Formulae

$$P = \chi_e \epsilon_0 E \qquad P = \frac{(\epsilon_r - 1)}{\epsilon_r} D.$$

$$D = \epsilon_0 \vec{E} + P \qquad \epsilon_r = 1 + \chi_e.$$

$$= \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_v \qquad \vec{B} = \vec{\nabla} \times \vec{A} \qquad \mu_0 = 4\pi \times 10^{-7}.$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J}.$$

$$\vec{m} = IS\, \hat{a}_n \qquad \vec{F} = I\, \vec{l} \times \vec{B}$$

$$\vec{T} = \vec{m} \times \vec{B} \qquad \vec{F} = q(\vec{v} \times \vec{B})$$

$$\vec{F} = q\, \vec{E}.$$

$$\boxed{RC = \frac{\epsilon}{\sigma}}$$

Coaxial Capacitor

$$Q = \epsilon \oint E \cdot ds = \epsilon E_\rho 2\pi \rho L$$

$$E = \frac{Q}{2\pi \epsilon \rho L} a_\rho.$$

$$V = -\int_2^1 E \cdot dl = -\int_b^a \frac{Q}{2\pi \epsilon \rho L} a_\rho \cdot d\rho\, a_\rho$$

$$V = \frac{Q}{2\pi \epsilon L} \ln b/a$$

$$\boxed{C = Q/V = \frac{2\pi \epsilon L}{\ln b/a}.} \qquad \boxed{R = \frac{\ln b/a}{2\pi \sigma L}}$$

Sperical capacitor

$$Q = \epsilon \oint \vec{E} \cdot ds = \epsilon E_r 4\pi r^2 \qquad E = \frac{Q}{4\pi \epsilon r^2} \hat{a}_r$$

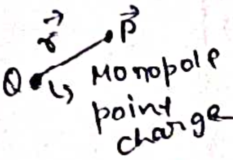
$$V = -\int_2^1 E \cdot dl = -\int_b^a \frac{Q}{4\pi \epsilon r^2} \hat{a}_r \cdot dr\, \hat{a}_r.$$

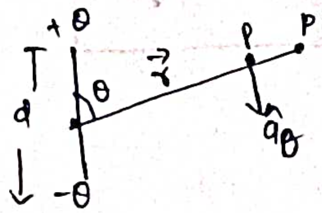
$$= \frac{Q}{4\pi \epsilon} \left[1/a - 1/b \right]$$

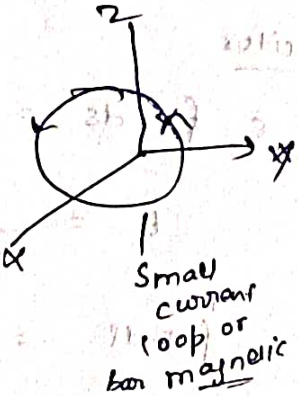
$$C = Q/V = \frac{4\pi \epsilon}{1/a - 1/b}.$$

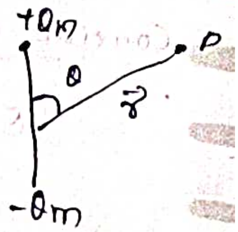
Comparision b/w Electric & magnetic monopole & dipole

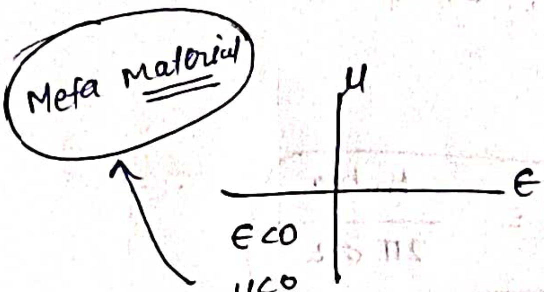
Electric	Magnetic
$V = \dfrac{Q}{4\pi\epsilon_0 r}$	does n't exist
$\vec{E} = \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r$	
$V = \dfrac{Q\cos\theta}{4\pi\epsilon_0 r^2}$	$\vec{A} = \dfrac{\mu_0 m \sin\theta\, \hat{a}_\phi}{4\pi r^2}$
$\vec{E} = \dfrac{(Q\vec{d})}{4\pi\epsilon_0 r^3}(2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta)$	$\vec{B} = \dfrac{\mu_0 m}{4\pi r^3}(2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta)$











Q. Magnetic - Materials

- Linear
- Non-linear

Magnetic Boundary Condition

$B_{1n} = B_{2n} = \delta q_2$.

$H_{1t} - H_{2t} = K$

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

$$(H_1 - H_2) \times a_{n_{12}} = K.$$

Magnetic Inductor

λ - flux-linkage

$\lambda = N\psi$

$\lambda \propto I$.

$\lambda = LI$

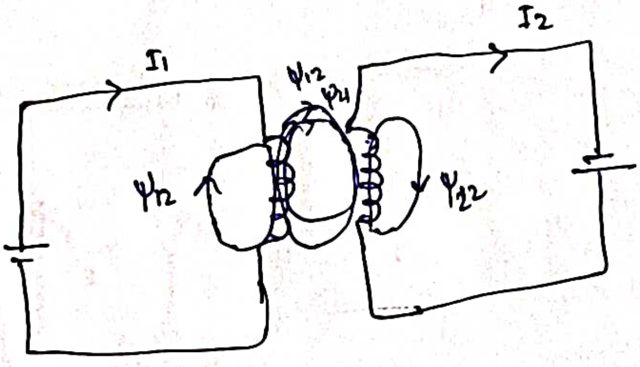
Inductance of circuit

$$L = \frac{\lambda}{I} = \frac{N\psi}{I}$$

magnetic Energy

$W_m = \frac{1}{2} L I^2$

$$\boxed{L = \frac{2W_m}{I^2}}$$



$$\boxed{\psi_{12} = \int_{S_1} B_2 \cdot ds}$$

Area of L.

pass through L due to 2.

magnetic field due to I_2.

S_1 due to area L.

λ_{12} : total flux linked.

$$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \psi_{12}}{I_2}.}$$

unit - henery.

$$\boxed{M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_1 \psi_{21}}{I_1}}$$

due to current I_1

In absence of ferromagnetic material,

$$M_{12} = M_{21}$$

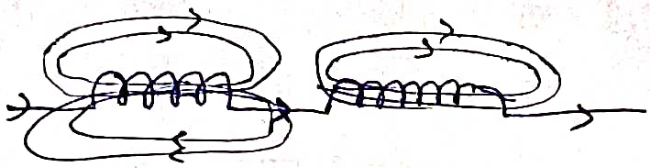
$$\psi_1 = \psi_{11} + \psi_{12} \qquad\qquad \psi_2 = \psi_{21} + \psi_{22}$$

self flux   Mutual flux

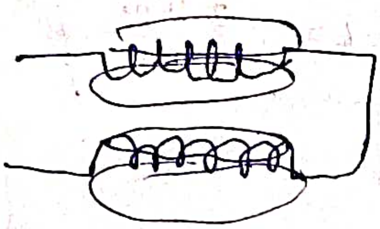
$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1 \psi_1}{I_1} \qquad\qquad L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2 \psi_2}{I_2}$$

$$W_m = W_1 + W_2 + W_{12}$$

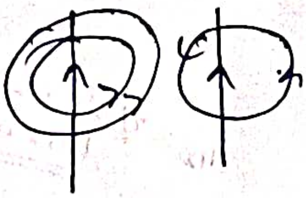
$$= \frac{1}{2} L I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2 .$$



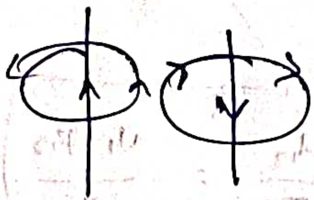
→ Magnetic Energy will be more



— opposite dirn. → Magnetic Energy will be less.



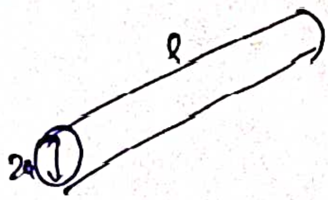
- Subtracted



- Add

Calculation of Self Inductance

1. Choose a suitable CS
2. Let the inductor carry current I
3. Determine B from Biot - Savart's law from Ampere's law if symmetry exists) and calculate ψ from $\psi = \int B \cdot ds$.
4. Finally find L from $L = \lambda / I = \frac{N\psi}{I}$.



$$W_m = \frac{1}{2} \int B \cdot H \, dv = \frac{1}{2} \varepsilon_0$$

Self-Inductance of an Infinitely long Solenoid

$$B = \mu H = \mu I n$$

$$\psi = BS = \mu I n S$$

4/10/23

Time Varying EM field $\rightarrow f(\vec{r}, t)$.

$\vec{H}$, $\vec{B}$, $\vec{E}$, $\vec{D}$ — $\vec{r}$

All are fn of $\vec{r}$ for static

Static - charge - Electrostatic field

Uniform motion - Magneostatic field.

faraday's laws ↳ rate of change flux

Transformer E.M.F
Motional E.M.F

$\lambda = N\psi$

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt}$$ — mathematical faraday's and lenz law.

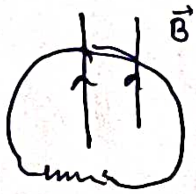
Transformer and Motional EMF

If $N = 1$

$$V_{emf} = \frac{-d\psi}{dt}$$

$$\int_{L} \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_{s}\vec{B}\cdot d\vec{s}$$

(A) Stationary loop in time varying Magnetic field



$$\int_{L} \vec{E}\cdot d\vec{l} = -\int_{S}\frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

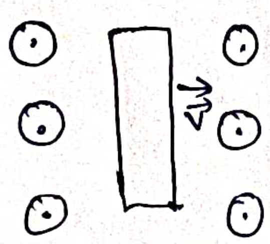
$$\int_{S} (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int_{S}\frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

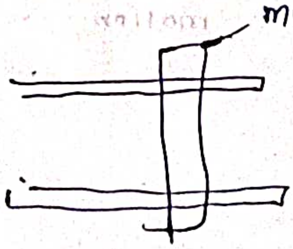
$\vec{\nabla}\times\vec{E} = 0$
for static
Electrostatic field is Conservative.

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$ — transformer emf.

(B) Moving loop in static $\vec{B}$ field

$\vec{B}(\vec{r})$ — static field





mettalic strip move

$\vec{F_m} = q(\vec{V} \times \vec{B})$

Strip is in motion
⇓
Charges also moves

$V_{emf} = \oint_L \vec{E_m} \cdot dl$

$$\boxed{E_m = \frac{\vec{F_m}}{q} = \vec{V} \times \vec{B}}$$

motional Emf.

$$V_{emf} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{s} = \oint_L \vec{E_m} \cdot d\vec{l}$$

$$\oint_L \vec{E_m} \cdot dl = \oint (\vec{u} \times \vec{B}) \cdot dl$$

$$\boxed{\int_S (\nabla \times \vec{E_m}) \cdot d\vec{s} = \int_S \vec{\nabla} \times (\vec{u} \times \vec{B})\, d\vec{s}}$$

(c) moving loop in time varying magnetic field.

$$V_{emf} = \int_L \vec{E} \cdot dl = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \oint_L (\vec{u} \times \vec{B}) \cdot dl$$

combinational Emf.

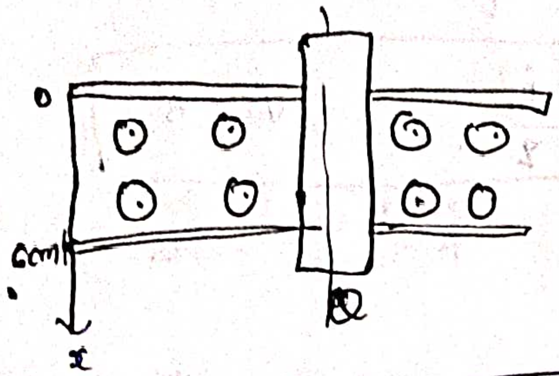
$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times (\vec{u} \times \vec{B})}$$

Q. A conducting bar can slide over two conducting rails as showns in fig. Calculate the Induced Voltage in the bar

(a) If the bar is stationed at $y = 8cm$, $\vec{B} = 4 \cos 10^6 t \, \hat{a}_z$ mWb/m^2

(b) If the bar slides at a velocity $\vec{u} = 20\, \hat{a}_y$ m/s and $\vec{B} = 4\, \hat{a}_z$ mWb/m^2

(c) If a bar slides at velocity $\vec{u} = 20\,\hat{a}_y$ m/s and $4\cos(10^6 t - y)\,\hat{a}_z$ mwb/m^2.



transformer em.

a).

$$V_{emf} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$d\vec{s} = dx\,dy\,\hat{a}_z$ $\quad \int_{x=0}^{6} \int_{y=0}^{0.8}$

$$= -\int \frac{\partial}{\partial t}(\ ) \cdot dx\,dy\,\hat{a}_z$$

$$= -\int_{x=0}^{6}\int_{y=0}^{0.8} 4\times10^6 \sin 10^6 t \; dx\,dy.$$

$$= +4\times10^6 \sin 10^6 t \times 0.8 \times 6.$$

$$= 19.$$

b) Motional Emf

$$V_{emf} = \int (\vec{u}\times\vec{B}) \cdot d\vec{l}$$

$$= \int (20\,\hat{a}_y \times 4\hat{a}_z) \cdot d\vec{l}$$

$$= -4.8 \text{ mV}.$$

c)

Maxwell's equation

Differential form | Integral form.

① $\vec{\nabla}\cdot\vec{D} = \rho_v$ $\qquad$ $\oint \vec{D}\cdot d\vec{S} = \int_v \rho_v \, dv.$

② $\vec{\nabla}\cdot\vec{B} = 0$ $\qquad$ $\oint \vec{B}\cdot d\vec{S} = 0.$

↳ Magnetic flux density is solenoidal
↓
non-existing of magnetic pole.

③ $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\qquad$ $\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}.$

faraday's law

(iv) $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\qquad$ $\oint \vec{H}\cdot d\vec{l} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{S}.$

conduction current density

for free space

$\rho_v = 0$

$\sigma = 0$ — conductivity.

$\longrightarrow J = \sigma E = 0$

$\vec{\nabla}\cdot\vec{D} = 0$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}.$

for static field

$\longrightarrow$ $\vec{D}$ $\vec{B}$ $\vec{E}$ $\vec{H}$

$\frac{\partial \vec{D}}{\partial t} = \frac{\partial \vec{B}}{\partial t} = \frac{\partial \vec{E}}{\partial t} = \frac{\partial \vec{H}}{\partial t} = 0$

$\vec{\nabla}\cdot\vec{D} = \rho_v$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = 0$

$\vec{\nabla}\times\vec{H} = \vec{J}$

Modified Ampere's Law

Static field Ampere's law : $\vec{\nabla} \times \vec{H} = \vec{J}$

$$\underbrace{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H})}_{= 0} = \vec{\nabla} \cdot \vec{J} \quad — ①$$

Continuity eqn $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$ — ②

contradiction

① & ② contradict

Modification:-

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_D$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J}_D$$

$$0 = -\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J}_D$$

$$\Rightarrow \vec{\nabla} \cdot \vec{J}_D = \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{D})$$

$$= \vec{\nabla} \cdot \frac{\partial}{\partial t}\vec{D}$$

$$\boxed{\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t}(\epsilon \vec{E}) = \epsilon \frac{\partial \vec{E}}{\partial t}.}$$

$\vec{J}_D$ exist when there is variation in $\vec{E}$.

