MA 102
Calculus
Tutorial–4

(1) Verify both forms of Green's Theorem for the vector field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C\colon r(t) = \cos(t)i + \sin(t)j, 0 \leq t \leq 2\pi$.

(2) Use Stokes' Theorem to evaluate $\oint_C \vec{F} \cdot \vec{dr}$ if $F = xz\ i + xy\ j + 3xz\ k$ and C is the boundary of the portion of the plane $2x+y+z = 2$ in the first octant, traversed counter clock wise as viewed from above.

(3) Find the flux of $F = xy\ i + yz\ j + xz\ k$ outward through the surface of the cube cut from the first octant by the planes $x = 1$,$y = 1$ and $z = 1$.

(4) Evaluate the line integral $\oint_C \vec{F} \cdot \vec{dr}$ when $\vec{F} = \langle z^2, y^2, xy\rangle$, C is the triangle defined by $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 2)$, and C is traversed counter clockwise as viewed from the origin.

(5) Evaluate the flux integral $\iint_S Curl(F) \cdot \vec{n}\ dS$ where $F = \langle 2z - y, x - z, y - x\rangle$ and S is the portion of the sphere $x^2 + y^2 + z^2 = 9$ with $z \geq y$ and $\vec{n}$ points away from the origin.

(6) Let S be the surface of the cube $D\colon 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$ and $F = (e^x + z)i + (y^2 - x)j - xe^y k$. Compute the outward flux $\iint_S F \cdot \vec{n}\ dS$.

(7) Use the divergence theorem to find the outward flux $\iint_S F \cdot \vec{n}\ dS$ of the vector field $F = x^3 i + y^3 j + z^3 k$ with D the region bounded by the sphere $S\colon x^2 + y^2 + z^2 = a^2$.