* Magnetisation in materials

$(1+\phi) = \chi_r$

Questions

Que-01 A conducting bar can slide over two conducting reels as shown in fig. Calculate the induced voltage in the bar if the bar is stationed at $y = 8cm$ & $\vec{B} = 4\cos 10^6 t\, \hat{a}_z$

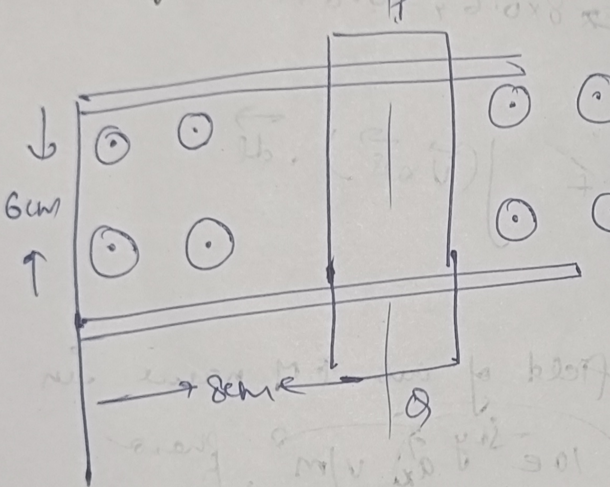
Sol

(a) $y = 8\,cm$ $\vec{B} = 4\cos 10^6 t\, \hat{a}_z$ mWb/m^2

(b) $\vec{v} = 20\,\hat{a}_y$ m/s and $\vec{B} = 4\,\hat{a}_z$ mWb/m^2

(c) $\vec{v} = 20\,\hat{a}_y$ m/s and $\vec{B} = 4\cos(10^6 t - y)\,\hat{a}_z$ mWb/m^2

Ans-



$\frac{\partial B}{\partial t} = 4\times10^6 \sin 10^6$

(a) $V_{emf} = \int -\frac{\partial B}{\partial t}\cdot ds$

$= \int_0^{0.08}\int_0^{0.06} -\frac{\partial(4\cos 10^6 t)}{\partial t}\, dx\, dy$

$= \int_0^{0.08}\int_0^{0.06} 4\times10^6 \sin 10^6 t \cdot dx\, dy$

$= 4\times10^6 \times \sin 10^6 t\,[0-0.048]$

$= 192\times10^2 \times \sin 10^6 t$

$=$

(b) $\vec{U} = 20\,\hat{a}_y$ m/s $\qquad \vec{B} = 4\,\vec{a}_z$ mWb/m²

Motional Emf

$$V_{emf} = \int (\vec{U}\times\vec{B})\cdot d\vec{l}$$

$$= \int_0^{0.06} 80\,\hat{a}_x\cdot dx$$

$$= 80\times[0.06]$$

$$= \cancel{80}\ 8\times 0.6 = 4.8 \text{ mV}$$

(c) $$V_{emf} = -\int \frac{dB}{dt}\cdot d\vec{s} + \int (\vec{U}\times\vec{B})\cdot d\vec{l}$$

Que-02 The Electric field of an EM wave in free space is $\vec{E}_s(y) = 10e^{-j4y}\,\hat{a}_x$ V/m. phasor form

Find (a) 'ω' so that E_s satisfy M.E

(b) the corresponding mag. field.

Ans: $\vec{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\,\hat{a}_\rho$ V/m

$$\text{Re}\left\{\frac{50}{\rho}e^{j(10^6 t + \beta z)}\,\hat{a}_\phi\right\}$$

$$\text{Re}\left\{\underbrace{\frac{50}{\rho}e^{j\beta z}\,\hat{a}_\phi}_{\vec{E}_s}\, e^{j10^6 t}\right\}$$

$\vec{E}(r,t) =$

$\vec{E}_s = E_{sx}(y)\,\hat{a}_x$

$\vec{\nabla}\times\vec{E}_s =$

$\vec{\nabla}\times\vec{E}_s =$

$\vec{B}_s =$

Que-03

$$\vec{\varepsilon}(r,t) = Re\{\vec{E}_s e^{j\omega t}\}$$

$$= Re\{10 e^{-j4y}\hat{a}_x\}$$

$$= Re\{10 e^{j(\omega t - 4y)})$$

$$= 10\cos(\omega t - 4y)\,\hat{a}_x$$

$$\vec{E}_s = E_{sx}(y)\,\hat{a}_x$$

$$\vec{\nabla}\times\vec{E}_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx}(y) & 0 & 0 \end{vmatrix}$$

$$= -\hat{a}_z \frac{\partial E_x}{\partial y} = -\hat{a}_z \frac{\partial}{\partial y}(10\cos(\omega t - 4y))$$

$$= 40 j e^{-4jy}$$

$$\vec{\Delta}\times\vec{E}_s = -j\omega\vec{B}_s$$

$$\boxed{\vec{B}_s = \cancel{10}\ \mu\vec{H}_s}$$

B

Problem 8-36 or 9.36

Que-03

Questions

Tutorial - 01

Que - 01 $\vec{H} = H_m e^{j(\omega t + \beta z)} \hat{a}_x$ in free space $\vec{E} = ?$ → wave is propagating along $-z$

