Nuclear Physics

Text books

1. **I. Kaplan, Nuclear Physics, Addison-Wesley, 2002**
2. **K. S. Krane, Introductory Nuclear Physics, John Wiley, 1987**
3. **S.N. Ghoshal, Nuclear Physics, S. Chand, 2010.**

Atomic nuclei are made up of neutrons and protons.

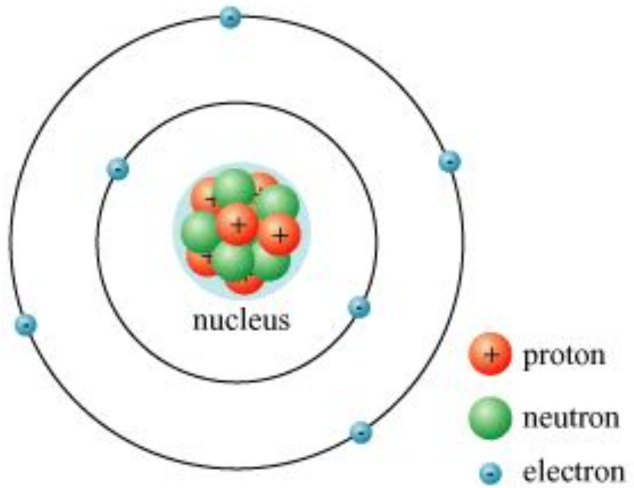
A nuclear species (nuclide) is characterized by the total amount of positive charge in the nucleus and by its total number of mass units.

General form of a nuclide(a specific nuclear species): ${}^{A}_{Z}X_{N}$

X: chemical symbol; Z: atomic number(or proton number)
N : neutron number; A: Mass number (or Number of nucleons)

Examples: ${}^{1}_{1}H_{0}$, ${}^{238}_{92}U_{146}$.

Since, N = A-Z, ${}^{238}U$ is also a valid way to indicate a nuclide.



Neutrons and protons are the two members of the family of **nucleons**. Thus a nucleus of mass number A contains A nucleons.

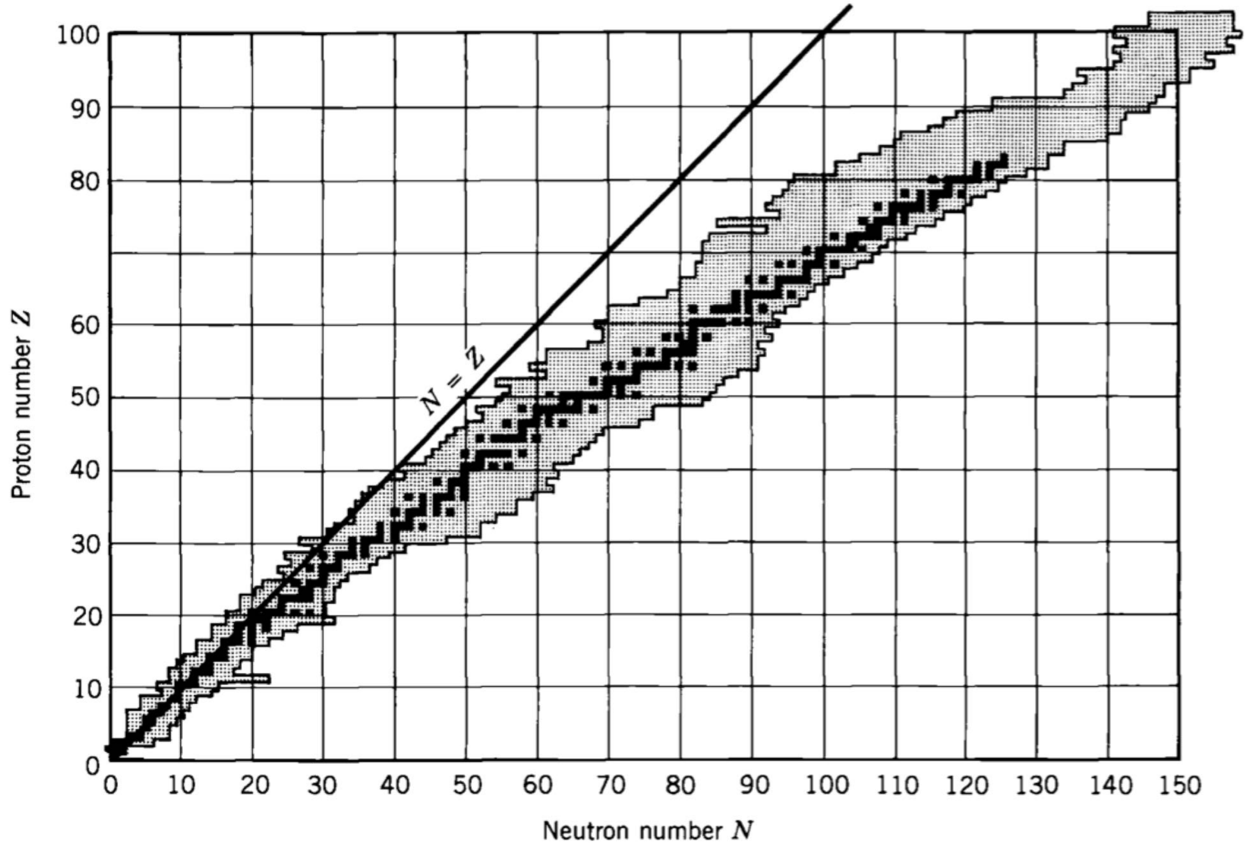
Nuclides with the same proton number (Z) but different neutron numbers (N) are called **isotopes.**

Ex: The element Cl has two stable isotopes: $^{35}\boldsymbol{Cl}$, $^{37}\boldsymbol{Cl}$

A sequence of nuclides with the same N but different Z are called **isotones.**
Ex: The stable isotones with N = 1 are $^{2}\boldsymbol{H}$, $^{3}\boldsymbol{He}$

Nuclides with the same mass number A are known as **isobars**.
Thus stable $^{3}\boldsymbol{He}$ and radioactive $^{3}\boldsymbol{He}$ are isobars.

Representation of stable and unstable(radioactive) nuclides



- Stable nuclei are shown in dark
- Known radioactive nuclei are in light shading

Units and dimensions

Length:

- In nuclear physics, lengths are of the order of $10^{-15}m$ = 1 femtometer (fm).
- 1 fm is also known an 1 fermi.
- Size range : From 1 fm for a single nucleon to about 7 fm for the heaviest nuclei.

