Properties of S matrix :

(*i*) *Symmetry property* : This property states that if a microwave junction satisfies reciprocity condition or if there are no active solid state devices in the circuit, then the junction is a linear passive circuit and the *s*-parameters are equal to their corresponding transposes.

i.e., $$S_{ij} = S_{ji} \quad \text{... (6.3)}$$

(*ii*) Scattering matrix is always a square matrix of $n \times n$ and the size of the matrix depends on number of ports of microwave circuit.

(*iii*) *Unity property* : The unity property states that the sum of the products of each term of any one row or column of the matrix S multiplied by its complex conjugate is unity, *i.e.,*

$$\sum_{i}^{n} S_{ij} \cdot S_{ij}^* = 1 \text{ for } j = 1, 2, 3, \ldots, n$$

$$= |S_{ij}|^2 \quad \text{... (6.4)}$$

since S_{ij} is symmetric for a lossless matched network power input is equal to power output.

i.e., $$[s]\,[s]^* = [I] \quad \text{... (6.4)}$$

(*iv*) The sum of the products of each term of any row (or column) multiplied by the complex conjugate of the corresponding terms of any other row (or column) is zero.

i.e., $$\sum_{i=1}^{n} S_{ik} \cdot S_{ij}^* = 0 \quad \text{.... [6.4(a)]}$$

(*v*) In order to move any terminal or reference planes from the junction by an electric distance $\beta_k\, l_k$, then each of the coefficient S_{ij} involving K will be multiplied by the factor $e^{-j\beta_k l_k}$.

6.6. DIRECTIONAL COUPLER

The directional coupler consists of two transmission lines or waveguides coupled together by fringing fields, or in case of a waveguide, by means of a small slot in such a manner that an incident wave on one line is partly transferred to the other with particular directional properties. It is a four port network having an input port, two mutually isolated output ports and one port isolated from the input port. Directional couplers are used as elements in power monitors, reflectometer and power dividers. The schematic diagram of a directional coupler is as shown in Fig. 6.11.

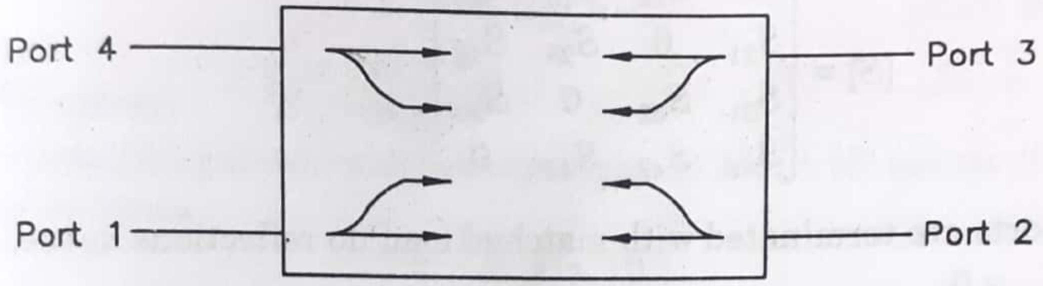


Fig. 6.11. Schematic diagram of a directional coupler.

In an ideal coupler when power enters in port 1 it will travel towards port 2 and a fixed part of it gets coupled to port 3 but no power is available at port 4. Here port 1 is called the input port, port 2 is the direct port, port 3 the coupled port and port 4 is the isolated port.

A directional coupler is characterized by the following parameters:

(*a*) **Directivity (*D*).** It is the ratio of power coupled to the auxillary arm to the power flowing in the uncoupled auxillary arm and is expressed in dB. Thus according to our schematic diagram of Fig. 6.11,

$$D = 10 \log \frac{P_3}{P_4} \text{ dB} \qquad \ldots(6.31)$$

ideally, no power is coupled to the port 4, therefore the directivity is infinite.

(*b*) **Coupling Factor (*C*).** Is the ratio of power at the input (entering) to the power coupled at the output in the auxillary arm and is expressed in dB.

