Shell Model
or
Independent Particle Model

Nucleons are also fermions & several nuclear properties vary periodically with Z and N in a manner reminiscent of the periodic variation of atomic properties with Z.

Stability is related to both high binding energy & high natural abundance.

The experimental facts indicate that especially stable nuclei result when either the number of protons Z or the number of neutrons N = A-Z is equal to one of the numbers: 2, 8, 20, 28, 50, 82 and 126. These numbers are commonly referred to as "magic numbers". Magic nuclei: those with N or Z with magic numbers. Doubly magic: both N&Z.

Semi-magic numbers:

Besides the magic numbers there are other numbers of Z or N at which B.E/A is high.

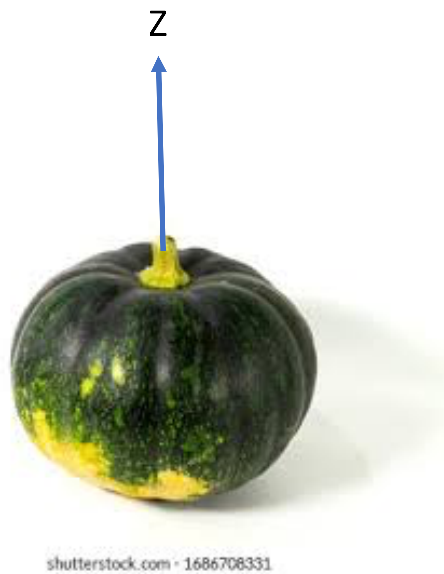
But the stability of nuclei are less than that of magic numbers.

These numbers are known as semi-magic numbers and include 14, 28, and 40.

Effect of magic numbers in nuclear structure

Nuclear electric quadrupole moments(Q) are measures of how much nuclear charge distributions depart from sphericity.

Q = 0



Q < 0

Q > 0

Example: Effect of magic numbers in nuclear structure

Nuclear electric quadrupole moments are measures of how much nuclear charge distributions depart from sphericity.

A nucleus ---

- which is spherical (are nuclei of magic N & Z) : has no quadrupole moment.
- shaped like a football : has a positive qudrupole moment.
- shaped like a pumpkin: has a negative quadrupole moment.

Shell model

According to the model:

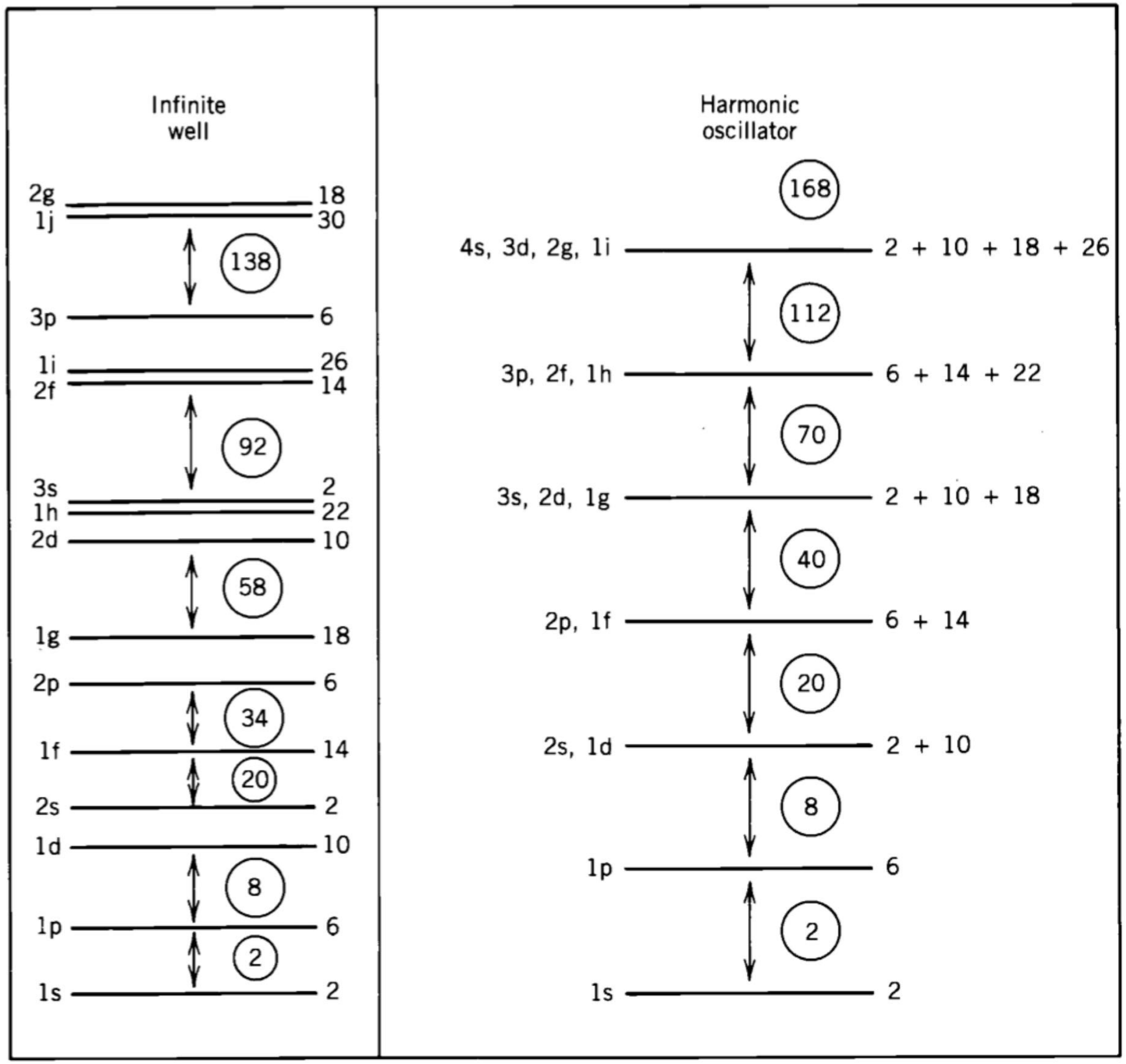
- Each nucleon moves in its orbit within the nucleus, independently of all other nucleons.

- The orbit is determined by a potential energy function V(r) which represents the average effect of all interactions with other nucleons and is the same for each particle.

- Each nucleon is regarded as an independent particle and the interaction between nucleons is considered to be a small perturbation on the interaction between a nucleon and the potential field.

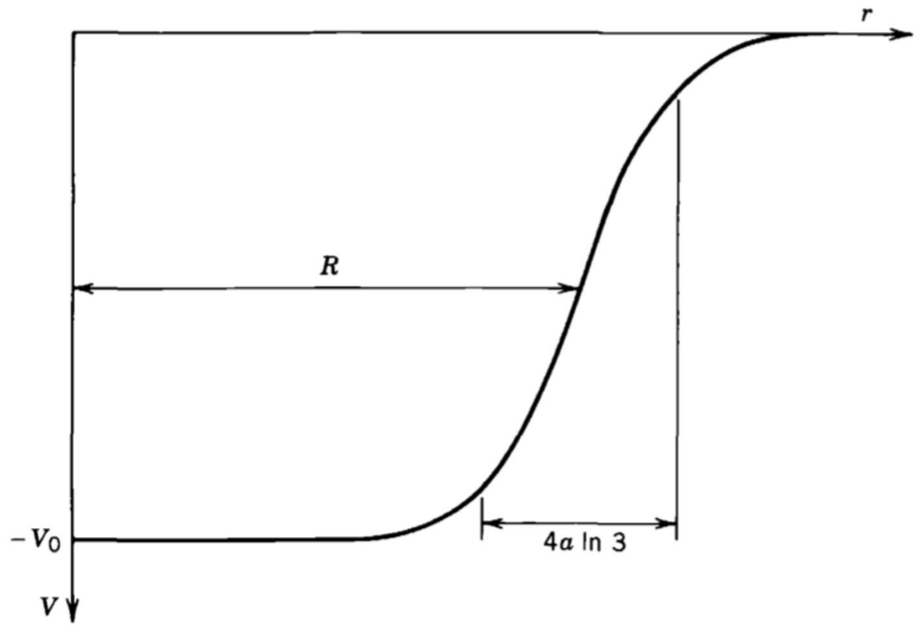
- The potential describing the nuclear attractions has a form between the square-well potential $V = -V_0$ and the oscillator potential $V = -V_0 + ar^2$

  (r: distance between the nucleon and the center of force and a is a constant.



Infinite well
2g 18
1j 30
138
3p 6
1i 26
2f 14
92
3s 2
1h 22
2d 10
58
1g 18
2p 6
34
1f 14
20
2s 2
1d 10
8
1p 6
2
1s 2
Harmonic oscillator
168
4s, 3d, 2g, 1i 2 + 10 + 18 + 26
112
3p, 2f, 1h 6 + 14 + 22
70
3s, 2d, 1g 2 + 10 + 18
40
2p, 1f 6 + 14
20
2s, 1d 2 + 10
8
1p 6
2
1s 2

Shell potential



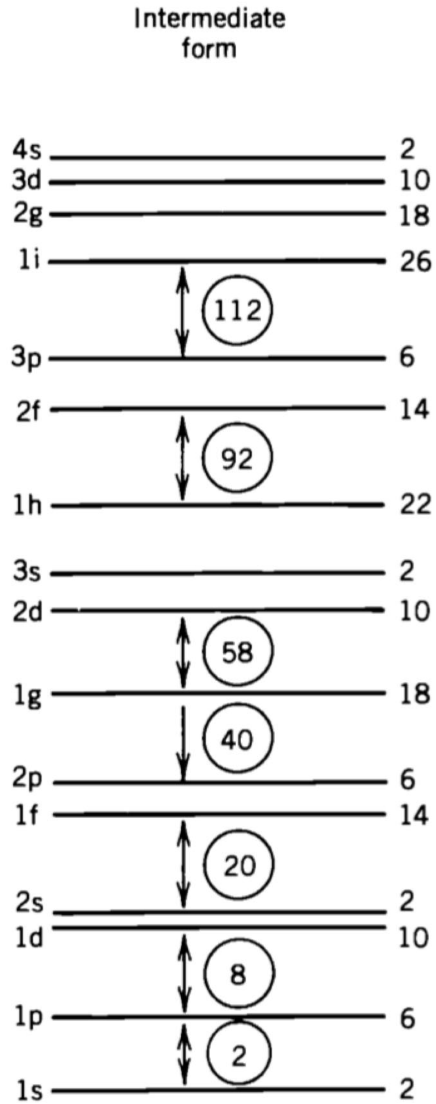
$$V(r) = \frac{-V_0}{1 + exp[(r - R)/a]}$$

$$R = 1.25\, A^{1/3}$$
$$a = 0.524\, fm$$
$$V_0 = 50\, MeV$$

The shell model of the nucleus is an attempt to account for the existence of magic numbers and certain other nuclear properties in terms of nucleon behavior in a common force field.

Solving Schrodinger's equation for a particle in a potential well V(r), it is found that stationary states of the system occur that are characterized by quantum numbers n, l and m_l.

However, the energy levels that come from such a calculation do not agree with the observed sequence of magic numbers.


Intermediate form
4s 2
3d 10
2g 18
1i 26
112
3p 6
2f 14
92
1h 22
3s 2
2d 10
58
1g 18
40
2p 6
1f 14
20
2s 2
1d 10
8
1p 6
2
1s 2

How magic numbers arise?

The problem of magic numbers was solved independently by Maria Goeppert-Mayer and J. H. D. Jensen in 1949.

The shell theory assumes that

- LS coupling holds only for the very lightest nuclei:

a) In this scheme, the intrinsic spin angular momenta $\boldsymbol{s_i}$ of neutrons or protons are coupled together into a total spin momentum $\boldsymbol{S}$.

b) The orbital angular momenta $\boldsymbol{l_i}$ are separately coupled together into a total orbital momentum L.

c) Then L and S are coupled to form a total angular momentum J of magnitude: $\sqrt{J(J+1)}\,\hbar$.

- The heavier nuclei exhibit jj coupling:

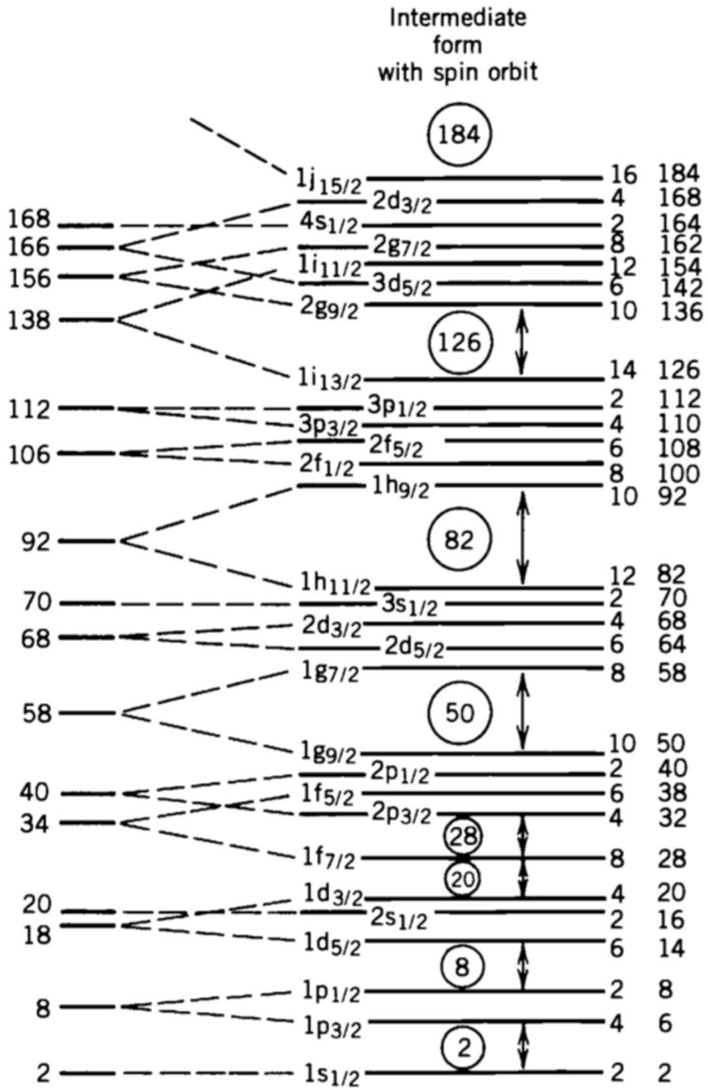
a) In this scheme, the $\boldsymbol{s_i}$ and $\boldsymbol{l_i}$ of each particle are first coupled to form a $\boldsymbol{j_i}$ For that particle of magnitude $\sqrt{j(j+1)}\ \hbar$.

b)The various $\boldsymbol{j_i}$ then couple together to form the total angular momentum **J**.

c) The jj coupling scheme holds for the great majority of nuclei.

The spin-orbit interaction splits each state of given j into 2j+1 substates.

Large energy gaps appear in the spacing of the levels at intervals are consistent with the notion of separate shells.



Intermediate form with spin orbit
184
126
82
50
28
20
8
2
1j15/2 16 184
2d3/2 4 168
168 4s1/2 2 164
166 2g7/2 8 162
156 1i11/2 12 154
3d5/2 6 142
138 2g9/2 10 136
1i13/2 14 126
112 3p1/2 2 112
3p3/2 4 110
106 2f5/2 6 108
2f1/2 8 100
1h9/2 10 92
92
1h11/2 12 82
70 3s1/2 2 70
2d3/2 4 68
68 2d5/2 6 64
1g7/2 8 58
58
1g9/2 10 50
40 2p1/2 2 40
1f5/2 6 38
34 2p3/2 4 32
1f7/2 8 28
20 1d3/2 4 20
18 2s1/2 2 16
1d5/2 6 14
1p1/2 2 8
8 1p3/2 4 6
2 1s1/2 2 2

The number of available nuclear states in each nuclear shell is, in ascending order of energy, 2, 6, 12, 8, 22, 32 and 44

Hence, the shells are filled when there are 2, 8, 20, 28, 50, 82 and 126 neutrons or protons in a nucleus.

The shell model explains:

(1) Magic numbers

(2) The existence of energy sublevels that can be occupied by two particles of opposite spin explains the tendency of nuclear abundances to favour even Z and even N.

(3) Predicts nuclear angular momenta:

(i) In even-even nuclei, all the protons and neutrons should pair off to cancel out one another's spin and orbital angular momenta. Thus, even-even nuclei have zero nuclear angular momenta.

(ii) In even-odd and odd-even nuclei, the half-integral spin of the single "extra" nucleon should be combined with the integral angular momenta of the rest of the nucleus. Hence, even-odd/odd-even nuclei have a half-integral total angular momentum

(iii) The odd-odd nuclei each have an extra neutron and an extra proton whose half-integral spins should yield integral total angular momenta.

The shell model explains:

(1) Magic numbers
(2) The existence of energy sublevels
(3) Predicts nuclear angular momenta:

Collective model

Collective or Unified model is proposed by Rainwater, Bohr and Mottleson.

This is the unification of liquid drop model and shell model which suggests that a number of nuclear properties could be explained by considering that both of these models are incomplete part of a more general model.

The liquid drop model—

- can account for the behavior of the nucleus as a whole, as in nuclear reactions & nuclear fission.

- describes the collective behavior of the nucleus and the excitation of the nucleus is treated as “collective modes of motion”(Ex: surface oscillations, elastic vibrations etc.,)

But many nuclear phenomena seem to show that nucleons behave as individual and nearly independent particles.

Hence there are two entirely different ways of regarding nuclei, with a basic Contradiction between them.

A reasonable conclusion is that the two different models or pictures are incomplete parts of a larger or more general model. This new model is "Collective model" .

The Collective model assumes that:

The particles within the nucleus exert a centrifugal pressure on the surface of the nucleus.

