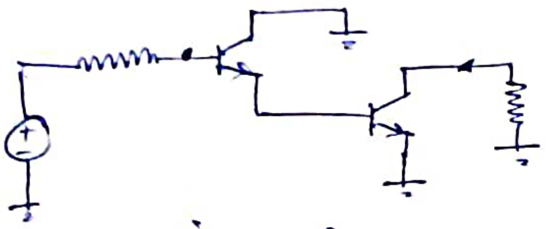
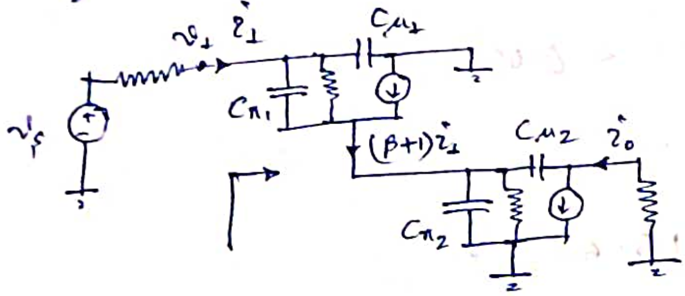
p.

Common collector- Common Emitter stage:





Absence of shunt Miller Capacitance.

$R_i \rightarrow$ as before, $\quad i_0 = \beta(\beta+1)i_1$

$= (\beta^2+\beta)\times \frac{v_1}{R_i}$

$G_m = \frac{100(101)}{523K} = 19.3m$

$\Rightarrow \frac{v_o}{v_s} = -G_m R_L \times \frac{R_i}{R_i+R_s}$

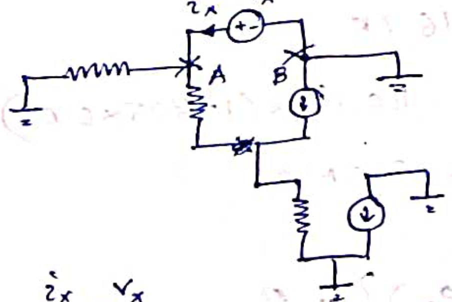
$\cong -49\ V/V$.

$R_{\pi_1} \rightarrow$ as before $= 43K$

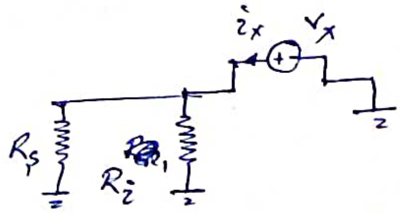
$R_{\pi_2} \rightarrow$ as before $= 1.5K$

$R_{\mu_2} \rightarrow$ as before $= 177.3K$

$R_{\mu_1} \rightarrow ? = 84K\Omega$



$R_{AB} = R_i$



$\Rightarrow R_{\mu_1} = R_i \| R_s$

$= 84K\Omega$

$\Rightarrow \tau = \sum R_i C_i = 43K\times 2.1p + 84K\times 0.4p + 1.5K\times 11.84pF + 177.3K\times 0.4pF$

$= 212.6ns$

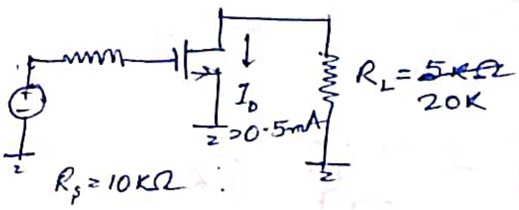
$\Rightarrow f_{-3dB} = \frac{1}{2\pi\times 212.6ns} = 748\ KHz$.

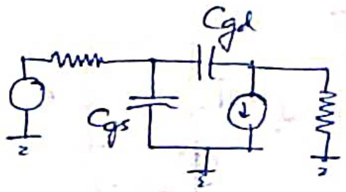
Cascode Amplifier:

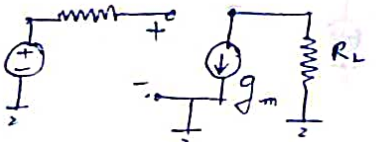
- Another case of avoiding shunt Miller Effect.

p. $C_{gs} = 90fF$

$C_{gd} = 14fF$ $\qquad K_n' = 60\mu A/V^2$, $\frac{W}{L} = \frac{100}{2}$



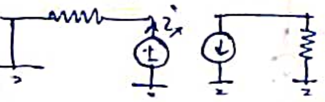




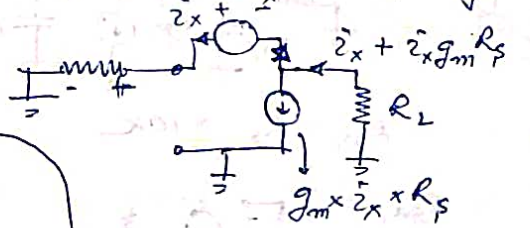
$$\frac{v_o}{v_s} = -g_m R_L = \cancel{-8.66}\ 34.6 \quad (20K)$$

$$g_m = \sqrt{2K_n'\left(\frac{W}{L}\right) \times I_D} = \sqrt{2.60\mu \times 50 \times 0.5m} = 1.73\ mA/V$$

$f_{-3dB} = ?$



$\Rightarrow R_{gs} = R_s \quad \therefore R_{gs}.C_{gs} = 10K \times 90fF = 900ps$



$$i_x R_s - v_x - (i_x + i_x g_m R_s) R_L = 0$$

$$\Rightarrow \frac{v_x}{i_x} = R_s + R_L + g_m R_s R_L = R_{gd}$$

$$= 10K + 20K + 1.73m\ 20K \times 10K$$

$$= \cancel{101.5K}\ 376K.$$

$\Rightarrow R_{gd} = 101.5K$

$\Rightarrow R_{gd}.C_{gd} = \cancel{1.421ns} = 5.24n$

$$\Rightarrow f_{-3dB} = \cancel{\frac{1}{2\pi(1.421n + 0.9n)}} = \cancel{68\ MHz}.$$

$$= \frac{1}{2\pi(5.4n + 0.9n)} = 25\ MHz.$$

$$\frac{1}{C_\pi . r_\pi + C_\mu . R'}$$

$$= \frac{1}{10p \times 2.5K + 1p \times 1.0125M}$$

$$r_\pi + r_o + g_m . r_\pi . r_o$$

$$= 2.5K + 10K + 40m \times 2.5K \times 10K$$

$$= 12.5K + 100 \times 10K = 1.0125M$$

$$= \frac{1}{6.52 \times 10^{-6}}$$

$$= \frac{10^6}{6.52}$$

$$= \frac{1000 \times 10^3}{6.52}$$

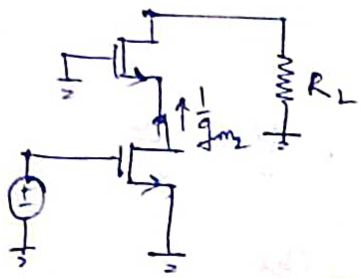
$$= 153\ KHz$$

$$g_m = \frac{I_C}{V_T} = \frac{1mA}{25mV} = \frac{1}{25} = 40mS$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40mS} = 2.5K\Omega$$

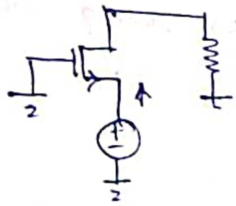
$$r_o = \frac{10}{1mA} = 10K$$

Cascode Amp :

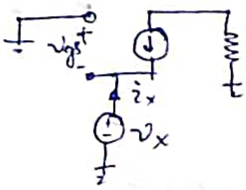


$$-g_{m_1} g_{m_2} (r_{o_1} \| R_{i_2}) (r_{o_2} \| R_L')$$

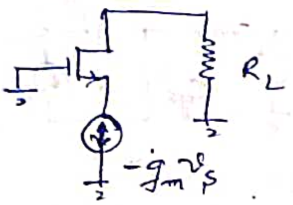
$$= -g_{m_1} g_{m_2} \cdot \frac{1}{g_{m_2}} \times R_L'$$

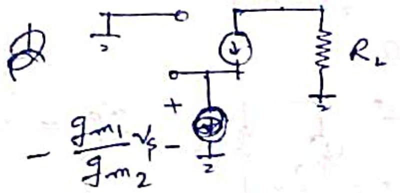


$$= -g_{m_1} R_L'$$



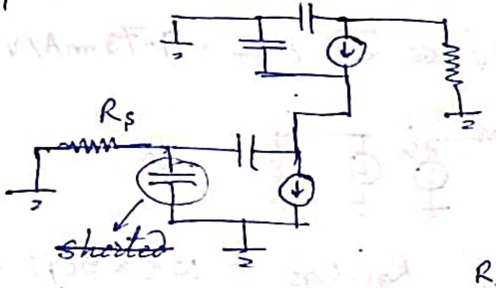
$$i_x = -g_m v_{gs}$$

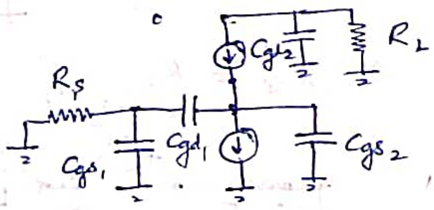


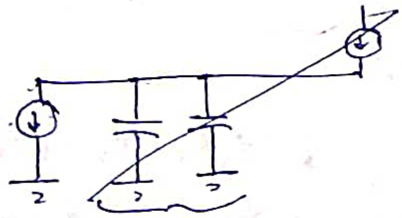


$$\frac{v_o}{v_i} = -g_{m_1} R_L$$

$f_{-3dB} = ?$



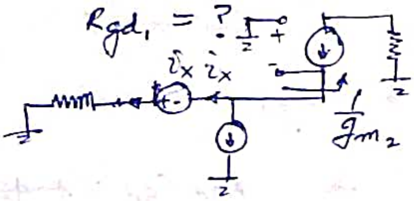




$C_{gd_1} + C_{gs_2}$

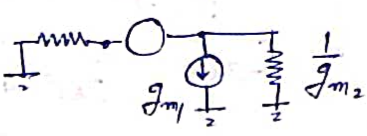
$R_{gs_1} = R_s = 10K$

$R_{gd_1} = ?$



$R_{gd_2} = R_L = 20K$

$R_{gs_2} = \frac{1}{g_{m_2}} = 0.56K$



$\Rightarrow R_{gd_1} = R_s + \frac{1}{g_{m_2}} + g_{m_1} \times \frac{1}{g_{m_2}} \times R_s$

$= 10K + 0.56K + 10K$

$= 20.56K$

$\Rightarrow \tau = 10K \times 90f$

$+ 0.56K \times 90f$

$+ 20.56K \times 14f$

$+ 20K \times 14f \quad = 1.51ns$

$\Rightarrow f_{-3dB} = \frac{1}{2\pi 1.51ns} = 104MHz.$

stability of feedback/amplifier
~~non~~-feedback

consider a single pole amplifier,

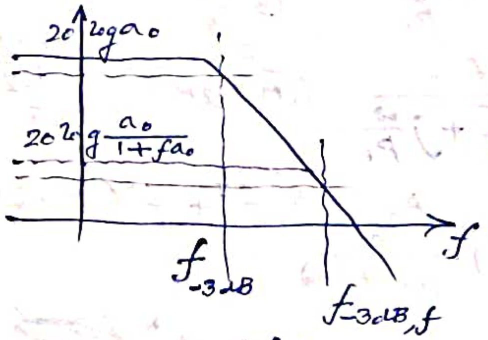
$$a(s) = \frac{a_0}{1+\frac{s}{p_1}} = \text{open loop gain.}$$

closed loop gain: $A(s) = \dfrac{a(s)}{1+f\,a(s)}$, assuming $f \rightarrow$ ~~only~~ a formed only by passive component.

$$= \frac{\frac{a_0}{1+\frac{s}{p_1}}}{1+f\frac{a_0}{1+\frac{s}{p_1}}}$$

$$= \frac{a_0}{1+\frac{s}{p_1}+f\cdot a_0} = \frac{a_0}{1+fa_0+\frac{s}{p_1}} = \frac{\frac{a_0}{1+fa_0}}{1+\frac{s}{p_1(1+fa_0)}}$$

clearly, gain falls by $1+fa_0$, BW increases by the same factor.



$f_{-3dB} = \dfrac{p_1}{2\pi}$

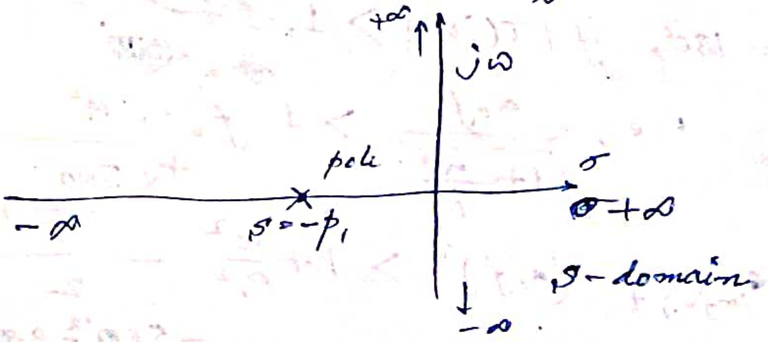
$f_{-3dB_f} = \dfrac{p_1(1+a_0f)}{2\pi}$

$\left(1-\frac{s}{p_1}\right)$

$= \sqrt{1+\frac{\omega^2}{p_1^2}}$

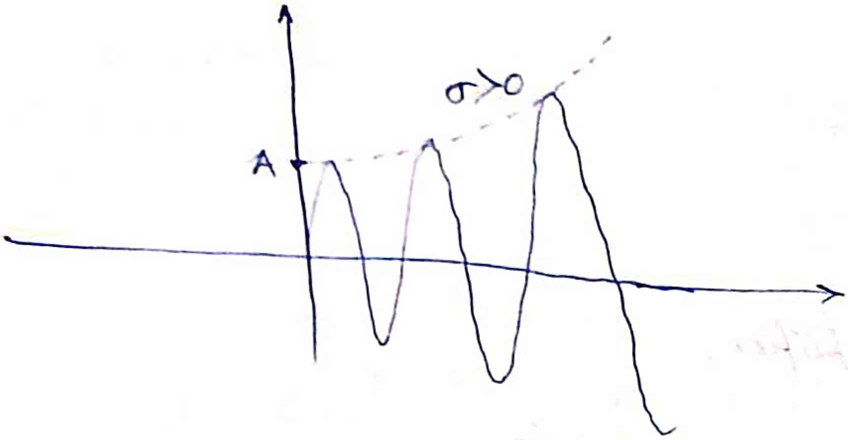
$\omega = p_1$

$\Rightarrow 2\pi f = p_1$



Let, $f(t) = A\cdot e^{st}$

$= A\cdot e^{\sigma t}\cdot e^{j\omega t}$



unstable : $\sigma > 0$,

$$T(s) = \frac{1}{s+a} = \frac{v_o(s)}{v_i(s)}$$

$$\Rightarrow v_o(s) = \frac{v_i(s)}{s+a}$$

$$\Rightarrow v_o(t) = \mathcal{L}^{-1}\left\{\frac{1}{s(s+a)}\right\} = \frac{1}{a}\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+a}\right\} = \frac{1}{a}\left(1 - e^{-at}\right)$$

$a(s)f = T(s)$

if ~~$T(s) =$~~ loop gain : $a(s).f$

if $|a(s).f|\big|_{\angle T = 180^\circ} > 1$,

i.e, let, $a(s)f = \cancel{R + jAI}$

$= Re\ L(j\omega) + j\,Im(j\omega)$

let, $a(s).f = \frac{a_o.f}{\cancel{(1+\frac{s}{p_1})(1+\frac{s}{p_2})}} = \cancel{\frac{K}{(1+j\frac{\omega}{p_1})(1+j}}$

$$= \frac{a_o f}{(1+\frac{s}{p_1})} = \frac{a_o f}{1 + \frac{\sigma + j\omega}{p_1}}$$

$$= \frac{a_o f}{1 + \frac{\sigma}{p_1} + j\frac{\omega}{p_1}}$$

$$A(s) = \frac{\frac{a_o}{1+fa_o}}{1 + \frac{s}{p_1(1+fa_o)}} = \frac{A_o}{1+\frac{s}{p_1(1+fa_o)}}$$

at at $\angle T(j\omega) = 180^\circ$, if $|T(j\omega)| > 1$,

i.e., $\frac{a_o f}{\sqrt{1+\frac{\omega^2}{p_1^2}}} > 1$

$$A(j\omega) = \frac{A_o}{1 + \frac{s}{p_1(1+fa_o)}}$$

$(a_o f)^2 > 1 + \frac{\omega^2}{p_1^2}$

$\Rightarrow v_o(t) = v_{in}(t) \times A_o . e^{-p_1(1+fa_o).t}$

$(1+\frac{s}{p_1})$

$$\frac{10}{(1+j\frac{4.17M}{2M})(1+j\frac{4.17M}{1M})}$$

$$= \frac{10}{e^{j64.37} . e^{j76.51}}$$

$= 1 . e^{-j140.88^\circ}$

$$\frac{a_o\beta}{\sqrt{(1+(\frac{f}{2M})^2)(1+(\frac{f}{1M})^2)}}$$

$\cancel{\frac{f^4}{(2M)^2}} -$

$\frac{f^4}{(1M)^2(2M)^2} + f^2\left(\frac{1}{2M^2} + \frac{1}{1M^2}\right) + 1 = 100$

$\Rightarrow f^4 + f^2(1M^2 + (2M)^2) - 99 \times 1M^2 \times (2M)^2 = 0$

$\Rightarrow$

$f^2 = \frac{-5M^2 + \sqrt{5\times10^{12} + 4.99.(1M)^2(2M)^2}}{2}$

$= \frac{-5\times10^{12} + 3.98\times10^{13}}{2}$

$f = 4.17\,MHz$

$$\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_2}\right)+fa_0$$

$$\Rightarrow \quad 1-\frac{\omega^2}{\omega_1\omega_2}+j\left(\frac{\omega}{\omega_1}+\frac{\omega}{\omega_2}\right)+fa_0=0$$

$$T(j\omega)=\frac{10}{\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_2}\right)\left(1+j\frac{\omega}{\omega_3}\right)}$$

$$A(j\omega)=\frac{a_0p_1}{s+p_1+fa_0}.$$

$$\Rightarrow \quad v_o(t)=v_{in}(t).\,a_0p_1.\,\cancel{e^{-\frac{t}{p_1+fa_0}}}\,e^{-(p_1+fa_0).t}$$

$$a(s)=\frac{a_0}{(s+p_1)(s+p_2)}$$

$$a(j\omega)f=\frac{a_0f}{1+\frac{s}{p_1}}=\frac{a_0fp_1}{s+p_1}$$

$$A(s)=\frac{a(s)}{1+fa(s)}=\frac{\frac{a_0}{(s+p_1)(s+p_2)}}{1+\frac{fa_0}{(s+p_1)(s+p_2)}}$$

$$s+p_1<a_0fp_1$$

$$\frac{s}{p_1}$$

Let, $(a_0f)^2=K.\left(1+\frac{\omega_0}{p_1}\right)^2$
where $K>1$

$$\Rightarrow a_0f\cong K.\frac{\omega_0}{p_1}$$

$$A(j\omega)=$$

$$\Rightarrow p_1=K.\frac{\omega_0}{a_0f}.$$

$$\frac{A_0}{1+\frac{j\omega_0}{p_1(1+fa_0)}}$$

$$\Rightarrow \quad A(s)=\frac{a_0}{(s+p_1)(s+p_2)+fa_0}$$

$$\Rightarrow \quad \frac{\omega^2}{\omega_1\omega_2}-j\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)\omega-(1+fa_0)=0$$

$$\frac{1}{(s-2)}\qquad a(s).f=\frac{1}{(s-2+j3)}$$

$$\Rightarrow \omega^2-j(\omega_1+\omega_2)\omega-(1+fa_0)=0$$

$$A(s)=\frac{1}{(s-2+j3)}$$

$$\Rightarrow v_{in}(t)=$$

$$\angle\frac{-3}{2+j(\omega+3)}$$

$$=-\tan^{-1}\frac{\omega+3}{2}$$

$$a_0(s)=\frac{1}{s+2+j3}\qquad =\frac{a_0}{\quad}.$$

let $\cancel{f=1}$ $f=-3$

$$\Rightarrow \quad a_0(s).f=\frac{-3}{s+2+j3}.$$

at, $\angle a_0(s).f=\cancel{\tan^{-1}\left(\frac{\omega}{\quad}\right.}$

$$-\tan^{-1}\frac{\omega+3}{2}=180^\circ$$

$$\Rightarrow \omega=-3.$$

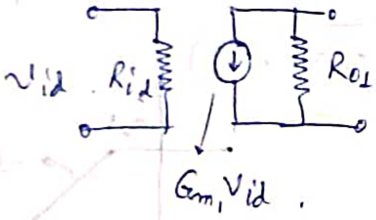
$$\Rightarrow \quad \frac{a_0(s)}{1+a_0(s)f}=\frac{\frac{1}{s+2+j3}}{1+\frac{1}{s+2+j3}}=\frac{1}{s+3+j3}$$

$$a_0(s).f=-\frac{3}{s+2+j3}$$

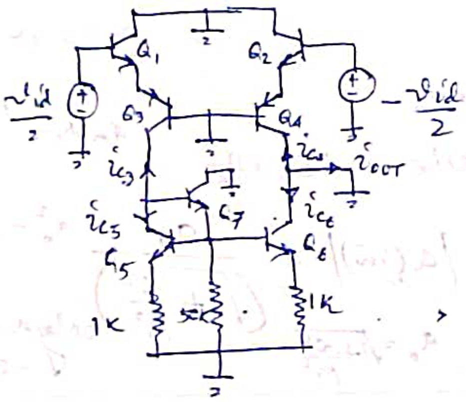
$$\Rightarrow \quad \frac{\frac{1}{s+2+j3}}{1-\frac{3}{s+2+j3}}=\frac{1}{s+2+j3-3}$$

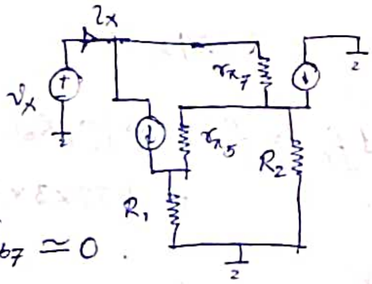
$$=\frac{1}{s-1+j3}$$

1st stage Equivalent ckt:



ac schematic:

