$B_{1n} = B_{2n} = \delta q_2$.

$H_{1t} - H_{2t} = K$

$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$

$(H_1 - H_2) \times a_{n12} = K$.

Magnetic Inductor

λ - flux-linkage

$\lambda = N\psi$

$\lambda \propto I$.

$\lambda = LI$

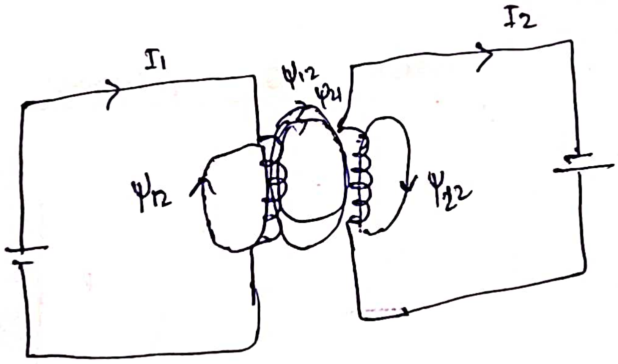
Inductance of circuit

$L = \frac{\lambda}{I} = \frac{N\psi}{I}$

magnetic Energy

$W_m = \frac{1}{2} L I^2$

$\boxed{L = \frac{2 W_m}{I^2}}$



λ_{12} : total flux linked.

$\boxed{\psi_{12} = \int_{S_1} B_2 \cdot ds}$ Area of L.

pass through L due to 2.

Magnetic field due to I_2.

S_1 due to area L.

$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \psi_{12}}{I_2}}$

unit - henery.

$\boxed{M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_1 \psi_{21}}{I_1}}$

due to current I_1

In absence of ferromagnetic material,

$$M_{12} = M_{21}$$

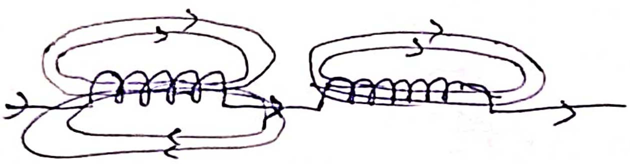
$$\psi_1 = \psi_{11} + \psi_{12} \qquad \psi_2 = \psi_{21} + \psi_{22}$$

Self flux   Mutual flux

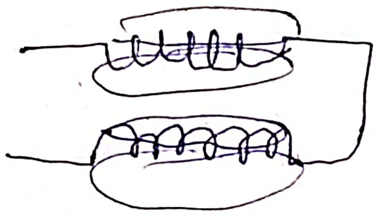
$$L_1 = \frac{\phi_{11}}{I_1} = \frac{N_1 \psi_1}{I_1} \qquad L_2 = \frac{\phi_{22}}{I_2} = \frac{N_2 \psi_2}{I_2}$$

$$W_m = W_1 + W_2 + W_{12}$$

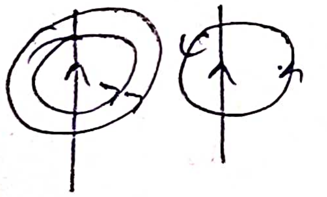
$$= \frac{1}{2} L I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2 .$$



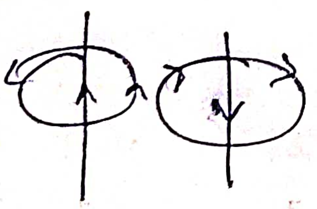
→ Magnetic Energy will be more.



– opposite dirn. → Magnetic Energy will be less.



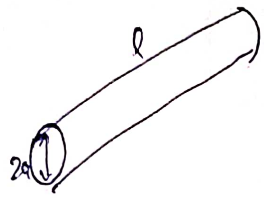
– Substracted



– Add

Calculation of Self Inductance

1. Choose a suitable CS
2. Let the inductor carry current I
3. Determine B from Biot-Savart's law (from Ampere's law if symmetry exists) and calculate ψ from $\psi = \int B \cdot ds$.
4. Finally find L from $L = \lambda/I = \frac{N\psi}{I}$.



$$W_m = \frac{1}{2}\int B \cdot H \, dv = \frac{1}{2} \epsilon_0$$

Self-Inductance of an Infinitely long solenoid

$$B = \mu H = \mu I n$$

$$\psi = BS = \mu I n S$$

4/10/23

Time Varying EM field $\rightarrow f(\vec{r}, t)$.

$\vec{H}$, $\vec{B}$, $\vec{E}$, $\vec{D}$ $(\vec{r})$ → All are fn of $\vec{r}$ for static

Static - charge - Electrostatic field

Uniform motion - Magnetostatic field.

faraday's laws ↳ rate of change flux

Transformer E.M.F
Motional E.M.F

$$\lambda = N\psi$$

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt}$$ — mathematical faraday's and lenz law.

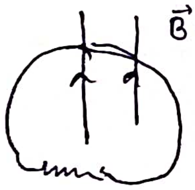
Transformer and Motional EMF

If $N = 1$

$$V_{emf} = \frac{-d\psi}{dt}$$

$$\int_L \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}$$

Ⓐ Stationary loop in time varying Magnetic field



$$\int_L \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

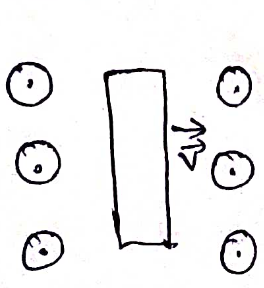
$$\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

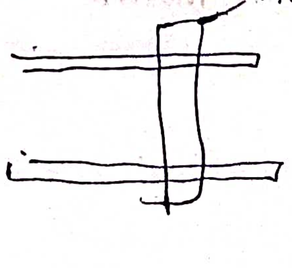
$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ — transformer emf.

$\vec{\nabla}\times\vec{E} = 0$
↓
for static
↓
Electro static field is Conservative.

Ⓑ Moving loop in static $\vec{B}$ field

$\vec{B}(\vec{r})$
↓
static field.





$\vec{F_m} = q(\vec{V} \times \vec{B})$

Strip is in motion
⇓
Charges also moves

$$V_{emf} = \oint_L \vec{E_m} \cdot dl$$

$$\boxed{E_m = \frac{\vec{F_m}}{q} = \vec{V} \times \vec{B}}$$

motional Emf

$$V_{emf} = -\frac{d}{dt}\oint_S \vec{B} \cdot d\vec{s} = \oint_L \vec{E_m} \cdot d\vec{l}$$

$$\oint_L \vec{E_m} \cdot dl = \oint (\vec{u} \times \vec{B}) \cdot dl$$

$$\boxed{\int_S (\nabla \times \vec{E_m}) \cdot d\vec{s} = \int_S \vec{\nabla} \times (\vec{u} \times \vec{B})\, d\vec{s}}$$

(c) moving loop in ~~the~~ time varying Magnetic field.

$$V_{emf} = \int_L \vec{E} \cdot dl = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \oint_L (\vec{u} \times \vec{B}) \cdot dl$$

combinational Emf.