So, nucleus may be deformed into a permanently nonspherical shape.

The surface may undergo oscillations (liquid-drop aspect).

The particles within the nucleus then move in a nonspherical potential.

Thus, nuclear distortion reacts on the particles and modifies somewhat the independent-particle aspect.

The nucleus is thus regarded as a shell structure capable of performing oscillations in shape & size.

The collective model can be made to describe such liquid-drop like properties as nuclear fission while at the same time preserving the shell model characteristics and in fact, improving on the earlier shell model.

Radioactivity

- In 1896, Becquerel discovered that crystals of a uranium salt emitted rays which were similar to x-rays in that they were highly penetrating, could affect a photographic plate, and induced electrical conductivity in gases.

- Becquerel's discovery -was followed by the identification by the Curies (1898), of two other radioactive elements, polonium and radium. The activity of radium as measured by the intensity of the emitted rays was found to be more than a million times that of uranium.

- By means of experiments in magnetic fields, it had been concluded that there are three kinds of radiations from naturally occurring radioactive substances.

- The emission of these radiation from a number of element takes place due to disintegration of nuclei and is called as Radioactivity decay.

- The elements/ nuclides emitting such radiation are called as radionuclides/radioisotopes.

- The early experimenters, Rutherford and his co-workers, distinguished three components in the radiations from radionuclides.

Experiment 1:

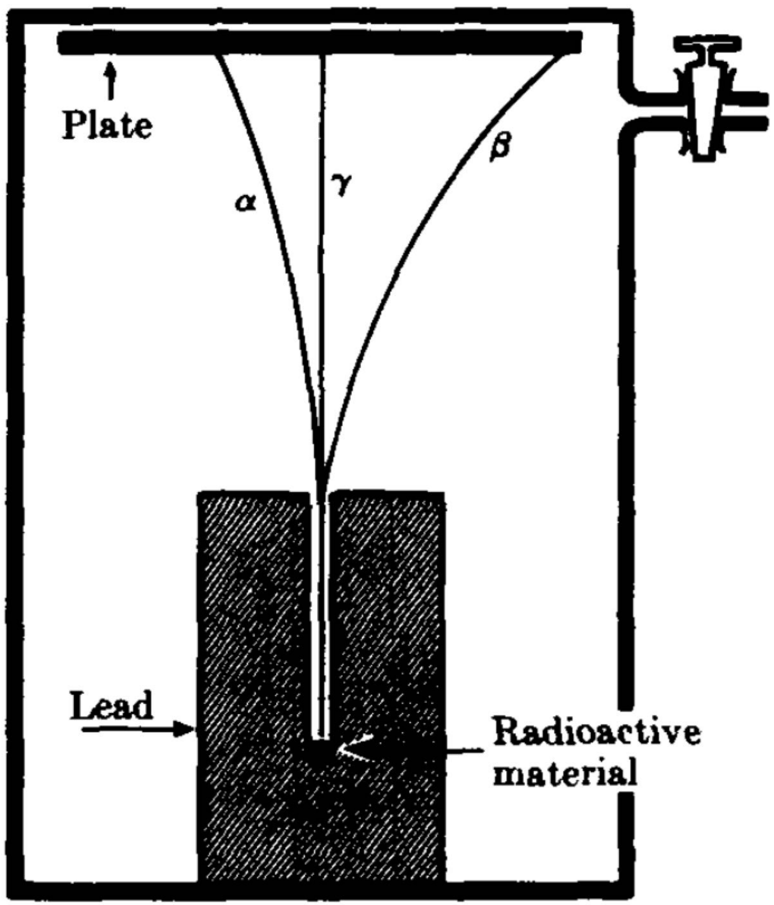
- In this experiment, a collimated beam of rays is supplied by a small piece of radioactive material at the bottom of a long groove in a lead block.

- A photographic plate is placed some distance above the lead block, and the air is pumped out of the chamber.

- A weak magnetic field is applied perpendicular to the plane of the diagram, and directed into the paper.

- When the plate is developed, two distinct spots are found, one in the direct line of the groove in the lead block, and one deflected to the right.

Experiment 1



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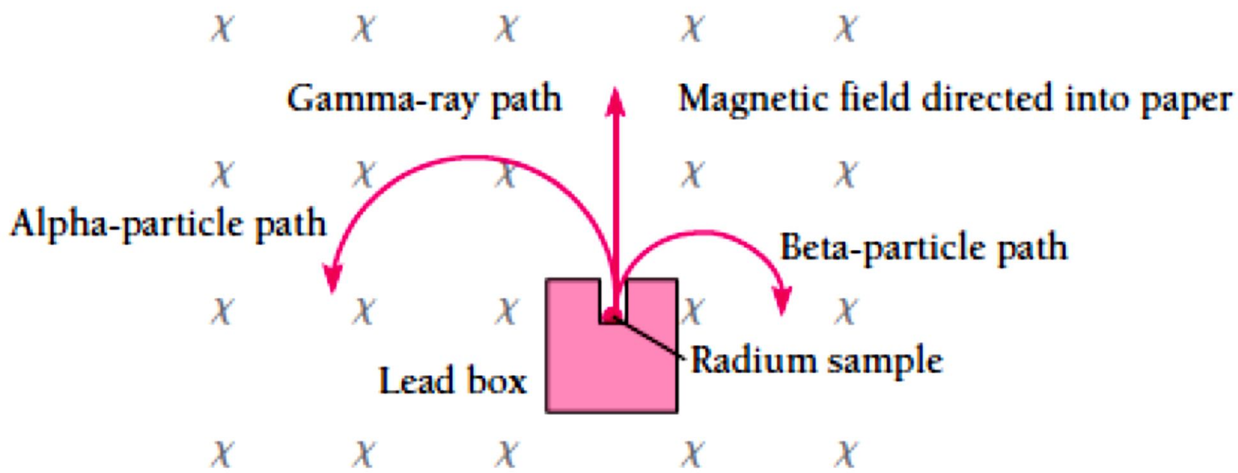
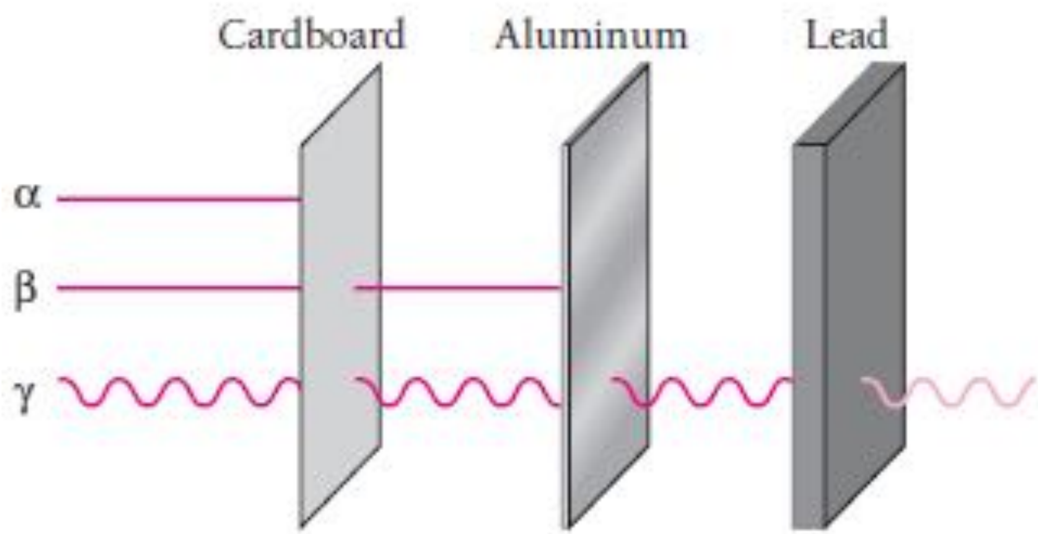

Gamma-ray path
Magnetic field directed into paper
Alpha-particle path
Beta-particle path
Radium sample
Lead box

Experiment 2:

In this experiment, one important property of these radiation was demonstrated.



- The alpha, beta particles and gamma ray were eventually identified as ${}_{2}^{4}He$ nuclei, electrons, and high-energy photons respectively.

- Later, positron emission and electron capture were added to the list of decay modes.

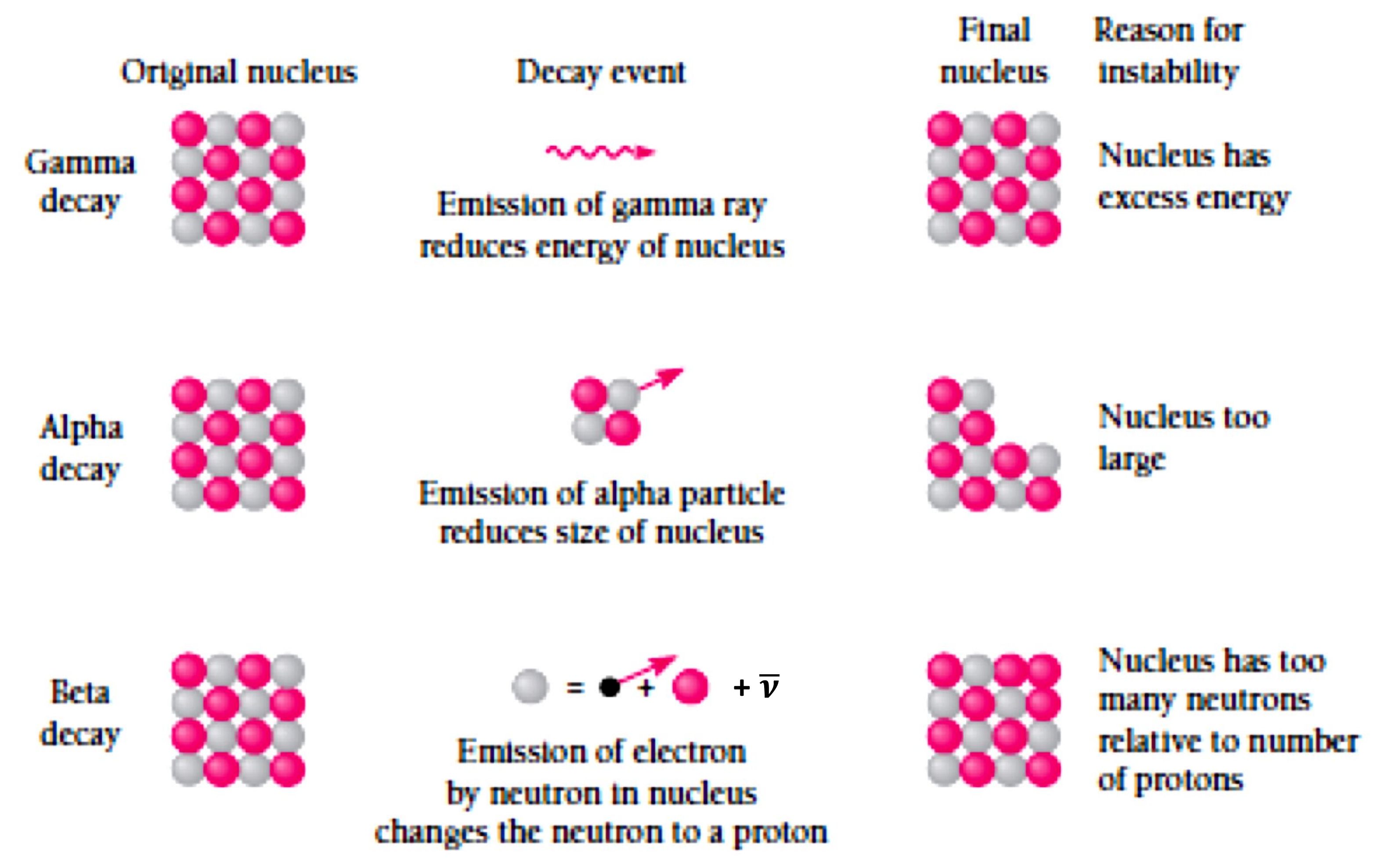
Table 12.1 Radioactive Decay†

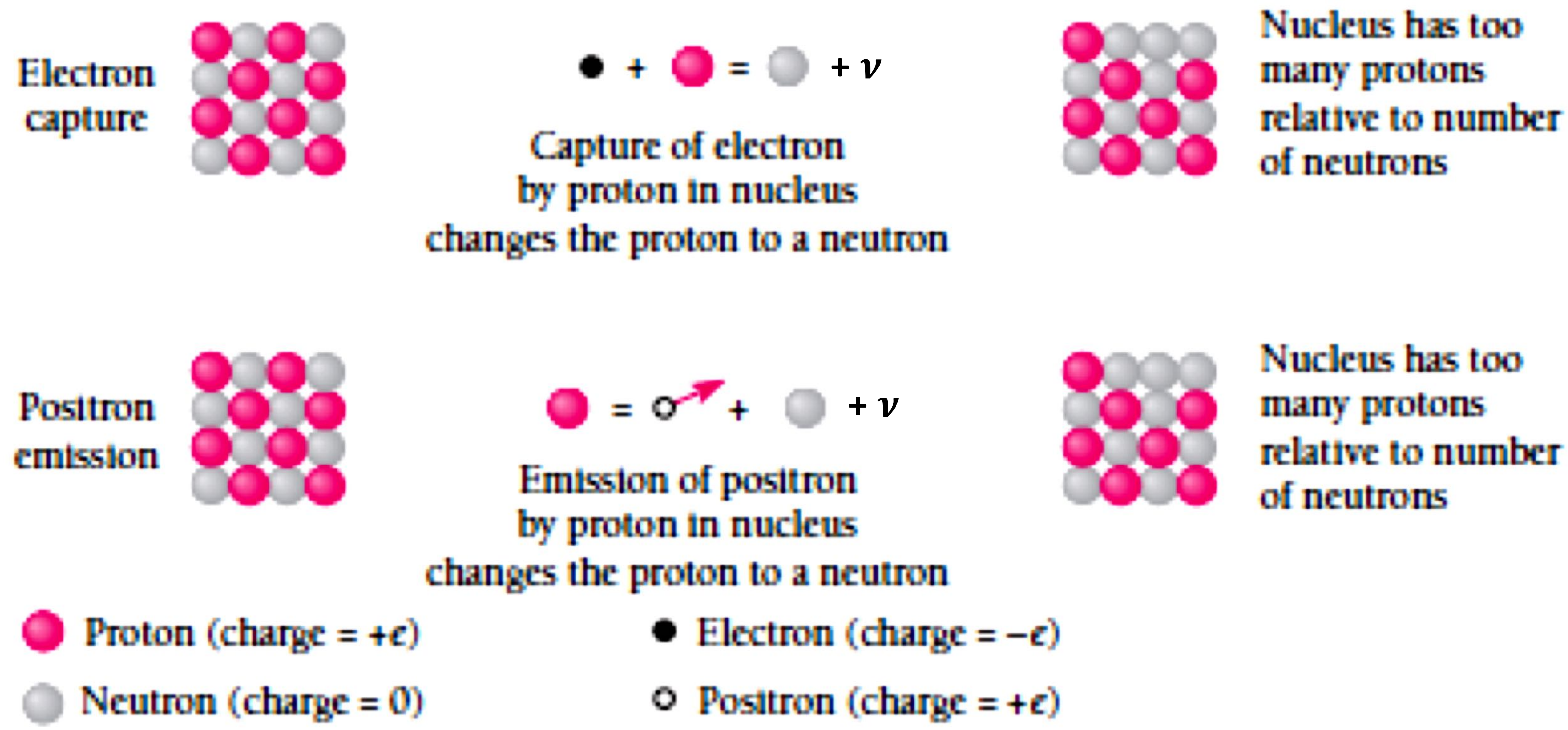
Decay	Transformation	Example
Alpha decay	${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$	${}_{29}^{238}U \rightarrow {}_{90}^{234}Th + {}_{2}^{4}He$
Beta decay	${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + e^{-} + \bar{\nu}$	${}_{6}^{14}C \rightarrow {}_{7}^{14}N + e^{-}$
Positron emission	${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + \nu$	${}_{29}^{64}Cu \rightarrow {}_{28}^{64}Ni + e^{+}$
Electron capture	${}_{Z}^{A}X + e^{-} \rightarrow {}_{Z-1}^{A}Y + \nu$	${}_{29}^{64}Cu + e^{-} \rightarrow {}_{28}^{64}Ni$
Gamma decay	${}_{Z}^{A}X^{*} \rightarrow {}_{Z}^{A}X + \gamma$	${}_{38}^{87}Sr^{*} \rightarrow {}_{38}^{87}Sr + \gamma$

†The * denotes an excited nuclear state and γ denotes a gamma-ray photon.

Why does Radioactivity occur?

Five ways by which an unstable nucleus can decay, to be stable:





Electron capture
+ = + ν
Capture of electron by proton in nucleus changes the proton to a neutron
Nucleus has too many protons relative to number of neutrons
Positron emission
= + + ν
Emission of positron by proton in nucleus changes the proton to a neutron
Nucleus has too many protons relative to number of neutrons
Proton (charge = +e)
Electron (charge = −e)
Neutron (charge = 0)
Positron (charge = +e)

Thus, **Radioactive decay** is a process by which the nuclei of a nuclide emit α, β or γ rays to be more stable.

Type of Radioactivity:

1. If the emission of radiation takes place due to disintegration of naturally occurring heavy elements, it is called as *natural radioactivity*

2. Radioactivity can also be induced by producing artificial transmutation of elements in laboratory and is called as *induced or artificial radioactivity*

Radioactive decay

- Assuming **λ** as the probability per unit time for the decay of each nucleus of a given nuclide, **λ** *dt* is the probability that any nucleus will undergo decay in a time interval *dt*.

