Tutorial 4

Find the magnitudes of **D** and **P** for a dielectric material in which $E = 0.15$ MV/m and $\chi_e = 4.25$.

Since $\varepsilon_r = \chi_e + 1 = 5.25$,

$$D = \varepsilon_0 \varepsilon_r E = \frac{10^{-9}}{36\pi}(5.25)(0.15 \times 10^6)$$

$$= 6.96\ \mu\text{C/m}^2$$

$$P = \chi_e \varepsilon_0 E = \frac{10^{-9}}{36\pi}(4.25)(0.15 \times 10^6)$$

$$= 5.64\ \mu\text{C/m}^2$$

7.1 Find the polarization **P** in a dielectric material with $\varepsilon_r = 2.8$ if $\mathbf{D} = 3.0 \times 10^{-7}$ **a** C/m².

Assuming the material to be homogeneous and isotropic,

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

Since $\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$ and $\chi_r = \varepsilon_r - 1$,

$$\mathbf{P} = \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)\mathbf{D} = 1.93 \times 10^{-7}\mathbf{a}\ \mathrm{C/m^2}$$

7.2 Determine the value of **E** in a material for which the electric susceptibility is 3.5 and $\mathbf{P} = 2.3 \times 10^{-7}\mathbf{a}\ \mathrm{C/m^2}$.

Assuming that **P** and **E** are in the same direction,

$$\mathbf{E} = \frac{1}{\chi_e \varepsilon_0}\mathbf{P} = 7.42 \times 10^3\mathbf{a}\ \mathrm{V/m}$$

7.3 Two point charges in a dielectric medium where $\varepsilon_r = 5.2$ interact with a force of 8.6×10^{-3} N. What force could be expected if the charges were in free space?

Coulomb's law, $F = Q_1Q_2/(4\pi\varepsilon_0\varepsilon_r d^2)$ shows that the force is inversely proportional to ε_r. In free space the force will have its maximum value.

$$F_{\max} = \frac{5.2}{1}(8.6 \times 10^{-3}) = 4.47 \times 10^{-2}$$

7.4 Region 1, defined by $x < 0$, is free space while region 2, $x > 0$, is a dielectric

A dielectric free-space interface has the equation $3x + 2y + z = 12$ m. The origin side of the interface has $\varepsilon_{r1} = 3.0$ and $E_1 = 2\mathbf{a}_x + 5\mathbf{a}_z$ V/m. Find $\mathbf{E}_2$.

The interface is indicated in Fig. 7.11 by its intersections with the axes. The unit normal vector on the free-space side is

$$\mathbf{a}_n = \frac{3\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{\sqrt{14}}$$

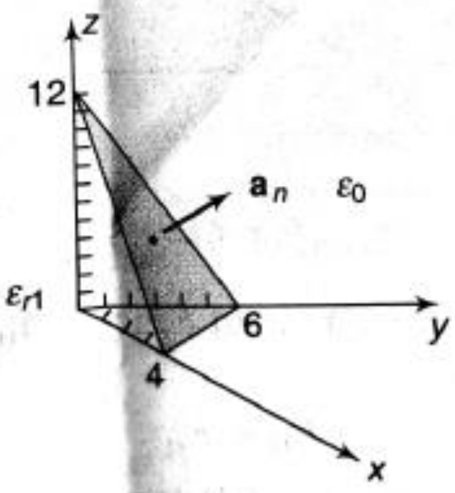

Fig. 7.11

The projection of $\mathbf{E}_1$ on $\mathbf{a}_n$ is the normal component of $\mathbf{E}$ at the interface.

$$\mathbf{E}_1 \cdot \mathbf{a}_n = \frac{11}{\sqrt{14}}$$

Then

$$\mathbf{E}_{n1} = \frac{11}{\sqrt{14}}\mathbf{a}_n$$
$$= 2.36\mathbf{a}_x + 1.57\mathbf{a}_y + 0.79\mathbf{a}_z$$
$$\mathbf{E}_{t1} = \mathbf{E}_1 - \mathbf{E}_{n1}$$
$$= -0.36\mathbf{a}_x - 1.57\mathbf{a}_y + 4.21\mathbf{a}_z = \mathbf{E}_{t2}$$
$$\mathbf{D}_{n1} = \varepsilon_0\varepsilon_{r1}\mathbf{E}_{n1}$$
$$= \varepsilon_0(7.08\mathbf{a}_x + 4.71\mathbf{a}_y + 2.37\mathbf{a}_z)$$
$$= \mathbf{D}_{n2}$$
$$\mathbf{E}_{n2} = \frac{1}{\varepsilon_0}\mathbf{D}_{n2}$$
$$= 7.08\mathbf{a}_x + 4.71\mathbf{a}_y + 2.37\mathbf{a}_z$$

