

### Problem

The input power in a two port directional coupler is  $1\text{ mW}$ , the coupler has a coupling factor of  $15\text{ dB}$  and a directivity of  $30\text{ dB}$ . Calculate the power in all the ports.

Soln

$$C = 10 \log \left( \frac{\text{Input power}}{\text{coupled off power}} \right)$$

$$\Rightarrow C = 10 \log \left( \frac{P_1}{P_4} \right)$$

$$\Rightarrow 15 = 10 \log \frac{P_1}{P_4}$$

$$\Rightarrow \frac{P_1}{P_4} = \text{antilog} \left( \frac{3}{2} \right) = 31.62$$

$$\therefore P_4 = \frac{1 \text{ mW}}{31.62} = 0.032 \text{ mW} = 32 \mu\text{W}$$

$$D = 10 \log \frac{P_4}{P_3} = 30$$

$$\Rightarrow \frac{P_4}{P_3} = \text{Anti log}(3) = 10^3$$

$$\Rightarrow P_4 = 10^3 P_3$$

$$\Rightarrow P_3 = \frac{P_4}{10^3} = \frac{32 \mu\text{W}}{10^3} = 0.032 \mu\text{W}$$

Then power in port 2

$$P_2 = \text{Input power} - (\text{power in coupled port} + \text{power in isolated port})$$

$$= P_1 - (P_3 + P_4)$$

$$= 1 - (0.032 + 0.00032)$$

$$= 0.968 \text{ mW}$$

Now to a Two port network

- ② The s-parameters of a two port network are given by

$$[S] = \begin{bmatrix} 0.2 \angle 0^\circ & 0.6 \angle 90^\circ \\ 0.6 \angle 90^\circ & 0.1 \angle 0^\circ \end{bmatrix}$$

- (a) Prove that the network is reciprocal but not lossless.  
(b) Find the return loss at Port 1 when Port 2 is short-circuited.

Sol<sup>n</sup>

Since A reciprocal device has the same transmission characteristics in either direction of a pair of ports & it is characterized by a symmetric scattering matrix;  
 $S_{ij} = S_{ji} \ (i \neq j)$  which results  $[S]_t = [S]$

Now in given matrix as  $[S]_t = [S]$ , so it is reciprocal.

but to prove it is lossless, given S-matrix needs to satisfy unitary property;

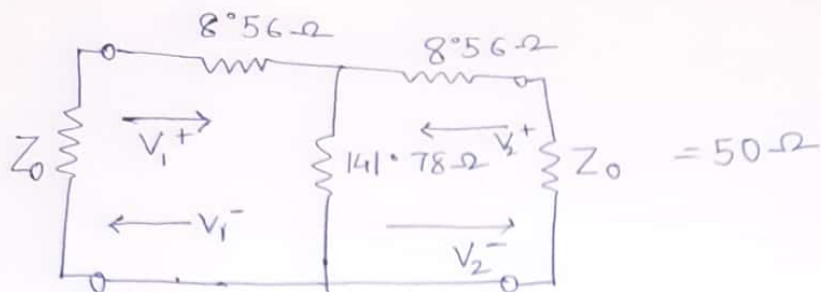
i.e.  $\sum_{n=1}^N S_{ni} \cdot S_{nj}^* = 1; \ i=j$  also,  $\sum_{n=1}^N S_{ni} \cdot S_{nj}^* = 0; \ i \neq j$

i.e.  $\left. \begin{aligned} S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* &= 1 \\ S_{21} \cdot S_{21}^* + S_{22} \cdot S_{22}^* &= 1 \end{aligned} \right\}$

but here,  $|S_{11}|^2 + |S_{12}|^2 = 0.2^2 + 0.6^2 \neq 1$

Again  $0.6^2 + 0.1^2 \neq 1$

So the network is not lossless. Reciprocal (Proved)



$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} = \frac{b_1}{a_1} = \frac{Z_{in1} - Z_0}{Z_{in1} + Z_0}$$

$$\begin{aligned} Z_{in1} &= 8.56 + 141.78 \parallel (8.56 + 50) \\ &= 8.56 + \frac{141.78 \times 58.56}{141.78 + 58.56} \\ &\approx 50 \text{ ohm} \end{aligned}$$

$$\therefore S_{11} = 0$$

For symmetry  $S_{22} = S_{11} = 0$ ,  $V_1^- = V_2^+ = 0$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

$$\text{Now } V_1 = V_1^+ + V_1^- = V_1^+$$

$$V_2 = V_2^+ + V_2^- = V_2^-$$

$$\begin{aligned} V_2^- = V_2 &= \left\{ \frac{V_1}{8.56 + 41.44} \times 41.44 \right\} \times \frac{50}{50 + 8.56} \\ &= 0.707 V_1 = 0.707 V_1^+ \end{aligned}$$

$(141.78 \parallel (50 + 8.56)) = 41.44$

$$\therefore S_{21} = \frac{V_2^-}{V_1^+} = \frac{V_2}{V_1} = 0.707$$

$$\therefore [S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix} \quad \underline{\text{Ans}}$$