Ans -

Maxwell's Eqⁿ

$\vec{\nabla} \cdot \vec{D} = \rho_v$

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla} \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$

free space

$\rho_v, \sigma = 0$

$J_c = 0$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_m e^{j(\omega t + \beta z)} & 0 & 0 \end{vmatrix}$$

$$= \frac{\partial}{\partial z} H_m e^{j(\omega t + \beta z)} \hat{a}_y$$

$$= H_m e^{j\omega t} \beta e^{j\beta z} \hat{a}_y$$

$$\vec{\nabla} \times \vec{H} = j\beta H_m e^{j(\omega t + \beta z)}$$

$\hat{a}_k = \hat{a}_E \times \hat{a}_H$

$-\hat{a}_z = \hat{a}_y \times \hat{a}_x$

In free space $J_c = 0$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\frac{\partial \vec{D}}{\partial t} = j\beta H_m e^{j\omega t} \cdot e^{j\beta z}$$

$$\partial \vec{D} = j\beta H_m e^{j\omega t} e^{j\beta z} \, dt$$

$$\vec{D} = \frac{j\beta H_m e^{j\omega t} e^{j\beta z}}{j\omega}$$

$$\vec{D} = \frac{\beta H_m}{\omega} e^{j(\omega t + \beta z)}$$

$\vec{D} = \epsilon_0 \vec{E}$

$$\boxed{\vec{E} = \frac{\beta H_m}{\epsilon_0 \omega} e^{j(\omega t + \beta z)} \hat{a}_y \ V/m}$$

Que:- $A = -\frac{\rho^2}{4} a_z$ Wb/m

Calculate the total magnetic flux crossing the surface $\phi = \frac{\pi}{2}$, $1 \le \rho \le 2\,m$, $0 \le z \le 5\,m$

Ans Method 1:

$$A = -\frac{\rho^2}{4} a_z \ \text{Wb/m}$$

$$B = \nabla \times A \qquad \begin{vmatrix} a_\rho & a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & -\frac{\rho^2}{4} \end{vmatrix}$$

$$B = \frac{1}{4}\frac{\partial \rho^2}{\partial \rho} a_\phi$$

$$= \frac{\rho}{2} a_\phi$$

$$\Psi = \int B \cdot dS = \int B \cdot \partial\rho\, \partial z\, a_\phi$$

$$= \int \frac{\rho}{2} \cdot \partial\rho \int dz$$

$$=$$

Que-01 $\vec{H} =$

Ans-

$\vec{\nabla} \times \vec{H} =$

$\vec{\nabla} \times$

In -

Que-02 In free space
$\vec{D} = D_m \sin(\omega t + \beta z)\, \vec{a}_x$

using maxwell's equations show that

$$\vec{B} = \frac{-\omega \mu_0 D_m}{\beta} \sin(\omega t + \beta z)\, \hat{a}_y$$

Ans $\vec{E} = \frac{\vec{D}}{\epsilon_0}$

$$= \frac{D_m}{\epsilon_0} \sin(\omega t + \beta z)\, \hat{a}_x$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{D_m}{\epsilon_0}\sin(\omega t + \beta z) & 0 & 0 \end{vmatrix}$$

$$= \frac{\partial}{\partial z} \frac{D_m}{\epsilon_0} \sin(\omega t + \beta z)\, \hat{a}_y + a_z \times 0$$

$$\vec{\nabla} \times \vec{E} = \frac{D_m}{\epsilon_0} \beta \cos(\omega t + \beta z)\, a_y$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\cancel{\frac{D_m}{\epsilon_0} \beta \omega \sin(\omega t + \beta z)}$$

$$\frac{\partial \vec{B}}{\partial t} = -\frac{D_m}{\epsilon_0} \beta \cos(\omega t + \beta z) \cdot \partial t\, \hat{a_y}$$

$$\boxed{\vec{B} = \frac{-D_m \beta \sin(\omega t + \beta z)}{\epsilon_0 \omega}\, \hat{a_y}}$$

$\boxed{\beta = \omega\sqrt{\mu_0 \epsilon_0}}$ in free

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\frac{\omega^2}{\beta^2} = \frac{1}{\mu_0 \epsilon_0}$$

$$\vec{B} = \frac{-D_m \beta}{\omega} \times \frac{\omega^2 \mu_0}{\beta^2}$$

$$\boxed{\vec{B} = \frac{-D_m \omega \mu_0}{\beta}}$$

Que-03 In a homoge...
find ϵ_r and ω ...

$\vec{H} = 1.0\, e^{j(\omega t - }$...

Ans wave ...

$\vec{E}$

$\vec{H}$

$\hat{a}_y$

$\vec{\nabla} \times \vec{E} =$

$\vec{\nabla} \times \vec{E} =$

$\boxed{\beta = \omega\sqrt{\mu_0 \epsilon_0}}$ in free space

$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ → apply only in free space.