So, from the schematic diagram of Fig. 6.11,

$$C = 10 \log \frac{P_1}{P_3} \text{ dB} \qquad \ldots(6.32)$$

(*c*) **Insertion Loss.** It is the loss arises due to insertion of the component over a line. It specifies the total output power from all the ports relative to the input power. The output power is less than input power for two reasons (*i*) some of the input power is absorbed and (*ii*) some gets reflected due to mismatch. Insertion loss is defined as :

$$\text{I.L.} = 10 \log \frac{P_2 + P_3 + P_4}{P_1} \text{ dB} \qquad \ldots(6.33)$$

(*d*) **Isolation.** It is another way of specifying directivity and it is equal to the sum of directivity and coupling. Mathematically it is defined as :

$$\text{Isolation} = 10 \log \frac{P_1}{P_4} \text{ dB} \qquad \text{...(6.34)}$$

Directional coupler is a reciprocal device like other devices of this chapter. Thus when power enters through port 2 then power gets coupled to port 4 and then port 3 becomes the isolated port.

The scattering matrix of the directional coupler is fourth order (because of 4 port) and when the ports are terminated with matched load is given by :

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix} \qquad \text{...(6.35)}$$

When ports are terminated with matched load no reflections hence, $S_{11} = S_{22} = S_{33} = S_{44} = 0$.

Also, adjacent ports are isolated from input port, thus :

$$S_{14} = S_{23} = S_{32} = S_{41} = 0$$

Thus S-parameter matrix of (6.35) is modified as

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{21} & 0 & 0 & S_{24} \\ S_{31} & 0 & 0 & S_{34} \\ 0 & S_{42} & S_{43} & 0 \end{bmatrix} \qquad \text{...(6.36)}$$

Again using symmetric property

$$S_{12} = S_{21}, S_{24} = S_{42}, S_{34} = S_{43}, S_{31} = S_{13}$$

Then matrix under (6.36) becomes

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} \qquad \text{...[6.36(a)]}$$

Since, $[S]\,[S^*] = I$ thus :

$$\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & S_{13}^* & 0 \\ S_{12}^* & 0 & 0 & S_{24}^* \\ S_{13}^* & 0 & 0 & S_{34}^* \\ 0 & S_{24}^* & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1 : |S_{12}|^2 + |S_{13}|^2 = 1 \qquad \text{...[6.36(b)]}$$

$$R_2C_2 : |S_{12}|^2 + |S_{24}|^2 = 1 \qquad \text{...[6.36(c)]}$$

$$R_3C_3 : |S_{13}|^2 + |S_{34}|^2 = 1 \qquad \text{...[6.36(d)]}$$

$$R_1C_4 : S_{12} \cdot S_{24}^* + S_{13} \cdot S_{34}^* = 0 \qquad \text{...[6.36(e)]}$$

Comparing equation [6.36(b)] with equation [6.36(c)] we get,

$$S_{13} = S_{24} \quad \text{...[6.36(f)]}$$

and equation [6.36(b)] with equation [6.36(d)]

$$S_{12} = S_{34} \quad \text{...[6.36(g)]}$$

Let us assume that S_{12} is real and positive = 'P'

$$S_{12} = S_{34} = P = S_{34}^* \quad \text{...[6.36(h)]}$$

Again from equation [6.36(e)] and [6.36(h)]

$$P \, . \, S_{24}^* + P \, . \, S_{13} = 0$$

$$P(S_{24}^* + S_{13}) = 0$$

As $P \neq 0, \quad \therefore \; S_{24}^* + S_{13} = 0$

If $S_{13} = jy$ then $S_{24}^* = -jy$ (must be imaginary)

Let $S_{13} = jq = S_{24}$

Therefore; $S_{12} = S_{34} = P$

Substituting these values in expression [6.36(a)], [S] matrix of a directional coupler becomes

$$[S] = \begin{bmatrix} 0 & P & jq & 0 \\ P & 0 & 0 & jq \\ jq & 0 & 0 & P \\ 0 & jq & P & 0 \end{bmatrix} \quad \text{...[6.36(i)]}$$

There are variety of ways in which auxillary arm may be coupled to the main guide to have a directional coupler. We will discuss some of them in brief.