and finally

$$\mathbf{E}_2 = \mathbf{E}_{n2} + \mathbf{E}_{t2}$$
$$= 6.72\mathbf{a}_x + 3.14\mathbf{a}_y + 6.58\mathbf{a}_z \text{ V/m}$$

(a) Show that the capacitor of Fig. 7.4(a) has capacitance

$$C_{eq} = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{d} + \frac{\varepsilon_0 \varepsilon_{r2} A_2}{d} = C_1 + C_2$$

(b) Show that the capacitor of Fig. 7.5(a) had reciprocal capacitance

$$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 \varepsilon_{r1} A/d_1} + \frac{1}{\varepsilon_0 \varepsilon_{r2} A/d_2}$$

$$= \frac{1}{C_1} + \frac{1}{C_2}$$

(a) Because the voltage difference V is common to the two dielectrics,

$$\mathbf{E}_1 = \mathbf{E}_2 = \frac{V}{d}\mathbf{a}_n \quad \text{and}$$

$$\frac{\mathbf{D}_1}{\varepsilon_0 \varepsilon_{r1}} = \frac{\mathbf{D}_2}{\varepsilon_0 \varepsilon_{r2}} = \frac{V}{d}\mathbf{a}_n$$

where $\mathbf{a}_n$ is the downward normal to the upper plate. Since $D_n = \rho_s$, the charge densities on the two sections of the upper plate are

$$\rho_{s1} = \frac{V}{d}\varepsilon_0\varepsilon_{r1} \qquad \rho_{s2} = \frac{V}{d}\varepsilon_0\varepsilon_{r2}$$

and the total charge is

$$Q = \rho_{s1}A_1 + \rho_{s2}A_2$$

$$= V\left(\frac{\varepsilon_0\varepsilon_{r1}A_1}{d} + \frac{\varepsilon_0\varepsilon_{r2}A_2}{d}\right)$$

Thus, the capacitance of the system, $C_{eq} = Q/V$, has the asserted form.

(b) [illegible]

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{d} \qquad C_2 = \frac{\varepsilon_0 \varepsilon_{r2} A_2}{d}$$

$$C_{eq} = C_1 + C_2 = \frac{\varepsilon_0}{d}(\varepsilon_{r1}A_1 + \varepsilon_{r2}A_2)$$

