2101024

MA102: Mathematics II - Multivariable Calculus

Mid Semester Examination, 2021-22

Time: 2 Hours | IIIT Guwahati | Marks: 40

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1. Verify the Stokes's theorem for the hemisphere $S: x^2+y^2+z^2=9, z \geq 0$, its bounding circle $C: x^2+y^2=9, z=0$, and the field $\mathbf{F}=y\hat{i}-x\hat{j}$. [6]

2. Verify the divergence theorem for the field $\mathbf{F}=x\hat{i}+y\hat{j}+z\hat{k}$ over the sphere $x^2+y^2+z^2=a^2$. [6]

3. Evaluate [6]

$$\int_0^3\int_0^4\int_{x=y/2}^{x=(y/2)+1}\left(\frac{2x-y}{2}+\frac{z}{3}\right)dx\,dy\,dz$$

by applying the transformation

$$u=(2x-y)/2, \quad v=y/2, \quad w=z/3$$

and integrating over an appropriate region in uvw-space.

4. Find the average value of $F(x,y,z)=xyz$ over the cube bounded by the coordinate planes and the planes $x=2, y=2$ and $z=2$ in the first octant. [6]

5. Find the maximum and minimum values of the function $f(x,y)=3x+4y$ on the circle $x^2+y^2=1$. [5]

6. The surfaces [6]

$$f(x,y,z)=x^2+y^2-2=0$$

and

$$g(x,y,z)=x+z-4=0$$

meet in an ellipse E. Find the parametric equations for the line tangent to E at the point $P_0=(1,1,3)$.

7. Find the curvature of a circle of radius a. [5]

$x^2+y^2=a^2$ $\quad \boxed{x\,dx - y\,\frac{dy}{2}}$ $\quad u = x - v$ $\quad du = dx - dv$