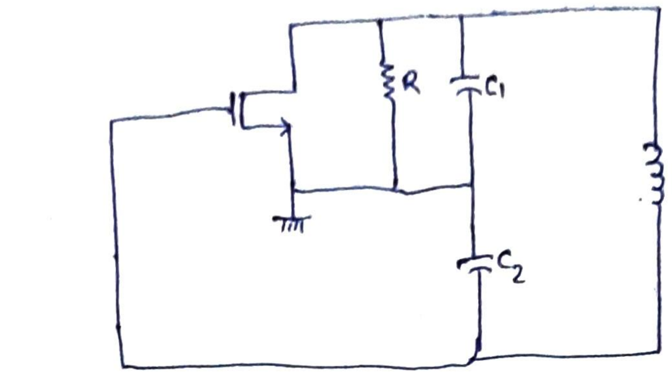
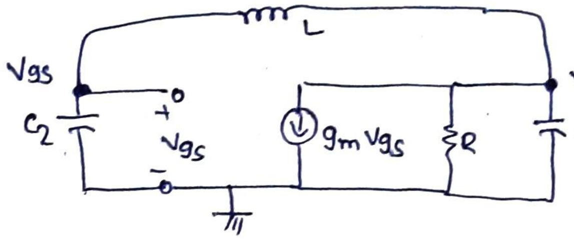
Colpitt's Oscillator





$$\frac{V_1 - V_{gs}}{LS}$$

$$\frac{V_1 - V_{gs}}{LS} + g_m V_{gs} + \frac{V_1}{R} + V_1 C_1 S = 0$$

$$\frac{V_1 - V_{gs}}{LS} = V_{gs} C_2 S$$

$$\Rightarrow \frac{V_1}{LS} = V_{gs}\left(\frac{1}{LS} + C_2 S\right)$$

$$V_1 = V_{gs}\left(1 + LC_2 s^2\right)$$

$$\frac{V_{gs}\left(1 + LC_2 s^2\right) - V_{gs}}{LS} + g_m V_{gs} + \frac{V_{gs}\left(1 + LC_2 s^2\right)}{R} + V_{gs} C_1 s \left(1 + LC_2 s^2\right) = 0$$

as $V_{gs} \neq 0$)

$$C_2 s + g_m + \left(1 + LC_2 s^2\right)\left(\frac{1}{R} + C_1 s\right) = 0$$

$s = j\omega$

$$j\omega C_2 + g_m + \left(1 - LC_2 \omega^2\right)\left(\frac{1}{R} + j\omega C_1\right) = 0.$$

$$g_m + \frac{\left(1 - LC_2 \omega^2\right)}{R} + j\left(\omega C_2 + \omega C_1 \left(1 - LC_2 \omega^2\right)\right) =$$

making Imaginary part = 0.

$$\omega C_2 = -\omega C_1 (1 - L C_2 \omega^2)$$

$$\omega C_2 + \omega C_1 - \omega^3 L C_1 C_2 = 0$$

$$\boxed{\omega_{osc} = \sqrt{\frac{(C_1 + C_2)}{L C_1 C_2}}}$$

$$\omega_0 = \sqrt{\frac{C_1 + C_2}{L C_1 C_2}}.$$

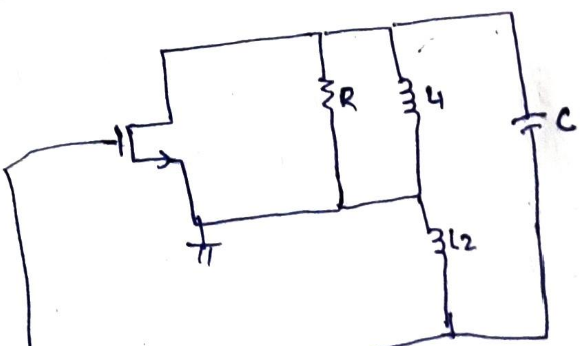
$$g_m + \frac{1 - L C_2 \left(\frac{C_1 + C_2}{L C_1 C_2}\right)}{R} = 0$$

