① A rectangular coil is placed in a field of $\vec{B} = (2\hat{a}_x + \hat{a}_y)$ wb/m^2. The coil is in y-z plane and has dimensions of $2m \times 2m$. It carries a current of 1A. Find the torque about the z-axis.

<u>Sol^n</u> $\vec{m} = IS\,\hat{a}_n = 1\times 4\,\hat{a}_x$

$\vec{T} = \vec{m}\times\vec{B} = 4\,\hat{a}_x \times (2\hat{a}_x + \hat{a}_y) = 4\,\hat{a}_z$ (N-m).

<u>Ans.</u>

② The vector magnetic potential, $\vec{A}$ due to a direct current in a conductor in free space is given by $\vec{A} = (x^2+y^2)\,\hat{a}_z$ (μ Wb/m^2). Determine the magnetic field produced by the current element at (1, 2, 3).

<u>Sol^n</u> $\vec{A} = (x^2+y^2)\,\hat{a}_z$ μWb/m^2

We have $\vec{B} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & x^2+y^2 \end{vmatrix}$

$= [(2y)\,\hat{a}_x - (2x)\,\hat{a}_y]\times 10^{-6}$

$\vec{B}\big|_{(1,2,3)} = (4\,\hat{a}_x - 2\,\hat{a}_y)\times 10^{-6}$

$\vec{H} = \dfrac{\vec{B}}{\mu_0} = \dfrac{1}{4\pi\times 10^{-7}}(4\hat{a}_x - 2\hat{a}_y)\,10^{-6}$

$= (3.978\,\hat{a}_x - 4.77\,\hat{a}_y)$ A/m.

(3) A current element 4 cm long is along y-axis with a current of 10 mA flowing in y-direction. Determine the force on the current element due to the magnetic field if the magnetic field $\vec{H} = \frac{5\hat{a}_x}{\mu}$ A/m

Solⁿ

$\vec{F} = I\vec{L} \times \vec{B}$; $I\vec{L} = 10\times10^{-3}\times0.04\,\hat{a}_y$

$\vec{B} = \mu\vec{H}$

$= \mu\frac{5\hat{a}_x}{\mu}$

$= 5\hat{a}_x$

$\therefore \vec{F} = 10\times10^{-3}\times0.04\,\hat{a}_y\times5\,\hat{a}_x$

$= -2.0\,\hat{a}_z$ mN

Ans.

(4) A charge of 12 C has velocity of $5\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z$ m/s. Determine $\vec{F}$ on the charge in the field of

(a) $\vec{E} = 18\hat{a}_x + 5\hat{a}_y + 10\hat{a}_z$ (V/m),

(b) $\vec{B} = 4\hat{a}_x + 4\hat{a}_y + 3\hat{a}_z$ (Wb/m²)

Solⁿ (a) $\vec{F}$ on Q due to $\vec{E}$ is

$\vec{F} = q\vec{E} = 12(18\hat{a}_x + 5\hat{a}_y + 10\hat{a}_z)$

$= 216\hat{a}_x + 60\hat{a}_y + 120\hat{a}_z$

or, $F = q|\vec{E}| = 12\sqrt{18^2 + 5^2 + 10^2} = 254.27$ (N)

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(b) The force $\vec{F}$ on the charge due to $\vec{B}$ is

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 5 & 2 & -3 \\ 4 & 4 & 3 \end{vmatrix}$$

$$\therefore \vec{F} = 12(18\hat{a}_x - 27\hat{a}_y + 12\hat{a}_z)$$

$$|\vec{F}| = 12\sqrt{324 + 729 + 144}$$

$$\therefore \boxed{F = 415 \cdot 17 \text{ N}}$$

(5) If $\vec{H} = y\cos 2x\, \hat{a}_x + (y + e^x)\hat{a}_z$, determine $\vec{J}$ at the origin.

Soln The differential form of Ampere's circuit law is

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\therefore \vec{J} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\cos 2x & 0 & y+e^x \end{vmatrix}$$

$$= \hat{a}_x - e^x\hat{a}_y - \cos 2x\, \hat{a}_z$$

$$\vec{J}\Big|_{(0,0,0)} = (\hat{a}_x - \hat{a}_y - \hat{a}_z)\ (\text{A}/\text{m}^2)$$

Ans

(6) If the magnetic flux density in a medium is given by $\vec{B} = \frac{1}{\rho}\cos\phi\ \hat{a}_\rho$, what is the flux crossing the surface defined by $-\frac{\pi}{4} < \phi \leq \frac{\pi}{4}$, $0 \leq z \leq 2m$.

