# Power Divider and Coupler

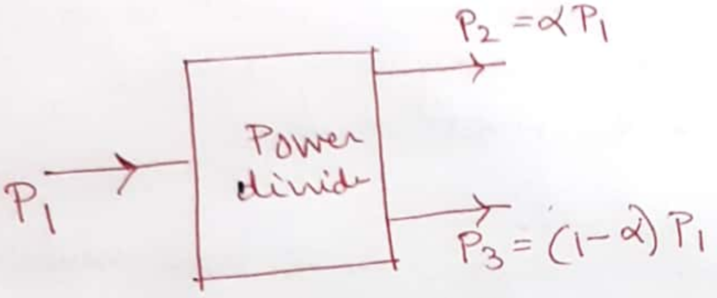


Fig: Power Division

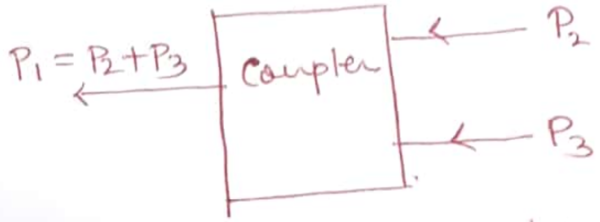


Fig: Power Combining

A three port network can not be lossless, reciprocal and matched at all ports:

or

It is not possible to construct a perfectly matched, lossless, reciprocal 3-port junction. Atleast one of the reflection coefficients must be different from zero in the reciprocal case.

If all ports are matched then $S_{ii} = 0$ & if the network is reciprocal then the S-matrix becomes,

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \quad \text{—— (2)}$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad \text{—— (1)}$$

If the network is also lossless then energy conservation requires that S-matrix be unitary

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{—— 3a}$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad \text{—— 3b}$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \quad \text{—— 3c}$$

$$S_{13}^* S_{23} = 0 \quad \text{—— 3d}$$

$$S_{23}^* S_{12} = 0 \quad \text{—— 3e}$$

$$S_{12}^* S_{13} = 0 \quad \text{—— 3f}$$

Eqn 3(d–f) show that atleast two of the three parameters ($S_{12}$, $S_{13}$, $S_{23}$) must be zero, but this condition is always inconsistent with one of eqns (3a–c), implying that a 3 port network cannot be lossless, reciprocal & matched [illegible]

If any one of these three condition is relaxed, then a physically realizable device is possible.

Check whether it is possible to design a lossless and reciprocal T-junction with two of its port being matched, while the third is not matched.

As For this assume that ports 1 & 2 are matched.
∴ S-matrix can be represented as follows:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

Since network is lossless; we can write the following equations;

$c_1 c_2$ $\quad S_{13} S_{23}^* = 0$ ——— a

$c_2 c_3$ $\quad S_{12}^* S_{13} + S_{23}^* S_{33} = 0$ ——— b

$c_3 c_1$ $\quad S_{23}^* S_{12} + S_{33}^* S_{13} = 0$ ——— c

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{——— d}$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad \text{——— e}$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1 \quad \text{——— f}$$

Eqns (d) & (e) reveal that

$$|S_{13}| = |S_{23}| \quad \text{——— g}$$

by applying (a) in (g) provides

$$S_{13} = S_{23} = 0 \quad \text{——— h}$$

Again substituting h in eqs d, e, f given

$$|S_{12}| = |S_{33}| = 1 \quad \text{——— i}$$

eq^ns (g)–(i) ensure the network will have scattering matrix,

$$[S] = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\theta} \end{bmatrix}$$

∴ it is possible to design a 3-port reciprocal & lossless network with any of its two ports matched.

<u>Check</u> whether it is possible to design a 3-port network with all its ports matched, provided the network is lossless but not reciprocal.

For such case 'S' matrix is represented as follow:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

network is still lossless,

$C_1C_2$ ∴ $S_{31}^* S_{32} = 0$ ——— a

$C_2C_3$ $S_{12}^* S_{13} = 0$ ——— b

$C_3C_1$ $S_{23}^* S_{12}^* = 0$ ——— c

$|S_{12}|^2 + |S_{13}|^2 = 1$ ——— d

$|S_{21}|^2 + |S_{23}|^2 = 1$ ——— e

$|S_{31}|^2 + |S_{32}|^2 = 1$ ——— f

Eq$^{ns}$ (a) – (f) can be satisfied simultaneously if any one of the following two condition is satisfied:

$S_{12} = S_{23} = S_{31} = 0$ ; $|S_{21}| = |S_{32}| = |S_{13}| = 1$

$S_{21} = S_{32} = S_{13} = 0$ ; $|S_{12}| = |S_{23}| = |S_{31}| = 1$

$$\therefore \; [S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

So, it can be proved that a three port network with all its ports matched can be designed if the network is lossy.

<u>S-matrix</u> of <u>3-port H-plane Tee</u>


$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- junction scattering coefficient $S_{13}$ and $S_{23}$ must be equal to the plane of symmetry;

$$\therefore \; S_{13} = S_{23}$$

- From symmetric property $S_{ij} = S_{ji}$

  $\therefore \; S_{12} = S_{21}$, $S_{31} = S_{13}$, $S_{23} = S_{32} = S_{13}$

- considering port 3 is perfectly matched reflection coefficient $S_{33} = 0$

$\therefore$ [S] matrix can be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

Now we have four unknowns,

$\therefore$ by using unitary property,

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{—— a}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{—— b}$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{—— c}$$

zero property

$$R_1 R_3 \quad S_{13} S_{11}^* + S_{13} S_{12}^* = 0 \quad \text{—— d}$$

from (c) $\quad |S_{13}| = \frac{1}{\sqrt{2}}$

comparing eqns (a) & (b);

$$S_{11} = S_{22}$$

from (d);

$$S_{13}(S_{11}^* + S_{12}^*) = 0$$

but $S_{13} \neq 0$, $\therefore S_{11}^* = -S_{12}^*$

or $S_{11} = -S_{12}$

or $S_{12} = -S_{11}$.

using eqn (a)

$$|S_{11}|^2 + |S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$\Rightarrow S_{11} = \frac{1}{2}$$

$$\therefore S_{12} = -\frac{1}{2}$$

$$\& \quad S_{22} = \frac{1}{2}$$

FILE NO.

$\therefore$ S matrix for H-plane tee;

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

Again $[B] = [S][A]$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{a_1}{2} - \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$$

$$b_2 = -\frac{a_1}{2} + \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}}$$

as port 3 is input, i.e. $a_3 \neq 0$;

$a_1 = 0$, $a_2 = 0$

$\therefore$ eqns becm

$$b_1 = \frac{a_3}{\sqrt{2}} ; \quad b_2 = \frac{a_3}{\sqrt{2}} , \quad b_3 = 0$$

$$\therefore \text{power gain} = 10 \log_{10}\left(\frac{P_1}{P_3}\right)$$

$$= 10 \log_{10} \frac{P_1}{2P_1}$$

$$= -3 \text{ dB}$$

$\therefore$ power coming out from port 1 and 2 is 3 dB less than input power at port 3.

Scanned by CamScanner

For E-plane, Tee.

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

Scanned by CamScanner