Control Systems

Subject Code: EC380

Lecture 4: Signal-Flow Graph Models

Surajit Panja
Associate Professor
Dept. of Electronics and Communication Engineering

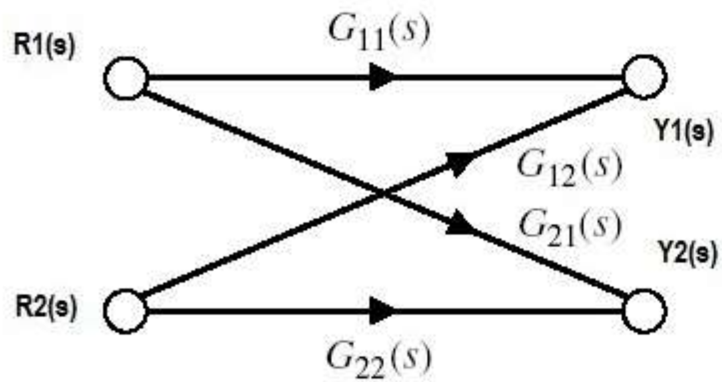
Indian Institute of Information Technology Guwahati
Bongora, Guwahati-781015

Signal-Flow Graphs

- A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations.

- When applying the signal flow graph method to analyses of control systems, we must first transform linear differential equations into algebraic equations in *s*.

- For complex systems, the block diagram method can become difficult to determine transfer function.

