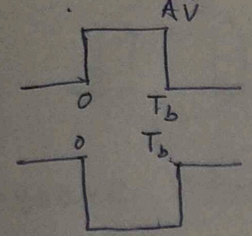
Digital Communication

Bandwidth for Baseband transmission

Bandwidth of PAM {one dimensional}



Polar NRZ

$$S_{BPAM}(f) = A^2 T_b \, sinc^2 (f T_b) = PSD_{NRZ}$$

ON-OFF signalling

0 ⟷ OFF – 0V
1 ⟷ ON – AV
0 to T_b

$$S_{ON-OFF}(f) = \frac{A^2 T_b}{4} sinc^2 (f T_b) + \frac{A^2}{4} \delta(f)$$

$$E_{avg\,PAM} = A^2 T_b = \frac{1}{2} A^2 T_b + \frac{1}{2} A^2 T_b$$

$$E_{avg\,ON-OFF} = \underbrace{\frac{A^2 T_b}{2}}_{\text{for bit 1}} + \underbrace{\frac{0}{2}}_{\text{for bit 0}}$$

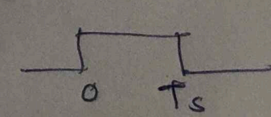
for $E_{avg\,PAM} = E_{avg\,ON-OFF}$ → A should change.

ie will be $A/\sqrt{2}$ in PAM case, but there peak will be different because of the dc component.

→ for $f = \frac{1}{T_b}$ ⇒ will get zero.

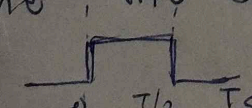
$$BW_{PAM} = \frac{1}{T_b} = R_b \text{ Hz}$$

$$BW_{ON-OFF} = \frac{1}{T_b}$$

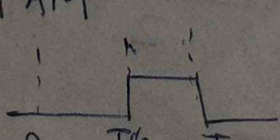


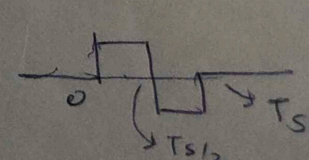
Q. Determine, the BW of 2D PAM

$\phi_1(t)$ →



$\phi_2(t)$



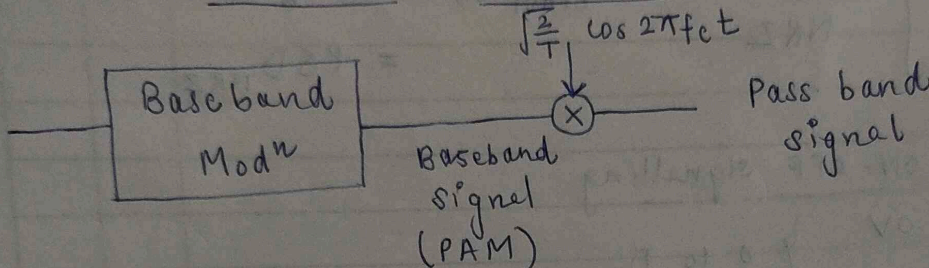


$$SER = 2Q\left(\sqrt{\frac{E_s}{N_0/2}}\right)$$

Symbol error rate.

$$= 2Q\left(\sqrt{\frac{E_{s\,avg}}{N_0/2}}\right)$$

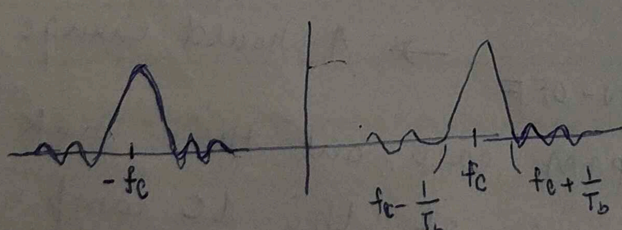
Pass band Modulation Scheme



$$\int_0^{T_b} A^2\cos^2 2\pi f_c t \, dt = \frac{A^2}{2} T_b = 1$$

$$A = \sqrt{\frac{2}{T_b}}$$

$$\left.\begin{aligned} 1 &\rightarrow \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \\ 0 &\rightarrow -\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \end{aligned}\right\} \quad 0 \le t \le T_b$$



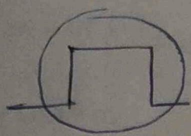
 $\times \cos 2\pi f_c t$

$$= p(f) * \left(\frac{\delta(f-f_c)}{2} + \frac{\delta(f+f_c)}{2}\right)$$

$$= \frac{p(f-f_c)}{2} + \frac{p(f+f_c)}{2}$$

{ low frequency ⟺ High frequency translation }

⇒ Antenna height reduction

 × { Gaussian filter } ⇒ Base band × Carrier

Passband schemes : BPSK, ASK, QPSK, OQPSK, QAM, MPSK, BFSK, MFSK, M-QAM, MSK etc.

Digital Bits —— [Mapper] —— (×) —— (×) —— Passband

(It maps symbol to diff. amplitude levels) $p_B(t)$ (⊓) carrier $\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$

BPAM → 0 → $-A, -\sqrt{E}$

1 → $+A, \sqrt{E}$

BPSK (Binary Pulse Shift Key).

Coherent BPSK

$$s_i(t) = \sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t + (i-1)\pi) \;;\; i=1,2$$

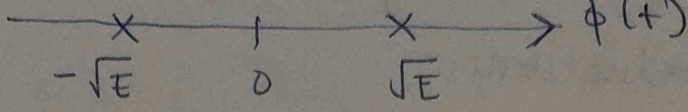
$$0 \to s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

$$1 \to s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t - \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

$$s_1(t) = -s_2(t) \Rightarrow 1 \text{ dimension.}$$

$$\phi(t) = \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t \;;\; 0 \le t \le T_b$$

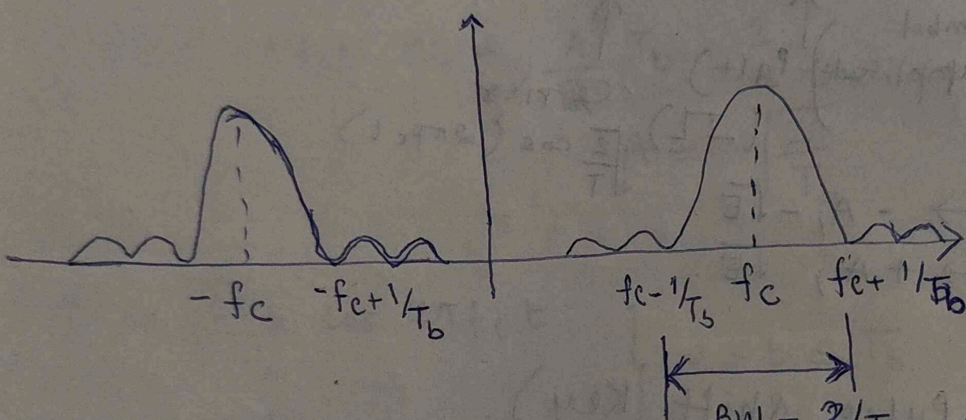
$$\left.\begin{aligned} s_1(t) &= \sqrt{E}\,\phi(t) \\ s_2(t) &= -\sqrt{E}\,\phi(t) \end{aligned}\right\} \text{BPAM.}$$



$$BER_{BPSK} = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$E_{avg\,BPSK} = \frac{E_b}{2} + \frac{E_b}{2} = E_b$$

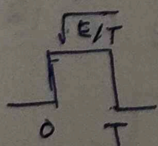
$$PSD_{BPSK} = \frac{E_b}{2}\left[\text{sinc}^2(T_b(f-f_c)) + \text{sinc}^2(T_b(f+f_c))\right]$$



$BW = 2/T_b$ {twice of bit rate}

$= 2R_b$

Amplitude Shift Key. (ON-OFF)

b_i		$p_i(t)$		$s(t)$	
0	$\longrightarrow$	0	$\longrightarrow$	0	$0 \le t \le T_b$
1	$\longrightarrow$		$\longrightarrow$	$\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t$	

$$E_{avg} = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot E_b = \frac{E_b}{2}$$

$$E_{avg\,BPSK} = E_b$$

PSD of ASK.

$$PSD = \frac{A^2 T_b}{4}\,\text{sinc}^2(f T_b) + \frac{A^2}{4}\delta(f)$$

$$(\text{ON-OFF})_{\text{Baseband}} \times \underbrace{\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t}$$

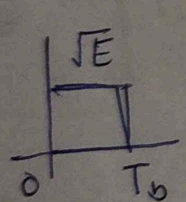
random process. $\Downarrow$ Deterministic process

$$PSD_{Base}(f) * \left[\frac{(\delta(f+f_c)+\delta(f-f_c))}{2}\sqrt{\frac{2}{T_b}}\right]^2$$

$$= PSD_B(f) * \left[\delta(f+f_c)+\delta(f-f_c)+2[\cancel{\delta(f-f_c)\,\delta(f+f_c)}] \to 0\right]\frac{1}{2T_b}$$

$$= PSD_B(f) * [\delta(f+f_c)+\delta(f-f_c)]\frac{1}{2T_b}$$

$$s(t) = \begin{cases} 0 & 0 < t \le T_b \quad \text{— '0'} \\ \sqrt{\frac{2E}{T}}\cos 2\pi f_c t & 0 < t \le T_b \quad \text{— '1'} \end{cases}$$

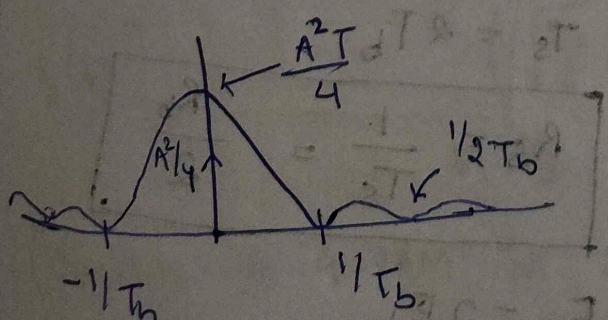


$\times \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t$

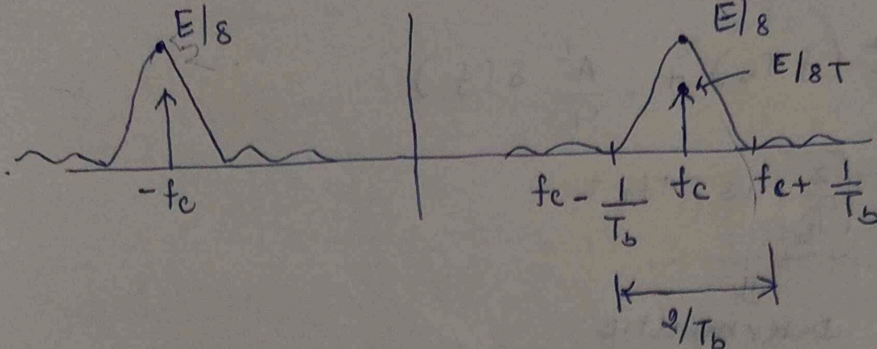
$0 \times \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t$

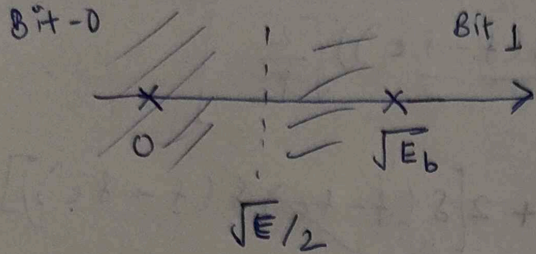
$\Downarrow$ PSD

$$\frac{A^2 T}{4}\,\text{sinc}^2(f T_b) + \frac{A^2}{4}\delta(f) \qquad \frac{1}{2T_b}[\delta(f-f_c)+\delta(f+f_c)]$$



$$* \frac{1}{2T_b}[\delta(f-f_c)+\delta(f+f_c)]$$



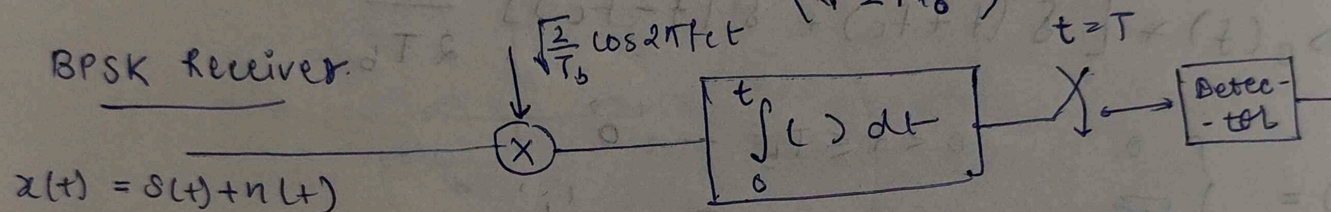


$$P_e = Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

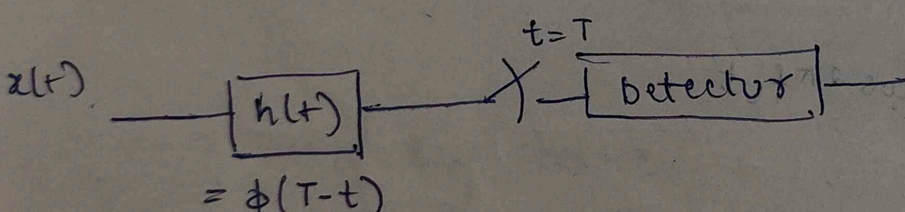
$$= Q\left(\frac{\sqrt{E_b}}{\sqrt{2N_0}}\right)$$

$$P_e = Q\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

BPSK Receiver



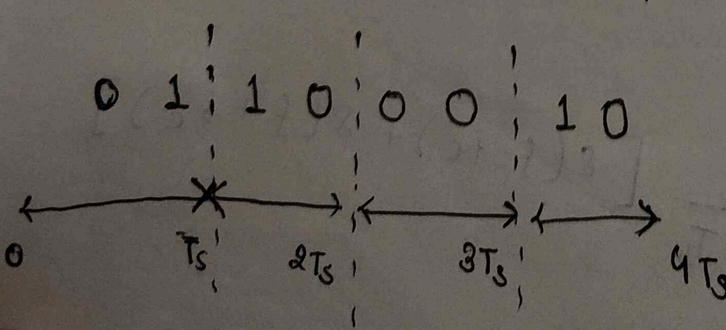
Coorelation.



$= \phi(T-t)$

$$\phi(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

QPSK {Quadrature Phase shift Keying}.



