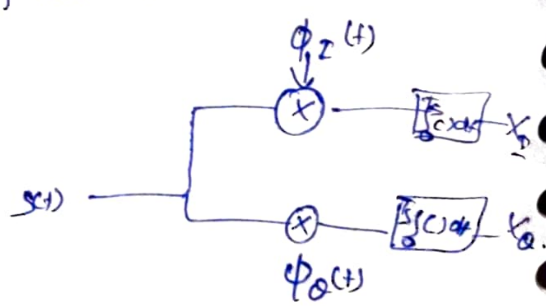
$$\boxed{R_s = \frac{R_b}{3}}$$

$SER = P_e | s_2(t) = 1 - P_c | s_2(t)$.

$$P_c | s_2(t) = 2 \int_{r=0}^{\infty} \int_{\theta=0}^{\pi/8} f_{r,\theta}(r,\theta) \, dr \, d\theta.$$

$r = \sqrt{x_I^2 + x_Q^2}$

$\theta = \tan^{-1}\left(\frac{x_Q}{x_I}\right)$



Assignment

H.W, Calculate exact probability

$r_1 = -\sqrt{E_s} + n_1$

$r_2 = n_2$

$E[n_1 n_2] :$

no. of neighbours

$$Pe|s_2(t) = 2\,Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$

$$d_{min} = \sqrt{\left(-\sqrt{E_s} + \sqrt{\frac{E_s}{2}}\right)^2 + \left(0 - \sqrt{\frac{E_s}{2}}\right)^2}$$

$E_s + \frac{E_s}{2} - \frac{2 E_s}{\sqrt{2}}$

$\frac{3E_s}{2} - \sqrt{2}E_s + \frac{E_s}{2}$

$\frac{4E_s}{2} - \sqrt{2}E_s$

$2E_s - \sqrt{2}E_s.$

$$d_{min} = \sqrt{2E_s - \sqrt{2}E_s.}$$

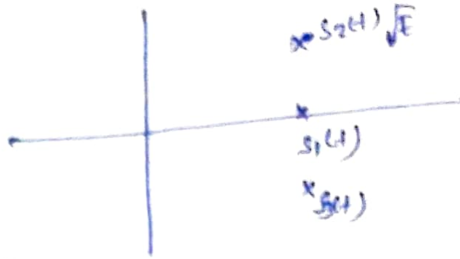
$$Pe|s_2(t) = 2\,Q\left(\sqrt{\frac{E(2-\sqrt{2})}{2N_0}}\right)$$

$$= 2\,Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right).$$

$\sqrt{E_s}\cos\frac{2\pi}{M}, \sqrt{E_s}\sin 2\pi/M.$

$\sqrt{E_s}, 0.$

$\left(\sqrt{E_s}\cos\frac{2\pi}{M}, \sqrt{E_s}\sin\frac{2\pi}{M}\right)$



$\sqrt{E_s}$

$$d_{min} = \sqrt{\left\{\sqrt{E_s}\left(\cos\frac{2\pi}{M} - 1\right)\right\}^2 + \left\{\sqrt{E_s}\sin\frac{2\pi}{M}\right\}^2}$$

$$- 2E_s\cos\frac{2\pi}{M} + E_s\sin^2\frac{2\pi}{M}$$

$$E_s \cos^2\frac{2\pi}{M} + E_s$$

$$2E_s - 2E_s\cos\frac{2\pi}{M}$$

$$2E_s\left(1 - \cos\frac{2\pi}{M}\right)$$

$$2E_s \times 2\sin^2 \pi/M$$

$$1 - \cos 2\theta = 2\sin^2\theta.$$

$$\boxed{d_{min} = 2\sqrt{E_s}\sin \pi/M.}$$

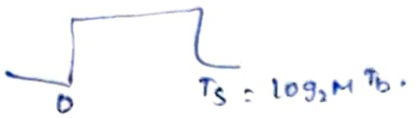
$$P_{e|s_i(t)} = 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin \pi/M\right).$$

Exact probabilities

$$\int_{r=0}^{\infty}\int_{-\pi/M}^{\pi/M} f_{r,\theta}(r,\theta)\, dr\, d\theta$$

$$\int_{-\pi/M}^{\pi/M} f_\theta(\theta)\, d\theta. \qquad \int_{r=0}^{\infty} f_{r,\theta}(r,\theta)\, dr = f_\theta(\theta).$$

$$PSD = |P(f)|^2 \sum_{k=-\infty}^{\infty} R_b(k)\, e^{j2\pi n T_b}$$



$$\times \sqrt{\frac{2}{T_s}} \cos 2\pi f_c t$$

Assumption
DMS - Discrete Memoryless Source
each symbol is Independent.

$$\boxed{(B\cdot W)_{M\text{-}PSK} = \frac{2}{T_s} = \frac{2}{\log_2 M\, T_b}}$$

QAM (Quadrature Amplitude Modulation).

M: no. of bits

$M \uparrow$ ↳ SER ↑ (QOS). → 10^{-5}

Quality of service depends on SER

(one error in 10^5 signal transmitted.

$M \downarrow$ ↳ B·W ↓

⇒ • It is a general type of Mod^n scheme.

- However, it is regular shape.
- M-ary QAM is a two dimensional signalling and it is a generalization of 2D - M-ary PAM.

  (It is a baseband version of 2D M-PAM).

$$S_i(t)_{MPSK} = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + (i-1)\pi/M\right). \quad i = 1, 2, 3 \ldots M.$$

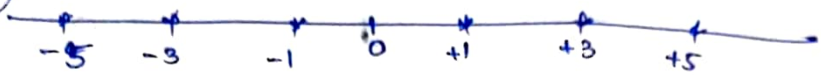
$$\left.\begin{aligned} \phi_I(t) &= \sqrt{\frac{2}{T_s}} \cos 2\pi f_c t. \\ \phi_Q(t) &= \sqrt{\frac{2}{T_s}} \sin 2\pi f_c t. \end{aligned}\right\} \quad 0 \le t \le T_s;$$

$$S_k(t) = \sqrt{\frac{2E_s}{T_s}}\, a_k \cos 2\pi f_c t - \sqrt{\frac{2E_s}{T_s}}\, b_k \sin 2\pi f_c t$$