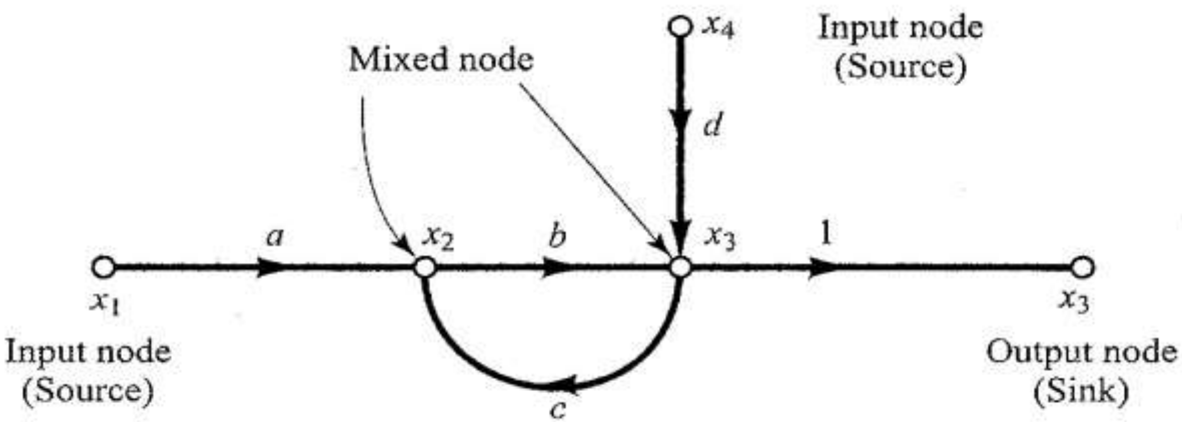


$$Y_1(s) = G_{11}(s) \cdot R_1(s) + G_{12}(s) \cdot R_2(s)$$

$$Y_2(s) = G_{21}(s) \cdot R_1(s) + G_{22}(s) \cdot R_2(s)$$

Definitions

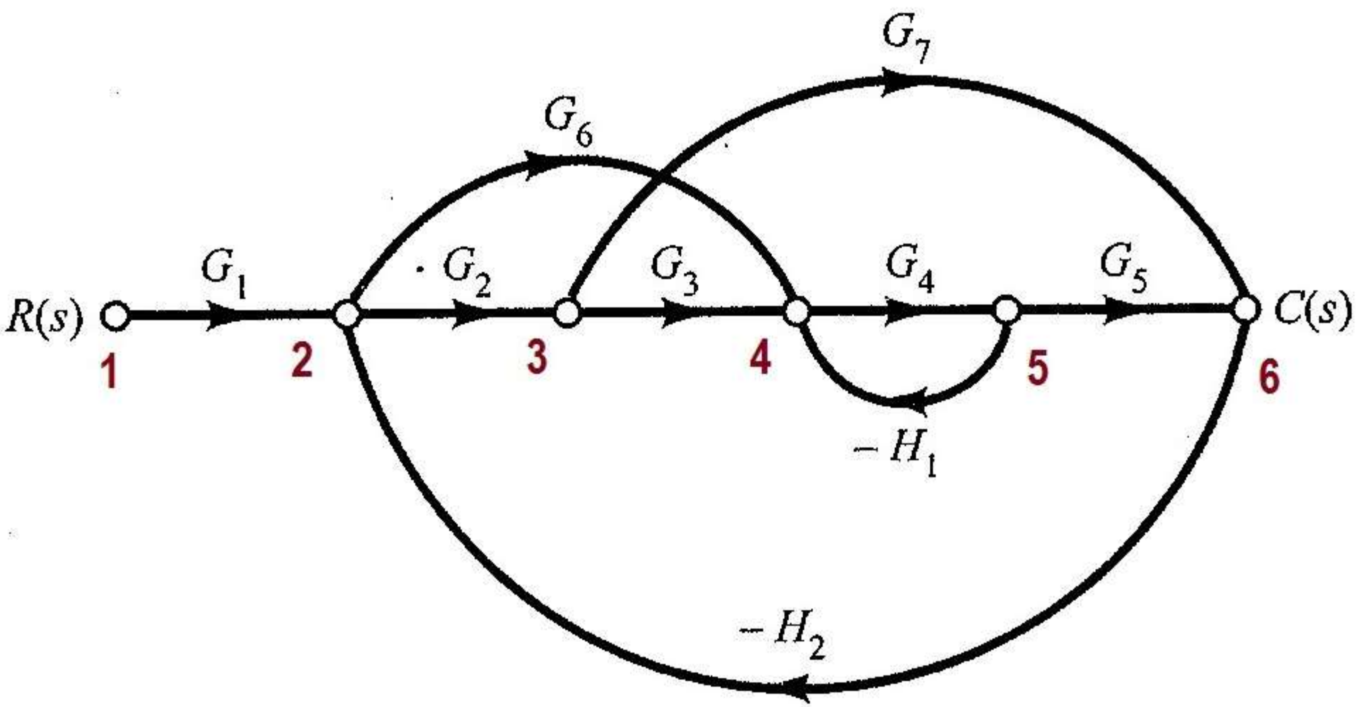
1. *Node.*: A node is a point representing a variable or signal.
2. *Transmittance.*: The transmittance is a real gain or complex gain between two nodes. Such gains can be expressed in terms of the transfer function between two nodes.
3. *Branch.*: A branch is a directed line segment joining two nodes. The gain of a branch is a transmittance.
4. *Input node or source.*: An input node or source is a node that has only outgoing branches. This corresponds to an independent variable.
5. *Output node or sink.*: An output node or sink is a node that has only incoming branches. This corresponds to a dependent variable.
6. *Mixed node:* A mixed node is a node that has both incoming and outgoing branches.
7. *Path:* A path is a traversal of connected branches in the direction of the branch arrows. If no node is crossed more than once, the path is open. If the path ends at the same node from which it began and does not cross any other node more than once, it is closed. If a path crosses some node more than once but ends at a different node from which it began, it is neither open nor closed.



Definitions

8. *Loop*: A loop is a closed path.
9. *Loop gain*: The loop gain is the product of the branch transmittances of a loop.
10. *Nontouching loops.* Loops are nontouching if they do not possess any common nodes.
11. *Forward path:* A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes more than once.

12. *Forward path gain.* A forward path gain is the product of the branch transmittances of a forward path.