- **Time:**

The Time scale of nuclear phenomena has an enormous range:

- Many nuclear reactions occur with a time scale of the order of $10^{-20}s$.
- Electromagnetic (γ) decays of nuclei occur within lifetimes of the order of $10^{-9}\ to 10^{-12}\ s$.
- Radioactive decays ($\alpha\ and\ \beta$) occur in minutes/hours/even millions of years.

Energy:

Nuclear energies are measured in millions of electron-volts (**MeV**).

- β and γ decay energies are in the range of 1 MeV.
- Low-energy nuclear reactions take place with kinetic energies of order 10 MeV.

- ## Mass:

- Nuclear masses are measured in terms of the unified atomic mass unit (a.m.u), u.

It is defined such that,

$$\mathbf{1\ u = \frac{1}{12}\ x\ mass\ of\ an\ atom\ of\ {}^{12}C} \quad .$$

$$= \frac{1}{12} \times 1.998467052 \times 10^{-26}\ kg \ = 1.66053886 \times 10^{-27}\ kg$$

In analyzing nuclear decays and reactions, we generally work with mass energies rather than with masses. The conversion of mass to energy is done using $E = m\,c^2$.

1 u = 931.502 MeV. (conversion factor)

In these units, $$\boldsymbol{c^2 = 9.314940 \times 10^8\ eV/u}$$

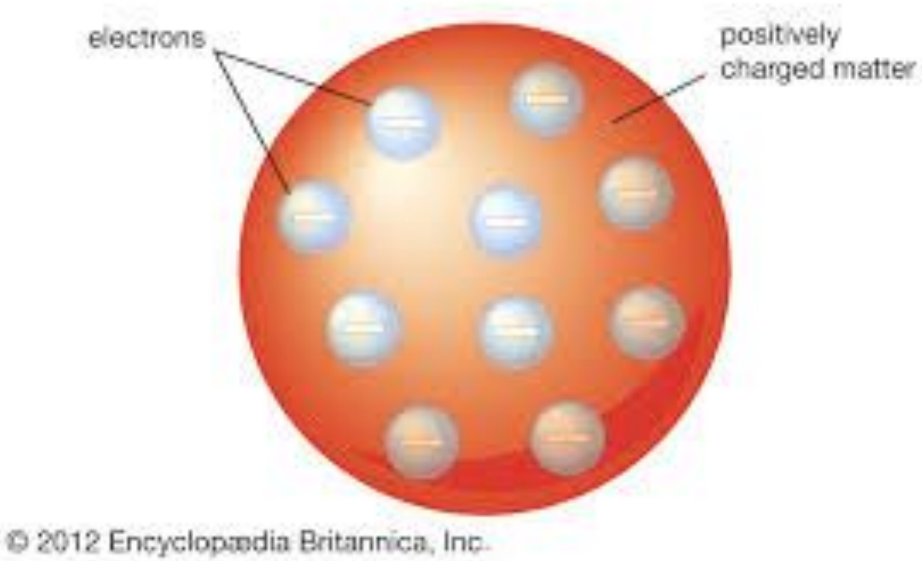
$$\boldsymbol{= 931.4940\ MeV/u}$$

Nucleons have masses of approximately 1 u.

∴ mass energies of nucleons ~ 1000 MeV

Particle	Mass (kg)	Mass (u)	Mass (MeV/c^2)
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	1.6750×10^{-27}	1.008665	939.57
Electron	9.1095×10^{-31}	5.486×10^{-4}	0.511
${}^{1}_{1}$H atom	1.6736×10^{-27}	1.007825	938.79

Plum pudding model of the atom by J.J. Thomson



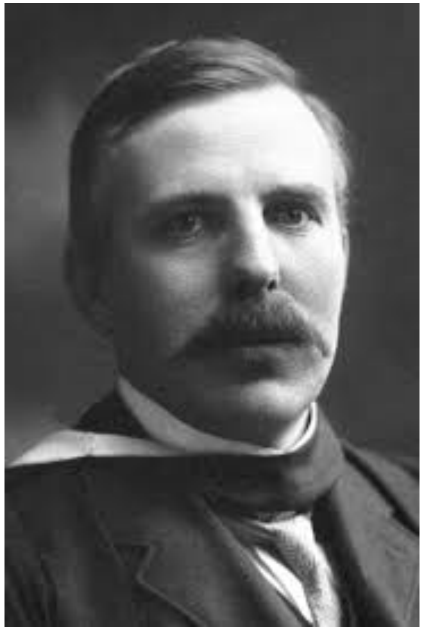
According to this model,

the electrons (the “plums”) were thought to vibrate about fixed points within this sphere of positive charge (the “pudding”)

Discovery of the Nucleus

Nobel Prize in Chemistry in 1908

“for his investigations into the disintegration of
The elements and the chemistry of radioactive substances”