$a_k = 2k - 1 - M_I$ ← In case of M-PAM

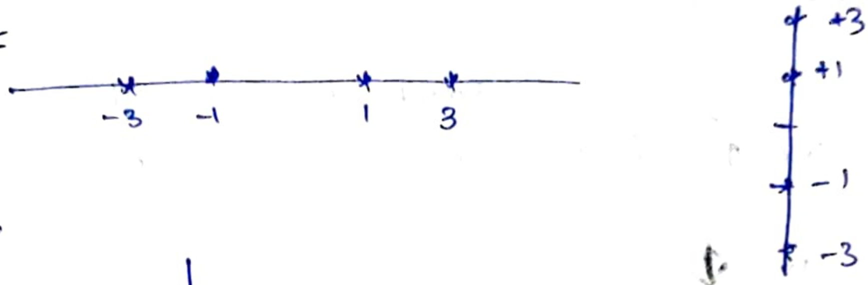
$b_k = 2k - 1 - M_Q$

d M-PAM M

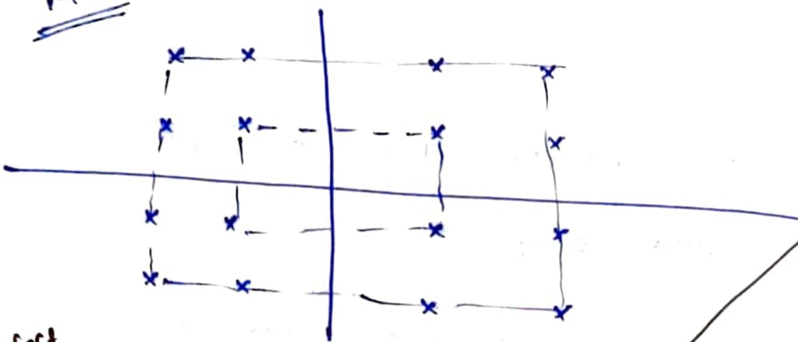


2-D PAM

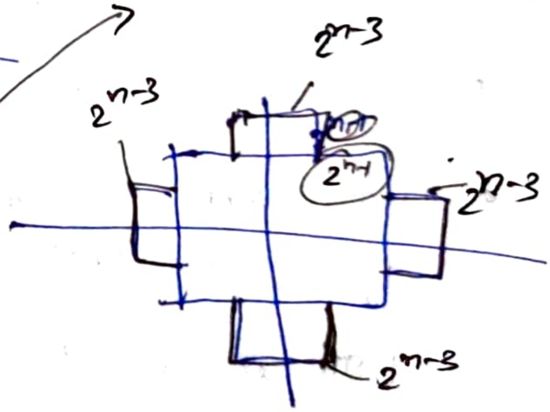
Square $\sqrt{M}$



M = 16

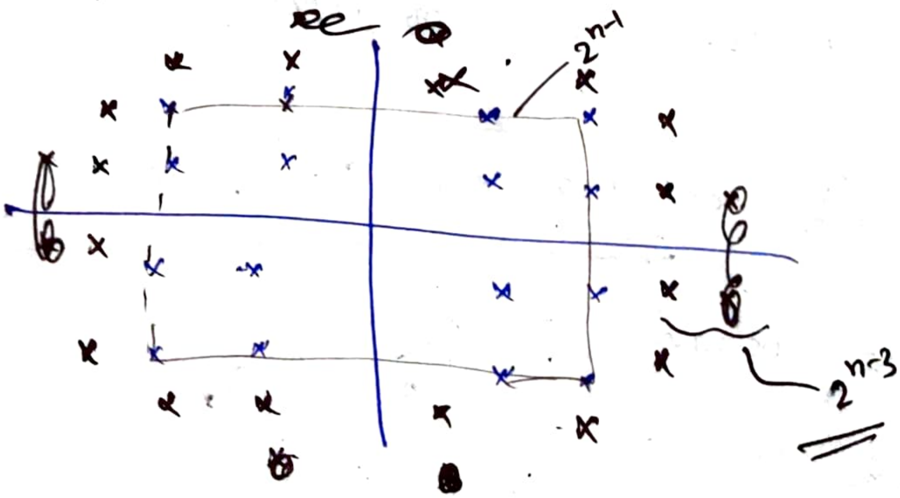


Rule to represent



for Imperfect $\sqrt{M}$

M = 32, bits = 5 bits per symbol



32 - Q PAM

$$\frac{2^n - 2^{n-1}}{4}$$

$$\frac{2^n(1 - 1/2)}{4}$$

$$2^n \cdot 1/8$$

$$2^{n-3}.$$

$\sqrt{M}$ - PAM in both dir^n

$$E_k = (a_k)^2 + (b_k)^2$$

$a_k = 2k - 1 - \sqrt{M}$

$b_k = 2k - 1 - \sqrt{M}$.

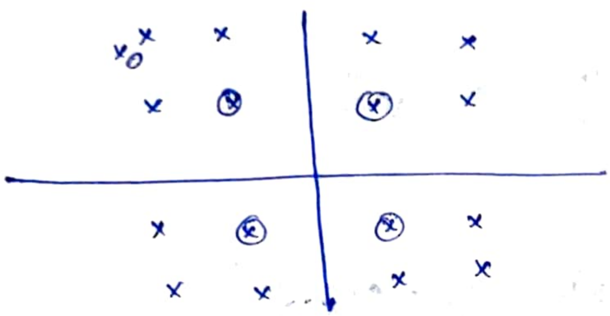
$$= (2k-1-\sqrt{m})^2 + (2k-1-\sqrt{m})^2$$

$$= 2\left(4k^2 + 1 + M - 2k + 2\sqrt{M} - 2k\sqrt{M}\right).$$

SER calculation

$\cancel{2} \times \frac{16-1}{\cancel{16}\ 8}$

$15/8$



Inner symbols : 4
↳ nearest neighbour = 4.

Outer symbols = 12
↳ nearest neighbour = 2.

} SER in terms of NN approximation

H.W

Exact SER Analysis (Square - QAM)

$P_e|s_1(t)$

$$= \int_{x_I=0}^{\infty} \int_{x_Q=0}^{\infty} f_{x_I, x_Q}\left(x_I, x_Q \mid s_1(t)\right) dx_I \, dx_Q$$

If x_I & x_Q are independent.

$$= \int_{x_I=0}^{\infty} \int_{x_Q=0}^{\infty} f_{x_I}\left(x_I \mid s_1(t)\right) f_{x_Q}\left(x_Q \mid s_2(t)\right) dx_I, dx_Q.$$

$$= \underbrace{\int_{x_I=0}^{\infty} f_{x_I}(x_I | s_1(t))\, dx_I}_{P_{c,I} | s_1(t)} \underbrace{\int_{x_Q=0}^{\infty} f_{x_Q}(x_Q | s_1(t))\, dx_Q}_{P_{c,Q} | s_1(t)}$$

$$= \left(1 - P_{e,I} | s_1(t)\right)\left(1 - P_{e,Q} | s_1(t)\right).$$

QAM is combination of 2 PAM.

⇒ From the discussion on 'SER M-PAM', the probability of error is,

$$P_{e,\,M\text{-}PAM} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2E_s}{N_0}}\right).$$

Hence for $\sqrt{M}$ PAM,

$$P_{e,\,\sqrt{M}\text{-}PAM} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_s}{N_0}}\right).$$

$$P_{e,\,QAM} = 1 - \left(1 - P_{e,I|s_1(t)}\right)\left(1 - P_{e,Q|s_1(t)}\right).$$

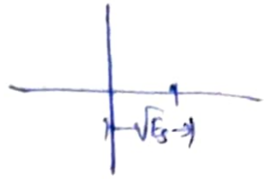
$$\boxed{P_{e,\,QAM} = -4\left(1 - \frac{1}{\sqrt{M}}\right)^2 Q^2\left(\sqrt{\frac{2\sqrt{E_s}}{N_0}}\right) + 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_s}{N_0}}\right).}$$

For high SNR, $E_s/N_0/2 \uparrow\uparrow$, $Q^2(\cdot)$ can be neglected.

$$\boxed{P_{e,\,QAM} \overset{HSNR}{=} 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_s}{N_0}}\right).}$$

$\underline{M=16}$ $\quad 4\left(1 - \frac{1}{4}\right) = \frac{4}{4} \times 3. = 3.$

Average Energy $= \frac{E_s}{M} \sum_{K=1}^{\sqrt{M}} \sum_{m=1}^{\sqrt{M}} a_k^2 + b_m^2$



$$\boxed{E_{avg.,M\text{-}QAM} = \frac{2 E_s (M-1)}{3}}$$

$$\boxed{B.W = \frac{2}{T_s}}$$

$T_s = \log_2 M \; T_b.$

$$\boxed{B.W = \frac{2}{\log_2 M \; T_b}}$$

$$\boxed{P_{e,\ cross\text{-}QAM} \simeq \underbrace{4\left(1 - \frac{1}{\sqrt{2M}}\right)}_{\text{Avg. nearest no. of neighbour.}} Q\left(\sqrt{\frac{2E_s}{N_0}}\right)}$$

$M = 32$
$M = 128$
$M = 512$

Relationship b/w f_c and T_s

$f_c = K/T_s$; K is any integer.

$f_c =$

Tutorial
16/10/23

$r_K = \sqrt{E_c}\, c_K + n_K$

$$r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \end{bmatrix}.$$

$c_1 = \{1,$

$$r_1 \big| c_1 = \begin{bmatrix} \sqrt{E_c} + n_1 \\ \sqrt{E_c} + n_2 \\ \vdots \\ \sqrt{E_c} + n_n \end{bmatrix} \begin{matrix} N(\sqrt{E_c}, \sigma^2) \\ \vdots \\ N(\sqrt{E_c}, \sigma^2). \end{matrix}$$

$$r|_{c_2} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} = \begin{bmatrix} \sqrt{E_c} + n_1 \\ \sqrt{E_c} \\ \sqrt{E_c} + n_w \\ -\sqrt{E_c} + n_{w+1} \\ \vdots \\ -\sqrt{E_c} + n_n \end{bmatrix} \begin{matrix} N(\sqrt{E_c}, \sigma^2) \\ \\ N(\sqrt{E_c}, \sigma^2) \\ N(-\sqrt{E_c}, \sigma^2) \\ \vdots \\ N(\end{matrix}$$

ML detector

$$x(t)|_{s_1(t)} = s_1(t) + n(t)$$

$$x|_{s_1(t)} = s_1 + n.$$

$$f_{x|s_1(t)} \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} f_{x|s_2(t)}.$$

$$f_{\underline{r}|c_1}(\,) \underset{c_2}{\overset{c_1}{\gtrless}} f_{\underline{r}|c_2}(\,)$$

$$f_{\underline{r}|c_1} = \left\{ \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \times e^{-\frac{(x-\sqrt{E_c})^2}{2\sigma^2}} \right\} e^{-\frac{(x-\sqrt{E_c})^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp \sum_{i=1}^{n} \frac{(r_i - \sqrt{E_c})^2}{2\sigma^2}$$

$$f_{r|c_2} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp \sum_{i=1}^{w} \frac{(r_i - \sqrt{E_c})^2}{2\sigma^2} \exp \sum_{i=w+1}^{n-w} \frac{(r_i+\sqrt{E_c})^2}{2\sigma^2}.$$

$$e^{-n\frac{(x-\sqrt{E_c})^2}{2\sigma^2}} \underset{c_2}{\overset{c_1}{\gtrless}} e^{-w\frac{(x-\sqrt{E_c})^2}{2\sigma^2}} \cdot e^{-(n-w)\frac{(x+\sqrt{E_c})^2}{2\sigma^2}}.$$

$$e^{-w(x-\sqrt{E_c})^2 + w(x+\sqrt{E_c})^2} \quad e^{-n\frac{(x+\sqrt{E_c})^2}{2\sigma^2}}.$$

$$-\omega\left(\cancel{x^2+E_c}-2x\sqrt{E_c}\right)+\omega\left(\cancel{(x^2+E_c)}+2x\sqrt{E_c}\right)$$

$$\frac{4\omega x\sqrt{E_c}}{\sigma^2}.$$

$$e^{\frac{4nx\sqrt{E_c}}{\sigma^2}} \gtrless e^{\frac{4\omega x\sqrt{E_c}}{\sigma^2}}.$$

$$\frac{4nx\sqrt{E_c}}{\sigma^2}$$

$\underline{P(x>0)}$

$$\sum_{i=\omega+1}^{n}\left\{(r_i-\sqrt{E_c})^2-(r_i+\sqrt{E_c})^2\right\} \underset{c_1}{\overset{c_2}{\gtrless}} 0$$

$$\sum_{i=\omega+1}^{n} -4r_i\sqrt{E_c} \underset{c_1}{\overset{c_2}{\gtrless}} 0$$

$$\boxed{\sum_{i=\omega+1}^{n} r_i \underset{c_2}{\overset{c_1}{\gtrless}} 0.}$$ — Sufficient Statistics

(b)

$$P_{e|c_1} = 1 - P_r\left[\sum_{i=\omega+1}^{n} r_i\Big|_{c_1} > 0\right].$$ — $N(\sqrt{E}(n-\omega), \sigma^2)$.