6.6.1. Bathe Hole Coupler (Single Hole)

Bathe hole coupler has a single centre offset hole in the common broad walls of the two guides and the coupling between port 1 and 3 is controlled by the offset of the hole *i.e.*, the angle θ between the axis of the two guides determines the amount of coupling. Also the frequency of operation and the size of hole determines the amount of coupling. The construction of Bathe hole coupler is depicted in Fig.6.12.

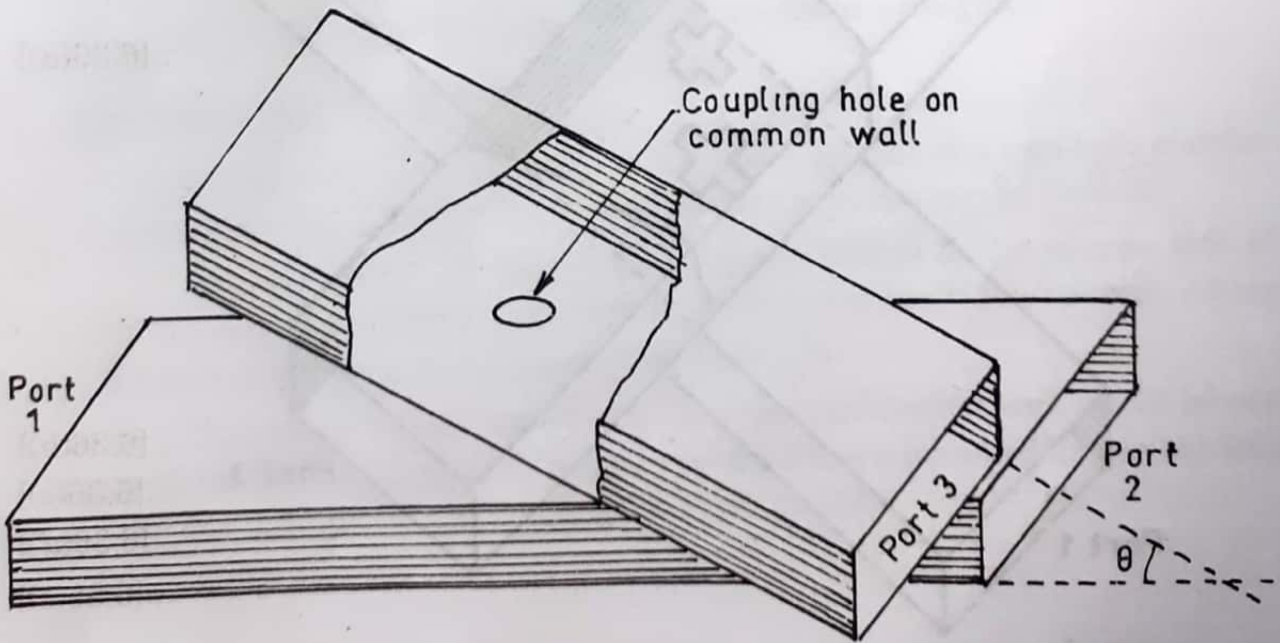


Fig. 6.12. Bathe hole coupler.

6.6.2. Double Hole Coupler

Two waveguides are connected to each other by two holes separated by a distance of $\lambda_g/4$ in the broad walls of the waveguide. This is also a Bathe hole type coupler. Fig. 6.13 shows the construction of a double hole coupler.

The wave travelling towards port 3 is due to the hole X and Y travel the same distances and hence add up constructively in port 3. In case of wave propagating in the reverse direction, the wave coming from hole Y has to travel $\lambda_g/2$ more distance than that coming from hole X. Therefore they add destructively (180° out of phase) in port 4 and as a result no power gets coupled to this port. In this coupler the directivity is a very sensitive function of frequency.