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times (\vec{u} \times \vec{B})}$$

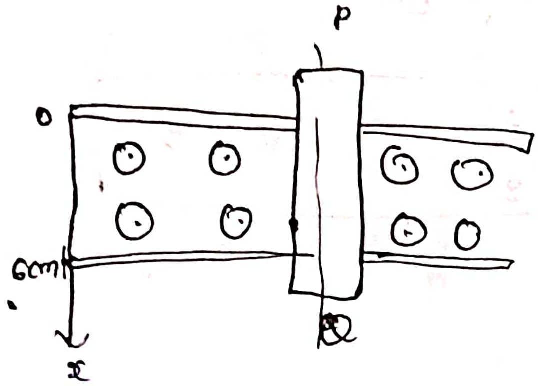
Q. A conducting bar can slide over two conducting rails as shown in fig. Calculate the Induced Voltage in the bar

(a) If the bar is stationed at $y = 8$ cm, $\vec{B} = 4\cos 10^6 t\, \hat{a}_z$ mWb/m^2

(b) If the bar slides at a velocity $\vec{u} = 20\, \hat{a}_y$ m/s and $\vec{B} = 4\, \hat{a}_z$ mWb/m^2

(Q) If a bar slides at velocity $\vec{u} = 20\,\hat{a}_y$ m/s and

$4\cos(10^6 t - y)\;\hat{a}_z$ mwb/m².



a). transforer em.

$$V_{emf} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$d\vec{s} = dx\,dy\,\hat{a}_z$

$$= -\int \frac{\partial}{\partial t}(\;)\cdot dx\,dy\,\hat{a}_z \qquad \int_{x=0}^{6}\int_{y=0}^{0.8}$$

$$= -\int_{x=0}^{6}\int_{y=0}^{0.8} 4\times10^6 \sin 10^6 t \;\; dx\,dy.$$

$$= +4\times10^6 \;\sin 10^6 t \;\times 0.8\times 6.$$

$$= 19.$$

b) motional Emf:

$$V_{emf} = \int(\vec{u}\times\vec{B})\cdot d\vec{l}$$

$$= \int(20\,\hat{a}_y\times 4\hat{a}_z)\cdot d\vec{l}$$

$$= -4.8\text{ mV}.$$

c)

Maxwell's equation

Differential form	Integral form
(1) $\vec{\nabla}\cdot\vec{D} = \rho_v$	$\oint \vec{D}\cdot d\vec{s} = \int_v \rho_v \, dv.$
(2) $\vec{\nabla}\cdot\vec{B} = 0$	$\oint \vec{B}\cdot d\vec{s} = 0.$
(3) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}.$
(iv) $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H}\cdot d\vec{l} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{s}.$

↳ Magnetic flux density is solenoidal → non-existence of magnetic pole.

faraday's law

Conduction current density

for free space

$\rho_v = 0$

$\sigma = 0$ — Conductivity.

$\to J = \sigma E = 0$

$\vec{\nabla}\cdot\vec{D} = 0$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}.$

for static field $\longrightarrow$ $\vec{D}\ \vec{B}\ \vec{E}\ \vec{H}$

$\frac{\partial \vec{D}}{\partial t} = \frac{\partial \vec{B}}{\partial t} = \frac{\partial \vec{E}}{\partial t} = \frac{\partial \vec{H}}{\partial t} = 0.$

$\vec{\nabla}\cdot\vec{D} = \rho_v$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = 0$

$\vec{\nabla}\times\vec{H} = \vec{J}$

Modified Ampere's Law

Static field

Ampere's law : $\vec{\nabla} \times \vec{H} = \vec{J}$

$$\underbrace{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H})}_{0} = \vec{\nabla} \cdot \vec{J} \quad \text{— (1)}$$

Continuity eq^n $\underbrace{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}_{\text{contradiction}}$ — (2)

(1) & (2) contradict

Modification:-

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_D$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J}_D$$

$$0 = -\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J}_D$$

$$\Rightarrow \quad \vec{\nabla} \cdot \vec{J}_D = \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{D})$$

$$= \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\boxed{\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t} (\epsilon \vec{E}) = \epsilon \frac{\partial \vec{E}}{\partial t}.}$$

$\vec{J}_D$ exist when there is variation in $\vec{E}$.

Time-harmonic field

$$z = x + jy \equiv |r| \angle \theta = r e^{j\theta}$$

Varies periodically & synasicdally

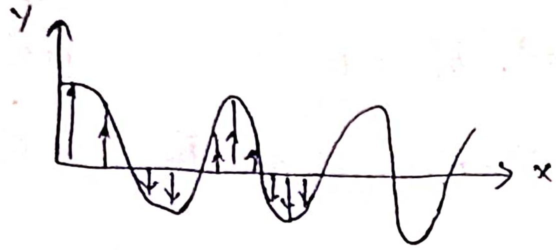
If a vector $A(\vec{r}, t)$ is a time harmonic field,

the phasor form of $\vec{A}$ is $\vec{A_s}(\vec{r})$

$\vec{A} = Re(\vec{A_s}\, e^{j\omega t})$

Eg. $\vec{A} = A_0 \cos(\omega t - \beta x)\, \hat{a}_y$

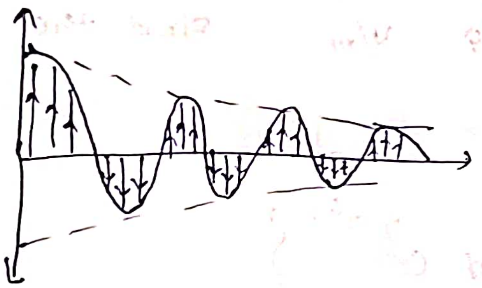
Exam Diagram



dir^n of $\vec{A}$ along y

Irrespective to time & variation Amplitude varies from $-A_0$ to A_0

$y = A_0 e^{-\alpha x} \cos(\omega t - \beta x)\, \hat{a}_y$ α: attuniation constant



$\vec{A} = A_y(x,t)\, \hat{a}_y$.

$\vec{A} = A_0 \cos(\omega t - \beta x)\, \hat{a}_y$

$= Re\left\{ A_0\, e^{j(\omega t - \beta x)}\, \hat{a}_y \right\}$

$= Re\left\{ \underbrace{A_0\, e^{-j\beta x}\, \hat{a}_y}_{\vec{A_s}}\, e^{j\omega t} \right\}$

phasor form $(\vec{A_s}) = A_0\, e^{-j\beta x}\, \hat{a}_y$.

~~phaso~~

$\frac{\partial \vec{A}}{\partial t} =$

Maxwell's eqn

Time Harmonic of Maxwell's eq.

Assuming time factor $e^{j\omega t}$.

$\vec{\nabla}\cdot\vec{D} = \rho_v$	$\vec{\nabla}\cdot\vec{D_s} = \rho_v$	$\oint_S \vec{D_s}\cdot d\vec{s} = \int \rho_s \, dv$
$\vec{\nabla}\cdot\vec{B} = 0$	$\vec{\nabla}\cdot\vec{B_s} = 0$	$\oint \vec{B_s}\cdot d\vec{s} = 0$
$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{\nabla}\times\vec{E_s} = -j\omega \vec{B_s}$	$\oint \vec{E_s}\cdot d\vec{l} =$
$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\vec{\nabla}\times\vec{H_s} = \vec{J_s} + j\omega \vec{D_s}$	

Q. $\vec{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\,\hat{a}_\phi$ V/m Find the phasor form of $\vec{E}$.

$$\vec{E} = \mathrm{Re}\left\{ \frac{50}{\rho} e^{j\beta z}\,\hat{a}_\phi \cdot e^{j10^6 t} \right\}.$$

$$\vec{E_s} = \frac{50}{\rho} e^{j\beta z}\,\hat{a}_\phi$$

Prob 8.36

Q. The electric field phasor of an EM wave in free space is $\vec{E_s}(y) = 10\, e^{-j4y}\,\hat{a}_x$ V/m.

Find (a) 'ω' so that $\vec{E_s}$ satisfy M.E.