$T_s = 2T_b$

$$\boxed{R_s = \frac{1}{T_s} = \frac{R_b}{2}}$$

$E_s = 2E_b$

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right) & ; \ 0 \le t \le T_s \\ 0 & \text{otherwise} \end{cases}$$

$i = 1, 2, 3, 4.$

$$s_1(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c t + \pi/4) \qquad 0 \le t \le T_s$$

$$= \sqrt{\frac{2E_s}{T_s}} \Big\{ \cos 2\pi f_c t \underbrace{(\cos \pi/4)}_{1/\sqrt{2}} - \sin 2\pi f_c t \underbrace{\sin \pi/4}_{1/\sqrt{2}} \Big\}$$

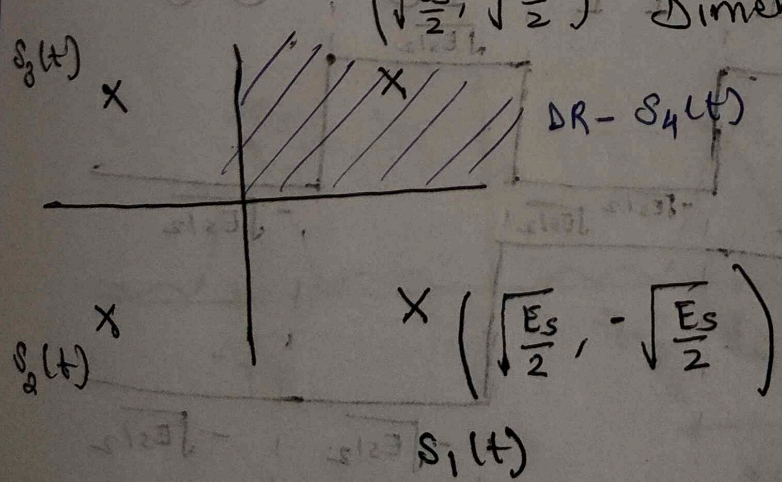
$$= \sqrt{\frac{2(E_s/2)}{T_s}} \{\cos 2\pi f_c t - \sin 2\pi f_c t\}$$

$$= \sqrt{\frac{E_s}{2}} \underbrace{\sqrt{\frac{2}{T_s}} \cos 2\pi f_c t}_{\phi_1(t)} - \sqrt{\frac{E_s}{2}} \underbrace{\sqrt{\frac{2}{T_s}} \sin 2\pi f_c t}_{\phi_2(t)}$$

$$= \sqrt{\frac{E_s}{2}} \phi_1(t) - \sqrt{\frac{E_s}{2}} \phi_2(t)$$

$$s_i(t) = \pm \sqrt{\frac{E_s}{2}} \overbrace{\phi_1(t)}^{\text{In-phase}} \pm \sqrt{\frac{E_s}{2}} \overbrace{\phi_2(t)}^{\text{Q-phase}}$$

$\left(\sqrt{\frac{E_s}{2}}, \sqrt{\frac{E_s}{2}}\right)$ Dimensionality $= 2$



(Similar to 4-PAM in 2D)

$$P_e|_{s_4(t)} = 1 - \int_0^{\infty}\int_0^{\infty} f_{x_1,x_2}(x_1,x_2)\, dx_1\, dx_2$$

$$= 1 - \int_0^{\infty} f_{x_1}(x_1)\, dx_1 \cdot \int_0^{\infty} f_{x_2}(x_2)\, dx_2$$

$$x_1|_{s_4(t)} \sim N\left(\sqrt{\frac{E_s}{2}}, \frac{N_0}{2}\right)$$

$$x_2|_{s_4(t)} \sim N\left(\sqrt{\frac{E_s}{2}}, \frac{N_0}{2}\right).$$

$$SNR = \frac{E_s/2}{N_0/2}$$

$$= 1 - \left(1 - Q\left(\sqrt{SNR}\right)\right)^2$$

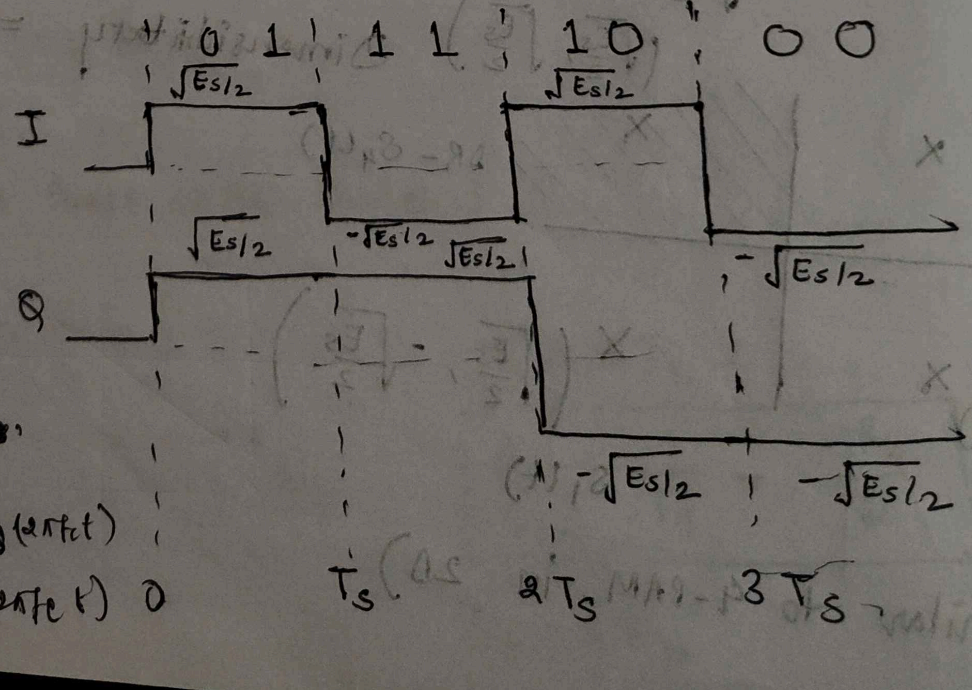
$$= 1 - \left(1 - Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right)^2$$

$$= 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right)$$

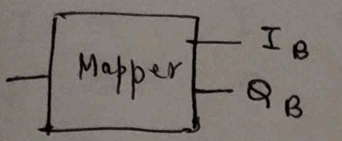
$$P_e|_{s_4(t)} = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right)$$

$$= P_e|_I + P_e|_Q - P_e|_I\, P_e|_Q$$

$01 \rightarrow s_4(t)$
$00 \rightarrow s_2(t)$
$10 \rightarrow s_3(t)$
$11 \rightarrow s_1(t)$

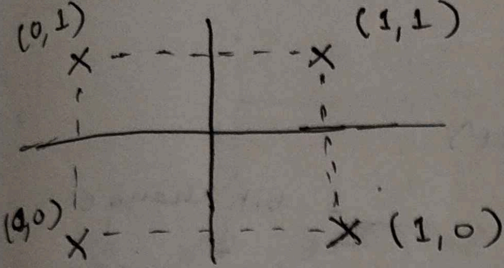


They are component or we can say waff.
$\times 2\sin(2\pi f_c t)$
$\times 2\sin(2\pi f_c t)$



$0 \rightarrow -\sqrt{E_s/2}$

$1 \rightarrow +\sqrt{E_s/2}$



$$(PSD)_{I\text{-}Pass} = PSD\left(\pm\sqrt{\frac{E_s}{T_b}}\cos 2\pi f_c t\right)$$

$$= PSD\left\{ \overset{\pm\sqrt{E_s/2}}{\sqcap} \times \sqrt{\frac{2}{T_s}}\cos 2\pi f_c t \right\}.$$

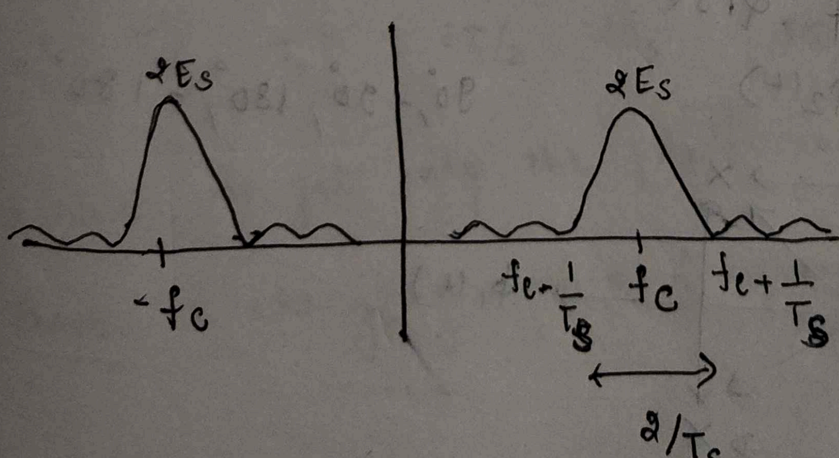
$$(PSD)_{Q\text{-}Pass} = PSD\left(\pm\sqrt{\frac{E_s}{T_b}}\sin 2\pi f_c t\right)$$

$$= PSD\left(\underset{T_s}{\overset{\pm\sqrt{E_s/2}}{\sqcap}} \times \sqrt{\frac{2}{T_s}}\sin 2\pi f_c t \right)$$

$$= PSD\left(\underset{0 \quad T_s}{\overset{\pm\sqrt{E_s/2}}{\sqcap}} \right) * PSD\left\{ \sqrt{\frac{2}{T_s}}\sin 2\pi f_c t \right\}.$$

$$\Downarrow$$

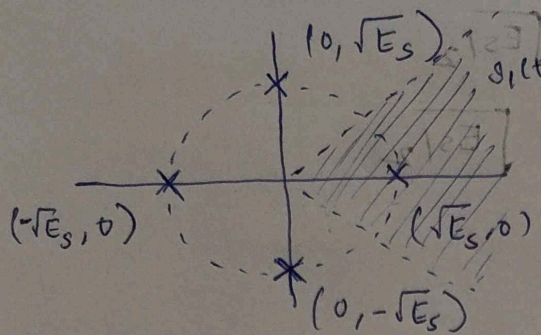
$$\delta(f-f_c) + \delta(f+f_c)$$



$$\boxed{A^2 T_b = E_s}$$

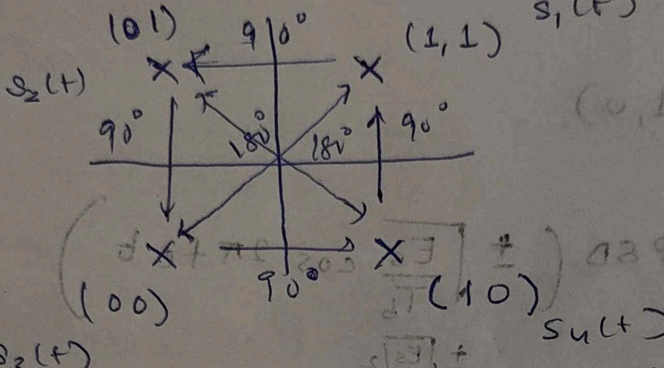
$BW = 2R_s$

$R_s = \frac{1}{T_s} = \frac{1}{2T_b}$

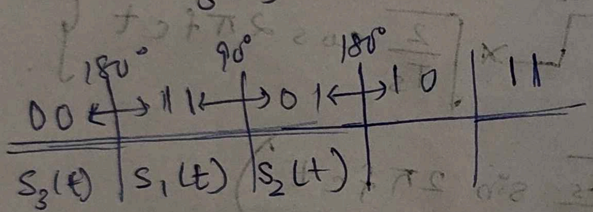


→ Another version of QPSK

1 → +ve
0 → −ve



1 bit change
↓
Gray coding

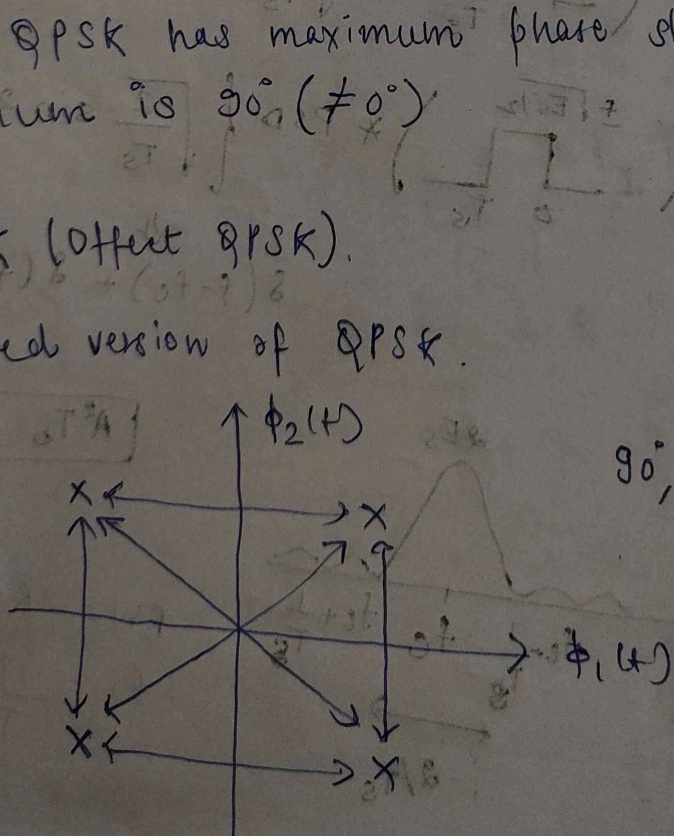


→ Gray coding does not solve all problems, the above example shows us that.

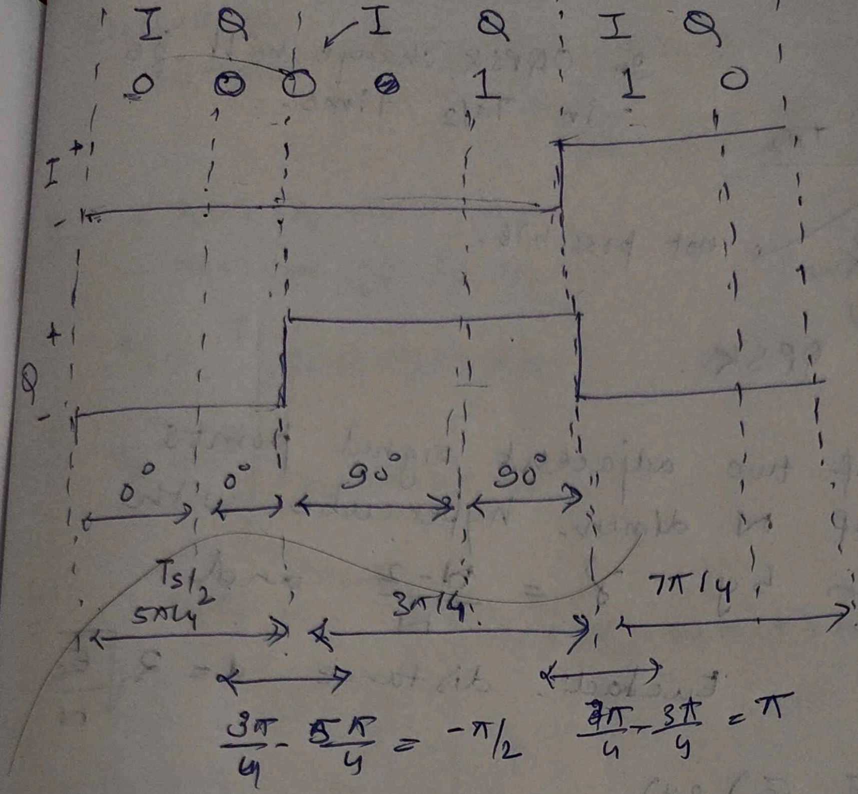
⇒ The QPSK has maximum phase shift of 180° and minimum is 90° (≠ 0°)

OQPSK (offset QPSK).

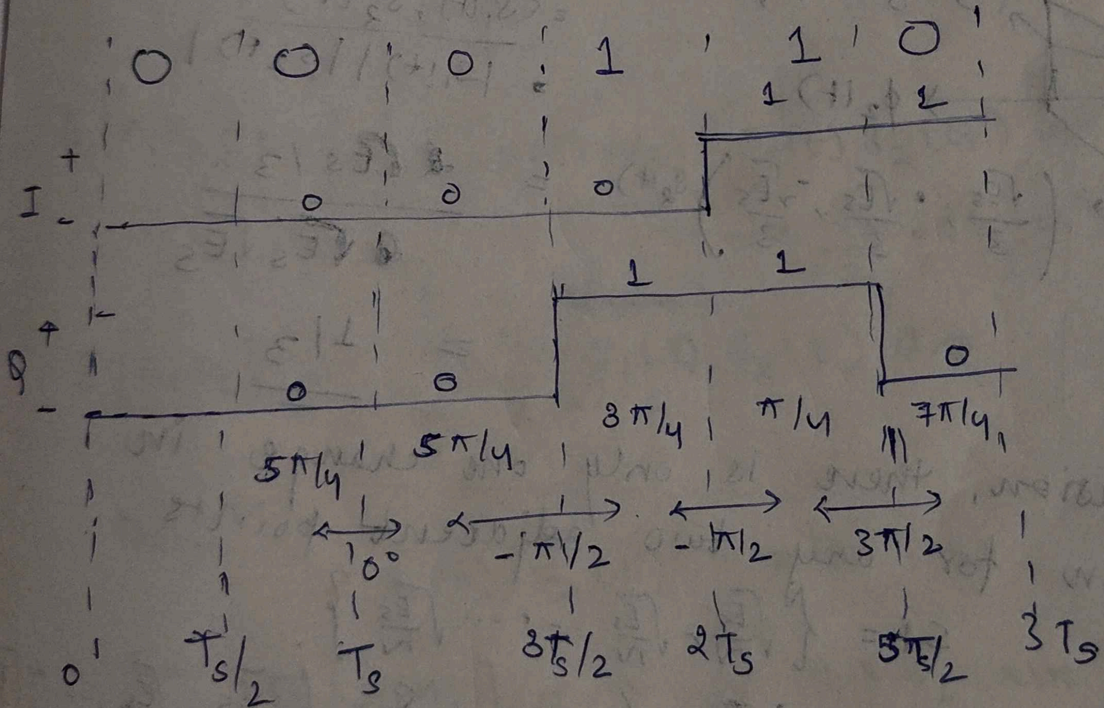
⇒ Modified version of QPSK.