$N(\sqrt{E_c}, \sigma^2)$.

$$\boxed{P_{e|c_1} = Q\left(\sqrt{\frac{E_c(n-\omega)}{\sigma^2}}\right) = P_{e|c_2}}$$

Individual term is r.v.

Summation will have mean $\sqrt{E_c}(n-\omega)$ & variance will be same σ^2.

To mimize the propability of error, ω should be equal to 0.

as it will Increase SNR and Q value "U".

18/10/23

Binary FSK (Frequency Shift Keying) (BFSK).

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t + \theta_i) ; & 0 \le t \le T_b. \\ 0 ; & \text{o.w} \end{cases}$$

$i = 1, 2.$

$\langle s_1(t), s_2(t) \rangle = 0$

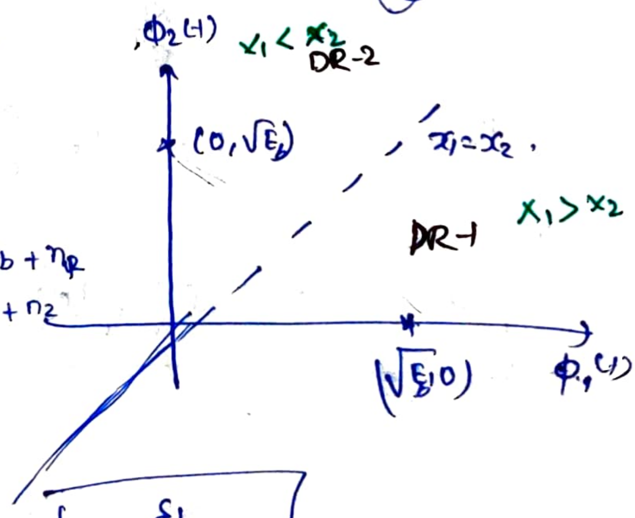
→ Two dimensional Signaling

$\cos(2\pi f_i t + \theta_i)$; θ_i = const then still it is orthogonal.

Prove H.W $\begin{cases} f_1 = f_0 \\ f_2 = 2f_0 \end{cases}$

$\boxed{f_0 = \frac{1}{T_b}}$

Choose f_i, & f_j such that $\langle \cos 2\pi f_i t, \cos 2\pi f_j t \rangle = 0.$



$x(t)\big|_{s_i(t)} = s_i(t) + n(t)$ ⇒ $\begin{cases} x_1 | s_1(t) = \sqrt{E_b} + n_1 \\ x_2 | s_1(t) = 0 + n_2 \end{cases}$

$x_1 | s_2(t) = n_1$

$x_2 | s_2(t) = \sqrt{E_b} + n_2.$

$$\boxed{x_1 \underset{s_2}{\overset{s_1}{\gtrless}} x_2.}$$

$$x_1 - x_2 \underset{s_2}{\overset{s_1}{\gtrless}} 0.$$

$\frac{N_0}{2} + 1^2 \frac{N_0}{2}$

$$Y|_{s(t)} \sim \mathcal{N}\left(\sqrt{E_b}, N_0\right)$$

$$Y|_{s_2(t)} \sim \mathcal{N}\left(-\sqrt{E_b}, N_0\right)$$

$\cancel{\sqrt{E_b}} + n_1 - n_2$

$\frac{N_0}{2} + (1)^2 + \frac{N_0}{2}$

$= N_0$

$E_b + E_b$

$= 2E_b$.

$$\boxed{Pe|_{FSK} = Q\left(\frac{\sqrt{2E_b}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right).}$$

~~Sundefe's~~ FSK

1	0	0	1
$\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_1 t$	$\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_2 t$	$\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_2 t$	$\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_1 t$

No discontinuity in this case as amplitude will be same.

~~Proakis book def^n~~

$$S_m(t) = \sqrt{\frac{2E_s}{T_b}}\cos\left[2\pi f_c t + 2\pi m \Delta f t\right]$$

$$= \sqrt{\frac{2E_s}{T_b}}\cos\left[2\pi(f_c + \Delta f)t\right].$$

Δf = frequency deviation

$\langle S_m(t), S_n(t)\rangle = 0 \quad m \neq n$

$f_m = f_c + m\Delta f$

$f_1 = f_c + \Delta f$

$f_2 = f_c + 2\Delta f$.

Simon Haykin.

Sunde's FSK (Binary FSK).

$$S(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t \pm \frac{2\pi}{2T_b}t\right)$$

Sign	Bit
$+ \rightarrow$	1
$- \rightarrow$	0

$$0 \rightarrow S_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t - \frac{\pi}{T_b}t\right); \quad 0 \le t \le T_b$$

$$1 \rightarrow S_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t + \frac{\pi}{T_b}t\right); \quad 0 \le t \le T_b.$$

HW

$\langle S_1(t) \cdot S_2(t)\rangle = 0$

i.e orthogonal

$$f_1 = f_c - \frac{1}{2T_b}$$

$$f_2 = f_c + \frac{1}{2T_b}$$

$$\boxed{\Delta f = |f_1 - f_2| = 1/T_b}$$

deviation.

$$S(t) = \sqrt{\frac{2E_b}{T_b}}\left[\cos 2\pi f_c t \;\cos\frac{\pi t}{T_b} \mp \sin 2\pi f_c t \;\sin\frac{\pi t}{T_b}\right].$$

$$= \underbrace{\sqrt{E_b}\cos\frac{\pi t}{T_b}}_{\text{In-phase}} \;\underbrace{\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t}_{\phi_I(t)} \mp \underbrace{\sqrt{E_b}\sin\frac{\pi t}{T_b}}_{\text{Q-phase}} \;\underbrace{\sqrt{\frac{2}{T_b}}\sin 2\pi f_c t}_{\phi_Q(t)}$$

PSD of BFSK :-

Both In-phase and Q-phase are base band equivalent.

PSD of In-phase,

$$PSD_{BP} = \frac{E_b}{4}\left[\delta\left(f-\frac{1}{2T_b}\right) + \delta\left(f+\frac{1}{2T_b}\right)\right].$$

PSD of Q-phase

$NRZ \times \sin\frac{\pi t}{T_b}$

$\cancel{\frac{E_b}{4}\,\mathrm{sinc}^2(f T_b) \;\; \frac{1}{2}\left[\delta\left(f-\frac{1}{2T_b}\right)\right]}$

$$\boxed{PSD_{BQ} = \frac{8E_b T_b \cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2 - 1)}}$$ ← H.W.

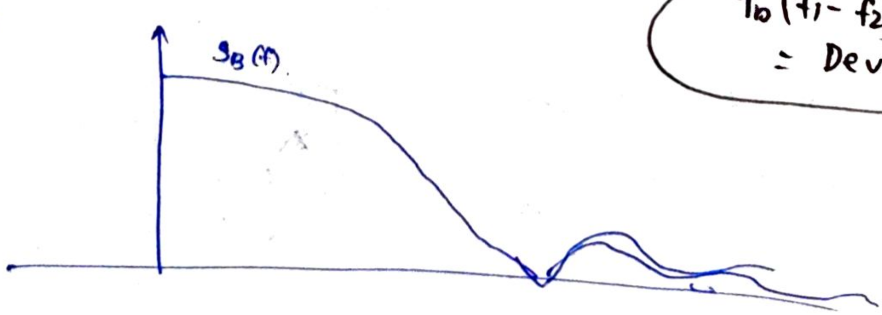
Sunde's FSK is possible to implement with single oscillator compared to BFSK.

$$PSD_{passband} = \frac{1}{4}\left[\delta(f-f_c) + \delta(f+f_c)\right] PSD_{BI}$$

$$+ \frac{1}{4}\left[\delta(f-f_c) + \delta(f+f_c)\right] PSD_{BQ}.$$

$\cos 2\pi f_c t$ & $\sin 2\pi f_c t$ have same PSD as we are taking square

$T_b(f_1 - f_2)$ = Deviation rati.



Digital Communication

① Signal representation

- Gram - Schmidt Orthogonilization procedure.

$$\psi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

$$\psi_2(t) = \frac{d_2(t)}{\sqrt{E_{d_2}}} \quad ; \quad d_2(t) = S_2(t) - \langle S_2(t), \psi_1(t) \rangle \psi_1(t).$$

$$S_m(t) = \sum_{n=1}^{N} S_{mn}\,\psi_n(t) \quad ; \quad S_{mn} = \int_{-\infty}^{\infty} S_m(t)\,\psi_n(t)\,dt$$

Binary Modulation Scheme

Also Binary pulse Amplitude mod.