Time -harmonic field

$$z = x + jy = |r| \angle \theta = r e^{j\theta}$$

Vary periodically & synosidally

If a vector $A(\vec{r}, t)$ is a time harmonic field, the phasor form of $\vec{A}$ is $\vec{A_s}(\vec{r})$

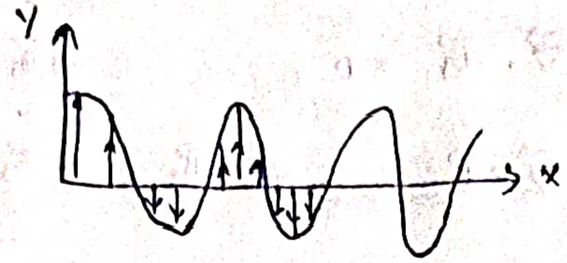
$\vec{A} = Re(\vec{A_s}\, e^{j\omega t})$

Eg.

$\vec{A} = A_0 \cos(\omega t - \beta x)\, \hat{a_y}$

dir^n of $\vec{A}$ along y

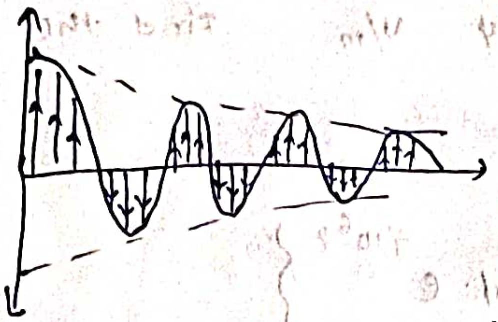
Exam Diagram



Amplitude varies from $-A_0$ to A_0

Irrespective to time & variation

$y = A_0 e^{-\alpha x} \cos(\omega t - \beta x)\, \hat{a_y}$ α : attuniation constant



$\vec{A} = A_y(x,t)\, \hat{a}_y$.

$\vec{A} = A_0 \cos(\omega t - \beta x)\, \hat{a_y}$

$= Re\left\{ A_0 e^{j(\omega t - \beta x)}\, \hat{a}_y \right\}$

$= Re\left\{ \underbrace{A_0 e^{-j\beta x}\, \hat{a_y}}_{\vec{A_s}}\, e^{j\omega t} \right\}$

phasor form $(\vec{A_s}) = A_0 e^{-j\beta x}\, \hat{a_y}$.

phaso

$\frac{\partial \vec{A}}{\partial t} =$

Maxwell's eqn

$\vec{\nabla}\cdot\vec{D} = \rho_v$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Time Harmonic of Maxwell's eq.

Assuming time factor $e^{j\omega t}$.

$\vec{\nabla}\cdot\vec{D_s} = \rho_v$ $\quad \oint_S \vec{D_s}\cdot d\vec{s} = \int \rho_v \, dv$

$\vec{\nabla}\cdot\vec{B_s} = 0$ $\quad \oint \vec{B_s}\cdot d\vec{s} = 0$

$\vec{\nabla}\times\vec{E_s} = -j\omega \vec{B_s}$ $\quad \oint \vec{E_s}\cdot d\vec{l} =$

$\vec{\nabla}\times\vec{H_s} = \vec{J_s} + j\omega \vec{D_s}$

Q. $\vec{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\,\hat{a}_\phi$ V/m Find the phasor form of $\vec{E}$.

$$\vec{E} = \text{Re}\left\{ \frac{50}{\rho} e^{j\beta z}\hat{a}_\phi \cdot e^{j10^6 t} \right\}.$$

$$\vec{E_s} = \frac{50}{\rho} e^{j\beta z}\,\hat{a}_\phi$$

Prob 8.36

Q. The electric field phasor of an EM wave in free space is $\vec{E_s}(y) = 10\, e^{-j4y}\,\hat{a}_x$ V/m.

Find (a) 'ω' so that $\vec{E_s}$ satisfy M.E.

(b) the corresponding mag. field.

$$\vec{E} = 10\cos(\omega t - 4y)\,\hat{a}_x$$

$$\vec{E_s} = \text{Re}\{\vec{E_s}\, e^{j\omega t}\}.$$

Normal (both) ↓ use Normal M.E

Given :- phasor form

↳ Maxwell eqn → phasor form

$\vec{\nabla} \cdot \vec{D_s} = 0$

$\vec{\nabla} \cdot \vec{B_s} = 0$

$\vec{\nabla} \times \vec{E_s} = -j\omega \vec{B_s}$

$\vec{\nabla} \times \vec{H_s} = j\omega \vec{D_s}$

$\vec{E_s} = E_{sx}(y) \hat{a}_y)$

$$\vec{\nabla} \times \vec{E_s} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx}(y) & 0 & 0 \end{vmatrix}$$

$$\left(-\frac{\partial}{\partial y} E_{sx}(y) \right) \hat{a}_z$$

$$0 - \frac{\partial}{\partial y} 10 e^{-j4y}$$

$$+40j\, e^{-j4y} \hat{a}_z$$

$$\vec{\nabla} \times \vec{E_s} = -j\omega \vec{B_s}$$

$$40j\, e^{-j4y} \hat{a}_z = -j\omega \vec{B_s}$$

$$\mu \vec{H_s} = \frac{-40}{\omega} e^{-j4y}$$

$$\vec{H_s} = \frac{-40}{\mu\omega} e^{-j4y} \hat{a}_z$$

Problem 8.36 text book

Wave eq^n in free space

$\rho_v = 0 \qquad \sigma = 0$

$\vec{\nabla} \cdot \vec{E} = 0$

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = \dfrac{-\partial \vec{B}}{\partial t}$

$\vec{\nabla} \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t}$

taking curl on both sides of eq^n (3)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 E = \frac{-\partial}{\partial t} \vec{\nabla} \times (\mu \vec{H})$$

$$\nabla^2 E = \mu \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

$$\nabla^2 E = \mu \frac{\partial}{\partial t}\left(\frac{\partial \vec{D}}{\partial t}\right)$$ from eq (4)

$$= \mu \epsilon \frac{\partial^2 E}{\partial t^2}$$ $(\vec{D} = \epsilon \vec{E})$

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$ (in free space $\mu = \mu_0$, $\epsilon = \epsilon_0$)

$$\boxed{\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}}$$

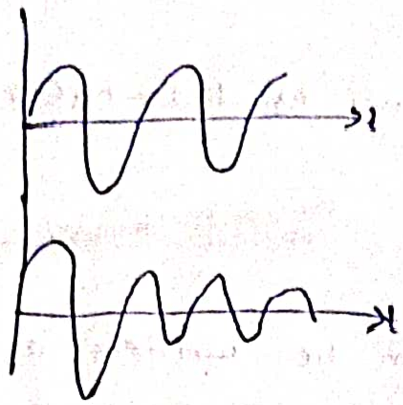
Wave eq^n in conducting medium

$\rho_v = 0 \qquad \sigma \neq 0 \qquad \vec{J} \neq 0$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 E}{\partial t^2} + \mu \sigma \frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 H}{\partial t^2} + \mu \sigma \frac{\partial \vec{H}}{\partial t}$$

wave propagation in lossy dielectric



— peak to peak amplitude fixed

Amp. decrease in exponential manner.

Consider a linear, homogenous, lossy, dielectric medium which is charge free.

$\rho_v = 0$

Assuming and suppressing time factor $e^{j\omega t}$,

Maxwells eq^n becomes.

$\vec{A}(r,t) = Re[\vec{A}_s(r)\, e^{j\omega t}]$.

$j\omega\, \vec{A}_s(r)$

$$\vec{\nabla} \cdot \vec{E_s} = 0 \quad — (1)$$

$$\vec{\nabla} \cdot \vec{H_s} = 0 \quad — (2)$$

$$\vec{\nabla} \times \vec{E_s} = -j\omega\mu \vec{H_s} \quad — (3)$$

$$\vec{\nabla} \times \vec{H_s} = (\sigma + j\omega\epsilon)\vec{E_s} \quad — (4)$$

$$\vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$\sigma \vec{E}$

Taking curl of both sides of eq (3),

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E_s}) = -j\omega\mu\, (\vec{\nabla} \times \vec{H_s}) \quad — (5)$$

$$\nabla^2 E_s = -j\omega\mu(\sigma + j\omega\epsilon)\vec{E_s} \quad — (6)$$

$$\boxed{\nabla^2 E_s - \gamma^2 \vec{E} = 0} \quad (7)$$

propagation constant

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) \quad (8)$$

$$\boxed{\nabla^2 H_s - \gamma^2 H_s = 0} \quad \text{--- (9)}$$

Eq^n (8) & (9) are known as Helmholtz's eqn on Simple vector

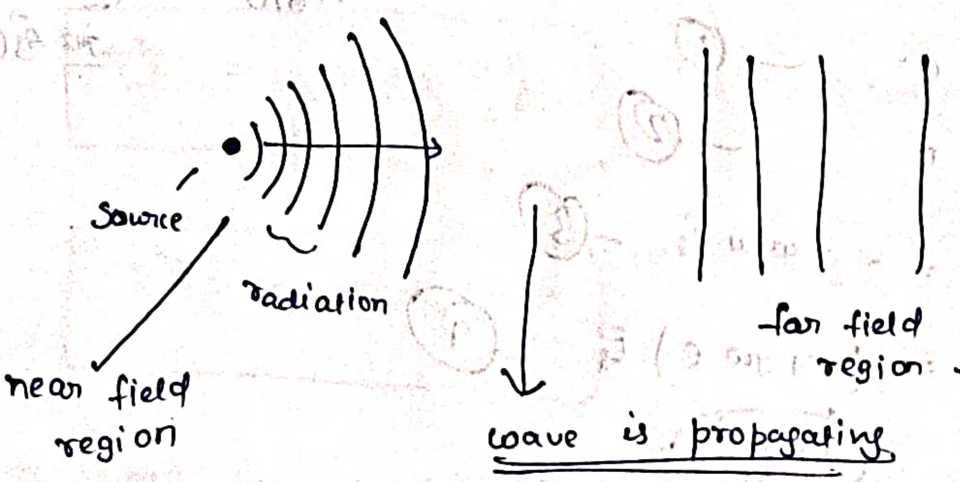
γ is a complex quantity can be written as,

$$\boxed{\gamma = \alpha + j\beta ;}$$

where α = attenuation const.

β = phase const.

Uniform plane wave (Def^n & property).



Mode TEM

Transverse electric & magnetic

TE mode

TM mode

TEM mode

TEM to z

↳ wave is propagating in z direction

w.r.to z dir^n of $\vec{E}$ and $\vec{H}$ are perpendicular.

$\vec{E} \perp z \Rightarrow E_z = 0$

$\vec{H} \perp z \Rightarrow H_z = 0$

TE to z i.e $E_z = 0$

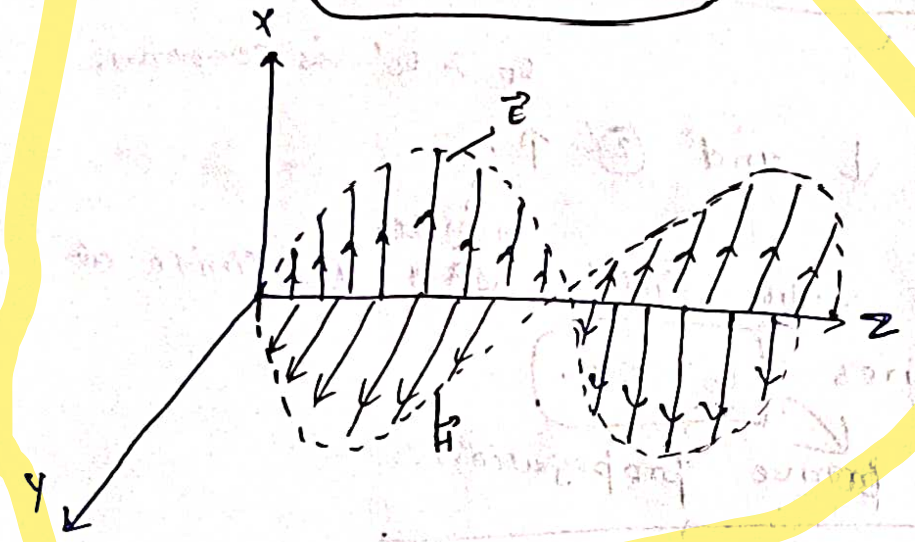
& TM to z i.e $H_z = 0$

TEM to z i.e $E_z = H_z = 0$

Q. Exam.

TEM propagation diagram

$$\boxed{\begin{aligned} \hat{a}_k &= \hat{a}_E \times \hat{a}_H \\ \hat{a}_z &= \hat{a}_x \times \hat{a}_y \end{aligned}}$$



TEM mode (same amplitude)

Right handed C.S.
⇓
'X-Y' using right hand thumb

Assume the wave propagation along $\hat{a}_z$ and E_s vector has only x-component.

$$\vec{E_s} = \vec{E}_{xs}(z)\,\hat{a}_x$$

(z) → fn of z as it is propagating in z.

phasor notation
⇓
fn of cordinate not time.

Substituting in eq (7)

$$\boxed{(\nabla^2 - \gamma^2)\,\bar{E}_{xs}(z) = 0}$$

$$\left(\underbrace{\frac{\partial^2}{\partial x^2}}_{=0} + \underbrace{\frac{\partial^2}{\partial y^2}}_{=0} + \frac{\partial^2}{\partial z^2}\right) E_{xs}(z) - \gamma^2 E_{xs}(z) = 0$$

$$\Rightarrow \boxed{\left(\frac{\partial^2}{\partial z^2} - \gamma^2\right) E_{xs}(z) = 0}$$

This is a scalar wave eqn and its solution is of the form,

$$\boxed{E_{xs}(z) = E_0 e^{-\gamma z} + E_0' e^{\gamma z}}$$

E_0 & E_0' is constant.

$z \uparrow \rightarrow$ ① $\downarrow$ and ② $\uparrow$.