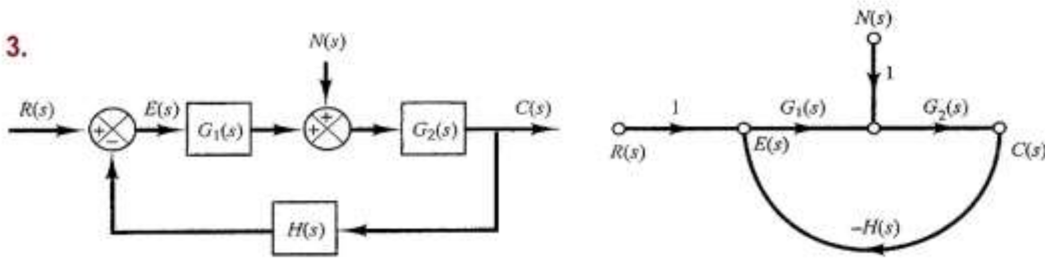


Signal Flow Graphs of Control Systems

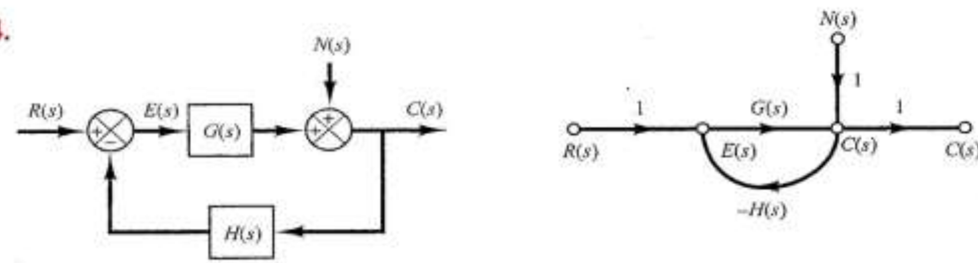
1.

2.

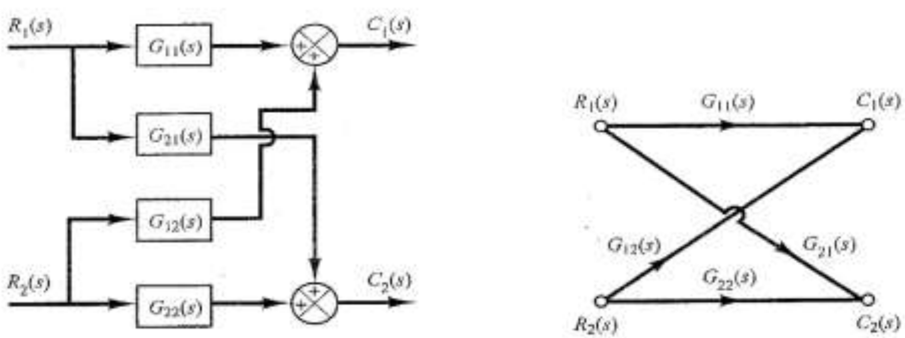
3.



4.



5.



Mason's Gain Formula

Mason's gain formula, which is applicable to the overall gain, is given by

$$P = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

where

P_k = path gain or transmittance of kth forward path

Δ = determinant of graph

= 1 − (sum of all individual loop gains) + (sum of gain products of all possible combinations of two nontouching loops) − (sum of gain products of all possible combinations of three nontouching loops) + ⋯

$$= 1 - \sum_a L_a + \sum_{b,c} L_b L_c - \sum_{d,e,f} L_d L_e L_f + \cdots$$

$\sum_a L_a$ = sum of all individual loop gains

$\sum_{b,c} L_b L_c$ = sum of gain products of all possible combinations of two nontouching loops