1) Binary Antipodal Signaling

$1 \rightarrow p(t)$
$0 \rightarrow -p(t).$

$R_b = 1/T_b.$

$$\psi(t) = \frac{p(t)}{\sqrt{E_p}}.$$ — Energy of $p(t)$.

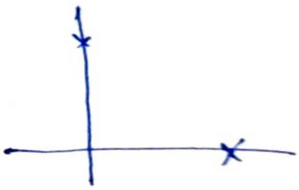
2) Binary Amplitude -Shift keying

$$S_1(t) = p(t) \cos 2\pi f_c t$$
$$S_2(t) = -p(t) \cos 2\pi f_c t$$
$$; \quad p(t) = \sqrt{\frac{2E_b}{T_b}}.$$

$$\psi(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t.$$

3) Binary orthogonal signaling,

$$\psi_1(t) = \frac{S_1(t)}{\sqrt{E_b}} \qquad \psi_2(t) = \frac{S_2(t)}{\sqrt{E_b}}.$$



4) Binary pulse position Modulation,

$$\int_0^{T_b} S_1(t)\,S_2(t)\,dt = 0 \quad ; \quad E_b = \int_0^{T_b} S_1^2(t)\,dt = \int_0^{T_b} S_2^2(t)\,dt.$$

$$S_1(t) = S_{11}\,\psi_1(t) + S_{12}\,\psi_2(t)$$
$$S_2(t) = S_{22}\,\psi_2(t) + S_{22}\,\psi_2(t)$$

5). Binary frequency -Shift keying.

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t \qquad 0 \le t < T_b$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t. \qquad 0 \le t < T_b$$

Strictly Stationary process

$$R_X(t_1, t_2) = R_X(t_2 - t_1)$$ → Auto correlation function depend only on the time difference $(t_2 - t_1)$.

$$C_X(t_1, t_2) = R_X(t_2 - t_1) - \mu_X^2.$$

Property of Autocorrelation fn:-

$$R_X(0) = E[X^2(t)].$$

$$R_X(\tau) = R_X(-\tau)$$ → even function.

$$R_X(\tau) = E[X(t)\,X(t-\tau)].$$

$$R(\tau) = \begin{bmatrix} R_X(\tau) & R_{XY}(\tau) \\ R_{YX}(\tau) & R_Y(\tau) \end{bmatrix}$$

$$R_{XY}(\tau) = R_{YX}(-\tau).$$

Ergodic Process

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E[X(t)]\,dt = \mu_X.$$

Power spectral density

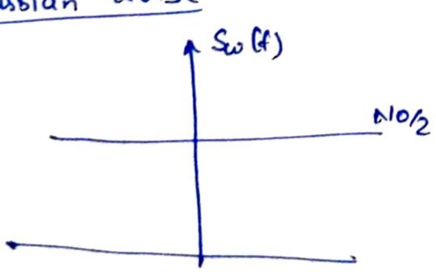
$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f\tau}\, d\tau.$$

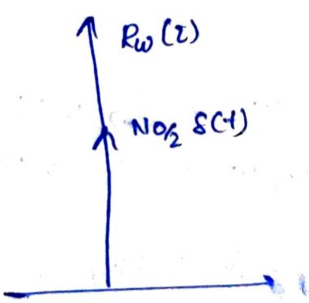
$$R_X(f) = \int_{-\infty}^{\infty} S_X(f)\, e^{+j2\pi f\tau}\, df.$$

$$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f)\, df.$$

$$S_X(f) = S_X(-f)$$ even fn.

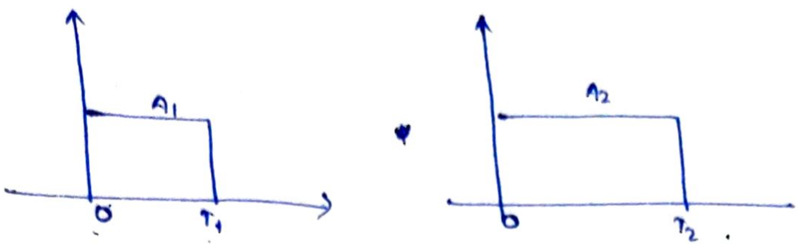
Gaussian Noise



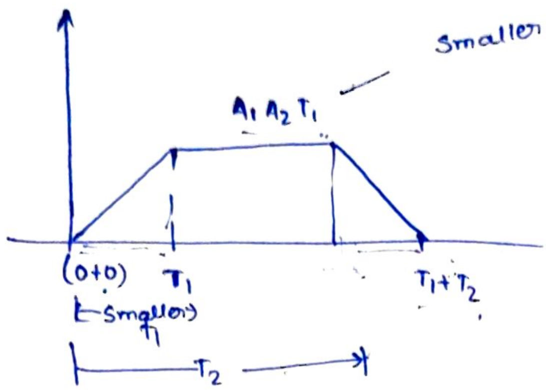


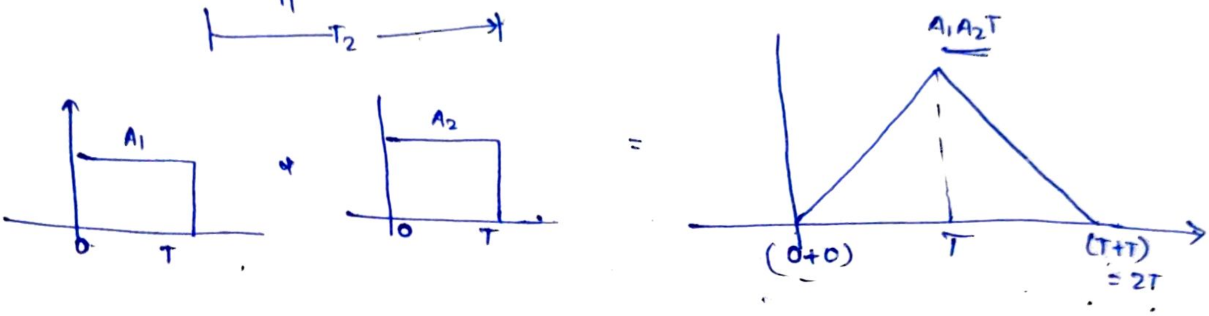
Convolution

$T_2 > T_1$



Smaller duration.





$$\boxed{\text{Deviation ratio} = T_b\,(f_1 - f_2).}$$

Phase transition in FSK

25/10/23

Tutorial

$$r_1 | s_1(t) = \int_0^{1.5} \{ s(t) + n(t) \}\, dt.$$

$$= \int_0^{1} s_1(t)\, dt + \int_0^{1.5} n(t)\, dt.$$

$$= 1 + n_1$$

$$E[n_1] = 0$$

$$Var[n_1] = \frac{N_0}{2} \int_0^{1.5} 1\, dt = 1.5\, N_0/2.$$

$$r_1 | S_2(t) = \int_0^{1.5} S_2(t)\, dt + \int_0^{1.5} n(t)\, dt$$

$$= 0.5 + n_1$$

$$r_2 | s_1(t) = 0 + \int_1^{2} n(t)\, dt.$$

$$r_2 | s_2(t) = 1 + \int_1^{2} n(t)\, dt.$$

$r_1 - r_2$ case comes when Both are Independent.

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$f_r | s_1(t) \underset{S_2(t)}{\overset{S_1(t)}{\gtrless}} f_r | S_2(t)$$

$$z = \begin{bmatrix} x \\ y \end{bmatrix}$$

X and Y are two gaussian R. V.

$$\boxed{f_{x,y}(x, y) = \left(\frac{1}{\sqrt{2\pi}}\right)^2 \frac{1}{\sqrt{\det(K)}}\, e^{-\frac{(z - E(z))^T K^{-1} (z - E(z))}{2}}}$$

covariance matrix

$$f_{XY}(x,y) = \left(\frac{1}{\sqrt{2\pi}}\right)^2 \frac{1}{\sqrt{\det(K)}}$$

$$K = \begin{bmatrix} var(x) & cov(X,Y) \\ cov(X,Y) & var(Y) \end{bmatrix}.$$

$$cov(X,Y) = E[(x-\mu_x)(Y-\mu_y)].$$

$$K = \begin{bmatrix} 1.5\,N_0/2 & N_0/4 \\ N_0/4 & N_0/2 \end{bmatrix}.$$

when S_1

$r_1 | s_1(t) = 1 + n_1$

$r_2 | s_1(t) = 0 + n_2.$

$\det K = \cancel{\quad}$

$$1.5\frac{N_0^2}{4} - \frac{N_0^2}{16}.$$

$$\frac{1.5\times 4 - 1}{16}$$

$$= \frac{5}{16} N_0^2.$$

$$cov(X,Y) = E[(r_1 - 1)(r_2 - 0)].$$

$$= E[r_1 r_2 - r_2].$$

$$= E[r_1 r_2] - E[r_2].$$

$$= E[(1+n_1)n_2]$$

$$= E[n_2 + n_1 n_2]$$

$$= E[n_1 n_2].$$

$$E[n_1 n_2] = E\left\{\int_0^{1.5} n(t)\,dt \int_1^2 n(u)\,du\right\}$$

$$= E\left\{\int_1^{1.5}\int_1 n(t)\,n(u)\,dt\,du\right\}.$$

$$= \int_1^{1.5}\int_1 E[n(t)\,n(u)]\,dt\,du = \frac{N_0}{2}\int_1^{1.5}\delta(t-u)\,dt\,du$$