Ernest Rutherford

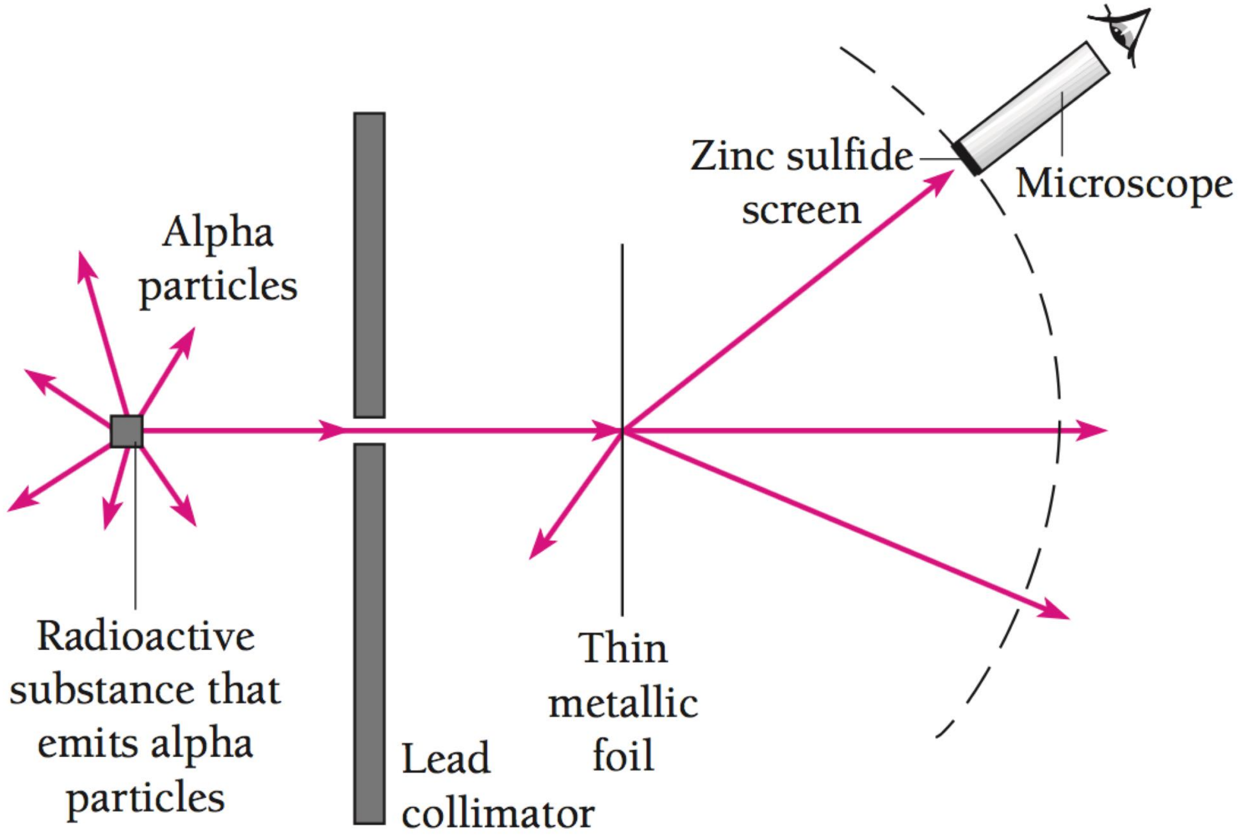
Rutherford's Scattering Experiment

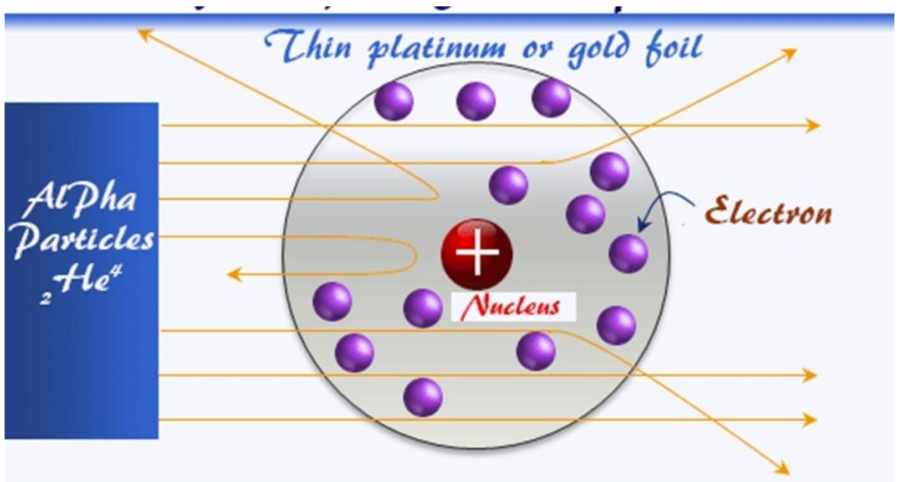
❖ In 1911 Ernest Rutherford proposed to study the scattering of energetic alpha (α) particles– ($^4\boldsymbol{He}$ Nuclei with charge +2e), by thin metal foils.

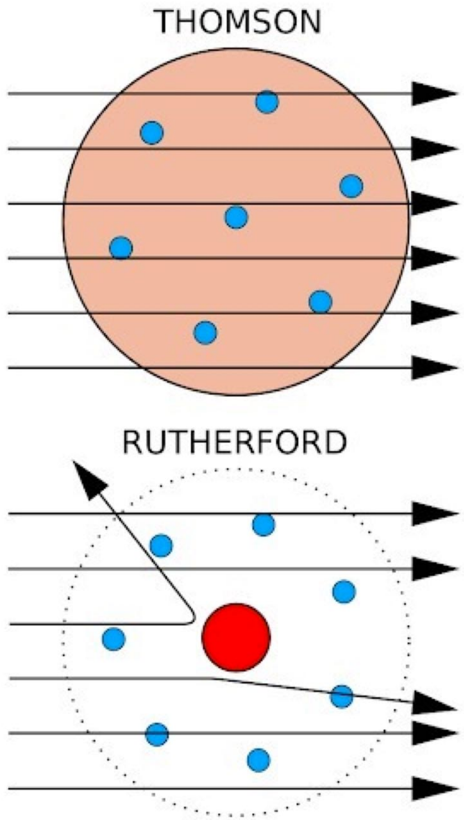
❖ **The alpha source:** Radon gas – a decay product of radium.