(b) the corresponding mag. field.

$$\vec{E} = 10\cos(\omega t - 4y)\,\hat{a}_x$$

$$\vec{E} = \mathrm{Re}\left\{ \vec{E_s}\, e^{j\omega t} \right\}.$$

Given :- phasor form

↳ Maxwell eqn → phasor form

Normal form ↓ use Normal M.E

$\vec{\nabla} \cdot \vec{D_s} = 0$

$\vec{\nabla} \cdot \vec{B_s} = 0$

$\vec{\nabla} \times \vec{E_s} = -j\omega \vec{B_s}$

$\vec{\nabla} \times \vec{H_s} = j\omega \vec{D_s}$

$\vec{E_s} = E_{sx}(y)\, \hat{a_y}$

$$\vec{\nabla} \times \vec{E_s} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx}(y) & 0 & 0 \end{vmatrix}$$

$$\left(-\frac{\partial}{\partial y} E_{sx}(y)\right) \hat{a_z}$$

$$0 - \frac{\partial}{\partial y} 10 e^{-j4y}$$

$$+40j\, e^{-j4y}\, \hat{a_z}$$

$$\vec{\nabla} \times \vec{E_s} = -j\omega \vec{B_s}$$

$$40j\, e^{-j4y}\, \hat{a_z} = -j\omega \vec{B_s}$$

Problem 8.36 textbook

$$\mu \vec{H_s} = \frac{-40}{\omega} e^{-j4y}$$

$$\vec{H_s} = \frac{-40}{\mu\omega} e^{-j4y}\, \hat{a_z}$$

Wave eqn in free space

$\rho_v = 0 \qquad \sigma = 0$

$\vec{\nabla} \times \vec{E} = 0$

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$

taking curl on both sides of eqn (3)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 E = \frac{-\partial}{\partial t} \vec{\nabla} \times (\mu \vec{H})$$

$$\nabla^2 E = \mu \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

$$\nabla^2 E = \mu \frac{\partial}{\partial t}\left(\frac{\partial \vec{D}}{\partial t}\right) \quad \text{from eq (9)}$$

$$= \mu \epsilon \frac{\partial^2 E}{\partial t^2} \quad (\vec{D} = \epsilon \vec{E})$$

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

(in free space $\mu = \mu_0$, $\epsilon = \epsilon_0$)

$$\boxed{\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}}$$

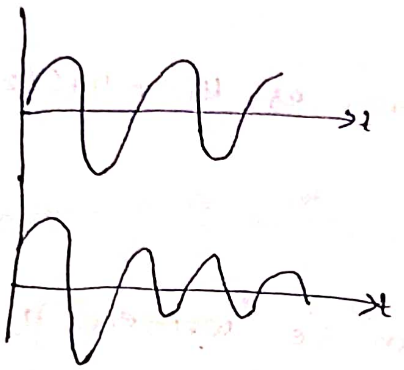
Wave eqn in conducting medium

$\rho_v = 0 \qquad \sigma \neq 0 \qquad \vec{J} \neq 0$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 E}{\partial t^2} + \mu \sigma \frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 H}{\partial t^2} + \mu \sigma \frac{\partial \vec{H}}{\partial t}$$

wave propagation in lossy dielectric



peak to peak amplitude fixed

Amp. decrease in exponential manner.

Consider a linear, homogenous, lossy dielectric medium which is charge free.

$\rho_v = 0$

Assuming and suppressing time factor $e^{j\omega t}$,

Maxwells eqn becomes.

$\vec{A}(r,t) = Re[\vec{A_s}(r)\, e^{j\omega t}]$

$j\omega \, \vec{A_s}(r)$

$$\vec{\nabla} \cdot \vec{E_s} = 0 \quad (1)$$

$$\vec{\nabla} \cdot \vec{H_s} = 0 \quad (2)$$

$$\vec{\nabla} \times \vec{E_s} = -j\omega\mu \vec{H_s} \quad (3)$$

$$\vec{\nabla} \times \vec{H_s} = (\sigma + j\omega\epsilon)\vec{E_s} \quad (4)$$

$$\vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$\sigma \vec{E}$

Taking curl of both sides of eq (3),

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E_s}) = -j\omega\mu (\vec{\nabla} \times \vec{H_s}) \quad (5)$$

$$\nabla^2 E_s = -j\omega\mu(\sigma + j\omega\epsilon)\vec{E_s} \quad (6)$$

$$\boxed{\nabla^2 E_s - \gamma^2 \vec{E} = 0} \quad (7)$$

propagation constant

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) \quad (8)$$

$$\boxed{\nabla^2 H_s - \gamma^2 H_s = 0} \quad \text{---(9)}$$

Eqn (7) & (9) are known as Helmholtz's eqn on Simple vector

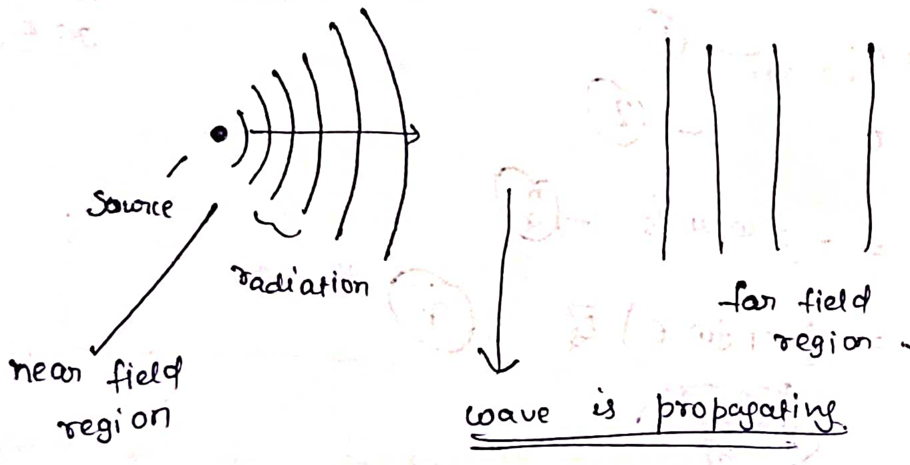
γ is a complex quanty can be written as,

$$\boxed{\gamma = \alpha + j\beta ;}$$

where α = attenuation const.

β = phase const.

Uniform plane wave (Defn & property).



Mode TEM

Transverse electric & magnetic

TE mode

TM mode

TEM mode

TEM to z

↳ wave is propagating in z direction

w.r.to z dirn of $\vec{E}$ and $\vec{H}$ are perpendicular.

$\vec{E} \perp z \Rightarrow E_z = 0$

$\vec{H} \perp z \Rightarrow H_z = 0$

TE ⊥ to z
i.e $E_z = 0$

&

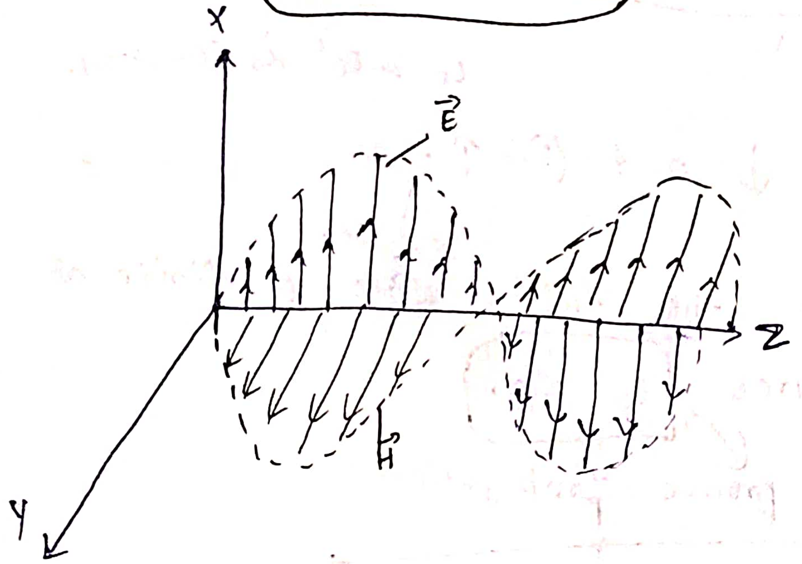
TM ⊥ to z
i.e $H_z = 0$

TEM ⊥ to z
i.e $E_z = H_z = 0$

Q Exam:

TEM propagation diagram

$$\boxed{\begin{aligned} \hat{a}_k &= \hat{a}_E \times \hat{a}_H \\ \hat{a}_z &= \hat{a}_x \times \hat{a}_y \end{aligned}}$$



TEM mode ~~for~~ (same amplitude)

— Right handed CS.

⇓

'X-Y' using right hand thumb

Assume the wave propagation along $\hat{a}_z$ and E_s vector has only x - component.