$$= \frac{N_0}{2}\times 0.5 = N_0/4.$$

$$f_{X,Y}(x,y) = \left(\frac{1}{\sqrt{2\pi}}\right)^2 \times \frac{4}{\sqrt{5}\,N_0} \times$$

$$K^{-1} = \begin{bmatrix} \frac{1.5\,N_0}{2} & \frac{N_0}{4} \\ \frac{N_0}{4} & \frac{N_0}{2} \end{bmatrix}^{-1}.$$

$$= \frac{1}{\frac{5}{16} N_0^2} \begin{bmatrix} \frac{N_0}{2} & -N_0/4 \\ -\frac{N_0}{4} & \frac{1.5\,N_0}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{N_0}{2\Delta} & -\frac{N_0}{4\Delta} \\ -\frac{N_0}{4\Delta} & \frac{1.5\,N_0}{2\Delta} \end{bmatrix}.$$

$$\{z - E(z)\}^T = \left(\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)^T$$

$$= \begin{bmatrix} r_1 - 1 & r_2 \end{bmatrix}$$

$$\cancel{(z-E[z])^T} = \begin{bmatrix} r_1 - 1 & r_2 \end{bmatrix}_{1\times 2} \begin{bmatrix} \frac{N_0}{2\Delta} & -\frac{N_0}{4\Delta} \\ -\frac{N_0}{4\Delta} & \frac{1.5\,N_0}{2\Delta} \end{bmatrix}_{2\times 2} \begin{bmatrix} r_1 - 1 \\ r_2 \end{bmatrix}.$$

$$\begin{bmatrix} a & -b \\ -b & 1.5a \end{bmatrix}.$$

$$= [a(r_1 - 1) - b(r_2) \quad -b(r_1 - 1)$$

$1 + n_1 - 1$

$n_1 \quad n_2.$

$$\begin{bmatrix} n_1 & n_2 \end{bmatrix} \begin{bmatrix} & \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}.$$

$$\begin{bmatrix} an_1 - bn_2 & -bn_1 + 1.5an_2 \end{bmatrix}_{1\times 2} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}_{2\times 1}.$$

$$\begin{bmatrix} an_1^2 - bn_2 n_1 \end{bmatrix}.$$

$$a = \frac{N_0}{2\Delta} = \frac{N_0 \times \cancel{16}^{8} N_0^2}{\cancel{2} \times 5} = \frac{8N_0^3}{5}$$

$$b = \frac{N_0 \times \cancel{16}^{4} N_0^2}{\cancel{4} \times 5} = \frac{4N_0^3}{5}$$

$$\frac{8N_0^3}{5} n_1^2 - \frac{4N_0^3}{5} n_1 n_2.$$

Prob. 10.3 prokslisn.

10.4

When s_2 is transmitted,

$$K = \begin{bmatrix} Var(x) & cov(x,y) \\ cov(y,x) & Var(y) \end{bmatrix}$$

$r_1 | s_2(t) = 0.5 + n_1$

$r_2 | s_2(t) = 1 + n_2$

$$= \begin{bmatrix} 1.5\frac{N_0}{2} & N_0/4 \\ N_0/4 . & N_0/2 \end{bmatrix}$$

$E[(r_1 - 0.5)(r_2 - 1))]$.

$= E[n_1 n_2]$.

$= E\left[\int_1^{1.5}\int_1^{1.5} n(t)\, n(u)\, dt\, du\right]$

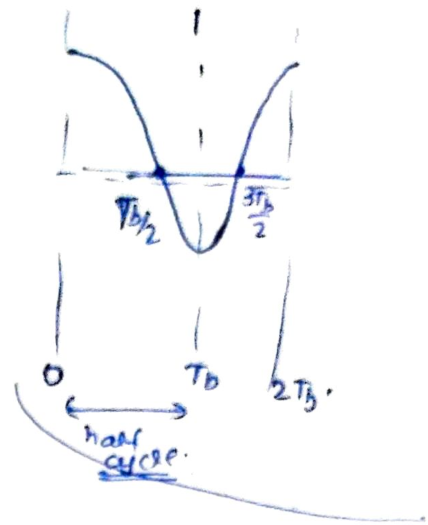
$= 0.5\ N_0/2$

$$f_{xy}(x,y)|_{s_2} = \left(\frac{1}{\sqrt{2\pi}}\right)^2 \times \frac{4}{\sqrt{5}N_0} \times \exp\left(e^{-\frac{(z-E(z))^T K^{-1}(z-E(z))}{2}}\right)$$

$$= \frac{1}{2\pi} \times \frac{4}{\sqrt{5}N_0}$$

$\cos \frac{\pi t}{T_b}$ $\qquad \cos(2\pi f_1 t)$

$$f_1 = \frac{1}{2T_b}$$



Phase transition in FSK

For Sunde's FSK, the phase is linear w.r.to t

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_c t \pm \frac{\pi t}{T_b} + \theta(0)\right].$$

$$\theta(t) = \pm \frac{\pi t}{T_b} + \theta(0)$$

$$\theta(t) - \theta(0) = \pm \frac{\pi t}{T_b}.$$

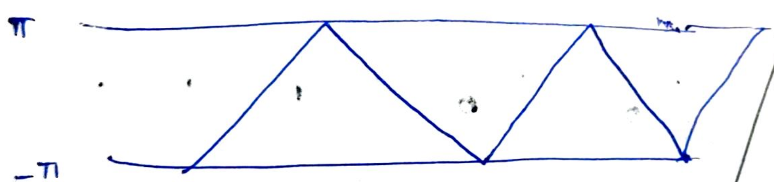
It doesn't have memory.

Irrespective of possible waveform it remains same.

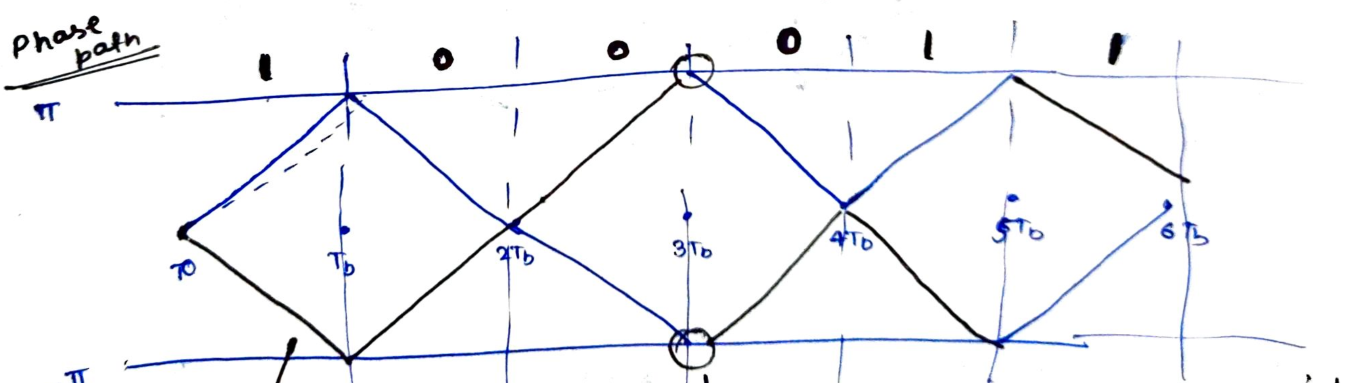
continuous phase.

⇓

Base band should be continuous.



Phase path



All the phase.

not discontinuous. (as π & $-\pi$ is same).

$2T_b$, $4T_b$, $6T_b$, phase $= 0°$ is not possible.

$\theta(t) - \theta(0) = \pm \frac{\pi h}{T_b}$.

$f_1 = f_c + \frac{1}{2T_b}$

$f_2 = f_c - \frac{1}{2T_b}$

$\Delta f = \frac{1}{T_b}$.

frequency deviation.

$f_1 = f_c + \frac{h}{2T_b}$

$f_2 = f_c - \frac{h}{2T_b}$

$\Delta f = \frac{h}{T_b}$. h such that Δf should decrease.

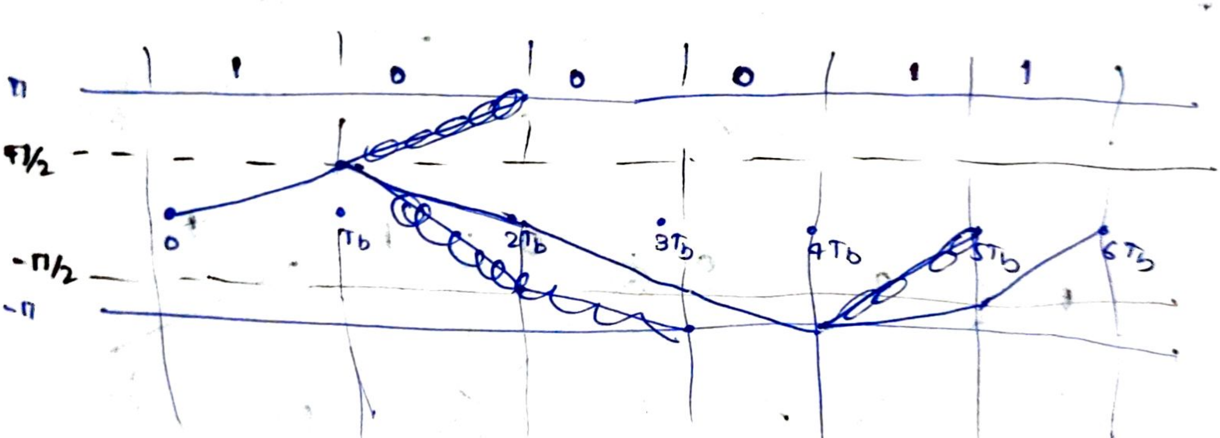
H.W

$h_{min} = 1/2$

for $h = 1/2$ prove that $s_1(t)$ and $s_2(t)$ are still FSK signal.