The fact that the field must be finite at infinity which requires $\boxed{E_0' = 0.}$

Assuming z dirⁿ of wave propagation.

for negative z dirⁿ, $\boxed{E_0 = 0}$

$$E_{xs}(z) = E_0 e^{-\gamma z}$$

$$\vec{E}(z,t) = Re\{E_{xs}(z) e^{j\omega t}\} \hat{a}_x$$

$$= Re\left[E_0 e^{-\gamma z} e^{j\omega t}\right] \hat{a}_x$$

$$= Re\left[E_0 e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right] \hat{a}_x$$

$$\boxed{\vec{E}(z,t) = \underbrace{E_0 e^{-\alpha z}}_{\text{decaying cor. to } z} \cos(\omega t - \beta z) \cdot \hat{a}_x} \quad [V/m]$$

dirⁿ of electric field

where α is attenuation constant.

$$\boxed{\vec{H}(z,t) = H_0\, e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_y .}$$

TEM mode in coaxial cable

$$\vec{\nabla} \times \vec{E}_s = -j\omega \mu_0 \vec{H}_s$$

$$= -j\omega \mu_0 \left(H_{sx}\hat{a}_x + H_{sy}\hat{a}_y + H_{sz}\hat{a}_z \right) \quad \text{—— (i)}$$

$$\vec{\nabla} \times \vec{E}_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{xs}(z) & 0 & 0 \end{vmatrix}$$

$$= + \hat{a}_y \frac{\partial}{\partial z} E_{xs}(z) \quad \text{—— (ii)}$$

from (i) & (ii),

$$\boxed{\frac{\partial}{\partial z} E_{xs} = -j\omega \mu_0 H_{sy}}$$

$$\frac{\partial}{\partial z}\left(E_0\, e^{-\gamma z}\right) = -j\omega \mu_0 H_{sy}$$

$$-\gamma E_0\, e^{-\gamma z} = -j\omega \mu_0 H_{sy}$$

$$\boxed{\frac{E_{xs}}{H_{sy}} = \frac{j\omega \mu_0}{\gamma} = \eta}$$

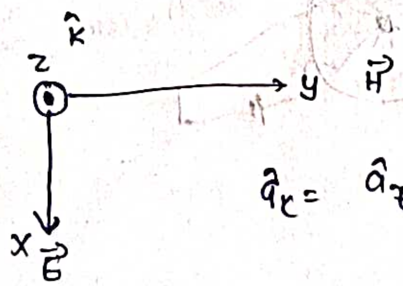
V/m

A/m

Complex

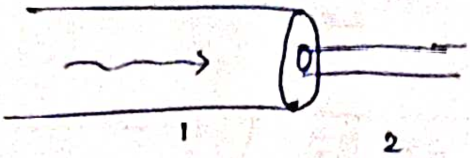
ohm. (Ω)

wave Impedance



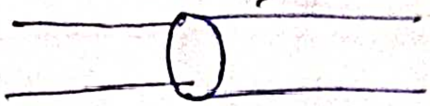
$$\hat{a}_k = \hat{a}_E \times \hat{a}_H$$

Case-1



— flow decreases

Case-II

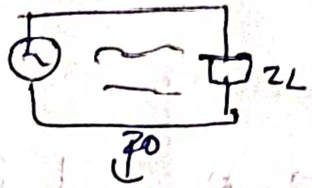


— Cross-section same

— flow properly

coaxial cable
w.r.to distance
capacitance varig

Electric field analogous to Voltage
Magnetic " " " Current



• Impedence mismatch

$\boxed{377\,\Omega \text{ - free space } \eta \text{ - value.}}$

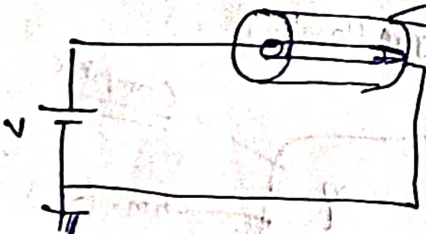
$$\vec{H}(z,t) = Re\{ H_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{a}_y \}$$

$$\boxed{\vec{H}(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\ \hat{a}_y}$$

$H_0 = \frac{E_0}{\eta}$

Instantenous form.

coaxial cable



wave prog. in coaxial cable TEM Mode

$\vec{E}$ dirn → radial dirn $(\hat{a}_\rho)$

$$\vec{E} = E_\rho(z)\,\hat{a}_\rho$$

$$\hat{a}_K = \hat{a}_E \times \hat{a}_H$$

$$\hat{a}_z = \hat{a}_\rho \times \hat{a}_\phi$$

η is complex (~~known as propagation constant~~)

$$\eta = |\eta| \angle \theta_n$$

$$\eta = |\eta| e^{j\theta_n} = \sqrt{\frac{j\omega\mu}{\sigma + j}}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} \quad ; \quad \tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$$

$$\vec{H} = Re\left[\frac{E_0}{|\eta| e^{j\theta_\eta}} e^{-\alpha z} e^{-j(\omega t - \beta z)} \hat{a}_y\right]$$

$$\boxed{\vec{H} = E_0 e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta) \hat{a}_y}$$

$$\gamma = \alpha + j\beta$$

where, α = attenuation constant

unit dB/m

np/m

neper

α:

- It is the measure of the spatial rate of decay of the wave in the medium.
- An attenuation of 1 np denotes a reduction of e^{-1} of the original amplitude and Increase of 1np indicates amplitue of wave is increased by e^{+1}.

$$1\,Np = 20 \log_{10} e$$
$$= 8.686\ dB.$$

β :- (phase constant or wave no)

- Measure of phase shift per unit length.
- unit : rad/m

Expression of α & β

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = (\alpha^2 - \beta^2) + 2j\alpha\beta \quad \text{--- (1)}$$

form eq (8)

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) \quad \text{--- (2)}$$

from (1) & (2)

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \quad \text{--- (3)}$$

$$2\alpha\beta = \omega\mu\sigma$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left\{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/2} - 1\right\}}} \quad (np/m).$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left\{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/2} + 1\right\}}} \quad (rad/m)$$

$$\vec{\nabla} \times \vec{H_s} = \vec{J_s} + j\omega\vec{D_s}$$

$$= \underbrace{\sigma\vec{E_s}}_{\vec{J_c}} + \underbrace{j\omega\epsilon\vec{E_s}}_{\vec{J_D}}$$

$$\boxed{\text{Dissipation factor} = \frac{\sigma}{\omega\epsilon} = \frac{\vec{J_c}}{\vec{J_D}}}$$

Good conductor, $J_C >> J_D$

i.e $\underline{DF >> 1}$.

for dielectric,

$$\frac{\sigma}{\omega \epsilon} << 1$$

$$DF << 1$$

$$DF = \frac{\sigma}{2\pi f \epsilon}$$

f low y DF - high

↓ conductor

for good conductor,

$$\frac{\sigma}{\omega \epsilon} >> 1$$

$$\boxed{\left.\begin{matrix}\alpha \\ \beta\end{matrix}\right\} \approx \sqrt{\frac{\omega \mu \sigma}{2}}}$$

$$\boxed{\alpha = \beta = \sqrt{\frac{2\pi f \mu \sigma}{2}} = \sqrt{\pi f \mu \sigma}}$$

phase velocity

$$\boxed{V_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu \sigma}}}$$

$$\eta = \sqrt{\frac{j\omega \mu}{\sigma}} = \sqrt{\frac{\omega \mu}{\sigma}} \angle 45^\circ$$ — $\vec{E}$ lead $\vec{H}$ by 45°.

Eg.

$$\vec{E} = E_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_x$$

$$\vec{H} = H_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_y$$

$$\hat{a}_k = \hat{a}_E \times \hat{a}_H$$

$$\hat{a}_z = \hat{a}_x \times \hat{a}_y.$$

$$H_0 = \frac{E_0}{\eta} = \frac{E_0}{|\eta| \, \theta_\eta}$$

$$\boxed{\vec{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_n) . \, \hat{a}_y}$$

Plane wave in free space

$\sigma = 0 \quad \rho_v = 0 \quad \epsilon = \epsilon_0 \quad \mu = \mu_0$

$\alpha = 0 \quad \beta = \omega\sqrt{\mu_0\epsilon_0}$

$\boxed{\beta = \frac{\omega}{c}} \quad \left[\because \sqrt{\mu_0\epsilon_0} = 1/c\right].$

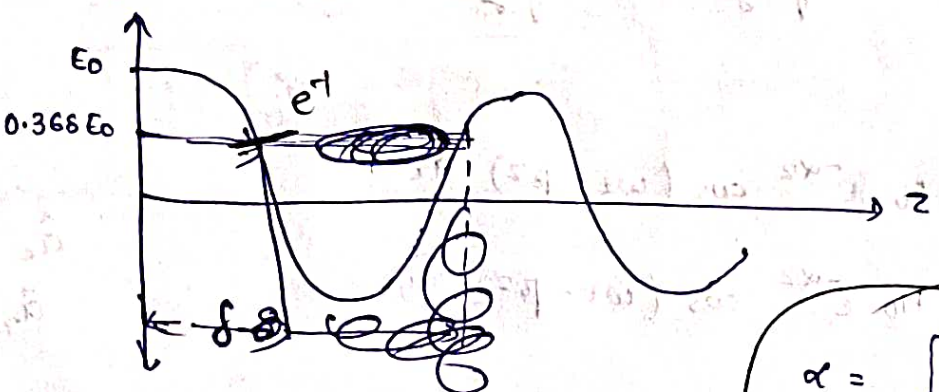
$$\boxed{\eta = \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\times\pi = 377\,\Omega.}$$

Intrinsic Impedence of freespace.

Plane wave in lossless dielectric

$$\frac{\sigma}{\omega\epsilon} \ll 1$$

The distance δ through which the wave amplitude decreases to a factor e^{-1} (i.e about 37 % of original amplitude) is called skin depth or peneteration depth of the medium.



$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$E_0\, e^{-\alpha\delta} = E_0\, e^{-1}$$

$$\alpha\delta = 1$$

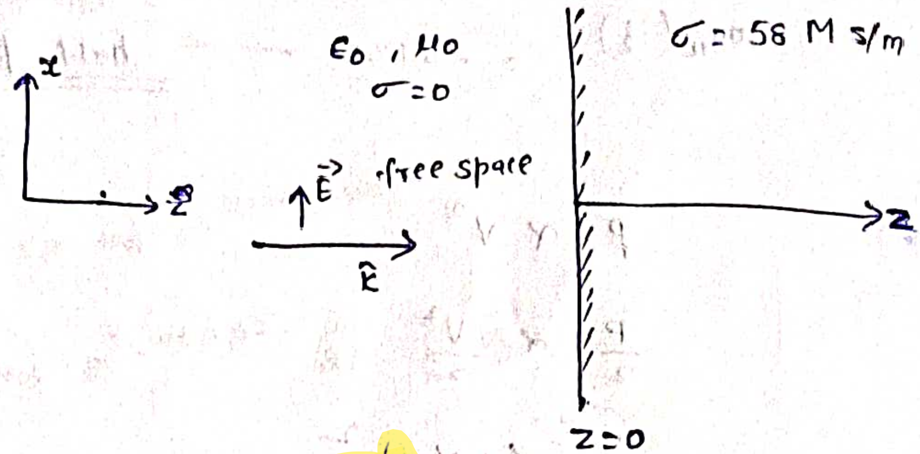
Exam imp

$$\boxed{\delta = 1/\alpha = \sqrt{\frac{2}{\omega\mu\sigma}}} = \frac{1}{\sqrt{\pi f \mu\sigma}}$$

σ not ϵ

Q. Assume a field $\vec{E} = 1.0\, e^{-\alpha z}\, e^{j(\omega t - \beta z)}\, \hat{a}_x (V/m)$ with $f = \frac{\omega}{2\pi} = 100\,MHz$ at the surface of copper conductor $\sigma = 58\, M\,s/m$ located at $z>0$ as shown in fig. Examine the attenuation of the wave propagating into the conducting medium.

Solⁿ :-



$\frac{\sigma}{\omega\epsilon} \gg 1.$

$|E| = 1.0\, e^{-\alpha z}$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 6.61\, \mu m.$$

After $6.61\, \mu m$ the field is attenuated to e^{-1} ie 36.8 % of the initial value.

As frequency ↑ — Skin depth ↓

⇓

That's why low freq. cable can't be used for high freq. Cable.

$G_v \text{ (dB)} = 20 \log_{10} V_o/V_i$

$G_p \text{ (dB)} = 10 \log_{10} P_o/P_i$

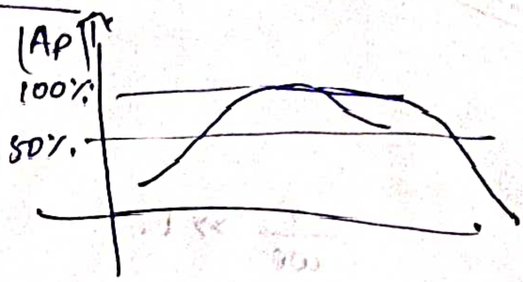
$\boxed{P = P_0 e^{-2\alpha z}}$ — Power attenuation is twice as ~~[illegible]~~ as $\vec{E}$ is analogy to $\vec{V}$.

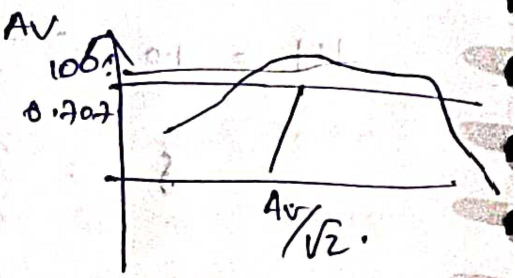
$10 \log_{10}(1/2) = -3\text{dB}$ — half power

$P \propto V^2$

$\frac{P}{2} \propto \frac{V^2}{2}$

$\propto \left(\frac{V}{\sqrt{2}}\right)^2$ —





Tutorial
12/10/23

Q. $\vec{H} = H_m e^{j(\omega t + \beta z)} \hat{a}_x$ in free space find $\vec{E}$.

Soln:-

$\hat{a}_K = \hat{a}_E \times \hat{a}_H$

free space, $\rho_v = 0$ $\sigma = 0$

$\vec{B} = \mu \vec{H}$, $\vec{D} = \epsilon \vec{E}$

$\vec{J_c} = \sigma \vec{E}$

from M.E.

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$= \frac{\partial}{\partial t} \epsilon_0 \vec{E} = \epsilon_0 \frac{\partial}{\partial t} \vec{E}$$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(z) & 0 & 0 \end{vmatrix}$$

$$= \hat{a}_y \frac{\partial}{\partial z} H_x(z)$$

$$= \hat{a}_y \, j\beta \, H_m \, e^{j(\omega t + \beta z)}$$

$$\epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \times \vec{H}$$

$$= \hat{a}_y \, j\beta \, H_m \, e^{j(\omega t + \beta z)}$$

$$\int \partial \vec{E} = \int \frac{j\beta}{\epsilon_0} H_m \, e^{j(\omega t + \beta z)} \, \partial t \quad \hat{a}_y$$

$$\vec{E} = \frac{\cancel{j}\beta \, H_m \, e^{j(\omega t + \beta z)}}{\cancel{j}\omega} \hat{a}_y \quad V/m$$

$$\vec{E} = \frac{\beta H_m}{\omega} e^{j(\omega t + \beta z)} \hat{a}_y \quad V/m.$$

Q. In free space, $\vec{D} = D_m \sin(\omega t + \beta z) \, \hat{a}_x$ using M.E show that $\vec{B} = \frac{-\omega \mu_0 D_m}{\beta} \sin(\omega t + \beta z)$ sketch the fields at $t=0$ along the z-axis assuming $D_m > 0$ and $\beta > 0$.