$\frac{\omega^2}{\beta^2} = \frac{1}{\mu_0 \epsilon_0}$ $\boxed{\frac{1}{\epsilon_0} = \frac{\omega^2 \mu_0}{\beta^2}}$

$\vec{B} = \frac{-D_m \beta}{\omega} \times \frac{\omega^2 \mu_0}{\beta^2} \sin(\omega t + \beta z)\, \hat{a_y}$

$$\boxed{\vec{B} = \frac{-D_m \omega \mu_0}{\beta} \sin(\omega t + \beta z)\, \hat{a_y}}$$

Que-03 In a homogeneous non-conducting region where $\mu_r = 1$ find ϵ_r and ω if $\vec{E} = 30\pi e^{j(\omega t - \frac{4}{3}y)}\hat{a_z}$ V/m and

$\vec{H} = 1.0\, e^{j(\omega t - \frac{4}{3}y)}\hat{a_x}$ A/m

Ans

wave propagation → $+y$

$\vec{E} \longrightarrow +z$

$\vec{H} \longrightarrow x$

$$\boxed{\hat{a_y} = \hat{a_z} \times \hat{a_x}}$$

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 30\pi e^{j(\omega t - \frac{4}{3}y)} \end{vmatrix}$$

$$= \frac{30\pi\, \partial e^{j(\omega t - \frac{4}{3}y)}}{\partial y}\hat{a_x} = 30\pi\left(-\frac{4}{3}\right) \times e^{j(\omega t - \frac{4}{3}y)}$$

$\vec{\nabla} \times \vec{H} = 80\pi x \left(-\frac{4}{3}\right) j x \, e^{j(\omega t - \frac{4}{3}y)} \hat{a}_x$

$= -40\pi \, e^{j(\omega t - \frac{4}{3}y)} \hat{a}_x$

$\frac{\partial \vec{H}}{\partial t} = j\omega \, e^{j(\omega t - \frac{4}{3}y)} \hat{a}_x$ A/m

$\vec{\nabla} \times \vec{E} = -\mu j\omega \, e^{j(\omega t - \frac{4}{3}y)} \hat{a}_x$

$-40\pi j = -\mu \times j\omega$

$40\pi \cancel{j} = \mu \times \cancel{j} \times \omega$

$\boxed{\omega = \frac{40\pi}{\mu}}$

$\mu = \mu_r \mu_0$

$\omega = \frac{40\pi}{\mu_0} = \frac{40\pi}{4\times10^{-7}\times\pi} = 10^8$

$\boxed{\omega = 10^8 \text{ rad/sec}}$

$\boxed{\frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}}$

$30\pi = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}}$

$30\pi = \frac{1}{\sqrt{\epsilon_r}} \sqrt{\frac{\mu_0}{\epsilon_0}}$

$\eta_0 = 120\pi$
$= 377\,\Omega$

$30\pi =$

$\epsilon_r =$

Que-04 A los

$\eta = 200 \angle 30^\circ \,\Omega$

that frequen

dielectric w

$\vec{H}$

Find $\vec{E}$ a

Ans or

$\eta =$

$E_0 =$

$=$

$E_0 =$

$\vec{E} =$

$\vec{E} =$

$\vec{E} =$

$\alpha =$

$\beta =$

$30\pi = \frac{1}{\sqrt{\epsilon_r}} 120\pi$

$\boxed{\epsilon_r = 16}$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma}} \quad \frac{\vec{E}}{\vec{H}}}$$

Que 04 A lossy dielectric has an intrinsic impedance of $\eta = 200 \angle 30° \Omega$ at a particular frequency ω. If at that frequency the plane wave propagating through the dielectric has the magnetic field component

$$\boxed{\vec{H} = 10 e^{-\alpha x} \cos(\omega t - \tfrac{1}{2}x) \hat{a}_y \ A/m}$$

Find $\vec{E}$ and α. Determine skin depth and polarization.

Ans orthogonal component of E and H given as η

$$\eta = \frac{E_0}{H_0} = 200 \angle 30°$$

$H_0 = 10$

$$E_0 = 200 H_0 e^{j\pi/6}$$

$$= 200 \times 10 \ e^{j\pi/6}$$

$$E_0 = 2000 e^{j\pi/6}$$

$$\boxed{\hat{a}_x = -\hat{a}_z \times \hat{a}_y}$$

$$\vec{E} = -E_0 e^{-\alpha x} \cos(\omega t - \tfrac{1}{2}x) \hat{a}_z$$

$$\boxed{\vec{E} = -E_0 e^{-\alpha x} \cos\left(\omega t - \tfrac{1}{2}x + \tfrac{\pi}{6}\right) \hat{a}_z} \ N/m$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1}{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1}\right]^{1/2}$$

$$\frac{\sigma}{\omega\epsilon} = \tan 2\theta_n = \tan 2\times\frac{\pi}{6} = \tan\ldots$$