- If a sample contains *N* undecayed nuclei, the number *dN* that decay in a time *dt* is the product of the number of nuclei *N* and the probability **λ** *dt* that each will decay in *dt*. That is,

$$dN = -N\lambda \, dt$$

where the minus sign is needed because *N* decreases with increasing *t*.

$$\frac{dN}{N} = -\lambda \, dt$$

On integrating each side:

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{0}^{t} dt$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\boldsymbol{N = N_0 e^{-\lambda t}}$$

Radioactive decay

- Radioactive decay is statistical in nature.
- There is no way to know in advance which nuclei will actually decay in a particular time span.
- If the sample is large enough, the actual fraction of it that decays in a certain time span will be very close to the probability for any individual nucleus to decay.

Activity

- *Activity* of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms decay.

- If N is the number of nuclei present in the sample at a certain time, its activity R is given by

$$\boldsymbol{R = -\frac{dN}{dt}} \qquad \textbf{Activity}$$

- The minus sign is used to make R a positive quantity since dN/dt is, of course, intrinsically negative.

<u>The unit of activity: Becquerel (Bq) and Curie (Ci)</u>

We know, the activity of a radioactive sample is defined as

$$R = -\frac{dN}{dt}$$

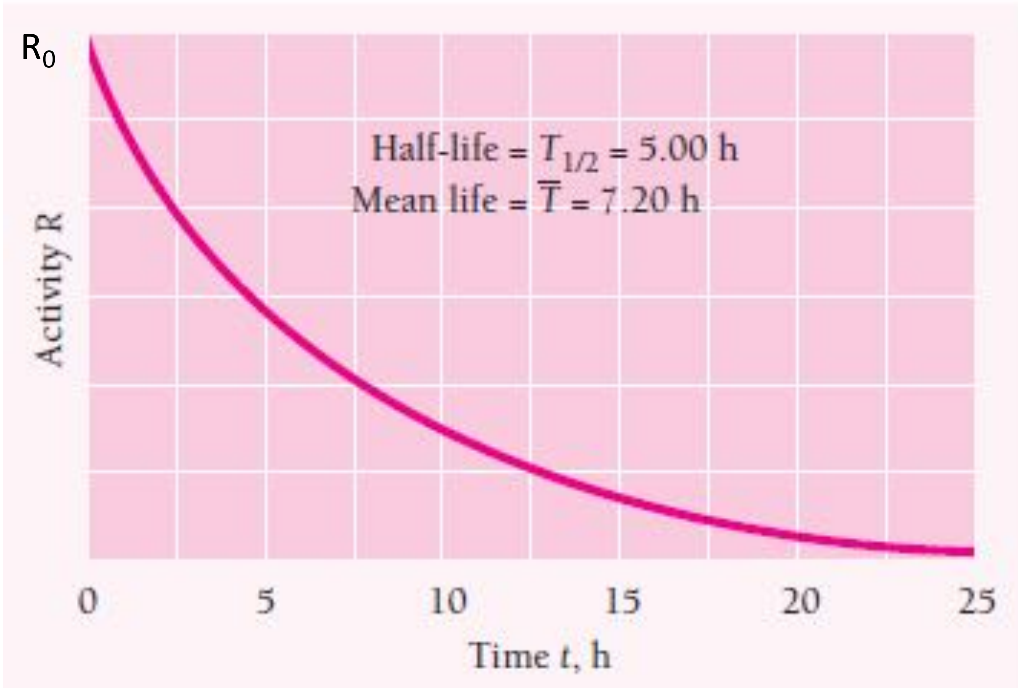
Using radioactive decay formula, $N = N_0 e^{-\lambda t}$, we can rewrite above expression as

$$R = -\frac{d\left(N_0 e^{-\lambda t}\right)}{dt}$$

$$R = \lambda N_0 e^{-\lambda t}$$

$$\boldsymbol{R = \lambda N}$$ **Activity**

Activity law/ Law of disintegration



Graph of R versus t for a typical radionuclide

Half-life$\left(T_{1/2}\right)$

- The half-life is the time needed for an initial activity to drop by half.
- After a half-life has elapsed, that is, when $t = T_{1/2}$, the activity of radionuclide drops R to $\frac{R_0}{2}$

$$\frac{R_0}{2} = R_0 e^{-\lambda T_{1/2}}$$

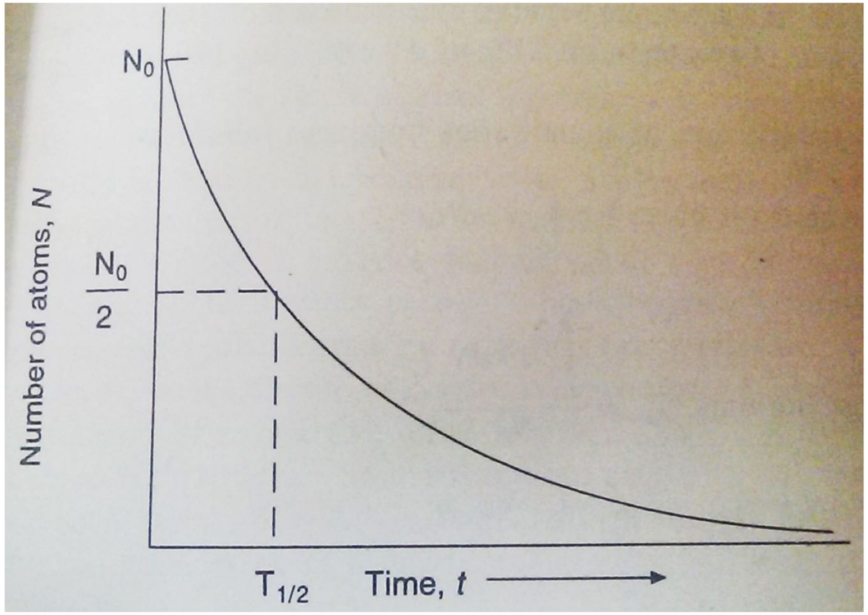
$$e^{\lambda T_{1/2}} = 2$$

Taking natural logarithms of both sides of this equation,

$$\lambda T_{1/2} = \ln 2$$

$$T_{1/2} = \frac{ln\,2}{\lambda} = \frac{0.693}{\lambda}$$

Half-life



N_0
$\frac{N_0}{2}$
Number of atoms, N
$T_{1/2}$
Time, t

Decay constant, λ

- The decay constant, λ, is defined as reciprocal of time during which number of atoms left are equal to 1/e times(~37%) of original number of atoms.

If , $N/N_0 = 1/e,$

then, $e^{-\lambda t} = e^{-1}$

or $\boldsymbol{\lambda = 1/t}$ **decay constant, λ**

Mean lifetime $\bar{T}$

The mean lifetime of a nuclide is the reciprocal of its decay probability per unit time

$$\bar{T} = \frac{1}{\lambda}$$

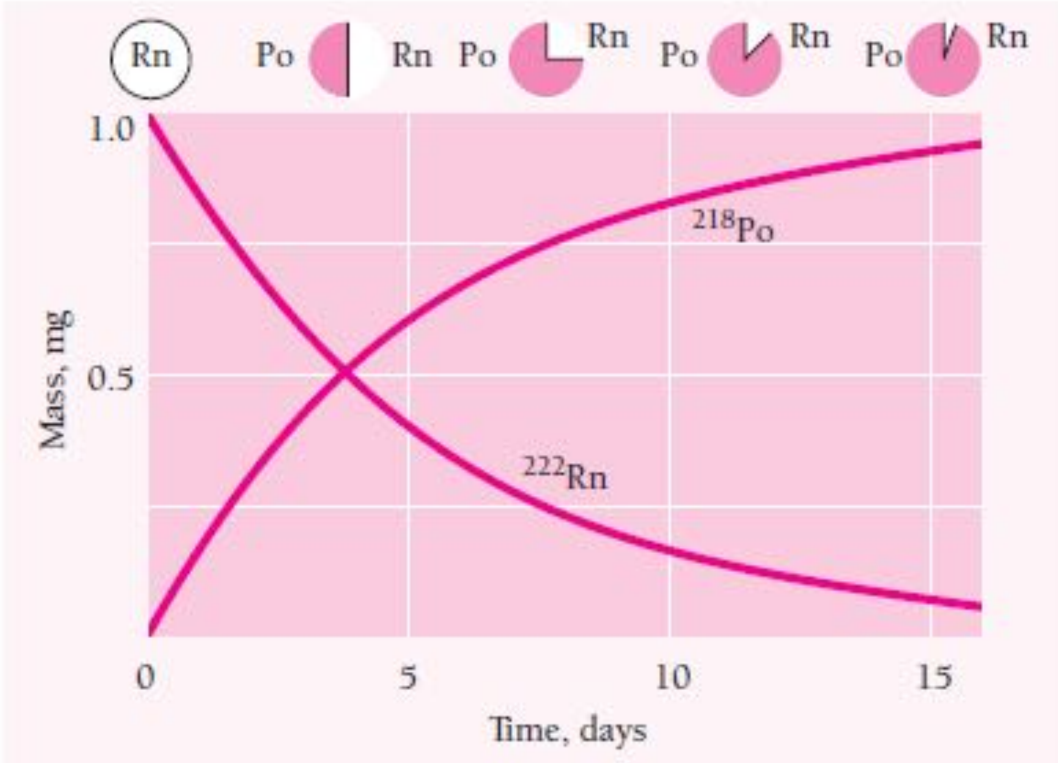
Mean lifetime

As, $\lambda = \frac{0.693}{T_{1/2}}$, hence,

$$\bar{T} = \frac{T_{1/2}}{0.693} = 1.44\, T_{1/2}$$

Ex. of Radioactive decay: The alpha decay of ^{222}Rn to ^{218}Po has a half-life of 3.8 d.

- Alpha decay of the gas radon, $^{222}_{86}R_n$, whose half-life is 3.82 days, to the polonium isotope $^{218}_{84}P_o$.
- If we start with 1.00 mg of radon in a closed container, 0.50 mg will remain after 3.82 days, 0.25 mg will remain after 7.64 days, and so on.



Exponential decay of a radionuclide.

$$N = N_0\left(\frac{1}{2}\right)^1 \quad \text{at } T_{1/2}$$

$$N = N_0\left(\frac{1}{2}\right)^2 \quad \text{at } 2T_{1/2}$$

$$N = N_0\left(\frac{1}{2}\right)^3 \quad \text{at } 3T_{1/2}$$

.

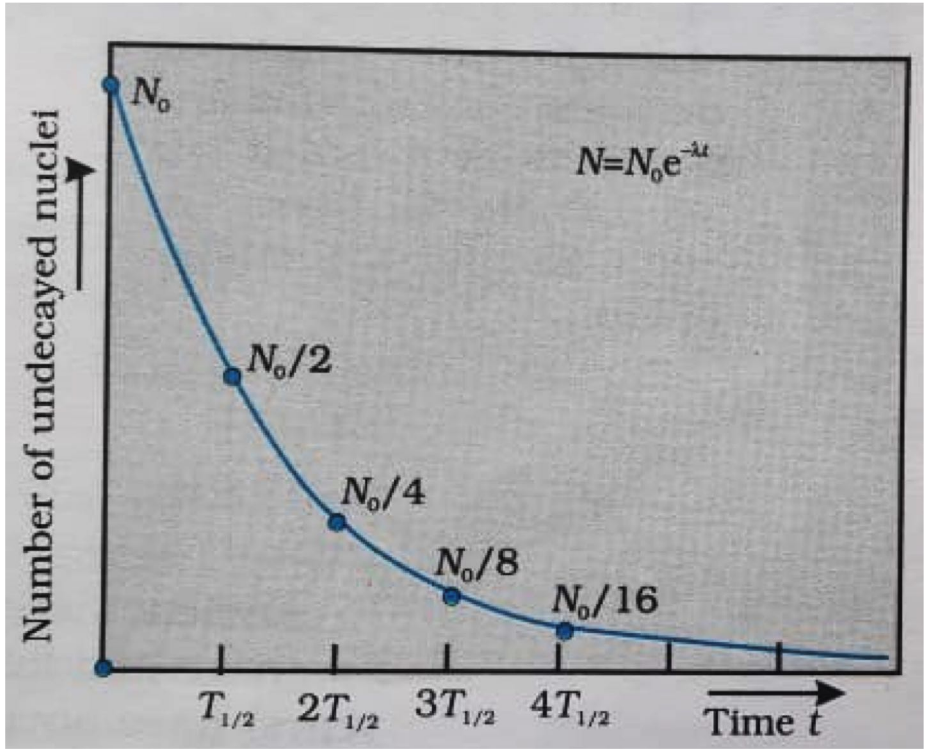
.

.

$$N = N_0\left(\frac{1}{2}\right)^n \quad \text{at } nT_{1/2}$$

$$\left(n = \frac{t}{T_{1/2}}\right)$$

$$N = N_0 e^{-\lambda t}$$



Radioactive Dating

- Using radioactivity, one can establish the ages of many geological and biological specimens.

- Because the decay of any particular radionuclide is independent of its environment, the ratio between the amounts of that nuclide and its stable daughter in a specimen depends on the latter's age.

- The greater the proportion of the daughter nuclide, the older the specimen.

- To date objects of biological origin, we will use **radiocarbon,** the beta-active carbon isotope $^{14}_{6}C$.

- When the age of a specimen is estimated from amount of $^{14}_{6}C$ radioisotope present in the specimen, it is called as **Carbon dating.**

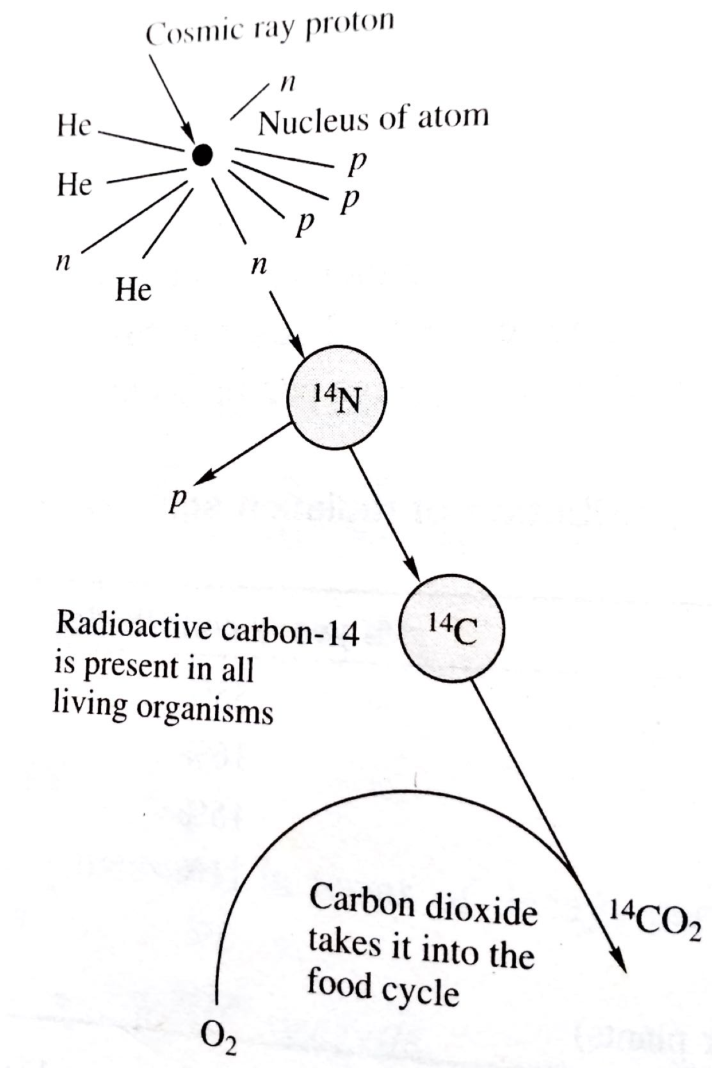
- Cosmic rays are high-energy atomic nuclei, chiefly protons, that circulate through the Milky Way galaxy.

- About 10^{18} of them reach the Earth each second.

- When they enter the Earth's atmosphere, they collide with the nuclei of atoms in their paths to produce showers of secondary particles.

- Among these, neutrons can react with nitrogen nuclei in the atmosphere to form radiocarbon with the emission of a proton:

$$ {}_0n^1 + {}_7N^{14} \rightarrow {}_6C^{14} + {}_1H^1 $$



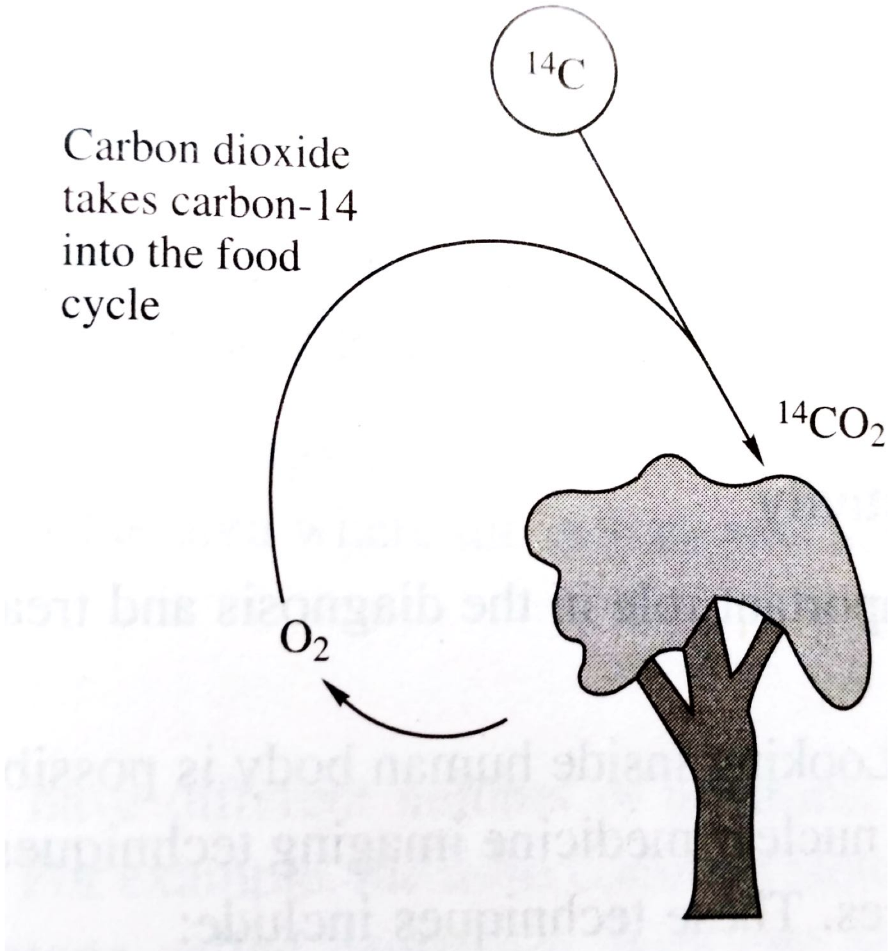
Absorption chain of ^{14}C in atmosphere

- After their formation, radiocarbon atoms combine with oxygen molecules to form CO_2 molecules.