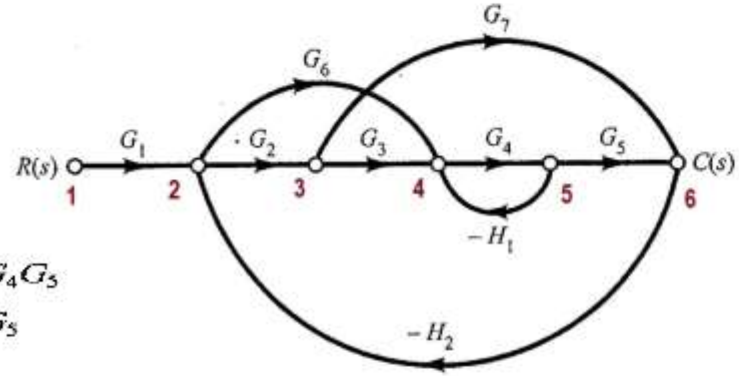
$\sum_{d,e,f} L_d L_e L_f$ = sum of gain products of all possible combinations of three nontouching loops

Δ_k = cofactor of the kth forward path determinant of the graph with the loops touching the kth forward path removed, that is, the cofactor Δ_k is obtained from Δ by removing the loops that touch path P_k

(Note that the summations are taken over all possible paths from input to output.)

In the following, we shall illustrate the use of Mason's gain formula by means of two examples.

Example-1



Forwardpath gains:

$$P_1 = G_1G_2G_3G_4G_5$$

$$P_2 = G_1G_6G_4G_5$$

$$P_3 = G_1G_2G_7$$

There are four individual loops, The gains of these loops are

$$L_1 = -G_4H_1$$

$$L_2 = -G_2G_7H_2$$

$$L_3 = -G_6G_4G_5H_2$$

$$L_4 = -G_2G_3G_4G_5H_2$$

Loop L_1 does not touch loop L_2. Hence, the determinant Δ is given by

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2 \qquad (3\text{–}82)$$

The cofactor Δ_1, is obtained from Δ by removing the loops that touch path P_1. Therefore, by removing L_1, L_2, L_3, L_4, and L_1L_2 from Equation (3–82), we obtain

$$\Delta_1 = 1$$

Similarly, the cofactor Δ_2 is

$$\Delta_2 = 1$$

The cofactor Δ_3 is obtained by removing L_2, L_3, L_4, and L_1L_2 from Equation (3–82), giving

$$\Delta_3 = 1 - L_1$$

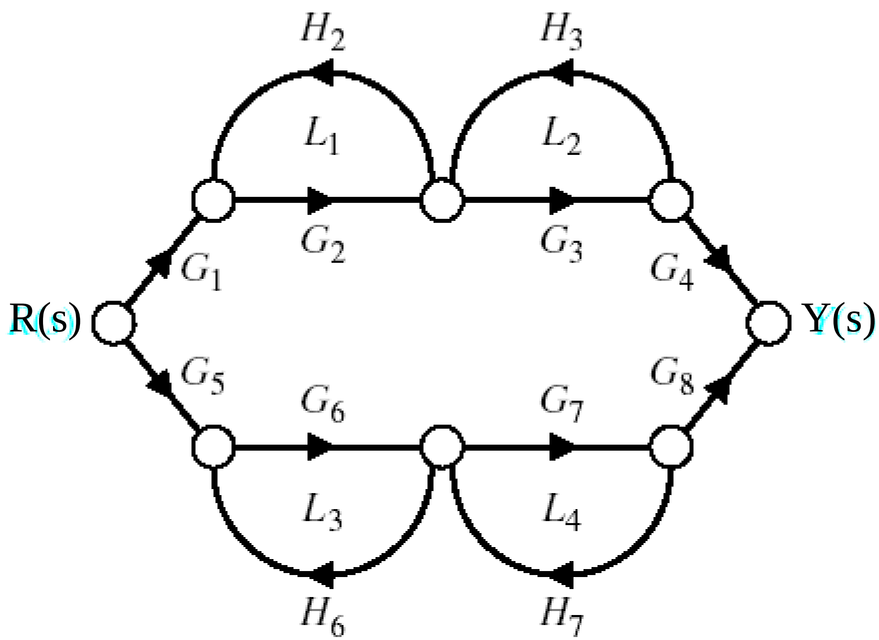
The closed-loop transfer function $C(s)/R(s)$ is then

$$\frac{C(s)}{R(s)} = P = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3)$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7(1 + G_4H_1)}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_4H_1G_2G_7H_2}$$