90°, −90°, 180°, −180°

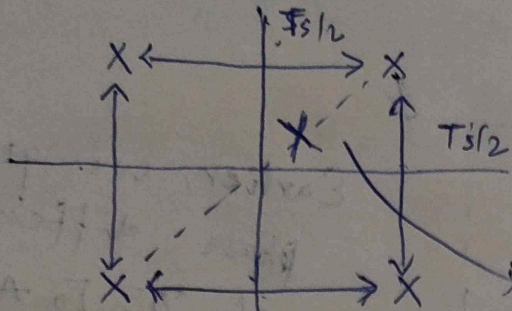


Earlier, we get phase difference of π. To Avoid it we use OPQSK (i.e. shift for delay) by some time).



→ Now, we get only the phase change between $-\pi/2$ & $\pi/2$

→ Phase is changing more frequently



not possible.

In OQPSK, change will be in Ts/2 time.

⇒ BER will be same as QPSK.

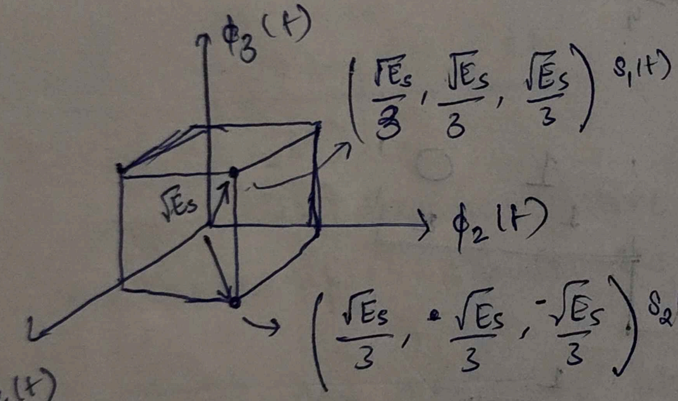
(TUT)

Q Show that corre. coeff of two adjacent signal points corresp. to vertices of N dimen. hypercube with centre at 0, is given by $\gamma = \frac{N-2}{N}$ and

Euclidi. distance $d = 2\sqrt{\frac{E_s}{N}}$

dim = N = 3

Sol^n



$$= \frac{\langle s_1(t), s_2(t) \rangle}{|s_1(t)|\,|s_2(t)|}$$

$$= \frac{E_s/3}{\sqrt{E_s}\sqrt{E_s}}$$

$$= \underline{\underline{1/3}}$$

for N dimension, there is only one change in the coordination for any two adjacent points.

for N dim, $s_1 = \left\{\sqrt{\frac{E_s}{N}}, \sqrt{\frac{E_s}{N}} \cdots\cdots \sqrt{\frac{E_s}{N}}\right\}$.

$s_2 = \left\{\sqrt{\frac{E_s}{N}}, \sqrt{\frac{E_s}{N}} \cdots -\sqrt{\frac{E_s}{N}}\right\}$

$\langle s_1(t), s_2(t) \rangle = (N-2)E_s/N$

$|s_1(t)| = \sqrt{E_s}$.

So,

$$\cos\theta = \frac{(N-2)E_s/N}{\sqrt{E_s}\sqrt{E_s}} = \frac{N-2}{N}$$

Eucladian distance $d = s_1 - s_2$

$= 2\sqrt{\frac{E_s}{N}}$ //

Q $\langle s_1(t), s_2(t) \rangle = 0$

$n(t) \sim \mathcal{N}\left(0, \frac{N_0}{2}\right)$

Prove that $E[n_1 n_2] = 0$

$n_1 = \int_0^T s_1(t)\, n(t)\, dt$

$n_2 = \int_0^T s_2(t)\, n(t)\, dt$

Sol^n

$$n_1 n_2 = \int_0^T \int_0^T s_1(t)\, s_2(\cancel{t})\, n(t)\, n(\cancel{t})\, dt\, du$$

$$E[n_1 n_2] = E\left[\int_0^T \int_0^T s_1(t)\, s_2(u)\, n(t)\, n(u)\, dt\, du\right]$$

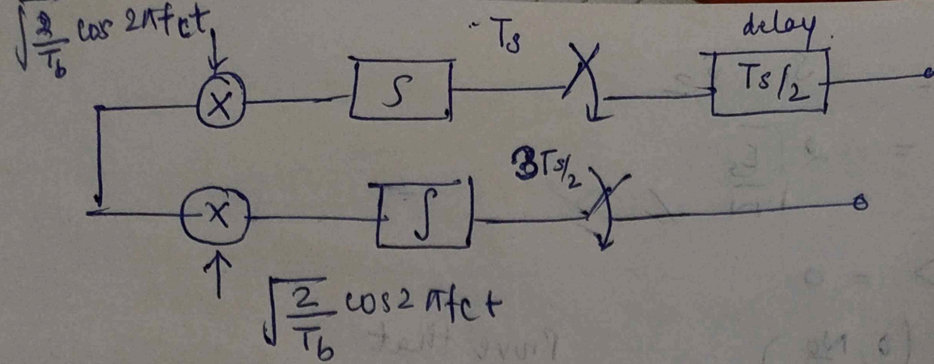
$$E[n_1 n_2] = \int_0^T \int_0^T \underbrace{E[n(t)\, n(u)]}_{\to \frac{N_0}{2}\delta(t-u)} s_1(t)\, s_2(u)\, dt\, du$$

$\cancel{\delta(t-u)}$

$$E[n_1 n_2] = \int_0^T \frac{N_0}{2}\, s_1(t)\, s_2(t)\, dt$$

Now, $\langle s_1(t), s_2(t) \rangle = 0$

So $E[n_1 n_2] = 0 \Rightarrow n_1$ & n_2 are independent



$\theta \rightarrow T_s$.

$T_s/2 \rightarrow T_s + T_s/2$

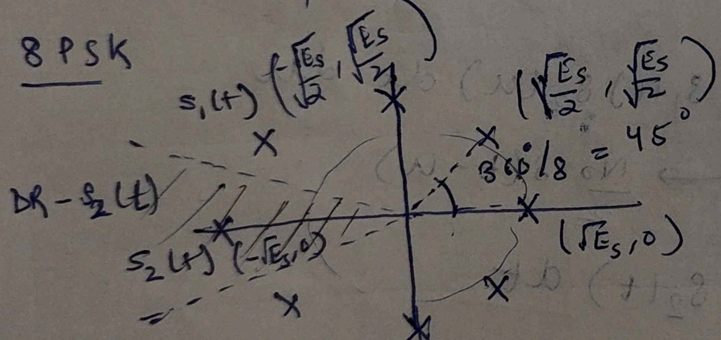
M-ary PSK (M-PSK)

$\Rightarrow$ QPSK $\rightarrow$ 4 symbols $\rightarrow$ 2 bits/symbol $\{90^\circ\}$

$\Rightarrow$ MPSK $\rightarrow$ M symbols $\rightarrow$ log M bits/symbol

QPSK $\rightarrow \frac{360^\circ}{4} = 90^\circ$ MPSK $\rightarrow \frac{360^\circ}{M}$

8 PSK



$T_s = 3T_b = (\log_2 M)\,T_b$
$= 3T_b$

$E_s = 3E_b$

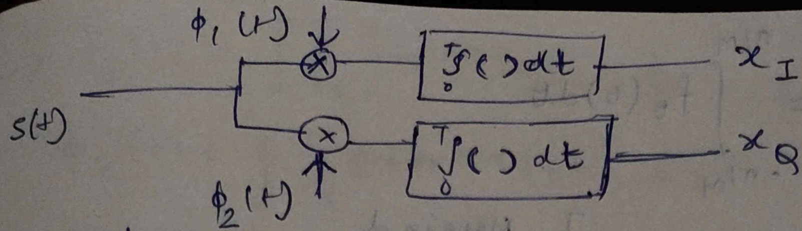
$\boxed{R_s = \frac{R_b}{3}}$ Dim = 2

$$SER = P_e|_{s_2(t)} = 1 - P_c|_{s_2(t)}$$

$$P_c|_{s_2(t)} = 2\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/8} f_{r,\theta}(r,\theta)\,dr\,d\theta$$

$$x = \sqrt{x_I^2 + x_Q^2}$$

$$\theta = \tan^{-1}\left(\frac{x_Q}{x_I}\right)$$



r follows rally
θ uniform distribution

Approx value

$d_{min} = \sqrt{E_S}\sqrt{2-\sqrt{2}}$

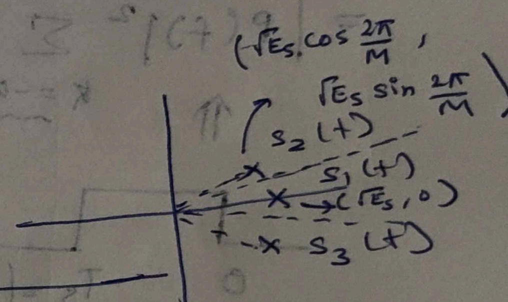
Nearest neighbour = 2.

So,

Approx value of SER $\cong 2Q\left(\dfrac{d_{min}}{\sqrt{2N_0}}\right)$

$\cong 2Q\left(\dfrac{\sqrt{E_S}\sqrt{2-\sqrt{2}}}{\sqrt{2N_0}}\right)$

$$\boxed{P_e|_{s_2(t)} \cong 2Q\left(\sqrt{\frac{E_S(2-\sqrt{2})}{2N_0}}\right)}$$



for M-case.

Nearest neighbour = 2.

$d_{min} = \sqrt{\left(1-\sqrt{E_S}\cos\frac{2\pi}{M}\right)^2 + \left(\sqrt{E_S}\sin\frac{2\pi}{M}\right)^2}$

$= \sqrt{1 + E_S\cos^2\frac{2\pi}{M} - \sqrt{E_S}\cos\frac{2\pi}{M} + E_S\sin^2\frac{2\pi}{M}}$

$= \sqrt{\cancel{1} - \sqrt{E_S}\cos\frac{2\pi}{M} + E_S}$

$= \sqrt{E_S}\sin\pi/M$

$P_e|_{s_1(t)} = 2Q\left(\sqrt{\frac{E_S}{N_0}}\sin\pi/M\right)$

$$\int_{r=0}^{\infty}\int_{-\pi/M}^{\pi/M} f_{r,\theta}(r,\theta)\,dr\,d\theta = \int_{-\pi/M}^{\pi/M} f_\theta(\theta)\,d\theta$$

$$\left.\int_{r=0}^{\infty} f_{r,\theta}(r,\theta)\,dr = f_\theta(\theta)\ \right]$$ Marginal pdf of "θ"

$$\int_{\langle x\rangle} f_{X,Y}(x,y)\,dx = f_Y(y) \qquad \int_{\langle y\rangle} f_{(x,y)}(x,y)\,dy = f_X(x)$$

$$f(x,y) = f_x(x)\cdot f_y(y)$$

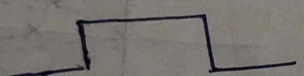
$M\uparrow$; SER $\uparrow$ (QoS)

$M\updownarrow$; BW $\downarrow$

$$= |P(f)|^2 \sum_{k=-\infty}^{\infty} R_b(k)$$

$\Uparrow$

$$\underset{\text{MPSK}}{S_i(t)} = \sqrt{\frac{2E_s}{T_s}}\cos\left(2\pi f_c t + (i-1)\frac{\pi}{M}\right)$$



0 $\qquad T_s = (\log_2 M)T_b$

Null-to-Null BW –

$$\boxed{\text{Band Width of M-PSK} = (BW)_{M\text{-}PSK} = \frac{2}{T_s} = \frac{2}{(\log_2 M)T_b}}$$

DMS → Discrete Memoryless Source.

↳ whatever signal generated does not depend on prev. signal.

QAM (Quadrature Amplitude Modulation).

→ It is a general type of mod^n scheme. However, it is regular shape.

→ M-ary QAM is a two dimensional signalling and it is a generalization of M-ary PAM.
(It has a baseband version of 2D – M-PAM)

$$\phi_I(t) = \sqrt{\frac{2}{T_s}} \cos 2\pi f_c t \quad \Big\} \; 0 \le t \le T_s$$

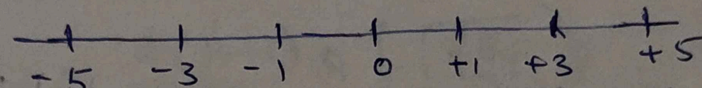
$$\phi_Q(t) = \sqrt{\frac{2}{T_s}} \sin 2\pi f_c t$$

$$s(t) = \sqrt{\frac{2E_s}{T_s}}\, a_K \cos 2\pi f_c t - \sqrt{\frac{2E_s}{T_s}}\, b_K \sin 2\pi f_c t$$

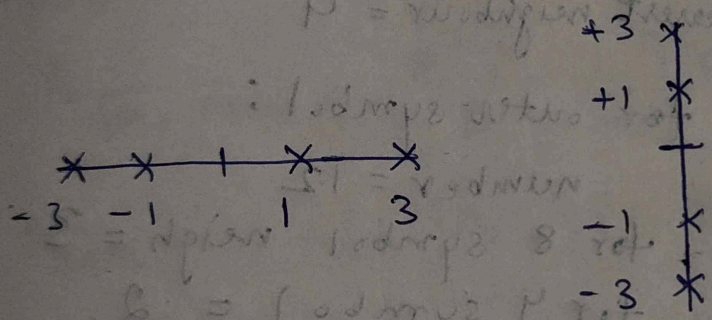
$a_K = 2K - 1 - M_I$ ← In case of M-PAM

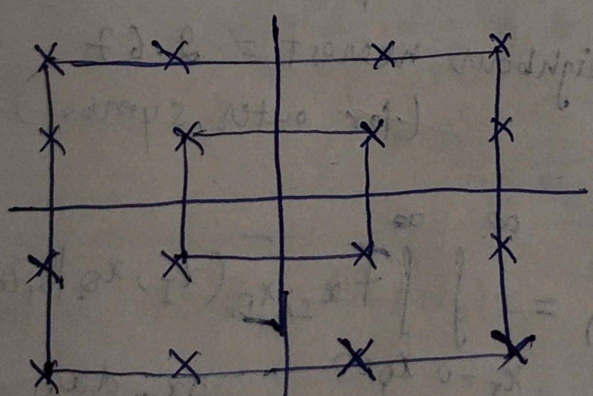
$b_K = 2K - 1 - M_Q$

MPAM



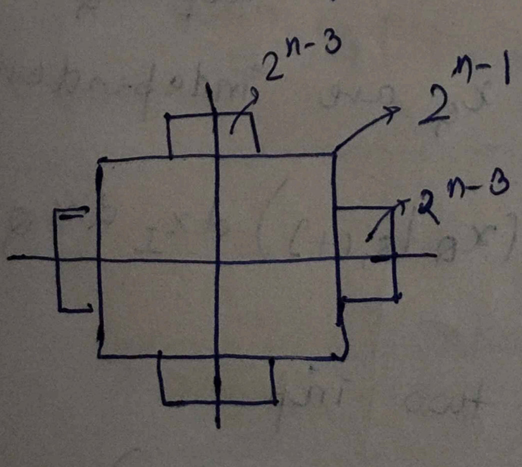
$M = 16 \Rightarrow \sqrt{M} = 4.$



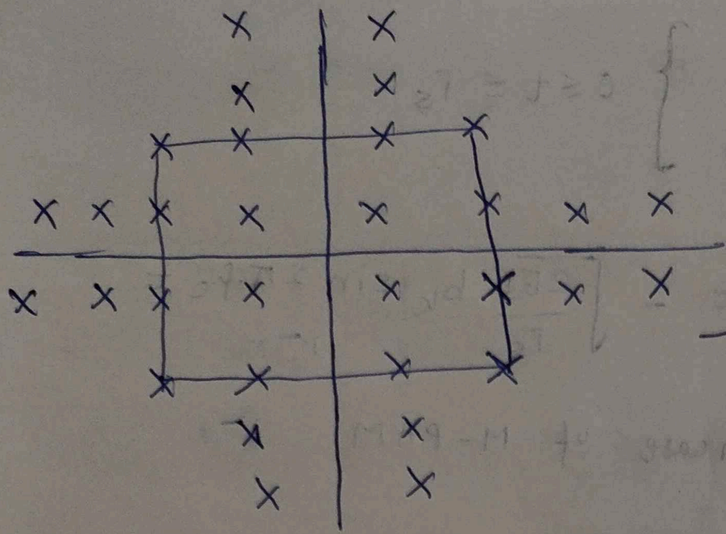


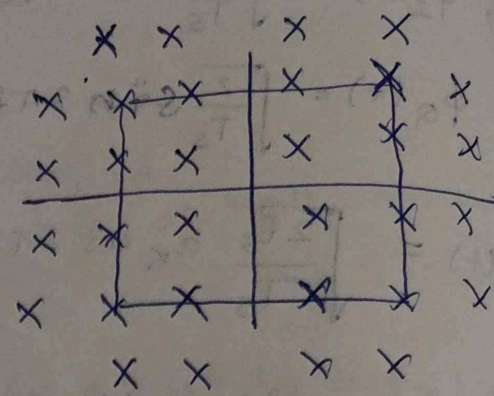
$M = 32,$

bits = 5 bits / symbol



$$\frac{2^n - 2^{n-1}}{4} = 2^{n-3}.$$

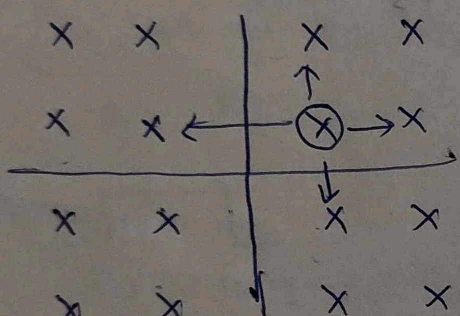




32-QAM constellation

SER Calculation {Square QAM}

P_e, 16-QAM



for inner symbol:

number = 4

nearest neighbour = 4

for outer symbol:

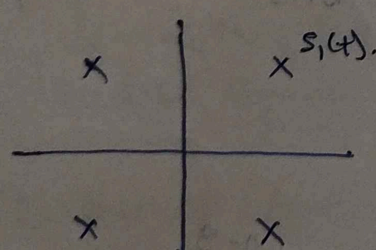
number = 12

for 8 symbol neigh = 3

for 4 symbol = 2.

neighbour nearest = 2.67 (for outer symbol)

Exact SER Analysis (Square-QAM)



$$P_e|_{s_1(t)} = \int_{x_I=0}^{\infty}\int_{x_Q=0}^{\infty} f_{x_I, x_Q}(x_I, x_Q | s_1(t)) \, dx_I \, dx_Q$$

If x_I and x_Q are independent

$$= \int_{x_I=0}^{\infty}\int_{x_Q=0}^{\infty} f_{x_I}(x_I | s_1(t)) \, f_{x_Q}(x_Q | s_1(t)) \, dx_I \, dx_Q$$

If X and Y are two inp.

then, $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$

$$= \int_{x_I=0}^{\infty} f_{x_I}(x_I | s_1(t))\, dx_I \cdot \int_{x_Q=0}^{\infty} f_{x_Q}(x_Q | s_1(t))\, dx_Q$$

$$\underbrace{\quad\quad\quad}_{Pe_I | s_1(t)} \quad \underbrace{\quad\quad\quad}_{Pc_Q | s_1(t)}$$

$$= (1 - Pe_I | s_1(t))(1 - Pe_Q | s_1(t))$$

$$\boxed{M_I = \sqrt{M} = M_Q}$$

⇒ from the discussion of SER M-PAM, the probability of error is

$$Pe_{M-PAM} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

Hence, for $\sqrt{M}$ PAM

$$Pe_{\sqrt{M}-PAM} = 2\left[1 - \frac{1}{\sqrt{M}}\right] Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

$$Pe_{QAM} = 1 - (1 - Pe_I)(1 - Pe_Q)$$

$$\boxed{Pe_{QAM} = -4\left(1 - \frac{1}{\sqrt{M}}\right)^2 Q^2\left(\sqrt{\frac{2E_s}{N_0}}\right) + 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_s}{N_0}}\right)}$$

for high SNR, $E_s/N_0/2$ ↑, $Q^2(\,)$ can be neglected

So,

$$\boxed{Pe_{QAM} \overset{HSNR}{\approx} 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_s}{N_0}}\right)}$$

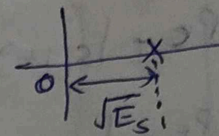
for $M = 16$,

Nearest neighbour = 3

$$\text{Avg. Energy} = \frac{E_s}{M} \sum_{k=1}^{\sqrt{M}} \sum_{m=1}^{\sqrt{M}} a_k^2 + b_m^2 = \frac{2E_s(M-1)}{3} = E_{avg\text{-}MQAM}$$

$$BW = 2/T_s = \frac{2}{(\log_2 M) T_b}$$

$$T_s = (\log_2 M) T_b$$



$$P_{e\,\text{cross-QAM}} \approx \underbrace{4\left(1 - \frac{1}{\sqrt{2M}}\right)}_{\text{Avg. Nearest no. of neighbour}} Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

f_c and T_s ? $\rightarrow$ f_c is a multiple of $1/T_s$.

$$\Rightarrow \boxed{f_c = \frac{k}{T_s}}, \quad k = 0, 1, 2 \ldots \text{ (complete cycle).}$$

$$\langle \phi_1(t), \phi_2(t) \rangle = 0$$

(Tut):

Q output of demodulator $r_k = \sqrt{E_c}\, c_k + n_k$, $k = \phi, 2, \ldots, n$

where $c_k = \pm 1$

$c_1 = [1, 1 \ldots 1]$

$c_2 = [\underbrace{1, 1 \ldots 1}_{w}\ \underbrace{-1 -1 \ldots -1}_{n-w}]$

Noise seq $\rightarrow$ Gaussian noise with AWGN noise $N_0/2$

Variance $= \sigma^2$

Sol^n

$$r|_{c_1} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}_{n \times 1} = \begin{bmatrix} \sqrt{E_c} + n_1 \\ \sqrt{E_c} + n_2 \\ \vdots \\ \sqrt{E_c} + n_n \end{bmatrix} \begin{matrix} \rightarrow N(\sqrt{E_c}, \sigma^2) \\ \rightarrow \\ \\ \rightarrow N(\sqrt{E_c}, \sigma^2) \end{matrix}$$

$$r|_{c_2} = \begin{bmatrix} r_1 \\ \vdots \\ r_w \\ \vdots \\ r_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} \sqrt{E_c} + n_1 \\ \vdots \\ \sqrt{E_c} + n_w \\ -\sqrt{E_c} + n_{w+1} \\ \vdots \\ -\sqrt{E_c} + n_n \end{bmatrix}_{n\times 1} \begin{matrix} \to N(\sqrt{E_c}, \sigma^2) \\ \\ \to N(-\sqrt{E_c}, \sigma^2) \end{matrix}$$

(a) -

<u>ML detector</u>

$$x(t)|_{s_1(t)} = s_1(t) + n(t)$$

$$\Downarrow$$

$$X|_{s_1(t)} = s_1 + n$$

$$f_{X|s_1(t)} \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} f_{X|s_2(t)}$$

$$f_{\underline{r}|c_1}(\quad) \underset{c_2}{\overset{c_1}{\gtrless}} f_{\underline{r}|c_2}(\quad)$$

$f_{\underline{r}|c_1} = ? \Rightarrow$ Joint probability of $\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$ They are independent Hence, will be equal to product

$$f_{\underline{r}|c_1} = \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{2\pi\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) \cdots$$

$$f_{\underline{r}|c_1} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left[-\frac{\sum_{i=1}^{n}(r_i - \sqrt{E_c})^2}{2\sigma^2}\right]$$

$$f_{\underline{r}|c_2} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left[-\frac{\sum_{i=1}^{w}(r_i - \sqrt{E_c})^2 + \sum_{i=w+1}^{n}(r_i + \sqrt{E_c})^2}{2\sigma^2}\right]$$

$$\underline{r}|_{c_1} \underset{c_2}{\gtrless} \underline{r}|_{c_2}$$

$$\cancel{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n} \exp\left[\quad\right] \gtrless \cancel{\left(\frac{1}{2\pi\sigma^2}\right)^n} \exp\left(- \quad + \frac{\sum_{i=W+1}^{n} (x_i+\sqrt{E})^2}{2\sigma^2}\right.$$

Taking ln on both sides.

We get,

$$-\left(\sum_{i=1}^{W} (x_i-\sqrt{E_c})^2 + \sum_{i=W+1}^{n} (x_i-\sqrt{E_c})^2\right) \underset{c_2}{\overset{c_1}{\gtrless}} -\left(\sum_{i=1}^{W} (r_i-\sqrt{E_c})^2 + \sum_{W+1}^{n} (r_i+\sqrt{E_c})^2\right)$$

Now,

$$\sum_{i=W+1}^{n} (r_i-\sqrt{E_c})^2 \underset{c_1}{\overset{c_2}{\gtrless}} \sum_{i=W+1}^{n} (r_i+\sqrt{E_c})^2$$

Now,

$$\sum_{i=W+1}^{n} \left\{(r_i-\sqrt{E_c})^2 - (r_i+\sqrt{E_c})^2\right\} \underset{c_1}{\overset{c_2}{\gtrless}} 0$$

$$\sum_{i=W+1}^{n} -4r_i\sqrt{E_c} \underset{c_1}{\overset{c_2}{\gtrless}} 0$$

$$-4\sqrt{E_c} \sum_{i=W+1}^{n} r_i \underset{c_1}{\overset{c_2}{\gtrless}} 0$$

$$\boxed{\sum_{i=W+1}^{n} r_i \underset{c_2}{\overset{c_1}{\gtrless}} 0} \Rightarrow \text{Sufficient stastics. (Decision Region).}$$

(b). $Pe|_{c_1} = 1 - Pr\left[\sum_{i=w+1}^{n} r_i|_{c_1} > 0\right]$

$\Downarrow$

$N(\sqrt{E_s}, \sigma^2)$

$$Pe|_{c_1} = 1 - \int_{w+1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\sqrt{E_s})^2}{2\sigma^2}}$$

$$Pe|_{c_1} = Q\left(\sqrt{\frac{E_s(n-w)}{\sigma^2}}\right) = Pe|_{c_2}$$

$$\sum_{i=w+1}^{n} r_i|_{c_1} \sim N\left(\sqrt{E_c}(n-w), \sigma^2\right)$$

$$= 1 - Pr[X_1 > 0]$$

$$= 1 - \left(1 - Q\left(\sqrt{\frac{E_c(n-w)}{\sigma^2}}\right)\right)$$

$$= Q\left(\sqrt{\frac{E_c(n-w)}{\sigma^2}}\right)$$

(c). for minimizing error (prop.)

then, $\boxed{w = 0}$ //

8.32 //

Binary Frequency Shift keying (BFSK).

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_i t & ; \; 0 \le t \le T_b \\ 0 & ; \; \text{otherwise} \end{cases}$$

$$\langle s_i(t), s_j(t) \rangle = 0$$

$\Rightarrow$ Two dimensional signaling.

$$f_1 = f_0$$
$$f_2 = 2f_0$$

if $s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t + \theta_i)$

We can take the same frequency.

$$f(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k f_0 t}$$

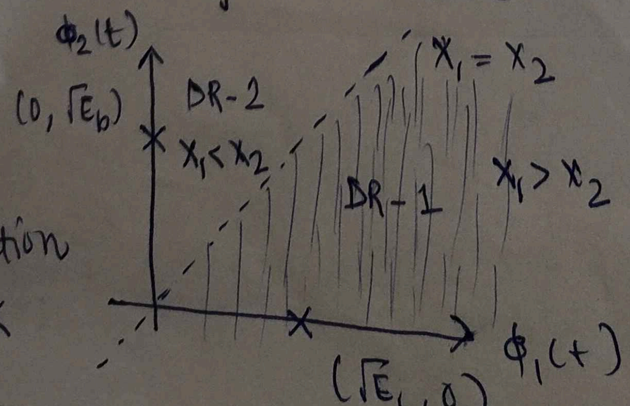
$$C_k = \frac{1}{T} \int_0^T e^{-j2\pi k f_0 t} f(t)\, dt$$

$$= \frac{1}{T} \langle e^{-j2\pi k f_0 t} f(t) \rangle$$

$$\langle \cos 2\pi f_i t, \cos 2\pi f_j t \rangle = 0$$

Chose f_i and f_j such that

$\theta_i = 0$



Constellation of FSK

$$x(t)|_{s_i(t)} = s_i(t) + n(t)$$

$$= \begin{cases} x_1 | s_1(t) = \\ x_2 | s_1(t) = \end{cases}$$

$$= \begin{cases} \sqrt{E_b} + n_1 \\ 0 + n_2 \end{cases}$$

$$\begin{cases} x_1 | s_2(t) = 0 + n_1 \\ x_2 | s_2(t) = \sqrt{E_b} + n_2 \end{cases}$$

$$\boxed{X_1 \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} X_2}$$

$$\boxed{\underbrace{X_1 - X_2}_{Y} \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} 0}$$

CPFSK
$\downarrow$
continuous.

$$s_m(t) = \sqrt{\frac{2E_s}{T_b}} \cos(2\pi f_c t + 2\pi m \Delta f\, t)$$

$$= \sqrt{\frac{2E_s}{T_b}} \cos(2\pi(f_c + m\Delta f)t]$$

$\Downarrow$ freq. deviation

$$Y|_{s_1(t)} \sim N\left(E[X_1|s_1(t) - X_2|s_1(t)],\ Var[X_1|s_1(t) - X_2|s_1(t)]\right)$$

$$\Downarrow$$

$$(\sqrt{E_b} - 0),\ N_0/2$$

$$Y|_{s_2(t)} \sim N(-\sqrt{E_b}, N_0/2)$$

$$P_e|_{FSK} = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{\sqrt{2E_b}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Sunde's FSK.

(Binary FSK).

$\theta_1 \neq \theta_2 \rightarrow$ discontinous - FSK.

$\langle s_m(t), s_n(t) \rangle = 0;\ m \neq n$

$f_m = f_c + m\Delta t$

$f_1 = f_c + \Delta f$

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{2\pi}{2T_b} t\right)$$

$$0 \rightarrow s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t - \frac{\pi}{T_b} t\right);\quad 0 \leq t \leq T_b$$

$$1 \rightarrow s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \frac{\pi}{T_b} t\right);\quad 0 \leq t \leq T_b$$

$$f_1 = f_c - \frac{1}{2T_b};\quad f_2 = f_c + \frac{1}{2T_b}$$

$\langle s_1(t), s_2(t) \rangle = 0.$

HW

$$\Delta f = |f_1 - f_2| = 1/T_b$$

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\left[\cos\frac{\pi t}{T_b}\cos 2\pi f_c t \mp \sin\frac{\pi t}{T_b}\sin 2\pi f_c t\right]$$

$$= \underbrace{\sqrt{E_b}\cos\frac{\pi t}{T_b}}_{\substack{\text{In-phase} \\ \text{(Baseband equ.)}}}\underbrace{\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t}_{\phi_1(t)} \mp \underbrace{\sqrt{E_b}\,\frac{\sin\pi t}{T_b}}_{\substack{\text{Q-phase} \\ \text{(Baseband equ.)}}}\underbrace{\sin 2\pi f_c t}_{\phi_2(t)}$$

PSD of BFSK

PSD of In phase.

$$PSD_{BI} = \frac{E_b}{4}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right]$$

PSD of Q-phase.