- Green plants take in CO_2 and water which they convert into carbohydrates in the process of photosynthesis, so that every plant contains some radiocarbon.

- Animals eat plants and thereby become radioactive themselves.

- Because the mixing of radiocarbon is efficient, living plants and animals all have the same ratio of radiocarbon to ordinary carbon ($_6C^{12}$).



- When plants and animals die, they no longer take in radiocarbon atoms and the absorption of $_6C^{14}$ is stopped.

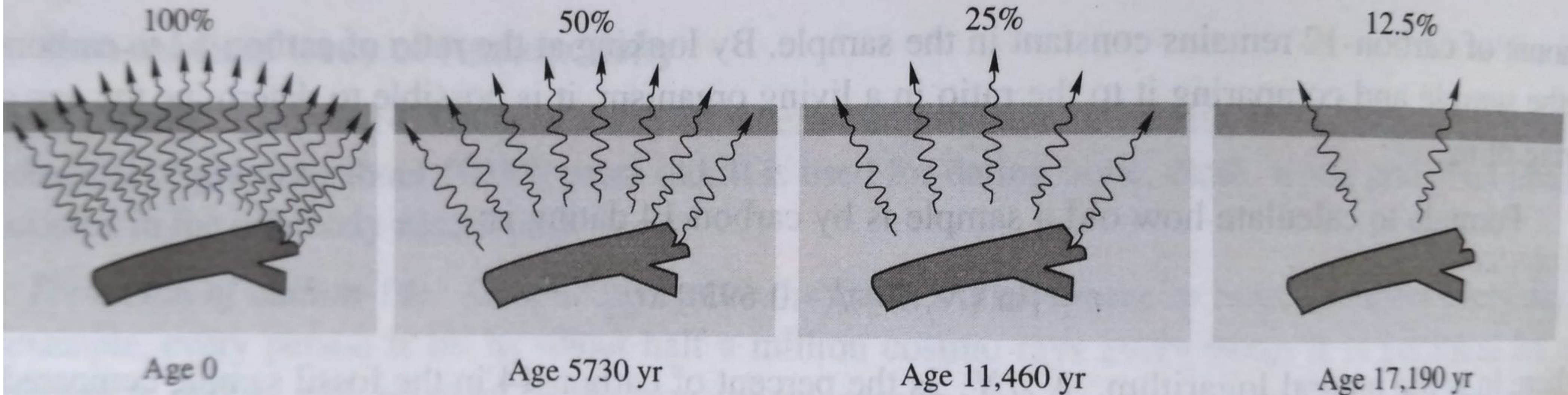
- But the radiocarbon, $_6C^{14}$ they contain keeps decaying away to $_7N^{14}$ by electron emission:

$$_6C^{14} \rightarrow \; _7N^{14} + e^{-}$$

- After 5760 years, then, they have only one-half as much radiocarbon left relative to their total carbon content as they had as living matter, after 11,520 years only one-fourth as much, and so on.

- By determining the proportion of radiocarbon($_6C^{14}$) to ordinary carbon ($_6C^{12}$) it is therefore possible to evaluate the ages of ancient objects and remains of organic origin.

- This method of carbon dating is extensively used to determine the age of old wood, paper, minerals, rocks etc.



. 7.11 *The concentration of carbon-14 is reduced with time.*

Radioactive Series

Most of the radionuclides found in nature are members of four radioactive series,
With each series consisting of a succession of daughter products all ultimately derived from a single parent nuclide.

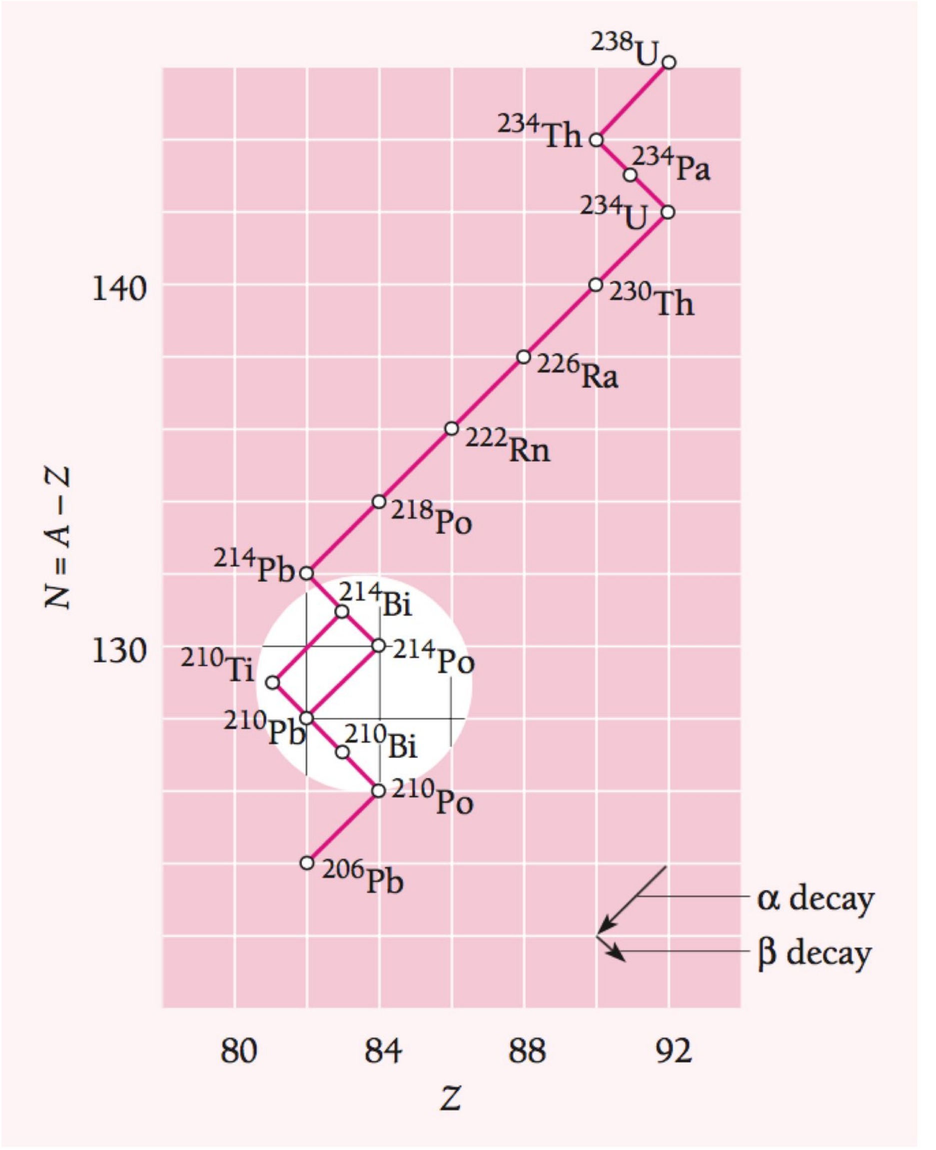
Alpha decay reduces the mass number of a nucleus by 4—Hence there are exactly four series.

Mass Numbers	Series	Parent	Half-Life, Years	Stable End Product
$4n$	Thorium	$^{232}_{90}Th$	1.39×10^{10}	$^{208}_{82}Pb$
$4n + 1$	Neptunium	$^{237}_{93}Np$	2.25×10^{6}	$^{209}_{83}Bi$
$4n + 2$	Uranium	$^{238}_{92}U$	4.47×10^{9}	$^{206}_{82}Pb$
$4n + 3$	Actinium	$^{235}_{92}U$	7.07×10^{8}	$^{207}_{82}Pb$

Nuclides whose mass numbers are all given by $A = 4n$, where n is an integer, can decay into one another in descending order of mass number.

Similarly, other series have mass numbers : $A = 4n + 1, 4n + 2, 4n + 3$

The half-life of neptunium is so short compared with the age of the solar system that members of this series are not found on the earth today.



238U
234Th
234Pa
234U
230Th
226Ra
222Rn
218Po
214Pb
214Bi
214Po
210Ti
210Pb
210Bi
210Po
206Pb
140
130
N = A − Z
80
84
88
92
Z
α decay
β decay

The intermediate members of each decay series have much shorter half-lives than their parent nuclide.

As a result, if we start with a sample of N_A nuclei of a parent nuclide A, after a period of time an equilibrium situation will come about in which each successive daughter B,C. . . decays at the same rate as it is formed.

Thus the activities R_a, R_b, R_c …. are all equal at equilibrium.

Since $R = \lambda N$ we have:

$$N_A\lambda_A = N_B\lambda_B = N_C\lambda_C = \ldots.. \quad (1)$$ **"Radioactive equilibrium"**

Each number of atoms N_a, N_b, N_c …. decreases exponentially with the decay constant λ_A of the parent nuclide, but equation (1) remains valid at any time.

Nuclear Reactions

When two nuclei come together, a nuclear reaction can occur that results in new nuclei being formed.

In the sun and other stars, whose internal temperatures range up to millions of kelvins, many nuclei present have high enough speeds for reactions to be frequent. Indeed, the reactions provide the energy that maintains these temperatures.

A typical nuclear reaction is written as

$$\mathrm{a} + X \longrightarrow Y + b$$

A compact way of indicating the same reaction : X(a,b)Y

(a: accelerated projectile, X: target (usually stationary in the lab),
Y & b: reaction products.)

Classification of nuclear reactions

- Scattering process : if the incident and outgoing particles are the same and correspondingly X and Y are the same nucleus.

  - *Elastic scattering*: if the KE of the system (X and a) before the event is the same as that of the system (Y and b) after the event.

  - *Inelastic scattering*: if Y or b is in an excited state from which it will generally decay quickly by γ emission.

- Knockout reaction: a & b are the same particle, but the reaction causes another nucleon to be ejected separately.

❖ Photo-disintegration: Photo transmutation

An energetic photon is absorbed by the target nucleus which disintegrates the nucleus.

Ex: γ-reaction

Photodisintegration or γ Reaction

(i) (γ, p) reaction

$$\gamma + {}_4\mathrm{Be}^9 \rightarrow {}_3\mathrm{Li}^8 + {}_1\mathrm{H}^1$$

$$\gamma + {}_{12}\mathrm{Mg}^{25} \rightarrow {}_{11}\mathrm{Na}^{24} + {}_1\mathrm{H}^1$$

(ii) (γ, n) reaction

$$\gamma + {}_1\mathrm{H}^2 \rightarrow {}_1\mathrm{H}^1 + {}_0n^1$$

$$\gamma + {}_{15}\mathrm{P}^{31} \rightarrow {}_{15}\mathrm{P}^{30} + {}_0n^1$$

$$\gamma + {}_4\mathrm{Be}^9 \rightarrow {}_4\mathrm{Be}^8 + {}_0n^1$$

❖ Radioactive capturing: neutron capturing:

The incident particle is captured by the target nucleus which disintegrates to produce a gamma ray.

Ex:*(n,γ)* reaction:

(n, γ) reaction or radiative capture

$$_0n^1 + {}_1H^1 \rightarrow {}_1H^2 + \gamma$$

$$_0n^1 + {}_{49}In^{115} \rightarrow {}_{49}In^{116} + \gamma$$

$$_0n^1 + {}_{92}U^{238} \rightarrow {}_{92}U^{239*} + \gamma$$

❖ Transfer reaction:

One or more nucleons are transferred to the target nucleus

1. Stripping reaction: Part of the incident nucleus combines with the target nucleus and remaining stripped nucleus (incident) continues with its original momentum in nearly same direction.

 Ex- *(d,p) reactions:*

(i) (*d*, *p*) reaction

$${}_1H^2 + {}_3Li^7 \rightarrow {}_3Li^8 + {}_1H^1$$

$${}_1H^2 + {}_6C^{12} \rightarrow {}_6C^{13} + {}_1H^1$$

$${}_1H^2 + {}_{15}P^{31} \rightarrow {}_{15}P^{32} + {}_1H^1$$

2. Pick-up reaction:

One or more nucleons are picked by the incident nucleus from the target nucleus

Ex: (p,d) reaction

(*p*, *d*) reaction

$$_1H^1 + {}_3Li^7 \rightarrow {}_3Li^6 \rightarrow + {}_1H^2$$

$$_1H^1 + {}_4Be^9 \rightarrow {}_4Be^8 + {}_1H^2$$

- Fission reaction: Neutrons produce fission reaction in heavy nuclei which disintegrate into two nuclei of comparable size along with emission of neutrons and release of energy.

- Fusion reaction: Lighter nuclei fuse to form heavy nuclei along with release of energy.

According to Bohr's theory (1936) : nuclear reactions involve two separate stages:

First stage: An incident particle (A) strikes a target nucleus (B) and the two combine to form a new nucleus, called a compound nucleus (C^*).

$$A + B. \longrightarrow C^* \longrightarrow D + E$$

Second stage: The compound nucleus (C^*) de-excites within a short time interval of about $10^{-16}s$ into a product nucleus D and emits a particle E and $\gamma - radiation.$

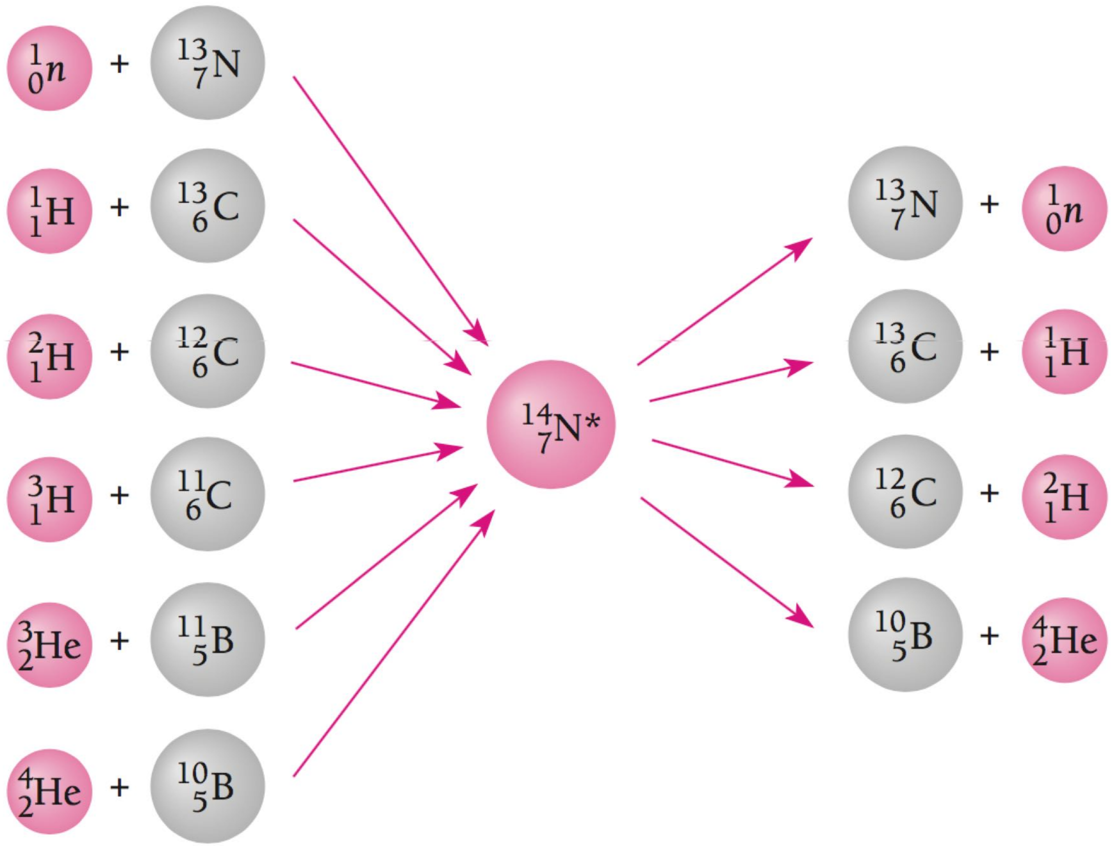
Its atomic and mass numbers are respectively the sum of the atomic numbers of the original particles and the sum of their mass numbers.

Compound nuclei have lifetimes on the order of $10^{-16}s$ (long relative to ($10^{-21}s$) a nuclear particle with an energy of several MeV would need to pass through a nucleus)

A compound nucleus has no memory of how it was formed, since its nucleons are mixed together regardless of origin and the energy brought into it by the incident particle is shared among all of them.

A given compound nucleus may therefore be formed in a variety of ways.

Depending on the excitation energy of a compound nucleus it may decay in one or more ways.