$$\boxed{\frac{\sigma}{\omega\epsilon} = \sqrt{3}}$$

$$\boxed{\theta_n = 30°}$$

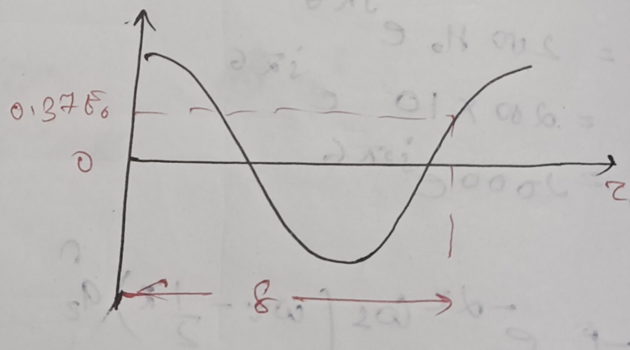
$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+3}-1}{\sqrt{1+3}+1}\right]^{1/2}$$

$$= \left(\frac{1}{3}\right)^{1/2} \qquad \beta = 1/2$$

$$\boxed{\alpha = \frac{1}{2\sqrt{3}}}$$

Skin depth

- The distance δ through which the wave amplitude dec to a factor e^{-1} (i.e 37% alone of the original amplitude) is known as skin depth or penetration depth of media.



$$E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

$$\alpha\delta = 1$$

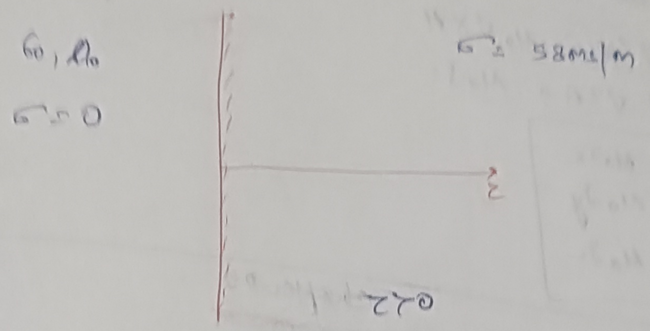
$$\boxed{\delta = \frac{1}{\alpha}}$$

$$\boxed{\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi f \mu\sigma}}}$$

Ques 5 $\vec{E} = 1.0 e^{-\alpha z} e^{j(\omega t - \beta z)} \vec{a}_x$ V/m

$f = \frac{\omega}{2\pi} = 100$ MHz at the surface of the Copper Conductor.
$\sigma = 58$ MS/m, $z > 0$ examine the attenuation of an EM wave propagating into the conducting medium.

Ans



$\frac{\sigma}{\omega \epsilon} \gg 1$ $|E| = 1.0 e^{-\alpha z}$

$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 6.61 \mu m$

Maxwell's equation

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \quad — ①$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial \vec{E}}{\partial t} \quad — ②$$

Dotting both sides of eq^n 2

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \sigma\vec{E}\cdot\vec{E} + \epsilon\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$$

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E}) + \vec{\nabla}\cdot(\vec{H}\times\vec{E}) = \sigma\vec{E}^2 + \vec{E}\cdot\epsilon\frac{\partial \vec{E}}{\partial t}$$

$$= \sigma E^2 + \frac{1}{2}\epsilon\frac{\partial}{\partial t}E^2$$

for lossless med.

$\eta_2 =$

$\vec{E_i} = -$

$\eta_0 =$

E_0

$\Gamma =$

$\vec{E_r}$

SWR C

Practice exercise 9.7

Q. Free space ($z \le 0$)

$\vec{H_i} = 10\cos(10^8 t - \beta z)\,\hat{a}_x$ mA/m, normal incident in loss-less medium $\epsilon = 2\epsilon_0$, $\mu = 8\mu_0$ ($z \geq 0$)

Determine the $\vec{H_r}$, $\vec{E_r}$, $\vec{H_t}$ and $\vec{E_t}$?

Ans-

$$\vec{a}_{k_i} = \vec{a}_{E_i} \times \vec{a}_{H_i}$$

$$\hat{a}_z = \quad \times \hat{a}_x$$

for freespace, $\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3}$

$\eta_1 = \eta_0 = 120\pi\ \Omega$

for lossless medium ($\alpha = 0$)

$\beta_2 = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_r\mu_0\epsilon_r\epsilon_0}$

$= \omega\sqrt{\mu_0\epsilon_0}\ 4$

$\beta_2 = \frac{\omega}{c}\times 4 = \frac{4}{3}$

$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{8\mu_0}{2\epsilon_0}} = 2\sqrt{\frac{\mu_0}{\epsilon_0}} = 2\eta_0 = 2\times120\pi$

$= 240\pi$

$\vec{E_i} = -\vec{E_0}i\cos(10^8t - \beta z)\,\hat{a}_y$

$\eta_0 = \frac{E_0 i}{H_0 i}$

$E_0 i = \eta_0 H_0 i$

$= 10\eta_0$

$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_0 - \eta_0}{3\eta_0} = \frac{1}{3}$

$\gamma = \frac{4}{3}$

$\vec{E_r} = E_{r0}\cos(10^8t + \beta z)\,\hat{a}_y$

$=$

$E_{r0} = \Gamma E_{i0}$

$\nabla \times \nabla \times E_s = -j\omega\mu \, \nabla \times H_s$

$\nabla(\underbrace{\nabla \cdot E_s}_{0}) - \nabla^2 E_s = -j\omega\mu(\sigma + j\omega\epsilon) E_s$

$$\boxed{\nabla^2 E_s - \gamma^2 E_s = 0}$$

$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

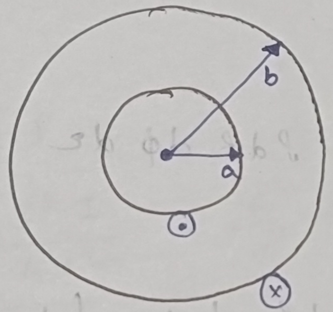
$\gamma \rightarrow$ propagation constant

$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$

$H(z,t) = H_0$

Tutorial

Que-01) Determine the self inductance of a coaxial cable.