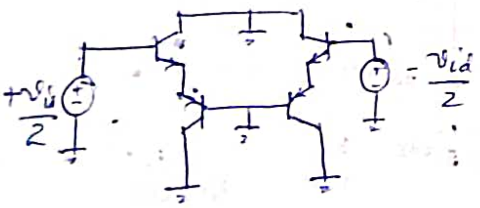


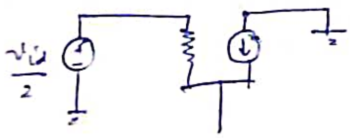


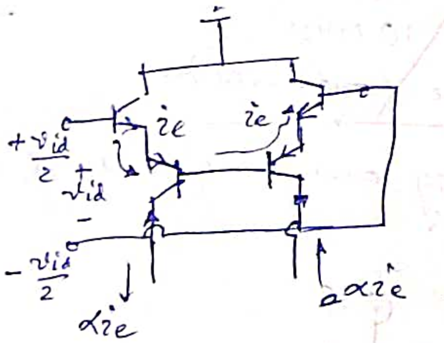
$i_{b7} \simeq 0$.

$$i_{oCT} = -(i_{c4} + i_{c6}) = -(i_{c4} - i_{c3})$$

$$\beta_{npn} = 200$$
$$\beta_{pnp} = 50$$

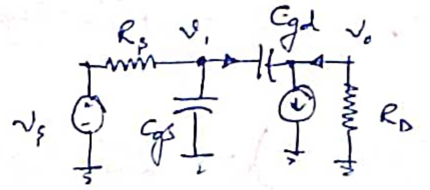






$$v_{id} = i_e \times 4 \cdot r_e$$
$$= i_e \times 4 \cdot \frac{r_\pi}{\beta+1}, \quad v_{id} = \cancel{\beta \cdot i_i}\ (\beta+1) i_i \times 4 \cdot \frac{r_\pi}{\beta+1}$$
$$= i_i \cdot 4 r_\pi$$
$$\Rightarrow \frac{v_{id}}{i_i} = 4 r_\pi = 4 \cdot \frac{\beta}{g_m}$$

CS amplifier



$$\cancel{v_o =}$$

$$(v_1 - v_o)\, sC_{gd} + \frac{v_o}{R_D} - g_m v_1 = 0$$

$$v_1 (sC_{gd} - g_m) = v_o \left(sC_{gd} + \frac{1}{R_D}\right)$$

$$v_1 = \frac{\cancel{sC_{gs}}\ \frac{1}{sC_{gs}}}{R_s + \frac{1}{sC_{gs}}} \times v_s = \frac{v_s \cdot 1}{(1 + sC_{gs}R_s)}$$

$$v_o = \frac{(sC_{gd} - g_m) \times v_s}{\left(sC_{gd} + \frac{1}{R_D}\right)(1 + sC_{gs}R_s)} \qquad \Rightarrow v_o = \frac{(sC_{gd} - g_m)}{\left(sC_{gd} + \frac{1}{R_D}\right)} v_1$$

$$= \frac{R_D (sC_{gd} - g_m) v_s}{(1 + sC_{gd}R_D)(1 + sC_{gs}R_s)}$$

$$\cancel{v_o = \frac{\left(sC_{gd} + \frac{1}{R_D}\right)}{\quad}}$$

g $$\frac{(v_s - v_1)}{R_s} = v_1 \cdot sC_{gs} + (v_1 - v_o)\, sC_{gd}$$

$$\frac{v_s}{R_s} = v_1 \left(\frac{1}{R_s} + sC_{gs} + sC_{gd}\right) - v_o sC_{gd}$$

$$v_1 \left(\frac{1}{R_s} + s(C_{gs} + C_{gd})\right) = \frac{v_s}{R_s} + v_o sC_{gd}$$

$$\Rightarrow v_o = \frac{(sC_{gd} - g_m)}{\left(sC_{gd} + \frac{1}{R_D}\right)} \times \frac{\left(\frac{v_s}{R_s} + v_o sC_{gd}\right)}{\left(\frac{1}{R_s} + s(C_{gs} + C_{gd})\right)}$$

$$\Rightarrow v_o \left\{ 1 - \frac{(sC_{gd} - g_m)\, sC_{gd}}{\left(sC_{gd} + \frac{1}{R_D}\right)\left(\frac{1}{R_s} + s(C_{gs} + C_{gd})\right)} \right\} = \frac{(sC_{gd} - g_m)\frac{v_s}{R_s}}{\left(sC_{gd} + \frac{1}{R_D}\right)\left(\frac{1}{R_s} + s(C_{gs} + C_{gd})\right)}$$

$$\Rightarrow v_o \left\{ s^2 C_{gd}^2 + s^2 C_{gs}C_{gd} + \frac{sC_{gd}}{R_s} + \frac{s(C_{gs} + C_{gd})}{R_D} + \frac{1}{R_s R_D} - s^2 C_{gd}^2 + s g_m C_{gd} \right\}$$

$$\Rightarrow \qquad = (sC_{gd} - g_m)\frac{v_s}{R_s}$$

$$\Rightarrow \left(s^2 C_{gs} C_{gd} + \frac{sC_{gd}}{R_s} + \frac{s(C_{gs}+C_{gd})}{R_D} + s\, g_m C_{gd} + \frac{1}{R_s R_D} \right) v_o = -\frac{v_s (g_m - s\, C_{gd})}{R_s}$$

$$\Rightarrow \left(s^2 C_{gs} C_{gd} R_s R_D + s\, C_{gd} R_D + s\,(C_{gs}+C_{gd}) R_s + s\, g_m C_{gd} R_s R_D + 1 \right) v_o = -\frac{v_s (g_m - s\, C_{gd})}{} R_D$$

$$\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right) v_o = -v_s (g_m - s\, C_{gd}) R_D.$$

$$\Rightarrow \left\{ \frac{s^2}{p_1 p_2} + s\left(\frac{1}{p_1}+\frac{1}{p_2}\right) + 1 \right\} v_o = -v_s (g_m - s\, C_{gd}) R_D$$

$$\frac{1}{p_1} + \frac{1}{p_2} = C_{gd} R_D + (C_{gs}+C_{gd}) R_s + g_m C_{gd} R_s R_D.$$

$$\frac{1}{p_1} \cdot \frac{1}{p_2} = C_{gs} C_{gd} R_s R_D$$

$$\frac{1}{p_1} = C_{gs} C_{gd} R_s R_D\, p_2$$

$\frac{1}{p_1} + \frac{1}{p_2} \simeq \frac{1}{p_1}$,

then $\frac{1}{p_1} = C_{gd} R_D + (C_{gs}+C_{gd}) R_s + g_m C_{gd} R_s R_D$

$$\Rightarrow \frac{1}{p_1} \approx g_m C_{gd} R_s R_D$$

$$\Rightarrow p_1 = \frac{1}{g_m C_{gd} R_s R_D}$$

$$\Rightarrow \frac{1}{p_1} \cdot \frac{1}{p_2} = C_{gs} C_{gd} R_s R_D$$

$$\Rightarrow \frac{1}{p_2} = C_{gs} \cancel{C_{gd} R_s R_D} \times \frac{1}{g_m \cancel{C_{gd} R_s R_D}}$$

$$\Rightarrow p_2 \approx \frac{g_m}{C_{gs}}$$

poles:

$$s^2\, 4.05\times10^{-19} + s\, 1.054\times10^{-8} + s\, 7.785\times10^{-9} + 1 = 0$$

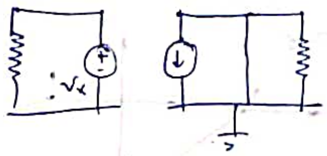
$$\Rightarrow s^2 \cdot 4.05\times10^{-19} + s\, 1.832\times10^{-8} + 1 = 0$$

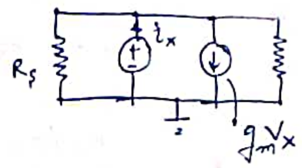
$$\Rightarrow s = \frac{-1.832\times10^{-8} \pm \sqrt{(1.83\times10^{-8})^2 - 4.4.05\times10^{-19}}}{2 \cdot 4.05\times10^{-19}}$$

$$= \frac{-1.832\times10^{-8} \pm 1.825\times10^{-8}}{8.1\times10^{-19}}$$

⇒ p_1 & p_2 = 13.75 MHz.

7.2 GHz.

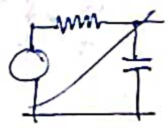


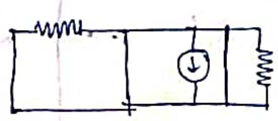


$$p_1 = -\frac{1}{R_s.\{C_{gs}+(1+g_mR_D)C_{gd}\}}$$

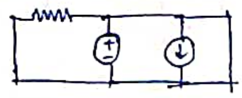
$$p_2 = -\frac{1}{R_D.C_{gd}}$$

$$= -\frac{1}{3k\times 15f} =$$



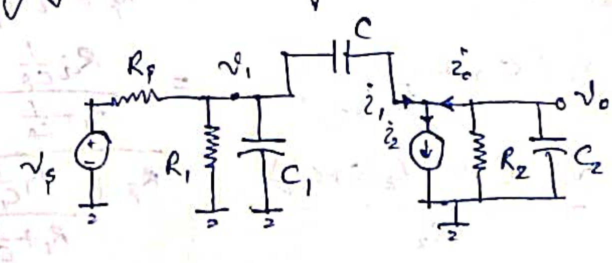


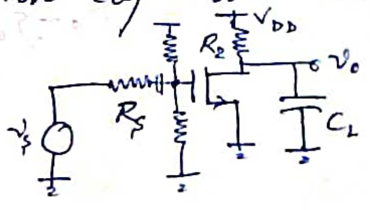
R_2C_2.



Theory of pole splitting:

a typical MOS amplifier with capacitive load:





$i_1 = (v_1 - v_0)sC$

$i_2 = g_m v_1$

$i_0 = -\frac{v_0}{R_2 \| \frac{1}{C_2 s}} = -\frac{v_0(1+R_2C_2s)}{R_2}$

@ $i_1 + i_0 - i_2 = 0$

$$v_1 = \frac{R_1 \| \frac{1}{C_1 s}}{R_s + R_1 \| \frac{1}{C_1 s}} \cdot v_s = \frac{\frac{R_1}{1+R_1C_1s}}{R_s + \frac{R_1}{1+R_1C_1s}}$$

$$= \frac{R_1}{R_s + R_1R_sC_1s + R_1}$$

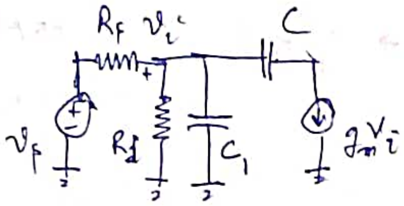
$$= \frac{R_1}{R_1 + R_s + sR_1C_1R_s} v_s$$

⇒ $(v_1 - v_0)sC - \frac{v_0(1+R_2C_2s)}{R_2} - g_m v_1 = 0$

⇒ $v_1(sC - g_m) = v_0 sC + \frac{v_0(1+R_2C_2s)}{R_2}$

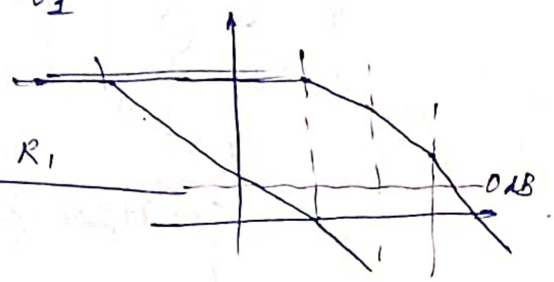
$= v_0 \frac{sCR_2 + 1 + R_2C_2s}{R_2}$

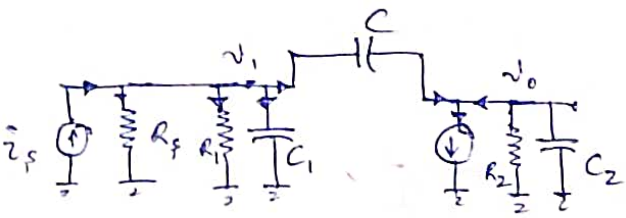
$$\Rightarrow v_1(sC - g_m) = v_o \frac{(1 + sCR_2 + sC_2R_2)}{R_2}.$$

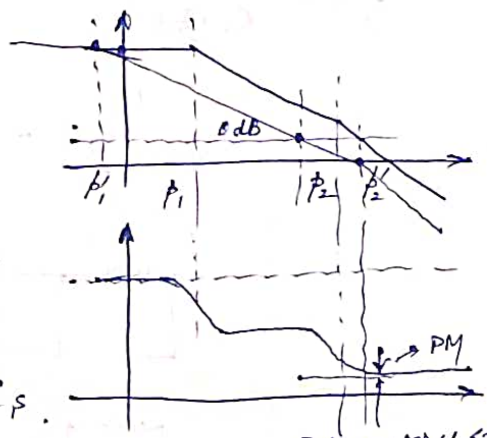


$$v_o \quad \frac{v_o}{v_1} = \frac{R_2(sC - g_m)}{1 + sR_2(C + C_2)} \cdot v_1$$

$$= \frac{R_2(sC - g_m)}{1 + sR_2(C + C_2)} \times \frac{R_1}{\quad}$$







$v_s = i_s . R_s.$ Let, $R_s \| R_1 = R_1'$

$$i_s = \frac{v_1}{R_1' \| \frac{1}{C_1 s}} + (v_1 - v_o) Cs.$$

PM → very small

$$\& \quad (v_1 - v_o)Cs - \frac{v_o}{R_2 \| \frac{1}{C_2 s}} = g_m v_1 \qquad v_1 = \frac{R_i \| \frac{1}{C_1 s}}{R_s + R_i \| \frac{1}{C_1 s}} v_s$$

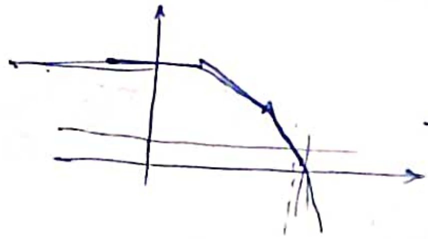
$$\Rightarrow v_1 Cs - g_m v_1 = v_o \left(Cs + \frac{1}{R_2 \| \frac{1}{C_2 s}}\right) \qquad = \frac{\frac{R_i \frac{1}{Cs}}{R_1 + \frac{1}{Cs}}}{R_s + \frac{R_1 \frac{1}{C_1 s}}{R_1 + \frac{1}{Cs}}}$$

$$= \frac{v_o \, (1 + sR_2(C + C_2))}{R_2}.$$

$$\Rightarrow v_o = \frac{v_1(Cs - g_m) R_2}{\{1 + sR_2(C + C_2)\}} \qquad = \frac{R_1 / R_1C_1s + 1}{R_s + \frac{R_1}{1 + R_1C_1 s}}$$

$$\& \quad \frac{v_1(1 + sR_1(C + C_1))}{R_1} = i_s + v_o Cs. \qquad = \frac{R_1}{R_s + R_1 + R_1C_1 s}$$

$$\Rightarrow v_1 = \frac{(i_s + v_o Cs) R_1}{1 + sR_1(C + C_1)}.$$



741 op-amp :

$$a(j\omega) = \frac{a_0}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})(1+\frac{s}{p_3})}$$

$p_1 = 30Hz$
$p_2 = 500Hz$
$p_3 = 10MHz$

$$\Rightarrow v_0 = \frac{(Cs-g_m)R_2}{\underbrace{\{1+sR_2(C+C_2)\}}_{=A}} \times \frac{(i_s+v_0Cs)R_1'}{\underbrace{\{1+sR_1'(C+C_1)\}}_{=B}}$$

$$= \frac{(Cs-g_m)R_2 \cdot i_s}{\{A\}\{B\}} + \frac{v_0 CR_1's(Cs-g_m)R_2}{\{A\}\{B\}}$$

$$\Rightarrow v_0\left\{1 - \frac{CR_1's(Cs-g_m)R_2}{\{A\}\{B\}}\right\} = \frac{(Cs-g_m)R_2 \cdot i_s}{\{A\}\{B\}}$$

$$\Rightarrow v_0\left\{s^2R_2R_1'(C+C_1)(C+C_2) + s\{R_2(C+C_2)+R_1'(C+C_1)\} + 1\right\} - s^2C^2R_1'R_2 + g_mR_2R_1'Cs = (Cs-g_m)R_2 i_s$$

$$\Rightarrow v_0\left\{s^2R_2R_1'(C_1C_2+C(C_1+C_2)) + s\{R_1'(C+C_1)+R_2(C+C_2)+g_mR_2'R_1'C\} + 1\right\} = (Cs-g_m)R_2 \cdot i_s$$

$$\Rightarrow v_0 = \frac{(Cs-g_m)R_2 \cdot i_s}{\{s^2R_2R_1'(C_1C_2+C(C_1+C_2)) + s\{R_1'(C+C_1)+R_2(C+C_2)+g_mR_2'R_1'C\}+1\}}$$

$$\frac{v_0}{i_s} = \frac{(Cs-g_m)R_2 i_s}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})} = \frac{(Cs-g_m)R_2 i_s}{\frac{s^2}{p_1p_2} + s(\frac{1}{p_1}+\frac{1}{p_2}) + 1}$$

if $p_1 \ll p_2$, $\frac{1}{p_1} + \frac{1}{p_2} \approx \frac{1}{p_1}$

$$\Rightarrow \frac{1}{p_1} \approx R_1'(C+C_1) + R_2(C+C_2) + g_mR_2R_1'C \approx g_mR_2R_1'C$$

& $\frac{1}{p_1p_2} = R_2R_1'\{C_1C_2 + C(C_1+C_2)\}$ $\Rightarrow$ $p_2 = \frac{g_m}{C}$

$\Rightarrow \; p_1 = \frac{1}{g_m R_2 R_1' C}$

$\& \; p_2 = \frac{g_m C}{C_1 C_2 + C(C_1 + C_2)}$

as C increases, p_1 & p_2 split further.

P: $T(jf) = \frac{a_0 \beta}{\left(1+j\frac{f}{f_1}\right)\left(1+j\frac{f}{f_2}\right)}$ $\quad \beta = 1, \; a_0 = 100\,dB$

$f_1 = 100\,Hz$

$f_2 = 120\,KHz$.

& $T(jf) = \frac{a_0 \beta\left(1+j\frac{f}{f_z}\right)}{\left(1+j\frac{f}{f_1}\right)\left(1+j\frac{f}{f_2}\right)}$ $\quad f_z = 100\,KHz$.

for $\beta = 1$ & $\beta = 10^{-3}$

Q. an amplifier has a phase margin of $20°$, what is the closed loop gain peak compared to the low freq. false value at f_0 i.e. $|T(jf_0)| = 1$

or closed loop gain is 9.2 dB

Q. $T(jf) = \frac{10 (= a_0\beta)}{\left(1+j\frac{f}{f_1}\right)\left(1+j\frac{f}{f_2}\right)\left(1+j\frac{f}{f_3}\right)}$

P.

An amplifier with low freq. fwd. gain: 40K, -ve real poles at 2 KHz, 200 KHz, 4 MHz.

It is connected in feedback with f = constant.

& closed loop gain: 400.

i) determine the phase margin.

~~ii)~~

$$A_{CL} = \frac{A_0}{1 + f.A_0} \qquad A_0 = 40K.$$

$$\Rightarrow \frac{40K}{1 + f 40K} = 400$$

$$\Rightarrow 1 + f.40K = \frac{40K}{400} = 100$$

$$\Rightarrow f.40K = 100 \Rightarrow f = 2.5 \times 10^{-3}$$

$$\text{loop gain} = A_0(j\omega).f = \frac{A_0 f}{\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_2}\right)\left(1+j\frac{\omega}{\omega_3}\right)}$$

~~the PM~~ when $|A_0(j\omega).f| = 1$,

$$\frac{A_0 f}{\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}\sqrt{1+\left(\frac{\omega}{\omega_2}\right)^2}\sqrt{1+\left(\frac{\omega}{\omega_3}\right)^2}} = 1$$

$$\Rightarrow 100 = \sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}\sqrt{1+\left(\frac{\omega}{\omega_2}\right)^2}\sqrt{1+\left(\frac{\omega}{\omega_3}\right)^2}$$

$$\therefore 20\log 100 = 10\log\left\{1+\left(\frac{\omega}{\omega_1}\right)^2\right\} + 10\log\left\{1+\left(\frac{\omega}{\omega_2}\right)^2\right\} + 10\log\left\{1+\left(\frac{\omega}{\omega_3}\right)^2\right\}$$

at $\omega = \omega_1$, $\omega_1 << \omega_2$, $\omega << \omega_3$.

$$\text{Loop gain} = \cancel{20\log}\ \frac{100}{\sqrt{1+1}\cdot 1} = \frac{100}{\sqrt{2}} = 70.7.$$

at $\omega = \omega_2$, $\omega_2 >> \omega_1$, $\omega_2 << \omega_3$.

then,
$$\text{loop gain} = \frac{A_0 f}{\sqrt{\left(\frac{\omega_2}{\omega_1}\right)^2}\cdot\sqrt{2}\cdot 1}$$

$$= \frac{A_0 f}{\frac{\omega_2}{\omega_1}\times\sqrt{2}} = \frac{A_0 f\,\omega_1}{\omega_2\cdot\sqrt{2}} = \frac{100\times 2K}{200K\cdot\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707.$$

$\therefore$ clearly, loop gain will be unity at ω_c, where $\omega_1 < \omega_c < \omega_2$

$\therefore \omega_c << \omega_3$.