Sol^n:-

$$\vec{E} = \underbrace{\frac{D_m}{\epsilon_0} \sin(\omega t + \beta z)}_{E_x(z).} \hat{a}_x$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z) & 0 & 0 \end{vmatrix} = \hat{a}_y \frac{\partial}{\partial z} E_x(z).$$

$$= \hat{a}_y \frac{D_m}{\epsilon_0} \cdot \beta \cos(\omega t + \beta z)$$

$$\int -\partial \vec{B} = \int \hat{a}_y \frac{D_m}{\epsilon_0} \beta \cos(\omega t + \beta z)\, \partial t$$

$$\vec{B} = -\frac{D_m}{\epsilon_0} \beta \frac{\sin(\omega t + \beta z)}{\omega} \hat{a}_y.$$

In free space, $\beta = \omega\sqrt{\mu_0 \epsilon_0}$

$$\frac{\beta^2}{\omega^2} = \mu_0 \epsilon_0$$

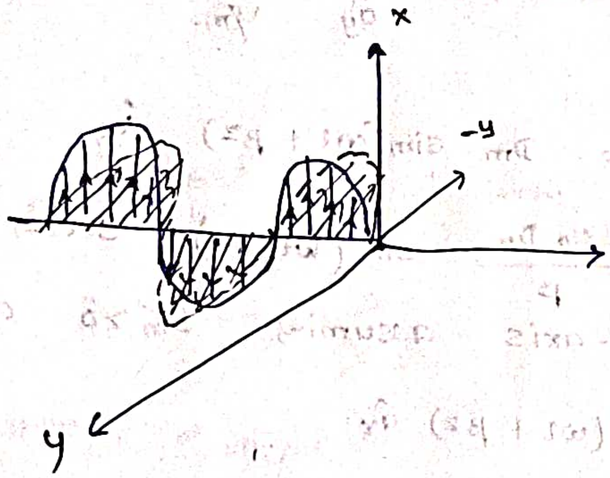
$$\Rightarrow \frac{1}{\epsilon_0} = \frac{\mu_0 \omega^2}{\beta^2}.$$

Imp.

$$\boxed{\vec{B} = -D_m \frac{\omega}{\beta} \mu_0 \sin(\omega t + \beta z)\, \hat{a}_y}$$

$t=0,$ $\vec{E} = E_0 \sin(\beta z)\, \hat{a}_x$

$\vec{H} = \frac{\vec{B}}{\mu_0} = -H_0 \sin(\beta z)\, \hat{a}_y$



Q. In a Homogeneous non-conducting medium $\mu_r = 1$. Find ϵ_r and ω if $\vec{E} = 30\pi\, e^{j(\omega t - 4/3 y)}\, \hat{a}_z$ (V/m)

and $\vec{H} = 1.0\, e^{j(\omega t - 4/3 y)}\, \hat{a}_x$ (A/m)

Soln: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}.$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ 0 & 0 & E_z(y) \end{vmatrix} = \hat{a}_x \frac{\partial E_z}{\partial y}.$$

$$= \hat{a}_x \quad 30\pi j \left(-\frac{4}{3}\right) e^{j(\omega t - 4/3 y)} \quad \text{—— (1)}$$

E_0

$$-\mu \frac{\partial \vec{H}}{\partial t} = -\mu \, j\omega \, e^{j(\omega t - 4/3 y)} \quad \text{—— (2).}$$

$$30\pi \left(-4/3\right)\cancel{j} = -\mu \cancel{j} \omega$$

$$\mu\omega = \cancel{30}\ 40\pi$$

$$\omega = \frac{40\pi}{\mu_0 \mu_r}$$

$$= \frac{40\pi}{\mu_0}.$$

$$\frac{\vec{E}}{\vec{H}} = \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\epsilon_r}} \times \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \text{—— } \underline{\underline{120\pi}}.$$

$$30\pi = \frac{1}{\sqrt{\epsilon_r}} \times 120\pi.$$

$$\epsilon_r = (4)^2 = 16.$$

Q. A lossy dielectric has an η = $200 \angle 30^\circ\ \Omega$ (intrinsic Impe) at a particular radian freq. ω. at that frequency the plance wave propagating through the dielectric has the magnetic field component $\vec{H} = 10\, e^{-\alpha x} \cos(\omega t - \frac{1}{2} x)\, \hat{a}_y$ A/m. Find $\vec{E}$ and α . determine skin depth and polarization.

Sol^n:-

$$\frac{E_0}{H_0} = \eta$$

$$E_0 = H_0 \eta$$

$$\hat{a}_k = \hat{a}_E \times \hat{a}_H$$

$$\hat{a}_y = \bigcirc \times \hat{a}_y$$

$$- \hat{a}_z$$

$$\vec{E} = 2000 \, e^{-\alpha x} \cos\left(\omega t - \tfrac{1}{2}x + \theta_\eta\right) \left(- \hat{a}_z\right)$$

due to multiplication.

$$\vec{E} = -2000 \, e^{-\alpha x} \cos\left(\omega t - \tfrac{1}{2}x + \pi/6\right) \hat{a}_z \ \text{V/m}.$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}.$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1}\right]^{1/2}.$$

Given :- dissipation factor

$$\frac{\sigma}{\omega\epsilon} = \tan 2\theta_\eta = \tan 60^\circ = \sqrt{3}.$$

DF >> 1

Good conductor

$$= \left[\frac{2-1}{2+1}\right]^{1/2} = \frac{1}{\sqrt{3}}.$$

$$\alpha = \beta/\sqrt{3}. \qquad \left[\because \text{Given } \beta = 1/2\right]$$

$$\alpha = 1/2\sqrt{3} \ \text{Np/m}.$$

$$\text{Skin depth } (\delta) = 1/\alpha = 2\sqrt{3} \ \text{m} = 3.46 \ \text{m}$$

Power and Poynting Vector

Maxwell's eq^n

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad -①$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad -②$$

Dotting both the sides of eq ②,

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \sigma \vec{E} \cdot \vec{E} + \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \quad -③$$

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) + \vec{\nabla} \cdot (\vec{H} \times \vec{E})$$

$$= \sigma E^2 + \vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$= \sigma E^2 + \frac{1}{2} \epsilon \frac{\partial}{\partial t} E^2 \quad -④$$

Dotting both side of eq^n ① w.r.to $\vec{H}$,

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t} \right) = -\frac{\mu}{2} \frac{\partial}{\partial t} (H^2) \quad -⑤$$

$$\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{H}) \, \partial V = -\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right] \partial V - \int_V \sigma E^2 \partial V$$

↓ diverge

Pointing Theorem

$$\boxed{\underbrace{\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S}}_{①} = -\underbrace{\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right] \partial V}_{②} - \underbrace{\int_V \sigma E^2 \partial V}_{③}}$$

Total power leaving the volume ①

② Rate of decrease in Energy stored in electric and magnetic field.

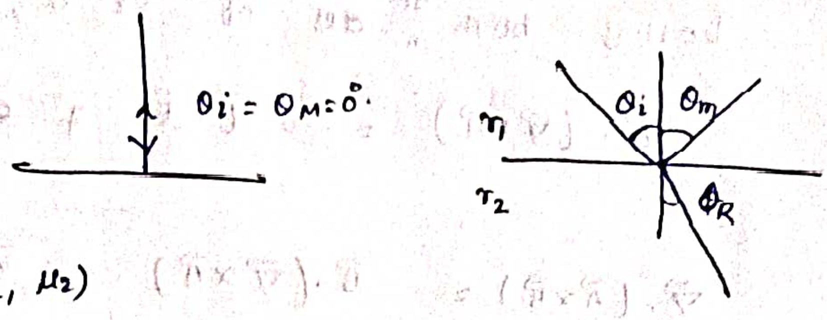
Ohomic power dissipation ③

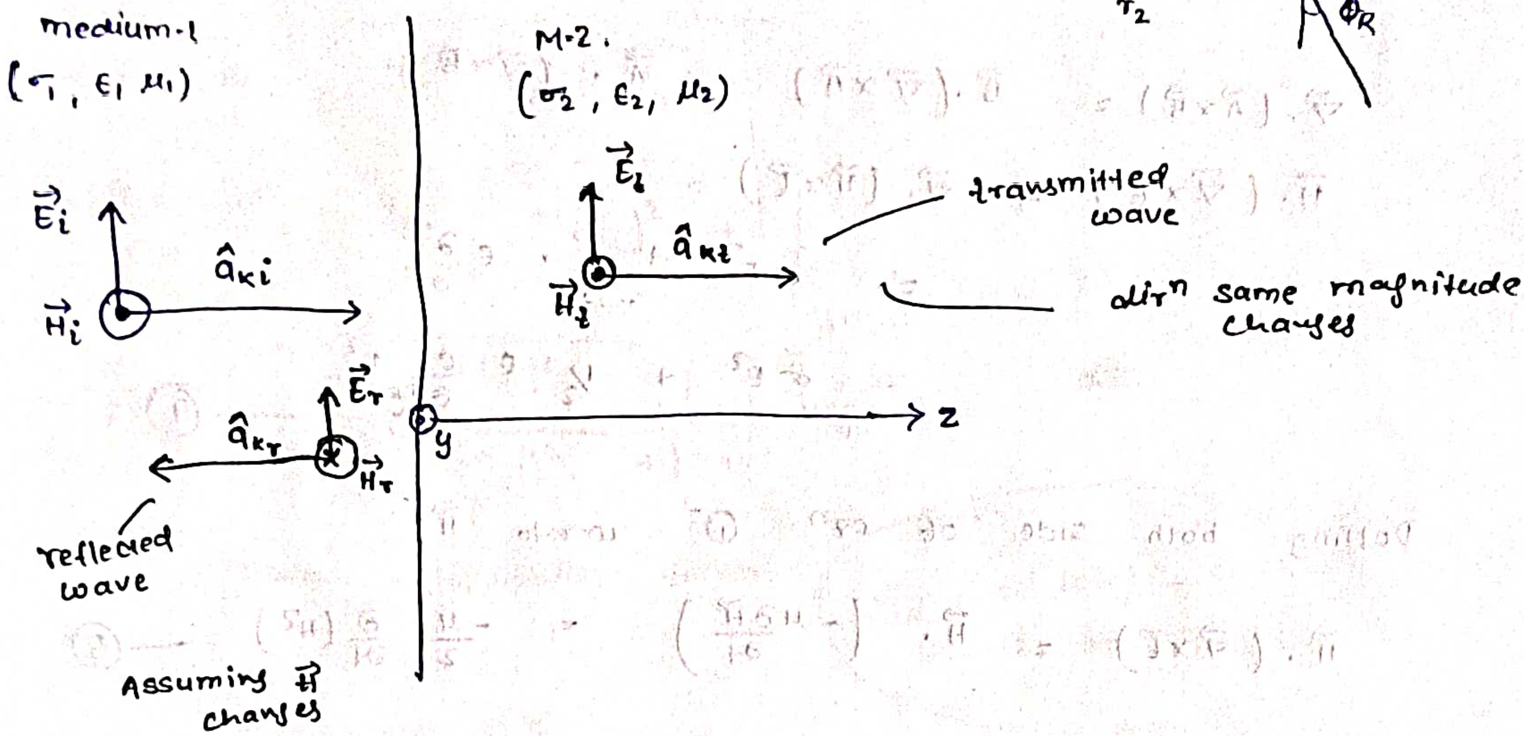
$\vec{S} = \vec{E} \times \vec{H}$

$\vec{W}/m^2 = V/m \quad A/m \Rightarrow \vec{W}/m^2$

Instantenious power density.

Reflection of plane wave at normal Incidence.





$\hat{a}_{ki} = \hat{a}_E \times \hat{a}_H$

$\hat{a}_z = \hat{a}_x \times \hat{a}_y$

Incident wave $(\vec{E}_{is}, \vec{H}_{is})$

$$\vec{E}_{is}(z) = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x$$

$$\vec{H}_{is}(z) = H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y$$

$$= \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \hat{a}_y$$

(a)

Reflected wave $(\vec{E}_{rs}, \vec{H}_{rs})$

$$\vec{E}_{rs}(z) = E_{ro}\, e^{+\gamma_1 z}\, (+\hat{a}_x)$$

$$\vec{H}_{rs}(z) = H_{ro}\, e^{\gamma_1 z}\, (-\hat{a}_y)$$

$$= \frac{E_{ro}}{\eta_1}\, e^{\gamma_1 z}\, (-\hat{a}_y)$$

Transmitted wave $(\vec{E}_{ts}, \vec{H}_{ts})$

$$\vec{E}_{ts}(z) = E_{t0}\, e^{-\gamma_2 z}\, \hat{a}_x$$

$$\vec{H}_{ts}(z) = H_{t0}\, e^{-\gamma_2 z}\, \hat{a}_y$$

$$\frac{E_{t0}}{\eta_2}$$

At interface, $z=0$, $\vec{E}$ and $\vec{H}$ should be continuous.

$$E_{1t}\, a_n = E_{2t}\, a_n$$

$$H_{2t}\, a_n = H_{2t}\, a_n$$

E_t'

E_r

+ve due to vector addition

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_t(0)$$

$$\Rightarrow E_{i0} + E_{r0} = E_{t0}$$

H_i

H_r

-ve due to vector substraction

$$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

$$\Rightarrow \frac{1}{\eta_1}(E_{i0} - E_{r0}) = \frac{E_{t0}}{\eta_2}$$

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0}$$

$$E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0}$$

gamma

$$\boxed{\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_2}}$$

reflection coefficient

Tau

$$\boxed{\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

transmission coefficient.

17/10/23

Case-1

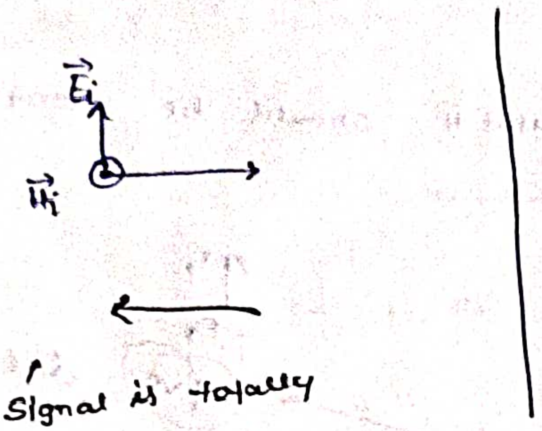
Medium-1 - Perfect dielectric

$\sigma = 0$

M-2 perfect ~~dielectric~~ conductor

$\sigma = \infty$ PEC

↳ perfect electric conductor



$\eta_2 = 0$

$\Gamma = -1$

180° phase shift.

PMC $\Gamma = +1$

Signal is totally reflected.

$$\vec{E}_{1s} = \vec{E}_{is} + \vec{E}_{rs}$$

total electric field

$$= \left(E_{i0}\, e^{-\gamma_1 z} + E_{r0}\, e^{\gamma_1 z} \right) \hat{a}_x$$

$$\Gamma = \frac{E_{r0}}{E_{i0}} = -1 \quad ; \quad \alpha = 0 \text{ for medium 1.} \quad \gamma_1 = \alpha + j\beta_1$$

$$\boxed{\vec{E}_{1s} = -E_{i0} \left(e^{j\beta_1 z} - e^{-j\beta_2 z} \right) \hat{a}_x}$$

$$\boxed{\vec{E}_{1s} = -2j\, E_{i0} \sin \beta_1 z\ \hat{a}_x}$$

total Electric field in M-1.

$$\vec{E}_1 = \text{Re}\left\{ \vec{E}_{1s}\, e^{j\omega t} \right\}.$$

$$= \text{Re}\left\{ -2j\, E_{i0} \sin\beta_1 z \left[\cos\omega t + j \sin\omega t \right] \right\}.$$

$$\boxed{\vec{E}_1 = 2\, E_{i0} \sin \beta_1 z \sin \omega t\ \hat{a}_x}$$

$$\boxed{\vec{H}_1 = \frac{2\, E_{i0}}{\eta_1} \cos \beta_1 z \cos \omega t\ \hat{a}_x}$$

Standing wave Ratio (SWR)

- Represented by S

$$\boxed{S = \frac{|\vec{E_1}|_{max}}{|\vec{E_1}|_{min}}}$$

SWR >1 or equal 1.