$\vec{B_1} = \frac{\mu I \rho}{2\pi a^2} \hat{a}_\phi \quad (0 \le \rho \le a)$

$= \frac{\mu I}{2\pi \rho} \hat{a}_\phi \quad (a \le \rho \le b)$

$L = \frac{2W_m}{I^2}$

$\vec{B} = \mu \vec{H}$

$L = \frac{\lambda}{I} = \frac{N\psi}{I}$

$\psi = \int_S \vec{B} \cdot ds$

Method 1:-

$W_m = \frac{1}{2} L I^2$

$W_m = \frac{1}{2} \int \vec{B} \cdot \vec{H} \, dV$

$L = \frac{2W_m}{I^2}$

$= \frac{1}{2\mu} \int |B|^2 \cdot dV$

$L_{in} = \frac{2}{I^2}\int |B|^2 dV$

$= \frac{2}{I^2}\int\left(\frac{\mu I \rho}{2\pi a^2}\right)^2 \rho\, d\rho\, d\phi\, dz$

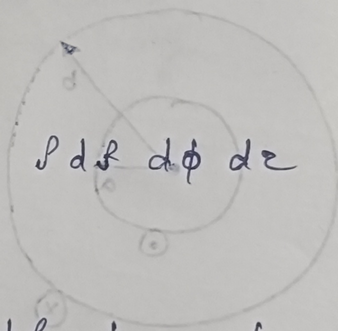
$= \frac{\mu^2}{2 I \pi^2 a^4}\int_0^a \rho^3\, d\rho \int_0^{2\pi} d\phi \int_{z=0}^{L} dz$

$= \frac{\mu^2}{2 I \pi^2 a^4}\left[\frac{\rho^4}{4}\right][2\pi][L]$

$= \frac{\mu a^{\cancel{4}}\, \cancel{2\pi}\, L}{4\, \cancel{8}\, I\, \cancel{\pi^2}\, a^4}$

$= \frac{\mu^2 L}{4 I \pi}$

For $a \le \rho \le b$



$L_{in} = \frac{2}{I^2}\int \frac{\mu^2 I^2}{4\pi^2 \rho^2}\rho\, d\rho\, d\phi\, dz$

$L_{in} = \frac{2}{I^2}\frac{\mu^2 I^2}{4\pi^2}\int_a^b \frac{1}{\rho}\, d\rho \int d\phi \int dz$

$= \frac{2\mu^2}{4\pi^2}\ln\left(\frac{b}{a}\right)\times 2\pi L$

$= \frac{\mu^2}{\pi^2}\ln\left(\frac{b}{a}\right)2\pi L = \frac{\mu^2}{\pi}\ln\left(\frac{b}{a}\right)L$

Method 2:-

$d\Phi$

Que-02 Show
L is

Ans-02

d

Assuming
conductor

J =

for

flux

Method 2:-

$d\phi_2 = \vec{B_2} \cdot d s_2$

$L = \frac{\phi}{I}$

Que-02 Show that internal inductance of a straight wire of length L is

$$L_{in} = \frac{\mu l}{8\pi} \text{ (Henry)}$$

$$L_{in}' = \frac{\mu}{8\pi} \text{ (per unit length) (Henry/L)}$$

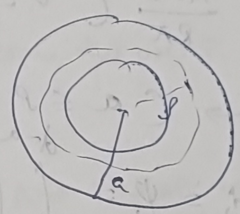
Ans-02

$$B_1 = \frac{\mu I \rho}{2\pi a^2} \, d\rho \, dz$$

$$d\phi_1 = B_1 \cdot ds$$

$$= B_1 \cdot d\rho \, dz \, \hat{a}_\phi$$

Assuming the current density is uniform over the entire conductor



$$J = \frac{I}{\pi a^2}$$

for radius ρ

$$\boxed{I_{enc} = \frac{\pi \rho^2 I}{\pi a^2} = \frac{\rho^2 I}{a^2}}$$

thus the total flux linkage within the differential flux element.

$$d\lambda_1 = \frac{I_{enc}}{I} \cdot d\phi_1$$

$$= \frac{\rho^2 I}{a^2} \times \frac{\mu I \rho}{2\pi a^2} \, d\rho \, dz$$

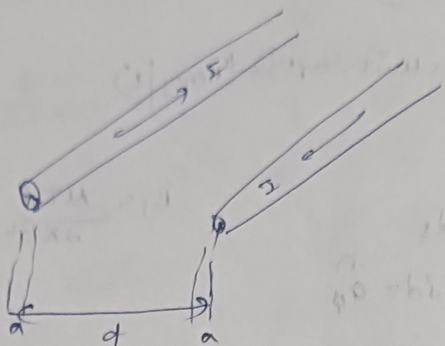
$$= \frac{\mu I}{2\pi a^4} \int \rho^3 \, d\rho \int dz$$

$$= \frac{\mu I}{2\pi a^4} \frac{a^4}{4} \times l = \frac{\mu I L}{8\pi}$$

$$L_{in} = \frac{\lambda_1}{I} = \frac{\mu l}{8\pi}$$

Que-03 Determine the Inductance L of two wire transmission line with separation distance d and each wire radius is a.