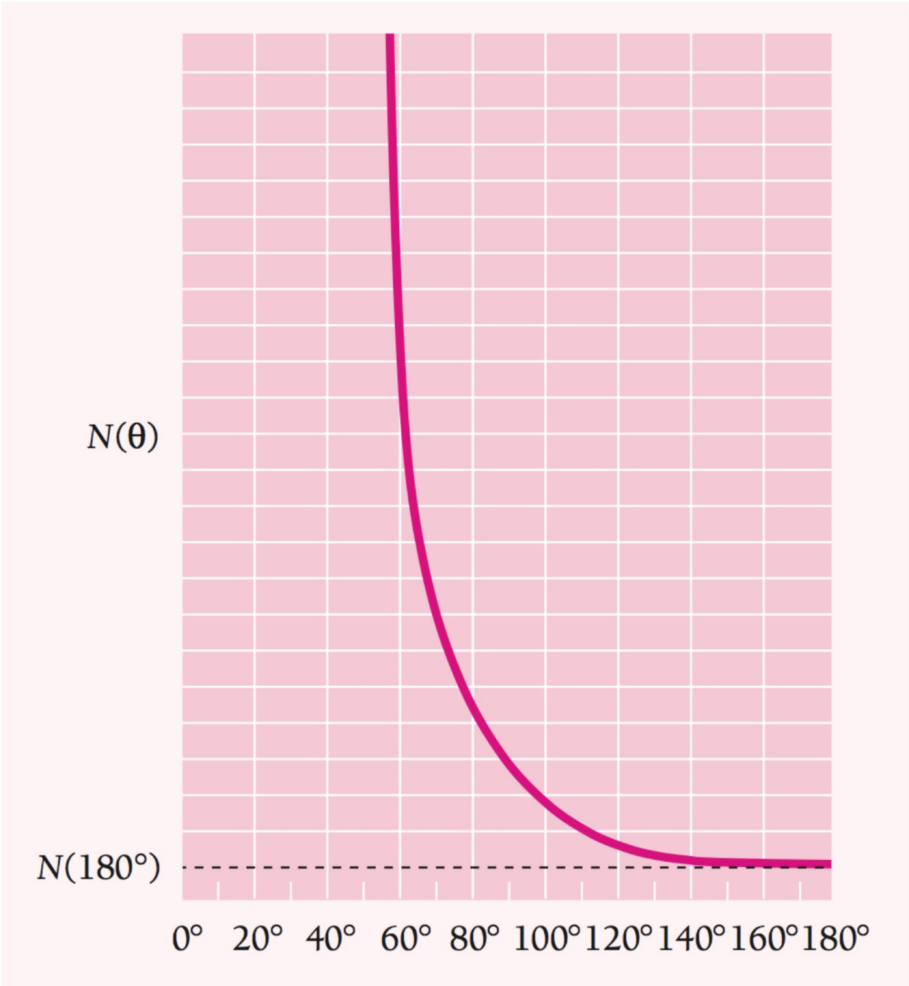
Radon gas emits α particles with an energy of about 5.5 MeV

α particles are relatively heavy (almost 8000 electron masses) and had high speeds of about $2 \times 10^7 m/s$.




Thin platinum or gold foil
AlPha Particles $_2He^4$
Electron
Nucleus


THOMSON
RUTHERFORD



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Nuclear Dimensions

Rutherford scattering gives a way to find an upper limit to nuclear dimensions.

An α particle will have its smallest R when it approaches a nucleus head on, which will be followed by a $180°$ scattering.

At the instant of closest approach, the initial KE is entirely converted to electric PE:

$$KE_{initial} = PE = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{R}$$ (charge of α particle : 2e & that of nucleus: Ze)

$$R = \frac{2Ze^2}{4\pi\varepsilon_0 KE_{initial}}$$ **"distance of closest approach"**

The maximum KE found in α particles of natural origin is 7.7 MeV = $1.2 \times 10^{-12} J$.

$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 N.m^2/C^2$, atomic number of gold (foil material) $Z = 79$

$$R(Au) = 3.0 \times 10^{-14} m$$

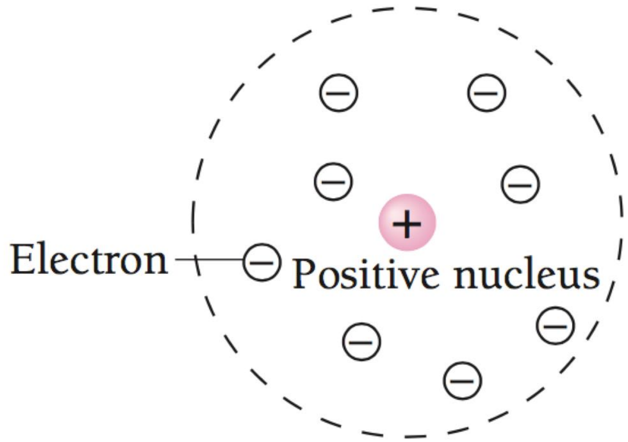
The radius of gold nucleus is therefore less than $3.0 \times 10^{-14} m$, well under 10^{-4}, the radius of the atom as a whole.

In 1911, Ernest Rutherford proposed that

- the positive charge of the atom is densely concentrated at the
center of the atom, forming its **nucleus**

- The nucleus is responsible for most of the mass of the atom.

- The radius of the nucleus must be smaller than the radius of an atom by a factor of about 10^4.

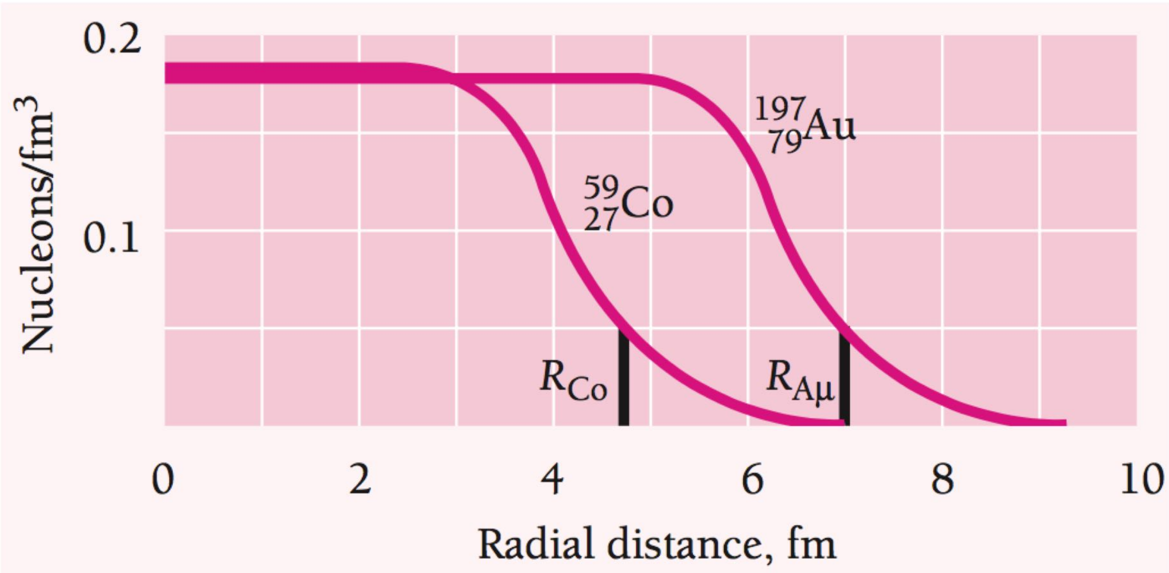


Rutherford's model of atom

General properties of Nucleus

Shape of the nucleus: From scattering experiments, it is observed that the shape of the nucleus is nearly spherical.