↳ it doesn't have fraction value.

$|\Gamma| \to 0$ to 1.

Γ Range -1 to 1

$$\boxed{|\Gamma| = \frac{S-1}{S+1}}$$

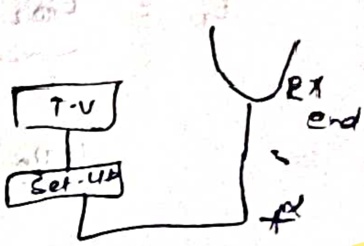
gamma.

$$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

$$\boxed{S_{dB} = 20\log_{10} S}$$

Vector-Network Analyzer

to measure Γ



Perfect Match.

$S=1$, $|\Gamma|=0$, $E_{r0}=0$;

Textbook Exam 9.7

Q. free space $(z\leq 0)$ a plane with $\vec{H_i} = 10\cos(10^8 t - \beta z)\,\hat{a}_x$ mA/m is incident normally on a lossless medium which $(z\geq 0)$ is characterized by $\epsilon = 2\epsilon_0$, $\mu = 8\mu_0$

determine, $\vec{H_r}$, $\vec{E_r}$, $\vec{H_t}$, $\vec{E_t}$.

Soln:-

$$\hat{a}_{k_i} = \hat{a}_{E_i} \times \hat{a}_{H_i}$$

$$\hat{a}_z = (-\hat{a}_y) \times \hat{a}_x$$

for free space, $\beta_1 = \omega/c$; $\omega = 10^8$

$$\beta_1 = \frac{10^8}{3\times 10^8} = 1/3.$$

$$\eta_1 = \eta_0 = 120\pi\ \Omega$$

For lossless medium,

$$\beta_2 = \omega\sqrt{\mu\epsilon}$$

$$= \omega\sqrt{\mu_r\mu_0\,\epsilon_r\epsilon_0}$$

$$\sqrt{\mu_0\epsilon_0} = 1/c$$

$$= \frac{\omega}{c} \times 4$$

$$\beta_2 = 4/3. \qquad \left[\because \omega/c = 1/3\right].$$

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{8\times\mu_0}{2\epsilon_0}}$$

$$= 2\sqrt{\frac{\mu_0}{\epsilon_0}} = 2\eta_0$$

$$= 240\pi\ \Omega.$$

$$\vec{E_i} = \underbrace{E_{oi}}_{10\eta_0} \cos(10^8 t - \beta z)\ (-\hat{a}_y)$$

$$\boxed{\eta_0 = \frac{E_{oi}}{H_{oi}}}$$

$$\eta_0 = \frac{E_{oi}}{H_{oi}} =$$

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\eta_0}{3\eta_0} = 1/3$$

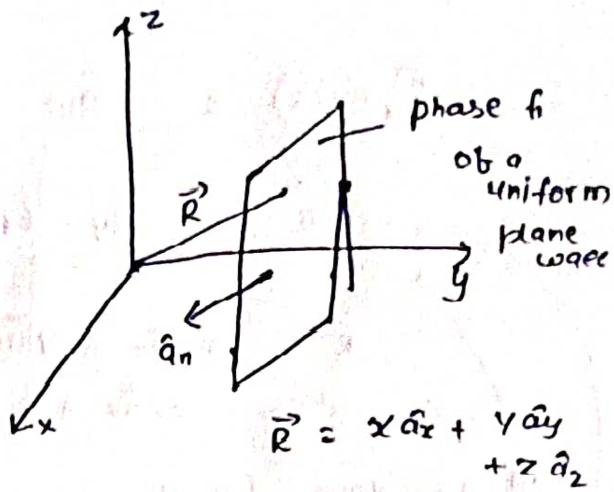
$$\tau = \frac{E_{to}}{E_{io}} = 4/3.$$

$$\vec{E_r} = \underbrace{E_{ro}}_{E_{io}/3} \cos(10^8 t + \beta z)\ \hat{a}_y$$

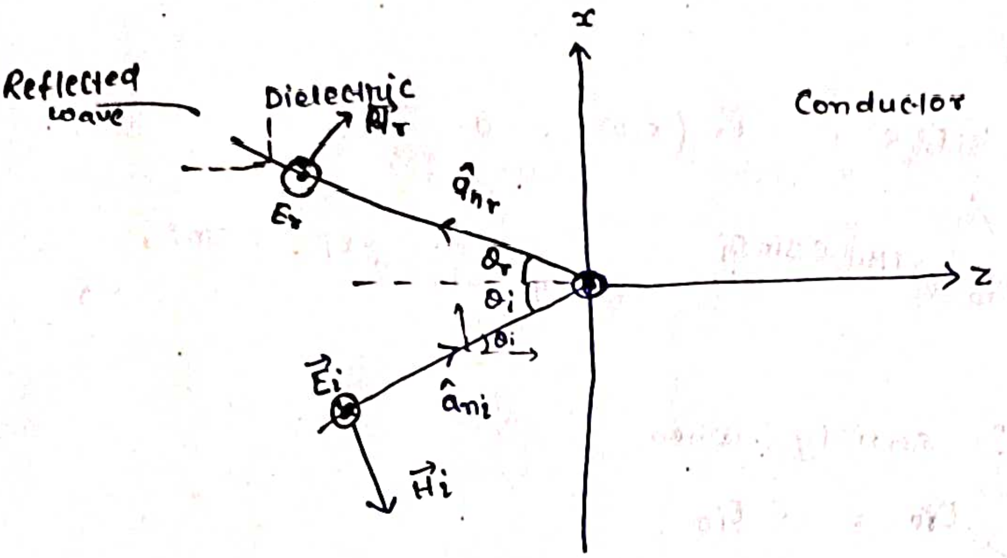
Oblique incidence at a plane conducting boundary

$$\boxed{\vec{E}(\vec{R}) = E_0 \, e^{-j\vec{K}\cdot\vec{R}}}$$

$$\boxed{\vec{K} = K_x \hat{a}_x + K_y \hat{a}_y + K_z \hat{a}_z = K \hat{a}_n}$$



Case-1 Perpendicular Polarization
($\vec{E} \perp$ to plane of Incidence)



vector incident on the normal dirn.

$$\hat{a}_{ni} = \hat{a}_x \sin\theta_i + \hat{a}_z \cos\theta_i \quad ; \quad \theta_i \text{ - angle of Incidence}$$

Incident wave

$$\vec{E}_i(x,z) = \hat{a}_y E_{io} \, e^{-j\beta_1 \hat{a}_{ni}\cdot\vec{R}}$$ — Generalized form to represent $\vec{E}$ in plane;

$$\boxed{\vec{E}_i(x,z) = \hat{a}_y E_{io} \cdot e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}}$$

$$\vec{H}_i(x,z) = \frac{1}{\eta_i}\left[\hat{a}_{ni} \times \vec{E}_i(x,z)\right]$$

$$\boxed{\vec{H}_i(x,z) = \frac{E_0}{\eta_i}\left(\hat{a}_z \sin\theta_i - \hat{a}_x \cos\theta_i\right) e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}}$$

medium,

$$\beta_2 = \omega\sqrt{\mu\epsilon}$$
$$= \omega\sqrt{\mu_r\mu_0\,\epsilon_r\epsilon_0}$$

Reflected wave,

$$\hat{a}_{nr} = \hat{a}_x \sin\theta_r - \hat{a}_z \cos\theta_r$$

$$\vec{E}_r(x,z) = \hat{a}_y\, E_{r0}\, e^{-j\beta_1 \hat{a}_{nr}\cdot\vec{R}}$$
$$= \hat{a}_y\, E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{R} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$
$$\vec{k} = k_x\,\hat{a}_x + k_y\,\hat{a}_y + k_z\,\hat{a}_z = k\,\hat{a}_n$$

At boundary $(z=0)$

$$\vec{E}_1(x,0) = \vec{E}_i(x,0) + \vec{E}_r(x,0) = 0.$$

$$\hat{a}_y\, E_{i0}\, e^{-j\beta_1 x\sin\theta_i} + \hat{a}_y\, E_{r0}\, e^{-j\beta_1 x\sin\theta_r} = 0$$

Above eq^n satisfy when,

$$E_{r0} = -E_{i0}$$
$$\theta_i = \theta_r$$

$$\boxed{\vec{E}_r(x,z) = -\hat{a}_y\, E_{i0}\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}}$$

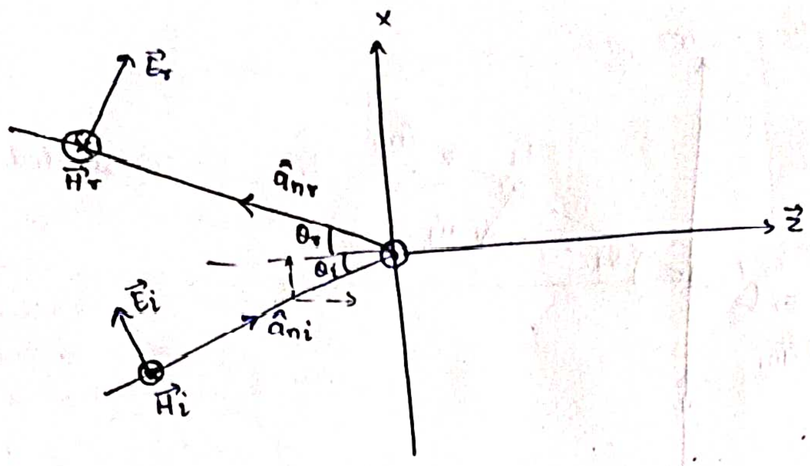
$$\boxed{\vec{H}_r(x,z) = \frac{1}{\eta_1}\left[\hat{a}_{nr} \times \vec{E}_r(x,z)\right].}$$

Total Electric field,

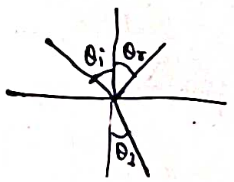
$$\vec{E}(x,z) = E_i(x,z) + E_r(x,z)$$

$$\boxed{\vec{E}(x,z) = -\hat{a}_y\, 2E_{i0}\sin(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i}}$$

Case-2 Parallel Polarization ($\vec{H}$ along y)



$$\vec{E_i} = E_{io}\left(\hat{a}_x \cos\theta_i - \hat{a}_z \sin\theta_i\right) e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)}$$



$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \frac{\beta_1}{\beta_2} = \frac{n_i}{n_t}}$$

$$n = \frac{c}{u_p}$$

Exam Imp.

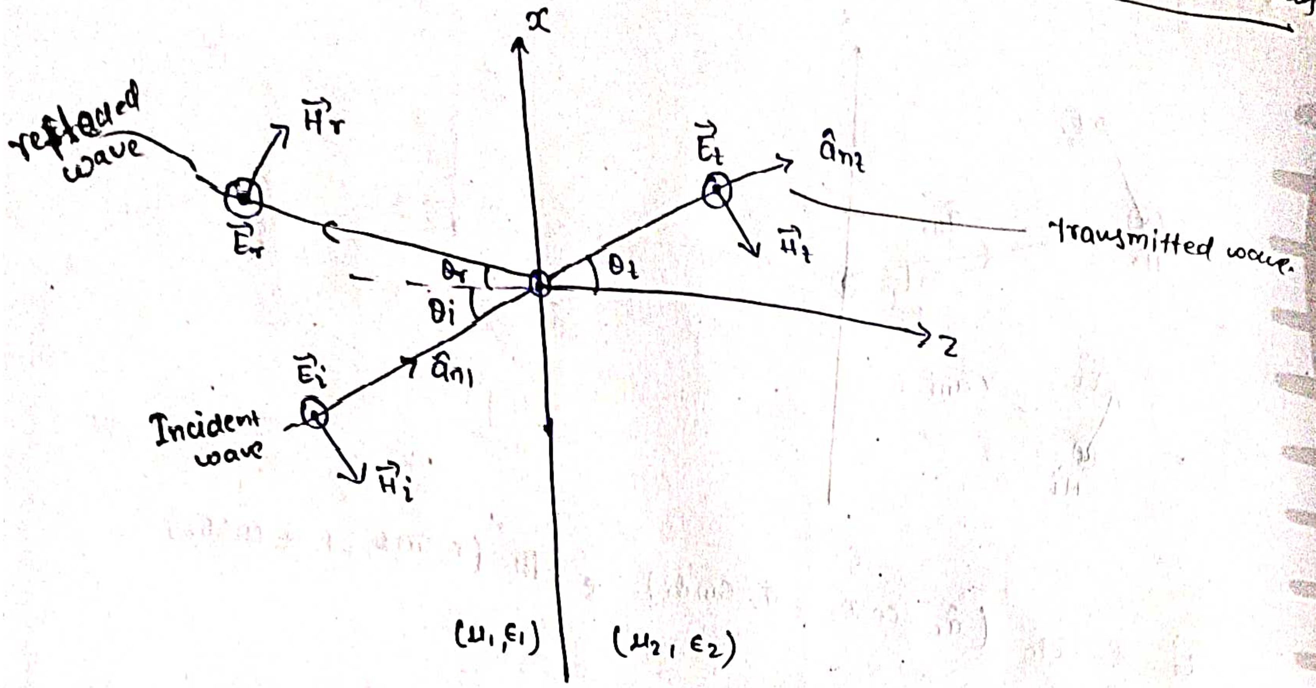
$$\boxed{\frac{n_1}{n_2} \neq \frac{\eta_1}{\eta_2} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}}$$

refractive Index — wave Impedent — $\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$\theta_i = \theta_c = 90°$

Critical angle

Perpendicular Polarization for two Dielectric - Dielectric Medium



Incident wave

$$\vec{E}_i(x,z) = \hat{a}_y \, E_{io} \, e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i(x,z) = \frac{E_{io}}{\eta_1} \left(-\hat{a}_x \cos\theta_i + \hat{a}_z \sin\theta_i \right) e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}$$

Refracted wave

Transmitted wave

$\beta_1 \to \beta_2$, $\theta_i \to \theta_t$.