Δf is minimum freq. devication for $h = 1/2$

It is called minimus FSK.



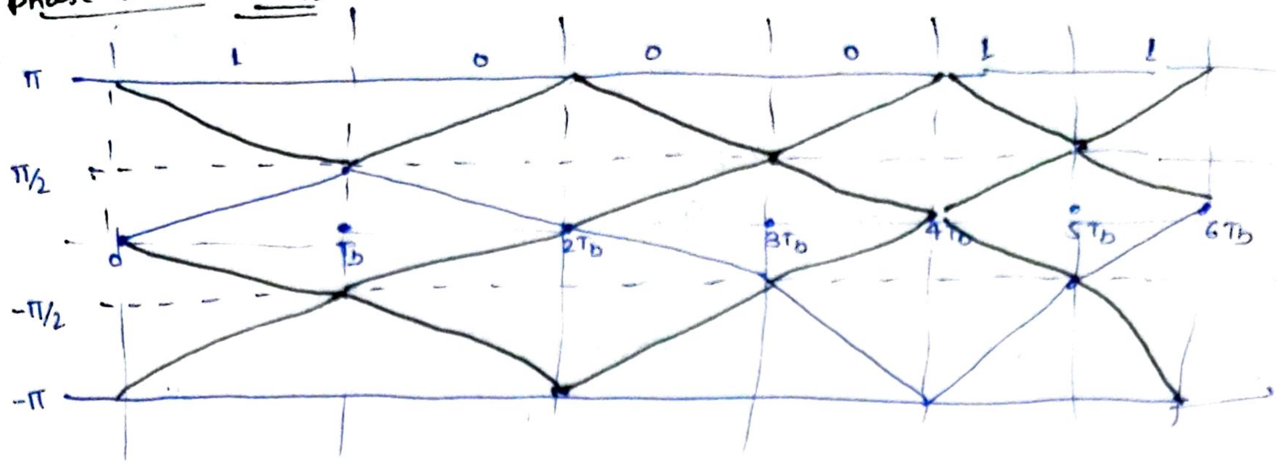
$\theta(t) - \theta(0) = \pm \frac{\pi t}{2T_b}$.

$\frac{\pi}{2} \times 3$

$3\pi/2$

At $t = T_b$.

Phase trellis $h = 1/2$



to decode phase transition ⇒ previous data should known.

There are two possible phase at any instant of multiple of 'Tb' times. Hence, it has memory. unlike FSK.

MSK (Minimum Shift Keying)

$$s(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_1 t + \theta(0)\right] & \text{for } 1 \\ \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_2 t + \theta(0)\right] & \text{for } 0 \end{cases}$$

$$f_1 = f_c + 1/4T_b$$

$$f_2 = f_c - 1/4T_b$$

$$f_c = \frac{1}{2}(f_1 + f_2)$$

$$\Delta f = f_1 - f_2 = 1/2T_b$$ – frequency separation reduced to ½ compared to FSK.

$$\boxed{\Delta f = \frac{R_b}{2}}.$$

$$\boxed{T_b (f_1 - f_2) = h}$$

Signal space diagram for MSK

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(t)) \cos 2\pi f_c t - \sqrt{\frac{2E_b}{T_b}} \sin(\theta(t)) \sin(2\pi f_c t)$$

Genearal case.

$$\theta(t) = \theta(0) \pm \pi/2T_b.$$

1 & 0 deceide by this.

Q-phase S_Q

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\left[\underbrace{\cos\left(\theta(0) \pm \frac{\pi t}{2T_b}\right)}_{\text{Inphase } S_I} \cos 2\pi f_c t - \sin\left(\theta(0) \pm \frac{\pi t}{2T_b}\right) \sin(2\pi f_c t)\right].$$

$$S_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos\theta(0) \cos\frac{\pi}{2T_b}t.$$

$\theta(0) \pm \pi/2T_b t \quad 2\pi f_c t\theta$

$\sin\theta(0)$ will vanish as at $\boxed{\theta(0) = \pm\pi}$

$\sin(\theta(0)) = 0$.

At 0, θ = either $+\pi$ or $-\pi$.

$$\boxed{S_I(t) = \pm\sqrt{\frac{2E_b}{T_b}} \cos\frac{\pi}{2T_b}t.}$$

If $\theta(0) = 0$, implies $+$

$\theta(0) = \pi$, implies , $-$

$$S_Q(t) = \sin(\theta(0)) \cancel{\cos\left(\frac{\pi t}{2T_b}\right)} \pm \cos\theta(0) \sin\left(\frac{\pi t}{2T_b}\right).$$

$\sin(\theta(0)) = 0$. $\rightarrow 0$

$$\boxed{S_Q(t) = \pm\sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{2T_b}\right) \cos(\theta(0)).}$$

$$\theta(t) = \theta((n-1)T_b) \pm \frac{\pi}{2T_b}t$$

$t = 3T_b$

$$\theta(3T_b) = \theta(2T_b) \pm \pi/2T_b \times 3T_b$$

$$= \theta(2T_b) \pm \frac{3\pi}{2}$$

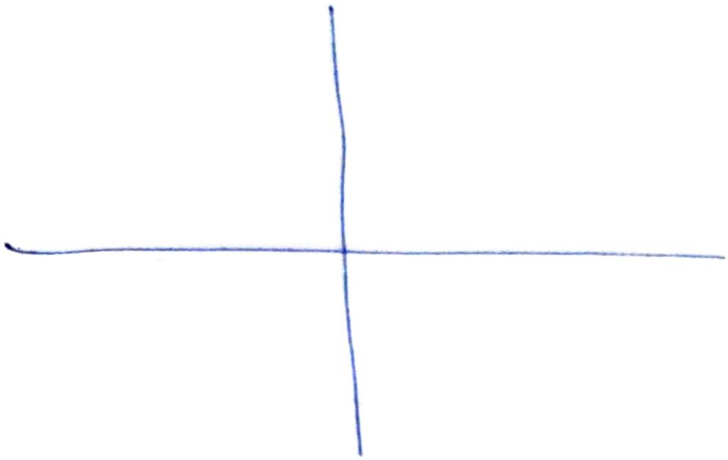
$$\boxed{\theta(nT_b) = \theta((n-1)T_b) \pm \frac{n\pi}{2}}$$

$\Rightarrow$ $\theta(0) = 0 \Rightarrow \theta(T_b) = \pi/2$; "1"

$\theta(0) = \pi \Rightarrow \theta(T_b) = \pi/2$; "0"

$\theta(0) = 0 \Rightarrow \theta(T_b) \ \pi/2$; 0

$\theta(0) = \pi \Rightarrow \theta(T_b) = -\pi/2$; "1"



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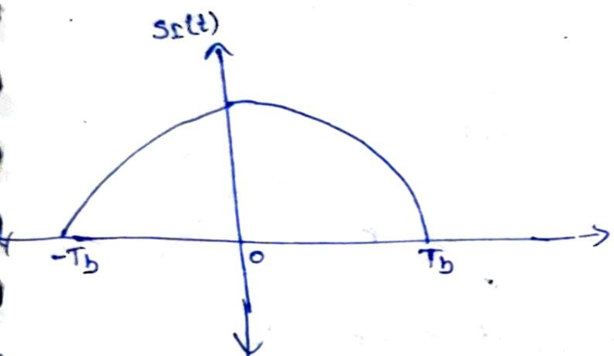
Assume $\theta(0) = 0^\circ$,

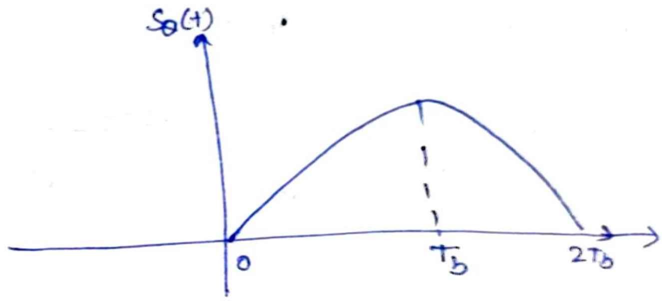
$$S_I = \sqrt{\frac{2E_b}{T_b}} \cos \frac{\pi}{2T_b} t.$$

; period = '$4T_b$' ;

; T = '$4T_b$' ;

$$S_Q = \pm \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi}{2T_b} t\right)$$





$\theta(0) = 0$

$b(t=0) = 0$

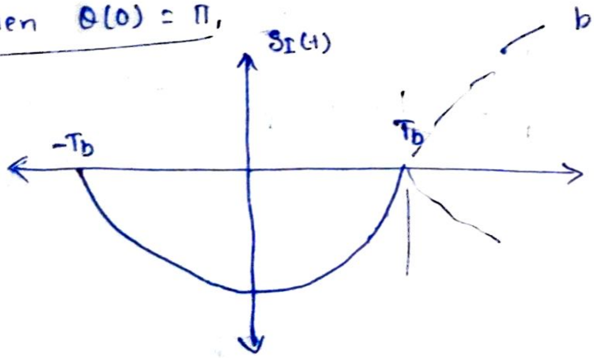
$b(t = T_b) = 0, 1.$

$b(t = -T_b) = 0, 1.$

depend on past bit.