Ans-03



$$\lambda_1 = \frac{\mu I L}{8\pi} \quad (0 \le \rho \le a) \longrightarrow \text{Internal}$$

$$\lambda_2 \rightarrow (a \le \rho \le d-a)$$

$$\lambda_2 = \psi_2 = \int_a^{d-a}\int_0^{l} \frac{\mu I}{2\pi\rho}\, d\rho\, dz$$

$$= \frac{\mu I}{2\pi} l \ln\left(\frac{d-a}{a}\right)$$

By symmetry same amount of flux is produced the current $-I_2$ in wire I_2. Hence the total flux $\psi_{total} = \lambda_{total} = 2(\lambda_1 + \lambda_2)$

$$= 2\left[\frac{\mu I l}{8\pi} + \frac{\mu I l}{2\pi} \ln\left(\frac{d-a}{a}\right)\right]$$

$$L_{ext} = \frac{\lambda_{total}}{I}$$

Que-04 Two

Find out

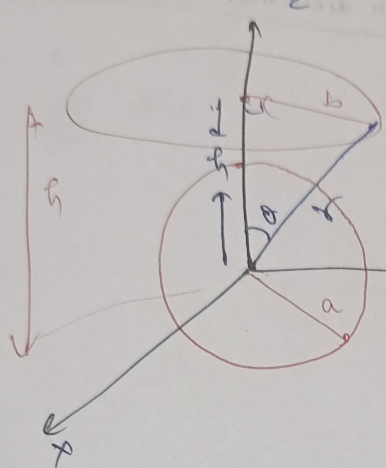
Ans M_{12}

$\vec{A_1} =$

$\vec{A_1}$

$\vec{A_1}$

Que-04 Two coaxial



$\sin\theta = \frac{b}{\sqrt{b^2+h^2}}$ $h >> a, b$

Find out the mutual Inductance between the wires.

Ans $M_{12} = \frac{\psi_{12}}{I_1} = \oint_{l_2} \vec{A_1} \cdot d\vec{l_2}$

$A \rightarrow$ magnetic vector potential

$\vec{A_1} = \frac{\mu I_1 \pi a^2 \sin\theta}{4\pi r^2} \hat{a}_\phi$

$A_1 \rightarrow$ for loop 1

$\vec{A_1} = \frac{\mu I_1 a^2 \sin\theta}{4 r^2} \hat{a}_\phi$

$r = \sqrt{h^2+b^2}$

$= \frac{\mu I_1 a^2}{4 r^2} \times \frac{b}{r}$

$\vec{A_1} = \frac{\mu I_1 a^2}{4(h^2+b^2)} \frac{b}{\sqrt{h^2+b^2}} = \frac{\mu I_1 a^2 b}{4 h^3} \hat{a}_\phi$