$$g_m + \frac{1 - \frac{(C_1 + C_2)}{C_1}}{R} = 0$$

$$\boxed{g_m R = C_2 / C_1}$$

$$g_m + \frac{\cancel{1} - \cancel{1} - C_2/C_1}{R} = 0$$

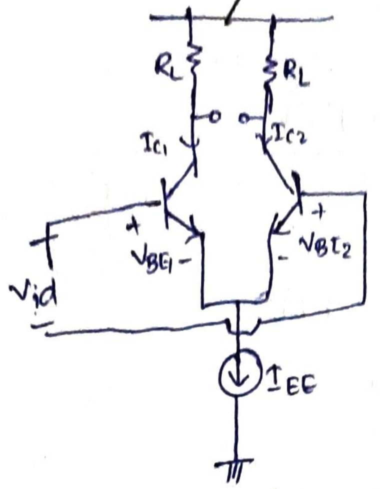
Hartley Oscillator



$$\boxed{\omega_0 = \sqrt{\frac{1}{C(L_1 + L_2)}}}$$

$$\boxed{g_m R = \frac{L_1}{L_2}}$$

Analog Multiplier / 2-Quadrant Multiplier



$$V_{id} - V_{BE_1} + V_{BE_2} = 0$$

$$I_{C1} = I_{S_1}\, e^{\frac{q V_{BE1}}{kT}}$$

$$I_{C_2} = I_{S_2}\, e^{\frac{q V_{BE_2}}{kT}}.$$

As $Q_1 \equiv Q_2$ $\quad I_{S_1} = I_{S_2}$ $\qquad I_S$ = saturation current.

$$V_{id} = V_{BE_1} - V_{BE_2}$$

$$= \frac{kT}{q} \ln \frac{I_{C1}}{I_{S_1}} - \frac{kT}{q} \ln \frac{I_{C2}}{I_{S2}}$$

$$V_{id} = V_T \ln \frac{I_{C1}}{I_{C_2}}$$

$\frac{kT}{q} = V_T = 26\,mV$ → Thermal Voltage.

$$I_{C1} + I_{C_2} = I_{EE}$$

$$I_{C2}\, e^{\frac{V_{id}}{V_T}} + I_{C2} = I_{EE}$$

$$I_{C2}\left(1 + e^{\frac{V_{id}}{V_T}}\right) = I_{EE}$$

$$I_{C_2} = \frac{I_{EE}}{1 + e^{\frac{V_{id}}{V_T}}}$$

$$I_{C_1} = \frac{I_{EE}}{1 + e^{-\frac{V_{id}}{V_T}}}$$

$$V_0 = \Delta I_C\, R_L$$

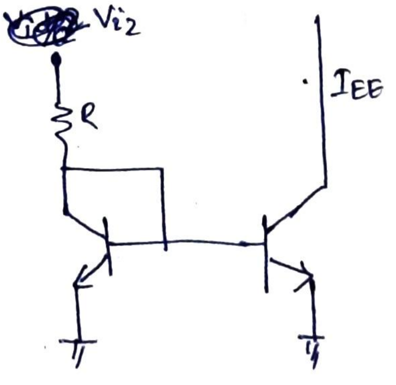
$$= \left(\frac{I_{EE}}{1 + e^{-\frac{V_{id}}{V_T}}} - \frac{I_{EE}}{1 + e^{\frac{V_{id}}{V_T}}}\right) R_L$$

$$I_{EE}\left\{\frac{1+e^{\frac{V_{id}}{V_T}}-1-e^{-V_{id}/V_T}}{\left(1+e^{\frac{-V_{id}}{V_T}}\right)\left(1+e^{V_{id}/V_T}\right)}\right\}R_L$$

$$= I_{EE}\left(\frac{e^{\frac{V_{id}}{V_T}}-e^{-V_{id}/V_T}}{2+e^{\frac{V_{id}}{V_T}}+e^{-V_{id}/V_T}}\right)R_L$$

$$V_o = I_{EE}\left(\frac{\left(e^{\left(\frac{V_{id}}{2T}\right)}\right)^2-\left(e^{\frac{-V_{id}}{V_T}}\right)^2}{\left(e^{\frac{V_{id}}{V_T}}\right)^2+\left(e^{\frac{-V_{id}}{V_T}}\right)^2+2e^{\frac{V_{id}}{2V_T}}e^{\frac{-V_{id}}{2V_T}}}\right)$$

$$\boxed{V_o = I_{EE}\,\frac{e^{\frac{V_{id}}{2V_T}}-e^{-\frac{V_{id}}{2V_T}}}{e^{\frac{V_{id}}{2V_T}}+e^{-\frac{V_{id}}{2V_T}}}\times R_L}$$