Signal-Flow Graph Models

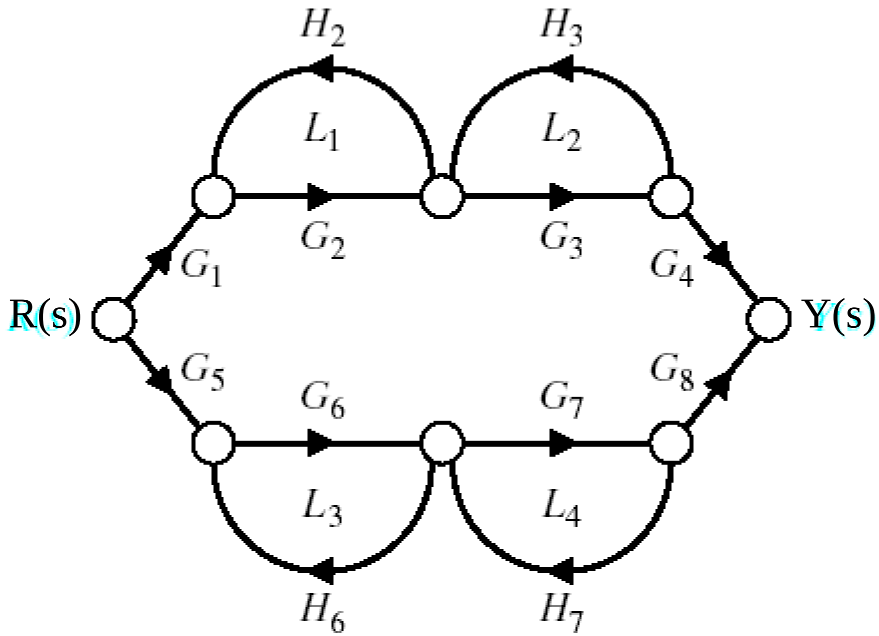
Example 2



Two-path interacting system.

Signal-Flow Graph Models

Example 2

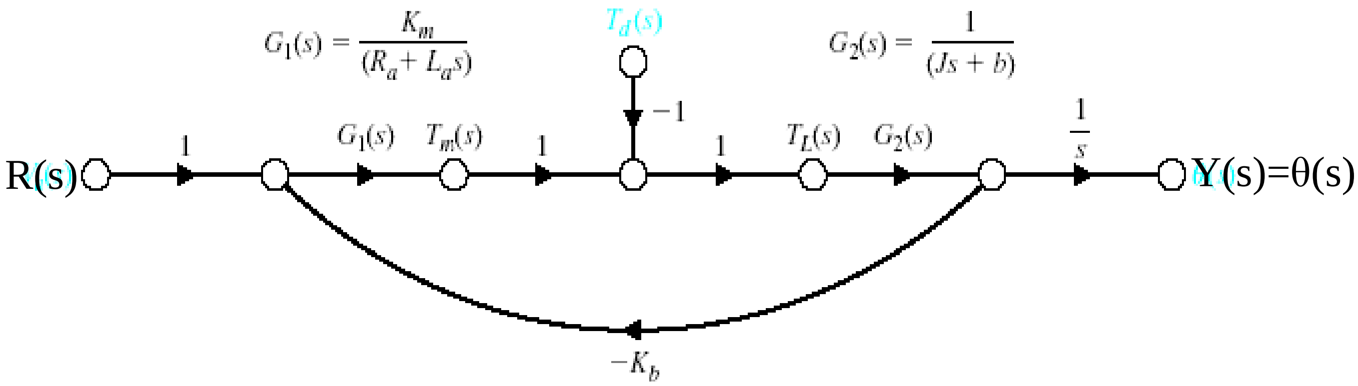


Two-path interacting system.