$\vec{E_s} = \vec{E}_{xs}(z)\,\hat{a}_x$

phasor notation ⇓ fn of cordinate not time.

fn of z as it is propagating in z,

Substituting in eq (7)

$$\boxed{(\nabla^2 - \gamma^2)\,\bar{E}_{xs}(z) = 0}$$

$$\left(\underset{=0}{\frac{\partial^2}{\partial x^2}} + \underset{=0}{\frac{\partial^2}{\partial y^2}} + \frac{\partial^2}{\partial z^2}\right) E_{xs}(z) - \gamma^2 E_{xs}(z) = 0$$

$$\Rightarrow \boxed{\left(\frac{\partial^2}{\partial z^2} - \gamma^2\right) E_{xs}(z) = 0}$$

This is a scalar wave eqn and its solution is of the form,

$$\boxed{E_{xs}(z) = E_0 e^{-\gamma z} + E_0' e^{\gamma z}}$$

E_0 & E_0' is constant.

$z \uparrow \rightarrow$ (1) $\downarrow$ and (2) $\uparrow$.

The fact that the field must be finite at infinity which requires $\boxed{E_0' = 0.}$

Assuming z dirn of wave propagation.

for negative z dirn, $\boxed{E_0 = 0}$

$$E_{xs}(z) = E_0 e^{-\gamma z}$$

$$\vec{E}(z,t) = \mathrm{Re}\{E_{xs}(z) e^{j\omega t}\} \hat{a}_x$$

$$= \mathrm{Re}\left[E_0 e^{-\gamma z} e^{j\omega t}\right] \hat{a}_x$$

$$= \mathrm{Re}\left[E_0 e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right] \hat{a}_x$$

dirn of electric field

$$\boxed{\vec{E}(z,t) = \underbrace{E_0 e^{-\alpha z}}_{\text{decaying cor. to } z} \cos(\omega t - \beta z) \cdot \hat{a}_x} \quad (V/m)$$

where α is attenuation constant.

$$\boxed{\vec{H}(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_y .}$$

TEM mode in coaxial cable

$$\vec{\nabla} \times \vec{E_s} = -j\omega \mu_0 \vec{H_s}$$

$$= -j\omega \mu_0 \left(H_{sx}\hat{a}_x + H_{sy}\hat{a}_y + H_{sz}\hat{a}_z \right) \quad \text{— (i)}$$

$$\vec{\nabla} \times \vec{E_s} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{xs}(z) & 0 & 0 \end{vmatrix}$$

$$= + \hat{a}_y \frac{\partial}{\partial z} E_{xs}(z) \quad \text{— (ii)}$$

from (i) & (ii),

$$\frac{\partial}{\partial z} E_{xs} = -j\omega \mu_0 H_{sy}$$

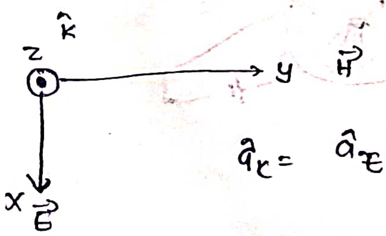
$$\frac{\partial}{\partial z}\left(E_0 e^{-\gamma z}\right) = -j\omega \mu_0 H_{sy}$$

$$-\gamma E_0 e^{-\gamma z} = -j\omega \mu_0 H_{sy}$$

$$\boxed{\frac{E_{xs}}{H_{sy}} = \frac{j\omega \mu_0}{\gamma} = \eta}$$

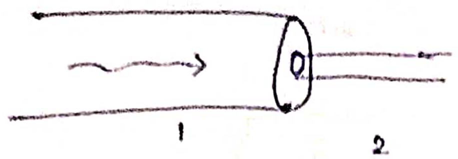
V/m (E_{xs}), A/m (H_{sy})

η — complex; ohm. (Ω); wave Impedance



$$\hat{a}_k = \hat{a}_E \times \hat{a}_H$$

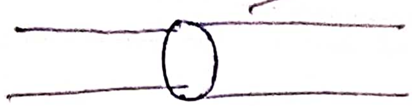
Case-I



— flow decreases

Cross-section same

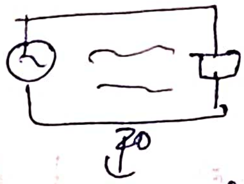
Case-II



— flow properly

coaxial cable
w.r.to distance
capacitance Varig

Electric field analogous to Voltage
Magnetic " " " Current



• Impedence mismatche

377 Ω – free space η - value.

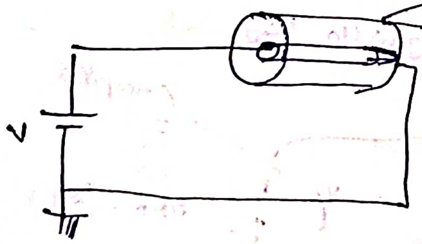
$$\vec{H}(z,t) = Re\left\{ H_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{a}_y \right\}$$

$$H_0 = \frac{E_0}{\eta}$$

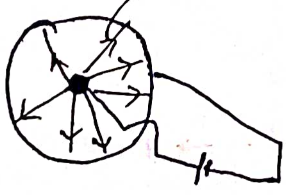
$$\boxed{\vec{H}(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_y}$$

Instantenous form.

coaxial cable



wave prog. in coaxial cable TEM Mode



$\vec{E}$ dirn → radial dirn $(\hat{a}_\rho)$

$$\vec{E} = E_\rho(z)\, \hat{a}_\rho$$

$$\hat{a}_k = \hat{a}_E \times \hat{a}_H$$

$$\hat{a}_z = \hat{a}_\rho \times \hat{a}_\phi$$

η is complex (~~known as propagation constant~~)

$\eta = |\eta| \angle \theta_n$

$\eta = |\eta| e^{j\theta_n} = \sqrt{\dfrac{j\omega\mu}{\sigma + j}}$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} \quad ; \quad \tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$$

$$\vec{H} = Re\left[\frac{E_0}{|\eta|\, e^{j\theta_\eta}}\, e^{-\alpha z}\, e^{-j(\omega t - \beta z)}\, \hat{a}_y\right]$$

$$\boxed{\vec{H} = H_0\, e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_y}$$

$\gamma = \alpha + j\beta$

where, α = attenuation constant

unit dB/m

np/m

neper

α:

- It is the measure of the spatial rate of decay of the wave in the medium.
- An attenuation of 1 np denotes a reduction of e^{-1} of the original amplitude and Increase of 1np indicates amplitue of wave is increased by e^{+1}.

$1\,Np = 20 \log_{10} e$

$= 8.686\ dB$.

β :- (phase constant or wave no)

- Measure of phase shift per unit length.
- unit : rad/m

Expression of α & β

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = (\alpha^2 - \beta^2) + 2j\alpha\beta \quad \text{—} ①$$

form eq ⑧

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) \quad \text{—} ②$$

from ① & ②

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \quad \text{—} ③$$

$$2\alpha\beta = \omega\mu\sigma$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left\{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/2} - 1\right\}}} \quad (np/m).$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left\{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/2} + 1\right\}}} \quad (rad/m)$$

$$\vec{\nabla} \times \vec{H_s} = \vec{J_s} + j\omega\vec{D_s}$$

$$= \underbrace{\sigma\vec{E_s}}_{\vec{J_c}} + \underbrace{j\omega\epsilon\vec{E_s}}_{\vec{J_D}}$$

$$\boxed{\text{Dissipation factor} = \frac{\sigma}{\omega\epsilon} = \frac{\vec{J_c}}{\vec{J_D}}}$$

Good conductor, $J_c >> J_D$

i.e. $DF >> 1$.

for dielectric,

$$\frac{\sigma}{\omega\epsilon} << 1$$

$$DF = \frac{\sigma}{2\pi f \epsilon}.$$

$$DF << 1$$

f low, DF - high
conductor.

for good conductor,

$$\frac{\sigma}{\omega\epsilon} >> 1$$

$$\boxed{\left.\begin{matrix}\alpha \\ \beta\end{matrix}\right\} \approx \sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$\boxed{\alpha = \beta = \sqrt{\frac{2\pi f \mu\sigma}{2}} = \sqrt{\pi f \mu \sigma}}$$

phase velocity

$$\boxed{V_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$ — $\vec{E}$ lead $\vec{H}$ by 45°.