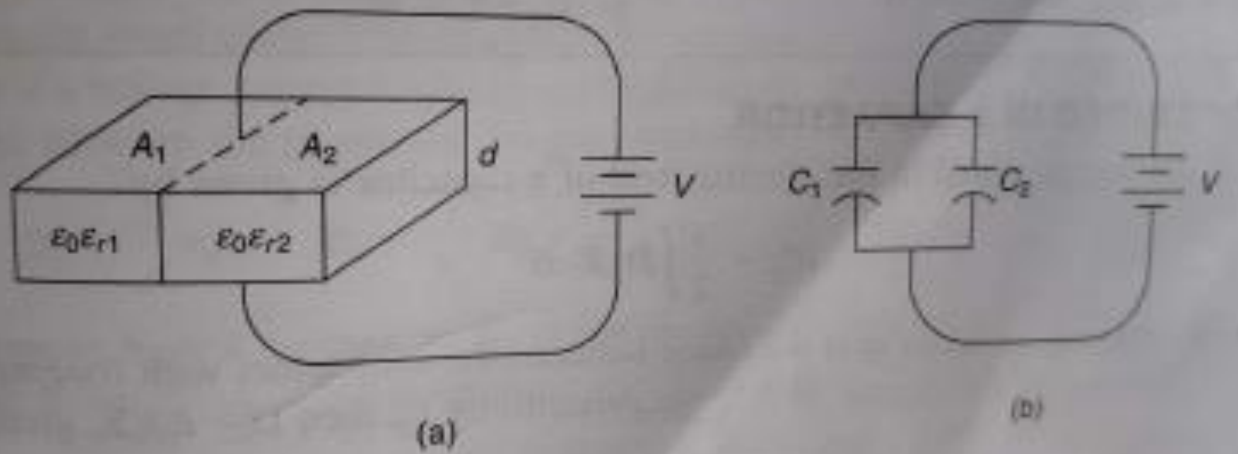

Fig. 7.4

When two dielectrics are present such that the interface is *normal to* **D** and **E**, as shown in Fig. 7.5(a), the equivalent capacitance can be obtained by treating the arrangement as two capacitors in series [Fig. 7.5(b)].

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{d_1} \qquad C_2 = \frac{\varepsilon_0 \varepsilon_{r2} A_2}{d_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\varepsilon_{r2} d_1 + \varepsilon_{r1} d_2}{\varepsilon_0 \varepsilon_{r1} \varepsilon_{r2} A}$$

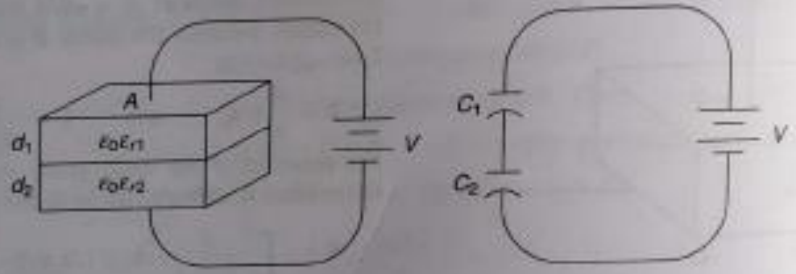

Fig. 7.5

The result can be extended to any number of dielectrics such that the interfaces are all normal to **D** and **E**: *the reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitances.*

Capacitance of parallel plate capacitor with two dielectrics:

Sol^n

The electric flux density

$D = D_n = \rho_s$ (C/m^2)

is same in both media

However the electric field intensity ($\bar{E}^n$) are different.

In 1st medium $E_1 = \dfrac{\rho_s}{\epsilon_{r1}\epsilon_0}$

" 2nd " $E_2 = \dfrac{\rho_s}{\epsilon_{r2}\epsilon_0}$

Assume 'V' potential diff the plates,

$$V = E_1 d_1 + E_2 d_2$$

$$= \frac{\rho_s d_1}{\epsilon_0 \epsilon_{r1}} + \frac{\rho_s d_2}{\epsilon_0 \epsilon_{r2}}$$

$\rho_s = \frac{q}{A}$

$$= \frac{q}{\epsilon_0 A}\left[\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}}\right]$$

$$C = \frac{q}{V} = \frac{\epsilon_0 A}{\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}}}$$

Now, the capacitance of the parallel plate capacitor with two dielectric media can be considered as two different capacitors connected in series

$$\frac{1}{C_{ser}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore C_{ser} = \frac{C_1 C_2}{C_1 + C_2}$$

$C_1 = \frac{\epsilon_{r1}\epsilon_0 A}{d_1}$

$C_2 = \frac{\epsilon_0 A \epsilon_{r2}}{d_2}$

$$\frac{1}{C_{ser}} = \frac{d_1}{\epsilon_{r1}\epsilon_0 A} + \frac{d_2}{\epsilon_{r2}\epsilon_0 A}$$

$$= \frac{d_1/\epsilon_{r1} + d_2/\epsilon_{r2}}{\epsilon_0 A}$$

$$\therefore C_{ser} = \frac{\epsilon_0 A}{\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}}}$$

7 Find the voltage across each dielectric in the capacitor shown in Fig. 7.18 when the applied voltage is 200 V.

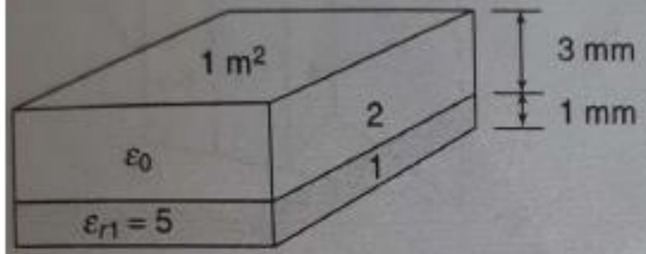

Fig. 7.18

$$C_1 = \frac{\varepsilon_0 5(1)}{10^{-3}} = 5000\varepsilon_0$$

$$C_2 = \frac{1000\varepsilon_0}{3}$$

and $$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$= 312.5\varepsilon_0 = 2.77 \times 10^{-9}\ \mathbf{F}$$