Radius of the nucleus



Radial distribution of density of nucleons

From scattering experiments it is found that volume of nucleus ∝ A

$$\frac{A}{\frac{4}{3}\pi R^3} \sim constant$$

(R; mean nuclear radius)

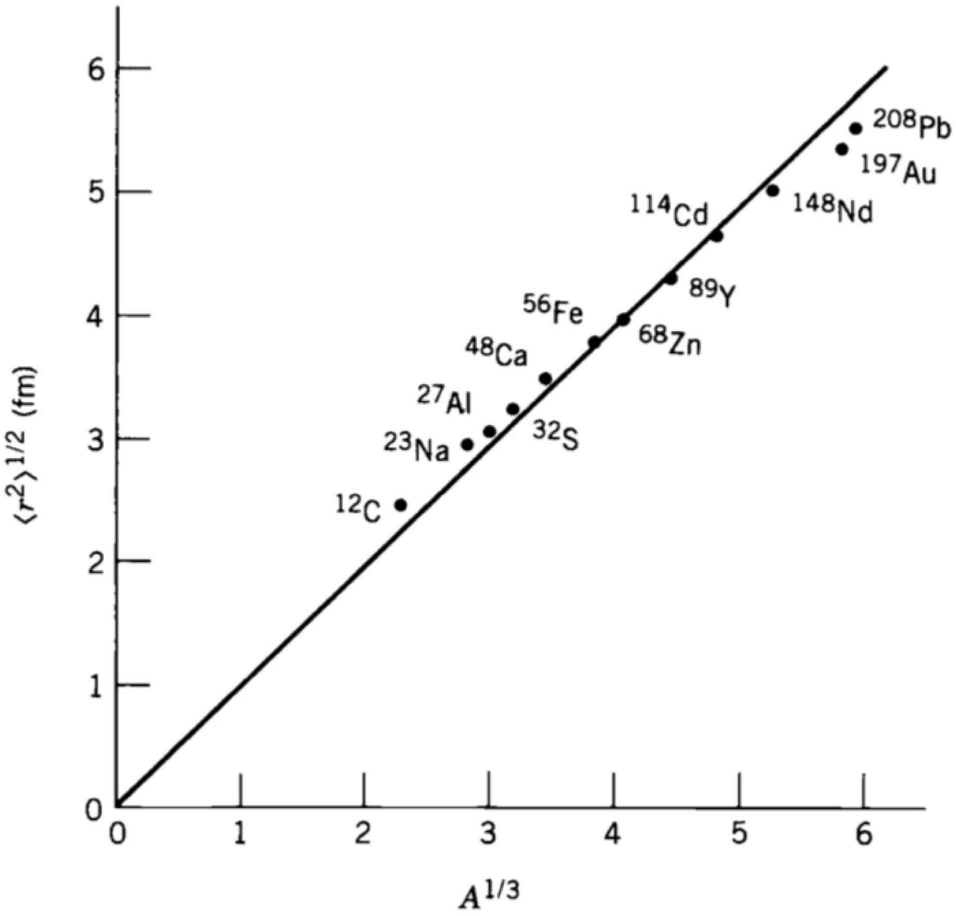
Thus $R \, \alpha \, A^{1/3}$

Defining proportionality constant

$$R = R_0 \, A^{1/3}$$

The density of nucleons is very nearly the same in the interiors of all nuclei.

RMS nuclear radius



- ❑ The slope of the straight line gives $R_0 \approx 1.23\ fm$
- ❑ The values of R_0 usually range from $1.0 \times 10^{-15}\ m\ to\ 1.5 \times 10^{-15}$ m)

Nuclear charge, spin & magnetic moment

The net nuclear charge of a nucleus with Z protons is $+Ze$.

Protons and neutrons, like electrons, are fermions with spin quantum numbers of $s = \frac{1}{2}$.

∴ Magnitude of spin angular momenta

$$S = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\,\hbar = \frac{\sqrt{3}}{2}\,\hbar$$

Spin magnetic quantum numbers :

$$m_s = \pm\frac{1}{2}$$

Nuclear spin & magnetic moment

Magnetic moments are associated with the spins of protons and neutrons. In nuclear physics, magnetic moments are expressed in **nuclear magnetons(μ_N):**

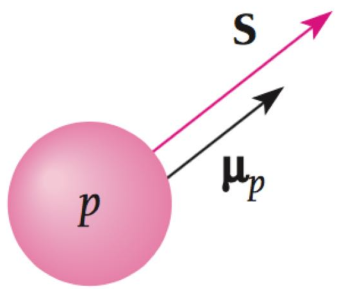
$$\mu_N = \frac{e\hbar}{2m_p} = 5.051 \times 10^{-27}\, J/T \qquad (m_p\text{: proton mass})$$

$$= 3.152 \times 10^{-8}\, eV/T$$

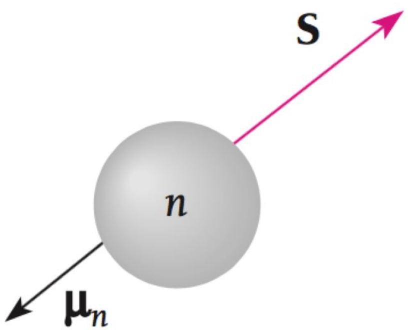
The nuclear magneton is smaller than the Bohr magneton by the ratio of the proton mass to the electron mass (which is 1836).

The z-component of spin magnetic moment of proton:

$$\mu_{pz} = \pm 2.793\ \mu_N$$



The spin magnetic moment μ_p of the proton is in the same direction as its spin angular momentum **S.** Hence $\pm$ sign is used for μ_{pz}.



The z-component of spin magnetic moment:

$$\mu_{nz} = \mp 1.913\ \mu_N$$

The spin magnetic moment μ_n of the neutron opposite to its spin angular momentum **S.** Hence $\mp$ sign is used for μ_{nz}.

Depending on whether m_s is $-\frac{1}{2}$ or $+\frac{1}{2}$ there are two possibilities for the signs of μ_{pz} and μ_{nz}.