Eg.

$$\vec{E} = E_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_x$$

$$\vec{H} = H_0 \, e^{-\alpha z} \cos(\omega t - \beta z) \, \hat{a}_y$$

$$\hat{a}_k = \hat{a}_E \times \hat{a}_H$$

$$\hat{a}_z = \hat{a}_x \times \hat{a}_y.$$

$$H_0 = \frac{E_0}{\eta} = \frac{E_0}{|\eta| \, \theta_\eta}$$

$$\boxed{\vec{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta). \, \hat{a}_y}$$

Plane wave in free space

$\sigma = 0 \quad \rho_v = 0 \quad \epsilon = \epsilon_0 \quad \mu = \mu_0$

$\alpha = 0 \quad \beta = \omega\sqrt{\mu_0 \epsilon_0}$

$$\boxed{\beta = \frac{\omega}{c}} \quad \left[\because \sqrt{\mu_0\epsilon_0} = 1/c\right].$$

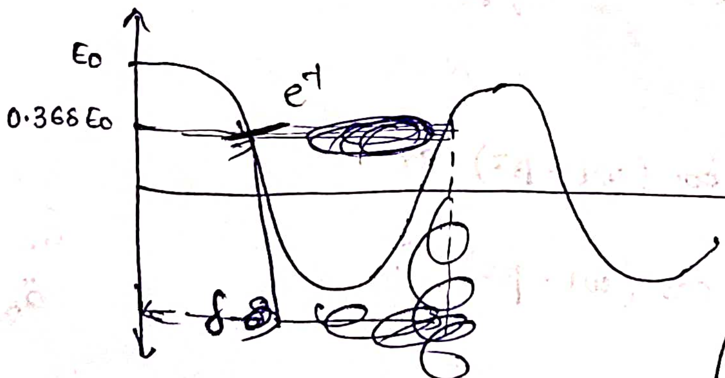
$$\boxed{\eta = \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120 \times \pi = 377\,\Omega.}$$

Intrinsic Impedence of free space

Plane wave in lossless dielectric

$$\frac{\sigma}{\omega\epsilon} << 1$$

The distance δ through which the wave amplitude decreases to a factor e^{-1} (i.e about 37 % of original amplitude) is called skin depth or peneteration depth of the medium.



$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$E_0\, e^{-\alpha\delta} = E_0\, e^{-1}$$

$$\alpha\delta = 1$$

Exam ~~ ~~

$$\boxed{\delta = 1/\alpha = \sqrt{\frac{2}{\omega\mu\sigma}}} = \frac{1}{\sqrt{\pi f \mu\sigma}}$$

σ not ϵ

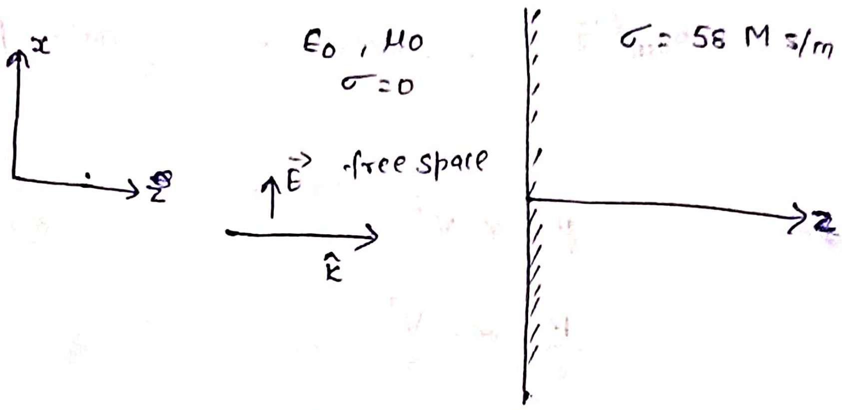
Q. Assume a field $\vec{E} = 1.0\, e^{-\alpha z} e^{j(\omega t - \beta z)}\, \hat{a}_x$ (V/m) with

~~for~~ $f = \frac{\omega}{2\pi} = 100\,MHz$ at the surface of copper conductor

$\sigma = 58\,M\,s/m$ located at $z>0$ as shown in fig.

Examine the attenuation of the wave propagating into the conducting medium.

<u>Soln</u> :-



$\frac{\sigma}{\omega \epsilon} \gg 1.$

$|E| = 1.0\, e^{-\alpha z}$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 6.61\,\mu m.$$

After $6.61\,\mu m$ the field is attenuated to e^{-1} ie

36.8 % of the initial value.

As frequency ↑ — Skin depth ↓

⇓

That's why low freq. cable can't be used for high freq. <u>Cable</u>.

$G_V\ (dB) = 20 \log_{10} V_o/V_i$

$G_P\ (dB) = 10 \log_{10} P_o/P_i$

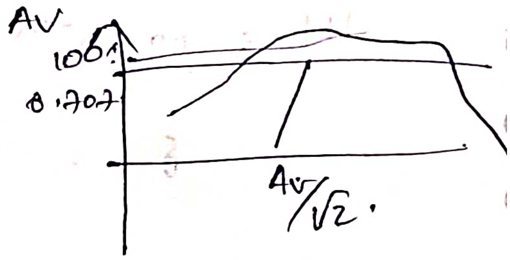
$$\boxed{P = P_o e^{-2\alpha z}}$$ — Power attenuation is twice as $\vec{E}$ as $\vec{E}$ is analogy to $\vec{V}$.

$10 \log_{10}(1/2) = -3dB$ — half power

$P \propto V^2$

$\frac{P}{2} \propto \frac{V^2}{2}$

$\propto \left(\frac{V}{\sqrt{2}}\right)^2$ —



Tutorial
12/10/23

Q. $\vec{H} = H_m e^{j(\omega t + \beta z)} \hat{a}_x$ in free space find $\vec{E}$.

Soln:- $\hat{a}_K = \hat{a}_E \times \hat{a}_H$

free space, $\rho_v = 0$ $\sigma = 0$

$\vec{B} = \mu \vec{H}$, $\vec{D} = \epsilon \vec{E}$

$\vec{J_c} = \sigma \vec{E}$

from M.E.

$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$

$= \frac{\partial}{\partial t} \epsilon_0 \vec{E} = \epsilon_0 \frac{\partial}{\partial t} \vec{E}$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(z) & 0 & 0 \end{vmatrix}$$

$$= \hat{a}_y \frac{\partial}{\partial z} H_x(z)$$

$$= \hat{a}_y \; j\beta \; H_m \; e^{j(\omega t + \beta z)}$$

$$\epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \times \vec{H}$$

$$= \hat{a}_y \; j\beta \; H_m \; e^{j(\omega t + \beta z)}$$

$$\int \partial \vec{E} = \int \frac{j\beta}{\epsilon_0} H_m \; e^{j(\omega t + \beta z)} \, \partial t \quad \hat{a}_y$$

$$\vec{E} = \frac{\cancel{j}\beta \; H_m \; e^{j(\omega t + \beta z)}}{\cancel{j}\omega} \quad \hat{a}_y \quad V/m$$

$$\vec{E} = \frac{\beta H_m}{\omega} \; e^{j(\omega t + \beta z)} \quad \hat{a}_y \quad V/m.$$

Q. In free space, $\vec{D} = D_m \sin(\omega t + \beta z) \; \hat{a}_x$ using M.E Show that $\vec{B} = \frac{-\omega \; \mu_0 D_m}{\beta} \sin(\omega t + \beta z)$. Sketch the fields at $t=0$ along the z-axis assuming $D_m > 0$ and $\beta > 0$.