$$\frac{Y(s)}{R(s)} = \frac{\left[G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot \left(1 - L_3 - L_4\right)\right] + \left[G_5 \cdot G_6 \cdot G_7 \cdot G_8 \cdot \left(1 - L_1 - L_2\right)\right]}{1 - L_1 - L_2 - L_3 - L_4 + L_1 \cdot L_3 + L_1 \cdot L_4 + L_2 \cdot L_3 + L_2 \cdot L_4}$$

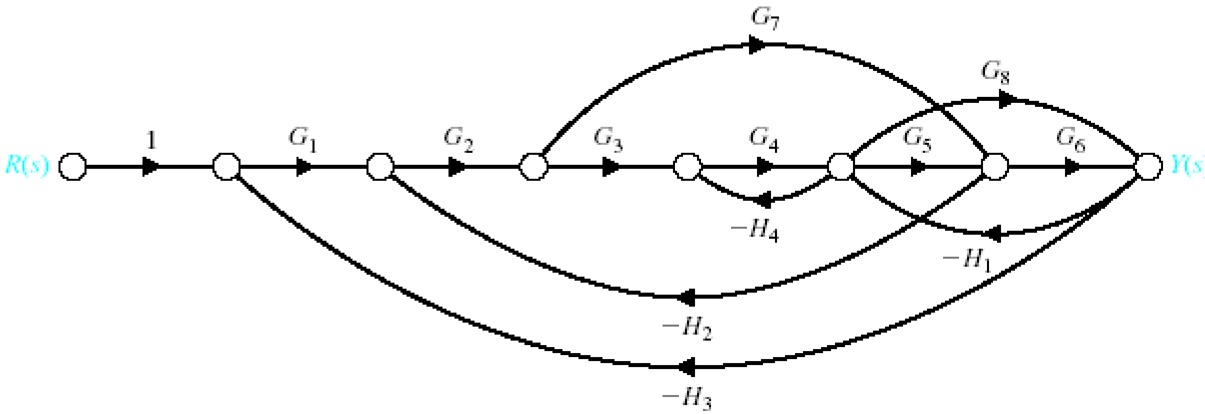
Signal-Flow Graph Models

Example 3



The signal-flow graph of the armature-controlled dc motor.

Signal-Flow Graph Models



Multiple-loop system.

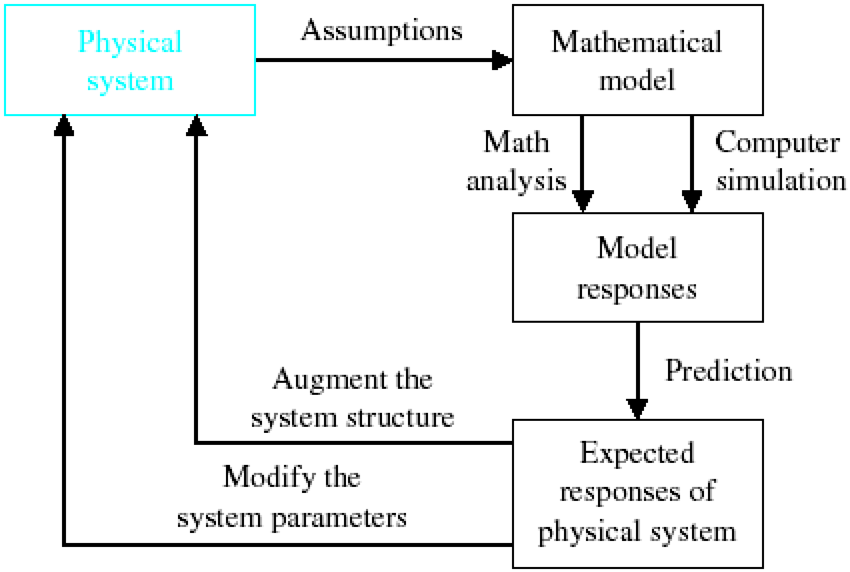
$$\frac{Y(s)}{R(s)} = \frac{P_1 + P_2 \cdot \Delta_2 + P_3}{\Delta}$$