$\therefore$ at $\omega = \omega_c$,

$$\text{loop gain} = \frac{A_0 f}{\sqrt{1+\left(\frac{\omega_c}{\omega_1}\right)^2}\cdot\sqrt{1+\left(\frac{\omega_c}{\omega_2}\right)^2}} = 1$$

$$\Rightarrow 100 = \sqrt{\left\{1+\left(\frac{\omega_c}{\omega_1}\right)^2\right\}\left\{1+\left(\frac{\omega_c}{\omega_2}\right)^2\right\}}$$

$$\Rightarrow 1 + \frac{\omega_c^4}{\omega_1^2\omega_2^2} + \omega_c^2\left(\frac{1}{\omega_1^2}+\frac{1}{\omega_2^2}\right) = 100^2.$$

$$\Rightarrow \frac{\omega_c^4}{\omega_1^2\omega_2^2} + \omega_c^2\left(\frac{\omega_1^2+\omega_2^2}{\omega_1^2\omega_2^2}\right) \cong 100^2$$

$$\Rightarrow \omega_c^4 + \omega_c^2(\omega_1^2+\omega_2^2) - 100^2\,\omega_1^2\omega_2^2 = 0$$

Consider the following $T(f)$

$$a_0 = \frac{\beta\times(1000)}{\left(1+j\frac{f}{1K}\right)\left(1+j\frac{f}{50K}\right)\times\left(1+j\frac{f}{1M}\right)}$$

find the value of β that yeilds a PM of 45°

$\beta = 0.0707$

$$T(f) = \frac{5\times10^5}{\left(1+j\frac{f}{1M}\right)\left(1+j\frac{f}{10M}\right)\left(1+j\frac{f}{100M}\right)}$$

Insert a dominant pole such that $PM = 45^\circ$

$f_{PD} = 2.83 Hz$

$$\omega_c^2 = \frac{-(\omega_1^2+\omega_2^2)+\sqrt{(\omega_1^2+\omega_2^2)^2+4.100^2\omega_1^2\omega_2^2}}{2}$$

$$\simeq \frac{-\omega_2^2+\sqrt{\omega_2^4+\cancel{400}_{4\times100^2}\cdot\omega_1^2\omega_2^2}}{2}$$

$$= \frac{-\omega_2^2+\omega_2\sqrt{\omega_2^2+\cancel{400}_{4\times100^2}\cdot\omega_1^2}}{2}$$

$$\Rightarrow f_c^2 = \cancel{\omega_2^2+\omega_2} = \frac{-f_2^2+f_2\cdot\sqrt{f_2^2+\cancel{400}_{4\times100^2}f_1^2}}{2}$$

$$= 396.07\times10^6$$

$$\Rightarrow f_c \simeq \cancel{20\,KHZ}\ 157.23\,KHZ$$

at ~~f2~~ $f_c = 20KHZ$,

$\omega_c \simeq 10.\omega_1$,

$\omega_c \simeq 0.1\omega_2$, & $\omega_c \ll \omega_3$

$\Rightarrow$ & $\omega_c \gg \omega_1$,

$\omega_c < \omega_2$

$\omega_c \ll \omega_3$

$\cancel{|T(j\omega)|} =$ Loop gain : $|A_0(j\omega).f|_{\omega=\omega_c}$

$$= \frac{A_0 f}{\left\{1+j\left(\frac{\omega_c}{\omega_1}\right)\right\}\left\{1+j\left(\frac{\omega_c}{\omega_2}\right)\right\}}$$

$$\angle A_0(j\omega).f = -\tan^{-1}\frac{\omega_c}{\omega_1} - \tan^{-1}\frac{\omega_c}{\omega_2}$$

$$= -\tan^{-1}10 - \tan^{-1}0.1$$

$$= -84.3^\circ - 5.7^\circ$$

$$= -90^\circ$$

$$= \frac{A_0 f}{\left\{1+j\frac{\omega_c}{\omega_2}\right\}\cdot j\frac{\omega_c}{\omega_1}}$$

$\angle A_0(j\omega).f$

$$= -90^\circ - \tan^{-1}\frac{\omega_c}{\omega_2}$$

$$= -90^\circ - \tan^{-1}\frac{157.2K}{200K}$$

$$= -90^\circ - 38.16^\circ$$

$$= -128.16^\circ$$

$$PM = 180^\circ - 128.16^\circ$$

$$= 51.84^\circ$$

if $A_{CL} = 100$,

$f = 0.01$,

$\therefore$ Loop gain$|_{low\ freq} = 40K \times 0.01 = 400$

$\therefore$ Loop gain$|_{\omega=\omega_2} = \dfrac{\cancel{100 \times 0.01}\ 400 \times \omega_1}{\omega_2 \times \sqrt{2}} = \dfrac{400}{100 \times \sqrt{2}}$

$= 2.82$.

$100 = \dfrac{40K}{1+f\,40K}$

ω_c in this case,

$\omega_c >> \omega_1, \quad \omega_c > \omega_2, \quad \omega_c < \omega_3.$

at $\omega = \omega_3$.

Loop gain$|_{\omega=\omega_3}$

$\dfrac{40K}{1+0.01 \times 40K}$

~~Loop gain =~~

$\omega_c^2 = \dfrac{-\omega_3^2 + \omega_3 . \sqrt{\omega_3^2 + 1600 . \omega_2^2}}{2}$

$= \dfrac{400}{\frac{\omega_3}{\omega_1} \times \frac{\omega_3}{\omega_2} \times \sqrt{2}}$

$100 = \dfrac{40K}{1+\beta\,40K}$

$1+\beta . 40K = \dfrac{40K}{100}$

$\simeq \dfrac{-(4M)^2 + 4M . \sqrt{(4M)^2 + \cancel{1600} (200K)^2}}{2}$ 4×400^2

$= \dfrac{400}{\sqrt{2} \times \frac{(4M)^2}{2K \times 200K}}$

$= \dfrac{400}{\sqrt{2} \times 40K}$

$\Rightarrow$ ~~$f_c = 3.14 MHz$~~. $\Rightarrow f_c =$

$\Rightarrow 1+\beta$

$\Rightarrow \omega_c >> \omega_2, \quad \omega_c >> \omega_1.$

$\therefore$ loop gain at $\omega = \omega_c$

Loop gain $= \dfrac{A_o f}{\left(\frac{\omega_c}{\omega_1}\right) \times \left\{\sqrt{1+\left(\frac{\omega_c}{\omega_2}\right)^2}\right\} \left\{\sqrt{1+\left(\frac{\omega_c}{\omega_3}\right)^2}\right\}}$

$= 1$

$\dfrac{A_o . f}{\left(1 + j\frac{\omega_c}{\omega}\right)} = 1$

$\Rightarrow$

$\Rightarrow \omega_c^4 + \omega_c^2 \omega_3^2 = \omega_2^2 \omega_3^2 \times 400^2 \times \left(\dfrac{\omega_1}{\omega_c}\right)^2$

$= \omega_2^2 \omega_3^2 \times$

Let, $\omega_c << \omega_3$, then, Loop gain $= \dfrac{A_o f}{\frac{\omega_c}{\omega_1} \times \sqrt{1 + \left(\frac{\omega_c}{\omega_2}\right)^2}}$

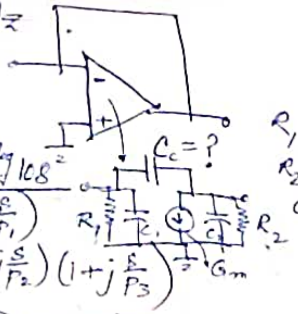
$\Rightarrow \left(\dfrac{\omega_c}{\omega_1}\right)^2 . \left(1 + \dfrac{\omega_c^2}{\omega_2^2}\right) = A_o f$

$\Rightarrow \dfrac{\omega_c^4}{\omega_1^2 \omega_2^2} + \dfrac{\omega_c^2}{\omega_1^2} = A_o f$

$\Rightarrow \omega_c^4 + \omega_c^2 . \omega_2^2 = A_o f . \omega_1^2 \omega_2^2$.

$\omega_{\text{unity gain}} = 5.7\,\text{MHz}$

Calculate C_c
so that $PM = 60^\circ$ & new $p_{d,2} \approx p_3$



$R_1 = 100K$
$R_2 = 10K$
$G_m = 1\,mA/V$
$C_1 = 1\,pF$
$C_2 = 2\,pF$

$\angle A' = -120^\circ$
$= -\tan^{-1}\frac{\omega}{p_2} - \tan^{-1}\frac{\omega}{p_3}$

$$A' = \frac{108\,dB}{\left(1+j\frac{\omega}{p_d}\right)\left(1+j\frac{\omega}{p_3}\right)}$$

$$T(s) = \frac{\text{antilog } 108}{\left(1+j\frac{s}{p_1}\right)\left(1+j\frac{s}{p_2}\right)\left(1+j\frac{s}{p_3}\right)}$$

$A_{of} = 400.$

$$\therefore f_c^2 = \frac{-f_2^2 \pm f_2\sqrt{f_2^2 + 4.400^2.f_1^2}}{2}$$

$\Rightarrow f_c = 375\,KHZ.$

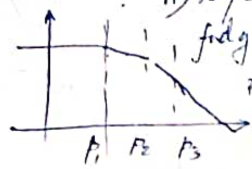
$$\therefore \text{Loop gain}\Big|_{f=f_c} = \frac{A_{of}}{\left(j\frac{\omega_c}{\omega_1}\right)\left(1+j\frac{\omega_c}{\omega_2}\right)}$$

$$\Rightarrow \angle\text{loop gain} = -90^\circ - \tan^{-1}\frac{375K}{200K}$$
$$= -90^\circ - 62^\circ$$
$$= -152^\circ$$

$\Rightarrow PM = -28^\circ$

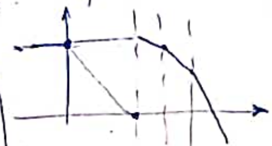
$\Rightarrow$ variation of f changes the P.M.
$\rightarrow$ gives the idea of phase-ed / freq. dependent feedback ckt.

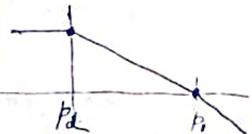
Q. An amplifier has a low freq. forward gain of 5000 & Xfr fn. has $p_1 = 300\,KHz$, $p_2 = 2\,MHz$, $p_3 = 25\,MHz$
i) Calculate dominant pole magnitude required to give unity gain compensation of 45° PM. What is the resulting bandwidth?
ii) Repeat if fdgain $= 20\,dB$, PM $= 45^\circ$



unity gain comp. $\therefore \beta = 1$

$\frac{A}{1+\beta A}$. $A\beta = A$





$A_1 \times p_d = p_1$
$\Rightarrow p_d = \frac{p_1}{A_1} = 60\,Hz$

BW:
$(1+A)\times p_d$
$\cong A.p_d \cong 300\,KHz$

2 stage CMOS opamp:

differential amplifier with active load
$+$
$\rightarrow$ common source amp with active load

$$T(s) = \frac{K}{\left(1+j\frac{s}{p_1}\right)}$$

$$|T(j\omega)| = \frac{K}{\sqrt{1+\frac{\omega^2}{p_1^2}}}$$

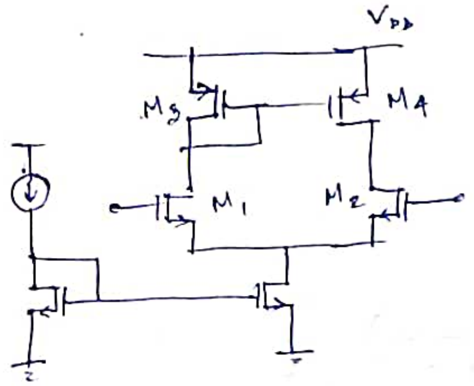
$\omega = \omega_c$, $|T(j\omega_c)| = 1$

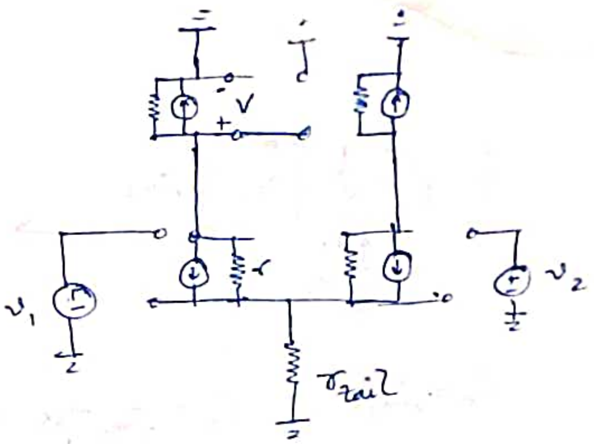
$$\Rightarrow 1 = \frac{K}{\sqrt{1+\frac{\omega_c^2}{p_1^2}}}$$

$\omega_c \gg p_1 \Rightarrow 1 = \frac{K}{\frac{\omega_c}{p_1}}$

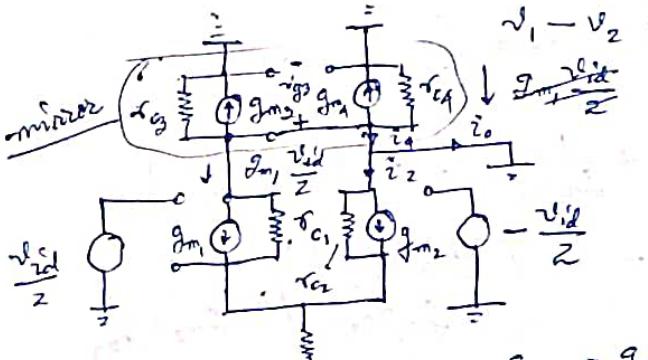
$\Rightarrow \frac{\omega_c}{p_1} = K \Rightarrow \omega_c = p_1 \times K$

Q. find $p_1' = ?$
if $p_2 = 2\,MHz$, $p_3 = 25\,MHz$ — consider unity gain feedback
i.e., p_2, p_3 do not move.

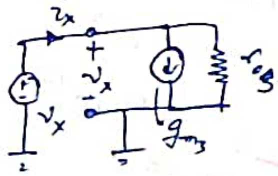




$$v_1 - v_2 = v_{id} = V_{CM} + \frac{v_{id}}{2}$$
$$\qquad - V_{CM} + \frac{v_{id}}{2}.$$



$g_{m_1} = g_{m_2}\ ,\ g_{m_3} = g_{m_4}$



$$i_x = g_m v_x + \frac{v_x}{r_{o_3}}$$
$$\Rightarrow \frac{v_x}{i_x} = r_{o_3} \| \frac{1}{g_m}$$

as, $\frac{1}{g_{m_3}} \ll r_{o_3}$,

$$v_{g3} = -g_{m_1} \frac{v_{id}}{2} \cdot \frac{1}{g_{m_3}}$$

$$\therefore\ i_4 = -g_{m_4} v_{g_3} = + g_{m_4} \cdot \frac{g_{m_1}}{g_{m_3}} \cdot \frac{v_{id}}{2},$$

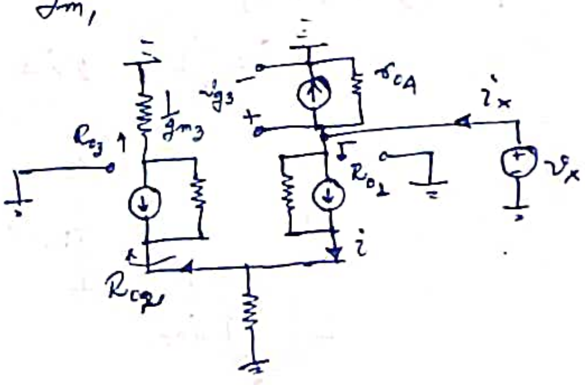
$$\&\ i_2 = g_{m_2}\left(-\frac{v_{id}}{2}\right) = -g_{m_2} \cdot \frac{v_{id}}{2}.$$

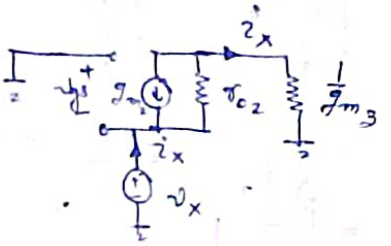
$$\therefore\ i_o = i_4 - i_2 = g_{m_4} \cdot \frac{g_{m_1}}{g_{m_3}} \cdot \frac{v_{id}}{2} + g_{m_2} \cdot \frac{v_{id}}{2}$$

$$= g_{m_1} \cdot \frac{v_{id}}{2} + g_{m_1} \frac{v_{id}}{2} = g_{m_1} \cdot v_{id}.$$

$$\Rightarrow G_m = g_{m_1}$$

$\&\ R_{out} = ?$





$\longrightarrow R_{o2}$

$\cancel{i_x = g_{m_2}}$

$$i_x + g_{m_2} v_{gs} + \frac{\left(i_x \frac{1}{g_{m_3}} - v_x\right)}{r_{o_3}} = 0$$

$$\Rightarrow i_x - g_{m_2} v_x + \frac{i_x}{g_{m_3} r_{o_3}} - \frac{v_x}{r_{o_3}} = 0.$$

$$\Rightarrow i_x \left(1 + \frac{1}{g_{m_3} r_{o_3}}\right) = v_x \left(g_{m_2} + \frac{1}{r_{o_3}}\right)$$

$$\Rightarrow \frac{v_x}{i_x} = \left(\frac{g_{m_3} r_{o_3} + 1}{g_{m_3} r_{o_3}}\right) \times \frac{1}{g_{m_2} + \frac{1}{r_{o_3}}}$$

$$\simeq \left(\frac{1}{g_{m_2}}\right) \parallel r_{o_3} \simeq \frac{1}{g_{m_2}}$$

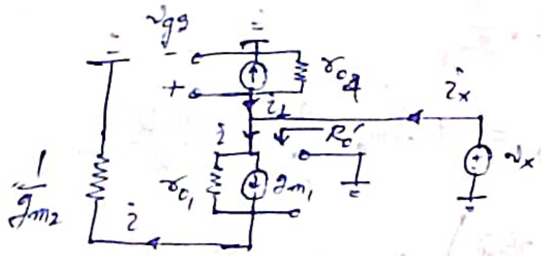
$$1 = \frac{5 \times 10^5}{\sqrt{1 + \left(\frac{f_c}{10^8}\right)^2}}$$

$$\Rightarrow 1 + \frac{f_c^2}{10^{16}} = 25 \times 10^{10}$$

$$\Rightarrow f_c^2 \approx 2.5 \times 10^{10} \times 10^{16}$$

$f_c =$

$\therefore$ equivalent ckt



$$\frac{v_x}{i_x} = R_o$$

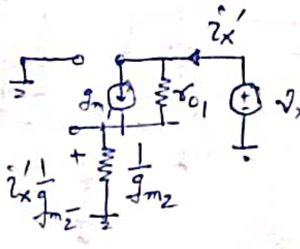
$$\therefore v_{gs} = i \times \frac{1}{g_{m_3}} \qquad \therefore i_1 = -g_{m_4} \times i \cdot \left(\frac{1}{g_{m_3}}\right) - \frac{v_x}{r_{o4}}$$

$$= -i - \frac{v_x}{r_{o4}}$$

$$\therefore i_x = i - i_1$$

$$= i + i + \frac{v_x}{r_{o4}} = 2i + \frac{v_x}{r_{o4}}$$

$$R_o' = \frac{v_x}{i}$$



$$\Rightarrow i_x' = g_{m_1}\left(-i_x' \cdot \frac{1}{g_{m_2}}\right) + \frac{v_x - i_x \frac{1}{g_{m_2}}}{r_{o_1}}$$

$$\Rightarrow i_x'\left(1 + \frac{g_{m_1}}{g_{m_2}} + \frac{1}{g_{m_2} r_{o_1}}\right) = \frac{v_x'}{r_{o_1}}$$

$$\Rightarrow i_x'\left(2 + \frac{1}{g_{m_2} r_{o_1}}\right) = \frac{v_x'}{r_{o_1}} \Rightarrow \frac{v_x'}{i_x'} = \cancel{\frac{r_{o_1}}{2}} \; r_{o_1}\left(2 + \frac{1}{g_{m_2} r_{o_1}}\right)$$

$$\approx 2 r_{o_1}$$

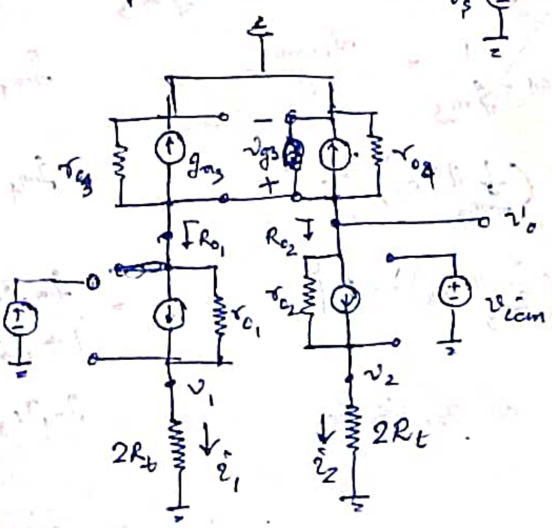
$$\Rightarrow i_x = 2 \cdot \frac{v_x}{R_o'} + \frac{v_x}{r_{o4}}$$

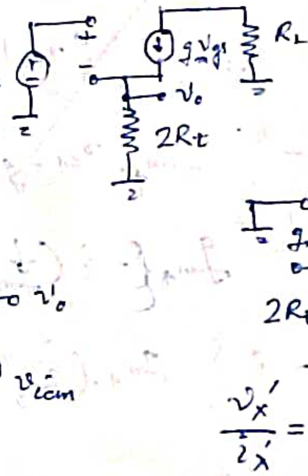
$$= 2 \cdot \frac{v_x}{2 r_{o_1}} + \frac{v_x}{r_{o4}} \Rightarrow \frac{v_x}{i_x} = r_{o_1} \parallel r_{o4}$$

$$\therefore \; \text{gain} = G_m \cdot R_o$$

$$\left.\begin{aligned} &= g_{m_1}(r_{o_1} \parallel r_{o4}) \\ &= g_{m_2}(r_{o_2} \parallel r_{o3}) \end{aligned}\right\} \rightarrow \text{differential gain}$$

Common mode gain:-





$$\Rightarrow$$

$$v_s - v_{gs} - g_m v_{gs} \cdot 2R_t = 0$$

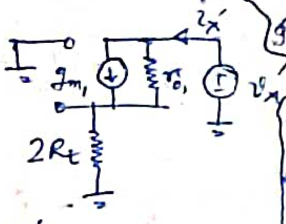
$$v_s - v_{gs} - v_o = 0$$

$$\Rightarrow v_{gs} = v_s - v_o$$

$$\& \; v_o = g_m v_{gs} \cdot 2R_t$$

$$\Rightarrow v_o = g_m (v_s - v_o) 2R_t$$

$$\Rightarrow \frac{v_o}{v_s} = \frac{g_m 2R_t}{1 + g_m 2R_t}$$



$$\frac{v_x'}{i_x'} = r_{o_1} \times \left\{1 + g_{m_1} 2R_t + \frac{2R_t}{r_{o_1}}\right\}$$

$$= \frac{\cancel{r_{o_1}}}{\cancel{r_{o_1}}}\left\{r_{o_1} + g_{m_1} 2R_t r_{o_1} + 2R_t\right\}$$

$$\simeq g_{m_1} r_{o_1} 2R_t$$

$$= R_{o2}$$

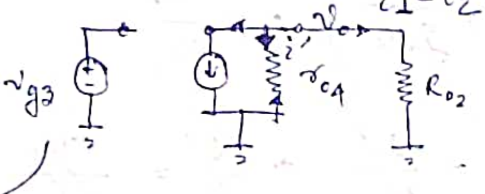
$$v_1 = v_2 \approx v_{icm} \Rightarrow i_1 = i_2 = \frac{v_{icm}}{2R_t}$$