The **D** filed within the capacitor is now found from

$$D_n = \rho_s = \frac{Q}{A} = \frac{CV}{A} = \frac{(2.77 \times 10^{-9})(200)}{1}$$

$$= 5.54 \times 10^{-7}\ \text{C/m}^2$$

Then,

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}} = 1.25 \times 10^4\ \text{V/m}$$

$$E_2 = \frac{D}{\varepsilon_0} = 6.25 \times 10^4\ \text{V/m}$$

from which

$$V_1 = E_1 d_1 = 12.5\ \text{V}$$

$$V_2 = E_2 d_2 = 187.5\ \text{V}$$

A parallel-plate capacitor with separation d = 1.0 cm has 29 KV applied

when free space is the only dielectric. Assume that air has a dielectric strength of 30 kV/cm. Show why the air breaks down when a thin piece of glass ($\varepsilon_r = 6.5$) with a dielectric strength of 290 kV/cm and thicknesses $d_2 = 0.20$ cm is inserted as shown in Fig. 7.20.

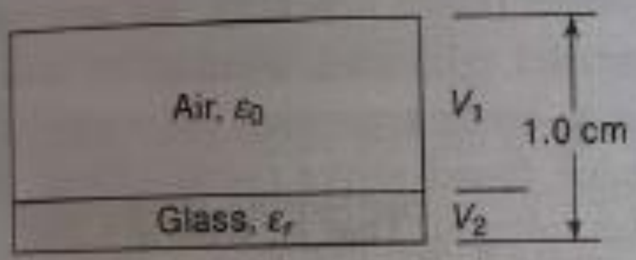

Fig. 7.20

The problem becomes one of two capacitors in series,

$$C_1 = \frac{\varepsilon_0 A}{8 \times 10^{-3}} = 125\varepsilon_0 A$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_r A}{2 \times 10^{-3}} = 3250\varepsilon_0 A$$

Then, as in Solved Problem 7.18,

$$V_1 = \frac{3250}{125 + 3250}(29) = 27.93 \text{ kV}$$

so that

$$E_1 = \frac{27.93 \text{ kV}}{0.80 \text{ cm}} = 34.9 \text{ kV/cm}$$

which exceeds the dielectric strength of air.

A free-space parallel-plate capacitor is charged by momentary connection to a voltage source V, which is then removed. Determine how W_E, D, E, C, and V change as the plates are moved apart to a separation distance $d_2 = 2d_1$ without disturbing the charge.

Relationship	**Explanation**
$D_2 = D_1$	$D = Q/A$
$E_2 = E_1$	$E = D/\varepsilon_0$
$W_{E2} = 2W_{E1}$	$W_E = \frac{1}{2}\int \varepsilon_0 E^2\, dv$, and the volume is doubled
$C_2 = \frac{1}{2} C_1$	$C = \varepsilon_A/d$
$V_2 = 2V_1$	$V = Q/C$

4 Find the capacitance of a parallel-plate capacitor containing two dielectrics, $\varepsilon_{r1} = 1.5$ and $\varepsilon_{r2} = 3.5$, each comprising one-half the volume, as shown in Fig. 7.16. Here, $A = 2\ \text{m}^2$ and $d = 10^{-3}$ m.