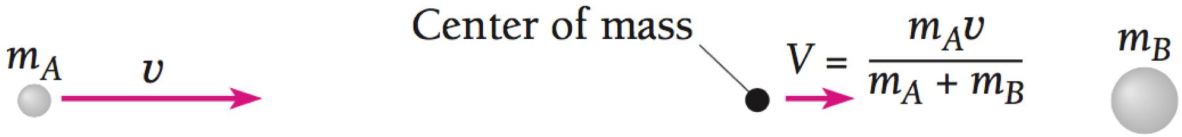


- $^{14}_{7}N^{*}$ can simply lose its excitation energy by emitting one or more gamma rays.

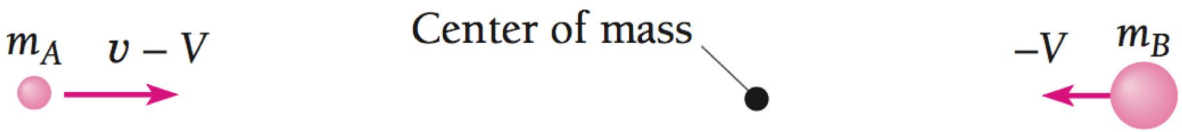
- It cannot decay by the emission of a triton $(^{3}_{1}H)$ or a helium-3 $\left(^{3}_{2}He\right)$ particle since it does not have enough energy to liberate them.

An excited nucleus is analogous to a drop of liquid

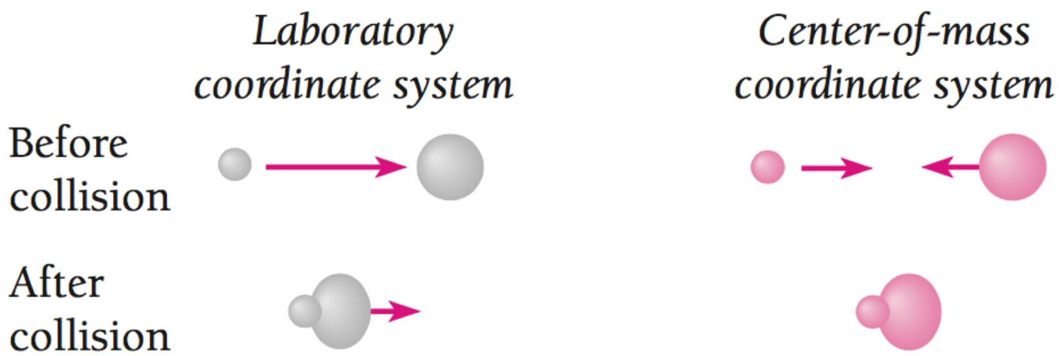
How?



(a) Motion in the laboratory coordinate system before collision



(b) Motion in the center-of-mass coordinate system before collision



(c) A completely inelastic collision as seen in laboratory and center-of-mass coordinate systems

Laboratory & center-of-mass coordinate systems.

If a particle of mass m_A and speed v approaches a stationary particle of mass m_B as viewed by an observer in the laboratory, the speed V of the center of mass is defined by the condition

$$m_A(v - V) = m_B V$$

$$\therefore \text{speed of center of mass: } V = \left(\frac{m_A}{m_A + m_B}\right) v$$

Total kinetic energy in lab system : $KE_{lab} = \frac{1}{2} m_A v^2$

In the center-of-mass, both particles are moving and contribute to the total kinetic energy:

$$KE_{cm} = \frac{1}{2} m_A (v - V)^2 + \frac{1}{2} m_B V^2$$

$$= \frac{1}{2} m_A v^2 + \frac{1}{2}(m_A + m_B) V^2$$

$$\boldsymbol{KE_{cm} = KE_{lab} - \frac{1}{2}(m_A + m_B) V^2}$$

KE in CM system: $$\boldsymbol{KE_{cm} = \left(\frac{m_B}{m_A + m_B}\right) KE_{lab}}$$

Thus we can regard $\boldsymbol{KE_{cm}}$ as the kinetic energy of the relative motion of the particles. When the particles collide, the maximum amount of KE that can be converted to excitation energy of the resulting compound nucleus while still conserving momentum is $\boldsymbol{KE_{cm}}$ which is always less than $\boldsymbol{KE_{lab}}$.

Q value of nuclear reaction

The Q value of the nuclear reaction

$$A + B \longrightarrow C + D$$

is defined as the difference between the rest energies of A and B and the rest energies of C and D:

$$Q = (m_A + m_B - m_c - m_D)c^2$$

If Q is a positive quantity, energy is given off by the reaction (exothermic reaction)
If Q is a negative quantity, enough kinetic energy KE_{cm} in the center-of-mass system must be provided by the reacting particles so that $KE_{cm} + Q \geq 0$ (endothermic reaction)
The minimum KE needed for endothermic reaction to occur is "threshold energy"

Conservation Laws:

1. Conservation of total energy

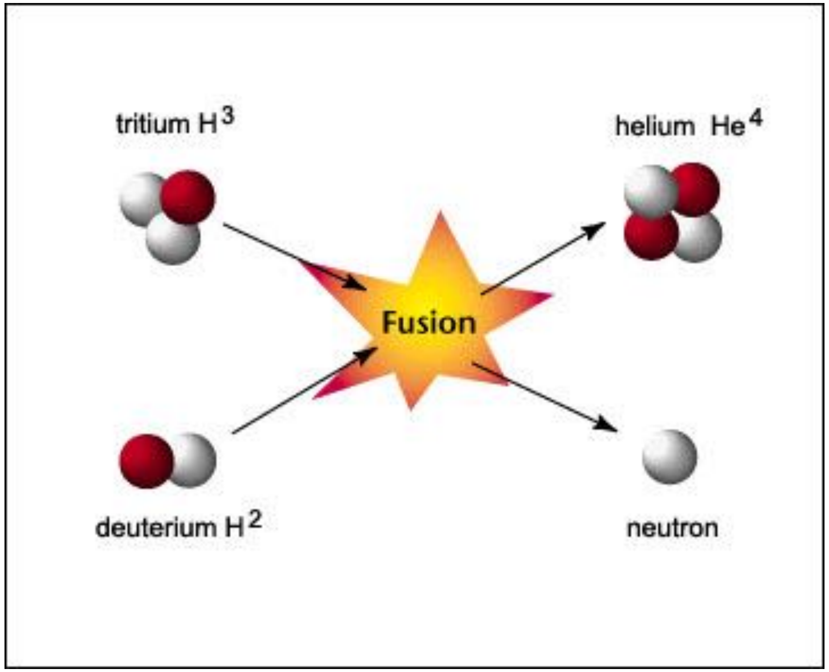
2. Conservation of linear momentum & angular momentum

3. Conservation of proton and neutron number: At high energies (> 1 GeV) we conserver total nucleon number, but at low energies (~ 10 MeV per nucleon or less) we conserve separately proton number and neutron number.

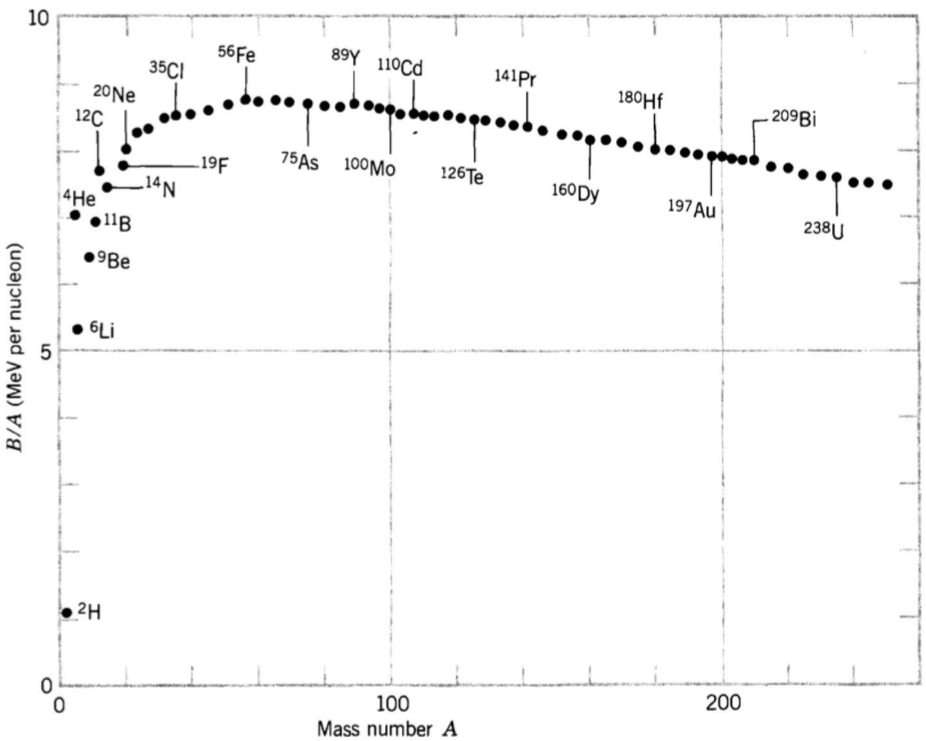
4. Conservation of charge

5. Conservation of parity: net parity before the reaction must equal the net parity after the reaction.

Nuclear Fusion



tritium H^3
helium He^4
Fusion
deuterium H^2
neutron



10
5
0
B/A (MeV per nucleon)
0
100
200
Mass number A
2H
6Li
9Be
11B
4He
14N
12C
19F
20Ne
35Cl
56Fe
75As
89Y
100Mo
110Cd
126Te
141Pr
160Dy
180Hf
197Au
209Bi
238U

Basic Fusion Processes

The most elementary fusion reaction,

$$p + p \rightarrow 2_{He}$$

is not possible due to the instability of 2_{He}.

Another elementary reaction is

$$2_H + 2_H \rightarrow 4_{He} + \gamma \qquad (Q = 23.8 \text{ MeV})$$

More likely reactions are thus:

$$2_H + 2_H \rightarrow 3_{He} + n \; (Q = 3.3 \text{ MeV})$$
$$2_H + 2_H \rightarrow 3_H + p \; (Q = 4.0 \text{ MeV})$$

"Deuterium-Deuterium (D-D) Reaction"

A reaction that forms 4_{He} would be likely to show a particularly large energy release:

$$2_H + 3_H \rightarrow 4_{He} + n \quad (\mathrm{Q} = 17.6\ \mathrm{MeV})$$

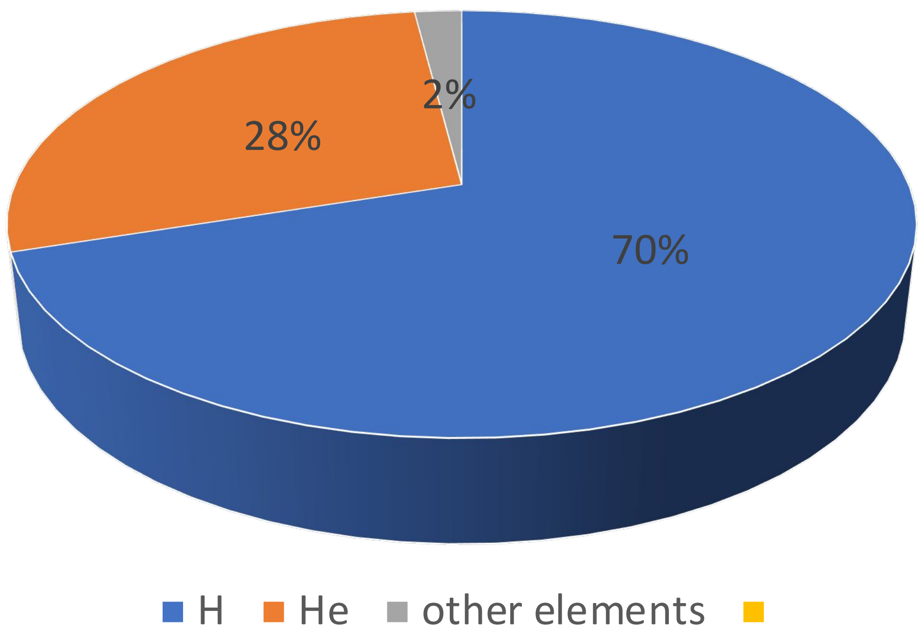
"Deuterium-Tritium (D-T) Reaction"

If the incident particles have negligibly small KE, the 4_{He} and n share 17.6 MeV consistent with linear momentum conservation and a monoenergetic neutron with energy 14.1 MeV emerges. This reaction often serves as a source of fast neutrons. Because of the large energy release, the D-T reaction has been selected for use in controlled fusion reactors.

Nuclear Fusion in Stars

- The basic process in the sun : fusion of hydrogen nuclei into helium nuclei.

Composition of the sun:



Nuclear Fusion in Stars

- The basic energy-producing process in the sun is the fusion of hydrogen nuclei into helium nuclei. The sun consists of 70 % hydrogen, 28% helium and 2% other elements. Most of the energy production takes place in the Sun's interior, where the temperature is ~ $1.5 \times 10^7 K$. Because such high temperatures are required to drive these reactions, they are called **thermonuclear fusion reactions.**

- At the $10^7 K$ temperature typical of the sun's interior, the average proton kinetic energy is only about 1 keV, whereas the barrier is about 1 MeV, a thousand times higher.

- The high temperature ensures that some nuclei have the energy needed to come close enough together to interact, which they do by tunneling through the electric potential barrier between them.

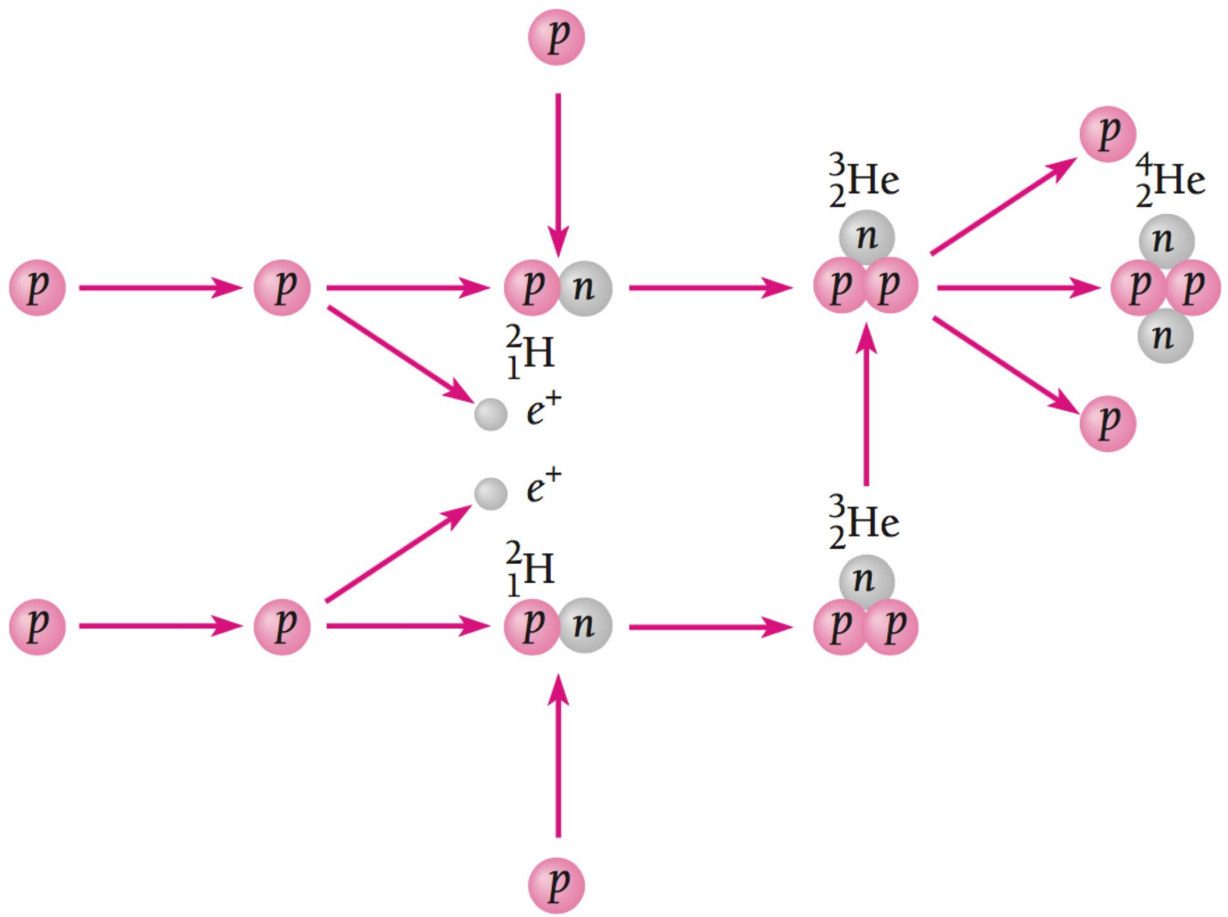
- The high density ensures that such collisions are frequent.

- Another condition for multistep cycles is a large reacting mass, such as that of the sun, since much time may elapse between the initial fusion of a particular proton and its eventual incorporation in an alpha particle.