$v_{g3} = \cancel{i \times} \cancel{g_m} - i \times \left(\frac{1}{g_{m_3}}\right)$ or $v_{g3} = -i_1 \left(\frac{1}{g_{m_3}} \| r_{o3}\right)$

$= -i_1 \cdot \dfrac{r_{o3}}{1 + g_{m_3} r_{o3}}$

$$\frac{\frac{1}{g_{m_3}} r_{o3}}{1 + \frac{1}{g_{m_3}} r_{o3}} = \frac{r_{o3}}{1 + g_{m_3} r_{o3}}$$

$i_1 = i_2$



$v_o = -g_{m_4} v_{g3} \cdot r_{o4}$

$\underline{= -g_{m_4}}$

$v_o =$

$\cancel{v_o = g_{m_4} v_{g3} \, i}$ $\quad v_o = i' \cdot r_{o4}$

$v_{g3} = -i \cdot \dfrac{1}{g_{m_3}}$

$-g_m v_{g3} \, i' - i_1 =$

$\Rightarrow i' = -(g_{m_4} v_{g3} + i$

$\cancel{i'} = i' + g_{m_4} v_{g3} + i = 0$

$\Rightarrow v_o = + \dfrac{g_{m_4}}{g_{m_3}} \cdot i_1 r_{o4}$

follow this

$\Rightarrow i' = -g_{m_4}\left(-i \dfrac{1}{g_{m_3}}\right) - i$

$= \dfrac{v_{icm}}{2R_t} \cdot r_{o4}$

$= + \dfrac{g_{m_4}}{g_{m_3}} \cdot i - i$

$r_{o4} \ll R_{o2}$

$\simeq 0$

$v_o = -(g_{m_4} v_{g3} + i_1) \cdot r_{o4}$

$= -\left(-g_{m_4} i_1 \dfrac{r_{o3}}{1 + g_{m_3} r_{o3}} + i_1\right) r_{o4} =$

$= \left\{ \dfrac{g_{m_4} r_{o3}}{1 + g_{m_3} r_{o3}} \cdot i_1 - i_1 \right\} r_{o4}$

$g_{m_3} = g_{m_4}$

$= \left\{ \dfrac{g_{m_3} r_{o3} - 1 - g_{m_3} r_{o3}}{1 + g_{m_3} r_{o3}} \right\} r_{o4} \, i_1$

$= -\dfrac{r_{o4}}{1 + g_{m_3} r_{o3}} \cdot \dfrac{v_{icm}}{2R_t}$

$= -\dfrac{v_{icm}}{2R_t \cdot g_{m_3}}$

$\cancel{-g_{m_4} (r_{o4} \| R_{o2}) \cdot v_{g3}}$ $\Rightarrow v_o = 0 \cdot v_{icm}$

$\dfrac{1}{g_{m_3}} \ll r_{o3}$

$\cancel{= -g_{m_4} (r_{o4} \| g_{m_1} r_{o1} 2R_t) v_{g3}}$

$\Rightarrow 1 \ll g_{m_3} r_{o3}$

$\cancel{= + g_{m_4} (r_{o4} \| g_{m_1} r_{o1} 2R_t) \cdot i \cdot \dfrac{1}{g_{m_3}}}$

$\cancel{= g_{m_4} \cdot r_{o4} \cdot \dfrac{1}{g_{m_3}} \times \dfrac{v_{icm}}{2R_t}}$

$g_{m_4} \left(\dfrac{\frac{r_{o3}}{g_{m_3}}}{\frac{1}{g_{m_3}} + r_{o3}} \right)$

$= -g_{m_4} \left\{ -i \left(\dfrac{1}{g_{m_3}} \| r_{o3} \right) \right\} - i$

$= g_{m_4} \dfrac{r_{o3}}{1 + g_{m_3} r_{o3}} - 1$

$= i \left\{ g_{m_4} \cdot \left(\dfrac{1}{g_{m_3}} \| r_{o3} \right) - 1 \right\}$

$= \dfrac{g_{m_4} r_{o3}}{1 + g_{m_3} r_{o3}} - 1$

$= \dfrac{g_{m_4} r_{o3} - 1 - g_{m_3} r_{o3}}{1 + g_{m_3} r_{o3}}$

$\Rightarrow v_o = i' \times r_{o4}$, as $r_{o4} \ll R_{o2}$

$=$

$v_o = r_{o4} \cdot \dfrac{v_{icm}}{2R_t} \cdot \dfrac{1}{1 + g_{m_3} r_{o3}}$

$-g_{m_4} \times v_{g3} - i' - i_1 = 0$

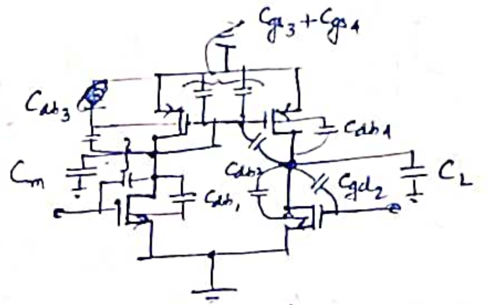
$= \dfrac{r_{o4} \, v_{ic}}{2R_t \cdot g_m}$

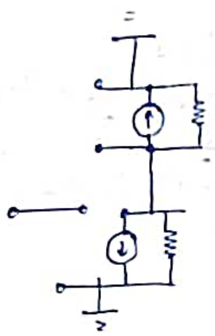
$\Rightarrow -g_{m_4}\left(-i_1 \dfrac{r_{o3}}{1 + g_{m_3} r_{o3}}\right) - \dfrac{v_o}{r_{o4}} - i_1 = 0$

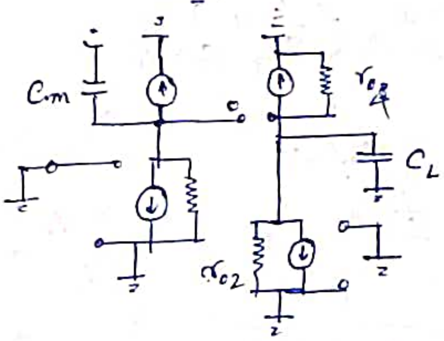
$\Rightarrow \dfrac{v_o}{r_{o4}} = i_1 \left\{ \dfrac{g_{m_4} r_{o3}}{1 + g_{m_3} r_{o3}} - 1 \right\} = i_1 \left\{ \dfrac{1}{1 + g_{m_3} r_{o3}} \right\}$

$= \dfrac{v_{icm}}{2 g_m R_t}$

frequency analysis:







$$C_m = C_{gd_1} + C_{db_1} + C_{gs_3} + C_{gs_4} + C_{db_3}$$

$$C_L = C_{gd_4} + C_{gd_2} + C_{db_2} + C_{db_4}$$

for C_{gd_1}, C_{gd_4} gate variation is much less than drain.

hence, gate terminals are approximated as ground terminals

$$\therefore \quad R_{C_m} = \frac{1}{g_{m_3}} \parallel r_{o_1}$$

$$\& \quad R_{C_L} = r_{o_2} \parallel r_{o_3}$$

$$\therefore 2\pi f_{p_1} = \frac{1}{\left(\frac{1}{g_{m_3}} \parallel r_{o_1}\right) \cdot C_m + \left(r_{o_2} \parallel r_{o_4}\right) C_L}$$

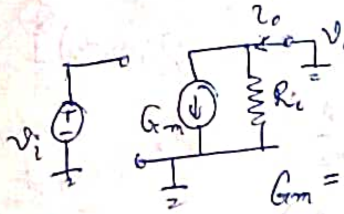
$\Rightarrow$ as $r_{o_2} \parallel r_{o_4} \gg \left(\frac{1}{g_{m_3}} \parallel r_{o_1}\right)$

$$\Rightarrow f_{p_1} = \frac{1}{2\pi (r_{o_2} \parallel r_{o_4}) C_L}$$

non-dominant pole:

$$\tau = \sum \frac{1}{R_i C_i} = \frac{1}{R_{C_L} \cdot C_L} + \frac{1}{R_{C_m} \cdot C_m}$$

$$\approx \frac{1}{R_{C_m} \cdot C_m} = \frac{1}{\left(\frac{1}{g_{m_3}} \parallel r_{o_1}\right) \cdot C_m} \approx \frac{g_{m_3}}{C_m}$$



$$\Rightarrow \quad f_{p_2} = \frac{g_{m_3}}{2\pi C_m}$$

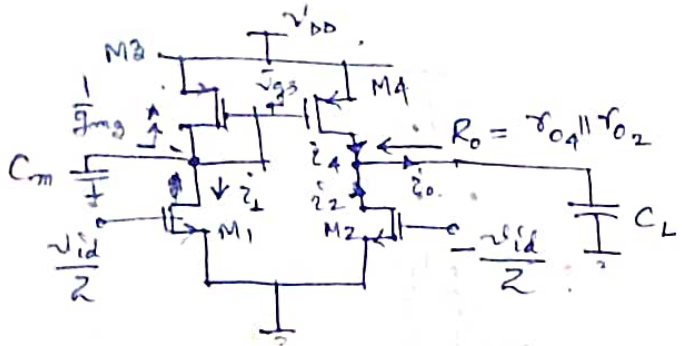
$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o = 0}$$

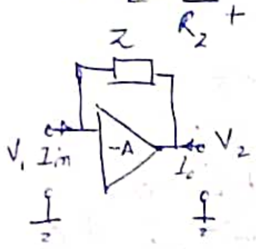
short circuit trans-conductance

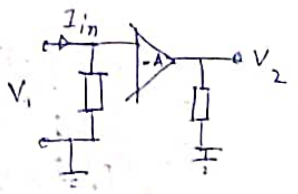
Determination of zero.

$$G_{m_1} V_{id}' + \frac{V_{i_2}}{R_1} + sC_1 V_{i_2} + sC_c (V_{i_2} - V_o) = 0$$

$$G_{m_2} V_{i_2} + \frac{V_o}{R_2} + sC_2 V_o + sC_c (V_o - V_{i_2}) = 0$$







$V_2 = -AV_1$.

$$I_{in} = \frac{V_1 - V_2}{Z} = \frac{V_1 + AV_1}{Z}$$

$$\Rightarrow \frac{V_1}{I_{in}} = \frac{Z}{1+A}$$

$$\frac{V_2 - V_1}{I_o} = Z_o \quad \text{or} \quad = \frac{V_2}{I_o}$$

$$\frac{V_2 + \frac{V_2}{A}}{I_o} = Z_o = \frac{V_2}{I_o}$$

$$\Rightarrow \frac{V_2}{I_o}\left(1 + \frac{1}{A}\right) = Z_o$$

$$\Rightarrow \frac{V_2}{I_o} = \frac{Z_o}{\left(1 + \frac{1}{A}\right)}$$

$$i = g_{m_1} \frac{v_{id}'}{2}, \quad v_{g3} = -g_{m_1} \frac{v_{id}}{2} \times \left(\frac{1}{g_{m_3}} \| \frac{1}{C_m s}\right)$$

$$= -g_{m_1} \frac{v_{id}}{2} \times \frac{\frac{1}{g_{m_3} C_m s}}{\frac{1}{g_{m_3}} + \frac{1}{C_m s}}$$

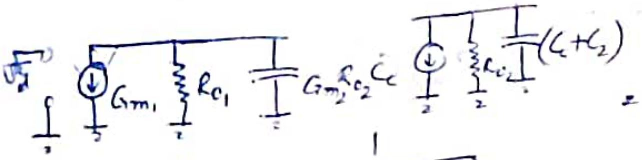
$$= -g_{m_1} \frac{v_{id}'}{2} \cdot \frac{1}{g_{m_3} + C_m s}.$$

$$C_c\left(1 + \frac{1}{G_{m_2} R_{o2}}\right)$$

$$\therefore i_4 = -g_{m_4} \cdot v_{g3} = + g_{m_1} g_{m_4} \cdot \frac{v_{id}}{2} \cdot \frac{1}{g_{m_3} + C_m s}$$

$$i_2 = g_m \frac{v_{id}}{2}. \quad \therefore i_o = i_4 + i_2$$

$$= \frac{g_{m_1} g_{m_4} v_{id}}{2(g_{m_3} + C_m s)} + g_{m_2} \frac{v_{id}'}{2}$$



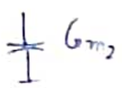
$$= \frac{g_{m_1} v_{id}}{2} \cdot \left\{\frac{g_{m_4}}{g_{m_3} + C_m s} + 1\right\}$$

$$\frac{1}{\tau} = \frac{1}{G_{m_1} R_{o_1} R_{o_2} C_c} + \frac{1}{R_{o_2}(C_c + C_2)}$$

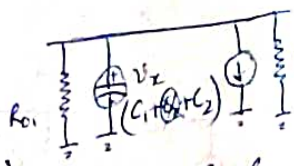
$$= \frac{g_{m_1} v_{id}'}{2} \cdot \left\{\frac{2g_{m_4} + C_m s}{g_{m_3} + C_m s}\right\}$$

$=$

$$v_o = i_o \times \left(R_o \| \frac{1}{C_L s}\right)$$

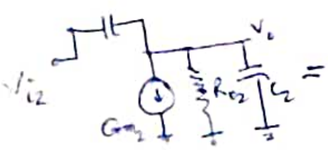


$$= i_o \frac{\frac{R_o}{C_L s}}{R_o + \frac{1}{C_L s}} = i_o \frac{R_o}{1 + R_o C_L s}$$



$$R_{eq} = R_{o_1} \| R_{o_2} \| \frac{1}{G_{m_2}}$$

$$, \tau \cong \frac{1}{\frac{1}{G_{m_2}} \times (C_c + C_1)}$$

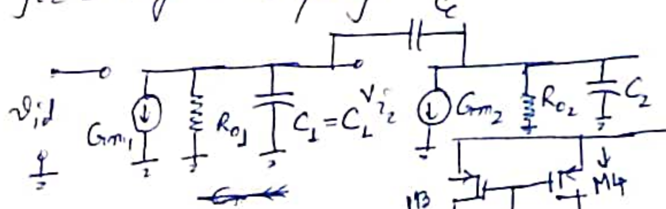


$$= \frac{g_{m_1} v_{id} R_o}{(1 + R_o C_L s)} \times \frac{\left(1 + \frac{C_m s}{2g_{m_4}}\right)}{\left(1 + \frac{C_m s}{g_{m_3}}\right)}$$

$$(V_{i_2} - V_o) C_c s = G_{m_2} V_{i_2} + \frac{V_o}{R_{o2} \| \frac{1}{C_2 s}}$$

$$V_{i_2}(C_c s - G_{m_2}) = V_o\left(C_c s + \frac{1}{R_{o2} \| \frac{1}{C_2 s}}\right)$$

for 2 stage CMOS opamp.

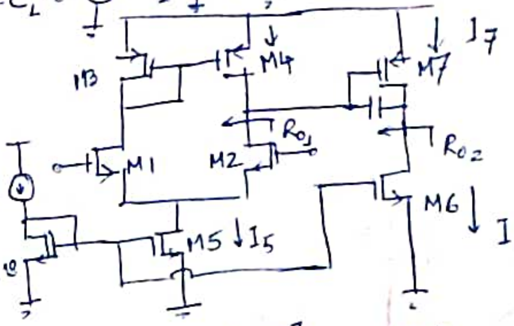


$C_2 \to 0.08pF$, parasitic

$C_2 \to 1pF$, load

~~$C_2 = 0.08pF$,~~

~~$C_L = 1pF$~~



$\therefore f_{P_1} = \dfrac{1}{2\pi R_o C_L}$

$f_Z = \dfrac{2 g_{m_4}}{2\pi C_m}$

$f_{P_2} = \dfrac{g_{m_3}}{2\pi C_m}$

$I_7 = \left(\frac{W}{L}\right)_7 / \left(\frac{W}{L}\right)_4 \times \left(\frac{I}{2}\right)$

$I_6 = \dfrac{(W/L)_6}{(W/L)_5} \cdot I$

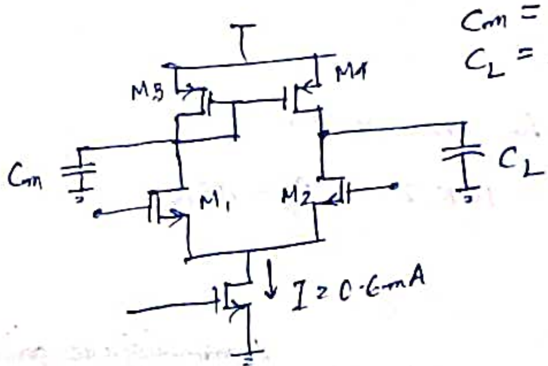
~~$I_5 = I$~~. now, $I_6 = I_7$ for no offset.

$\Rightarrow \dfrac{(W/L)_7}{(W/L)_4} \times \dfrac{I}{2} = \dfrac{(W/L)_6}{(W/L)_5} \times I$

p. ~~A current mirror loaded~~ if $M_5 \equiv M_6$, $(W/L)_7 = 2 \times (W/L)_4$

$C_m = 0.1pF \to$ Cap at the input of the mirror

$C_L = 0.2pF \to$ Cap at the output.



$V_{ov_{1,2}} = 0.3V$ $\quad V_{An} = |V_{Ap}| = 9V$,

$V_{ov_{3,4}} = 0.5V$ $\quad \therefore r_{o_{1,2,3,4}} = \dfrac{V_A}{(I/2)}$

$= \dfrac{9}{(0.3m)} = 30K\Omega$

$V_{ov} = V_{GS} - V_{TH}$

$\frac{I}{2} = \frac{1}{2} K_n' \left(\frac{W}{L}\right) . (V_{GS} - V_{TH})^2$

find the poles & zeros,
find the differential gain

$= 0.3m = \frac{1}{2} K_n' \left(\frac{W}{L}\right) 0.3^2 \Rightarrow K_n'\left(\frac{W}{L}\right) = \dfrac{0.6m}{(0.3)^2}$

$= 6.67 mA/V^2$

$\Rightarrow g_{m_1} = \sqrt{2K_n'\left(\frac{W}{L}\right)\left(\frac{I}{2}\right)}$

$= \sqrt{2 \times 6.67m \times 0.3m}$ = ~~4 mA/V~~ $= g_{m_2}$

$= 2mA/V$

$K_p'\left(\frac{W}{L}\right) = \dfrac{0.6m}{(0.5)^2}$

$= 2.4mA/V^2$

~~$r_{o_1} = r_{o_2}$~~

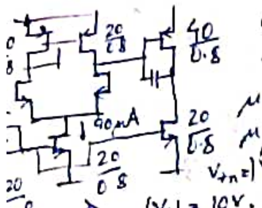
$g_{m_{3,4}} = \sqrt{2 K_p'\left(\frac{W}{L}\right).(I/2)}$

$= \sqrt{2 . 2.4m . 0.3m} = \sqrt{1.44\mu}$

$= 1.2mA/V$.

$K_n' = 90\mu A/V^2$

$K_p' = 30\mu A/V^2$



$C_{db} = 5fF$

$C_{gs} = C_{gd} = 4fF/(\mu m)^2$

$\mu_n C_{ox} = 160\mu A/V^2$

$\mu_p C_{ox} = 40\mu A/V^2$

$V_{tn} = |V_{tp}| = 0.7V$

$|V_A| = 10V$.

find the poles & zeros

$f_{p_1} = \dfrac{1}{2\pi(R_1 C_c G_{m_2} R_2)}$

$f_{p_2} = \dfrac{G_{m_2} C_c}{2\pi(C_1 C_2 + C_c(C_1 + C_2))}$

$f_z = \dfrac{G_{m_2}}{2\pi C_c}$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 74$

$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 80$

$f_{P_1} = \dfrac{1}{\{C_L \times (r_{o_2} \| r_{o_4})\} \times 2\pi}$, $\quad f_{P_2} = \dfrac{g_{m_3}}{2\pi C_m}$

$= \dfrac{1}{2\pi \times 0.2p \times 15K}$ $\quad = \dfrac{1.2m}{2\pi . 0.1pF}$

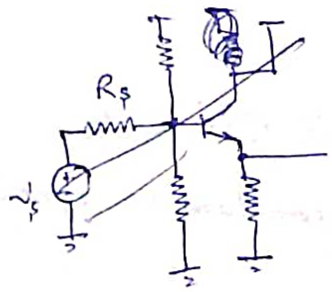
$= 53 MHz$ $\quad = 1.9 GHz$

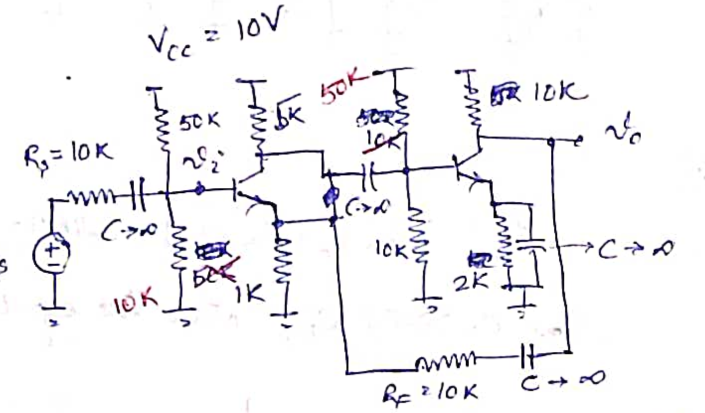
$f_Z = 3.8 GHz$

Gain $= g_{m_1} . (r_{o_2} \| r_{o_4})$

$= 60 V/V$

ii) ~~seriese~~ shunt ckt .