The magnetic moment of a nucleus $\boldsymbol{\mu_I} \propto$ Total angular momentum of nucleus $\boldsymbol{I}$

It can be represented as

$$\boldsymbol{\mu_I = \gamma_I \frac{h}{2\pi} I}$$

$$\boldsymbol{= g_I \mu_N I}$$

Where γ_I : nuclear gyromagnetic ratio; g_I:nuclear g-factor; I: Total angular momentum of nucleus

Angular momentum

Each nucleon may be pictured as having an angular momentum associated with orbital motion within the nucleus.

For a single nucleon:

Orbital angular momentum (l) is a vector whose greatest possible component in any given direction is an integral multiple of $h/2\pi$.

Spin contributes $\pm\frac{1}{2}h/2\pi$

Total angular moment i about a given direction, with $\boldsymbol{i = l \pm s}$

$\therefore$ $\boldsymbol{i}$ is half-integral

For a nucleus with more than one particle:

Resultant total angular momentum of the nucleus is

$$\boldsymbol{I} = \boldsymbol{L} \pm \boldsymbol{S}$$

(L: Total orbital angular momentum; S: Total spin angular momentum)

L is an integral multiple of $h/2\pi$
For nucleus with
(i) even number of nucleons: S is an even half-integral multiple of $h/2\pi$
(ii) odd number of nucleons: S is an odd half-integral multiple of $h/2\pi$

∴ **Even-A nuclei:** $\boldsymbol{I}$ **= integral multiple of** $\boldsymbol{h/2\pi}$
odd-A nuclei: $\boldsymbol{I}$ **= an odd half-integral multiple of** $\boldsymbol{h/2\pi}$

PARITY, π

The nucleus can be considered like *a quantum-mechanical system.* Parity is an important wave-mechanical property of nucleus.

A nucleon is described by a function of its three space coordinates and the value of its spin.

To a good approximation,

Wave function of a nucleus = function of the space coordinate x function of spin orientation

Consider the transformation of the space coordinates from (x, y, z) to $(-x, -y, -z)$.

This transformation of coordinates is equivalent to a reflection of the nucleus'position about the origin of the x, y, z system of axes.

- The parity of a WF representing a nuclear state is said to be *even or positive* if spatial part of a WF is unchanged when space coordinates (x,y,z) are replaced by (-x,-y,-z).

$$\psi\ (x,y,z) = +\psi\ (-x,-y,-z)$$

Symmetry behaviour of the wave function under reflection (in space)

- When reflection of nuclear WF changes the sign of the spatial part of the WF, the nucleus is said to be have *odd or negative* parity

$$\psi\ (x,y,z) = -\psi\ (-x,-y,-z)$$

Asymmetric behaviour of the wave function under reflection (in space)

PARITY

Relation between angular momentum (L) and parity:

$$Parity\ \pi = (-1)^L$$

- If L is odd, parity is odd.
- If L is even, parity is even.

Note: Selection rules for many nuclear transitions involves conditions on parity & L.

Nuclear Binding Energy

The missing energy that keeps a nucleus together

Mass of a nucleus:

- If a nucleus ${}^{A}_{Z}X$ consists of Z protons(${}_1H^1$ atom) and N neutrons.
- The mass of the nucleus would be just the sum of the masses of these constituent nucleons, i.e. ,

$$M(Z,N) = Z\,m({}^{1}_{1}H) + N\,m(n)$$

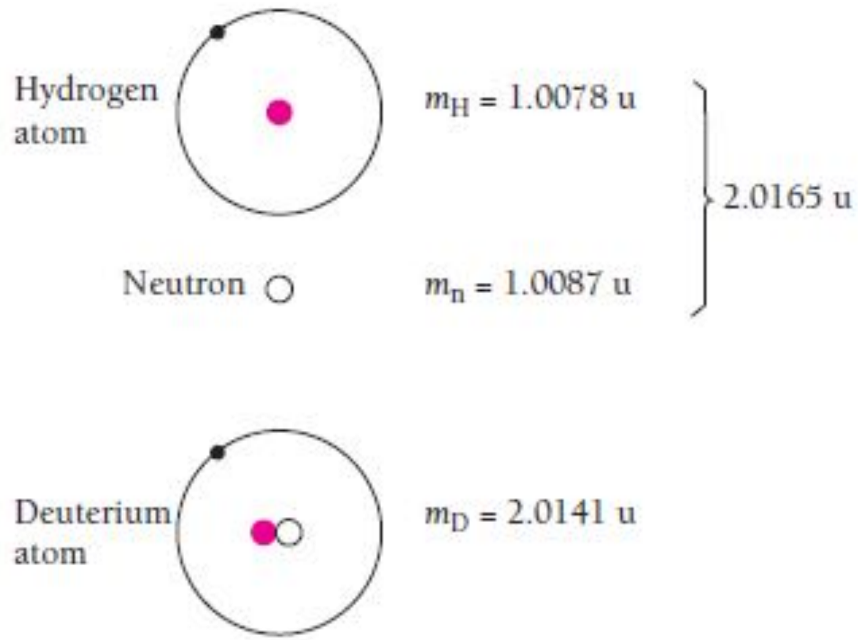
where $m({}^{1}_{1}H)$ is the atomic mass of ${}^{1}_{1}H$, $m(n)$ is the neutron mass, and $m\left({}^{A}_{Z}X\right)$ is atomic mass of ${}^{A}_{Z}X$, all in atomic mass units.

- A nucleus is not a simple collection of protons and neutrons (nucleons), but they strongly combine with each other through a strong interaction.

Missing mass:

- The hydrogen isotope deuterium, ${}_1H^2$, has a neutron as well as a proton(${}_1H^1$ atom) in its nucleus.

- The expected mass of the deuterium atom to be equal to that of an ordinary ${}_1H^1$ atom plus the mass of a neutron:



Mass of ${}_1^1H$ atom	1.007825 u
+ mass of neutron	+1.008665 u
Expected mass of ${}_1^2H$ atom	2.016490 u

Mass defect:

- The measured mass of the $_1H^2$ atom is only 2.014102 u, which is 0.002388 u *less* than the combined masses of a $_1H^1$ atom and a neutron.

- This "missing" mass is called as **mass defect, Δm** and it might corresponds to energy given off when a nucleus is formed from a free proton and neutron.