$NRZ \times \sin\frac{\pi t}{T_b} \Leftarrow$ kind of BPSK

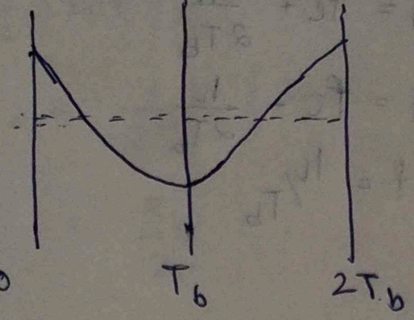
$NRZ \times \cos 2\pi f_c t$

$$PSD_{BQ} = \frac{8E_bT_b\cos^2(\pi T_b f)}{\pi^2(4T_b^2f^2 - 1)}$$

$$PSD_{passband} = \frac{1}{4}\left(\delta(f-f_c) + \delta(f+f_c)\right) * PSD_{BI}$$

$$+ \frac{1}{4}\left(\delta(f-f_c) - \delta(f+f_c)\right) * PSD_{BQ}$$

$$\boxed{T_b(f_1 - f_2) = \text{deviation Ratio}}$$



$\cos \frac{\pi t}{T_b}$.

Time period $= 2T_b$

Phase Transition in FSK.

For Sunde's FSK the phase is a linear phase wrt to 't'.

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b} + \theta(0)\right)$$

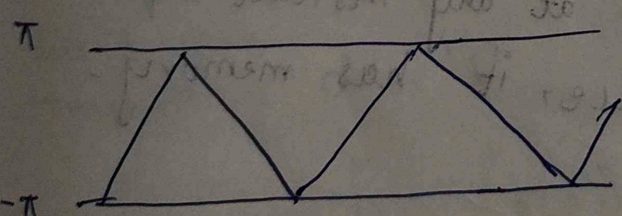
$$\theta(t) = \pm \frac{\pi t}{T_b} + \theta(0)$$

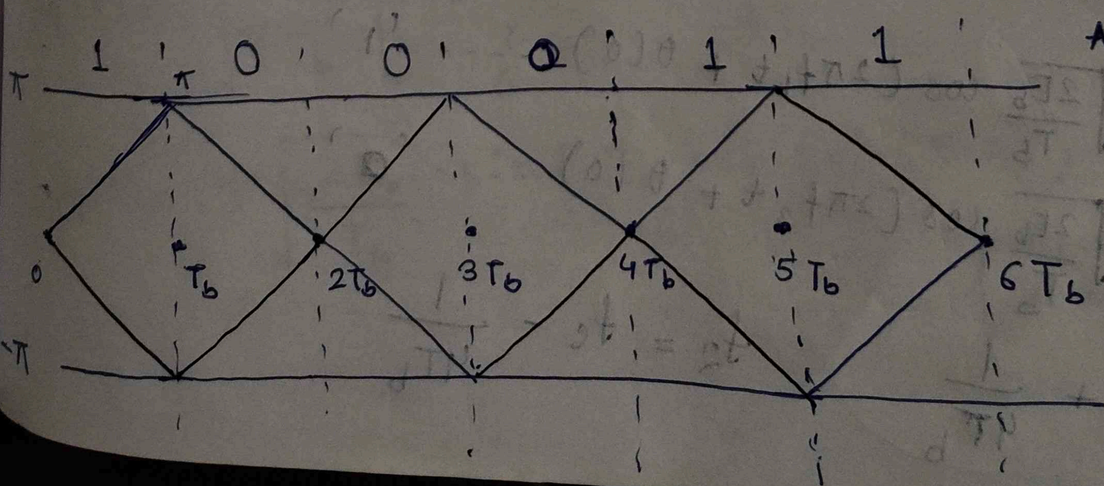
$$\theta(t) - \theta(0) = \pm \frac{\pi t}{T_b}$$

$\because \left(\cos \frac{\pi t}{T_b}\right)$ or $\left(\sin \frac{\pi t}{T_b}\right)$ i.e. baseband signal is continuous in 't'.

$\Downarrow$

continous signal.





At even T_b
$\Downarrow$
0.

At odd T_b
$\Downarrow$
π

has no memory.

$\theta(t) - \theta(0) = \pm \frac{\pi h}{T_b}$

$f_1 = f_c + \frac{h}{2T_b}$

$f_2 = f_c - \frac{h}{2T_b}$

$\Delta f = h/T_b$

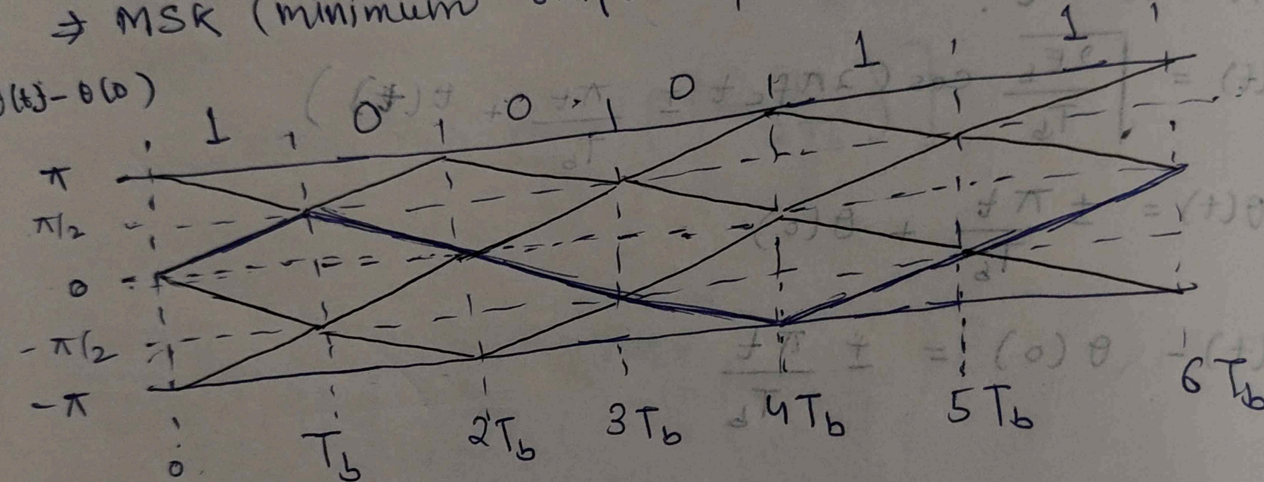
$h = \frac{1}{2}$ {H.W.}

Assign.

For $h = 1/2$, prove that $s_1(t)$ and $s_2(t)$ are still FSK signal.

Δf is minimum frequency deviation for $h = 1/2$.

$\Rightarrow$ MSK (minimum shift key).



$\Rightarrow$ MSK has memory.

At 0, two phase $0, \pi$

At T_b, two phase $-\pi/2, \pi/2$

$\Rightarrow$ There are two possible phase at any instant of multiple of 'T_b' times. Hence, it has memory unlike FSK.

$$s(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t + \theta(0)) & \text{---- '1'} \\ \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t + \theta(0)) & \text{---- '0'} \end{cases}$$

$f_1 = f_c + \frac{1}{4T_b}$ $\qquad$ $f_2 = f_c - \frac{1}{4T_b}$

$f_c = \frac{1}{2}(f_1 + f_2)$

$f_1 - f_2 = \frac{1}{2T_b}$ $\left\{ \begin{array}{l} \text{Incase of FSK} \\ f_1 - f_2 = 1/T_b = R_b \end{array} \right.$

$= \frac{R_b}{2}$

Signal space Diagram of MSK.

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(t)) \cos 2\pi f_c t - \sqrt{\frac{2E_b}{T_b}} \sin(\theta(t)) \sin 2\pi f_c t$$

$$\theta(t) = \theta(0) \pm \frac{\pi}{2T_b}$$

Now)

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \Big[\overbrace{\cos\left(\theta(0) \pm \frac{\pi}{2T_b}\right) \cos 2\pi f_c t}^{S_I \text{ Inphase}} - \underbrace{\sin\left(\theta(0) \pm \frac{\pi}{2T_b}\right) \sin 2\pi f_c t}_{\text{Q-phase } S_Q} \Big]$$

$$s_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos \theta(0) \cos \frac{\pi}{2T_b} t \qquad \left\{ \begin{array}{l} \sin\theta(0) = 0. \\ \text{for } 0, \pi \end{array} \right\}$$

$$= \pm \sqrt{\frac{2E_b}{T_b}} \cos \frac{\pi}{2T_b} t \quad ; \quad \begin{array}{l} \theta(0) = 0 \Rightarrow \text{'+'} \\ \theta(0) = \pi \Rightarrow \text{'-'} \end{array}$$

$\pm$ doesn't mean transmitting '0' or '1'

$$s_Q(t) = \sqrt{\frac{2E_b}{T_b}} \Big[\underbrace{\sin(\theta(0))}_{0} \cos \frac{\pi}{2T_b} t \pm \sin\left(\frac{\pi t}{2T_b}\right) \cos \theta(0) \Big]$$

$$s_Q(t) = \pm \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi}{2T_b} t\right) \cos(\theta(0))$$

$\left\{ \begin{array}{l} \text{depend on } \theta(0) \\ \text{only} \end{array} \right\}$

$\theta(t) = \theta((n-1)T_b) \pm \frac{\pi}{2T_b} t$

DPSK is used for non-coherent BPSK

$\Rightarrow \theta(0) = 0 \Rightarrow \theta(T_b) = \pi/2$; '1'

$\Rightarrow \theta(0) = \pi \Rightarrow \theta(T_b) = \pi/2$; '0'

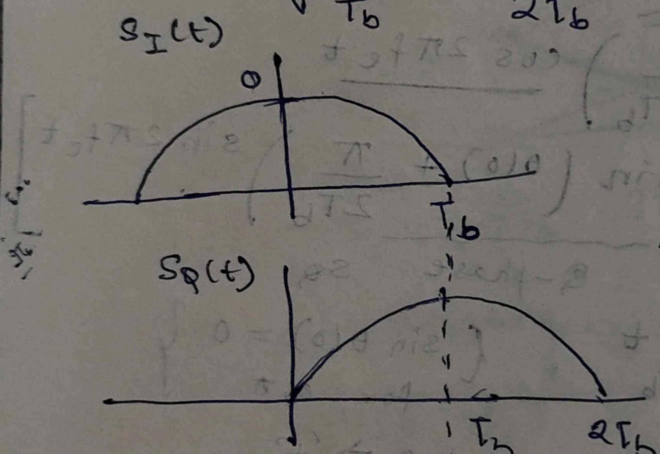
$\Rightarrow \theta(0) = 0 \Rightarrow \theta(T_b) = \pi/2$; '0'

$\Rightarrow \theta(0) = \pi \Rightarrow \theta(T_b) = -\pi/2$; '1'

Assume $\theta = 0$

$s_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos \frac{\pi}{2T_b} t$; period = '$4T_b$'

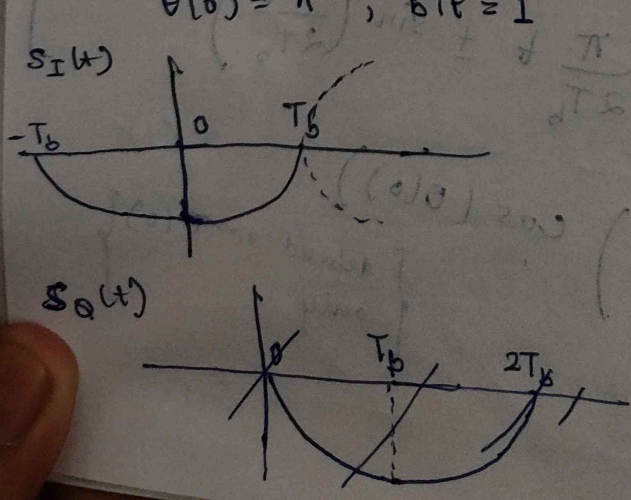
$s_Q(t) = \pm\sqrt{\frac{2E_b}{T_b}} \sin \frac{\pi t}{2T_b}$; period = '$4T_b$'



$\theta(0) = 0$
$b(t=0) = 0$
$b(t = T_b) = 0, 1$
$b(t = -T_b) = 0, 1$

$$\boxed{\theta(nT_b) = \theta((n-1)T_b) \pm \frac{\pi}{2} n}$$

(c) $\theta(0) = \pi$; bit = 1

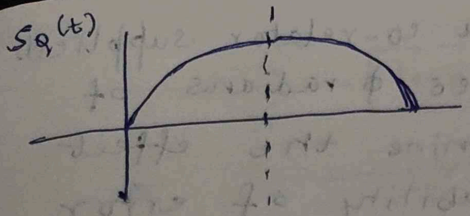


$s_I(t)$ depend on prev bit and doesn't on the current bit

$s_Q(t) = \pm\sqrt{\frac{2E_s}{T_b}} \cos\theta(0) \sin \frac{\pi t}{2T_b}$

$s_Q(t) = (-)(-) \sin \frac{\pi t}{2T_b}$

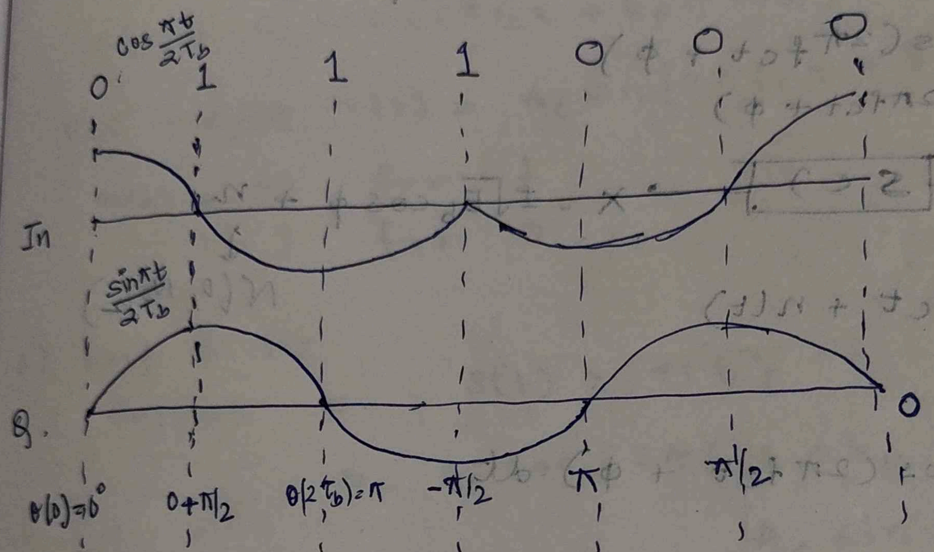
$= \sin \pi t / 2T_b$



$\cos\left(\theta(0) \pm \frac{\pi}{2T_b} t\right)$

In phase Even $T_b \to 0^\circ$ (pos.)

spread $\to 2T_b$



$-\pi/2 \to$ (neg)

$\pi/2 \to$ (pos).

Q-phase

In phase

$\pi \to$ (neg).

$0 \to$ (pos).

$$= \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{2T_b} + \theta(0)\right) ; \quad + \to 1, \quad - \to 0$$

$$= \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \, \underbrace{\cos\left(\theta(0) \pm \frac{\pi t}{2T_b}\right)}_{s_{BI}(t)} - \sqrt{\frac{2E_b}{T_b}} \sin 2\pi f_c t \, \underbrace{\sin\left(\theta(0) \pm \frac{\pi t}{2T_b}\right)}_{s_{BQ}(t)}$$

$$s_{BI}(t) = \sqrt{E_b} \cos\left(\theta(0) \pm \pi t / 2T_b\right)$$

$$s_{BQ}(t) = -\sqrt{E_b} \sin\left(\theta(0) \pm \pi t / 2T_b\right)$$

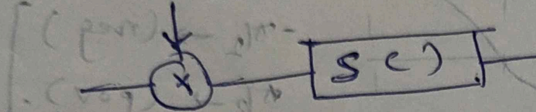
$$s_{BI}(t) = \sqrt{E_b} \cos \theta(0) \cos \frac{\pi t}{2T_b}$$

$$s_{BQ}(t) = \mp \sqrt{E_b} \cos(\theta(0)) \sin \frac{\pi t}{2T_b}$$

Simon (Tut).

6.2 A PSK signal is applied to a co-relator supplied with phase reference that lies ϕ radians of the exact carrier phase. Determine the effect of phase error on av. probability of error.