$\beta = 120$, assume $V_{A_{1,2}} = 0$, $I_{B_1}, I_{B_2} = 0$, $V_{BE_1}, = V_{BE_2} = 0.7V$

find g_{m_1}, g_{m_2}, r_{π_1}, r_{π_2}, (2×4)

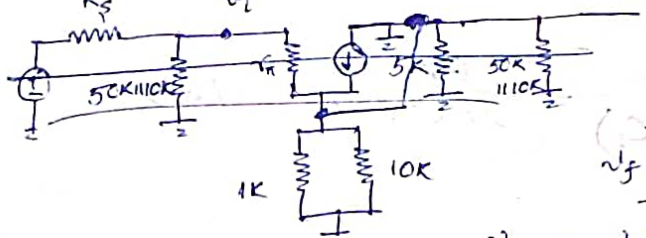
find the feedback factor f . (2)

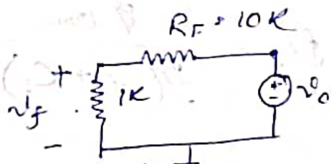
$f =$

$I_{C_1} \simeq \frac{5-0.7}{1K}$

$= 4.3mA$

$g_{m_1} = \frac{I_{C_1}}{V_T} = 165.3m$





$I_{C_2} = \frac{5-0.7}{2K}$

$= 2.15m$

$g_{m_2} = 82m$

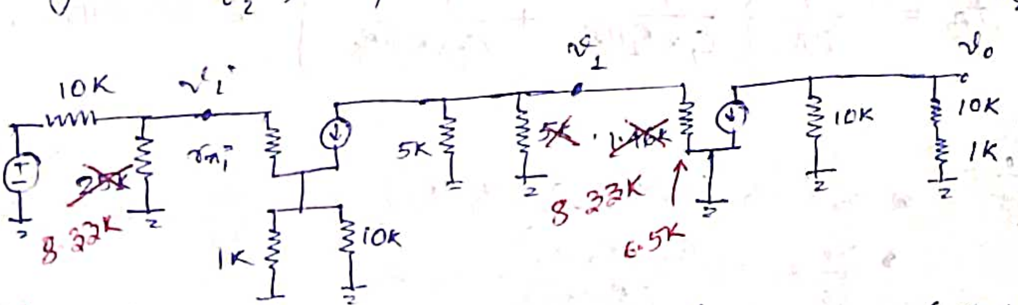
find the small signal ~~voltage~~ closed loop voltage gain . $\frac{v_o}{v_i}$, $\frac{v_o}{v_s}$ ~~(3+3)~~ ~~(5)~~ (4+2)

$r_{\pi_1} = \frac{\beta}{g_{m_1}} =$

726Ω

$r_{\pi_2} = \frac{120}{82m} =$

$1.46K\Omega$



$\frac{v_1}{v_i} = \frac{(5K \| 8.33K \| \ldots}{909}$

$r_{\pi_1} = 3.26K$

$r_{\pi_2} = 6.5K$

$\frac{v_1}{v_i} = -\frac{(5K\|5K\|1.46K)}{(1K \| 10K)}$, $\frac{v_o}{v_1} = -g_{m_2} \times (10K \| 11K)$

$= -\frac{3.12K \ldots}{909}$

$= -82m \times 5.23K$

$= -\frac{2.108K}{909}$

$= -428$.

$= -2.32$

$\frac{v_o}{v_1} = -g_{m_2} \times (10K \| 11K)$ $= -\frac{922}{909} = -1.01$

$= -18.44m \times 5.23K$

$= -96.44$ ∴ $\left(\frac{v_o}{v_i}\right)_{OL} = +433$ V/V, $f = \frac{1K}{11K} = 0.09$

$\left(\frac{v_o}{v_i}\right)_{OL} = +223.7$ V/V

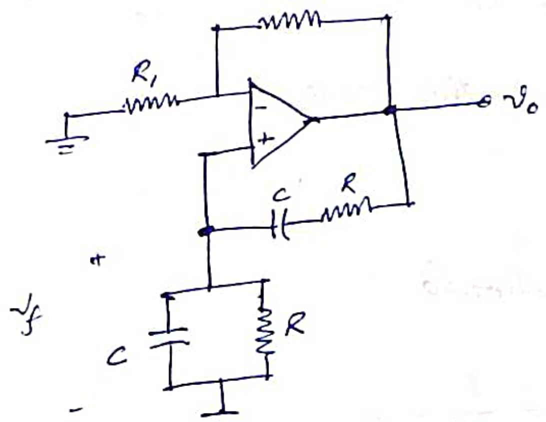
$\left(\frac{v_o}{v_i}\right)_{CL} = \cancel{\frac{433}{1+433 \times 0.09}} = \cancel{10.8}$ V/V $= \frac{223.7}{1+223.7 \times 0.09}$

$= 10.48$ V/V

$(1+A\beta) = (1+223.7 \times 0.09) = 21.133$

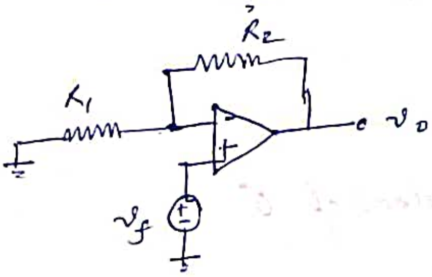
Wien Bridge Oscillator

$$\frac{\frac{R}{Cs}}{R+\frac{1}{Cs}} = \frac{R}{1+RCs}$$



$$v_f = \frac{\frac{1}{Cs} \parallel R}{R \parallel \frac{1}{Cs} + R + \frac{1}{Cs}} \times v_o$$

$$= \frac{\frac{R}{1+RCs}}{\frac{R}{1+RCs} + R + \frac{1}{Cs}} \times v_o$$



$$= \frac{RCs}{RCs + R(1+RCs)Cs + (1+RCs)} v_o$$

$$= \frac{RCs}{RCs + RCs + R^2C^2s^2 + 1 + RCs}$$

$$= \frac{RCs}{3RCs + R^2C^2s^2 + 1} \cdot v_o$$

$$= \frac{1}{3 + \left(RCs + \frac{1}{RCs}\right)} \cdot v_o$$

$$\Rightarrow \frac{0 - v_f}{R_1} = \frac{v_f - v_o}{R_2}$$

$$\Rightarrow -\frac{v_f}{R_1} - \frac{v_f}{R_2} = -\frac{v_o}{R_2}$$

$$\Rightarrow v_f\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = v_o \frac{1}{R_2}$$

$$\Rightarrow v_f \cdot \frac{R_1 + R_2}{R_1} = v_o$$

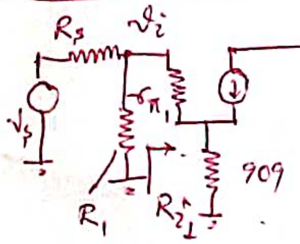
$$\Rightarrow v_o = v_f \cdot \left(1 + \frac{R_2}{R_1}\right)$$

$$\therefore \text{loop gain} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{3 + \left(RCs + \frac{1}{RCs}\right)}$$

at $s = j\omega$,

$$= \frac{\left(1 + \frac{R_2}{R_1}\right)}{3 + \left(RCj\omega_0 - j\frac{1}{RC\omega_0}\right)}$$

$$= \frac{\left(1 + \frac{R_2}{R_1}\right)}{3 + j\left(\omega_0 RC - \frac{1}{\omega_0 RC}\right)}$$



$R_{i1} = r_{\pi_1} + (\beta+1)0.909K$
$= 3.26K + 109.98K$
$= 113.25K$

$$\Rightarrow v_i = \frac{(R_1 \parallel R_{i1})}{(R_1 \parallel R_{i1}) + R_s} \cdot v_s = \frac{7.75K}{7.75K + 10K} \cdot v_s = 0.43 \cdot v_s$$

$$\frac{v_i}{v_s} \simeq \frac{R_1}{R_1 + R_s}$$

as $R_{if} \gg R_1$

$\Rightarrow = 0.45$

$R_{if} = R_{i1}(1 + A\beta)$
$= 113.25K \times 21.13$
$= 2.4M$

$$\cancel{\frac{v_o}{v_s} = \frac{v_o}{v_i} \cdot \frac{v_i}{v_s}}$$

$$\left(\frac{v_o}{v_s}\right)_{CL} \simeq \left(\frac{v_o}{v_i}\right)_{CL} \times \frac{v_i}{v_s} = 10.48 \times 0.45 = 4.76$$

F. for the Wien Bridge oscillator,

the phase shift by op-amp and resistors : $-\frac{\pi}{10}$ at $\omega = \frac{1}{RC}$.

find the freq. of oscillation in this case.

In this case, RC circuit should

at $\omega = \frac{1}{RC}$, $\frac{v_f}{v_o} = \frac{1}{3 + j \times 0}$.

hence, $\angle \frac{v_f}{v_o} = 0°$

Loop phase : $-\frac{\pi}{10}$ instead of 0°.

$\therefore \angle \frac{v_f}{v_o} = +\frac{\pi}{10}$, for oscillation.

i.e., $\angle \frac{1}{3 + j\left(RC\omega - \frac{1}{RC\omega}\right)} = \frac{\pi}{10}$.

$\Rightarrow -\tan^{-1}\left(\frac{RC\omega - \frac{1}{RC\omega}}{3}\right) = \frac{\pi}{10}$

$\Rightarrow -\frac{RC\omega - \frac{1}{RC\omega}}{3} = \tan\frac{\pi}{10}$

$= \cancel{0.3249}\ 0.1$

$\Rightarrow -RC\omega + \frac{1}{RC\omega} = \cancel{+0.97}\ 0.3$

$\Rightarrow -R^2C^2\omega^2 + 1 = -0.97\,RC\omega$

$\Rightarrow +R^2C^2\omega^2 - 0.97\,RC\omega - 1 = 0$

$\cancel{\omega = -0.97}$

$\omega RC = \frac{0.97 + \sqrt{0.97^2 + 4.1}}{2}$

$= \frac{0.97 + 2.22}{2} = 1.6$ $\Rightarrow \omega RC = 1.6$

$\Rightarrow \omega = \frac{1.6}{RC}$

Loop gain :

Loop phase : $\phi = -\tan^{-1}\left(\dfrac{\omega RC - \frac{1}{\omega RC}}{3}\right)$

$\dfrac{d\tan}{d\theta}$.

$\Rightarrow \cot\theta$

$\dfrac{d}{dx}\tan^{-1}x$

$\therefore$ change of loop phase w.r.t. freq. :

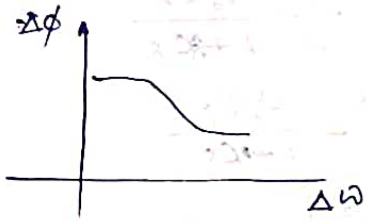
$\dfrac{d}{d\theta}\tan^{-1}\theta x$

$= \dfrac{1}{1+x^2}\cdot\dfrac{\partial\theta}{\partial x}$

$$\frac{\partial\phi}{\partial\omega} = -\frac{d}{d\omega}\tan^{-1}\left(\frac{\omega RC - \frac{1}{\omega RC}}{3}\right)$$

$$= \frac{-1}{1+\left(\frac{\omega RC - \frac{1}{\omega RC}}{3}\right)^2} \times \frac{1}{3}\cdot\frac{d}{d\omega}\left(\omega RC - \frac{1}{\omega RC}\right)$$

$$= -\frac{1}{1+\left(\frac{\omega RC - \frac{1}{\omega RC}}{3}\right)} \times \frac{1}{3}\cdot\left(RC + \frac{1}{\omega^2 RC}\right)$$



$$\therefore \left.\frac{\partial\phi}{\partial\omega}\right|_{\omega=\frac{1}{RC}} = -\frac{1}{1}\times\frac{1}{3}\times(RC+RC)$$

$$= -\frac{2}{3}RC$$

$\therefore$ at $\omega = \frac{1}{RC}$, change of phase : $-\dfrac{\pi}{10}$

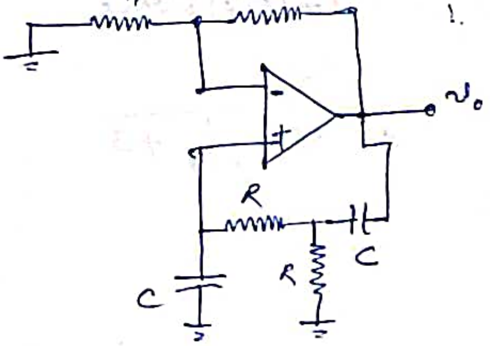
$$\Rightarrow \frac{\Delta\phi}{\Delta\omega} = -\frac{2}{3}RC \Rightarrow +\frac{\pi}{10}\times\frac{1}{\frac{2}{3}RC} = \Delta\omega$$

$\pi^c \longrightarrow 180^\circ$

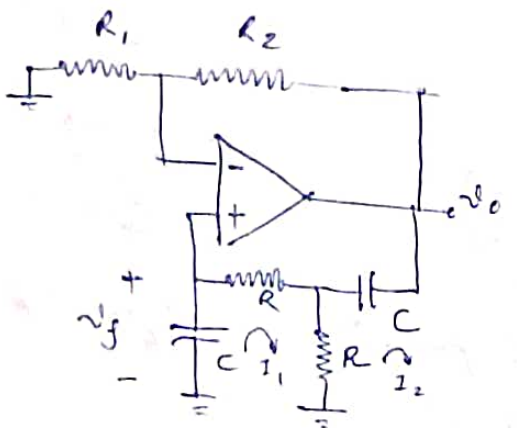
$1^c \longrightarrow \frac{180}{\pi}$

$\therefore 0.1 \longrightarrow \frac{180}{\pi}\times 0.1$

p.



$f_{osc} =$



$$I_1 \frac{1}{Cs} + I_1 R + (I_1 - I_2) R = 0$$

$$(I_2 - I_1) R + I_2 \frac{1}{Cs} + v_o = 0$$

$$\Rightarrow I_2 \left(R + \frac{1}{Cs}\right) = -v_o + I_1 R$$

$$I_2 \left(\frac{RCs+1}{Cs}\right) = I_1 R - v_o$$

$$\Rightarrow I_2 = \frac{(I_1 R - v_o) Cs}{1 + RCs}$$

$$\Rightarrow I_1 \left(\frac{1}{Cs} + 2R\right) = \frac{(I_1 R - v_o) Cs}{1 + RCs} \times R$$

$$\Rightarrow I_1 \left(\frac{2RCs + 1}{Cs}\right) = \frac{I_1 R^2 Cs}{1 + RCs} - \frac{v_o RCs}{1 + RCs}$$

$$\Rightarrow I_1 \left\{\frac{1 + 3RCs + 2(RCs)^2 - R^2C^2s^2}{Cs(1 + RCs)}\right\} = -\frac{v_o RCs}{1 + RCs}$$

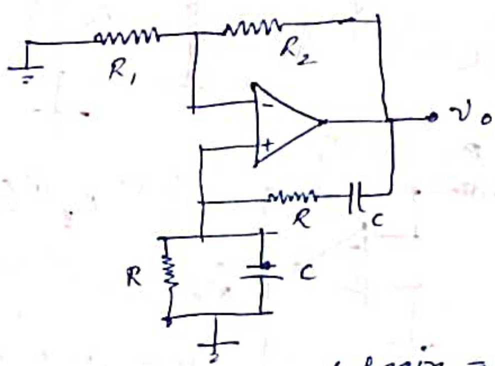
$$\Rightarrow I_1 \left\{\frac{1 + 3RCs + (RCs)^2}{Cs}\right\} = -v_o RCs$$

$$\Rightarrow I_1 = -\frac{v_o RCs \cdot Cs}{1 + (RCs)^2 + 3RCs}$$

$$\Rightarrow v_f = -I_1 \times \frac{1}{Cs} = +\frac{v_o RCs}{1 + (RCs)^2 + 3RCs}$$

$$= \frac{v_o}{RCs + \frac{1}{RCs} + 3}$$

p.



If the op-amp & R_1, R_2 produces a phase lag of $-\frac{\pi}{10}$ at $\omega = \frac{1}{RC}$

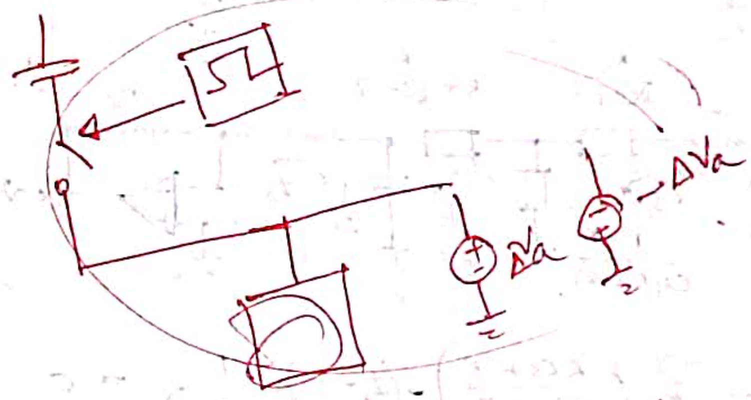
~~R =~~

$$\text{loop gain} = \frac{\left(1+\frac{R_2}{R_1}\right)}{3+j\left(\omega RC-\frac{1}{\omega RC}\right)}$$

$$\angle \text{loop gain} = -\tan^{-1}\left(\frac{\omega RC-\frac{1}{\omega RC}}{3}\right)$$

find the new value of R_x, for which oscillation at $\omega = \frac{1}{RC}$ is recovered.

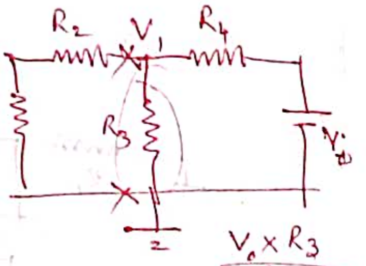
what is the new value of $\frac{R_2}{R_1}$ at the recovered oscillation frequency?

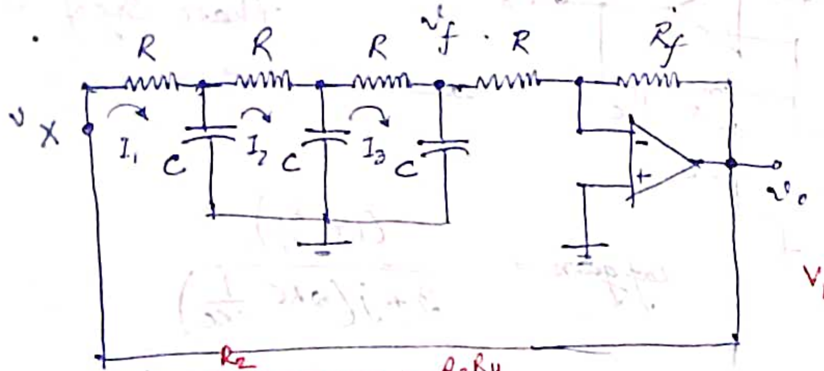


phase shift oscillator :

$$\Rightarrow V_f = \frac{R_1}{R_1+R_2+\frac{R_3R_4}{R_3+R_4}} \times \frac{V_o R_3}{R_3+R_4}$$

$$= \frac{R_1R_3}{R_1R_3+R_2R_4+R_3R_4} \times V_o$$



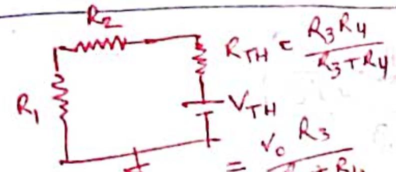


$$\frac{V_o \times R_3}{R_3+R_4}, \quad \frac{R_3R_4}{R_3+R_4}$$

$$R_3 \parallel (R_1+R_2)$$

$$\frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$$

$$V_1 = \frac{\frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \times V_o}{\frac{R_3(R_1+R_2)}{R_1+R_2+R_3} + R_4}$$

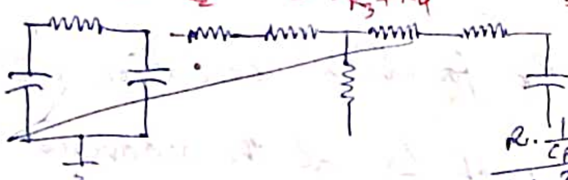


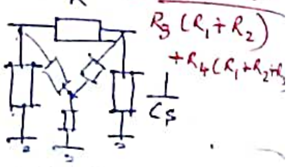
$$= \frac{V_o R_3}{R_3+R_4}$$

$$V_f = \frac{R_1}{R_1+R_2} \times V_1$$

$$= \frac{R_1}{\cancel{R_1+R_2}} \times \frac{R_3\cancel{(R_1+R_2)}V_o}{R_3(R_1+R_2)+R_4(R_1+R_2+R_3)}$$

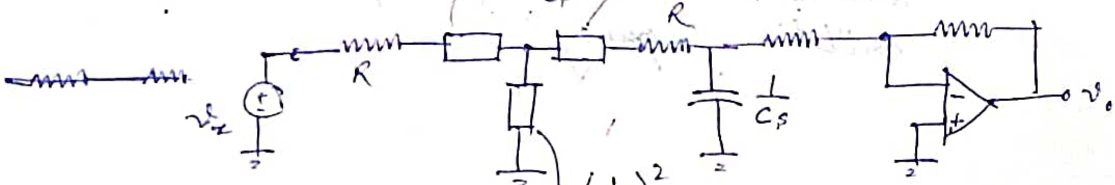
$$\rightarrow = \frac{R_3(R_1+R_2)V_o}{R_3(R_1+R_2)+R_4(R_1+R_2+R_3)}$$





$$= \frac{R_1R_3V_o}{R_1R_3+R_2R_3+R_1R_4+R_4R_2+R_4R_3}$$

$$\frac{R \cdot \frac{1}{C_f}}{R+\frac{2}{C_f}} \qquad \frac{R \cdot \frac{1}{C_f}}{R+\frac{2}{C_f}}$$