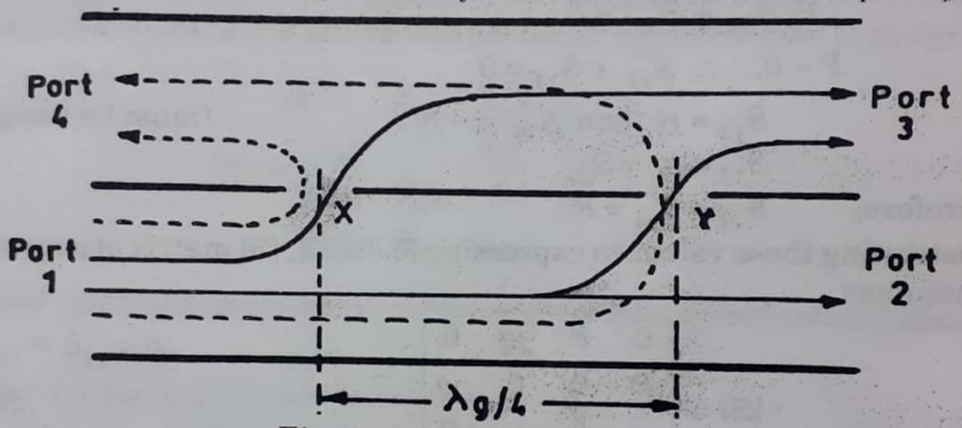


Fig. 6.13. Double hole coupler.

Scanned by CamScanner

6.6.5. Multihole Couplers

Its basic principle of operation is same as that of a two-hole coupler. By varying the hole dimensions a wide frequency range can be covered.

For high power operation the holes are drilled in the narrow side of the waveguide. Fig. 6.16 describes the construction of multihole coupler, where the distance between two successive holes are $\lambda_g/4$.

Though directional coupler is a four port device/network but in laboratory it can be used as a 3-port network by terminating a matched load in the isolated port (*i.e.* port 4) as shown in Figs. 6.17 (*a*) and (*b*).