Replacing with current Mirror

condition Must satisfy
i) $V_{i2} > V_{BE}$
ii) $V_{id} \ll 2V_T$

$\downarrow$

2-Quadrant Multiplier

$\downarrow$

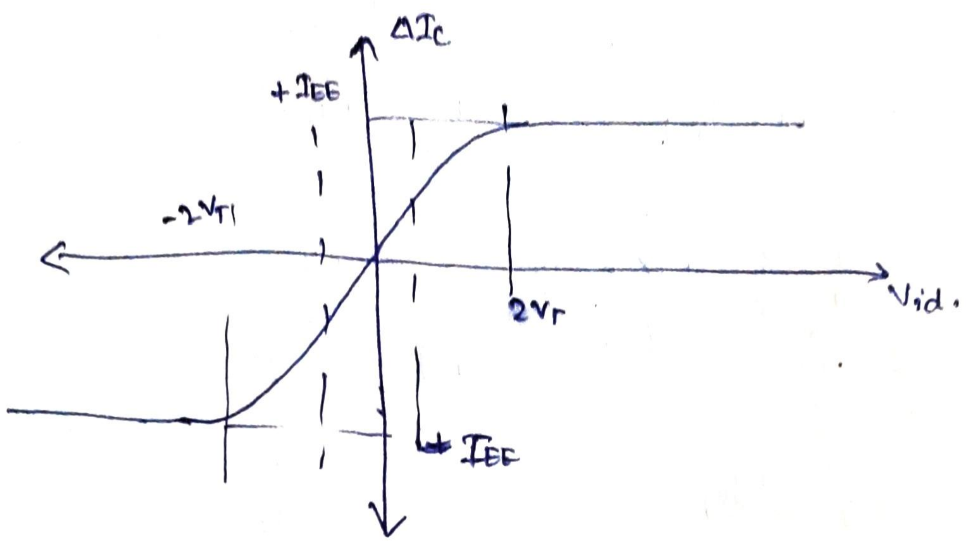
V_{i2} has to be +ve

& V_{id} can be +ve or -ve.

$$V_o = I_{EE}\tanh\left(\frac{V_{id}}{2V_T}\right)\times R_L$$

$$\simeq I_{EE}\left(\frac{V_{id}}{2V_T}\right)\times R_L$$

$$\boxed{V_o \simeq \frac{V_{i2}-V_{EE}}{R}\times\frac{V_{id}}{2V_T}\times R_L}$$



Q. For the 2-quadrant multiplier, find the magnitude of dc differential voltage that is required to cause the slope of (ΔIc vs Vid) to change by 1 % from that of origin.

Soln:-

$$\frac{\partial \Delta I_C}{\partial V_{id}} = ??$$

$$\Delta I_C = I_{EE} \tanh \frac{V_{id}}{2V_T}.$$

$$m_1 - m_0 = 0.01\, m_0.$$

$$\Delta I_C = I_{EE} \frac{e^{\frac{V_{id}}{2V_T}} - e^{-\frac{V_{id}}{2V_T}}}{e^{\frac{V_{id}}{2V_T}} + e^{-\frac{V_{id}}{2V_T}}}$$

$$= I_{EE} \frac{e^{\frac{V_{id}}{V_T}} - 1}{e^{\frac{V_{id}}{V_T}} + 1}$$

$$\frac{\partial \Delta I_C}{\partial V_{id}} = I_{EE} \frac{\partial}{\partial V_{id}} \left(e^{\frac{V_{id}}{V_T}} - 1\right) \times \frac{1}{\left(e^{V_{id}/V_T} + 1\right)} + I_{EE} \frac{\partial}{\partial V_{id}} \left(1 + e^{\frac{V_{id}}{V_T}}\right)^{-1} \left(e^{\frac{V_{id}}{V_T}} - 1\right)$$

$$= I_{EE} \frac{\frac{1}{V_T} e^{\frac{V_{id}}{V_T}}}{\left(e^{\frac{V_{id}}{V_T}} + 1\right)} - I_{EE} \left(1 + e^{\frac{V_{id}}{V_T}}\right)^{-2} \times \frac{1}{V_T} \times \left(e^{\frac{V_{id}}{V_T}} - \right.$$