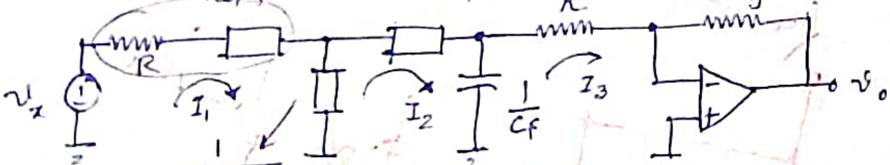


$$\frac{\left(\frac{1}{C_f}\right)^2}{R+\frac{2}{C_f}}$$

$$\frac{R^2C_f+3R}{RC_f+2}$$

$$\frac{R}{RC_f+2} \qquad R+\frac{R}{RC_f+2}$$

=



$$\frac{R^2C_f+3R}{RC_f+2}$$

$$\frac{0-v_o}{R_f} = I_3$$

$$\Rightarrow v_o = -I_3R_f$$

$$v_x - I_1\left(\frac{R^2C_f+3R}{RC_f+2}\right) - (I_1-I_2)\frac{1}{C_f(RC_f+2)} = 0$$

$$-(I_2-I_1)\frac{1}{C_f(RC_f+2)} - I_2 \cdot \left(\frac{R^2C_f+3R}{RC_f+2}\right) - (I_2-I_3)\frac{1}{C_f} = 0$$

$$-(I_3-I_2)\frac{1}{C_f} - I_3(R+R_f) = 0$$

$$v_o = -I_3R_f$$

$$v_2 - I_1\left(\frac{R^2Cs+3R}{RCs+2} + \frac{1}{Cs(RCs+2)}\right) = -I_2\,\frac{1}{Cs(RCs+2)}$$

$$\Rightarrow \quad v_x = I_1\left(\frac{R^2C^2s^2+3RCs+1}{(RCs+2)\,Cs}\right) - I_2\,\frac{1}{Cs(RCs+2)} + I_3\cdot 0$$

$$I_1\cdot\frac{1}{Cs(RCs+2)} - I_2\left(\frac{1}{Cs(RCs+2)} + \frac{R^2Cs+3R}{RCs+2} + \frac{1}{Cs}\right) + I_3\cdot\frac{1}{Cs} = 0$$

$$\Rightarrow I_1\cdot\frac{1}{Cs(RCs+2)} - I_2\left(\frac{1+R^2C^2s^2+3RCs+RCs+2}{Cs(RCs+2)}\right) + I_3\cdot\frac{1}{Cs} = 0$$

$$\Rightarrow I_1\,\frac{1}{Cs(RCs+2)} - I_2\left(\frac{R^2C^2s^2+4RCs+3}{Cs(RCs+2)}\right) + I_3\cdot\frac{1}{Cs} = 0$$

$$-I_3\left(\frac{1}{Cs} + R+R_f\right) + I_2\cdot\frac{1}{Cs} = 0$$

$$\Rightarrow \begin{bmatrix} \frac{R^2C^2s^2+3RCs+1}{(RCs+2).Cs} & -\frac{1}{Cs(RCs+2)} & 0 \\ \frac{1}{Cs(RCs+2)} & -\frac{R^2C^2s^2+4RCs+3}{Cs(RCs+2)} & \frac{1}{Cs} \\ 0 & \frac{1}{Cs} & -\frac{(R+R_f)Cs+1}{Cs} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} v_x \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \cancel{I_2 =}$ $\cancel{I_3 =}$ $\Delta = \dfrac{R^2C^2s^2+3RCs+1}{\cancel{(RCs+2)}.Cs} \times \cancel{R^2C^2s^2+4RCs}+3$

$$= \begin{bmatrix} a & -b & 0 \\ b & -a\cdot c & c \\ 0 & c & -d \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} v_x \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \Delta = \cancel{a \times (b \; a - c}$$

$$= a.\left\{ (a+c)d - c^2 \right\} - b.(-bd)$$

$$= a\left\{ (a+c).d - c^2 \right\} + b^2.d$$

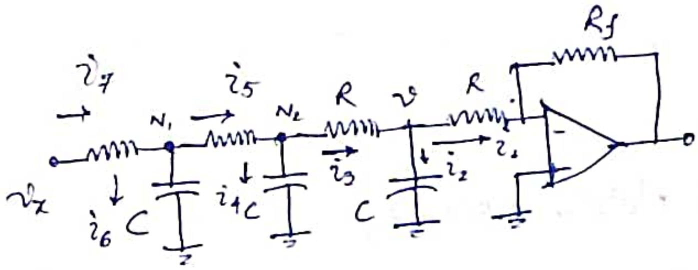
$$\cancel{I_3 =}$$

$$I_3 = \frac{\begin{bmatrix} a & -b & v_x \\ b & -(a+c) & 0 \\ 0 & c & 0 \end{bmatrix}}{\Delta}$$

$$= \frac{v_x . bc}{a\left\{(a+c)d - c^2\right\} + b^2 d}$$

$$= \frac{v_x . \frac{1}{C_s(RC_s+2)} . \frac{1}{C_s}}{\frac{R^2C_s^2 + 3RC_s + 1}{C_s(RC_s+2)} \times \left\{ \frac{R^2C_s^2 + 4RC_s + 3}{C_s(RC_s+2)} \times \frac{(R+R_f)C_s + 1}{C_s} - \frac{1}{(C_s)^2} \right\} + \frac{1}{(C_s)^2(RC_s+2)^2} \times \frac{(R+R_f)C_s +}{C_s}}$$

$$= \frac{v_x \frac{1}{(C_s)^2(RC_s+2)}}{\frac{R^2C_s^2 + 3RC_s + 1}{C_s(RC_s+2)} \times \left\{ \frac{\quad}{(C_s)^2(RC_s+2)} \right\}}$$



$$i_1 = \frac{v}{R}, \quad i_2 = vsC$$

$$i_3 = \frac{v}{R} + vsC$$

$$= v\left(\frac{1}{R} + sC\right) = \frac{v(RCs+1)}{R}.$$

$$\therefore \cancel{v_{N_2} =} \quad v_{N_2} - i_3 R = v$$

$$\Rightarrow v_{N_2} = v + i_3 R = v + \frac{(RCs+1)\cdot v}{\cancel{R}} \times \cancel{R}$$

$$= v + v(RCs+1)$$

$$= v(RCs+2)$$

$$\Rightarrow i_4 = v(RCs+2) \times sC$$

& $i_5 = i_4 + i_3$

$$= v\frac{(RCs+1)}{R} + v(RCs+2).sC$$

$$= v\left\{\frac{RCs+1}{R} + R(Cs)^2 + 2Cs\right\}$$

$$= v\cdot\left\{\frac{RCs+1+R^2C^2s^2+2RCs}{R}\right\}$$

$$= v\cdot\left\{\frac{R^2C^2s^2+3RCs+1}{R}\right\}$$

& $v_{N_1} - i_5 R = v_{N_2}$

$$= v(RCs+2)$$

$$\Rightarrow v_{N_1} = i_5 \cdot R + v(RCs+2)$$

$$= \cancel{2}\, v\left\{\frac{R^2C^2s^2+3RCs+1}{R}\right\} \times R$$

$$+ v(RCs+2)$$

$$= v\cdot\left\{R^2C^2s^2+4RCs+3\right\}.$$

$$\therefore i_6 = v_{N_1} Cs \quad \therefore i_7 = i_6 + i_5.$$

$$= v\left\{R^2C^2s^2+4RCs+3\right\}Cs + v\left\{\frac{R^2C^2s^2+3RCs+1}{R}\right\}$$

& $v_x - i_7 \cdot R = v_{N_1}$

$$= v\left\{R^2C^3s^3 + 4RC^2s^2 + 3Cs + \frac{R^2C^2s^2+3RCs+1}{R}\right\}$$

$$= v\left\{\frac{R^3C^3s^3 + 4R^2C^2s^2 + 3RCs + R^2C^2s^2 + 3RCs + 1}{R}\right\}$$

$$= v\left(\frac{R^3C^3s^3 + 5R^2C^2s^2 + 6RCs + 1}{R}\right)$$

$$\Rightarrow v_x = i_7 R + v_{N_1}$$

$$= i_7 . R + v\{R^2C^2s^2 + 4RCs + 3\}$$

$$= v(R^3C^3s^3 + 5R^2C^2s^2 + 6RCs + 1 + R^2C^2s^2 + 4RCs + 3)$$

$$= v(R^3C^3s^3 + 6R^2C^2s^2 + 10RCs + 4)$$

$$\Rightarrow \frac{v-0}{R} = \frac{0-v_o}{R_f}$$

$$\Rightarrow \frac{v}{R} = -\frac{v_o}{R_f} \Rightarrow v = - v_o \times \frac{R}{R_f}$$

$$\Rightarrow v_x = - v_o . \frac{R}{R_f} . (R^3C^3s^3 + 6R^2C^2s^2 + 10RCs + 4)$$

$$\Rightarrow \frac{v_o}{v_x} = \frac{\left(-\frac{R_f}{R}\right)}{R^3C^3s^3 + 6R^2C^2s^2 + 10RCs + 4}$$

$$(j\omega)^3 = j^3\omega^3 = j(j^2)\omega^3 = -j\omega^3$$

at $s = j\omega$

$$= \frac{\left(-\frac{R_f}{R}\right)}{-jR^3C^3\omega^3 - 6R^2C^2\omega^2 + j10RC\omega + 4}$$

$$= \frac{\left(-\frac{R_f}{R}\right)}{(4 - 6R^2C^2\omega^2) + j(10RC\omega - R^3C^3\omega^3)}$$

$$-\tan^{-1}\frac{10RC\omega - R^3C^3\omega^3}{4 - 6R^2C^2\omega^2} = \pi$$

if $10RC\cancel{\omega} = R^{\cancel{3}2}C^{\cancel{3}2}\omega^2$,

$$\Rightarrow \omega^2 = \frac{1}{10R^2C^2} \Rightarrow \omega = \frac{1}{\sqrt{10}} . \frac{1}{RC}$$

at $\omega = \frac{1}{\sqrt{10}.RC}$, attenuation by phase shift ckt :

$$4 - 6.\cancel{R^2C^2}.\frac{1}{10\cancel{R^2C^2}}$$

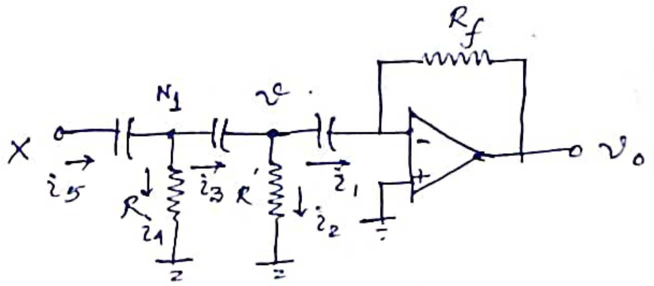
$$= 4 - \frac{\cancel{6}^3}{\cancel{10}_5} = \left(\frac{17}{5}\right).$$

$\omega\ |A\beta| = 1$

$$\therefore \frac{R_f}{R} = \frac{17}{5}$$

$$\Rightarrow 5R_f = 17R$$

RC phase shift oscillator :



$i_1 = v . sC \qquad i_2 = \frac{v}{R} \quad \Rightarrow i_3 = i_1 + i_2 = v\left(sC + \frac{1}{R}\right)$

$= v\left(\frac{RCs+1}{R}\right)$

& $\quad \cancel{(v_{N_1} - v)} \qquad v_{N_1} - i_3 \times \frac{1}{Cs} = v$

$\Rightarrow v_{N_1} = i_3 \cdot \frac{1}{Cs} + v$

$= v\left(\frac{RCs+1}{RCs}\right) + v$

$= v\left(\frac{2RCs+1}{RCs}\right)$

$\therefore \quad i_4 = \frac{v_{N_1}}{R} = \frac{v(2RCs+1)}{R^2Cs}$

$\Rightarrow \quad i_5 = i_4 + i_3 = \frac{v(2RCs+1)}{R^2Cs} + v\frac{(RCs+1)}{R}$

$= \frac{v}{R}\left\{\frac{2RCs+1}{RCs} + RCs+1\right\} \qquad \cancel{= \frac{v(RCs+1)}{R}\left(\frac{1}{RCs}+1\right)}$

$= \frac{v}{R}\cdot\left\{\frac{2RCs+1+R^2C^2s^2+RCs}{RCs}\right\} \qquad \cancel{= \frac{v(RCs+1)^2}{R^2Cs}} \quad = v\cdot\frac{R^2C^2s^2+3RCs+1}{R^2Cs}$

$\Rightarrow \quad v_x - i_5\frac{1}{Cs} = v_{N_1}$

$\Rightarrow \quad v_x = \frac{v(RCs+1)^2}{R^2Cs}\cdot\frac{1}{Cs} + v.\frac{(2RCs+1)}{RCs}$

$= v.\left\{\frac{(RCs+1)^2}{R^2C^2s^2} + \frac{2RCs+1}{RCs}\right\} = \frac{R^2C^2s^2+2RCs+1+2R^2C^2s^2+RCs}{R^2C^2s^2} \times v$

$= v \times \frac{3R^2C^2s^2+3RCs+1}{R^2C^2s^2}$

$$v \cdot sC = \frac{0 - v_o}{R_f}$$

$$\Rightarrow \quad v = -\frac{v_o}{R_f C . s}$$

$$\Rightarrow \quad v_x = \frac{i_s}{Cs} + v_N,$$

$$= v \cdot \frac{R^2C^2s^2 + 3RCs + 1}{R^2C^2s^2} + v \cdot \frac{2RCs+1}{RCs}$$

$$= v \cdot \left\{ \frac{R^2C^2s^2 + 3RCs + 1 + 2R^2C^2s^2 + RCs}{R^2C^2s^2} \right\}$$

$$= v \left\{ \frac{3R^2C^2s^2 + 4RCs + 1}{R^2C^2s^2} \right\}.$$

$$= -\frac{v_o}{R_f C . s} \times \frac{3R^2C^2s^2 + 4RCs + 1}{R^2C^2s^2}$$

$$\Rightarrow \quad \frac{v_o}{v_x} = \frac{-R^2C^2s^2 . R_f . C . s}{3R^2C^2s^2 + 4RCs + 1}.$$

$$\cancel{-R^2C^2\omega^2 . R_f C}$$

~~at s = jω, =~~ $-$

$$= -\frac{RCs . R_f C . s}{3RCs + 4 + \frac{1}{sRC}}$$

$$= \frac{RR_f C^2 \omega^2}{4 + j\left(3\omega RC - \frac{1}{\omega RC}\right)}$$

at $3\omega RC = \frac{1}{\omega RC}$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{3} . RC}$$

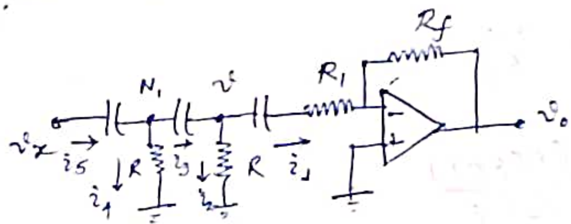
$$2\pi f = \frac{1}{\sqrt{3}\, RC}$$

at $\omega = \frac{1}{\sqrt{3}RC}$,

$$|A\beta| = 1, \quad = \frac{R R_f C^2 \cdot \frac{1}{3R^2C^2}}{4}$$

$$= \frac{R_f}{12R} = 1 \Rightarrow \frac{R_f}{R} = 12$$

Actual phase-shift ckt:



$$i_1 = \frac{v}{R + \frac{1}{Cs}} = \frac{vCs}{RCs+1} \qquad i_2 = \frac{v}{R}$$

$$i_3 = v\left(\frac{1}{R} + \frac{Cs}{RCs+1}\right) = v\left(\frac{RCs+1+RCs}{R(RCs+1)}\right)$$

$$= v\left\{\frac{2RCs+1}{R(RCs+1)}\right\}$$

$$v_{N_1} - \frac{i_3}{Cs} = v \Rightarrow v_{N_1} = v + \frac{v\{2RCs+1\}}{RCs(RCs+1)}$$

$$= v\left\{\frac{R^2C^2s^2 + RCs + 2RCs + 1}{RCs\,(RCs+1)}\right\}$$

$$= v\left\{\frac{R^2C^2s^2 + 3RCs + 1}{RCs\,(RCs+1)}\right\}$$

$$\Rightarrow i_4 = \frac{v_{N_1}}{R} = v\left\{\frac{R^2C^2s^2 + 3RCs + 1}{R^2Cs\,(RCs+1)}\right\}$$

$$\Rightarrow \cancel{v_x - i} \quad \Rightarrow i_5 = i_4 + i_3 = v\left\{\frac{R^2C^2s^2+3RCs+1}{R^2Cs(RCs+1)} + \frac{2RCs+1}{R(RCs+1)}\right\}$$

$$\Rightarrow v_x - \frac{i_5}{Cs} = v_{N_1} \qquad = v\left\{\frac{R^2C^2s^2+3RCs+1+2R^2C^2s^2+RCs}{R^2Cs\,(RCs+1)}\right\}$$

$$= v\left\{\frac{3R^2C^2s^2 + 4RCs + 1}{R^2Cs\,(RCs+1)}\right\}$$

$$\Rightarrow v_x = \left\{\frac{3R^2C^2s^2+4RCs+1}{R^2C^2s^2(RCs+1)} + \frac{R^2C^2s^2+3RCs+1}{R^2Cs\,(RCs+1)}\right\} \times v$$

$$= \left\{\frac{3R^2C^2s^2+4RCs+1 + R^3C^3s^3+3R^2C^2s^2+RCs}{R^2C^2s^2\,(RCs+1)}\right\} v$$

$$= \left\{\frac{R^3C^3s^3 + 6R^2C^2s^2 + 5RCs + 1}{R^2C^2s^2\,(RCs+1)}\right\} v$$

now,
$$\frac{v_i\, Cs}{RCs+1} = -\frac{v_o}{R_f}$$

$$\Rightarrow v_i = -v_o\frac{(RCs+1)}{R_f Cs}$$

$$\Rightarrow v_x = \frac{R^3C^3s^3 + 6R^2C^2s^2 + 5RCs + 1}{R^2C^2s^2\,(RCs+1)} \times -v_o \times \frac{(RCs+1)}{R_f Cs}$$

$$\Rightarrow \frac{v_o}{v_x} = -\frac{R^2C^2s^2 \cdot R_f \cdot C \cdot s}{R^3C^3s^3 + 6R^2C^2s^2 + 5RCs + 1}$$

$$= \frac{RCs \cdot R_f Cs}{R^2C^2s^2 + 6RCs + 5 + \frac{1}{RCs}}$$

at $s = j\omega$, $\dfrac{v_o}{v_x} = \dfrac{+\omega^2 R R_f C^2}{(5-\omega^2R^2C^2) + j\left(\omega RC6 - \frac{1}{\omega RC}\right)}$

if $\omega = \dfrac{1}{\sqrt{6}\cdot RC}$.

at $\omega = \dfrac{1}{\sqrt{6}} \cdot \dfrac{1}{RC}$, ~~phase shift attenuation~~ :

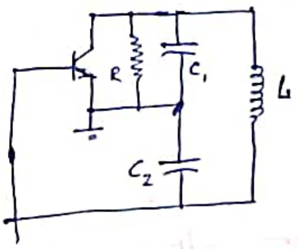
as $|A\beta| = 1$, $\cancel{\left(5 - \frac{1}{6}\right) = \frac{29}{6}}$.

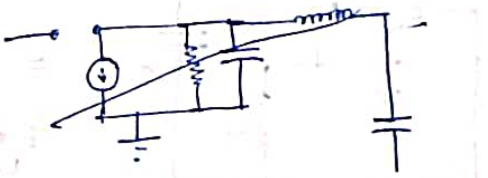
we must have,
$$\frac{\frac{1}{6R^2C^2} \cdot RC^2 \cdot R_f}{\left(5 - \frac{1}{6}\right)} = 1.$$

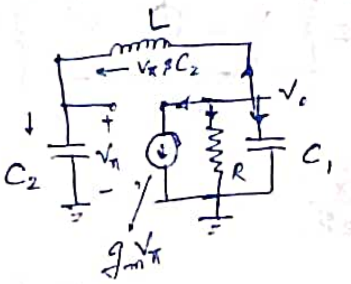
$$\Rightarrow \frac{\frac{1}{6} \cdot \frac{R_f}{R}}{\frac{29}{6}} = 1.$$

$$\Rightarrow R_f = 29R.$$

Colpitt's Oscillator







$$\cancel{(V_\pi - V_o)}$$

$$\frac{(V_o - V_\pi)}{Ls} = V_\pi s C_2$$

$$\Rightarrow V_o - V_\pi = V_\pi L C_2 s^2$$

$$\Rightarrow V_o = V_\pi (1 + LC_2 s^2)$$

also, $g_m V_\pi + \frac{V_o}{R} + V_o . s C_1 + V_\pi s C_2 = 0$.

$$\Rightarrow \quad g_m V_\pi + V_\pi s C_2 + V_\pi (1 + LC_2 s^2)\left(\frac{1}{R} + s C_1\right) = 0$$

$\Rightarrow$ as $V_\pi \neq 0$,

$$g_m + s C_2 + (1 + LC_2 s^2)\left(\frac{1}{R} + s C_1\right) = 0$$

$$\Rightarrow \quad g_m + sC_2 + \frac{1}{R} + sC_1 + \frac{LC_2}{R} s^2 + LC_1C_2 s^3 = 0$$

$$\Rightarrow s^3 LC_1C_2 R + LC_2 s^2 + sR(C_1 + C_2) + g_m R + 1 = 0$$

at $s = j\omega$,

$$-j\omega^3 LC_1C_2R - LC_2\omega^2 + j\omega R(C_1 + C_2) + g_m R = 0$$

$$\Rightarrow 1 + g_m R - LC_2\omega^2 + j\{\omega R(C_1 + C_2) - \omega^3 LC_1C_2R\} = 0 .$$

$$\omega_o (C_1 + C_2) = \omega_o^3 LC_1C_2$$

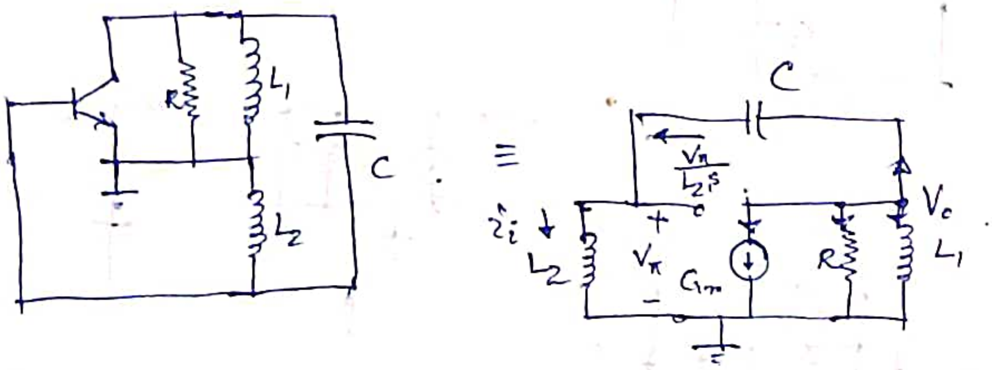
$$\Rightarrow \omega_o^2 = \left(\frac{C_1 + C_2}{C_1C_2}\right) \times .$$

$$\Rightarrow \quad 1 + g_m R = LC_2 . \omega^2$$

$$= \cancel{L}\cancel{C_2} \times \frac{(C_1 + C_2)}{C_1\cancel{C_2}} \times \frac{1}{\cancel{L}}$$

$$\Rightarrow 1 + g_m R = 1 + \frac{C_2}{C_1} \Rightarrow g_m R = \frac{C_2}{C_1}$$

Hartley Oscillator:



$$i_i = \frac{V_\pi}{L_2 s}.$$

$$(V_o - V_\pi)Cs = \frac{V_\pi}{L_2 s}. \Rightarrow V_o Cs = V_\pi\left(Cs + \frac{1}{L_2 s}\right)$$

$$\Rightarrow V_o = \frac{V_\pi\left(L_2 C s^2 + 1\right)}{L_2 C s^2}$$

also,

$$G_m V_\pi + \frac{V_o}{R} + \frac{V_o}{L_1 s} + \frac{V_\pi}{L_2 s} = 0$$

$$\Rightarrow \left(G_m + \frac{1}{L_2 s}\right)V_\pi + V_o\left(\frac{1}{R} + \frac{1}{L_1 s}\right) = 0$$

$$\Rightarrow \left(G_m + \frac{1}{L_2 s}\right)V_\pi + \frac{V_\pi(L_2 C s^2 + 1)}{L_2 C s^2}\left(\frac{L_1 s + R}{L_1 R s}\right) = 0$$

$\Rightarrow$ as $V_\pi \neq 0$,

$$\frac{G_m L_2 s + 1}{L_2 s} + \frac{(L_2 C s^2 + 1)(L_1 s + R)}{L_2 L_1 R C s^3} = 0$$

$$\Rightarrow \frac{(G_m L_2 s + 1) L_1 R C s^2 + L_2 L_1 C s^3 + L_2 R C s^2 + L_1 s + R}{L_2 L_1 R C s^3} = 0$$

$$\Rightarrow G_m L_2 L_1 R C s^3 + L_2 L_1 C s^3 + L_1 R C s^2 + L_2 R C s^2 + L_1 s + R = 0$$

$$-j(G_m R+1)L_2L_1C\,\omega^3 - \omega^2(L_1RC+L_2RC)+R + j\omega L_1 = 0$$

$$\Rightarrow R-\omega^2(L_1RC+L_2RC)+j\{\omega L_1-(G_mR+1)L_2L_1C\omega^3\}=0$$

$R = \omega^2(L_1RC+L_2RC)$ & $\omega L_1 = (G_mR+1)L_2L_1C\omega^3$.

at $\omega = \omega_0$

$\Rightarrow 1 = \omega^2(L_1C+L_2C)$ & $L_1 = (G_mR+1)L_2L_1C\times\omega_0^2$

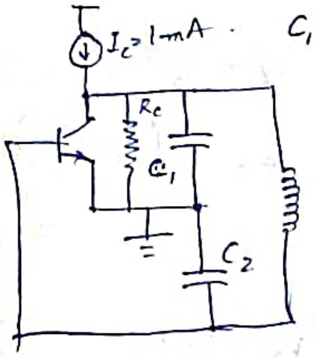
$\Rightarrow \omega_0 = \sqrt{\dfrac{1}{C(L_1+L_2)}}$ $\quad \cancel{L_1} = (G_mR+1)L_2\cancel{L_1}\cancel{C}\times\dfrac{1}{\cancel{C}(L_1+L_2)}$

$$\Rightarrow 1 = (G_mR+1)\times\frac{L_2}{(L_1+L_2)}$$

$$\Rightarrow G_mR+1 = \frac{L_1+L_2}{L_2}$$

$$\Rightarrow G_mR = \frac{L_1}{L_2}$$

P. a BJT is biased at $I_c = 1mA$



$C_1 = 0.01\mu F$ $\quad R_c = 2K\Omega$, $\quad \omega_0 = 10^6$ rad/s.

