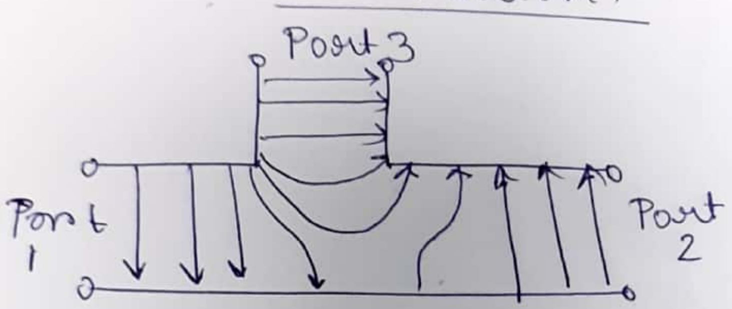
For <u>E-plane, Tee.</u>

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

<u>Derivation.</u>



→ Port 3 is the E-arm through which TE_{10} mode is made to propagate,

→ matched port with the source then reflection coefficient $S_{33} = 0$.

→ Since o/p ports 1 & 2 are out of phase by 180° with the input port 3,

$\therefore S_{23} = -S_{13}$

→ Now from symmetry property

$S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{23} = S_{32}$

finally S-matrix of a three port E-plane Tee becomes:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

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using Unitary Property,

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{--- a}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{--- b}$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{--- c}$$

from (c) $|S_{13}| = \frac{1}{\sqrt{2}}$ — d

$C_3 C_1^*$ Again $S_{13} S_{11}^* - S_{13} S_{12}^* = 0$ — e

$$\Rightarrow S_{13}(S_{11}^* - S_{12}^*) = 0$$

$$S_{13} \neq 0$$

$$\therefore S_{11}^* = S_{12}^* \longrightarrow f.$$

Comparing a & b

$S_{11} = S_{22}$

$$\Rightarrow S_{11} = S_{12}$$

$$\therefore S_{11} = S_{12} = S_{21} \quad \text{--- g}$$

now ~~for~~ substitute in (a)

$$|S_{11}|^2 + |S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$\Rightarrow 2|S_{11}|^2 = \frac{1}{2}$$

$$\Rightarrow S_{11} = \frac{1}{2}$$

$$\therefore [S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

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③

now $[b] = [S][a]$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\therefore b_1 = \frac{1}{2} a_1 + \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3$$

$$b_2 = \frac{1}{2} a_1 + \frac{1}{2} a_2 - \frac{1}{\sqrt{2}} a_3$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 - \frac{1}{\sqrt{2}} a_2$$

If $a_1 = a_2 = 0$ & $a_3 \neq 0$,
then $b_1 = \frac{1}{\sqrt{2}} a_3$ & $b_2 = -\frac{1}{\sqrt{2}} a_3$

$b_3 = 0$

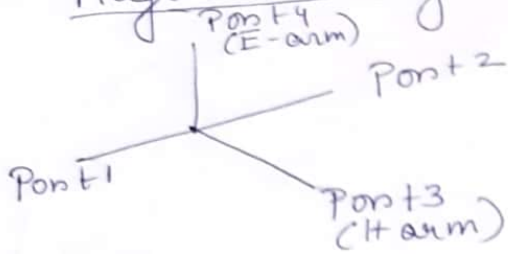
i.e. an i/p power in port 3 divides equally betn port 1 & 2 but 180° out of phase with each other. Thus E-plane Tee acts as a 3dB splitter.

Microwave Hybrid circuits:

Hybrid means mixture of two different types of things.

Eg: Hybrid T junction, Rat race coupler, branchline hybrid.

Magic Tee (Hybrid Tee Junction)



$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Port 1 & Port 2 are collinear arms.

For H-plane Tee,

$$S_{23} = S_{13}$$

For E-plane Tee.

$$S_{24} = -S_{14}$$

but i/p. of port 3 cannot come out through port 4;

$$S_{34} = S_{43} = 0$$

Now four symmetry property;

$S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{23} = S_{32} = S_{13}$

$S_{34} = S_{43}$, $S_{24} = S_{42} = -S_{14}$, $S_{41} = S_{14}$

As ports 3 & ports 4 are perfectly matched w.r.t. input sources; hence no reflection

i.e. $S_{33} = S_{44} = 0$

S-matrix of a magic tee can be written as.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

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(iii)

Using unitary property

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ —— a

$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ —— b

$|S_{13}|^2 + |S_{13}|^2 = 1$ —— c

$|S_{14}|^2 + |S_{14}|^2 = 1.$ —— d

from eqns (c) & (d)

$S_{13} = \frac{1}{\sqrt{2}} = S_{14}$ —— (e)

comparing a & b ; $S_{11} = S_{22}$ —— (f)

using the value of (e) & (f) in (a)

$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$

$\Rightarrow |S_{11}|^2 + |S_{12}|^2 = 0$, which implies

$S_{11} = S_{12} = 0$

Again from eqn (f) $S_{22} = 0$

then [S] matrix of a 4 port Magic Tee,

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4)$; $b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)$

$b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4)$; $b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)$

If $a_3 \neq 0$; $a_1 = a_2 = a_4 = 0$,

then $b_1 = \frac{a_3}{\sqrt{2}}$, $b_2 = \frac{a_3}{\sqrt{2}}$; $b_3 = b_4 = 0$

Thus when port 3 i/P, power will be divided equally in port 1 & 2.

In the second case when, port 4 i/P i.e. $a_4 \neq 0$;

$a_1 = a_2 = a_3 = 0$,

then $b_1 = \frac{a_4}{\sqrt{2}}$, $b_2 = -\frac{a_4}{\sqrt{2}}$

$b_3 = b_4 = 0$

When port 4 is i/P then power will be divided equally but in out of phase between ports 1 & 2.