Solⁿ:-

$$\vec{E} = \underbrace{\frac{D_m}{\epsilon_0} \sin(\omega t + \beta z)}_{E_x(z)} \; \hat{a}_x$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z) & 0 & 0 \end{vmatrix} = \hat{a}_y \frac{\partial E_x(z)}{\partial z}.$$

$$= \hat{a}_y \; \frac{D_m}{\epsilon_0} \cdot \beta \; \cos(\omega t + \beta z)$$

$$\int -\partial \vec{B} = \int \hat{a}_y \frac{D_m}{\epsilon_0} \beta \cos(\omega t + \beta z)\, \partial t$$

$$\vec{B} = -\frac{D_m}{\epsilon_0} \frac{\beta \sin(\omega t + \beta z)}{\omega} \hat{a}_y.$$

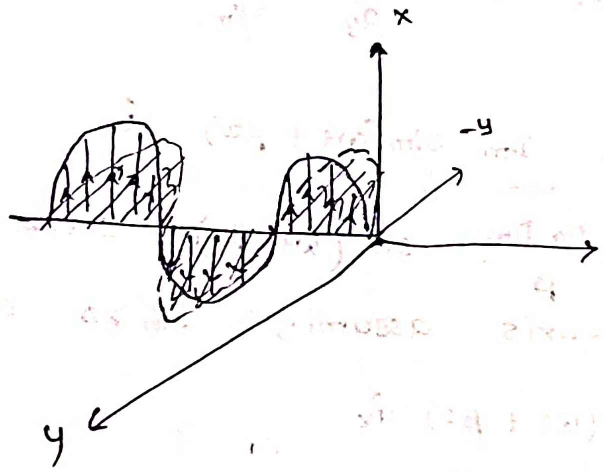
In free space,

$$\left.\begin{aligned} \beta &= \omega\sqrt{\mu_0 \epsilon_0} \\ \frac{\beta^2}{\omega^2} &= \mu_0 \epsilon_0 \\ \Rightarrow \frac{1}{\epsilon_0} &= \frac{\mu_0 \omega^2}{\beta^2}. \end{aligned}\right\}$$ Imp:

$$\boxed{\vec{B} = -D_m \frac{\omega}{\beta} \mu_0 \sin(\omega t + \beta z) \hat{a}_y}$$

$t = 0$, $\vec{E} = E_0 \sin(\beta z)\, \hat{a}_x$

$\vec{H} = \frac{\vec{B}}{\mu_0} = -H_0 \sin(\beta z)\, \hat{a}_y$



Q. In a Homogeneous non-conducting medium $\mu_r = 1$. Find ϵ_r and ω if $\vec{E} = 30\pi\, e^{j(\omega t - 4/3\, y)}\, \hat{a}_z$ (V/m)

and $\vec{H} = 1.0\, e^{j(\omega t - 4/3\, y)}\, \hat{a}_x$ (A/m)

Soln:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}.$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ 0 & 0 & \vec{E}_z(y) \end{vmatrix} = \hat{a}_x \frac{\partial E_z}{\partial y}.$$

$$= \hat{a}_x\, 30\pi j\left(-\frac{4}{3}\right) e^{j(\omega t - 4/3 y)} \quad \text{—— (1)}$$

$$-\mu\frac{\partial \vec{H}}{\partial t} = -\mu\, j\omega\, e^{j(\omega t - 4/3 y)} \quad \text{—— (2).}$$

$$30\pi\left(-4/3\right)\cancel{j} = -\mu \cancel{j}\omega$$

$$\mu\omega = \cancel{30}\, 40\pi$$

$$\omega = \frac{40\pi}{\mu_0 \mu_r}$$

$$= \frac{40\pi}{\mu_0}.$$

$$\frac{\vec{E}}{\vec{H}} = \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \frac{1}{\sqrt{\epsilon_r}}\times\sqrt{\frac{\mu_0}{\epsilon_0}} \text{ —— } \underline{\underline{120\pi}}.$$

$$30\pi = \frac{1}{\sqrt{\epsilon_r}}\times 120\pi$$

$$\epsilon_r = (4)^2 = 16.$$

Q A lossy dielectric has an η = $200\angle 30°\ \Omega$ (Intrinsic Impe) at a particular radian freq. ω. If at that frequency the dielectric has the plance wave propagating through the dielectric has the magnetic field component $\vec{H} = 10\, e^{-\alpha x}\cos(\omega t - \frac{1}{2}x)\,\hat{a}_y$ A/m. Find $\vec{E}$ and α determine skin depth and polarization,

Sol^{n}:-

$$\frac{E_0}{H_0} = \eta$$

$$E_0 = H_0\eta$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ 0 & 0 & \vec{E}_z(y) \end{vmatrix} = \hat{a}_x \frac{\partial E_z}{\partial y}.$$

$$= \hat{a}_x \; 30\pi j\left(-\frac{4}{3}\right) e^{j(\omega t - 4/3 y)} \quad \text{———} (1)$$

$$-\mu\frac{\partial \vec{H}}{\partial t} = -\mu \; j\omega \; e^{j(\omega t - 4/3 y)}$$

$$\text{———} (2).$$

$$30\pi\left(-4/3\right)\cancel{j} = -\mu \cancel{j}\omega$$

$$\mu\omega = \cancel{30}\; 40\pi$$

$$\omega = \frac{40\pi}{\mu_0 \mu_r}$$

$$= \frac{40\pi}{\mu_0}.$$

$$\frac{\vec{E}}{\vec{H}} = \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \frac{1}{\sqrt{\epsilon_r}} \times \sqrt{\frac{\mu_0}{\epsilon_0}} \text{ —— } \underline{\underline{120\pi}}.$$

$$30\pi = \frac{1}{\sqrt{\epsilon_r}} \times 120\pi$$

$$\epsilon_r = (4)^2 = 16.$$

Q A lossy dielectric has an η = 200 $\angle 30^\circ$ Ω (Intrinsic Impe) at a particular radian freq. ω. If at that frequency the plane wave propagating through the dielectric has the magnetic field component $\vec{H} = 10\, e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)\, \hat{a}_y$ A/m. Find $\vec{E}$ and α determine skin depth and polarization.

Soln:-

$$\frac{E_0}{H_0} = \eta$$

$$E_0 = H_0\eta$$

$\hat{a}_x : \hat{a}_E \times \hat{a}_H$

$\hat{a}_y : \quad \times \hat{a}_y$

$-\hat{a}_z$

$$\vec{E} = 2000\, e^{-\alpha x} \cos\left(\omega t - \tfrac{1}{2}x + \theta_\eta\right)\left(-\hat{a}_z\right)$$

due to multiplication.

$$\vec{E} = -2000\, e^{-\alpha x} \cos\left(\omega t - \tfrac{1}{2}x + \pi/6\right) \hat{a}_z \quad V/m.$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}.$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1}\right]^{1/2}$$

Given :- dissipation factor $\frac{\sigma}{\omega\epsilon} = \tan 2\theta_\eta = \tan 60° = \sqrt{3}.$

DF >> 1

Good conductor

$$= \left[\frac{2-1}{2+1}\right]^{1/2} = \frac{1}{\sqrt{3}}.$$

$\alpha = \beta/\sqrt{3}$. [∵ Given $\beta = 1/2$)

$\alpha = 1/2\sqrt{3}$ Np/m.

Skin depth $(\delta) = 1/\alpha = 2\sqrt{3}$ m.

$= 3.46$ m.

12/10/16

Power and Poynting Vector

Maxwell's eqⁿ

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad — ①$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad — ②$$

Dotting both the sides of eq ②,

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \sigma \vec{E} \cdot \vec{E} + \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \quad — ③$$

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) + \vec{\nabla} \cdot (\vec{H} \times \vec{E})$$

$$= \sigma E^2 + \vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$= \sigma E^2 + \frac{1}{2} \epsilon \frac{\partial}{\partial t} E^2 \quad — ④$$

Dotting both side of eqⁿ ① w.r.t $\vec{H}$;

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) = -\frac{\mu}{2} \frac{\partial}{\partial t} (H^2) \quad — ⑤$$

$$\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{H}) \, \partial v = -\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2\right] \partial v - \int_V \sigma E^2 \partial v$$

↓ diverge

Pointing Theorem

$$\boxed{\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2\right] \partial v - \int_V \sigma E^2 \partial v}$$

① ② ③

Total power leaving the volume ①

② Rate of decrease in Energy stored in electric and magnetic field.