$$e^{\frac{V_{id}}{V}}$$

$$= \frac{I_{EE}}{V_T}\left[\frac{1}{(1+e^{V_{id}/V_T})} - \frac{(e^{V_{id}/V_T}-1)}{(1+e^{V_{id}/V_T})^2}\right] e^{V_{id}/V_T}$$

$$= \frac{I_{EE}}{V_T} e^{V_{id}/V_T}\left[\frac{1+e^{V_{id}/V_T} - e^{V_{id}/V_T} + 1}{(1+e^{V_{id}/V_T})^2}\right]$$

$$\frac{\partial \Delta I_C}{\partial V_{id}} = \frac{I_{EE}}{V_T}\frac{2e^{V_{id}/V_T}}{(1+e^{V_{id}/V_T})^2}\Bigg|_{V_{id}=0} = \frac{I_{EE}}{2V_T}$$

$\hookrightarrow m_0.$

$$m_1 - m_0 = -0.01\, m_0$$

$$\frac{I_{EE}}{V_T}\frac{2e^{V_{id}/V_T}}{(1+e^{V_{id}/V_T})^2} = -1.01\frac{I_{EE}}{2V_T}$$

$$e^{V_{id}/V_T} = x$$

$$\frac{4x}{(1+x)^2} = -1.01$$

$x = 1.2$

$x = 0.8$

$$-4x = 1.01\,(1+x)^2$$

$$1.01\,x^2 + (2x)1.01 + 1.01 + 4x = 0$$

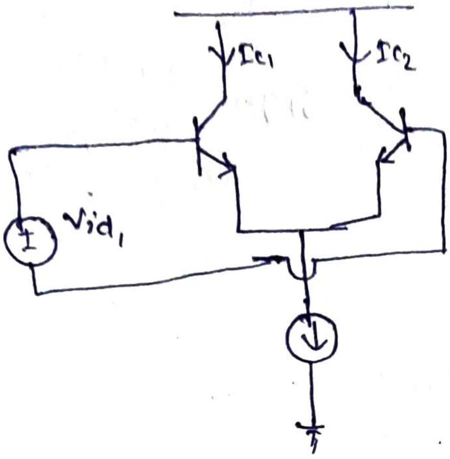
$$1.01\,x^2 + (0.1)\,2x + 1.01 = 0$$

$$1.01\,x^2 + 0.2x + 1.01 = 0.$$

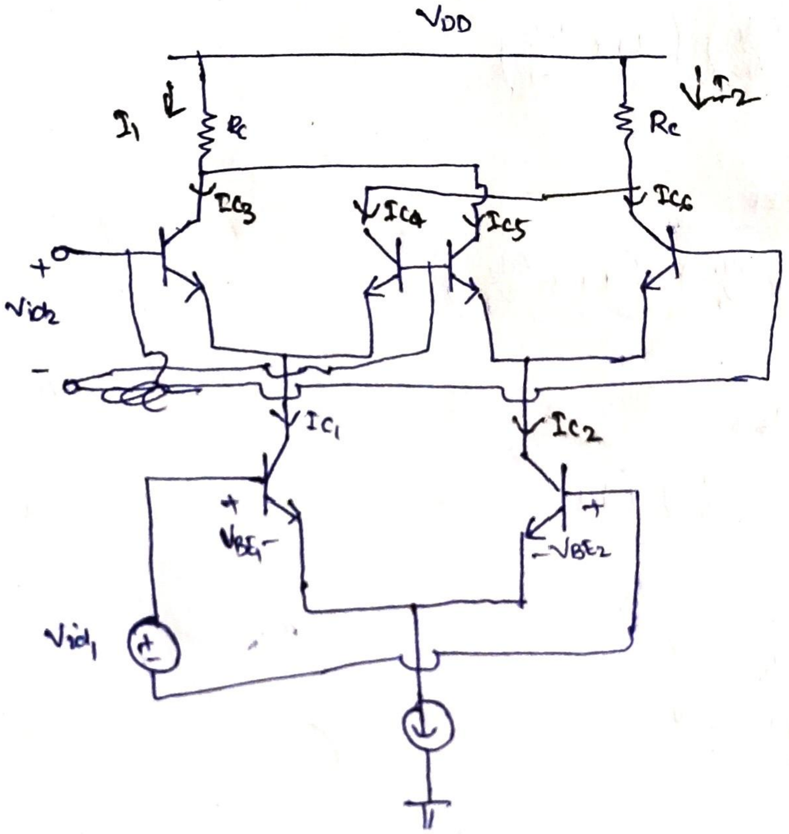
$$1.01x^2 + 6.02x + 1.01 = 0 \quad x = \frac{-0.2 \pm \sqrt{0.04 - 4\times 1.0201}}{2}$$