Ans. $\phi(t) = \cos(2\pi f_c t + \phi)$



$x = \pm\sqrt{E_b}\cos\phi + n$, $\quad n \to N(0, N_0/2)$

$x(t) = \pm\sqrt{\dfrac{2E_b}{T_b}}\cos 2\pi f_c t + n(t)$

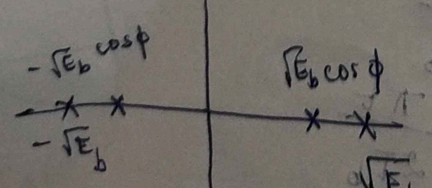
$$n = \sqrt{\frac{2}{T_b}}\int_0^{T_b} n(t)\cos(2\pi f_c t + \phi)\,dt$$

$E[n] = 0$

$$E\left[(n - E[n])^2\right] = \int_0^{T_b}\int_0^{T_b} \underbrace{E[n(t)\,n(u)]}_{\frac{N_0}{2}\delta(t-u)} \cos(2\pi f_c t + \phi)\cos(2\pi f_c u + \phi)\,dt\,du$$

$$= \int_0^{T_b} \frac{N_0}{2}\cos(2\pi f_c t + \phi)\cos(2\pi f_c t + \phi)\,dt$$

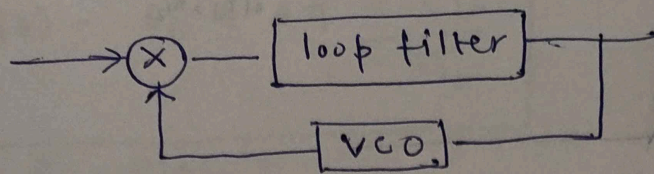
$$= \frac{N_0}{2}\,\frac{2}{T_b}\int_0^{T_b} \underbrace{\cos^2(2\pi f_c t + \phi)\,dt}_{1}$$



$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\cos\phi\right)$$

for $\phi = \pi/2 \Rightarrow$ At zero. (Null effect).

simon
1.3.



$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$

VCO output $r(t) = A_c \sin[2\pi f_c t + \theta(t)]$

$m(t) = +1$ for 1

$= -1$ for 0

(a) Loop filter output $= s(t) \times r(t)$

$= A_c \cos(2\pi f_c t + k_p m(t)) \times A_c \sin(2\pi f_c t + \theta(t))$

$= A_c^2 \cos(2\pi f_c t + k_p m(t)) \sin(2\pi f_c t + \theta(t))$

$= \frac{A_c^2}{2}\left[\sin(4\pi f_c t + k_p m(t) + \theta(t)) - \sin(k_p m(t) - \theta(t))\right]$

$= -\frac{A_c^2}{2} \sin(k_p m(t) - \theta(t))$

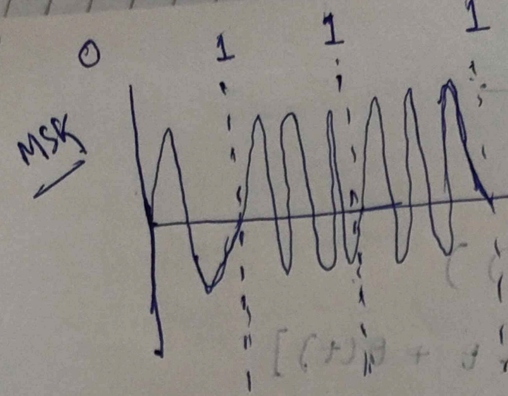
(b). Now, $\theta(t) = 0$

$y(t) = -\frac{A_c^2}{2} \sin(k_p m(t))$ $\quad y(t) \propto m(t)$ h.

Since $m(t) = +1$ or -1

So,

$= -\frac{1}{2} m(t) A_c^2 \sin k_p$



MSK constellation

$$S_{BI}(t) = \sqrt{E_b}\cos\theta(0)\cos\frac{\pi t}{2T_b}$$

$$S_{BQ}(t) = -\sqrt{E_b}\cos\theta(0)\sin\left(\pm\frac{\pi t}{2T_b}\right)$$

$$S_I(t) = \sqrt{E_b}\cos\theta(0)\left[\cos\frac{\pi t}{2T_b}\cos 2\pi f_c t\left(\sqrt{\frac{2}{T_b}}\right)\right] \rightarrow \phi_1(t)$$

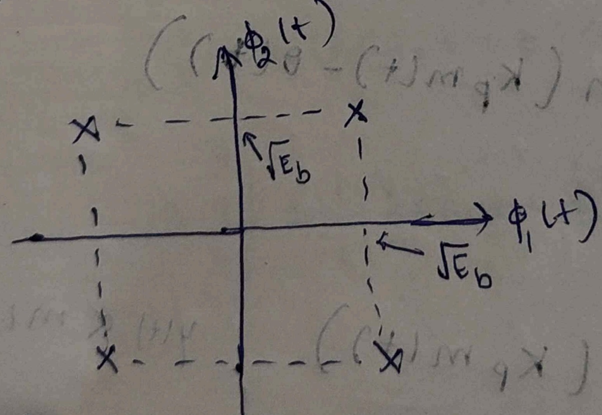
$$S_Q(t) = -\sqrt{E_b}\cos\theta(0)\left[\sin\left(\pm\frac{\pi t}{2T_b}\right)\sqrt{\frac{2}{T_b}}\sin 2\pi f_c t\right] \rightarrow \phi_2(t)$$

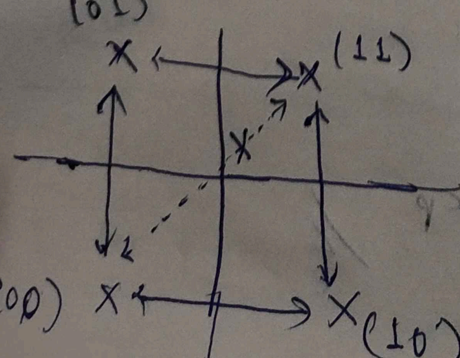
$s_1 = \sqrt{E_b}\cos\theta(0) \quad (t) \rightarrow 1$

$s_2 = \mp\sqrt{E_b}\cos\theta(0) \quad (t) \rightarrow 0$

$0 \le t \le 2T_b$

(H.W.)





$$P_e = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

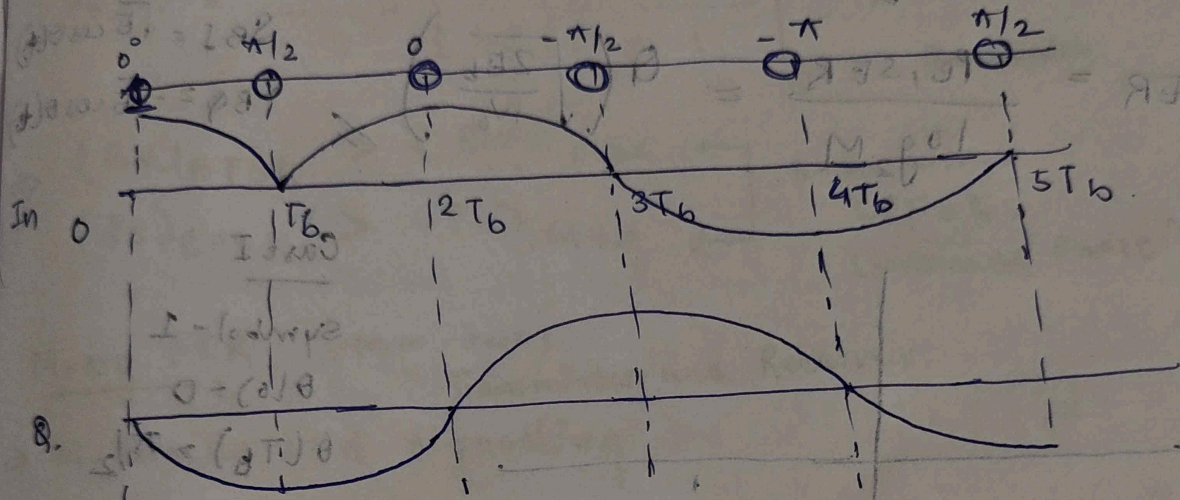
$$\boxed{\theta(t) = \theta_l \pm 2\pi \frac{t - lT_b}{4T_b}}$$

$0 < t \leq T_b$

$lT_b < t < (l+1)T_b$

$\theta_l = \theta(lT_b)$

$S_{BI}(t) = \sqrt{E}\cos\theta(t) = \sqrt{E}\cos\left(\theta(0) + \frac{\pi}{2T_b}t\right)$



$S_{BQ}(t) = -\sqrt{E}\sin\theta(t) = -\sqrt{E}$

At $2T_b$, in case MSK it is continous but in the case of QPSK it may be discontinous -

$S_{BI}(t) = \pm\sqrt{E}\cos\frac{\pi t}{2T_b}$; $-T_b \leq t \leq T_b$

$\cos\theta(0)$

$S_{BQ}(t) = -\sqrt{E}\sin\frac{\pi t}{2T_b}$; $0 \leq t \leq 2T_b$.

$\sin\theta(T_b)$

$$BER = \frac{SER}{\log_2 M.}$$

For 2 time period

$E_b = \frac{E_s}{2} \Rightarrow E_s = 2E_b$

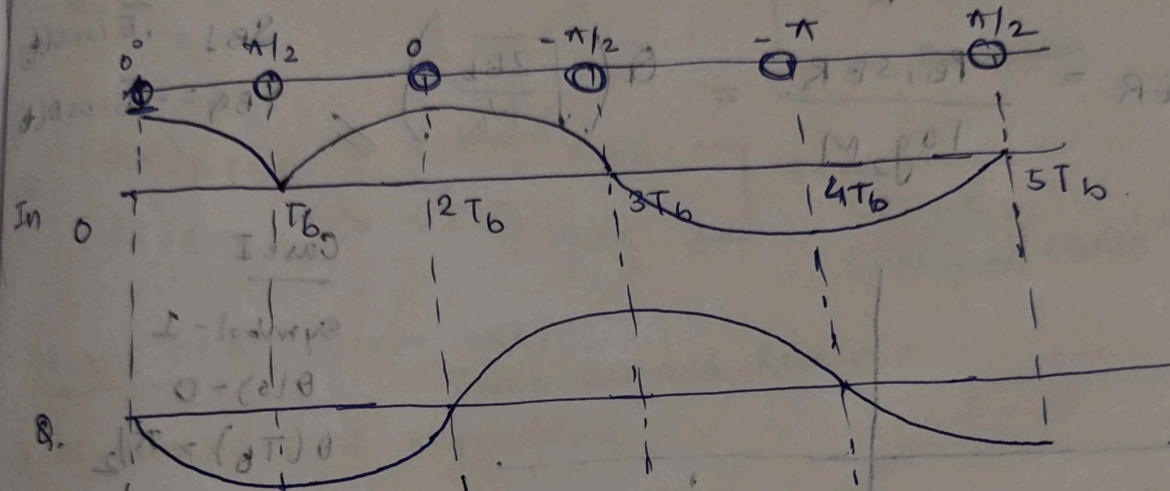
$$\boxed{\theta(t) = \theta_l \pm 2\pi \frac{t - lT_b}{4T_b}}$$

$0 < t \leq T_b$

$lT_b < t < (l+1)T_b$

$\theta_l = \theta(lT_b)$

$$S_{BI}(t) = \sqrt{E}\cos\theta(t) = \sqrt{E}\cos\left(\theta(0) + \frac{\pi}{2T_b}t\right)$$



$$S_{BQ}(t) = -\sqrt{E}\sin\theta(t) = -\sqrt{E}$$

At $2T_b$, in case MSK it is continous but in the case of QPSK it may be discontinous -

$$S_{BI}(t) = \pm\sqrt{E}\cos\frac{\pi t}{2T_b} \; ; \; -T_b \leq t \leq T_b$$

$\cos\theta(0)$

$$S_{BQ}(t) = -\sqrt{E}\sin\frac{\pi t}{2T_b} \; ; \; 0 \leq t \leq 2T_b.$$

$\sin\theta(T_b)$

For 2 time period

$$E_b = \frac{E_s}{2} \Rightarrow E_s = 2E_b$$

$$BER = \frac{SER}{\log_2 M.}$$

$$P_e = 2Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = 2Q\left(\frac{2\sqrt{E_s/2}}{\sqrt{2N_0}}\right).$$

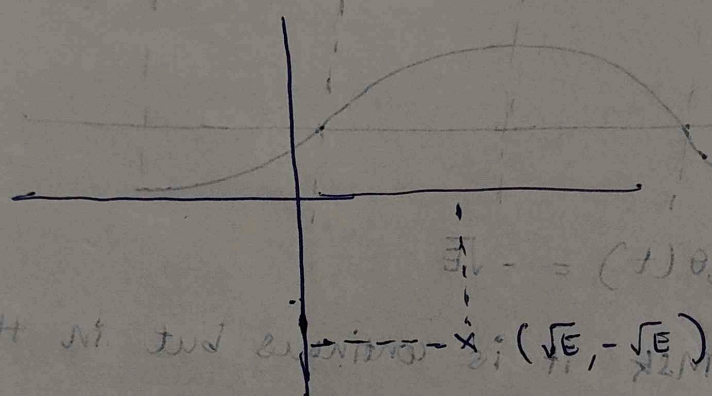
$$= 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) \qquad E_s = 2E_b.$$

$$P_{e_{SER}} = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_{e,BER} = \frac{P_{e,SER}}{\log_2 M} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$S_{BI} = \sqrt{E}\cos\theta(t)$

$S_{BQ} = -\sqrt{E}\cos\theta(t)$



Case I

symbol - 1

$\theta(0) = 0$

$\theta(T_b) = \pi/2$

Case II

$\theta(0) = 0$

$\theta(T_b) = -\pi/2$

PSD of MSK.

$$s(t) = \sqrt{\frac{2E_b}{T_b}}\left[\cos 2\pi f_c t \cos\theta(t) - \sin 2\pi f_c t \sin\theta(t)\right]$$

$$S_{BI}(t) = \sqrt{\frac{2E_b}{T_b}}\ \sqrt{E_b}\cos\theta(t)$$

$$S_{BQ}(t) = -\sqrt{E_b}\sin\theta(t)$$

$$\theta(t) = \theta(0) \pm \frac{\pi t}{2T_b}$$

$+ \longleftrightarrow$ bit '1'

$- \longleftrightarrow$ bit '0'

$$S_{BI}(t) = \sqrt{E_b}\underbrace{\cos\theta(0)}_{NRZ\ \pm}\cos\frac{\pi t}{2T_b} \quad ; \ -T_b \le t \le T_b.$$

$$S_{BQ}(t) = \sqrt{E_b}\underbrace{\sin\theta(0)}_{\pm}\sin\frac{\pi t}{2T_b} \quad ; \ 0 \le t \le 2T_b$$

$$\Rightarrow \left(\frac{|P(f)|^2}{T}\sum_{k=-\infty}^{\infty} R_b(k)\right) * \left(\delta\left(f - \frac{1}{4T_b}\right) + \delta\left(f + \frac{1}{4T_b}\right)\right)$$

$$\text{F.T.}\left(\cos\frac{\pi t}{2T_b}\right) \longleftrightarrow \frac{\delta\left(f-\frac{1}{4T_b}\right)+\delta\left(f+\frac{1}{4T_b}\right)}{2}$$

$$\text{F.T.}\left(\sin\frac{\pi t}{2T_b}\right) \longleftrightarrow \frac{\delta\left(f-\frac{1}{4T_b}\right)-\delta\left(f+\frac{1}{4T_b}\right)}{2j}$$

$$S_B(f) = \frac{32E_b}{\pi^2}\left[\frac{\cos 2\pi T_b f}{16T_b^2f^2-1}\right]^2 \text{ ; MSK.}$$

~~Bandwidth~~

⇒ $(BW)_{BFSK} > (BW)_{MSK}$

$(Pe)_{BFSK} > (Pe)_{MSK}$

Both are CPFSK. (Continous Phase).

M-ary FSK. (from book). Transmitter and Receiver

⇒ M-dimensional signalling

Non-coherent Detection of FSK.