$r_o = 100K\Omega$,

coil: Q : 100. $\quad$ find C_2 & L.

$$Q = \omega_0 RC.$$

$100 = 10^6\times R.C_1$ $\qquad C_1 = 0.01\mu F.$

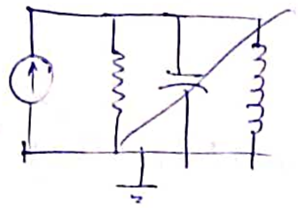
$$\Rightarrow R = \frac{100}{10^6\times 0.01\times10^{-6}} = 10K\Omega$$

$\therefore R_{eq} = R_c \| r_o \| R.$

$= 2K \| 100K \| 10K.$

$\simeq 2K\|10K = 1.67K\Omega.$

Q factor: $2\pi \times$



$$\omega_0^2 = \frac{C_1 + C_2}{C_1 C_2} \times \frac{1}{L}$$

$$10^{12} = \frac{0.01 + 0.64}{0.01 \times 0.64} \times \frac{1}{L} \times \frac{1}{10^{-6}}$$

$\Rightarrow L = 101.56 \times 10^{6} \times 10^{-12}$

$\simeq 101.56\,\mu H$.

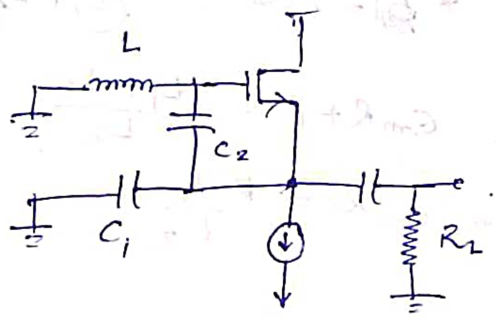
$\Rightarrow g_m R = \frac{C_2}{C_1}$

$\Rightarrow \frac{I_C}{V_T} \cdot (1.67 k\Omega) = \frac{C_2}{0.01\mu}$

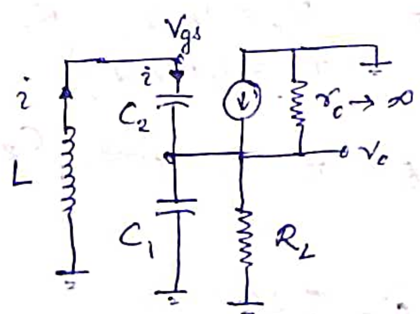
$\Rightarrow C_2 = \frac{1m}{26m} \times 0.01\mu \times 1.67k$

$= 0.64 \mu F$.

P.



find frequency of oscillation:



~~$i = V_{gs} \cdot C_2 s$.~~

$(V_{gs} - V_o) C_2 s = i$

$= \frac{0 - V_{gs}}{Ls}$.

$\Rightarrow (V_{gs} - V_o) C_2 s = -\frac{V_{gs}}{Ls}$

$\Rightarrow V_{gs}\left(C_2 s + \frac{1}{Ls}\right) = V_o C_2 s$

$\Rightarrow V_o = \left(\frac{L C_2 s^2 + 1}{L C_2 s^2}\right) V_{gs}$

$$g_m V_{gs} + i = V_o\left(C_1 s + \frac{1}{R}\right)$$

$$\Rightarrow g_m V_{gs} - \frac{V_{gs}}{Ls} = V_o\left(C_1 s + \frac{1}{R}\right)$$

$$\Rightarrow \left(\frac{g_m L s - 1}{Ls}\right) V_{gs} - V_o\left(C_1 s + \frac{1}{R}\right) = 0$$

$$\Rightarrow \left(\frac{g_m L s - 1}{Ls}\right) V_{gs} - \left(\frac{L_2 C s^2 + 1}{L_2 C s^2}\right)\left(\frac{R C_1 s + 1}{R}\right) V_{gs} = 0$$

$$(V_{gs} - V_o) C_2 s = \frac{0 - (V_{gs} + V_o)}{Ls}$$

$$\Rightarrow V_{gs}\left(C_2 s + \frac{1}{Ls}\right) = -\frac{V_o}{Ls} + V_o C_2 s$$

$$\Rightarrow V_{gs}\left(\frac{LC_2 s^2 + 1}{Ls}\right) = V_o\left(\frac{LC_2 s^2 - 1}{Ls}\right)$$

$$\Rightarrow V_{gs} = V_o\left(\frac{LC_2 s^2 - 1}{LC_2 s^2 + 1}\right)$$

$$V_{gs} \cdot C_2 s = \frac{0 - (V_{gs} + V_o)}{Ls}$$

$$\Rightarrow V_{gs}\left(C_2 s + \frac{1}{Ls}\right) = -\frac{V_o}{Ls}$$

$$\Rightarrow V_{gs}(LC_2 s^2 + 1) = -V_o$$

$$\Rightarrow V_o = -V_{gs}(LC_2 s^2 + 1)$$

$$\cancel{g_m V_{gs} =} \quad g_m V_{gs} + (V_{gs} - V_o) C_2 s = V_o\left(C_1 s + \frac{1}{R_L}\right)$$

$$\Rightarrow (g_m + C_2 s) V_{gs} = V_o\left(C_2 s + C_1 s + \frac{1}{R_L}\right)$$

$$(g_m + C_2 s) V_{gs} = \frac{(LC_2 s^2 + 1)}{LC_2 s^2 - 1} \frac{(R_L(C_1 + C_2)s + 1)}{R_L} V_{gs}$$

$$\Rightarrow (g_m + C_2 s)\left(R_L \times (LC_2 s^2 - 1)\right) V_{gs} - (LC_2 s^2 + 1)(R_L(C_1 + C_2)s + 1) V_{gs} = 0$$

$V_{gs} \neq 0$,

$$\Rightarrow g_m R_L L C_2 s^2 - g_m R_L + R_L L C_2^2 s^3 - R_L C_2 s - \{LC_2 R_L (C_1 + C_2) s^3 + LC_2 s^2 + R_L(C_1 + C_2)s + 1\} = 0$$

$$\Rightarrow g_m R_L L C_2 s^2 - LC_2 s^2 - g_m R_L - 1 + R_L L C_2^2 s^3 - LC_2 R_L C_1 s^3 - LC_2^2 R_L s^3 + R_L C_2 s$$

$$g_m V_{gs} + V_{gs}.C_2 s = V_o\left(C_1 s + \frac{1}{R_L}\right)$$

$$= -\frac{V_{gs}\,(LC_2 s^2+1)(R_L C_1 s+1)}{R_L}$$

as $V_{gs} \neq 0$,

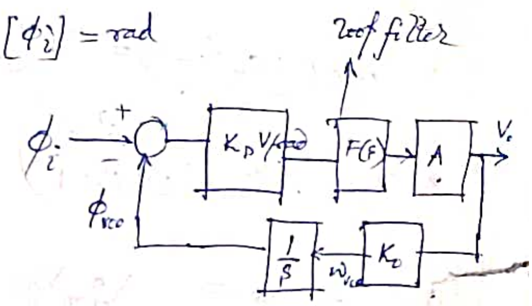
$$(g_m + C_2 s)R_L + (LC_2 s^2+1)(R_L C_1 s+1) = 0$$

$$\Rightarrow g_m R_L + R_L C_2 s + LC_1C_2R_L s^3 + LC_2 s^2 + R_L C_1 s + 1 = 0$$

$$\Rightarrow (g_m R_L + 1) - j\omega^3 LC_1C_2R_L + j\omega R_L(C_1+C_2) - \omega^2 LC_2 = 0$$

$$\Rightarrow g_m R_L + 1 - \omega^2 LC_2 + j\left\{\omega R_L(C_1+C_2) - \omega^3 LC_1C_2R_L\right\} = 0$$

$[\phi_i] = \text{rad}$



$$\rightarrow \omega(C_1+C_2) = \omega^3 LC_1C_2$$

$$\Rightarrow \omega^2 = \frac{C_1+C_2}{LC_1C_2}$$

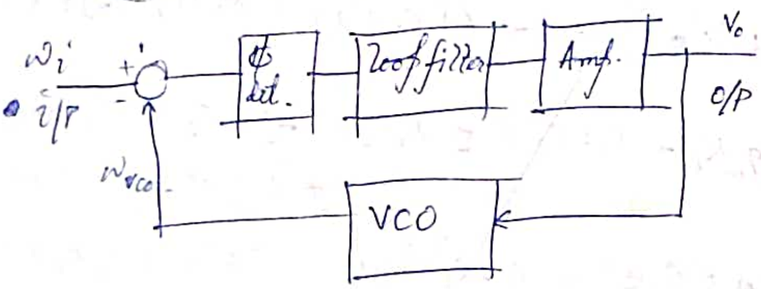
$$g_m R_L + 1 - \frac{C_1+C_2}{\cancel{L}C_1\cancel{C_2}}.\cancel{L}\cancel{C_2} = 0$$

$$\Rightarrow g_m R_L + \cancel{1} = \cancel{1} + \frac{C_2}{C_1} \quad \Rightarrow g_m R_L = \frac{C_2}{C_1}$$

$\phi_{vco} = \int_0^t \omega_{vco}\, dt$

$\omega_{vco} = K_o V_o + \omega_o$

ω_o = free running freq. when $V_o = 0$



Locked signal

$\omega_i = \omega_{vco}$

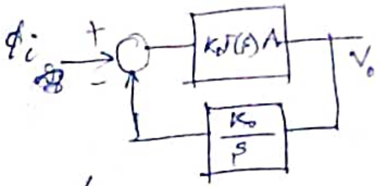
$\omega_{vco} = f(V_o) = K.V_o$

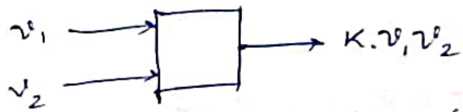
when $V_o = 0$,

$\omega_{vco} = \omega_o$ = free running freq. PLL condition

$$\omega_i = \underbrace{\omega_{vco} \pm \Delta\omega}_{\text{capture range}}$$

Analog Multiplier:

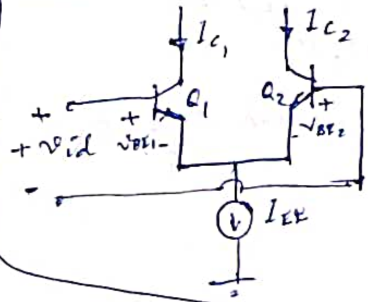




$$\frac{V_o}{\phi_i} = \frac{K_D F(s) A}{1 + \frac{K_D K_o F(s) A}{s}} = \frac{s K_D F(s) A}{s + K_D K_o F(s) A}$$

for expressing V_o vs I/P freq., ω_i $\quad \omega_i = \frac{d\phi_i}{dt}$

$\Rightarrow \omega_i(s) = s\phi_i(s)$



$$v_{id} - V_{BE_1} + V_{BE_2} = 0$$

$$V_{BE_1} = V_T \ln \frac{I_{c_1}}{I_{s_1}}$$

$$V_{BE_2} = V_T \ln \frac{I_{c_2}}{I_{s_2}}$$

$$\frac{V_o(s)}{s\phi_i(s)} = \frac{K_D F(s) A}{s + K_D K_o F(s) A} = \frac{V_o(s)}{\omega_i(s)}$$

$$= \frac{K_D K_o F(s) A}{s + K_D K_o F(s) A} \cdot \frac{1}{K_o}$$

$$= \left(\frac{K_V}{s + K_V}\right) \frac{1}{K_o}$$

$$\Rightarrow v_{id} - V_T \ln \frac{I_{c_1}}{I_{s_1}} + V_T \ln \frac{I_{c_2}}{I_{s_2}} = 0$$

as $Q_1 \equiv Q_2$, $I_{s_1} = I_{s_2}$

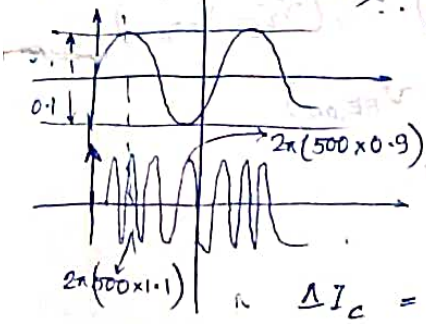
$$\Rightarrow v_{id} = V_T \ln \frac{I_{c_1}}{I_s} - V_T \ln \frac{I_{c_2}}{I_s}$$

$$= V_T \ln \frac{I_{c_1}}{I_{c_2}}$$

$$\Rightarrow \frac{I_{c_1}}{I_{c_2}} = e^{\frac{v_{id}}{V_T}}$$

e.g.:

$\omega_i = 2\pi 500 (1 + 0.1 \sin 2\pi 100 t)$



$\&$ $I_{c_1} + I_{c_2} \approx I_{EE}$

$$\Rightarrow I_{c_2} e^{\frac{v_{id}}{V_T}} + I_{c_2} = I_{EE}$$

$$\Rightarrow I_{c_2} = \frac{I_{EE}}{1 + e^{\frac{v_{id}}{V_T}}} \quad \& \quad I_{c_1} = \frac{I_{EE}}{1 + e^{-\frac{v_{id}}{V_T}}}$$

$$\Delta I_c = I_{c_1} - I_{c_2} = I_{EE} \left(\frac{\cancel{1} + e^{+\frac{v_{id}}{V_T}} - \cancel{1} - e^{-\frac{v_{id}}{V_T}}}{1 + 1 + e^{\frac{v_{id}}{V_T}} + e^{-\frac{v_{id}}{V_T}}} \right)$$

ω_{osc} $K_o \times V_o(t)$

$= 0.031 \times (2\pi \times 1K) \times \sin(2\pi 100 t - 51^\circ)$

$= 0.031 \times 2\pi \times 1K . \sin(2\pi 100 t - 51^\circ)$

$$= I_{EE} \left\{ \frac{\left(e^{+\frac{v_{id}}{2V_T}} + e^{-\frac{v_{id}}{2V_T}}\right)\left(e^{+\frac{v_{id}}{2V_T}} - e^{-\frac{v_{id}}{2V_T}}\right)}{\left(e^{\frac{v_{id}}{2V_T}} + e^{-\frac{v_{id}}{2V_T}}\right)^2} \right\}$$

$$\tanh \frac{v_{id}}{2V_T} = \frac{e^{\frac{v_{id}}{V_T}} - 1}{e^{\frac{v_{id}}{V_T}} + 1}$$

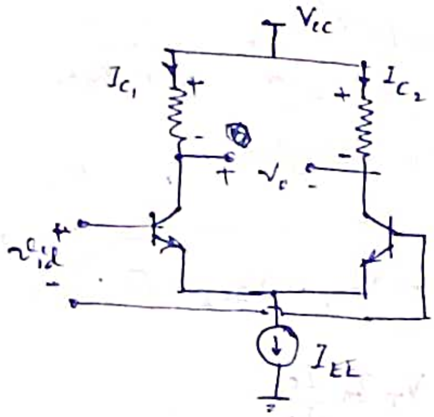
$$= I_{EE} . \frac{e^{+\frac{v_{id}}{2V_T}} - e^{-\frac{v_{id}}{2V_T}}}{e^{-\frac{v_{id}}{2V_T}} + e^{+\frac{v_{id}}{2V_T}}} = I_{EE} . \tanh \frac{v_{id}}{2V_T}$$

if $v_{id} \gg V_T$, $e^{\frac{v_{id}}{V_T}} \gg 1$,

$\tanh \frac{v_{id}}{2V_T} \longrightarrow 1$

$\|^{ly}$, if $v_{id} \ll V_T$, $\tanh \frac{v_{id}}{2V_T} \to -1$

Complete ckt.

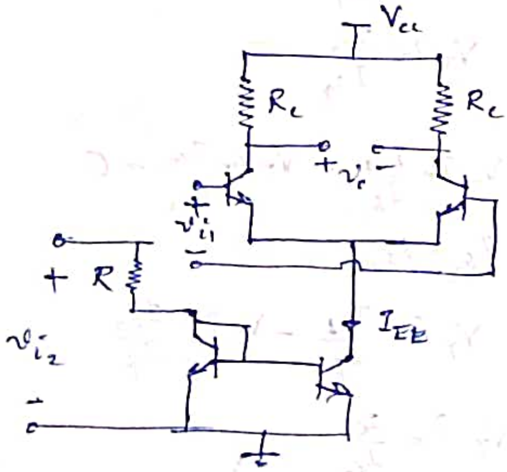


$v_o + I_{c_1}R_C - I_{c_2}R_c = 0$

$\Rightarrow v_o = -(I_{c_1} - I_{c_2})R_c$

$= -\Delta I_c R_c$

$= -I_{EE} \cdot \tanh \frac{v_{id}}{2V_T} \cdot R_c$



$I_{EE} = \frac{v_{i2} - V_{BE,ON}}{R}$

$\Rightarrow v_o = -\frac{R_c}{R} \cdot (v_{i2} - v_{BE,ON}) \tanh \frac{v_{id}}{2V_T}$

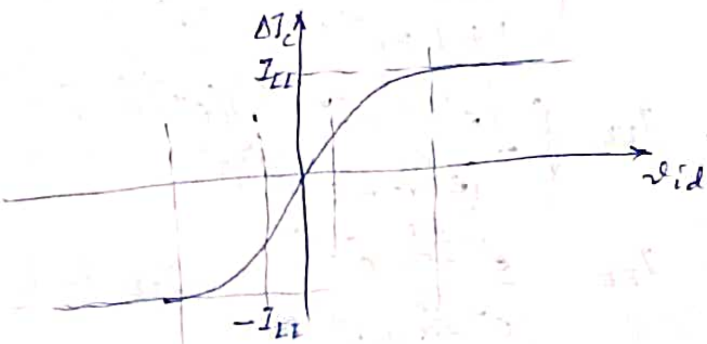
$\simeq -\frac{R_c}{R} \cdot (v_{i2} - v_{BE,ON}) \frac{v_{id}}{2V_T}$

$= \left(-\frac{R_c}{R} \times \frac{1}{2V_T}\right) \cdot v_{i_1} (v_{i2} - v_{BE,ON})$

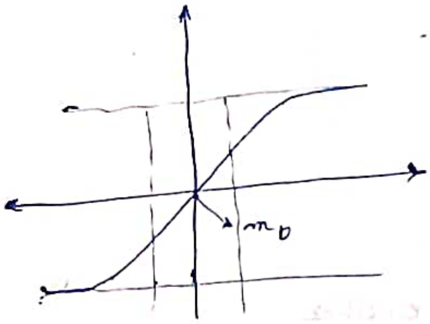
$= K_0 \cdot v_{i_1} (v_{i2} - v_{BE,ON})$

if $v_{i2} < v_{BE,ON}$, I_{EE} becomes zero.

slope of ΔI_c vs v_{id}



P for the 2 quadrant multiplier,, find the magnitude of the dc differential voltage required to cause the slope of the transfer curve (ΔI_c vs v_{id}) to change by 1% from that of the origin.



$$\Delta I_c = I_{EE} \cdot \frac{e^{\frac{v_{id}}{V_T}}-1}{e^{\frac{v_{id}}{V_T}}+1}$$

$$\therefore \frac{d\Delta I_c}{dv_{id}} = I_{EE}\left\{ \frac{e^{\frac{v_{id}}{V_T}}}{V_T\left(e^{\frac{v_{id}}{V_T}}+1\right)} - \frac{\left(e^{\frac{v_{id}}{V_T}}-1\right)}{\left(e^{\frac{v_{id}}{V_T}}+1\right)^2}\times\frac{e^{\frac{v_{id}}{V_T}}}{V_T}\right\}$$

$$= \frac{I_{EE}}{V_T}\left\{\frac{e^{\frac{v_{id}}{V_T}}}{e^{\frac{v_{id}}{V_T}}+1} - \frac{e^{\frac{v_{id}}{V_T}}\left(e^{\frac{v_{id}}{V_T}}-1\right)}{\left(e^{\frac{v_{id}}{V_T}}+1\right)^2}\right\}$$

$$= \frac{I_{EE}}{V_T}\cdot\left\{\frac{e^{\frac{v_{id}}{V_T}}\left(1+e^{\frac{v_{id}}{V_T}}\right) - e^{\frac{v_{id}}{V_T}}\left(e^{\frac{v_{id}}{V_T}}-1\right)}{\left(e^{\frac{v_{id}}{V_T}}+1\right)^2}\right\}$$

$$= \frac{I_{EE}}{V_T}\cdot\left\{\frac{e^{\frac{v_{id}}{V_T}}+\cancel{e^{\frac{2v_{id}}{V_T}}}-\cancel{e^{\frac{2v_{id}}{V_T}}}+e^{\frac{v_{id}}{V_T}}}{\left(1+e^{\frac{v_{id}}{V_T}}\right)^2}\right\}$$

$$= \frac{I_{EE}}{V_T}\frac{2e^{\frac{v_{id}}{V_T}}}{\left(1+e^{\frac{v_{id}}{V_T}}\right)^2}$$

$$\therefore m_0 = \left.\frac{d\Delta I_c}{dv_{id}}\right|_{v_{id}=0} = \frac{I_{EE}}{2V_T}$$

now, $m_1 - m_0 = -0.01 m_0 \Rightarrow m_1 = \cancel{1.01 m_0}\ 0.99 m_0$

$$\therefore \frac{\cancel{I_{EE}}\cdot 2e^{\frac{v_{id}}{V_T}}}{V_T\left(1+e^{\frac{v_{id}}{V_T}}\right)^2} = \overset{0.99}{\cancel{1.01}}\times\frac{\cancel{I_{EE}}}{2V_T}$$

Let, $e^{\frac{v_{id}}{V_T}} = x$

$$\therefore \frac{2x}{(1+x)^2} = \frac{\cancel{1.01}\ 0.99}{2} \Rightarrow 4x = 1.01\{x^2+2x+1\}$$

$\Rightarrow 1.01x^2+2.02x+1.01-4x=0$

$\Rightarrow 1.01x^2-1.98x+1.01=0$

$\Rightarrow x^2-1.96x+1=0$

$\Rightarrow x = \dfrac{1.96 \pm \sqrt{1.96^2 - 4}}{}$

$\Rightarrow x = \dfrac{2.04 \pm \sqrt{4.16 - 4}}{2}$

$= 1.22$ or 0.82

$e^{\frac{v_{id}}{V_T}} = 1.22$

$\Rightarrow \dfrac{v_{id}}{V_T} = \pm 0.2 V_T$

P. a sinusoidal differential voltage is applied to a emitter coupled pair.

find the maximum allowable v_{id}, such that 3rd harmonic at the output is $\leq 1\%$ of the fundamental.