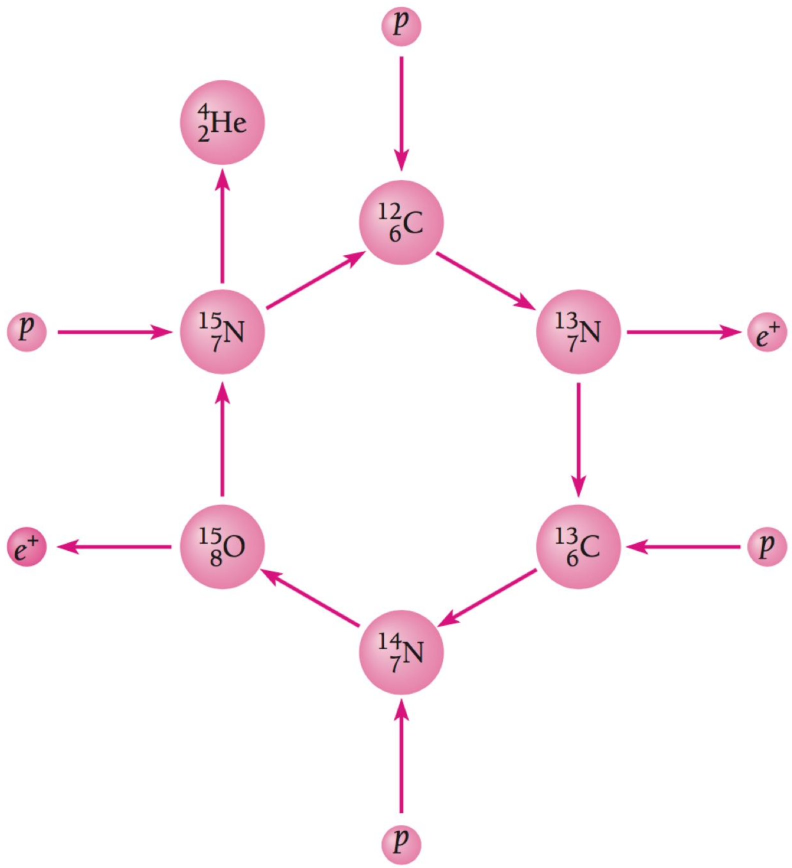
- There are two different reaction sequences in which fusion of hydrogen nuclei into helium nuclei occurs:

  - The Proton-proton cycle
  - The carbon cycle

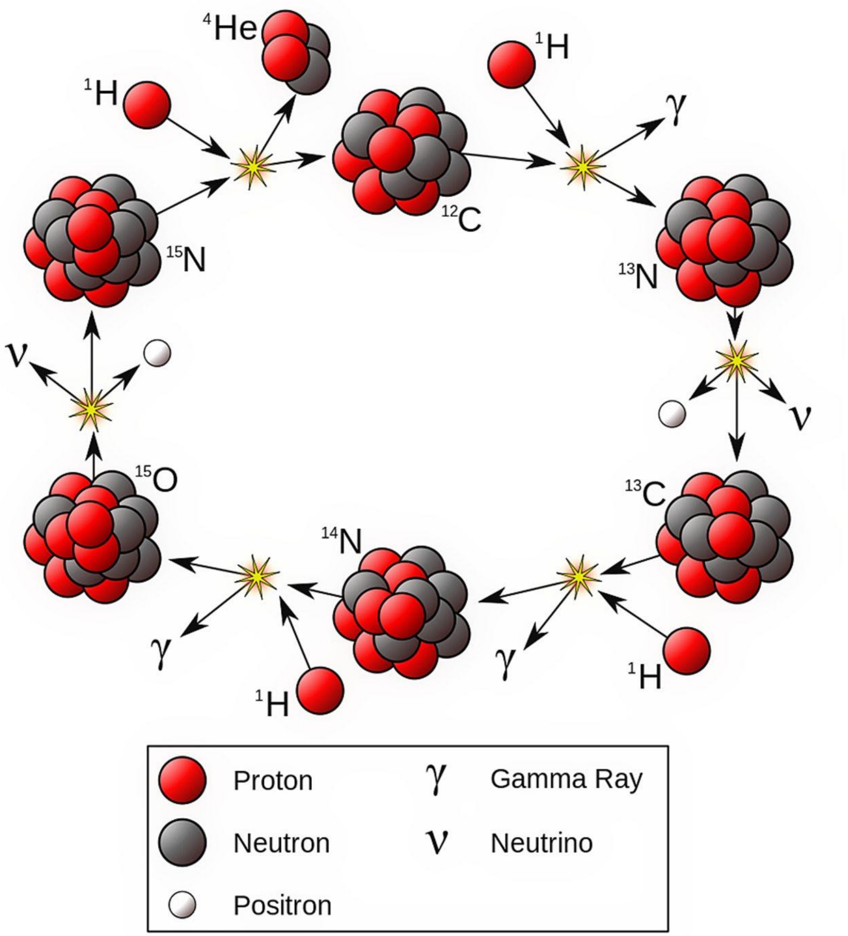
The Proton-Proton cycle



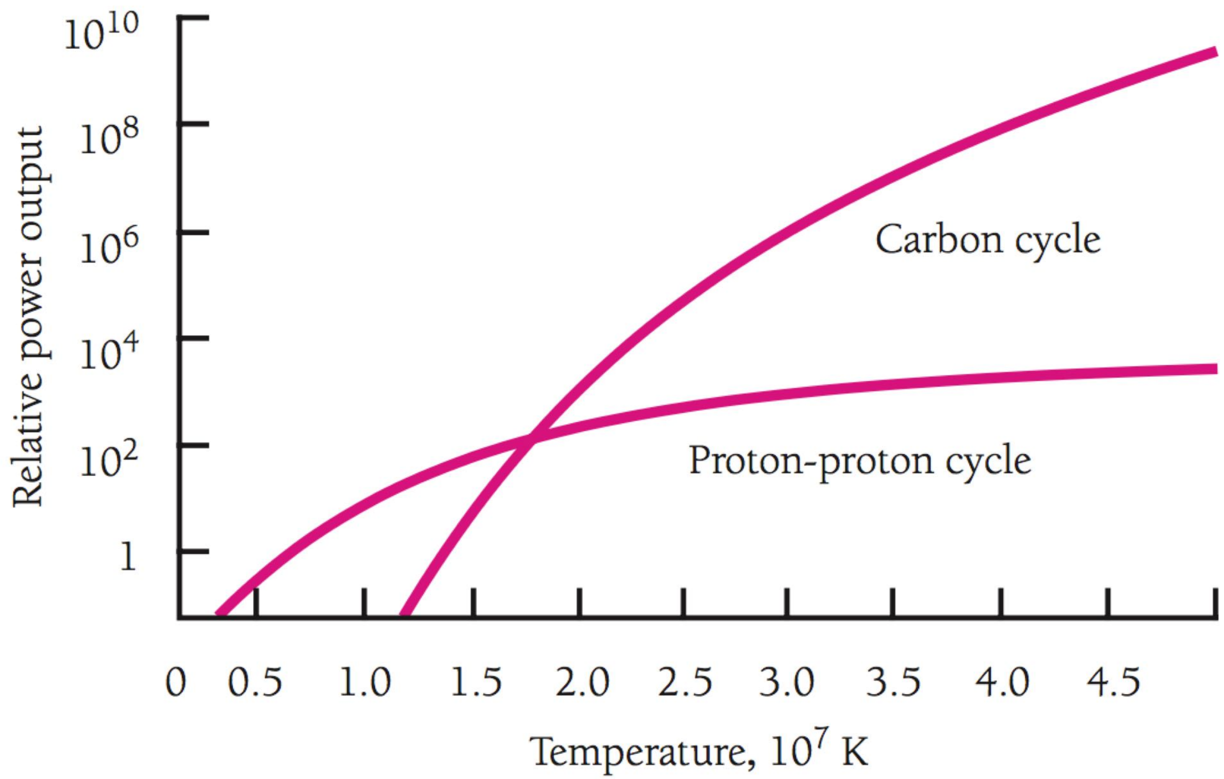
The carbon cycle



The carbon cycle



Rates of energy generation



Formation of Heavier Elements

When all the hydrogen in a star's core has become helium, gravitational contraction compresses the core and raises its temperature to be 10^8 K needed for helium fusion to begin. This involves the combination of three alpha particles to form a carbon nucleus with the evolution of 7.5 MeV:

$$ {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{8}_{4}\mathrm{Be} + \gamma $$

$$ {}^{4}_{2}\mathrm{He} + {}^{8}_{4}\mathrm{Be} \rightarrow {}^{12}_{6}\mathrm{C} + \gamma $$

Triple-alpha reaction

Because the beryllium isotope is unstable and breaks apart into two alpha particles with a half-life of only $6.7 \times 10^{-17}\ s$, the second reaction must take place immediately after the first.

The smallest stars do not get hot enough (over $10^7\ K$) to go beyond hydrogen fusion, and helium fusion is as far as a star with the sun's mass gets. But in heavier stars, core temperatures can go even higher, and fusion reactions that involve carbon then become possible. Some examples are:

$$^{4}_{2}\mathrm{He} + ^{12}_{6}\mathrm{C} \rightarrow ^{16}_{8}\mathrm{O}$$

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{24}_{12}\mathrm{Mg}$$

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{20}_{10}\mathrm{Ne} + ^{4}_{2}\mathrm{He}$$

The heavier the star, the higher the eventual temperature of its core, and the larger the nuclei can be formed.

In stars more than about 10 times as massive as the sun, the iron isotope $^{56}_{26}Fe$ is reached. Any reaction between a $^{56}_{26}Fe$ nucleus and another nucleus will therefore lead to the breakup of the iron nucleus, not to the formation of a still heavier one. Nuclides beyond $^{56}_{26}Fe$ originate through the successive capture of neutrons, with beta decays when needed for appropriate neutron/proton ratios. The neutrons are liberated in such sequences:

$$^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma$$

$$^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + e^{+} + \nu$$

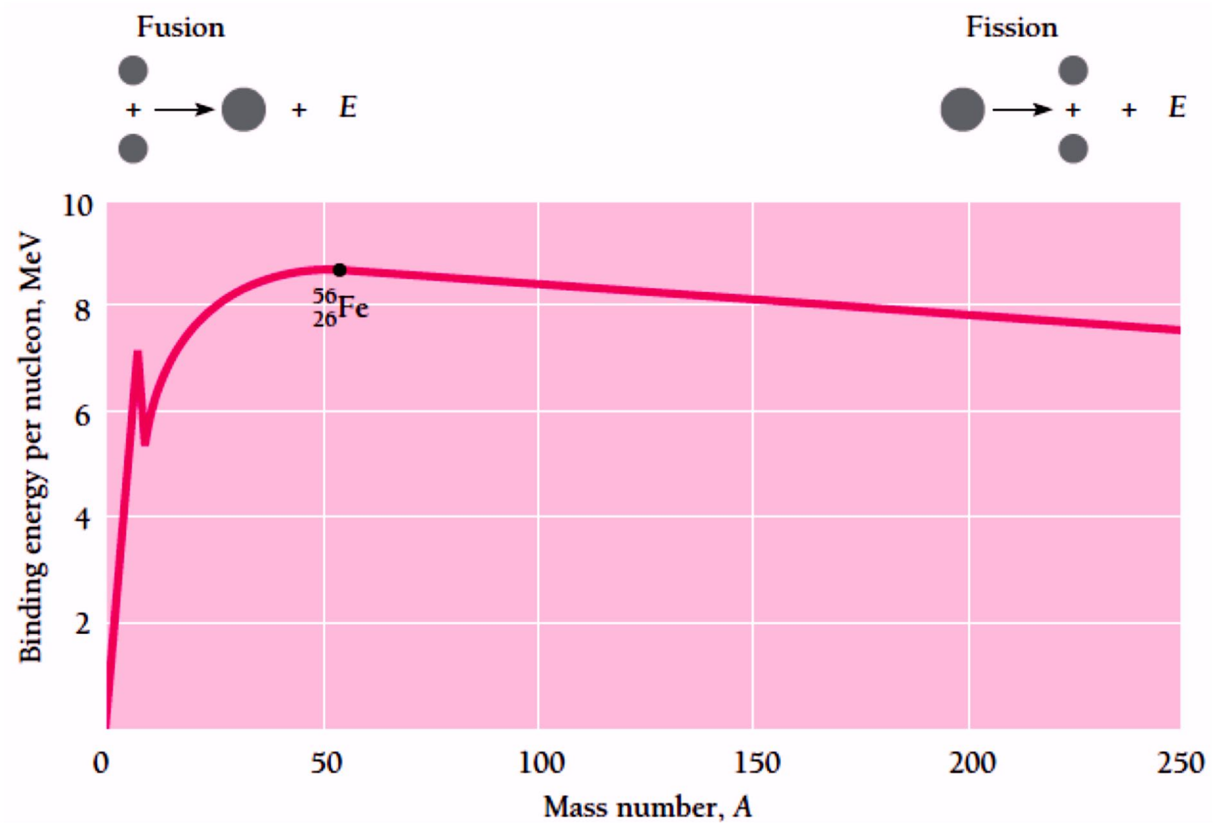
$$^{4}_{2}\mathrm{He} + {}^{13}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + {}^{1}_{0}n$$

Neutron-capture reactions in a stellar interior can build up nuclides as far as ${}^{209}_{83}Bi$ the largest stable nucleus, but no further. The density of neutrons there is not sufficient for them to be captured in rapid enough succession by nuclei of A > 209 before such nuclei decay.

When a very massive star has reached the end of its fuel supply, its core collapses and a violent explosion follows that appears in the sky as a supernova.

NUCLEAR FISSION

Divide and conquer

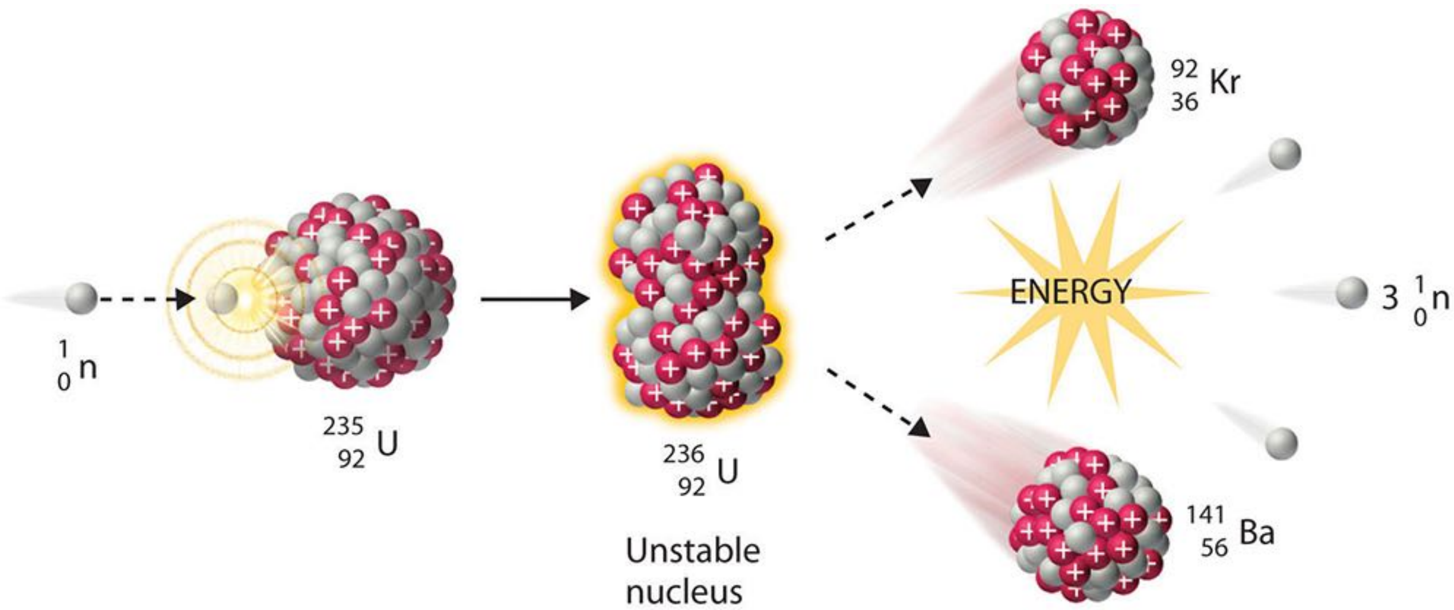


Fusion
+
+
E
Fission
+
+
E
10
8
6
4
2
0
50
100
150
200
250
Binding energy per nucleon, MeV
Mass number, A
$^{56}_{26}Fe$

- Following the discovery of the neutron by Chadwick in 1932, it was a natural next step to study the effects of exposing various nuclei to neutrons.

- Further work revealed that many intermediate-mass nuclei formed in the bombardment of uranium by neutrons and the energy released following neutron capture was very large, of the order of 100 MeV.

- In 1938, Lise Meitner observed that a nucleus of the uranium isotope $^{235}_{92}U$ undergoes fission when struck by a neutron (following neutron Capture).