⇒ We assumed that carrier phase at the R_x perfectly known (synced) with phase transmitter local oscillator

↳ We can achieve almost synced phase by employing PLL.

$$s_i(t) = \underbrace{\sqrt{\frac{2\delta E_b}{T_b}}\cos 2\pi f_c t}_{\text{Pilot symbol}} \pm \underbrace{\sqrt{\frac{2E_b(1-\delta)}{T_b}}\cos 2\pi f_c t}_{\text{Actual Msg signal}}$$

⇓

PLL

(To track the phase of carrier by PLL).

⇒ Ideally, tracking the phase perfectly is extremely difficult.

⇒ In case of Non-coherent detection, the exact phase tracking by PLL may not be required.

Q 10.15

$\Delta f = 1/T_b$.

$f_1 = f_c + \Delta f \rightarrow 1/2T_b$

$f_2 = f_c - \Delta f$

$\Delta f = \frac{1}{4T_b} \Rightarrow$ MSK.

$\Delta f = \frac{0.715}{T}$

$f_1 = f_c + \frac{0.715}{T}$

$f_2 = f_c - \frac{0.715}{T}$

$P_e = Q\left(\sqrt{\frac{E_b(1-\rho)}{N_0}}\right)$

ρ = correlation coeff.

$-1 \leq \rho < 1$

BPSK $\rho = -1$.

FSK $\rho = 0$.

$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi(f_c + \Delta f)t)$

$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi(f_c - \Delta f)t)$

$\rho = \frac{\langle s_1(t), s_2(t) \rangle}{|s_1(t)|\,|s_2(t)|}$

$|s_1(t)| = \sqrt{E_b}$

$|s_2(t)| = \sqrt{E_b}$

$\langle s_1(t), s_2(t) \rangle = \frac{2E_b}{T_b} \int_0^{T_b} \cos(2\pi f_c t + 2\pi \Delta f t) \cos(2\pi f_c t - 2\pi \Delta f t)\, dt$

$= \frac{2E_b}{2T_b} \left[\int_0^{T_b} \cos(4\pi \Delta f t)\, dt + \int_0^{T_b} \cos \cancel{4\pi f_c t}\, dt \right]$

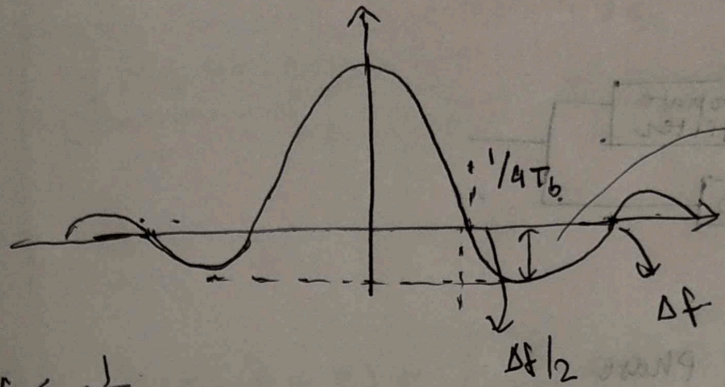
0.

$f_c \rightarrow \frac{n_c}{T_b}$

$= \frac{E_b}{T_b} \frac{\sin(4\pi \Delta f T_b)}{4\pi \Delta f}$

$$\rho = \frac{\frac{\cancel{E_b}}{\cancel{\quad}}\left(\frac{\sin 2\pi (2\Delta f) T_b}{4\pi \Delta f T_b}\right)}{\cancel{E_b}} = \frac{\sin 2\pi(2\Delta f)T_b}{4\pi\Delta f T_b}.$$

$\Downarrow$ sinc



sinc $(4\Delta f T_b)$ = -0.216.

$\Delta f < \frac{1}{4T_b}$

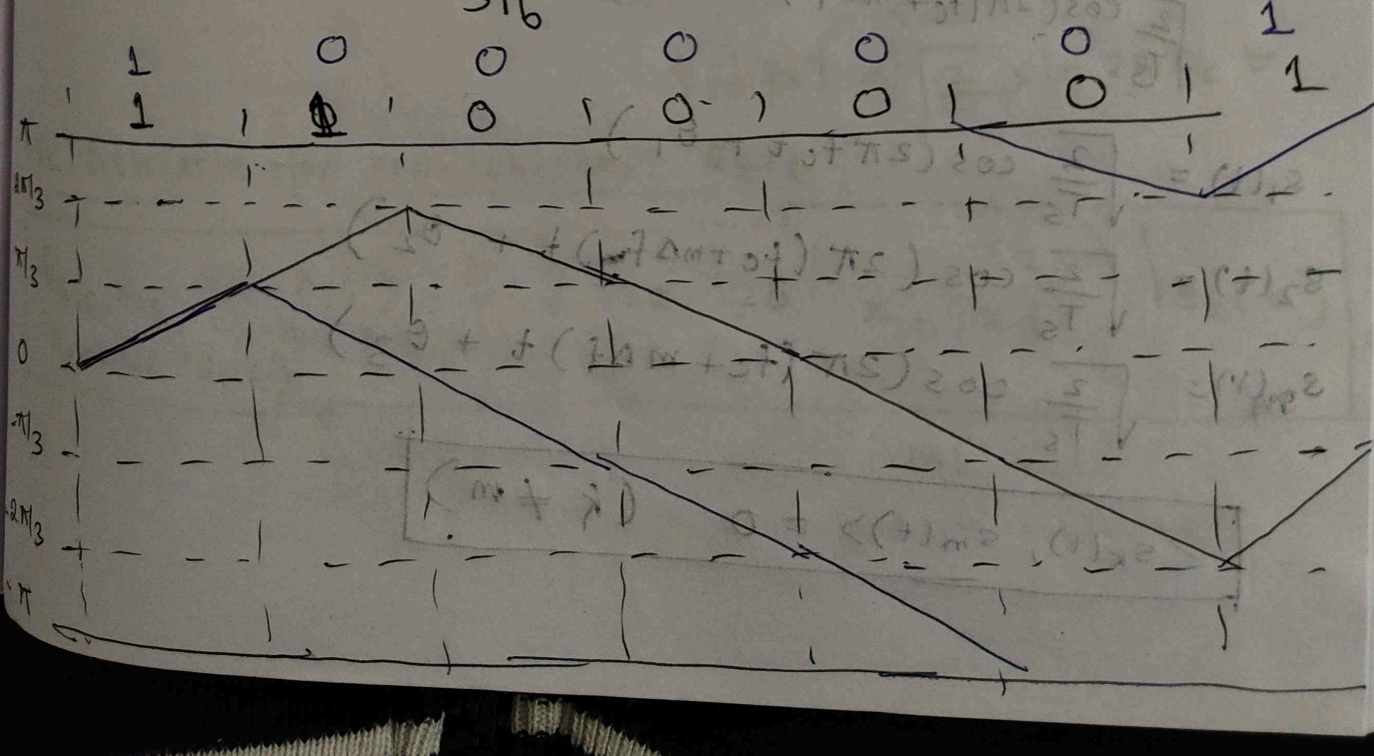
$\Delta f = \frac{1}{4T_b}$ for which $s_1(t)$ & $s_2(t)$ are orthogonal & it is minimum

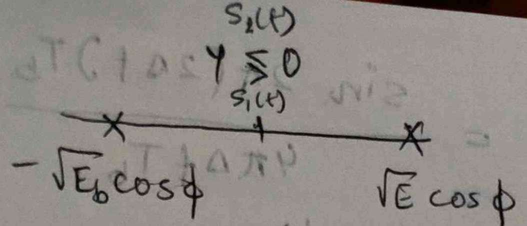
Q. Draw the phase trallis for $h = 1/3$. $\theta(0) = 0^\circ$

$\theta(t) = \theta(0) \pm \frac{\pi h}{T_b}$; $1 \rightarrow$ '+', $0 \rightarrow$ '−'

Sol^n 1 1 0 0 0 0 1

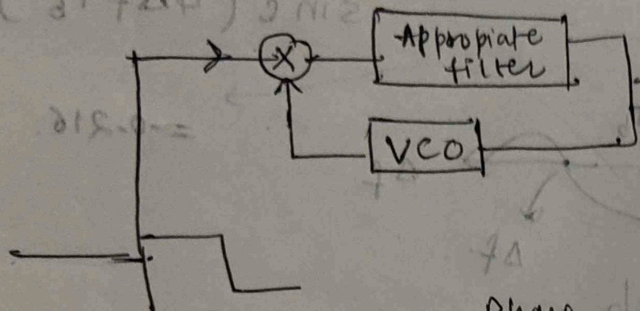
$\Rightarrow$ $\theta(t) = \theta(0) \pm \frac{\pi}{3T_b}$

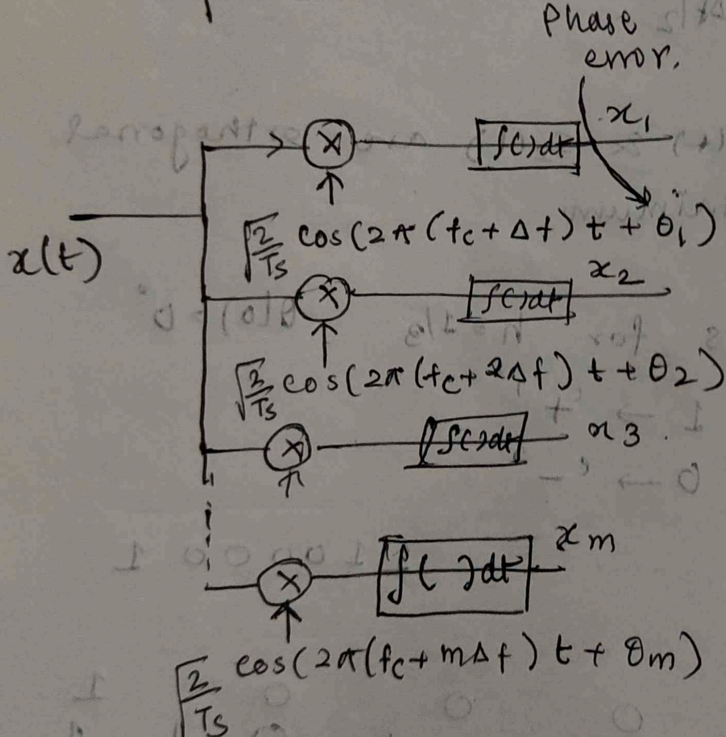




$$Q\left(\sqrt{\frac{2(E_b/2)}{N_0}}\cos\phi\right)$$

M-ary FSK receiver.



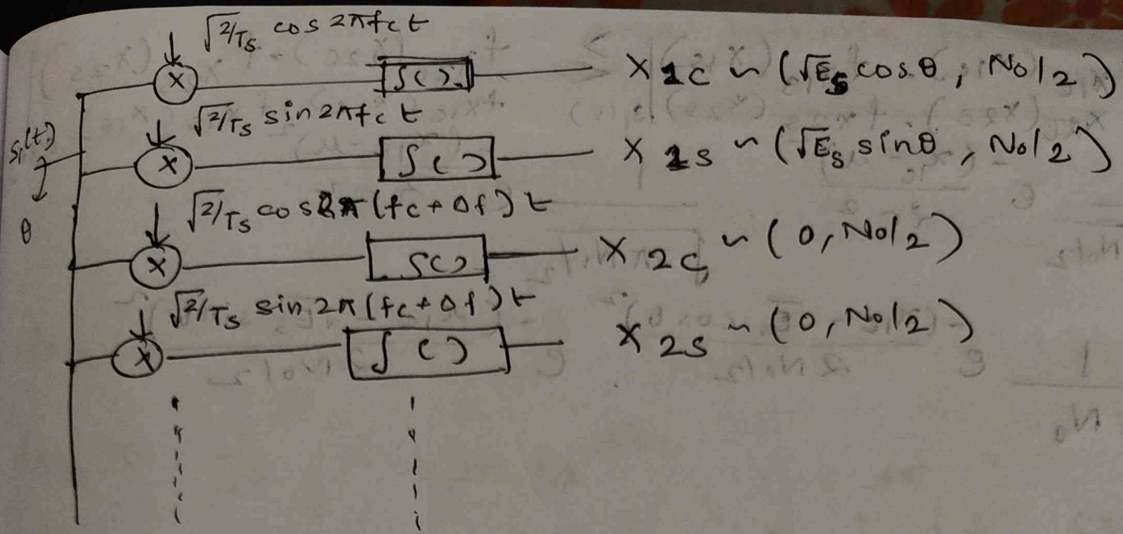


$$s_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t + \theta_1)$$

$$s_2(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi(f_c + m\Delta f)t + \theta_2)$$

$$s_m(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi(f_c + m\Delta f)t + \theta_3)$$

$$\boxed{\langle s_k(t), s_m(t)\rangle = 0 \quad (k \neq m)}$$



$$\sqrt{\frac{2E_s}{T_s}}\cos(2\pi f_c t+\theta) = \sqrt{\frac{2E_s}{T_s}}\cos 2\pi f_c t \cos\theta$$
$$-\sqrt{\frac{2E_s}{T_s}}\sin 2\pi f_c t \sin\theta$$

For the BFSK case.

$s_1(t)$.

$$X_{1c} = \sqrt{E_b}\cos\theta + n_{1c}$$
$$X_{1s} = \sqrt{E_b}\sin\theta + n_{1s}$$
$$X_{2c} = n_{2c}$$
$$X_{2s} = n_{2s}$$

$$\boxed{n(t) = n_c\,\phi_{1c}(t) + n_s\,\phi_{1s}(t)}$$

$$\boxed{n(t) = \sum_{i=-\infty}^{\infty} n_i\,\phi_i(t).}$$

$$\Downarrow$$

$$\phi_{1c}(t) = \sqrt{\frac{2}{T_s}}\cos 2\pi f_c t$$
$$\phi_{1s}(t) = \sqrt{\frac{2}{T_s}}\sin 2\pi f_c t$$

ML Detection for non-coherent case

$$\boxed{f_{X_{1c},X_{1s} \mid X_{2c},X_{2s},\,s_1(t)}(x_{1c},x_{1s} \mid x_{2c},x_{2s}) \gtrless f_{X_{2c},X_{2s} \mid X_{1c},X_{1s},\,s_2(t)}(x_{2c},x_{2s} \mid x_{1c},x_{1s})}$$

ML rule

detection Rule.

$$f_{x_{1c}}(x_{1c}) \cdot f_{x_{1s}}(x_{1s}) \cdot \cancel{f_{x_{2c}}(x_{2c}) \cdot f_{x_{2s}}(x_{2s})}\Big|_{s_1(t)} \gtrless \cancel{f_{x_{2c}}(x_{2c}) \cdot f_{x_{2s}}(x_{2s})}\; f_{x_{1c}}(x_{1c}) \cdot f_{x_{1s}}(x_{1s})\Big|_{s_2(t)}$$

$$\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(x_{1c}-\mu)}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(x_{1s}-\mu)^2}{2\sigma^2}}$$

$$\frac{1}{\pi N_0} e^{-\frac{(x_{1c}-\sqrt{E_s}\cos\theta)^2}{2N_0/2}} \cdot e^{-\frac{(x_{1s}-\sqrt{E_s}\sin\theta)^2}{2N_0/2}}.$$

$\underline{s_2(t).}$

$$x_{1c} = n_{1c}$$
$$x_{1s} = n_{1s}$$
$$x_{2c} = \sqrt{E_b}\cos\theta_2 + n_{2c}$$
$$x_{2s} = \sqrt{E_b}\sin\theta_2 + n_{2s}$$

$\Rightarrow$ θ_1 & θ_2 are random variables

Now,

ML detection.

$$f_{x|s_1(t),\theta_1} > f_{x|s_2(t),\theta_2}$$

$$f_{x|s_1(t),\theta_1} \Rightarrow f_{x|s_1(t)}.$$

$$\int_{\langle x\rangle} f_{y|x} f(x)\, dx = f_y(y)$$

$$f_{x|s_1(t)} = \int_{\langle\theta_1\rangle} f_{x|s_1(t),\theta_1} f_{\theta_1}(\theta_1)\, d\theta_1$$