David Chain book

At boundary,

$$\vec{E}_{iy}(x,0) + \vec{E}_{ry}(x,0) = E_{t,y}(x,0)$$

$$\vec{H}_{ix}(x,0) + H_{rx}(x,0) = \vec{H}_{t,x}(x,0)$$

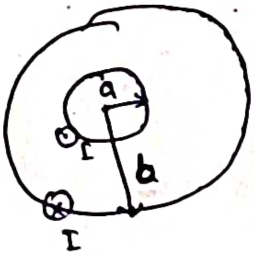
Tutorial

$$\boxed{L = \lambda/I = \frac{N\psi}{I}}$$

$$\boxed{L = \frac{2W_m}{I^2}}$$

Q. Determination of self Inductance of a Coaxial cable

For 10-15 marks derive B_1 & B_2.



$$\vec{B}_1 = \frac{\mu I \rho}{2\pi a^2}\hat{a}_\phi \quad \text{- Region 1} \quad (0 \le \rho \le a)$$

$$\vec{B}_2 = \frac{\mu I}{2\pi \rho}\hat{a}_\phi \quad \text{- Region-2} \quad (a \le \rho \le b)$$

Method-1.

$$W_m = \frac{1}{2}\int \vec{B}_i \cdot \vec{H}\, dv = \frac{1}{2\mu}\int_V |\vec{B}|^2 dv$$

R-1.

$$W_m = \frac{1}{2\mu}\left(\frac{\mu I}{2\pi a}\right)^2 \int_{\rho=0}^{a} \rho^2 \rho\, d\rho \cdot \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{l} dz$$

$$= \frac{\mu I^2}{8\pi^2 \cancel{a^4}} \frac{\cancel{a^4}}{4} \times 2\pi \times l$$

$$= \frac{\mu I^2 \cancel{a^2} L}{16\pi}$$

$$L_{in} = \frac{2W_m}{I^2} = \frac{\mu l}{8\pi}$$

For region-2

$$W_m = \frac{1}{2\mu} \int_V \left(\frac{\mu I}{2\pi\rho}\right)^2 \rho\, d\rho\, d\phi\, dz$$

$$= \frac{1}{2\mu} \left(\frac{\mu I}{2\pi}\right)^2 \int_{\rho=a}^{b} \frac{1}{\rho^2} \rho\, d\rho \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{l} dz$$

$$\left[\ln \rho\right]_a^b \quad 2\pi \quad l$$

$$= \frac{\mu I^2 L}{4\pi}$$

$$\boxed{L_{ext} = \frac{2W_m}{I^2} = \frac{\mu L}{2\pi} \ln b/a.}$$

$$L = L_{int} + L_{ext} = \frac{\mu l}{8\pi} + \frac{\mu l}{2\pi} \ln b/a$$

Method-2

Region-2

$$L_{ext} = \lambda/I = \frac{N\psi}{I}.$$

$$d\psi_2 = \vec{B_2} \cdot dS_2$$

$$= \frac{\mu I}{2\pi\rho} \hat{a}_\phi \cdot d\rho\, dz\, \hat{a}_\phi$$

$$= \frac{\mu I}{2\pi\rho} d\rho\, dz.$$

$$\psi_2 = \int_{\rho=a}^{b} \int_{z=0}^{l} \frac{\mu I}{2\pi\rho} d\rho\, dz$$

$$= \frac{\mu I}{2\pi} \ln b/a \cdot l.$$

$$= \frac{\mu I}{2\pi} \ln b/a \cdot l.$$

$$L_{ext} = \frac{N\psi}{I} = N \frac{\mu I l}{2\pi} \ln b/a.$$

$$L_{in} = \lambda / I = \frac{N\varphi}{I}$$

We know,

$$d\varphi_1 = \vec{B}_1 \cdot d\vec{s}$$

$$= \frac{\mu I \rho}{2\pi a^2} \hat{a}_\phi \cdot d\rho \, dz \, \hat{a}_\phi$$

$$= \frac{\mu I \rho}{2\pi a^2} d\rho \, dz$$

Assuming current density is uniform over entire conductor

$$\frac{I}{\pi a^2} = \frac{I_{enc}}{\pi \rho^2}$$

$$I_{enc} = \frac{\pi \rho^2}{\pi a^2} I.$$

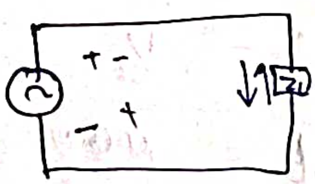
Thus the total flux linkage within differential element is $d\lambda_1 = d\varphi_1 \cdot \frac{I_{enc}}{I}$

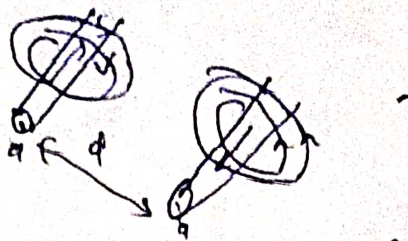
$$\lambda_1 = \frac{\mu I}{2\pi a^4} \int_{\rho=0}^{a} \rho^3 \, d\rho \int_{z=0}^{l} dz$$

$$= \frac{\mu I}{8\pi} l$$

$$L_{in} = \frac{\lambda}{I} = \frac{\mu I}{8\pi} \text{ (Henery / Unit length).}$$

Self Inductance for two-wire transmission Line





— If current same dir^n
↳ cancel each other

Internal will be same
Linkage $\lambda_1 = \frac{\mu I l}{8\pi}$

$$\lambda_2 = \psi_2 = \int_{\rho=a}^{d-a} \int_{z=0}^{L} \frac{\mu I}{2\pi\rho} \, d\rho \, dz$$

$$= \frac{\mu I l}{2\pi} \ln \frac{d-a}{a}.$$

By symmetry,
the same amount of flux is produced by the current
$-I$. Hence the flux linkage,

$$\lambda = 2(\lambda_1 + \lambda_2)$$

$$L' = \frac{L}{l} = \frac{\mu}{\pi}\left[\frac{1}{4} + \ln d/a\right].$$

Self Inductance of an Infinitely long Solenoid

$$\boxed{\vec{B} = \mu\vec{H} = \mu I n}$$ $n = N/l$

$\psi = B \cdot S = \mu I n S$

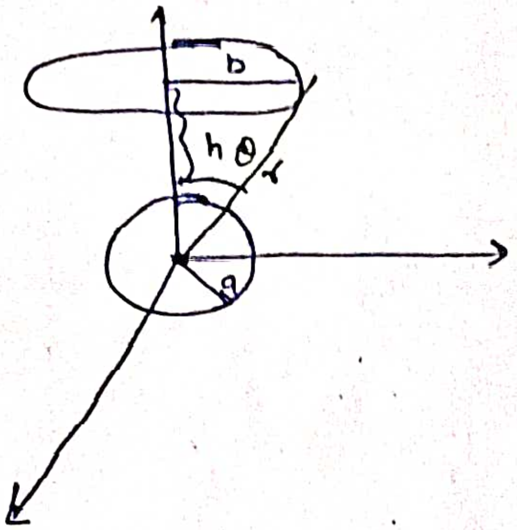
$\lambda = N\psi = \mu I n S \cdot N.$

$= \mu I \frac{N}{l} S \cdot N$

$L = \lambda / I$

$$\boxed{L = \mu \frac{N^2}{l} S}$$

$\boxed{\text{Inductance per unit length} = \mu N^2 S \text{ (Henry-per unit length)}}$



$h >> ab$

Q. Find the mutual Inductance b/w the wires.

$$\vec{A_1} = \frac{\mu I_1 \pi a^2 \sin\theta}{4\pi r^2} \hat{a}_\phi \quad ; \quad \sin\theta = b/r = \frac{b}{\sqrt{h^2+b^2}}$$

$$= \frac{\mu I_1 \pi a^2 \frac{b}{\sqrt{h^2+b^2}}}{4\pi (h^2+b^2)} \hat{a}_\phi$$

$$= \frac{\mu I_1 a^2 b}{4 h^3} \hat{a}_\phi$$

$$= \frac{\mu I_1 a^2 b}{4 h^3} \hat{a}_\phi$$

$$\psi = \int_S \vec{B} \cdot d\vec{s} = \int_S (\vec{\nabla} \times \vec{A_1}) \cdot d\vec{s}$$

$$\psi_{12} = \oint_{l_2} \vec{A_1} \cdot \vec{dl_2}$$

$$= \frac{\mu \pi I_1 a^2 b}{2 h^3}$$

$$M_{12} = \frac{\psi_{12}}{I_1} = \frac{\mu \pi a^2 b^2}{2 h^3}$$

solve **Imp.**

Q. Find the mutual inductance of two co-planar concentric circular loop of radius 2m and 3m.

$h = 0$
$\theta = 90^\circ$
$r = b$

2.632 μHenry.

25/10/23

David chain - Read

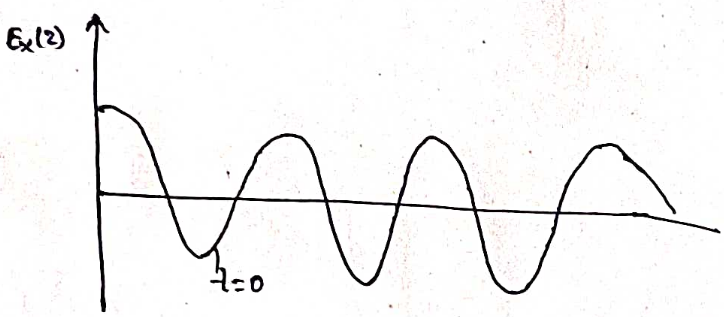
Concept of phase and group velocity

$$\vec{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$$

Assuming $\alpha = 0$

$$\vec{E}(z,t) = E_0 \cos(\omega t - \beta z)\, \hat{a}_x \quad \leftarrow \text{Instantaneous form}$$

$$\vec{E}(z,0) = E_0 \cos(\beta z)\, \hat{a}_x$$



$$\omega t - \beta z = \text{constant}$$

$$\boxed{V_p = \frac{dz}{dt} = \frac{\omega}{\beta}}$$

In free space,

$$\beta = \omega\sqrt{\mu_0\epsilon_0}$$

$$\boxed{V_p = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \cancel{\frac{\omega}{\beta}} = c}$$

Velocity of propagation of an equiphase point

= speed of light.

Group Velocity :-

$\beta = \omega\sqrt{\mu\epsilon}$; lossless medium.

In lossy dielectric medium,
↳ β is not proportional to ω.

phase constant is not a linear fn of ω for lossy di ~~such~~ so wave with different freq. will propagate with diff. phase velocity.

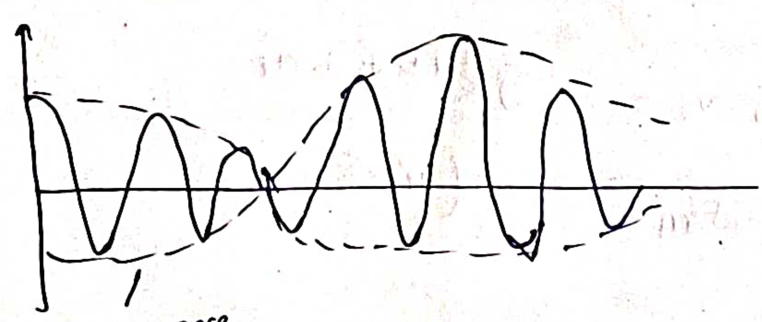
⇓

Diff. phase velocity causes distortion in signal wave shape, the phenomena of signal distortion is caused by the dependence of phase velocity and frequency.

$$\vec{E}(z,t) = E_0 \cos[(\omega_0 + \Delta\omega)t - (\beta_0 + \Delta\beta)z] + E_0 \cos[(\omega_0 - \Delta\omega)t - (\beta_0 - \Delta\beta)z].$$

$$= 2E_0 \cos(t\Delta\omega - z\Delta\beta) \cos(\omega_0 t - \beta z)$$

$\Delta\omega << \omega_0$



$t\Delta\omega - z\Delta\beta = \text{constant}$

$$\boxed{V_g = \frac{dz}{dt} = \frac{\Delta\omega}{\Delta\beta}}$$

Group velocity.

$$\boxed{V_p = \frac{\omega_0}{\beta_0} = \frac{dz}{dt}}$$

$$\boxed{V_p = \frac{\omega}{\beta}}$$

$$\boxed{V_g = \frac{d\omega}{d\beta}}$$

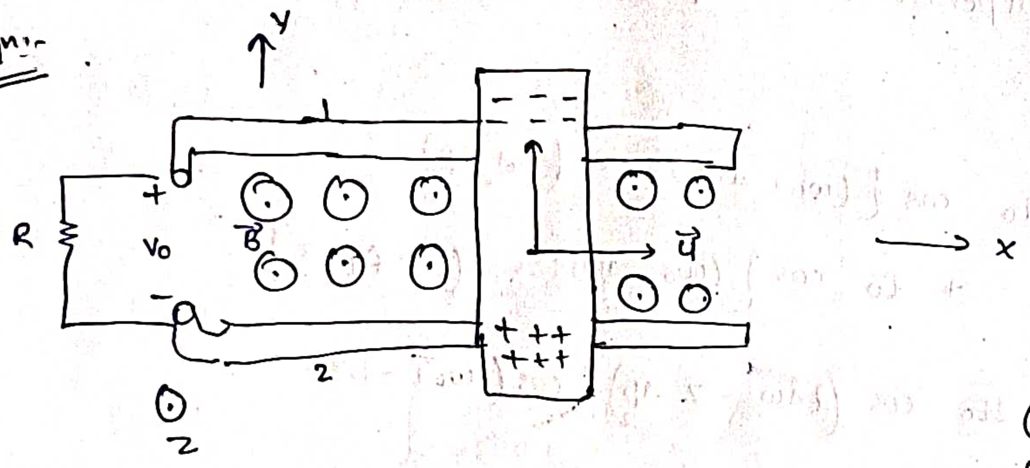
~~Q. A metal bar~~

Faraday disk — Example 7.2

Q. A metal bar slides over a pair of conducting rails in a uniform magnetic field $\vec{B} = \hat{a}_z B_0$ with a constant velocity u as shown in fig. Determine the open circuit voltage V_0 that appears across the terminals 1 and 2. Assuming that a resistance R is connected b/w the terminal. Find the Electric power dissipated.

Calculate the mechanical power required to move the bar with velocity $\vec{u}$.

Soln:



(a) $V_0 = ?$

(b) $P_e = ?$

(c) $\vec{P}_m = ?$

from motional Emf,

$$V_0 = V_1 - V_2 = \oint (\vec{u} \times \vec{B}) \cdot \vec{dl}$$

$$\vec{F}_m = -q(\vec{u} \times \vec{B})$$

$$I = \frac{u B_0 h}{R}$$

$$P_e = I^2 R = \frac{(u B_0 h)^2}{R}$$

$$P_m = \vec{F} \cdot \vec{u} \quad (c)$$

$$\vec{F_m} = I \int_{l'} d\vec{l}' \times \vec{B}$$

$$F = -\vec{F_m}$$

Magnetic circuit
Maglev technology
time-varying potential

Tutorial

Average Power

$\hat{a}_z \quad \hat{a}_x \times \hat{a}_y$

$$\mathcal{P} = \underset{(V/m)}{\vec{E}} \times \underset{(A/m)}{\vec{H}}$$

W/m^3

→ Instant. power density.

suppose $\vec{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$

$\vec{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_y$

$$\mathcal{P}(z,t) = \frac{E_0^2}{2|\eta|} e^{-2\alpha z}\, 2 \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta)\, (\hat{a}_x \times \hat{a}_y)$$

$$= \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \left[\cos\theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)\right] \hat{a}_z$$

$$\mathcal{P}_{avg.}(z) = \frac{1}{T}\int_0^T \mathcal{P}(z,t)\, dt$$

in free space $\eta = \eta_0$
$= 120\pi\ \Omega$
$= 377\ \Omega.$

$$\boxed{\mathcal{P}_{ave}(z) = \frac{1}{2} \mathrm{Re}\left[\vec{E_s} \times \vec{H_s}^*\right]}$$ conjugate form

Q. In Free space, $\vec{E}(z,t) = 50 \cos(\omega t - \beta z)\, \hat{a}_x$ V/m. Find the avg. power passing a circular area of radius 2.5 m in the plane z = constant.