$\tanh x = x - \dfrac{x^3}{3} \ldots$

$\Delta I_C = I_{EE} \cdot \tanh\left(\dfrac{v_{id}}{2V_T}\right)$

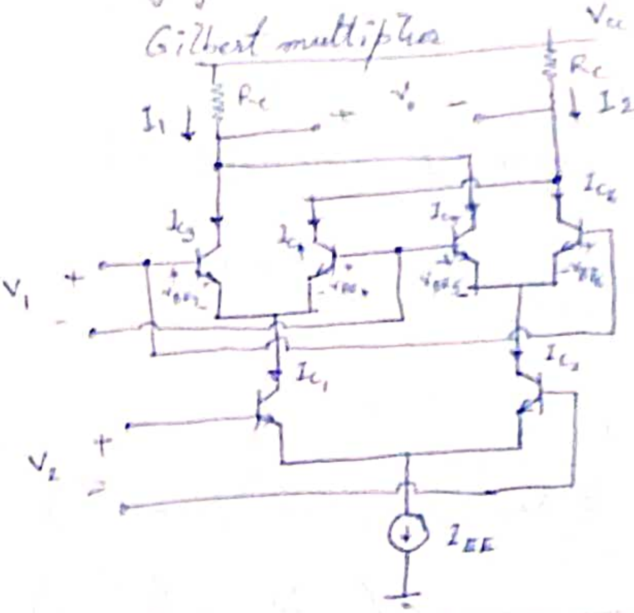
$= I_{EE} \cdot \left\{ \dfrac{v_{id}}{2V_T} - \dfrac{1}{3}\left(\dfrac{v_{id}}{V_T}\right)^3 \right\}$

Let, $v_{id} = V_m \sin \omega_0 t$

$\Delta I_C = \dfrac{I_{EE}}{2V_T}\left[V_m \sin \omega_0 t - \dfrac{V_m^3}{48V_T^2}(3\sin \omega_0 t - \sin 3\omega_0 t)\right]$

Remedy of 2 qdrnt multiplier:

Gilbert multiplier



$$V_1 - V_{BE_3} + V_{BE_4} = 0$$

$$\Rightarrow V_1 = V_T \ln \frac{I_{c_3}}{I_S} - V_T \ln \frac{I_{c_4}}{I_S}$$

$$= V_T \ln \frac{I_{c_3}}{I_{c_4}}$$

$$\Rightarrow \frac{I_{c_3}}{I_{c_4}} = e^{\frac{V_1}{V_T}}$$

$$\& \; I_{c_3} + I_{c_4} = I_{c_1}$$

$$\Rightarrow I_{c_3} + I_{c_3} e^{-\frac{V_1}{V_T}} = I_{c_1}$$

$$\Rightarrow I_{c_3} = \frac{I_{c_1}}{1 + e^{-\frac{V_1}{V_T}}}$$

$$I_{c_4} = \frac{I_{c_1}}{1 + e^{\frac{V_1}{V_T}}}$$

also,

$$-V_1 - V_{BE_5} + V_{BE_6} = 0$$

$$\Rightarrow V_1 = V_{BE_6} - V_{BE_5}$$

$$= V_T \ln \frac{I_{c_6}}{I_{c_5}} \Rightarrow I_{c_6} = I_{c_5} e^{\frac{V_1}{V_T}}$$

$$\& \; I_{c_6} + I_{c_5} = I_{c_2} \Rightarrow I_{c_5} = \frac{I_{c_2}}{1 + e^{\frac{V_1}{V_T}}}$$

$$I_{c_6} = \frac{I_{c_2}}{1 + e^{-\frac{V_1}{V_T}}}$$

$$I_{c_1} = \frac{I_{EE}}{1 + e^{-\frac{V_2}{V_T}}}, \quad I_{c_2} = \frac{I_{EE}}{1 + e^{\frac{V_2}{V_T}}}$$

$$I_{c_3} = \frac{I_{EE}}{\left(1 + e^{-\frac{V_2}{V_T}}\right)\left(1 + e^{-\frac{V_1}{V_T}}\right)}, \quad I_{c_4} = \frac{I_{EE}}{\left(1 + e^{-\frac{V_2}{V_T}}\right)\left(1 + e^{\frac{V_1}{V_T}}\right)}$$

$$I_{c_5} = \frac{I_{EE}}{\left(1 + e^{\frac{V_1}{V_T}}\right)\left(1 + e^{\frac{V_2}{V_T}}\right)}, \quad I_{c_6} = \frac{I_{EE}}{\left(1 + e^{\frac{V_2}{V_T}}\right)\left(1 + e^{-\frac{V_1}{V_T}}\right)}$$

$$\therefore I_1 = I_{c_3} + I_{c_5} = \frac{I_{EE}}{\left(1 + e^{-\frac{V_2}{V_T}}\right)\left(1 + e^{-\frac{V_1}{V_T}}\right)} + \frac{I_{EE}}{\left(1 + e^{\frac{V_2}{V_T}}\right)\left(1 + e^{\frac{V_1}{V_T}}\right)}$$

$$\Rightarrow \quad I_1 = I_{EE} \times \left\{ \frac{1}{\left(1+e^{-\frac{V_2}{V_T}}\right)\left(1+e^{-\frac{V_1}{V_T}}\right)} + \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)\left(1+e^{\frac{V_1}{V_T}}\right)} \right\}$$

$$= I_{EE} \times \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)\left(1+e^{\frac{V_1}{V_T}}\right)} \times \left(e^{\frac{V_2}{V_T}} \cdot e^{\frac{V_1}{V_T}} + 1\right)$$

$||^{ly}$, $$I_2 = I_{EE} \times \left\{ \frac{1}{\left(1+e^{-\frac{V_2}{V_T}}\right)\left(1+e^{\frac{V_1}{V_T}}\right)} + \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)\left(1+e^{-\frac{V_1}{V_T}}\right)} \right\}$$

$$= I_{EE} \times \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)\left(1+e^{-\frac{V_1}{V_T}}\right)} \times \left(e^{\frac{V_2}{V_T}} \cdot e^{-\frac{V_1}{V_T}} + 1\right)$$

$$\Rightarrow \quad \Delta I = I_1 - I_2 =$$

$$I_{EE} \times \left\{ \frac{\left(e^{\frac{V_2}{V_T}} \cdot e^{\frac{V_1}{V_T}} + 1\right)}{\left(1+e^{\frac{V_2}{V_T}}\right)\left(1+e^{\frac{V_1}{V_T}}\right)} - \frac{\left(e^{\frac{V_2}{V_T}} \cdot e^{-\frac{V_1}{V_T}} + 1\right)}{\left(1+e^{\frac{V_2}{V_T}}\right)\left(1+e^{-\frac{V_1}{V_T}}\right)} \right\}$$

$$= I_{EE} \times \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)} \left\{ \frac{\left(e^{\frac{V_2}{V_T}} \cdot e^{\frac{V_1}{V_T}} + 1\right)\left(1+e^{-\frac{V_1}{V_T}}\right) - \left(e^{\frac{V_2}{V_T}} \cdot e^{-\frac{V_1}{V_T}} + 1\right)\left(1+e^{\frac{V_1}{V_T}}\right)}{\left(1+e^{\frac{V_1}{V_T}}\right)\left(1+e^{-\frac{V_1}{V_T}}\right)} \right\}$$

$$= I_{EE} \times \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)} \left(\frac{\cancel{e^{\frac{V_2}{V_T}}} + \cancel{1} + e^{-\frac{V_1}{V_T}} + e^{\frac{V_2}{V_T}} \cdot e^{\frac{V_1}{V_T}} - \left(\cancel{e^{\frac{V_2}{V_T}}} + \cancel{1} + e^{\frac{V_1}{V_T}} + e^{\frac{V_2}{V_T}} \cdot e^{-\frac{V_1}{V_T}}\right)}{\left(e^{\frac{V_1}{2V_T}} + e^{-\frac{V_1}{2V_T}}\right)^2} \right)$$

$$= I_{EE} \times \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)} \times \frac{e^{\frac{V_2}{V_T}} \cdot e^{\frac{V_1}{V_T}} + \left(e^{-\frac{V_1}{V_T}} - e^{\frac{V_1}{V_T}}\right) - e^{\frac{V_2}{V_T}} \cdot e^{-\frac{V_1}{V_T}}}{\left(e^{\frac{V_1}{2V_T}} + e^{-\frac{V_1}{2V_T}}\right)^2}$$

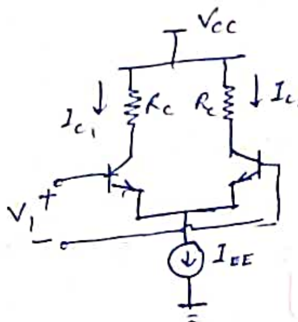
$$= I_{EE} \times \frac{1}{\left(1+e^{\frac{V_2}{V_T}}\right)} \times \frac{\left(e^{-\frac{V_1}{V_T}} - e^{\frac{V_1}{V_T}}\right)\cancel{\left(e^{\frac{V_2}{V_T}} + 1\right)}\left(1 - e^{\frac{V_2}{V_T}}\right)}{\left(e^{\frac{V_1}{2V_T}} + e^{-\frac{V_1}{2V_T}}\right)^2}$$

$$= I_{EE} \cdot \frac{\left(1-e^{\frac{V_2}{V_T}}\right)}{\left(1+e^{\frac{V_2}{V_T}}\right)} \times \frac{\left(e^{-\frac{V_1}{V_T}}-e^{\frac{V_1}{V_T}}\right)}{\left(e^{\frac{V_1}{2V_T}}+e^{-\frac{V_1}{2V_T}}\right)^2}$$

$$= I_{EE} \cdot \frac{e^{\frac{V_2}{2V_T}}\left(e^{-\frac{V_2}{2V_T}}-e^{\frac{V_2}{2V_T}}\right)}{e^{\frac{V_2}{2V_T}}\left(e^{-\frac{V_2}{2V_T}}+e^{\frac{V_2}{2V_T}}\right)} \times \frac{e^{-\frac{V_1}{2V_T}}-e^{\frac{V_1}{2V_T}}}{\left(e^{\frac{V_1}{2V_T}}+e^{-\frac{V_1}{2V_T}}\right)}$$

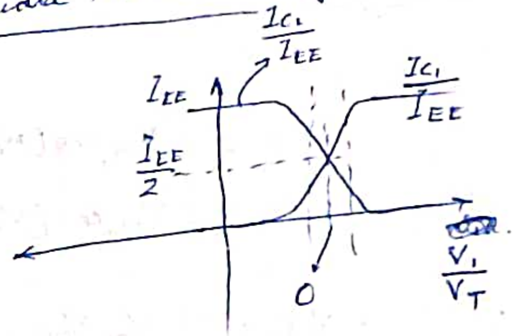
$$= I_{EE} \cdot \tanh\left(\frac{V_1}{2V_T}\right) \tanh\left(\frac{V_2}{2V_T}\right).$$

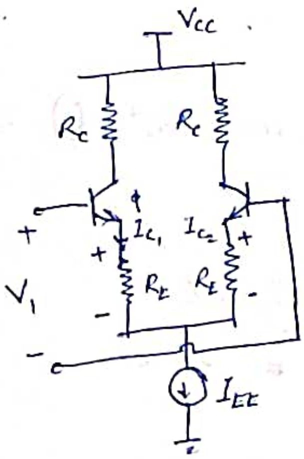
Consider a differential amplifier: how to increase the linear range



$$I_{C_1} = \frac{I_{EE}}{1+e^{-\frac{V_1}{V_T}}}$$

$$I_{C_2} = \frac{I_{EE}}{1+e^{\frac{V_1}{V_T}}}.$$





$$V_1 - V_{BE_1} - I_{C_1} \cdot R_E + I_{C_2} R_E + V_{BE_2} = 0$$

$$\Rightarrow V_1 = V_{BE_1} - V_{BE_2} + (I_{C_1} - I_{C_2}) R_E$$

$$= V_T \ln \frac{I_{C_1}}{I_{C_2}} + \Delta I_C \cdot R_E.$$

& $I_{C_1} + I_{C_2} = I_{EE}$

$$V_T \ln \frac{I_{C_1}}{I_{C_2}} = V_1 - \Delta I_C \cdot R_E$$

$$\Rightarrow \frac{I_{C_1}}{I_{C_2}} = e^{\frac{V_1 - \Delta I_C R_E}{V_T}}$$

$$\Rightarrow I_{C_1} + I_{C_1} \cdot e^{-\frac{V_1 - \Delta I_C R_E}{V_T}} = I_{EE}$$

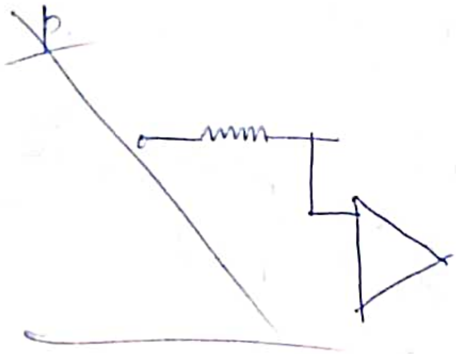
$$\Rightarrow I_{C_1} = \frac{I_{EE}}{1+e^{-\frac{V_1}{V_T}} e^{\frac{\Delta I_C R_E}{V_T}}}$$

$$I_{C_2} = \frac{I_{EE}}{1+e^{\frac{V_1}{V_T}} e^{-\frac{\Delta I_C R_E}{V_T}}}$$

$\Delta I_C = I_{EE} \times$

if $I_{C_1} \simeq I_{EE}$, $\quad V_1 = V_T \ln$

$$I_{C_1} = \frac{I_{EE}}{1+e^{-\frac{(V_1 - I_{EE} R_E)}{V_T}}}.$$



$$\tanh x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}.$$

$$\therefore \cancel{\tanh} = \frac{e^{2x}-1}{e^{2x}+1}$$

$$\cancel{e^{2}}$$

<u>multiplier as phase detector</u>:

$$v_1 = V_{m_1}\cos(\omega_c t)$$

$$v_2 = V_{m_2}\cos(\omega_c t+\theta)$$

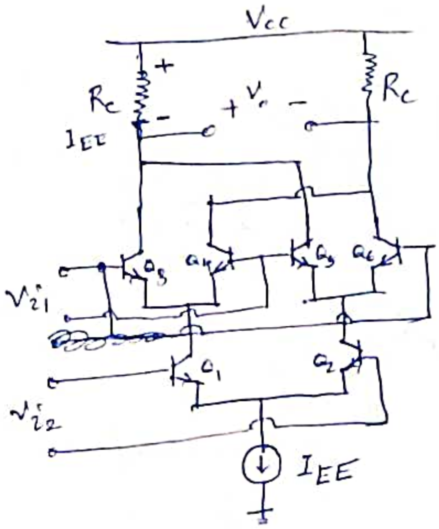
$$\Rightarrow v_1 v_2 = \frac{V_{m_1}V_{m_2}}{2}\cdot\left\{\cos(\omega_c t+\omega_c t+\theta) + \cos\theta\right\}.$$

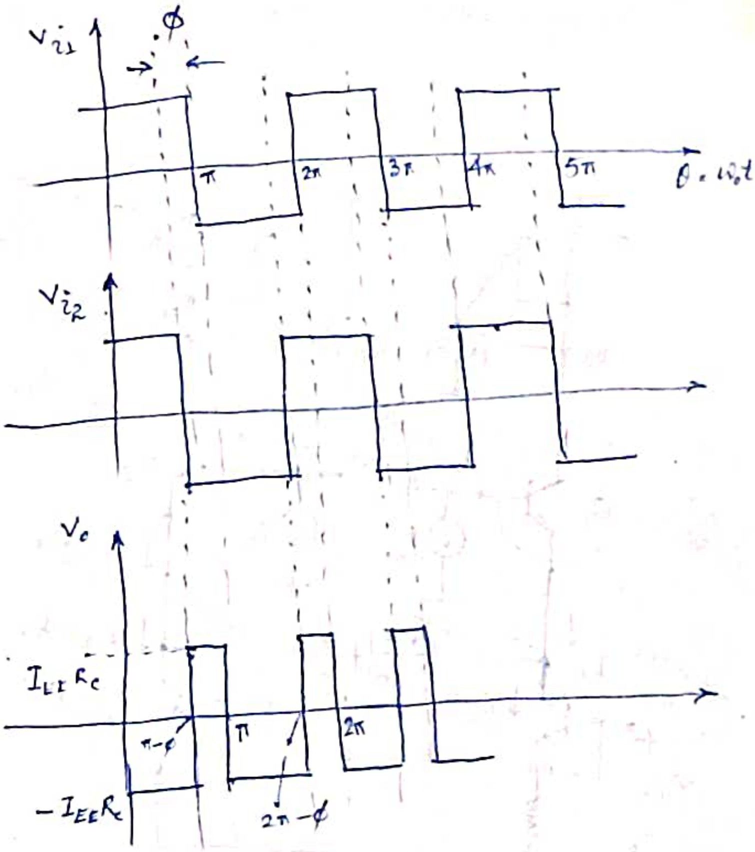
phase difference: θ.

$$\Rightarrow v_1 . v_2 = \underbrace{\frac{V_{m_1}V_{m_2}}{2}.\cos\theta}_{\text{dc component}} + \frac{V_{m_1}V_{m_2}}{2}.\cos(2\omega_c t+\theta)$$

If, the signals are large square wave signals, then, the output dc component will be a ~~direct~~ linearly related to the phase difference.

as in this case, transistors work as switches.





when V_{i_1}, V_{i_2} high,

Q_1, Q_3, Q_6 will be on.

as I_{EE} will not go through Q_2, ..

$Q_1, Q_3 \to ON$, $V_o = -I_{EE} \cdot R_c$.

during ϕ, $V_{i_1} \to$ high, $V_{i_2} \to$ low

$Q_2 \to ON$, $Q_6 \to ON$,

$\Rightarrow V_o = +I_{EE} \cdot R_c$.

when both $V_{i_1}, V_{i_2} \to$ low,

$Q_2 \to ON$, $Q_5 \to ON$,

$\Rightarrow V_o = -I_{EE} \cdot R_c$

when $V_{i_2} \to$ high,

$V_{i_1} \to$ low,

$Q_1 \to ON$, $Q_4 \to ON$,

$V_o = +I_{EE} \cdot R_c$

$$V_{avg} = V_{dc} = \frac{1}{2\pi} \int_0^{2\pi} V_o(t)\, d\theta$$

$$= \frac{1}{2\pi} \int_0^{\pi-\phi} (-I_{EE} \cdot R_c)\, d\theta + \int_{\pi-\phi}^{\pi} I_{EE} \cdot R_c\, d\theta + \int_{\pi}^{2\pi-\phi} (-I_{EE} \cdot R_c)\, d\theta + \int_{2\pi-\phi}^{2\pi} I_{EE} R_c\, d\theta$$

$$= \frac{1}{2\pi} I_{EE} \cdot R_c \left\{ (\phi - \pi) + \phi + (\phi - \pi) + \phi \right\}$$

$$= \frac{1}{2\pi} \cdot I_{EE} \cdot R_c \left\{ 4\phi - 2\pi \right\} = I_{EE} R_c \left\{ \frac{2\phi}{\pi} - 1 \right\}.$$