MA 102

Calculus

Tutorial–1

Note: $i = \vec{i}, j = \vec{j}, k = \vec{k}$

(1) Give a geometric description of the set of points in space whose coordinates satisfy the given pairs of equations.
   (a) $x^2 + y^2 = 4, z = -2$
   (b) $x^2 + y^2 + z^2 = 1, x = 0$

(2) Describe the sets of points in space whose coordinates satisfy the given inequalities or combinations of equations and inequalities.
   (a) The circle of radius 1 centered at $(-3, 4, 1)$ and lying in a plane parallel to the xy-plane.
   (b) The solid cube in the first octant bounded by the coordinate planes $x = 2$, $y = 2$ and $z = 2$.
   (c) Find an equation for the set of all points equidistant from the point $(0, 0, 2)$ and the xy-plane.

(3) Express each vector as a product of its length and direction.
   (a) $2i + j - 2k$,
   (b) $9i - 2j + 6k$,

(4) Express each vector in the form $v = v_1 i + v_2 j + v_3 k$.
   (a) $5u - w, u = (1, 1, -1), w = (2, 0, 3)$,
   (b) $-2u + 3w, u = (-1, 0, 2), w = (1, 1, 1)$,

(5) The midpoint of line segment PQ where $P = (1, 4, 5), Q = (4, -2, 7)$.

(6) Find the measures of the angles between the diagonals of the rectangle whose vertices are $A = (1, 0), B = (0, 3), C = (3, 4), D = (4, 1)$.

(7) Show that squares are the only rectangles with perpendicular diagonals.

(8) Which of the following are always true, and which are not always true? Give reasons for your answers.
   (a) $-u \times v = -(u \times v)$.
   (b) $(u \times u).u = 0$.

(9) Find a unit vector perpendicular to plane PQR, where $P(2, -2, 1)$, $Q(3, -1, 2)$, $R(3, -1, 1)$.

(10) Find the volume of the parallelepiped (box) determined by $u = i + j - 2k$, $v = -i - k$, and $w = 2i + 4j - 2k$.

(11) Find parametrizations for the line segments joining the points $(1, 1, 0)$, $(1, 1, 1)$.

(12) Find the distance from the point $(2, 1, -1)$ to the line $x = 2t, y = 1 + 2t, z = 2t$.

(13) The plane through $(1, -1, 3)$ parallel to the plane $3x + y + z = 7$.

(14) Find parametrizations for the lines in which the planes $x - 2y + 4z = 2$, $x + y - 2z = 5$.

(15) Find the distance from the point $(0, 0, 0)$ to the plane $3x + 2y + 6z = 6$.

(16) Find a plane through the origin that meets the plane $M : 2x + 3y + z = 12$ in a right angle. How do you know that your plane is perpendicular to M?

(17) Give the position vectors of particles moving along $y = x^2 + 1$ in the xy-plane at $x = -1, 0, 1$. Find the particle's velocity and acceleration vectors at the stated positions.

(18) Find parametric equations for the line $r(t) = cos(t)i + sin(t)j + sin(2t)k$ that is tangent to the given curve at $t = \frac{\pi}{2}$.

(19) Show that if u, v, and w are differentiable vector functions of t, then $\frac{d}{dt}(u \cdot v \times w) = \frac{du}{dt} \cdot v \times w + u \cdot \frac{dv}{dt} \times w + u \cdot v \times \frac{dw}{dt}$.

(20) Evaluate $\int_0^{\frac{\pi}{3}} [sec(t)tan(t)i + tan(t)j + 2sin(t)cos(t)k]dt$.

(21) Find the arc length parameter along the curve $r(t) = (cos(t) + tsin(t))i + (sint - tcos(t))j$, $\frac{\pi}{2} \leq t \leq \pi$.

(22) Find T, N, and κ for the plane curve $r(t) = ln(sec(t))i + tj$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$.

(23) Let $r(t) = cos(t)i + sin(t)j + tk$. Find T, N, and B at the given value of $t = 0$. Then find equations for the osculating, normal, and rectifying planes at $t = 0$.

(24) Show that the torsion of the helix $r(t) = a\ cos(t)i + a\ sin(t)j + btk, a, b \geq 0$ is $\tau = \frac{b}{a^2+b^2}$.