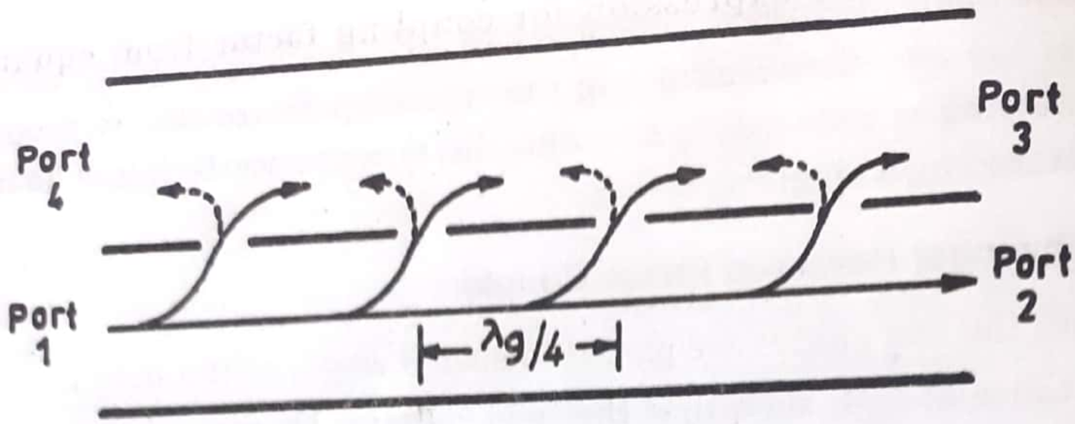
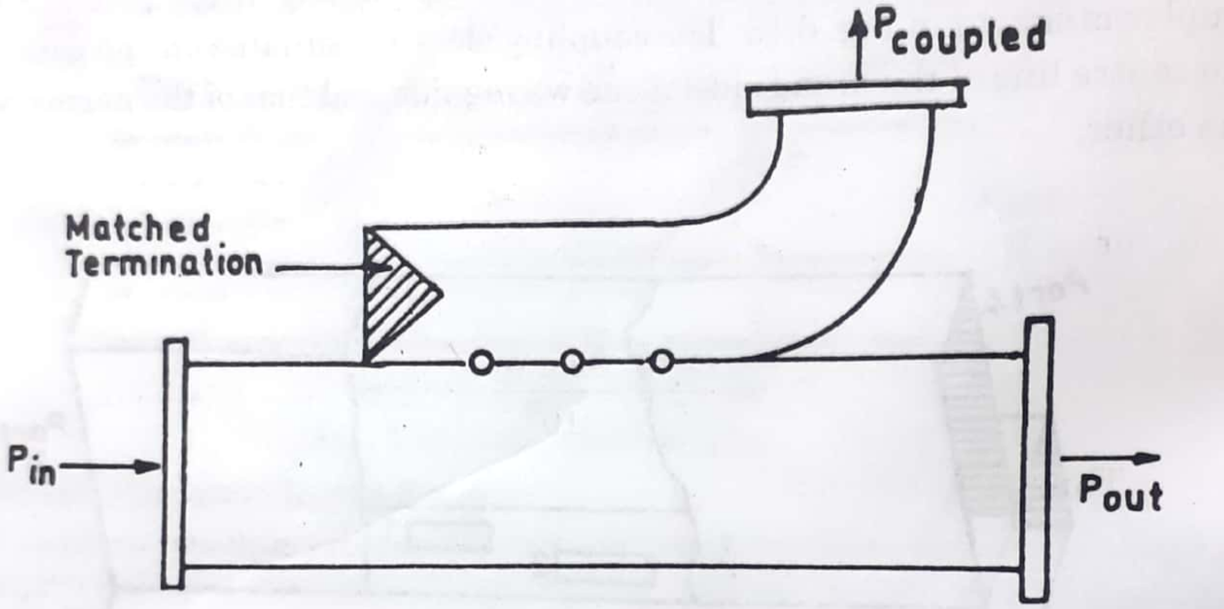
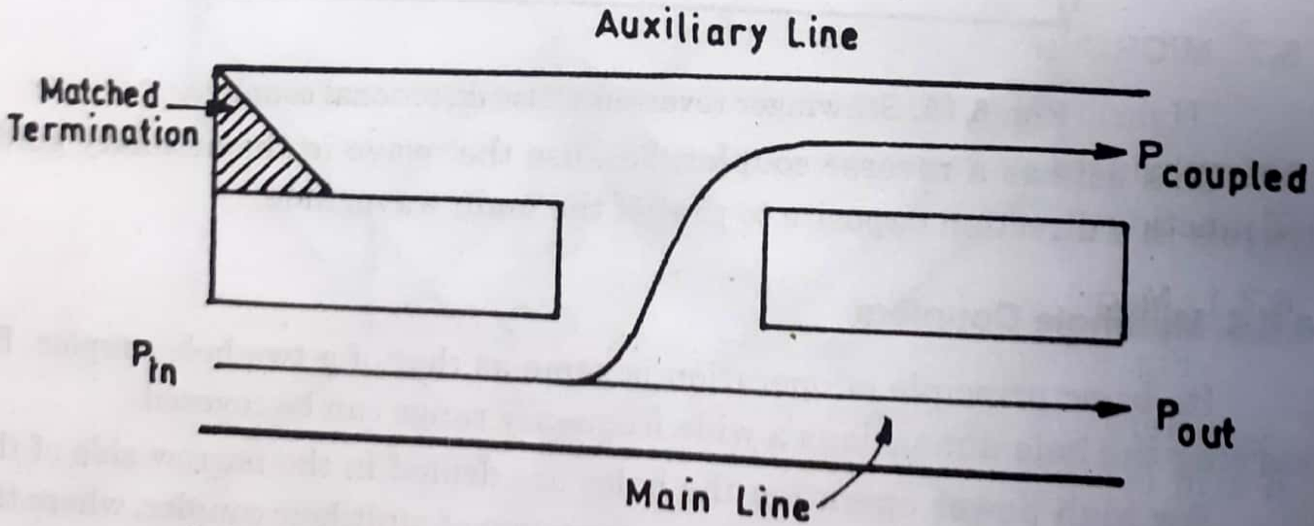


Fig. 6.16. Multihole coupler.



(*a*) Directional coupler as a 3-port device.



(*b*) Power coupling in a directional coupler used as 3 port.

Fig. 6.17. Directional coupler setup.

Example 6.1. *The input pow...*