Sol^n:-

$$\vec{H}(z,t) = \frac{50}{|\eta|} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_y.$$

In free space, $\theta_\eta = 0$, $|\eta| = 120\pi$

$$\vec{E}(z,t) = Re[\vec{E_s}\, e^{j\omega t}].$$

$$= Re\left[50\, e^{j(\omega t - \beta z)}\hat{a}_x\right].$$

$$= Re\left[50\, e^{-j\beta z}\,\hat{a}_x\, e^{j\omega t}\right].$$

$$\vec{E_s} = 50\, e^{-j\beta z}\,\hat{a}_x$$

$$\vec{H_s} = \frac{50}{120\pi}\, e^{-j\beta z}\,\hat{a}_y$$

$$P_{avg} = \frac{1}{2} Re\left[50\, e^{-j\beta z}\hat{a}_x \times \frac{50}{120}\, e^{j\beta z}\,\hat{a}_y\right.$$

$$= \frac{50^2}{2\times 120}\,\hat{a}_z$$

$$\boxed{P_{avg.} = \frac{E_0^2}{2\eta_0}\,\hat{a}_z} \boxed{= \frac{H_0^2\eta_0}{2}\,\hat{a}_z}$$

$$P_{ave\,(W)} = \int_S P_{ave}(z)\cdot d\vec{s}$$

$$= \int \frac{50^2}{2\times 120\cancel{\pi}}\,\hat{a}_z \cdot \cancel{\pi} r^2\,\hat{a}_z$$

$$= \frac{50^2}{240}\times(2.5)^2$$

$$= 65.1\ W.$$

Q. $\vec{E}(x,t) = \hat{a}_y\, 24\pi \cos(\omega t - k_0 x)$ V/m - in free space. In this field consider a square area 10 cm × 10 cm on a plane $x+y=1$. Total time avg. power passing through square Area

$\vec{E} \times \vec{H}$

$\hat{a}_x \quad \hat{a}_y \quad \hat{a}_z$

$$P_{avg.} = \frac{E_0^2}{2\eta_0}\,\hat{a}_x$$

$$= \frac{24^2\times\pi^2}{2\times 120\pi} = 2.4\pi\ \hat{a}_x.$$

$\vec{ds} = |\vec{ds}|\ \hat{a}_n$

$\hat{a}_n = \frac{\nabla f}{|\nabla f|}$ $\qquad \hat{a}_n = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$

$f(x,y) = x + y - 1$

$|\nabla f| = \sqrt{1+1} = \sqrt{2}$ $\qquad \frac{10 \times 10 \times 10^{-4}}{10^{-2}}$

$P_{avg.} = \oint 2.4\pi\ \hat{a}_x \cdot \frac{\hat{a}_x}{\sqrt{2}} \times 10^{-2}$ $\qquad \partial/\partial x\ \hat{a}_x$

$= \int \frac{2.4\pi}{\sqrt{2}} \times 10^{-2}$

$= 5.33 \times 10^{-2}$

$= 53.3\ mW$

Q. In free space, $\vec{H} = 0.2 \cos(\omega t - \beta x)\ \hat{a}_z$ A/m. in free space. Calculate total power passing through (a) square plate of side 10cm on the plane $x+y=1$
(b) a circular disc of radius 5 cm on the plane $x=1$.

Sol^n:- $\qquad \vec{E} \times \vec{H}$ $\qquad \hat{a}_x = \hat{a}_y \times \hat{a}_z$

$P_{ave} = \frac{H_0^2\ \eta_0}{2}\ \hat{a}_x$

$= \frac{(0.2)^2 \times 120\pi}{2}\ \hat{a}_x$

$\hat{a}_n = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$

$|\vec{S}|\ \hat{a}_n = 10^{-2}\ \hat{a}_n$

(a) $P_{avg} = \int \frac{4 \times 10^{-2} \times 120\pi}{2} \times 10^{-2} \times \frac{1}{\sqrt{2}}$

$= 533.14 \times 10^{-4}$

$|S|\ \hat{a}_n = \pi r^2\ \hat{a}_x$

$= 5.3314\ \mu W$

(b) $P_{avg} = \int \frac{4 \times 10^{-2} \times 120\pi}{2} \times \pi\ 5^2 \cdot 6 = 59.2\ mW$

$\omega\sqrt{\mu\epsilon}$

Q. TEM wave of freq. $f = 14\ GHz$ in a Homogenous medium, where, $\mu_r = 1$.

$$\vec{E} = E_p\, e^{j(\omega t - 280\pi y)}\ \hat{a}_z\ (V/m)$$

$$\vec{H} = 3\, e^{j(\omega t - 280\pi y)}\ \hat{a}_x\ (A/m)$$

$$c = 3\times10^8\ m/s$$

$$\eta_0 = 120\pi$$

Calculate $\epsilon_r = ?$ $E_p = ?$

Solⁿ:-

$$\beta = \omega\sqrt{\mu\epsilon} \qquad ;\ \beta = 280\pi \text{ given}$$

$$280\pi = \omega\sqrt{\epsilon_r}\sqrt{\mu_0\epsilon_0}$$

$$= 2\pi f \times \sqrt{\epsilon_r} \times 1/c$$

$$\sqrt{\epsilon_r} = \frac{\cancel{280\pi}^{140} \times 3\times\cancel{10^8}^{10}}{\cancel{2\pi}\times\cancel{14}\times\cancel{10^9}} = 3$$

$$\epsilon_r = 9.$$

$$\frac{E_0}{H_0} = \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$= \sqrt{\frac{\mu_r}{\epsilon_r}\times\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{\mu_r}{\epsilon_r}}\times\sqrt{\frac{\mu_0}{\epsilon_0}} - \eta_0$$

$$\frac{E_p}{\cancel{3}} = \frac{1}{\cancel{3}}\times 120\pi$$

$$E_p = 120\pi.$$

Q.

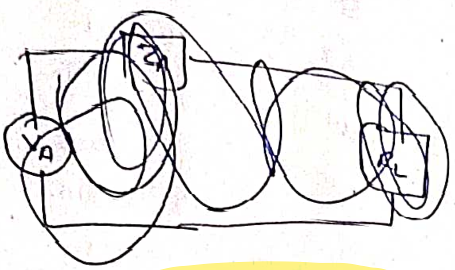
Fundamentals of Antenna

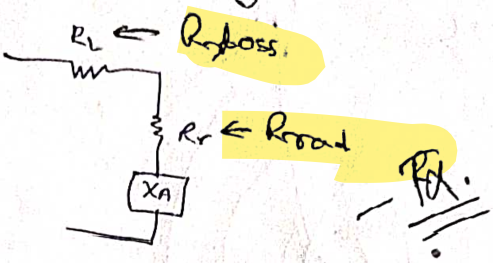
Types of Antennas

L.E. Frenzel
Principles of Electronic communication Systems.

R_L – ohmic losses
R_r – power radiated by Antenna
X_A – P

Antenna components.

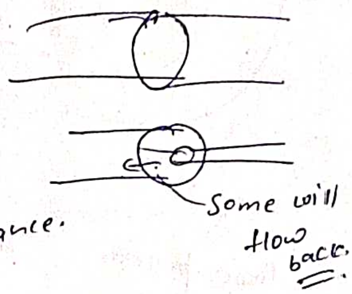




$$Z_A = R_A + jX_A.$$

$R_A = R_L + R_r$

$jX_A = 0$ when resonance.

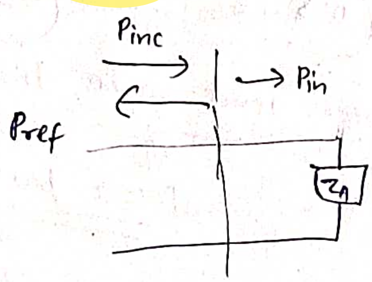


Matched

$Z_0 = Z_A$.
50 Ω

SWR = 1 very difficult to achieve.

$\Gamma = 0$



$P_{in} = P_{inc} - P_{ref}$.

Radiation Efficiency

$$= \frac{P_{rad}}{P_{in}} \times 100\%$$

$$= \frac{R_{Rad}}{R_{Rad} + R_L} \times 100\%$$

80 – 85% good antenna.

$P_{in} \; (\uparrow) = P_{inc} - P_{ref} \; (\downarrow)$ ← If $Z_0 = Z_A$.

$P_{in} = P_{R_L} + P_{Radiation}$.
$(\uparrow) \quad (\downarrow) \quad (\uparrow)$

loss is less.

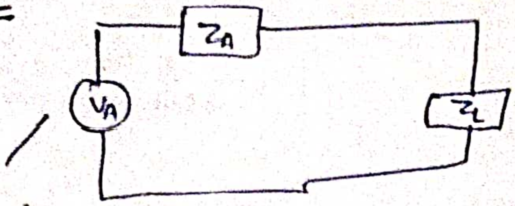
lium,

$\beta_2 = \omega\sqrt{\mu\epsilon}$

$= \omega\sqrt{\mu_r\mu_0\,\epsilon_r\epsilon_0}$

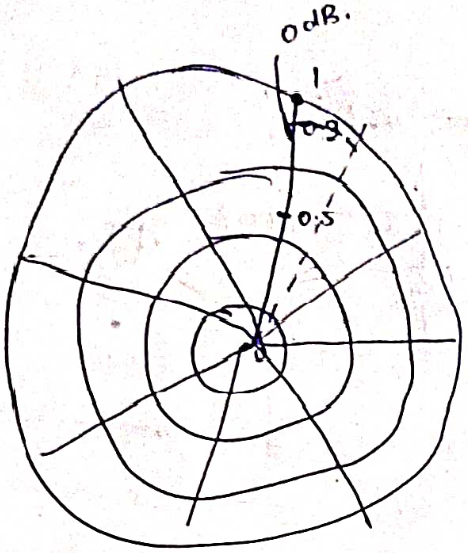
$\sqrt{\mu_0\epsilon_0} = 1/c$

R_x



thevinin
eq. voltage

normalized power
$p = P/P_{max}$



θ	$P(\omega)$	P_{nor}	$E\ (V/A)$
$0°$	10	1	
$10°$	9	0.9	
$20°$	8	0.8	
$30°$	7	0.7	
⋮	⋮	⋮	
$360°$			

Power pattern   Field pattern

$10\log_{10} \frac{P_{out}}{P_{in}}$ — in dB

$P \propto E^2$
$\propto H^2$.

0.5 — power pattern.
-3 dB

HPBW
Half power
Beam width.

} ← power level varies from 50% to 10%.
||
power pattern

Field pattern

-27 $\xrightarrow{-3dB}$ -30 dB
27 $\xrightarrow{-3dB}$ 24 dB

$P \propto V^2$

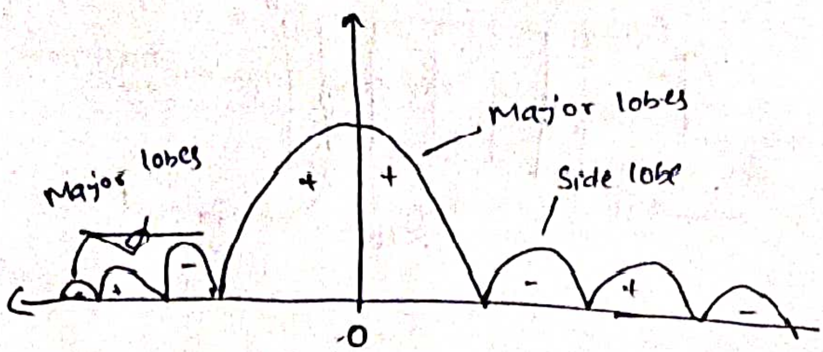
$\frac{P}{2} \propto \frac{V^2}{2} = \left(\frac{V}{\sqrt{2}}\right)^2$

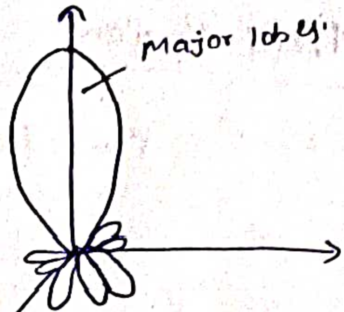
HPBW — 0.707 — field patter

Radiation Pattern Lobes

Major lobes

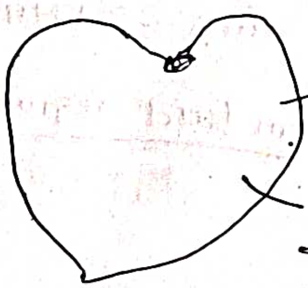
Minor lobes
- Back lobes
- Side lobes





Radiation pattern

- Isotropic — uniformly in all dir^n
- Directional — radiating in particular direction.
  - point to point communication
- Omni directional



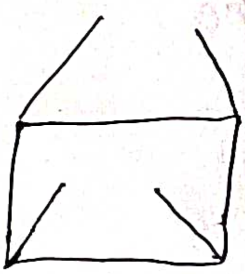
omnidirectional pattern — like circle in particular plane.

half apple.

Principal E- and H-plane pattern

P. E plane — dir^n on $\vec{E}$, dir^n in which wave is propagating

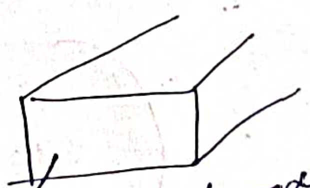
P. H plane — dir^n on $\vec{H}$,



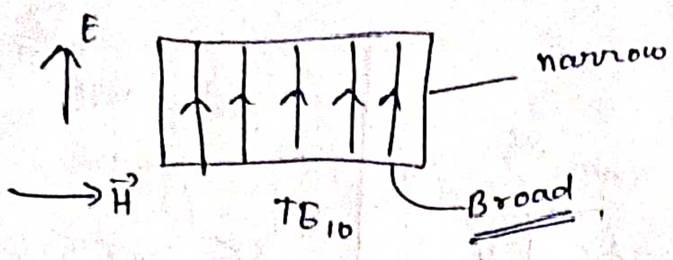
↓
Impedance matching.

Open ckt
↓
Impedance will be mismatched.

poor radiator



Guided media



$\theta = 0$ to $90°$

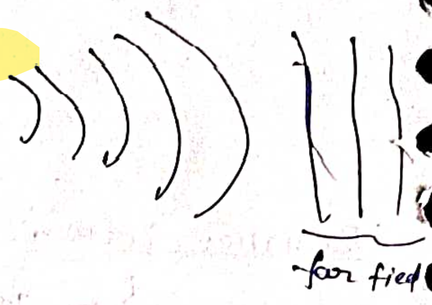
xz - E plane

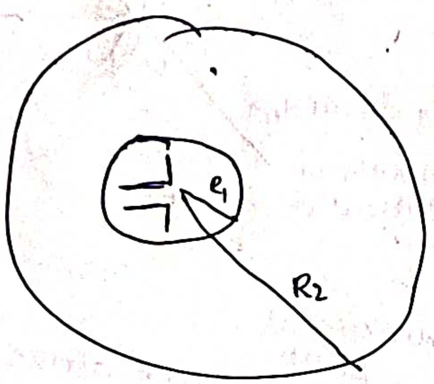
yz - H plane

$\theta = 0$

Field Regions

Far field region, wave propagate linear.