$P_1 = G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_5 \cdot G_6$ $\quad P_2 = G_1 \cdot G_2 \cdot G_7 \cdot G_6$ $\quad P_3 = G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot G_8$

$$\Delta = 1 - \left(L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8\right) + \left(L_5 \cdot L_7 + L_5 \cdot L_4 + L_3 \cdot L_4\right)$$

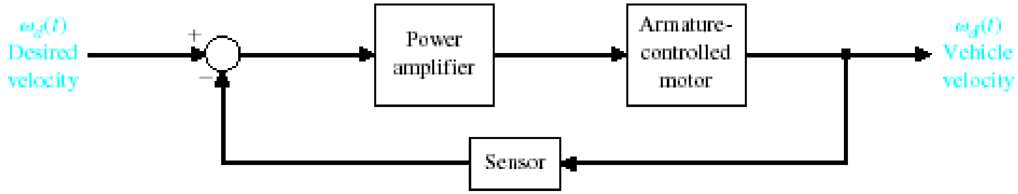
$\Delta_1 = \Delta_3 = 1$ $\quad \Delta_2 = 1 - L_5 = 1 + G_4 \cdot H_4$

Design Examples

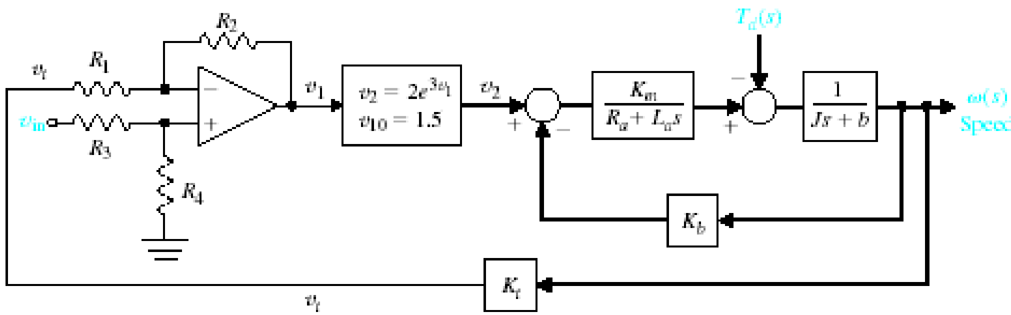


Analysis and design using a system model.

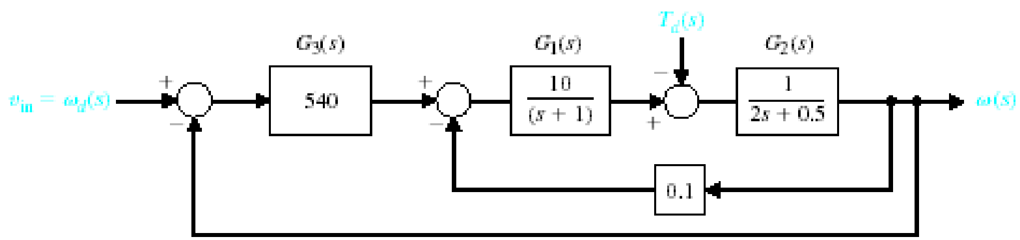
Design Examples: Speed control of an electric traction motor.



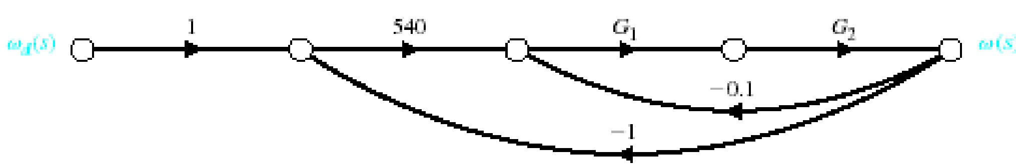
(a)



(b)



(c)



(d)

Thank You