$$x = \frac{6.02 \pm \sqrt{3.9204 - 4.0804}}{2} = -0.1 \pm 0.5\sqrt{-4.0404}$$

$$= 1.98 \pm 0.4 \qquad = -0.1 \pm j$$



$$\Delta I_C = I_{EE} \tanh \frac{V_{id_1}}{2V_T}$$



$$V_{id_1} = V_{BE_1} - V_{BE_2}$$

$$V_{id_2} = V_{BE_3} - V_{BE_4},$$

$$V_{id_2} = V_{BE_6} - V_{BE_5}$$

$$I_{C_1} = I_{C_2}\, e^{\frac{V_{id}}{V_T}}$$

$$I_{C_1} = \frac{I_{EE}}{1 + e^{-\frac{V_{id_1}}{V_T}}}, \qquad I_{C_2} = \frac{I_{EE}}{1 + e^{V_{id}/V_T}}$$

$$I_{C_3} = \frac{I_{C_1}}{1 + e^{-V_{id_2}/V_T}} \qquad I_{C_4} = \frac{I_{C_1}}{1 + e^{V_{id_2}/V_T}}.$$

$$I_{C5} = \frac{I_{C2}}{1 + e^{+\frac{V_{id2}}{V_T}}} \qquad I_{C6} = \frac{I_{C2}}{1 + e^{-V_{id}/2}}.$$

Sign changes because

$V_{id2} = V_{BE6} - V_{BE5}$

$$I_{C3} = \frac{I_{EE}}{\left(1 + e^{-\frac{V_{id1}}{V_T}}\right)\left(1 + e^{-\frac{V_{id2}}{V_T}}\right)}$$

$$I_{C4} = \frac{I_{EE}}{\left(1 + e^{-\frac{V_{id1}}{V_T}}\right)\left(1 + e^{\frac{V_{id2}}{V_T}}\right)}$$

$$I_{C5} = \frac{I_{EE}}{\left(1 + e^{\frac{V_{id1}}{V_T}}\right)\left(1 + e^{\frac{V_{id2}}{V_T}}\right)}$$

$$I_{C6} = \frac{I_{EE}}{\left(1 + e^{\frac{V_{id1}}{V_T}}\right)\left(1 + e^{-V_{id2}/V_T}\right)}$$

$V_{id1} = V_1$

$V_{id2} = V_2$.

$$I_1 = I_{C3} + I_{C5}$$

$$= I_{EE}\left[\frac{1}{\left(1 + e^{-V_1/V_T}\right)\left(1 + e^{-V_2/V_T}\right)} + \frac{1}{\left(1 + e^{V_1/V_T}\right)\left(1 + e^{V_2/V_T}\right)}\right]$$

$$\boxed{I_1 = I_{EE}\,\frac{1}{\left(1 + e^{V_1/V_T}\right)\left(1 + e^{V_2/V_T}\right)} \times \left(e^{V_1/V_T} \times e^{V_2/V_T} + 1\right)}$$

$$\boxed{I_2 = I_{EE}\,\frac{1}{\left(1 + e^{-V_1/V_T}\right)\left(1 + e^{V_2/V_T}\right)}\left(e^{-V_1/V_T}\, e^{-V_2/V_T} + 1\right).}$$

$$\boxed{\Delta I = I_1 - I_2 = I_{EE} \quad \tanh \frac{V_1}{2V_T} \quad \tanh \frac{V_2}{2V_T}.}$$

Gazayman Book,

$$\boxed{\Delta I \simeq I_{EE} \left(\frac{V_1}{2V_T}\right)\left(\frac{V_2}{2V_T}\right).}$$