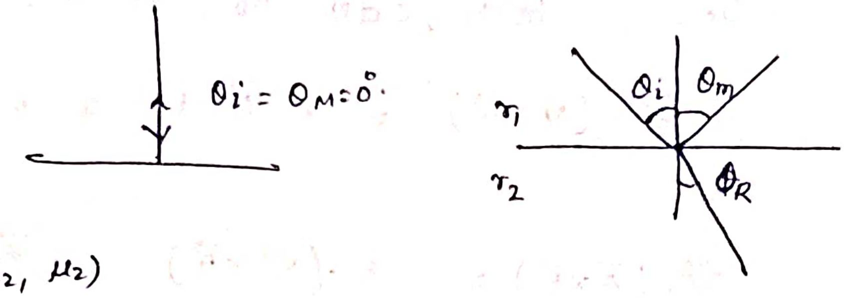
Ohomic power dissipation ③

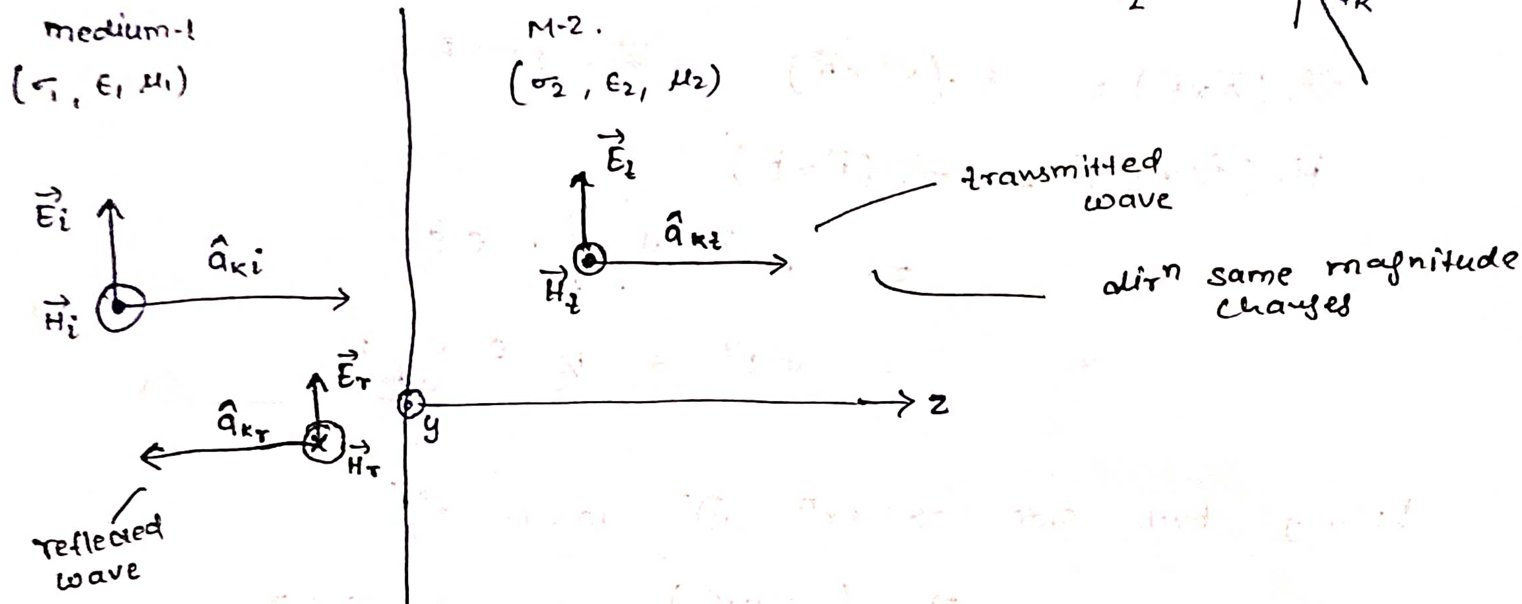
$\vec{S} = \vec{E} \times \vec{H}$

$W/m^2 = V/m \quad A/m \quad W/m^2$

Instantenious power density.

Reflection of plane wave at normal Incidence.





Assuming $\vec{H}$ changes

$\hat{a}_{ki} = \hat{a}_E \times \hat{a}_H$

$\hat{a}_z = \hat{a}_x \times \hat{a}_y$

Incident wave $(\vec{E}_{is}, \vec{H}_{is})$

$$\left.\begin{aligned}\vec{E}_{is}(z) &= E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x \\ \vec{H}_{is}(z) &= H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y\end{aligned}\right\} \text{(a)}$$

$$= \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \hat{a}_y$$

Reflected wave $(\vec{E}_{rs}, \vec{H}_{rs})$

$$\vec{E}_{rs}(z) = E_{ro}\, e^{+\gamma_1 z}\, (+\hat{a}_x)$$

$$\vec{H}_{rs}(z) = H_{ro}\, e^{\gamma_1 z}\, (-\hat{a}_y)$$

$$= \frac{E_{ro}}{\eta_1}\, e^{\gamma_1 z}\, (-\hat{a}_y)$$

Transmitted wave $(\vec{E}_{ts}, \vec{H}_{ts})$

$$\vec{E}_{ts}(z) = E_{t0}\, e^{-\gamma_2 z}\, \hat{a}_x$$

$$\vec{H}_{ts}(z) = H_{t0}\, e^{-\gamma_2 z}\, \hat{a}_y$$

$$\frac{E_{t0}}{\eta_2}$$

At interface, $z=0$, $\vec{E}$ and $\vec{H}$ should be continuous.

$$E_{1t}\,an = E_{2t}\,an$$

$$H_{2t}\,an = H_{2t}\,an$$

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_t(0)$$

E_t'

E_r

+ve due to vector additio

$$\Rightarrow E_{i0} + E_{r0} = E_{t0}$$

H_i

H_r

$$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

-ve due to vector subsinaltn

$$\Rightarrow \frac{1}{\eta_1}(E_{i0} - E_{r0}) = \frac{E_{t0}}{\eta_2}$$

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0}$$

$$E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0}$$

gamma

$$\boxed{\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_2}}$$ reflection coefficient

Tau

$$\boxed{\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}}$$ transmission coefficient.

17/10/23

Case-1

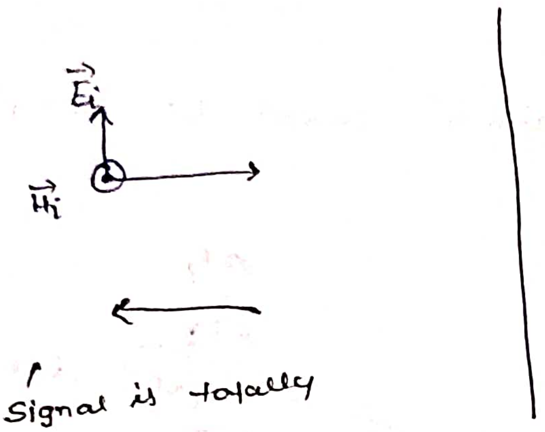
Medium-1 - Perfect dielectric

M-2 perfect ~~dielectric~~ conductor

$\sigma = 0$

$\sigma = \infty$

PEC → perfect electric conductor



$\eta_2 = 0$

$\Gamma = -1$ — 180° phase shift.

PMC $\Gamma = +1$

Signal is totally reflected.

$$\vec{E}_{1s} = \vec{E}_{is} + \vec{E}_{rs}$$

total electric field

$$= \left(E_{io}\, e^{-\gamma_1 z} + E_{ro}\, e^{\gamma_1 z}\right) \hat{a}_x$$

$\Gamma = \frac{E_{ro}}{E_{io}} = -1$; $\alpha = 0$ for medium 1.