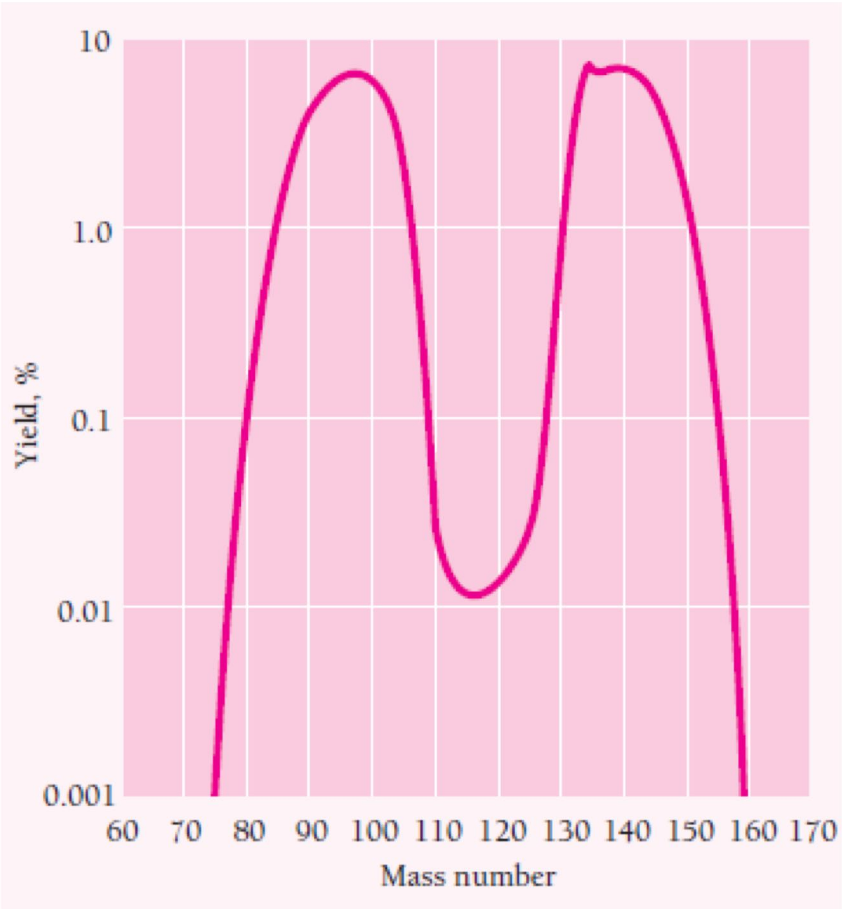


$^{1}_{0}n$
$^{235}_{92}U$
$^{236}_{92}U$
Unstable nucleus
$^{92}_{36}Kr$
ENERGY
$3\ ^{1}_{0}n$
$^{141}_{56}Ba$

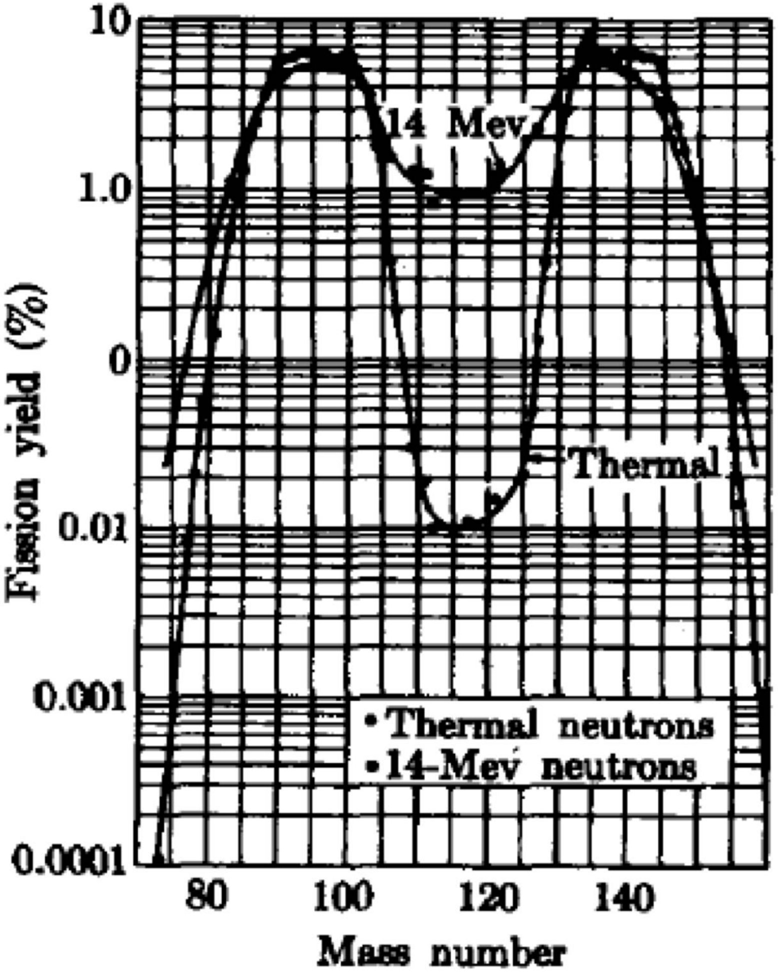
- The new nuclei that result from fission are called fission fragments.
- Usually fission fragments are of unequal size.
- Because heavy nuclei have a greater neutron/proton ratio than lighter ones, the fragments contain an excess of neutrons. To reduce this excess, two or three neutrons are emitted by the fragments as soon as they are formed, and subsequent beta decays bring their neutron/proton ratios to stable values.
- A typical fission reaction is

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^{*} \rightarrow ^{141}_{56}B_{e} + ^{92}_{36}Kr + ^{1}_{0}n + ^{1}_{0}n + ^{1}_{0}n$$

Mass Distribution of Fragments



The distribution of mass numbers in the fragments from the fission of ${}^{235}_{92}U$



Fission yield from $^{235}_{92}U$

- A heavy nucleus undergoes fission when it has enough excitation energy (5 MeV or so) to oscillate violently.

- $^{235}_{92}U$ are able to split in two merely by absorbing an additional neutron.

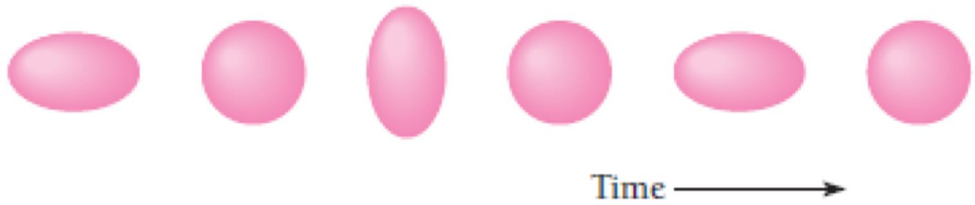
- Nuclei, like $^{238}_{92}U$, need more excitation energy for fission than the binding energy released when another neutron is absorbed. Such nuclei undergo fission only by reaction with fast neutrons whose kinetic energies exceed about 1 MeV.

Nuclear fission can be understood on the basis of the liquid-drop model of the nucleus.

- When a liquid drop is suitably excited, it may oscillate in a variety of ways.

- The drop in turn becomes a prolate spheroid, a sphere, an oblate spheroid, a sphere, a prolate spheroid again, and so on.

- The restoring force of its surface tension always returns the drop to spherical shape, but the inertia of the moving liquid molecules causes the drop to overshoot sphericity and go to the opposite extreme of distortion.



The oscillations of a liquid drop.

- Nuclei exhibit surface tension, and so can vibrate like a liquid drop when in an excited state. They also are subject to disruptive forces due to the mutual repulsion of their protons.

- When a nucleus is distorted from a spherical shape, the short-range restoring force of surface tension must cope with the long-range repulsive force as well as with the inertia of the nuclear matter.

- If the degree of distortion is small, the surface tension can do this, and the nucleus vibrates back and forth until it eventually loses its excitation energy by gamma decay.

- If the degree of distortion is too great, however, the surface tension is unable to bring back together the now widely separated groups of protons, and the nucleus splits into two parts.

Nuclear fission according to the liquid-drop model.

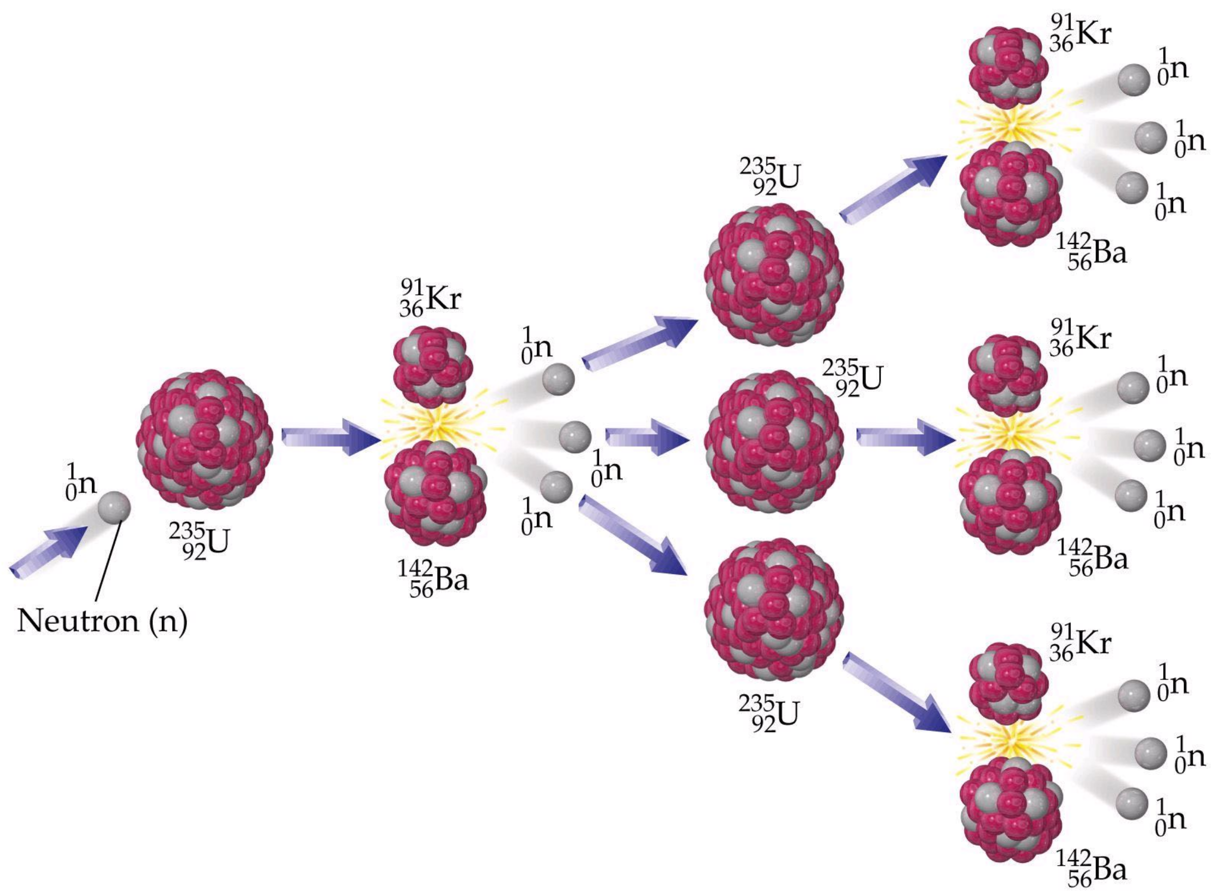
Nuclear chain reaction

- Soon after its discovery, the applicability of fission for obtaining large total energy releases was realized.

- Every neutron-induced fission produces the two heavy fragments and several neutrons which can themselves induce a new self-substaining sequence of fissions. This is chain reaction of fissions, which occurs very rapidly.

- If precisely one neutron per fission causes another fission, energy will be released at a constant rate. (which is the case in a nuclear reactor).

- If the frequency of fissions increases, the energy release will be so rapid that an explosion will occur (which is the case in an atomic bomb).



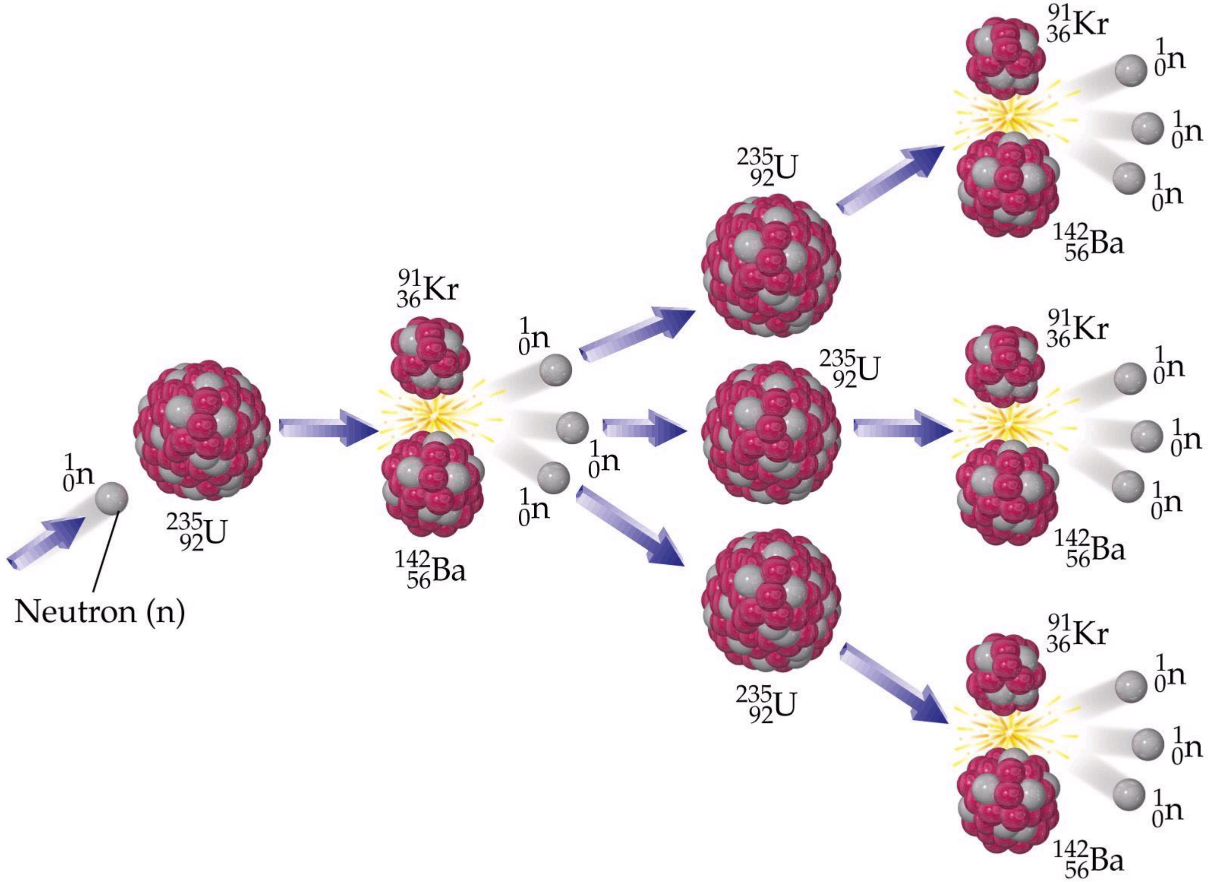
If two neutrons from each fission in an atomic bomb induce further fissions in 10^{-8} s, a chain reaction starting with a single fission will give off $2\mathrm{x}10^{13}$ J of energy in less than 10^{-6} s.

Nuclear (fission) Reactors

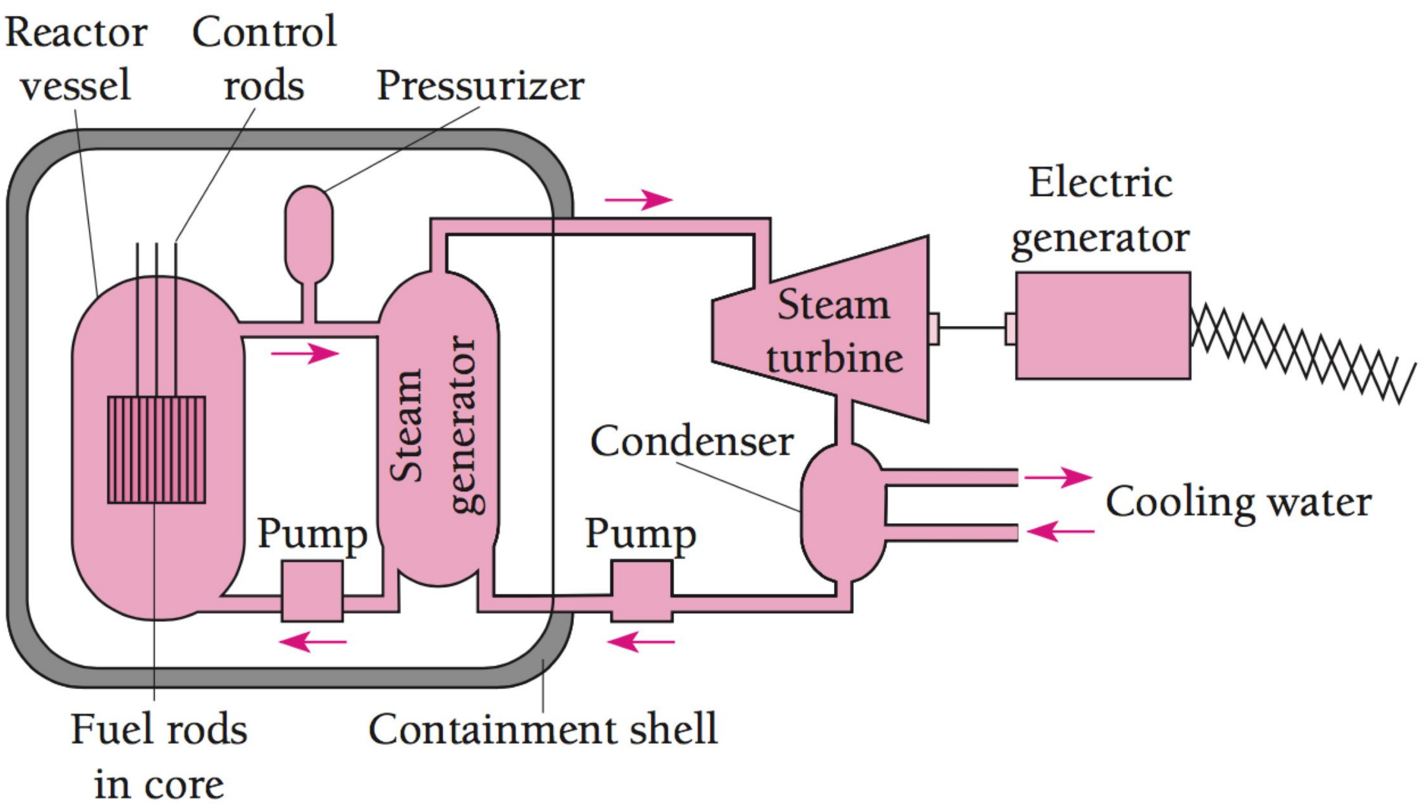
A nuclear reactor is a very efficient source of energy:

The fission of 1 g of 235_U /day evolves energy at a rate of about 1 MW, whereas 2.6 tons of coal/day must be burned in a conventional power plant to produce 1 MW.