- The mass defect Δm of a nucleus, ${}_Z^AX$ of Z protons and N neutrons is defined by

$$\Delta m = Z\, m({}_1^1H) + N\, m(n) - m\left({}_Z^AX\right)$$

where $m({}_1^1H)$ is the atomic mass of ${}_1^1H$, $m(n)$ is the neutron mass, and $m\left({}_Z^AX\right)$ is atomic mass of ${}_Z^AX$, all in atomic mass units.

- The atomic masses of the nuclei can be measured with Mass spectrometer.

The nuclear binding energy:

- The energy equivalent of the missing mass, corresponds to energy given off when a ${}_1H^2$ nucleus is formed from a free proton and neutron is

$$
\begin{aligned}
E_b &= \Delta m \times 931.5\ MeV/u \\
&= (0.002388)\,(931.5\ MeV/u) \\
&= 2.224\ MeV
\end{aligned}
$$

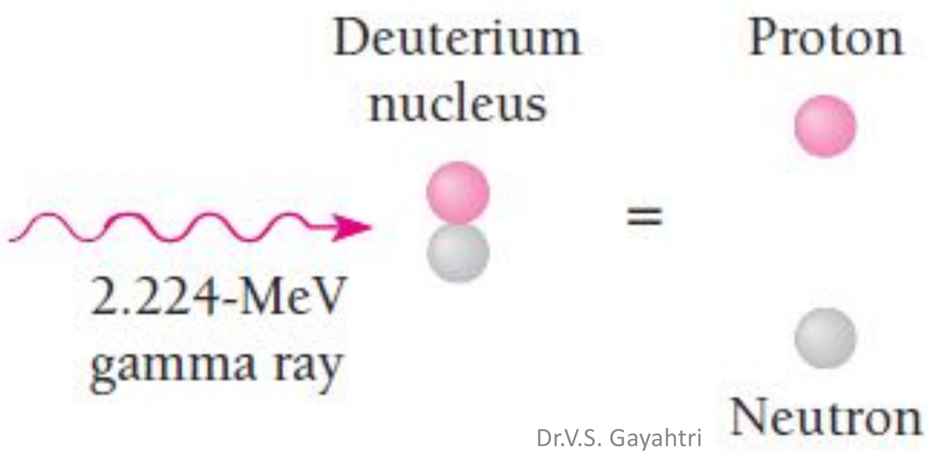
- Like ${}_1H^2$ nucleus, all nuclei have less measured mass than the combined masses of the nucleons they are composed of.

- The energy equivalent of the missing mass of a nucleus is called the **nuclear binding energy of the nucleus.**

- The greater its binding energy, the more the energy that must be supplied to break up the nucleus.

To test the interpretation of the missing mass:

- For same, we can perform experiments to see how much energy is needed to break apart a deuterium, ${}_1H^2$ nucleus into a separate neutron and proton.
- The required energy to break is indeed equal to its binding energy i.e. 2.224 MeV.
- When less energy than 2.224 MeV is given to a ${}_1H^2$ nucleus, the nucleus stays together.
- When the added energy is more than 2.224 MeV, the extra energy goes into kinetic energy of the neutron and proton as they fly apart.



Expression for nuclear binding energy:

The binding energy E_b in MeV of the nucleus ${}^{A}_{Z}X$, which has $N = A - Z$ neutrons, is given as:

$$E_b = \Delta m \times 931.5\ MeV/u$$

.

$$E_b = \left[Z\ m({}^{1}_{1}H) + N\ m(n) - m\left({}^{A}_{Z}X\right)\right] \times (931.5\ MeV/u)$$