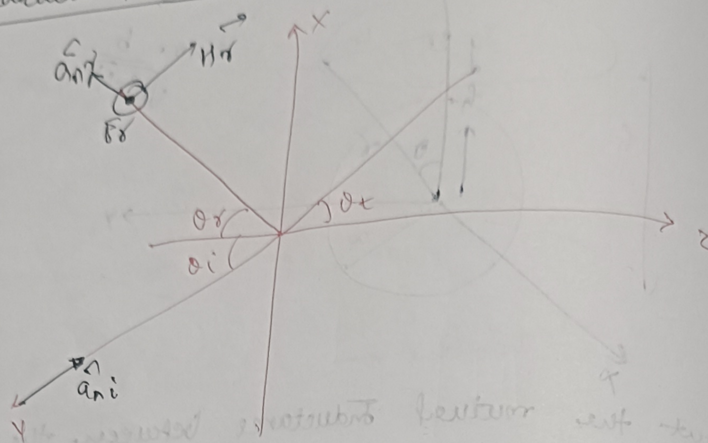
$\vec{dl_2} = \rho\, d\rho\, d\phi$

$\rho \rightarrow (0 \rightarrow b)$

$\phi \rightarrow (0 \rightarrow 2\pi)$

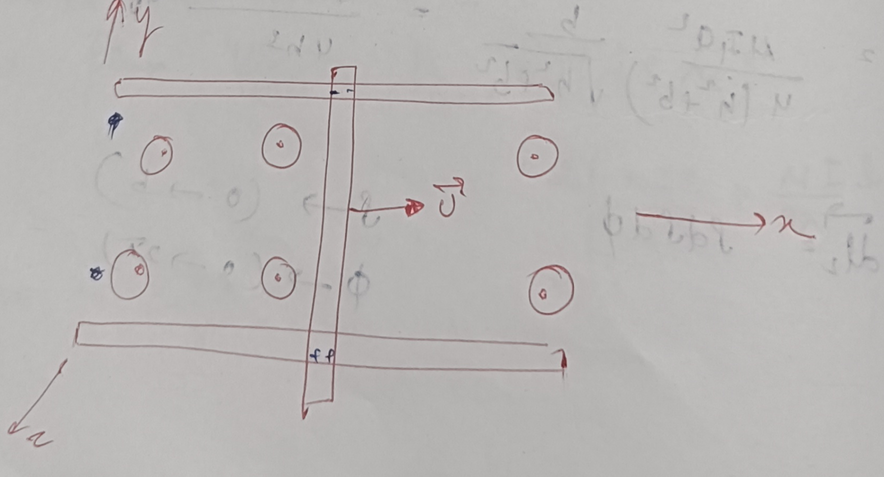
For coplanar $\theta = 90^\circ$
$\sigma = 6$

* Perpendicular polarisation for dielectric - dielectric interface



Que- A metal bar slides over a pair of conducting rails in a uniform magnetic field $\vec{B} = \hat{a}_z B_0$ with a constant velocity $\vec{v}$ as shown in figure. Determine the open circuit voltage V_0 that appears across the terminal 1 & 2. Assuming that a current is connected between the terminal. Find the electric dissipated.
Calculate the mechanical power required to move a sliding bar with the velocity $\vec{v}$?

Ans



$f_m = -q(\vec{V} \times \vec{B})$
$= -q(\hat{a}_x \times \hat{a}_z)$
$= q\, \hat{a}_y (VB)$
$qE = qVB\, \hat{a}_y$
$E = VB\, \hat{a}_y$
$\int dV =$

Que- Calculate the radiation intensity of a half wave

$\cos^{\circ} 0^{\circ} =$

Tutorial

Que-01 Calculate the directionality of an antenna the power pattern of which is given by

$$U(\theta,\phi) = \begin{cases} \sin^2\theta \sin\phi & 0<\phi<\pi,\ 0<\theta<\pi \\ 0 & 0\le\theta\le\pi,\ \pi\le\phi\le 2\pi \end{cases}$$

Ans- Total power radiated by the antenna.

$$\boxed{P_{rad} = \int_{\Omega} U(\theta,\phi)\, d\Omega \text{ watt}}$$ Imp

$$= \int_0^{\pi}\int_0^{\pi} \sin^2\theta \sin\phi \cdot (\sin\theta \cdot d\theta)\cdot d\phi$$

$$= \int_0^{\pi} \frac{1-\cos 2\theta}{2}\cdot d\theta \int \sin\phi \cdot d\phi$$

$$= \frac{\pi}{2} - \left[\frac{\sin 2\theta}{2}\right]_0^{\pi} \left[-\cos\phi\right]_0^{\pi}$$

$$= \frac{\pi}{2} \text{ watt.}$$

$$\boxed{\text{Directionality} \quad D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{rad}}}$$ Important

$$= \frac{4\pi \sin\theta \sin\phi}{\pi}$$

The max. value of directionality is 4

$$D_{dB} = 10\log_{10} 4 = 6.02\,dB$$

Max. when $\theta = \phi = \pi/2$

Que-02 Calculate the bandwidth ... an antenna. The power pattern

$U(\theta,\phi) = \{$ ...

Ans-02 In X-Y plane $\theta =$ ...

$U(\frac{\pi}{2},\phi) =$ ...

The angles along which ... value is gi...

$\sin\phi =$ ...

$\phi =$ ...

3-dB beam width

In Y-Z plane, $\phi =$ ...

by $U(\theta,\frac{\pi}{2}) =$ ...

Imp

Que-03 Ex-12.1

Ans-03 Magnetic ... is 2 km away

(a) For

$|H_{\phi s}|$

Que-02 Calculate the bandwidth in x-y and y-z plane of an antenna. The power pattern is given by

$$U(\theta,\phi) = \begin{cases} \sin^2\theta \sin\phi, & 0 \le \theta \le \pi, 0 \le \phi \le \pi \\ 0, & 0 \le \theta \le \pi, \pi < \phi \le 2\pi \end{cases}$$

Ans-02 In X-Y plane $\theta = \frac{\pi}{2}$

$U(\frac{\pi}{2}, \phi) = \sin\phi$

The angles along which the power is half the maximum value is given by the solution of

$\sin\phi = 0.5$

$\phi = 30^\circ, 150^\circ$

3-dB beam width in X-Y plane is $150^\circ - 30^\circ = 120^\circ$

↳ (BWHP)

In Y-Z plane, $\phi = \frac{\pi}{2}$, therefore power pattern is given by $U(\theta, \frac{\pi}{2}) = \sin^2\theta$.

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \frac{1}{\sqrt{2}}$ $\theta = 45^\circ, 135^\circ$

Beamwidth in Y-Z plane is 90°.

Imp. Ex-12.1 → sadiku.