$L = d/2$

rad. freq. = 1 MHz.

far field region = ?

far field

$d = \frac{c}{f}$

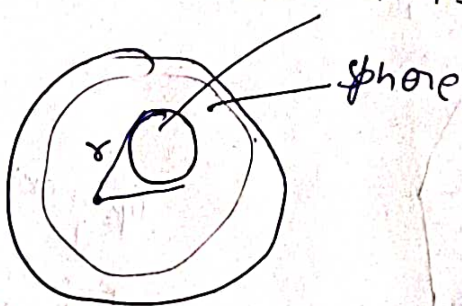
$$\boxed{ffd = \frac{2D^2}{d}}$$

D = largest dimesion of antenna

$d = c/f$.

Radian and Steradian

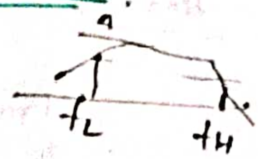
Area = πr^2,



Q.

Power density $= \frac{1}{2} \, Re \left[\vec{E_s} \times \vec{H_s}^{*} \right]$ — conjugate

W/m^2 V/m A/m



$$\vec{E} = E_\theta \hat{a}_\theta + E_\phi \hat{a}_\phi$$

$$\vec{H} = H_\theta \hat{a}_\theta + H_\phi \hat{a}_\phi$$

$$\vec{E} \times \vec{H} =$$

Book.

Sadiku — Ch-1 & Ch-2 ~~Batteries~~

CA — Balanis — Anteena theory, 2.

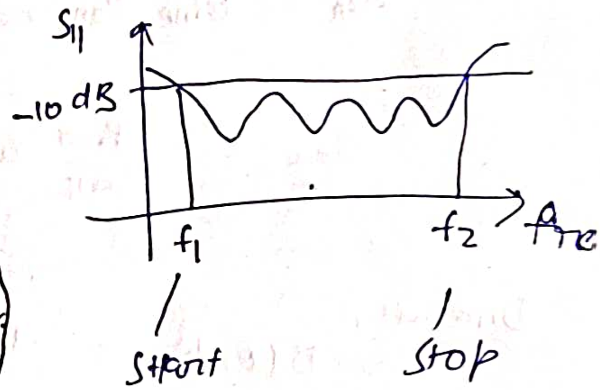
Harish & Sachinanda

Bandwidth

$$B.\omega = \frac{f_U}{f_L}$$

gamma — $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$



Impedence B.ω — $$\boxed{B.\omega = \frac{f_2 - f_1}{f_c} \times 100\%}$$

Centre frequency — $$f_c = \frac{f_1 + f_2}{2}$$

medium,

Beam Solid Angle, Directivity and Gain

$$P_{rad} = \oint_{\Omega} S_{rad}(r,\theta,\phi)\, a_r\, dA \quad W$$

radi

pointing vector

$dA = r^2 \sin\theta\, d\theta\, d\phi$

$$P_{rad} = \oint_{\Omega} U(\theta,\phi)\, d\Omega \quad W$$

$d\Omega = \sin\theta\, d\theta\, d\phi$

Ω_A - Beam solid angle

$$U(\theta,\phi) = U_{max}(\theta,\phi)\, P_n$$

$$P_n = \frac{U(\theta,\phi)}{U_{max}(\theta,\phi)}$$

$$\Omega_A = \theta_{1HP}\, \theta_{2HP} \quad sr$$

$$U_{avg} = \frac{P_{rad}}{4\pi} \quad W/sr$$

Directivity

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}} = 4\pi \frac{U(\theta,\phi)}{P_{rad}}$$

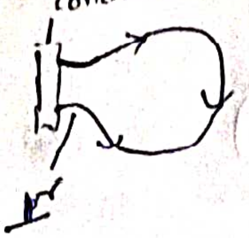
$$D = 4\pi \frac{U_{max}}{P_{rad}} = \frac{4\pi}{\Omega_A}$$

Balanis 2nd chapter

$$\boxed{P_{dBm} = 10 \log_{10}\left(\frac{P}{1\times 10^{-3}}\right) \ dBm}$$

$$\boxed{P_{dBW} = 10 \log_{10}\left(\frac{P}{1}\right) \ dBW}$$

time varying current

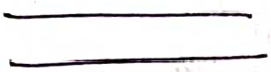


Single wire

to create radiation there must be a time-varying current or an acceleration (or deceleration) of charge

$$I \frac{dI_z}{dt} = Iq_l \frac{dv_z}{dt} = Iq_l a_z$$

Two-wire

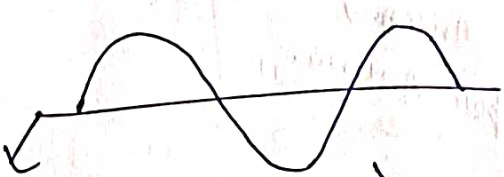


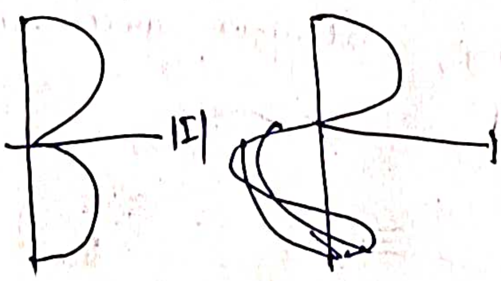
→ transmission line
⇓
Signal don't radiate.

Exam

Q. Draw current distribution network for antenna.

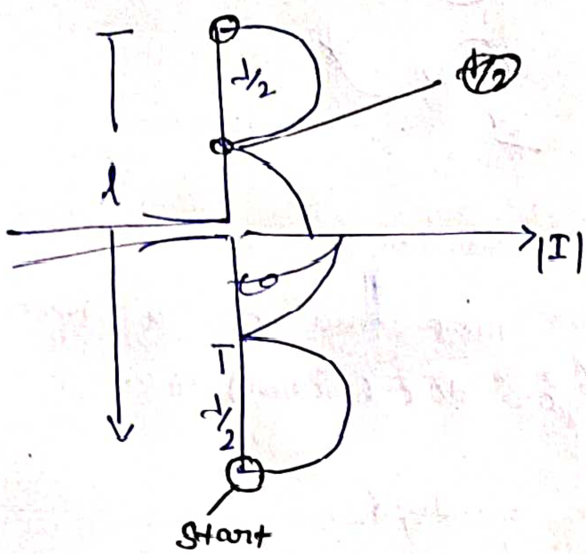
why?





$\frac{3\lambda}{2} > l > \frac{\lambda}{2}$

medium,



• First start from end complete $\lambda/2$.

Hertz dipole

$L < \lambda/50$. — change in current distribution

an infinitesimally small current element is

λ :- full length dipole

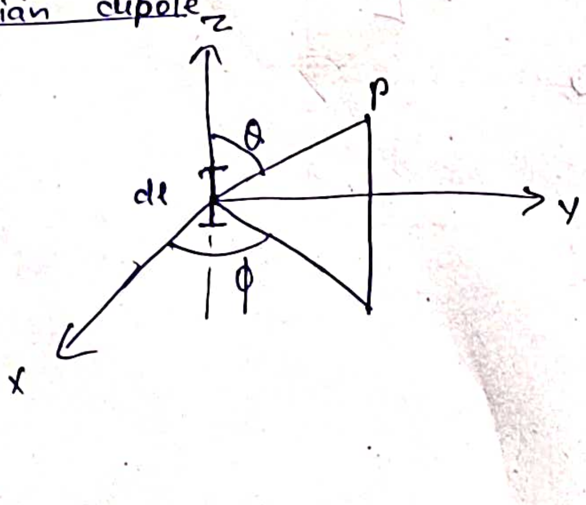
$\lambda/4$ - Quarter length dipole.

Dipole Antenna
monopole Antenna.

Wire Anteenas

L thin Conducting, Straight and curve

Hertzian dipole



θ & ϕ should same

the dipole orientation changed.

$$\vec{A} = \frac{\mu [I] \, dl}{4\pi r} \hat{a}_z$$

$[I]$ - retarded current

$$[I] = I_0 \cos(\omega t - \beta r)$$
$$= Re\left\{ I_0 e^{j(\omega t - \beta r)} \right\}.$$

in which $\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda}$ and $u = \frac{1}{\sqrt{\mu \epsilon}}$

Velocity

Tutorial

Polarization

↳ describes shape, orientation and sense of orientation in the dirn of propagation.

$$E = a_\theta E_\theta + a_\phi E_\phi$$

$f(r, \theta, \phi)$

$$\bar{E}(r, \theta, \phi, t) = a_\theta \, Re\left[E_\theta e^{j\omega t} \right] + a_\phi \, Re\left[E_\phi e^{j\omega t} \right].$$

$$E_\theta = A e^{j\alpha} \qquad E_\phi = B e^{j\beta}.$$

$$a_\theta \, A \cos(\omega t + \alpha) + a_\phi \, A \cos(\omega t + \beta).$$

Linear polarization

$B = 0$

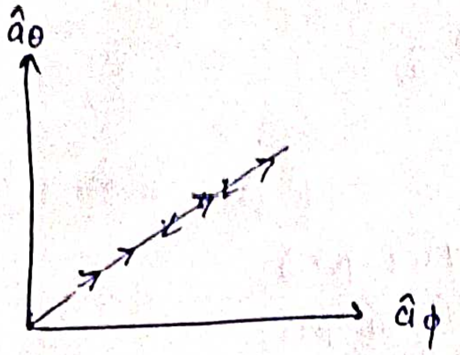
— traces along a particular dirn

(St. line along the a_θ dirn)

96 A & $B \neq 0$

$\alpha = \beta$. $\qquad a_\theta A \cos(\omega t) + a_\phi B \cos(\omega t)$

medium,



$\vec{E}(r,t) = (A\hat{a}_\theta + B\hat{a}_\phi)\cos\omega t$ — $\alpha = \beta = 0$

Circular polarization

$A = B$

$\beta = \alpha - \pi/2$

dir^n $\perp$ — vertical

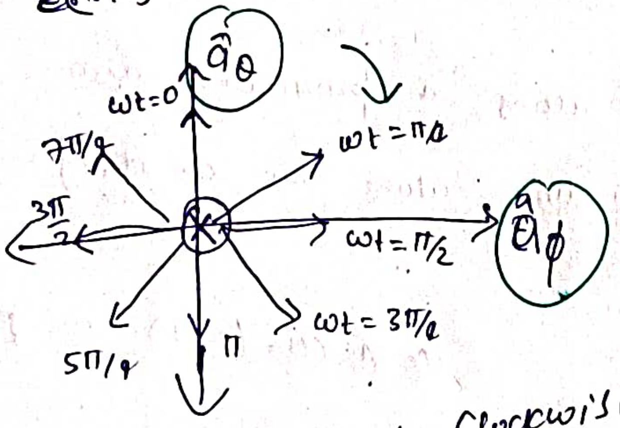
dir^n $\parallel$ — Horizontal.

$\vec{E}(r,t) = A\left[\cos(\omega t+\alpha)\,\hat{a}_\theta + \hat{a}_\phi \sin(\omega t+\alpha)\right]$

RCP — Right handed ⇓ $\vec{E}$ – clockwise

LCP — left handed. $\vec{E}$ – anticlockwise

$\vec{E}(r,t) = \cos\omega t\,\hat{a}_\theta + \sin\omega t\,\hat{a}_\phi$

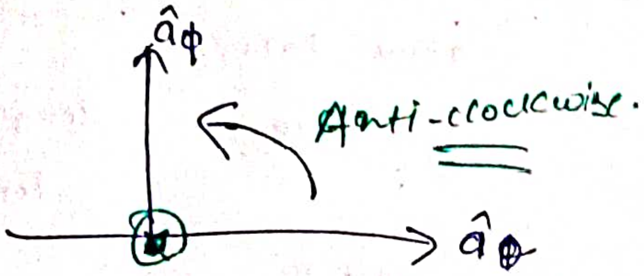


θ to ϕ ⇓ Right hand Rule

Rotating Clockwise ⇓ Right handed circular polarization

Imp. IG dirn of r changes

⊙



Q.

phasor form

$\downarrow$

Eg. 2.11 & 2.12

Inst. form

$a_\theta j + a_\phi$

$\downarrow$

$Re\left[(a_\theta j + a_\phi)e^{j\omega t}\right]$.

$\vec{E}_b(t) = -a_\phi \sin \omega t + a_\phi \cos \omega t$.

LP $\longrightarrow$ sum of (LCP + RCP).

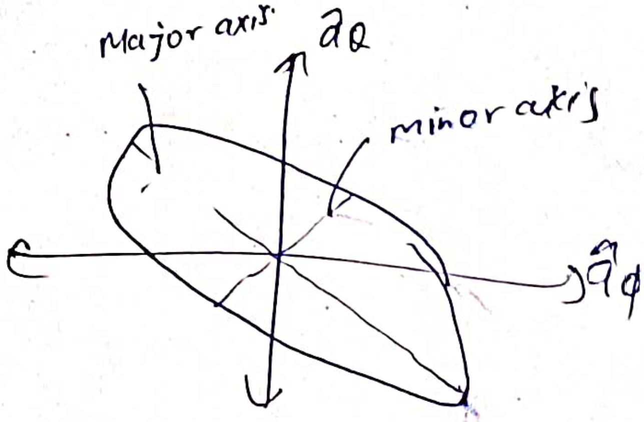
$E = a_\theta j E_0$.

$a_\theta = \frac{1}{2}\left[(a_\theta + a_\phi j) + (a_\theta - a_\phi j)\right]$

$E = E_0 \times \frac{1}{2}\left[\underbrace{(a_\theta + a_\phi j)}_{LCP} \underbrace{(a_\theta + a_\phi j)}_{RCP}\right]$.

Elliptical polarization

$A \neq B \neq 0$ $\quad \alpha \neq \beta$.



$\sqrt{\mu_r \mu_0}$ $\epsilon_r \epsilon_0$

axial Ratio,

$$AR = \frac{\text{length of major axis}}{\text{length of minor axis}}$$

At any point in space, the tip of the electric field of an elliptically pl. wave traces an ellipse as a function of t

for linear $AR = \infty$

for circular $AR = 1$

Q. $\vec{E}(t) = a_\theta \cos \omega t - a_\phi\, 2\cos(\omega t + \pi/4)$.

calculate for different value of $\omega t = 0$ — values

$\omega t = \pi/8$

$\omega t = \pi/4$.

plot the value

clockwise — Right handed

Anti-clockwise — left handed

Harish and Sachinanda Antenna theory

parameters

problems

Q. which TEM travelling in + x-axis

(a) $\vec{E}$ $\vec{H}$

$\vec{E}$	$\vec{H}$
$+6\hat{y}$	$-4\hat{z}$
$-2\hat{y}$	$-3\hat{z}$
$2\hat{z}$	$+2\hat{y}$
$-3\hat{y}$	$+4\hat{z}$

Q. losses dielectric medium.

$$\vec{E}(z,t) = a_y \, 2 \cos\left(10^8 t - z/\sqrt{2}\right) \text{ V/m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1/\sqrt{2}}$$

$$= 2\sqrt{2}\pi = 8.88 \text{ m.}$$

Q 2010

$E_0 = 1\text{V/m}$

$\epsilon_r = 4.$

Avg. power density $= \frac{1}{2\eta}|E_0|^2$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r \mu_0}{\epsilon_0 \epsilon_r}}.$$

Q. $\vec{E}_i = 24 \cos(3\times10^8 t - \beta y)\, \hat{a}_z$ V/m.

free space | dielectric

$y \geq 0$

$\mu = \mu_0$

$\epsilon = 9\epsilon_0.$

Reflected Magnetic field

$H_r.$