Each fission in 235_U releases an average of 2.5 neutrons, so no more than 1.5 neutrons/fission can be lost for a self-sustaining chain reaction to occur.



Basic design of a nuclear power plant



Essential elements of reactors

1. Fuel or fissile material:

- The most commonly used fuels are natural uranium (0.72 % 235_U), enriched uranium(>0.72 % 235_U), 239_{Pu} and 233_U .

- Natural uranium contains only 0.7% of the fissionable isotope 235_U. The more abundant 238_U readily captures fast neutrons but usually does not undergo fission as a result.

- 238_U has only a small cross section for the capture of slow neutrons, whereas the cross section of 235_U for slow neutron-induced fission is 582 barns.

- Slowing down the fast neutrons that are liberated in fission thus helps prevent their unproductive absorption by 238_U and at the same time promotes further fissions in 235_U.

2. A moderator : A substance whose nuclei absorb energy from fast neutrons in collisions without much tendency to capture the neutrons.

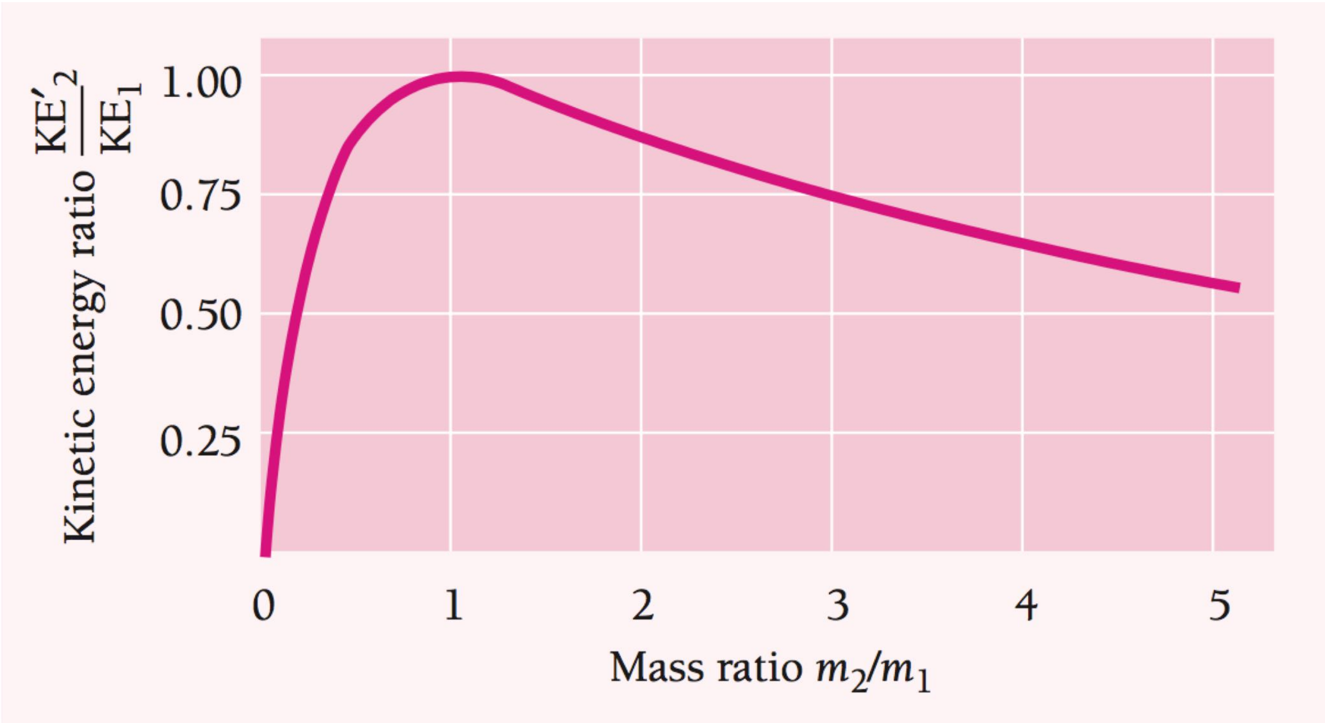
- To slow down fission neutrons, the uranium in a reactor is mixed with a moderator.

- The majority of today's commercial reactors use light water both as moderator and as coolant. Since proton nuclei have masses almost identical with that of the neutron, light water is an efficient moderator.

- As protons tend to capture neutrons to form deuterons in the reaction $1_H(n,\gamma)2_H$.

∴ light-water reactors can't use natural uranium for fuel but need **enriched uranium** whose 235_U content has been increased to about 3%.

Energy transfer in an elastic head-on collision



$$\frac{KE'_2}{KE_1} = 4(m_2/m_1)/(1+m_2/m_1)^2$$

The fuel for a water-moderated reactor consists of uranium oxide (UO_2) pellets sealed in long, thin tubes.

3. Core : Fuel elements + moderators.

4. Control rods : Rods of cadmium or boron, which are good absorbers of slow neutrons, can be slid in and out of the reactor core to adjust the rate of the chain reaction.

5. A reflector surrounding core – to reduce neutron leakage and thereby reduce the critical size of a reactor.

6. Containment vessel – to prevent the escape of radioactive fission products, some of which are gases.

7. Shielding – to prevent neutrons and γ rays from causing biological harm to operating personnel.

8. Coolant – An essential element of the reactor to remove heat from the core. Without it the heat generated would melt the core(called “meltdown”).

9. A control system – allows operator to control the power level and to keep it constant during normal operation.

10. Emergency systems – designed to prevent runaway operation in the event of failure of the control or coolant systems.

The energy given off in a reactor becomes heat.

The water, which acts as both moderator and coolant, circulates around the fuel in the core is kept at a high pressure, about 155 atmospheres, to prevent boiling.

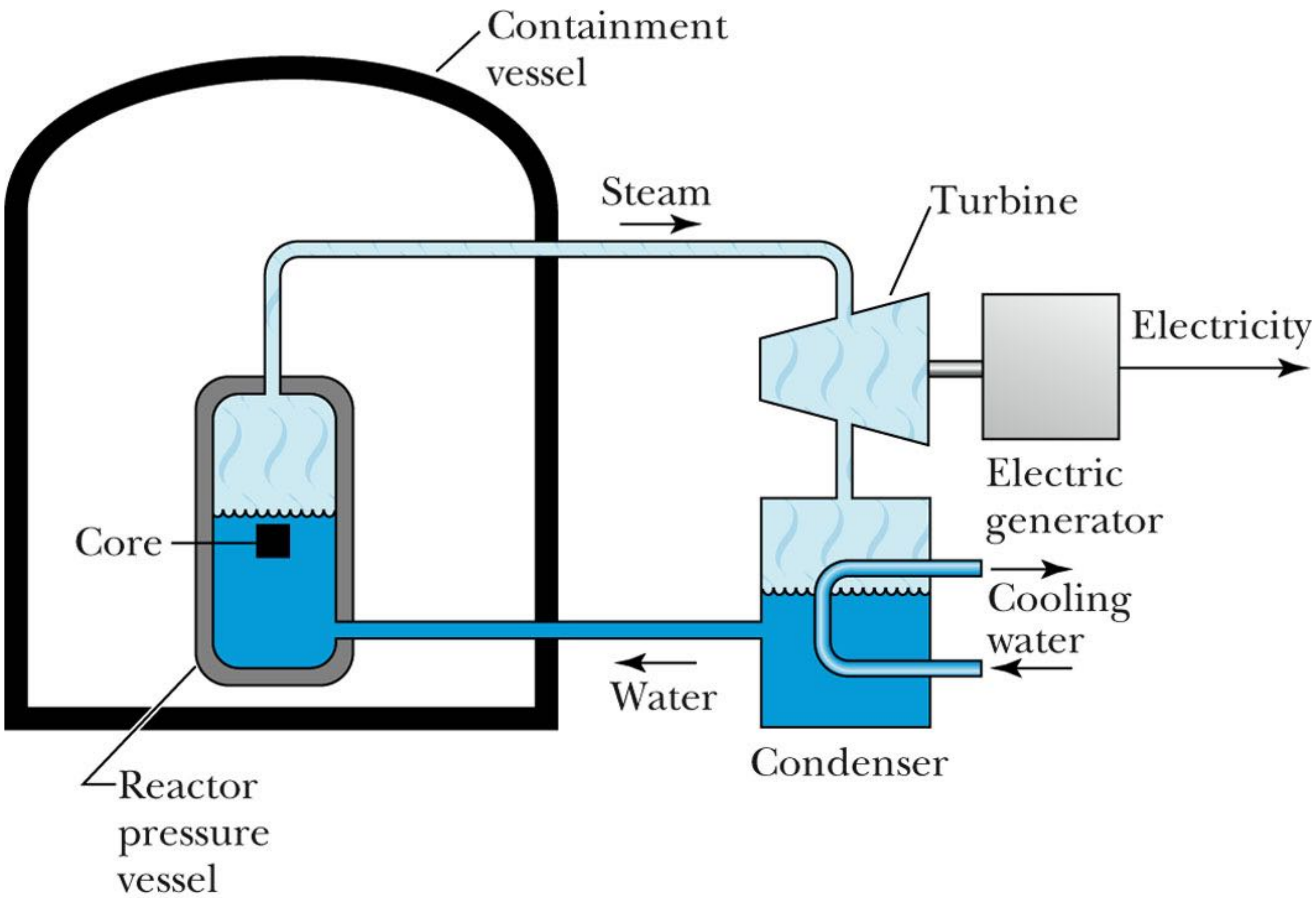
It is passed through a heat exchanger to produce steam that drives a turbine.

Types of Reactors

Depending on different kinds of nuclear fuel and cooling materials, there may be various types of nuclear reactors.

- *The boiling-water reactor*
- *The pressurized-water reactor*
- *Gas-cooled reactors*
- *Heavy-water reactor*
- *Fast breeder reactors*

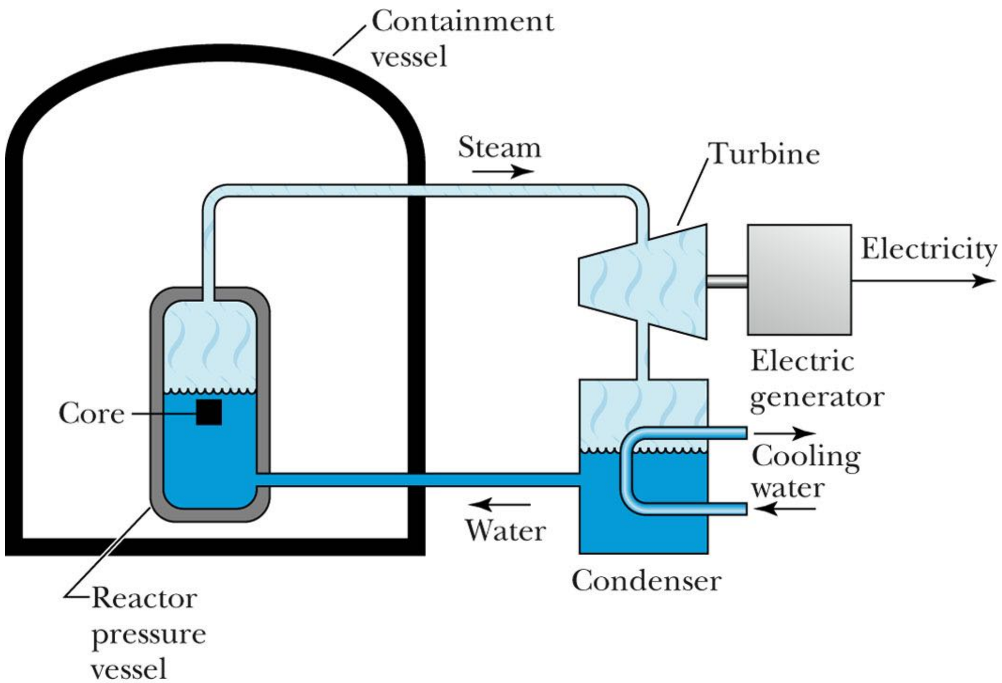
Principle sketch of a boiling-water reactor



- The heat generated in the nuclear fuel is transferred to the cooling water, which is pumped upward along the rods.

- The water can start to boil by this. The steam is subsequently led to a steam turbine, in which the blades are driven and a rotation is induced.

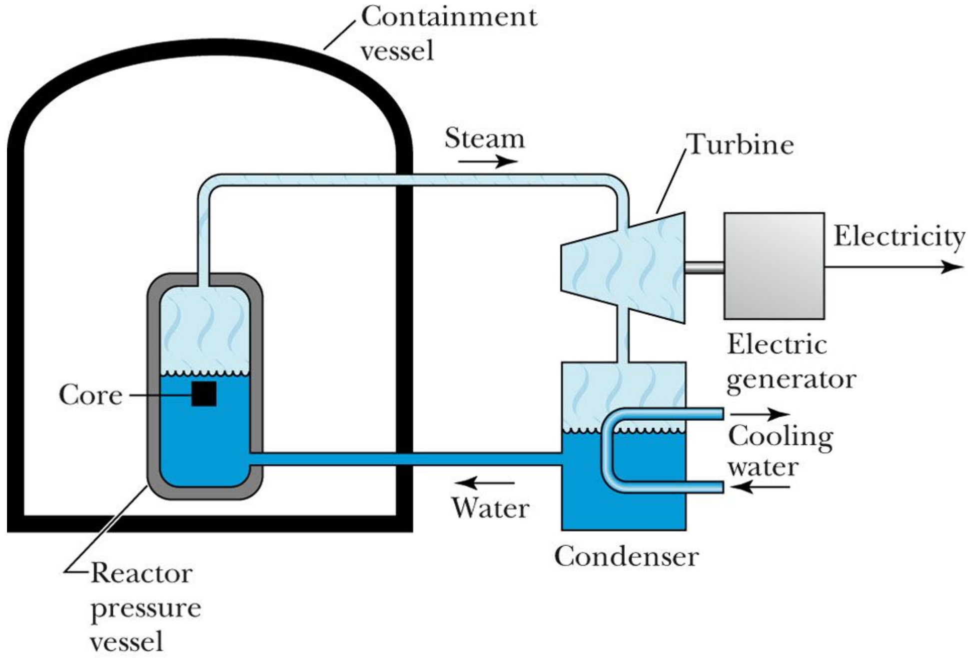
- The turbine shaft subsequently drives an electro generator, which generates the electric energy and supplies it to the electrical power network.



- The expanded steam from the turbine is condensed, after which the water is pumped through the core again.

- Cooling water, which is drawn from river or seawater, is used for condensation of the steam, or one applies a cooling tower.

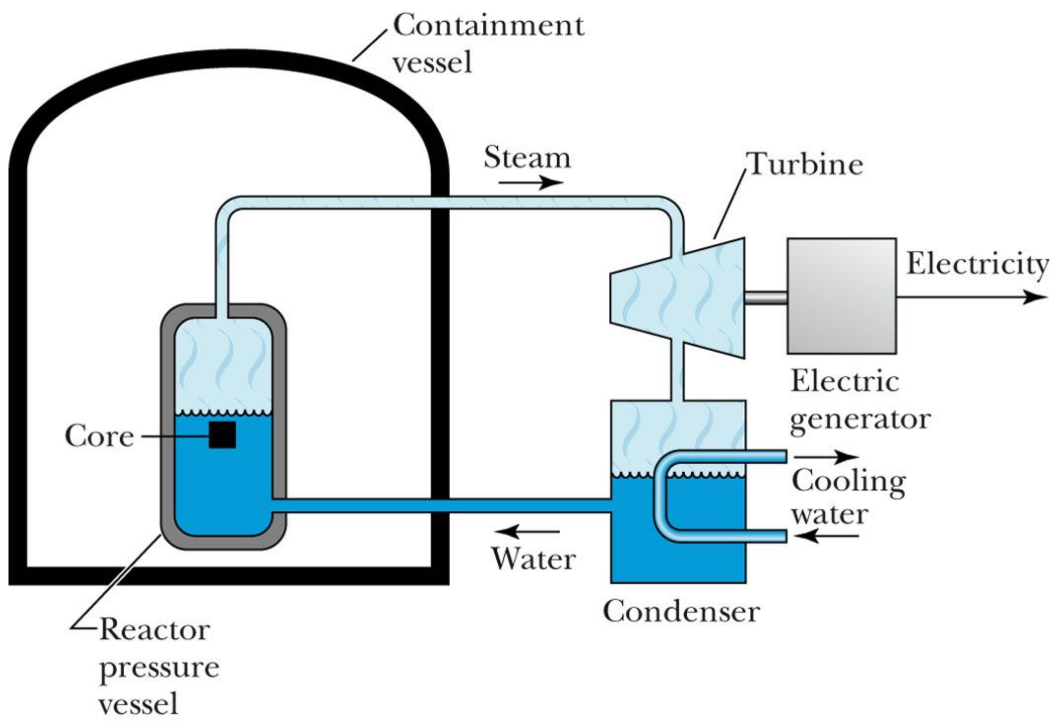
- The water in a boiling-water or pressurized-water reactor not only serves as coolant, but also for slowing down the neutrons in their energy.



- The neutrons released during fission have a high energy, whereas the chance of causing a new fission is larger if the neutrons have a low energy.

- By collisions with light nuclei such as hydrogen in water, the neutrons lose energy, so that the water also works as moderator.

- Finally, the neutrons get an energy distribution that corresponds with the heat movement of the atomic nuclei.



Breeder reactor

Some non-fissionable nuclides can be transmutated into fissionable ones by absorbing neutrons.