S_I doesn't depend on present bit

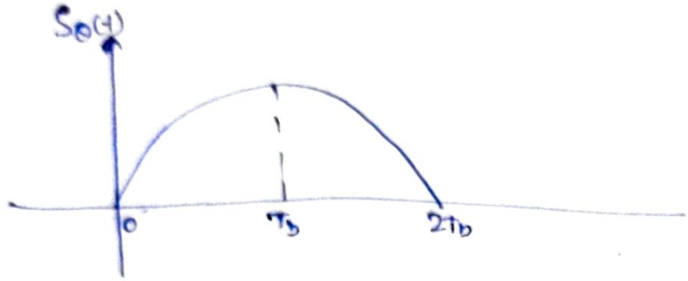
When $\theta(0) = \pi$,



based on bit pattern up or down

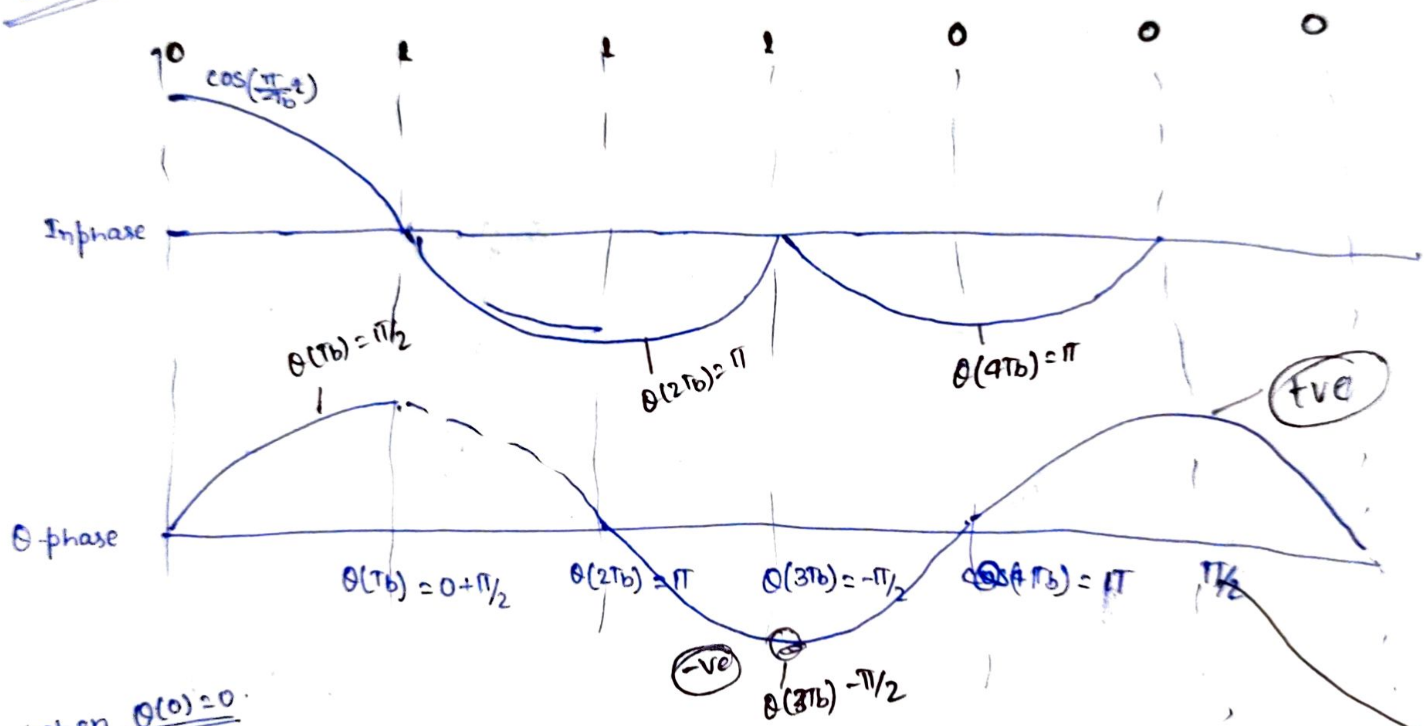
$S_Q(t) = \pm\sqrt{\frac{2E_b}{T_b}}\cos\theta(0)\ \sin\frac{\pi t}{2T_b}$

$S_Q(t) = \underset{\text{bit 1}}{(-)}\ \underset{\theta(0)=\pi}{(-)}\ \sqrt{\frac{2E_b}{T_b}}\ \sin\left(\frac{\pi t}{2T_b}\right).$



$+ \rightarrow$ '1'
$- \rightarrow$ '0'.

$\theta(0)=0$



when $\theta(0)=0$.

$S_I = \sqrt{\frac{2E_b}{T_b}}\cos\left(\frac{\pi}{2T_b}t\right)$ '+' $\longrightarrow$ '1'

$S_Q = -\sqrt{E_b}\ \sin\left(\pm\frac{\pi}{2T_b}t\right)$ '−' $\longrightarrow$ '0'.

$S_Q = \mp\sqrt{E_b}\ \sin\left(\frac{\pi}{2T_b}t\right).$; '−' $\longrightarrow$ 1, '+' $\longrightarrow$ 0 — due to sign change.

$\theta(0) = -\pi/2$

$\cos\left(+\pi/2 + \frac{\pi}{2T_b}t\right)$

$\cos\left(\pi/2 + \frac{\pi}{2T_b}t\right).$

$\cos(90-\theta).$

$\sin\left(\frac{\pi}{2T_b}t\right)$

$\sin\left(\pi/2 + \frac{\pi}{2T_b}t\right).$

At even times

0° ⟶ Inphase

+ve

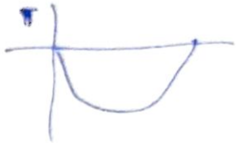
b/w two time period

-ve

$+\cos\left(\frac{\pi}{2T_b}t\right)$

0° Inphase - always +ve

π Inphase - negtive

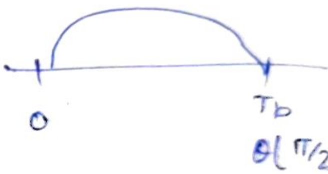


π/2 +ve

−π/2

odd times

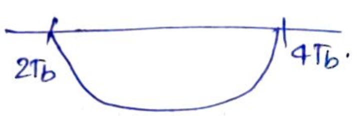
$\theta = +\pi/2$ ⟶ Q-phase +ve



$\theta(T_b) = -\pi/2$

$T_b.$ $\theta(-\pi/2)$

$\theta(3T_b) = \pi/2$



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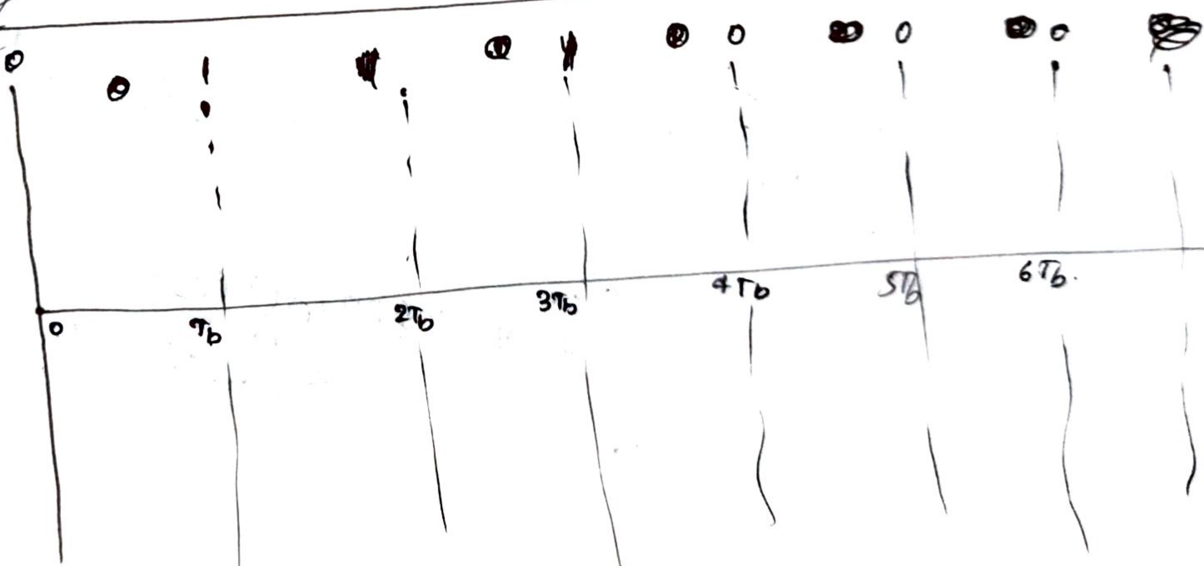


Q. PSK signal

Q. A PSK signal is applied to a correlat

ϕ —

$$\int_0^{T_b} \underbrace{\sqrt{E_b}\cdot\phi_1(t)} * \cos(2\pi f_c t + \phi)$$

$$\int_0^{T_b} \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \times .\cos(2\pi f_c t) * \cos\phi - \sin 2\pi f_c t \cdot \sin\phi$$

$$\sqrt{2E_b} \int_0^{T_b} \sqrt{\frac{2}{T_b}}\cos^2 2\pi f_c t \cdot \cos\phi - 0$$

$$= \sqrt{E_b}\cdot\cos\phi \quad -0$$

$$= \sqrt{E_b}\cos\phi + n(t).$$

mean $= \sqrt{E_b}\cos\phi$. $N_0/2$.