<u>Sol^n</u> $\vec{B} = \frac{1}{\rho}\cos\phi\ \hat{a}_\rho$

By defination $\Phi = \int_S \vec{B}\cdot\vec{ds}$

$$= \int_S \frac{1}{\cancel{\rho}}\cos\phi\ \hat{a}_\rho \cdot \cancel{\rho}\, d\phi\, dz\ \hat{a}_\rho$$

$$= \int_{z=0}^{2} dz \int_{\phi=-\pi/4}^{\pi/4} \cos\phi\ dz$$

$$= 2.83\ (Wb)$$ <u>Ans</u>

(7) A charged particle of mass 1 kg and charge 2C starts at the origin with zero initial velocity in a region where $\vec{E} = 3\,\hat{a}_z$ (V/m). Find the followings:

(a) The force on the particle

(b) The time it takes to reach point (0, 0, 12 m)

(c) It's velocity and accelaration at 'p'

(d) It's K.E. at P.

Soln (a) $\vec{F} = q\vec{E} = 6\,\hat{a}_z$ N Ans

(b) $m\frac{d\vec{v}}{dt} = 6\hat{a}_z$ $\qquad \frac{dv_x}{dt} = 0$

$\frac{dv_z}{dt} = 6\,\hat{a}_z$ $\qquad \frac{dv_y}{dt} = 0$

$\Rightarrow v_z = 6t + c$

at $t=0$, initial velocity $=0$,

$\therefore c = 0$

$\therefore \vec{v} = 6t\,\hat{a}_z$ m/s. Ans

(c)

$\frac{dz}{dt} = 6t$

$\therefore z = \cancel{6}^{3}\frac{t^2}{\cancel{2}} + A$

$t=0$, $x=0$, $y=0$, $z=0$ (at origin)

$\therefore A = 0$,

$z = 3t^2$

$\therefore$ when $z = 12$, $t^2 = 4$

$\therefore \Rightarrow t = 2$ sec Ans

$v\,|(0,0,12\,m) = 6\cdot 2 = 12\,\hat{a}_z$ m/s.

$f = 6\,\hat{a}_z$ m/s^2 Ans

$K.E = \frac{1}{2}mv^2 = \frac{1}{2}(12)^2 = 6\cdot 12 = 72$ J Ans

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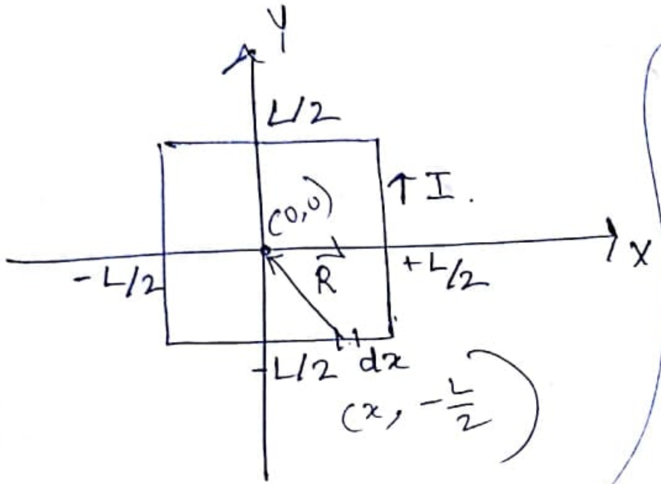
(8) Find $\vec{H}$ at the centre of a square current loop of side 'L'.



for $0 \le x \le L/2$, $y = -L/2$

$$\vec{dH} = \frac{I\, \vec{dl} \times \hat{a}_R}{4\pi R^2} \quad (A/m)$$

$$= \frac{I\, dx\, \hat{a}_x \times \left[-x\,\hat{a}_x + \frac{L}{2}\hat{a}_y\right]}{4\pi\left[x^2 + \left(\frac{L}{2}\right)^2\right]^{3/2}}$$

$\therefore \vec{H} =$ By symmetry, each half-side contributes the same amount of $\vec{H}$ at the centre.

total field at origin

$$\therefore = 8 \int_0^{L/2} \frac{I\, dx \left(\frac{L}{2}\right) \hat{a}_z}{4\pi\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{3/2}}$$

$$= \frac{2\sqrt{2}\, I}{\pi L}\, \hat{a}_z$$

(or $\hat{a}_n$

where $\hat{a}_n$ is the unit normal to the plane of the loop as given by the usual right-hand rule.

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(9) In the region $0 < \rho < 0.5m$, the cylindrical co-ordinates, the current density is
$\vec{J} = 4.5 \, e^{-2\rho} \, \hat{a}_z$ (A/m²)
& $\vec{J} = 0$ elsewhere. Use Ampere's law to find $\vec{H}$.

Solⁿ

$$\oint \vec{H} \cdot \vec{dl} = 2\pi\rho H_\varphi = I_{enc}$$

$$= \int_s \vec{J} \cdot \vec{ds}$$

$$= \int_{\varphi=0}^{2\pi} \int_{\rho=0}^{\rho} 4.5 e^{-2\rho} \hat{a}_z \cdot \rho \, d\rho \, d\varphi \, \hat{a}_z$$

$$\boxed{\vec{H} = \frac{0.297}{\rho} \hat{a}_\varphi} \; (A/m)$$