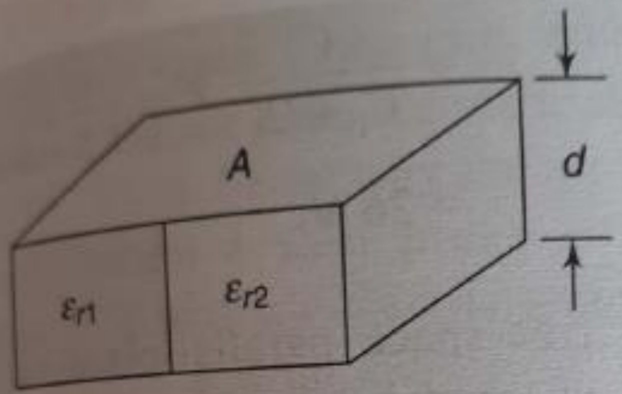

Fig. 7.16

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{d} = \frac{(8.854 \times 10^{-12})(1.5)1}{10^{-3}}$$

$$= 13.3 \text{ nF}$$

Similarly, $C_2 = 31.0$ nF. Then

$$C = C_1 + C_2 = 44.3 \text{ nF}$$

7.8 GAUSS'S LAW IN PRESENCE OF A DIELECTRIC

Gauss's law gets modified in the presence of a dielectric. This can be demonstrated by taking an example of a dielectric introduced into a parallel-plate capacitor but the results are applicable for all dielectric mediums. Two parallel-plate capacitors of plate area A, one with and the other without a dielectric, both having charge q on the plates are shown in Fig. 7.9. The electric field between the plates can be obtained by applying Gaussian Law on the Gaussian surface as shown in Fig. 7.9.

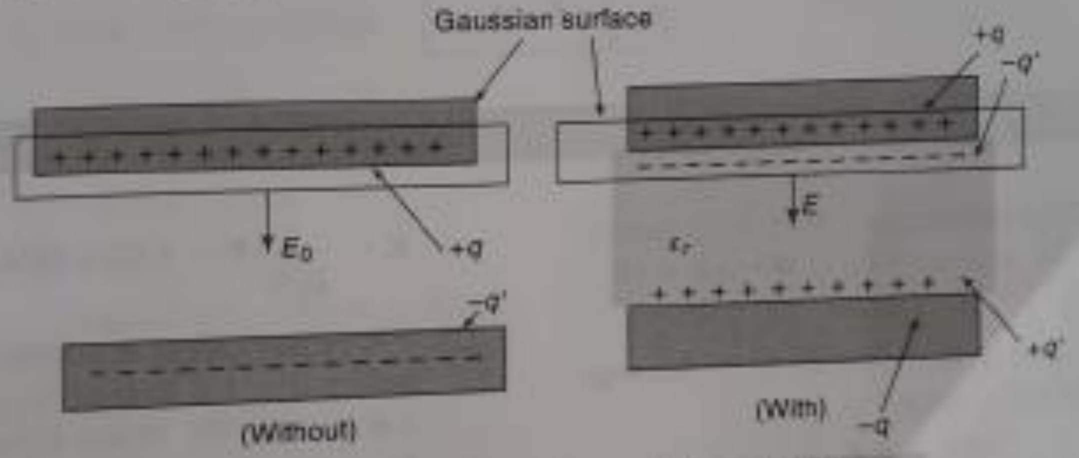

Fig. 7.9 *Capacitors without and with a dielectric*

$$\varepsilon_0 \oint E \cdot dA = \varepsilon_0 E_0 A = q \tag{7.1}$$

$$\boxed{E_0 = \frac{q}{\varepsilon_0 A}} \tag{7.2}$$

For the capacitor with dielectric, electric field can be obtained by using the same Gaussian surface. However, the surface is enclosed by two types of charge, i.e., $+q$ on the top plate and an induced charge $-q'$ on the top face of the dielectric. The charge on the conducting plate is said to be *free charge* because it can move with change in the electric potential of the plate. The induced charge on the surface of the dielectric is not a free charge because it cannot move from the surface

$$\varepsilon_0 \oint \mathbf{E}.d\mathbf{A} = \varepsilon_0 E_0 A = q - q'$$

$$\boxed{E_0 = \frac{q - q'}{\varepsilon_0 A}}$$

The effect of the dielectric is to weaken the original field E_o by a factor κ as

$$\boxed{E = \frac{E_0}{\kappa} = \frac{q}{\kappa \varepsilon_0 A}}$$

Comparing the equations (7.4) and (7.5),

$$q - q' = \frac{q}{\kappa}$$

The magnitude q' of the induced surface charge is less than that of the free charge q and is [illegible] no dielectric is present. By substituting (7.6) in (7.3), Gauss' law in presence of dielectric is

$$\boxed{\varepsilon_0 \oint \kappa \mathbf{E}.d\mathbf{A} = q}$$

SOLVED PROBLEMS