We will consider the worst case scenario for the phase θ_1 & θ_2 i.e. θ_1 and θ_2 follows uniform distribution

$$f_{\theta_1}(\theta_1) = \frac{1}{2\pi} \left\{ \begin{array}{l} -\pi \le \theta_1 \le \pi \\ \text{or } 0 \le \theta_1 \le 2\pi \end{array} \right\}.$$

$$f_{x|s_1(t)} = \frac{1}{2\pi} \int_0^{2\pi} f_{x|s_1(t), \theta_1} \, d\theta_1$$

$$f_{x|s_2(t)} = \frac{1}{2\pi} \int_0^{2\pi} f_{x|s_2(t), \theta_2} \, d\theta_2$$

$$f_{x|s_1(t)} = f_{x_{2c}}(x_{2c}) \cdot f_{x_{2s}}(x_{2s}) \frac{1}{2\pi} \int_0^{2\pi} f_{x_{1c}}(x_{1c}) f_{x_{1s}}(x_{1s}) \, d\theta_1$$

$$f_{x|s_2(t)} = f_{x_{1c}}(x_{1c}) \cdot f_{x_{1s}}(x_{1s}) \frac{1}{2\pi} \int_0^{2\pi} f_{x_{2c}}(x_{2c}) f_{x_{2s}}(x_{2s}) \, d\theta_2$$

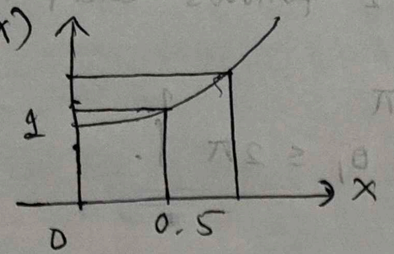
So

$$f_{x|s_1(t), \theta_1(t)} = \left(\frac{1}{\sqrt{2\pi \frac{N_0}{2}}} \right)^4 \operatorname{Exp}\left[- \frac{(x_{1c} - \sqrt{E_b}\cos\theta_1)^2 + (x_{1s} - \sqrt{E_b}\sin\theta_1)^2 + x_{2c}^2 + x_{2s}^2}{2 N_0/2} \right]$$

$$f_{x|s_1(t)} = (\quad)^4 \operatorname{Exp}\left[- \frac{x_{2c}^2 + x_{2s}^2 + x_{1c}^2 + x_{1s}^2 + E_b}{2 N_0/2} \right] \underbrace{\frac{1}{2\pi} \int_0^{2\pi} \operatorname{Exp}\left[- \frac{-(2x_{1c}\sqrt{E_b}\cos\theta_1 + 2x_{1s}\sqrt{E_b}\sin\theta_1)}{2 N_0/2} \right] d\theta_1}_{J_1}$$

$$J_1 = \frac{1}{2\pi} \int_0^{2\pi} \exp\left[\frac{2\sqrt{E_b}(x_{1c}\cos\theta_1 + x_{1s}\sin\theta_1)}{N_0} \right] d\theta_1$$

$I_0 \rightarrow$ Modified Bessel function of order zero.



$$f_{x|s_2(t)} = \left(-/-\right) \mathrm{Exp}\left[-\frac{x_{2c}^2 + x_{2s}^2 + x_{1c}^2 + x_{1s}^2 + E_b}{2N_0/2}\right].$$

$$\underbrace{\frac{1}{2\pi}\int_0^{2\pi} \mathrm{Exp}\left[-\frac{-\left(2x_{2c}\sqrt{E_b}\cos\theta_2 + 2x_{2s}\sqrt{E_b}\sin\theta_2\right)}{2N_0/2}\right] d\theta_2}_{I_0\left(\sqrt{\frac{E_b(x_{2c}^2 + x_{2s}^2)}{N_0}}\right)}$$

Apply ML Rule.

$$f_{x|s_1(t)} \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} f_{x|s_2(t)}$$

$$I_0\left[\sqrt{\frac{E_b(x_{1c}^2 + x_{1s}^2)}{N_0/2}}\right] \gtrless I_0\left[\sqrt{\frac{E_b(x_{2c}^2 + x_{2s}^2)}{N_0/2}}\right]$$

$f(x_1) \gtrless f(x_2) \Rightarrow x_1 \gtrless x_2$

$\leftarrow$ monotonic increasing.

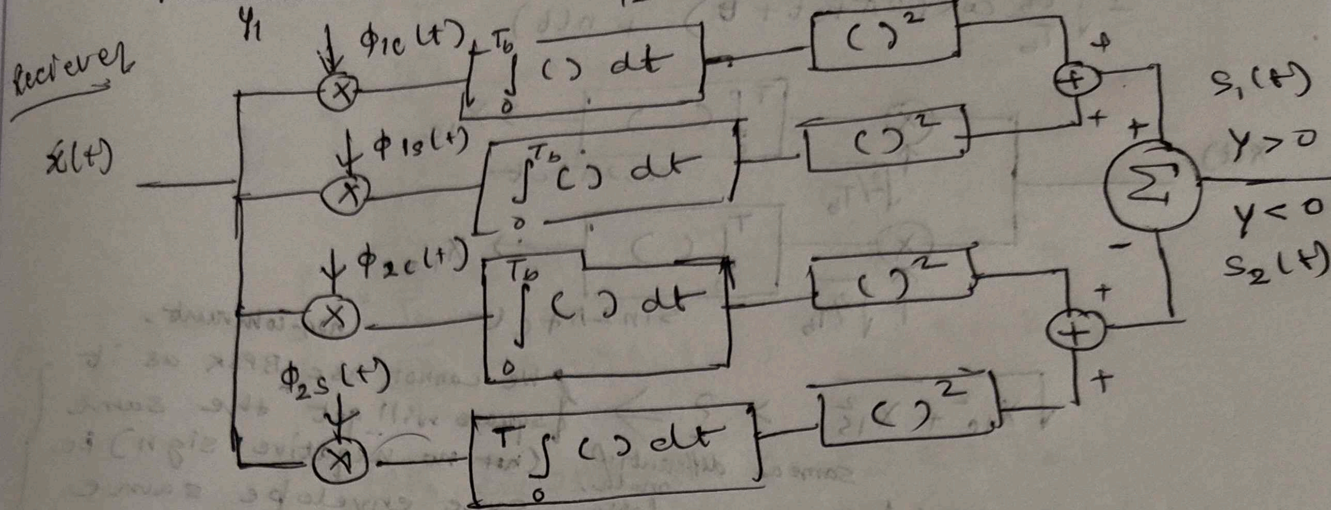
sufficient statistics

$$\boxed{\underbrace{\sqrt{x_{1c}^2 + x_{1s}^2}}_{\text{Envelope of } X_1} \gtrless \underbrace{\sqrt{x_{2c}^2 + x_{2s}^2}}_{\text{Envelope of } X_2}}$$

Best optical Receiver

Non coherent decision Region.

$$\boxed{\underbrace{X_{1c}^2 + X_{1s}^2}_{y_1} \gtrless \underbrace{X_{2c}^2 + X_{2s}^2}_{y_2}}$$



$y_1 = X_{1c}^2 + X_{1s}^2 \Rightarrow f_{r,\theta}(r,\theta) =$

$y_2 = X_{2c}^2 + X_{2s}^2$

Rice Distribution

$f_{y_1}(y_1)$ Rayleigh Distribution.

$f_{y_2}(y_2)$

$$(X_{1c}^2 + X_{1s}^2) - (X_{2c}^2 + X_{2s}^2) \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} 0$$

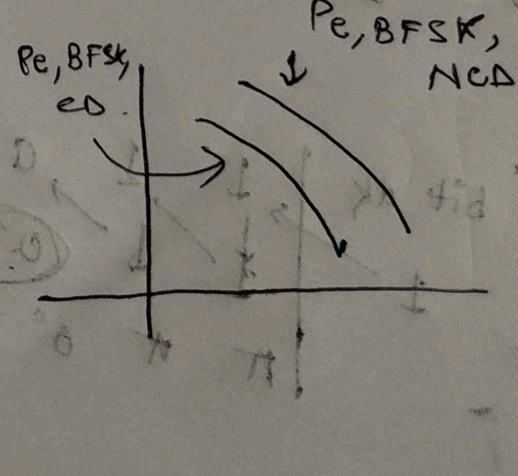
$s_1(t), \Rightarrow X_{1c}|_{\theta} = \sqrt{E_b}\cos\theta + n_1 \;;\; \sim N(\sqrt{E_b}\cos\theta, N_0/2)$

$X_{1s}|_{\theta} = \sqrt{E_b}\sin\theta + n_1 \;;\; \sim N(\sqrt{E_b}\sin\theta, N_0/2)$

$P_e|_{BFSK} = \frac{1}{2}\exp\left[-E_b/2N_0\right]$; NCD

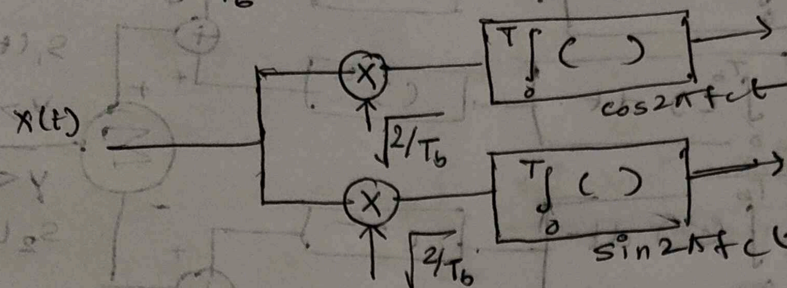
$P_{e,BFSK} = Q\left(\sqrt{\frac{E_b}{\cancel{2}N_0}}\right)$; CD.

$$\boxed{P_{e,BFSK,NCD} > P_{e,BFSK,CD}.}$$



BPSK : $\pm\sqrt{\frac{E_b}{T_b}}\cos 2\pi f_c t$

$x(t) = \pm\sqrt{\frac{E_b}{T_b}}\cos(2\pi f_c t + \theta) + n(t)$

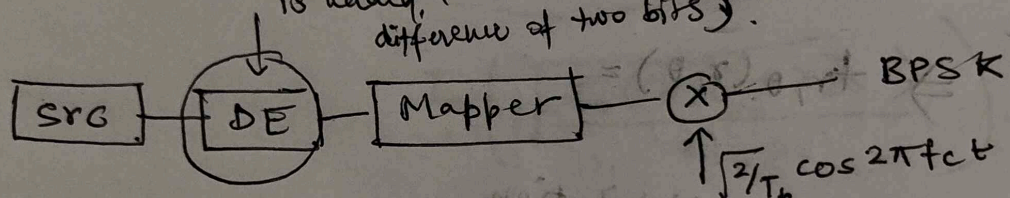


non-coherent.

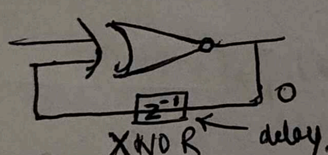
$\sqrt{X_{1c}^2 + X_{1s}^2} > ?$ $\Rightarrow$ { We cannot use BPSK as it square will get the same (~~not~~ no negative sign) i.e. same envelope same in QPSK. }

same as differential mode.

is added (we are now taking difference of two bits).



DE



(or we can use XOR).

DPSK (Differentially Encoded PSK).

$t=0$

1 1 0 0 1 0

d_K	d_{K-1}	output
0	0	1
0	1	0
1	0	0
1	1	1.

π $t=0$

1 1 1 0 1 1 → $0°$

$0°$ $0°$ $0°$ BPSK. $0°$

for this   for this

Input bit d_K		1	1	0	1	1	1	0	0	1	1
d_K	1	1	1	0	0	0	0	1	0	0	0
phase		π	π	$0°$	$0°$	$0°$	$0°$	π	$0°$	$0°$	$0°$

1 $\quad s_2(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ 0 \le t \le T_b \\ \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ T_b \le t \le 2T_b \end{cases}$

$$= \begin{cases} -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ 0 \le t \le T_b \\ -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ T_b \le t \le 2T_b \end{cases}$$

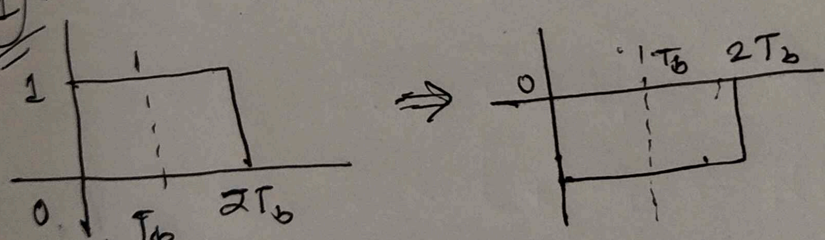
0 $\quad s_1(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ 0 \le t \le T_b \\ -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ T_b \le t \le 2T_b \end{cases}$

$$= \begin{cases} -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ 0 \le t \le T_b \\ \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & ; \ T_b \le t \le 2T_b \end{cases}$$

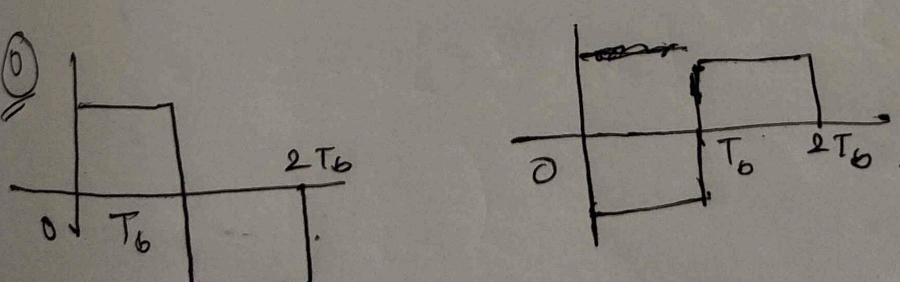
for 1 $\rightarrow$ same either 0 0 or 1 1.
for 0 $\rightarrow$ different 0 1 or 1 0.

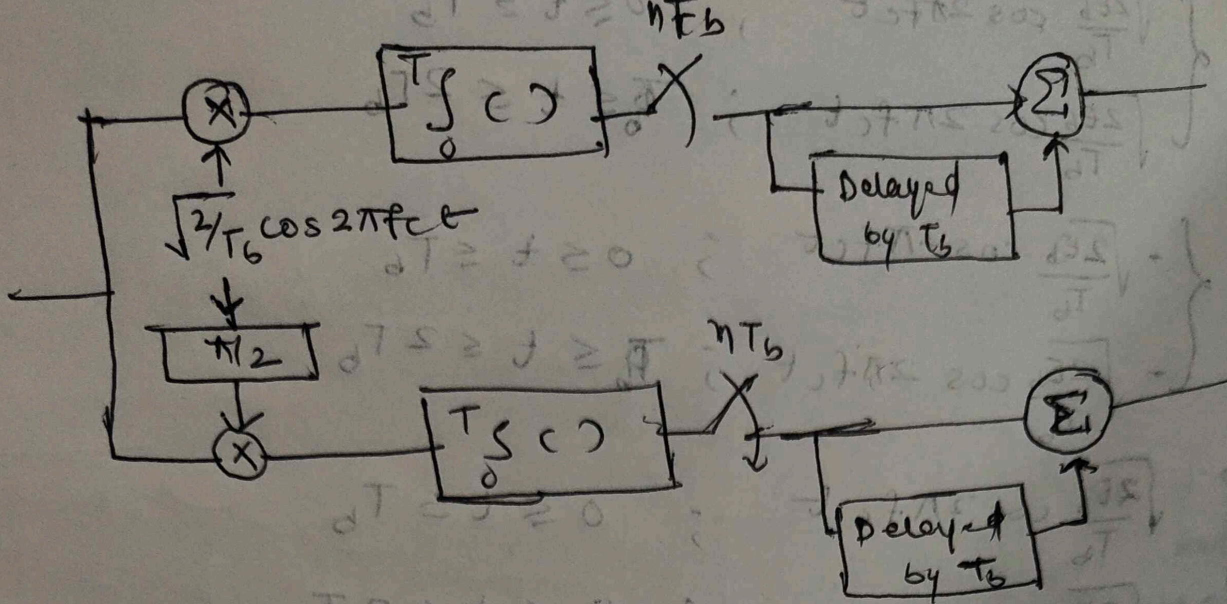
We are using memory (hence change of error occurrence).

1



0




nTb
Delayed by Tb
√2/Tb cos 2πfc t
π/2
nTb
Delayed by Tb