Que-03

Ans-03 Magnetic field strength $5\mu A/m$, $\theta = \frac{\pi}{2}$ which is 2 km away from antenna in air.

(a) For a Hertzian dipole

$$|H_{\phi s}| = \frac{I_s \beta dl \sin\theta}{4\pi r}$$

where $dl = \frac{\lambda}{25}$ or $\beta dl = \frac{2\pi}{\lambda} \times \frac{\lambda}{25} = \frac{2\pi}{25}$.

$$5\times10^{-6} = \frac{I_0 \times \frac{2\pi}{25}(1)}{4\pi \times 2\times10^{3}} = \frac{I_0}{10^5}$$

$$\boxed{I_0 = 0.5A}$$

$$\boxed{P_{rad} = 40\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_0^2}$$ Imp

$$= \frac{40\pi^2 (0.5)^2}{(25)^2} = 15.8 \text{ mW}$$

(b) $$|H_{\phi s}| = \frac{I_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$5\times10^{-6} = \frac{I_0}{2\pi r} = \frac{I_0}{2\pi(2\times10^3)}$$

$$\boxed{I_0 = 20\pi \text{ mA}}$$

$$P_{rad} = \frac{1}{2} I_0^2 R_{rad}$$

$= 144$ mW Ans

Que-04 An electric field strength of 10 μV/m is to be measured at an observation point $\theta = \frac{\pi}{2}$, 500 km from a half-wave (resonant) dipole antenna operating in air at 50 MHz

(a) what is the length of the dipole

(b) Calculate the current that must be fed to the antenna.

(c) Find the avg power radiated by the antenna, ~~determine the standing wave~~

(d) If a transmission line with $Z_0 = 75\,\Omega$ is connected to the antenna, determine the standing wave ratio.

Ans- (a) $\lambda = \frac{c}{f} = \frac{3\times}{50}$

length of the dipole, $l =$

(b) $$\boxed{|E_{\theta s}| = \frac{\eta_0 I_0 \cos(}{2\pi r}}$$

$$I_0 = \frac{|E_{\theta s}|\, 2\pi r \sin}{\eta_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}$$

(c) $\boxed{R_{rad} \simeq 73\,\Omega}$

$P_{rad} = \frac{1}{2}$

(d) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$= \frac{73 + j}{73 + j}$

$= \frac{-2}{14}$

$= \frac{4}{1}$

$= 0$

$S = \frac{1 + |\Gamma|}{}$

Que-05 $W_{rad} =$

The radiated power to ϕ & even a constan

Ans (a) $\lambda = \frac{c}{f} = \frac{3\times10^8}{50\times10^6} = 6\,m$

Length of the dipole, $l = \frac{\lambda}{2} = 3m$

(b) $$\boxed{|E_{\theta s}| = \frac{\eta_0 I_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}}$$

$$I_0 = \frac{|E_{\theta s}|\, 2\pi r \sin\theta}{\eta_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}$$

(c) $\boxed{R_{rad} \simeq 73\Omega}$

$$P_{rad} = \frac{1}{2} I_0^2 R_{rad}$$

(d) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (Z_L = Z_{in} \text{ in this case})$$

$$= \frac{73 + j42.5 - 75}{73 + j42.5 + 75}$$

$$= \frac{-2 + j42.5}{148 + j42.5}$$

$$= \frac{42.55 \angle 92.65^\circ}{153.98 \angle 16.02^\circ}$$

$$= 0.2763 \angle 76.67^\circ$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Power density

$$\mathcal{P}_r\ C_0 \frac{1}{r^2} \cos^4\theta$$

Que-05 $W_{rad} = W_{average} \simeq$

The radiated power density is symmetrical with respect to ϕ & exist, $0 \le \theta \le \pi/2$, $0 \le \phi \le 2\pi$. C_0 is a constant. The power radiated by antenna &

Measured

of coax

MHz

una.

Determine

is

ually

Maximum ~~density~~ directivity of antenna.

Ans- $P_{rad} = 1.256\,C_0$

Max directivity = 10 dB

$$P_{rad} = \iint W_{rad} \cdot ds$$

$$= \int_0^{\pi/2}\int_0^{2\pi} C_0 \frac{1}{\cancel{r^2}} \cos^4\theta \cdot \cancel{r^2} \sin\theta \, d\theta \cdot d\phi$$

$$= C_0 \int_0^{\pi/2} \cos^4\theta \sin\theta \cdot d\theta \int_0^{2\pi} d\phi$$

$\cos\theta = t$

$\frac{dt}{d\theta} = \sin\theta$

$$= - C_0 \left[\frac{\cos^5\theta}{5}\right]_0^{\pi/2} \times 2\pi$$

$$= 2\pi \, C_0 \left[+\frac{1}{5}\right] =$$

$$\text{Directivity} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi (r^2 \vec{W}_{rad})}{P_{rad}}$$

$$= \frac{2\cancel{4\pi}\cancel{r^2} C_0 \times \frac{1}{\cancel{r^2}} \cos^4\theta}{\frac{\cancel{2\pi} C_0}{5}} = 10\, C_0 \cos\theta$$