$-\sqrt{E_b}\cos\phi$ $\qquad$ $+\sqrt{E_b}\cos\phi$

$$P_e = Q\left(\sqrt{\frac{2E_b\cos\phi}{N_0}}\right)$$

$$\int_0^{T_b} n(t) * \cos 2\pi f_c(t)\cos\phi + \int_0^{T_b}\sin(2\pi f_c t\sin\phi$$

$$n = \int_0^{T_b} n(t)\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t + \phi)\, dt$$

$$E[n] = 0.$$

$$E\left[\left[n - E(n)\right]^2\right] = E\left[\int_0^{T_b}\int_0^{T_b} n(t)\, n(u)\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t + \phi)\sqrt{\frac{2}{T_b}}\cos(2\pi f_c u + \phi)\cdot dt\, du\right]$$

$$= \int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\,\delta(t-u)\cos(2\pi f_c t + \phi)\cos(2\pi f_c u + \phi)\, dt\, du.$$

$$= \frac{2}{T_b}\int_0^{T_b} \frac{N_0}{2}\cos^2(2\pi f_c t + \phi)\, dt.$$

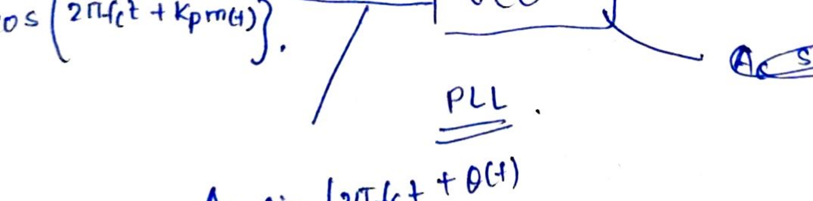
$$= \frac{N_0}{2}\cdot\frac{2}{T_b}\int_0^{T_b}\cos^2(2\pi f_c t + \phi)\, dt.$$

$\phi = \pi/2$.

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\cos\phi\right)$$

$\phi = \pi/2$, $Q(0) = 1/2$. — Max^m probability of error.

6.3 Book



$A_c\cos(2\pi f_c t + K_p m(t))$.

~~$A_c \sin(2\pi f_c t + \theta(t))$~~

PLL.

$A_c \sin(2\pi f_c t + \theta(t))$

$$A_c\cos(2\pi f_c t + K_p m(t)) \cdot A_c\sin(2\pi f_c t + \theta(t))$$

$$= \frac{A_c^2}{2}\left\{\sin(2\pi f_c t + K_p m(t) + 2\pi f_c t + \theta(t)) + \sin(K_p m(t) - \theta(t))\right\},$$

$$= -\frac{A_c^2}{2} \sin(K_p m(t)) - \theta(t) \}.$$

$$y(t) = \frac{A_c^2}{2} \cos(K_p m(t)) \cdot \sin\theta(t) - \frac{A_c^2}{2} \sin(K_p m(t)) \cdot \cos\theta(t)$$

$\underline{\theta(t) = 0}$

$$= -\frac{A_c^2}{2} \sin(K_p m(t))$$

$$= -\frac{1}{2} m(t) A_c^2 \sin(K_p) \qquad [\because m(t) = \pm 1].$$

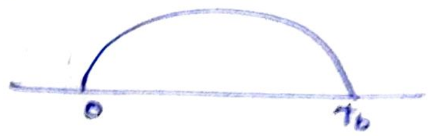
$\underline{y(t) \propto m(t)}$

1/11/23

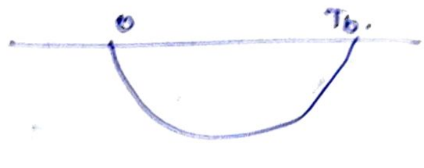
So - without considering -

- so

$\theta(T_b) = -\pi/2$



$\theta(T_b) = \pi/2$



Inphase

0 — +ve cycle

π — -ve cycle

Qphase - θ

$-\pi/2$ — +ve cycle

$+\pi/2$ — -ve cycle.

MSK

T_b - time - BER different

$2T_b$ - time — BER different

we calculate,

0 to $2T_b$ — Better BER

MSK constellation

$$S_{BI}(t) = \sqrt{E_b}\ \cos(\theta(0))\ \cos\frac{\pi t}{2T_b}$$

$$S_{BQ}(t) = -\sqrt{E_b}\ \cos(\theta(0))\ \sin\left(\pm\frac{\pi t}{2T_b}\right)$$

$$S_I = S_{BI}(t) \times \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t).$$

$$S_Q = S_{BQ}(t) \times \sqrt{\frac{2}{T_b}}\sin(2\pi f_c t).$$

let

$$\phi_1(t) = \cos\frac{\pi t}{2T_b}\ \sqrt{\frac{2}{T_b}}\ \cos(2\pi f_c t)$$

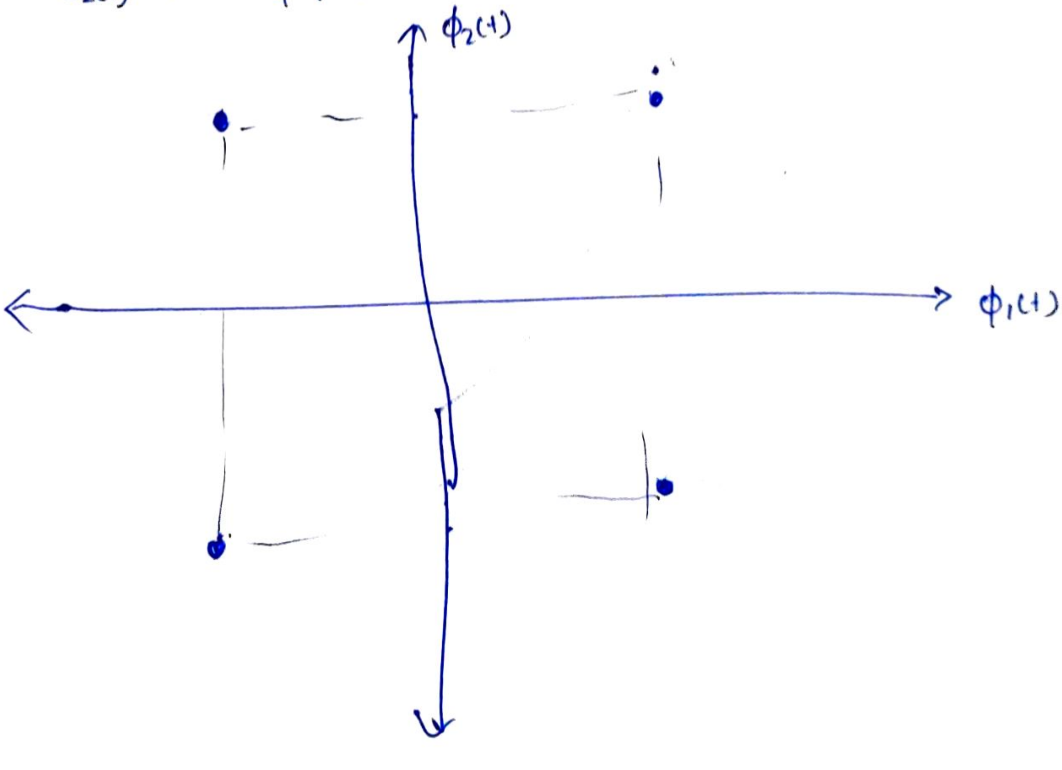
$$\phi_2(t) = \sin\left(\pm\frac{\pi t}{2T_b}\right)\sqrt{\frac{2}{T_b}}\ \sin(2\pi f_c t)$$

H.W

ϕ_1 & $\phi_2(t)$ are orthogonal on $0 \le t \le 2T_b$.

$$S_1(t) = \sqrt{E_b}\ \cos\theta(0)$$

$$S_2(t) = \mp\sqrt{E_b}\ \cos\theta(0)$$



$$\theta(t) = \theta(0) \pm \frac{\pi}{2T_b} t.$$

$$\theta(t) = \theta((n-1)T_b) \pm \frac{\pi}{2T_b} t. \qquad nT_b \le t \le (n+1)T_b.$$

$t = 0$ to $2T_b$

$\theta(0) = \pi$, $\theta(T_b) = -\pi/2$

$S_1 = -\sqrt{E_b}$ $S_2 =$