$\gamma_1 = \alpha + j\beta_1$

$$\boxed{\vec{E}_{1s} = -E_{io}\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right) \hat{a}_x}$$

$$\boxed{\vec{E}_{1s} = -2j\, E_{io} \sin\beta_1 z\ \hat{a}_x}$$

total Electric field in M-1.

$$\vec{E}_1 = Re\left\{\vec{E}_{1s}\, e^{j\omega t}\right\}.$$

$$= Re\left\{-2j\, E_{io} \sin\beta_1 z \left[\cos\omega t + j \sin\omega t\right]\right\}.$$

$$\boxed{\vec{E}_1 = 2 E_{io} \sin\beta_1 z \sin\omega t\ \hat{a}_x}$$

$$\boxed{\vec{H}_1 = \frac{2 E_{io}}{\eta_1} \cos\beta_1 z \cos\omega t\ \hat{a}_x}$$

Standing wave Ratio (SWR)

- Represented by S

SWR > 1 or equal 1.

↳ it doesn't have fraction value.

$$\boxed{S = \frac{|\vec{E_1}|_{max}}{|\vec{E_1}|_{min}}}$$

$|\Gamma| \Rightarrow 0$ to 1.

Γ Range -1 to 1

$$\boxed{|\Gamma| = \frac{S-1}{S+1}}$$

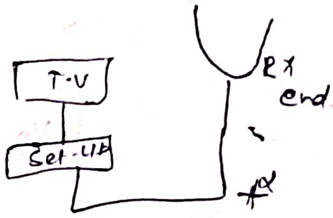
gamma.

$$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

Vector - Network Analyzer

to measure Γ

$$\boxed{S_{dB} = 20 \log_{10} S}$$



Perfect Match.

$S=1$, $|\Gamma|=0$, $E_{ro} = 0$;

Texbook (Exam) 9.7

Q. free space $(z \le 0)$ a plane with $\vec{H_i} = 10 \cos(10^8 t - \beta z)\, \hat{a}_x$ mA/m is incident normally on a lossless medium which is $(z \ge 0)$ is characterized by $\epsilon = 2\epsilon_0$, $\mu = 8\mu_0$

determine, $\vec{H_r}$, $\vec{E_r}$, $\vec{H_t}$, $\vec{E_t}$.

Soln:-

$$\hat{a}_{K_i} = \hat{a}_{E_i} \times \hat{a}_{H_i}$$

$$\hat{a}_z = (-\hat{a}_y) \times \hat{a}_x$$

for free space, $\beta_1 = \omega/c$; $\omega = 10^8$

$$\beta_1 = \frac{10^8}{3 \times 10^8} = 1/3.$$

$$\eta_1 = \eta_0 = 120\pi \ \Omega$$

For lossless medium,

$$\beta_2 = \omega\sqrt{\mu\epsilon}$$

$$= \omega\sqrt{\mu_r\mu_0\,\epsilon_r\epsilon_0}$$

$$= \frac{\omega}{c}\times 4 \qquad \sqrt{\mu_0\epsilon_0} = 1/c$$

$$\beta_2 = 4/3. \qquad \left[\because \omega/c = 1/3\right].$$

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{8\times\mu_0}{2\,\epsilon_0}}$$

$$= 2\sqrt{\frac{\mu_0}{\epsilon_0}} = 2\eta_0$$

$$= 240\pi\ \Omega.$$

$$\vec{E_i} = \underbrace{E_{oi}}_{10\eta_0}\cos(10^8 t - \beta z)\,(-\hat{a}_y) \qquad \boxed{\eta_0 = \frac{E_{oi}}{H_{oi}}}$$

$$\eta_0 = \frac{E_{oi}}{H_{oi}} =$$

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\eta_0}{3\eta_0} = 1/3$$

$$\tau = \frac{E_{to}}{E_{io}} = 4/3.$$

$$\vec{E_r} = \underbrace{E_{ro}}_{E_{io}/3}\cos(10^8 t + \beta z)\,\hat{a}_y$$

7/11/23 Hertzian dipole

(Book) Sadiku.

$\frac{1}{r^3}$

If $\beta r >> 1$

$\frac{1}{r^2}$

$\frac{1}{r} > \frac{1}{r^2} > \frac{1}{r^3}$ - r is much larger.

$\frac{1}{r}$

term $\left(\frac{1}{r}\right)$ — far field component.

exists on far field:

$\Downarrow$

$E_r = 0$

$E_\phi = \frac{1}{r}(\quad)$ exist. , H_ϕ

Only

Near field of Hertzian dipole,

$\vec{H} = H_\phi \hat{a}_\phi$

$\vec{E} = E_r \hat{a}_r + E_\theta \hat{a}_\theta$

Far field $(1/r)$.

$\hookrightarrow \vec{H} = H_\phi \hat{a}_\phi$

$\vec{E} = E_\theta \hat{a}_\theta$

two orthogonal component

$\Downarrow$

θ & ϕ.

$E_\theta = \eta H_\phi$

$\boxed{\frac{E_\theta}{H_\phi} = \eta}$

Q. Exam.

time average power density $= \frac{1}{2} Re(\vec{E} \times \vec{H}^*)$

$= \frac{1}{2} Re(E_\theta \hat{a}_\theta \times \vec{H}_\phi^* \hat{a}_\phi)$

$= \frac{1}{2} \eta |H_\phi|^2 \hat{a}_r$

Prad $= \int P_{avg} . ds$

$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{I_0^2 \beta^2 \eta \, dl^2}{32\pi^2 r^2} \sin^2\theta \, r^2 \sin\theta \, d\theta \, d\phi$

$= \eta \frac{I_0^2 \pi^2}{3} \left(\frac{dl}{\pi}\right)^2$

Radiation resistance,

$$I = I_0 \cos(\omega t).$$

Hertization diople is poor radiator

$$P_{rad} = I_{rms}^2 R_{rad}$$

$$= \frac{1}{2} I_0^2 R_{rad}$$

$$\boxed{R_{rad} = \frac{2 \times P_{rad}}{I_0^2}}$$

Half wave dipole Antenna

- total length ($\lambda/2$)
  - difference in path length (r, r').
- Combination of several dipoles. A Hertzian.

$$r - r' = z\cos\theta$$

$$r' = r - z\cos\theta.$$

Q. Dalculate the directivity and radiation Intensity for half-wave anteena.

Soln:-

Hertzian & some formulae should be remembered

Radiation Intensity $= r^2 S(\theta, \phi)$ → time avg power density.

$$S = \frac{1}{2} \operatorname{Re}(\vec{E} \times \vec{H}^*)$$

$$= \frac{1}{2} \eta |\vec{H}|^2 \hat{a}_r \quad \text{or} \quad \frac{1}{2\eta} |\vec{E}| \hat{a}_r.$$

$$\frac{\eta I_0^2 \cos^2(\frac{\pi}{2}\cos\theta)}{8\pi^2 r^2 \sin^2\theta} \hat{a}_r$$

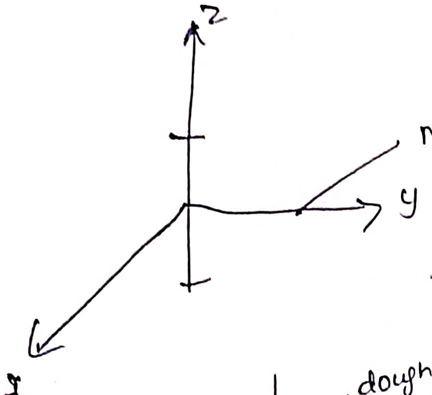
$$U = \frac{\eta}{2} \left[\frac{I_0}{2\pi}\right]^2 \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$$

$$D(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{rad}} \quad 36.56 I_0^2$$

$$= 1.642 \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}.$$

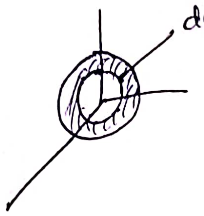
$D(\pi/2, \phi) = 1.642$ $\qquad$ $D_{max} = 1.642$ at $\theta = \pi/2$.

So, diopole is located along the z-axis



max^m directivity along y

↳ Independent of ϕ.



doughnut shape.

$D_{dB} = 10\log_{10}(1.642)$

$= 2.15$ dB.

fixed for half wave dipole.