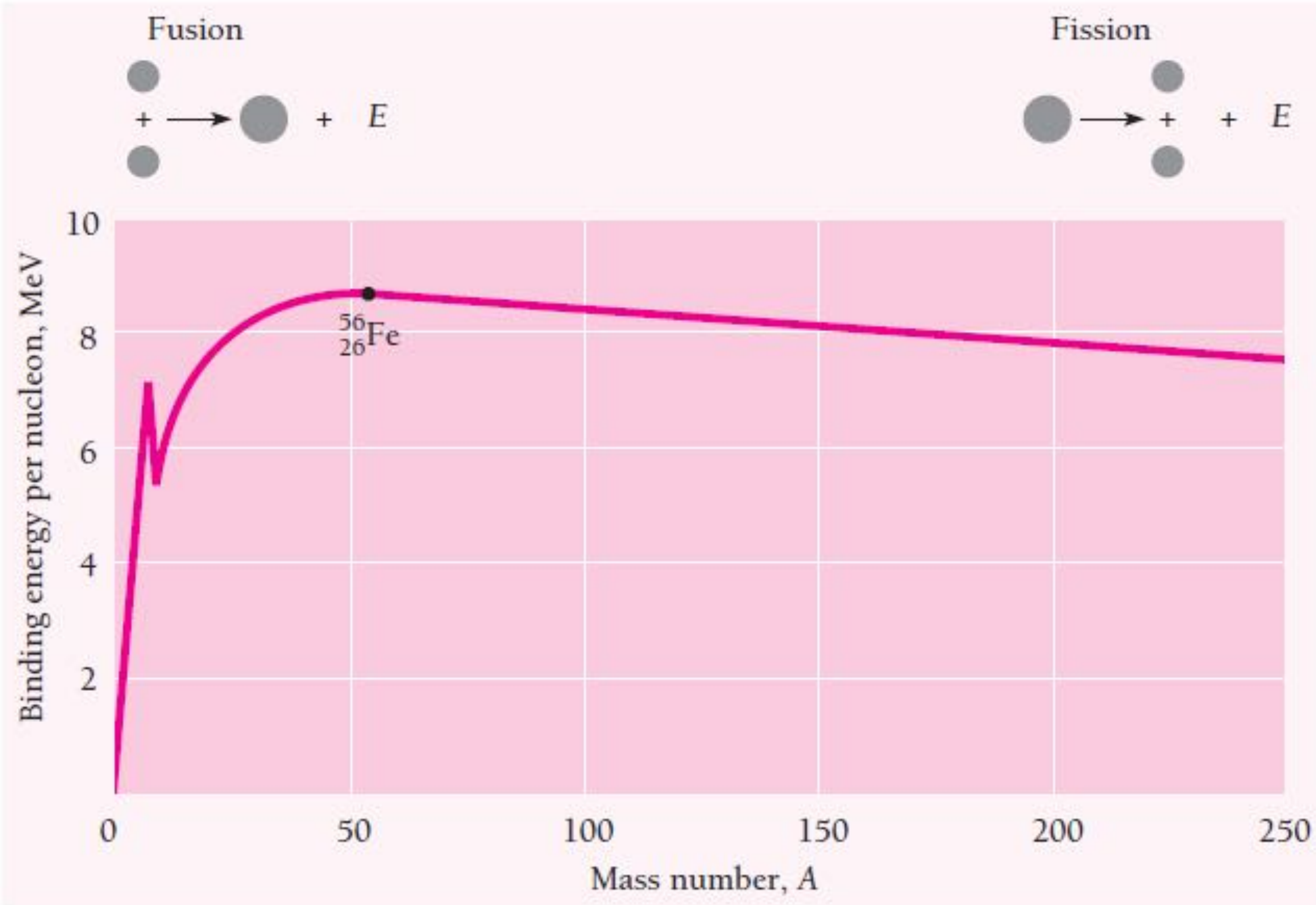
Binding Energy per Nucleon

The **binding energy per nucleon** for a given nucleus is an average found by dividing its total binding energy by the number of nucleons it contains.

For Ex.

The binding energy per nucleon for ${}_{1}^{2}H$ is (2.224 MeV)2 = 1.1 MeV/nucleon

The binding energy per nucleon for ${}_{83}^{200}Bi$ is (1640 MeV)/209 = 7.8 MeV/nucleon.



Binding energy per nucleon as a function of mass number.

- The binding energy per nucleon against the number of nucleons in various atomic nuclei is plotted.

- The greater the binding energy per nucleon, the more stable the nucleus is.

- The peak at A =4 corresponds to the exceptionally stable $^{4}_{2}He$ nucleus, which is the alpha particle.

- The graph has its maximum of 8.8 MeV/nucleon when the total number of nucleons is 56. The nucleus that has 56 protons and neutrons is $^{56}_{26}Fe$, an iron isotope. This is the most stable nucleus of them all, since the most energy is needed to pull a nucleon away from it.

Two remarkable conclusions drawn from the curve:

1. If we can somehow split a heavy nucleus into two medium-sized ones (**nuclear fission)**, each of the new nuclei will have *more* binding energy per nucleon than the original nucleus did. The extra energy will be given off, and it can be a lot.

For Ex.

- If the uranium nucleus ${}^{235}_{92}U$ is broken into two smaller nuclei, the binding energy difference per nucleon is about 0.8 MeV. The total energy given off is therefore 0.8/235= 188MeV.

- This is a truly enormous amount of energy to be produced in a single atomic event.

2. Joining two light nuclei together to give a single nucleus of medium size also means *more* binding energy per nucleon in the new nucleus.

For Ex.

- If two deuterium, ${}_{1}^{2}H$ nuclei combine to form a heliun, ${}_{2}^{4}He$ nucleus, over 23 MeV is released. Such a process, called **nuclear fusion,** is also a very effective way to obtain energy.
- In fact, nuclear fusion is the main energy source of the sun and other stars.

Nuclear Forces

Particle exchange can produce either attraction or repulsion

Nuclear Forces

A molecule is held together by the exchange of electrons between adjacent atoms.

What holds the nucleons together?

Heisenberg (1932) suggested that the nuclear forces are exchange forces in which the electrons (or positrons) and neutrinos shift back & forth between two nucleons.

Drawback:

Calculations based on beta-decay showed that the nuclear forces that should result from the electron-neutrino interchange were weaker by a factor of about 10^{-14} than those required by a square-potential with a range of the order of $10^{-13} cm$.

Yukawa's Meson Theory of Nuclear Forces

The Japanese physicist Hideki Yukawa(1935) proposed:

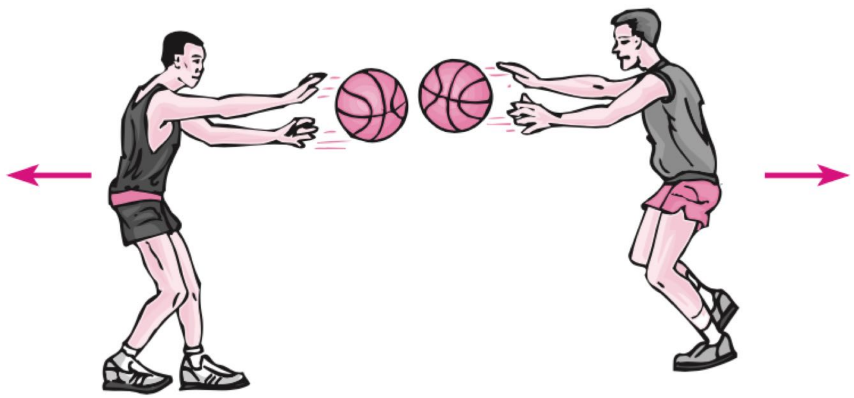
Particles intermediate in mass between electrons and nucleons are responsible for nuclear forces---they are called pions

Pions: Members of a class of elementary particles collectively called mesons.
They may be charged (π^+, π^-) or neutral (π^0).

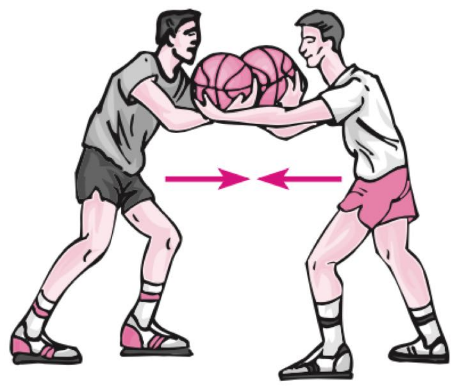
According to Yukawa’s theory:

- Every nucleon continually emits and reabsorbs pions.
- If another nucleon is nearby, an emitted pion may shift across to it instead of returning to its parent nucleon.
- The associated transfer of momentum is equivalent to the action of a force.
- Nuclear forces are repulsive at very short range as well as being attractive at greater nucleon-nucleon distances.

Forces due to particle exchange: An analogy



Repulsive force due to particle exchange



Attractive force due to particle exchange

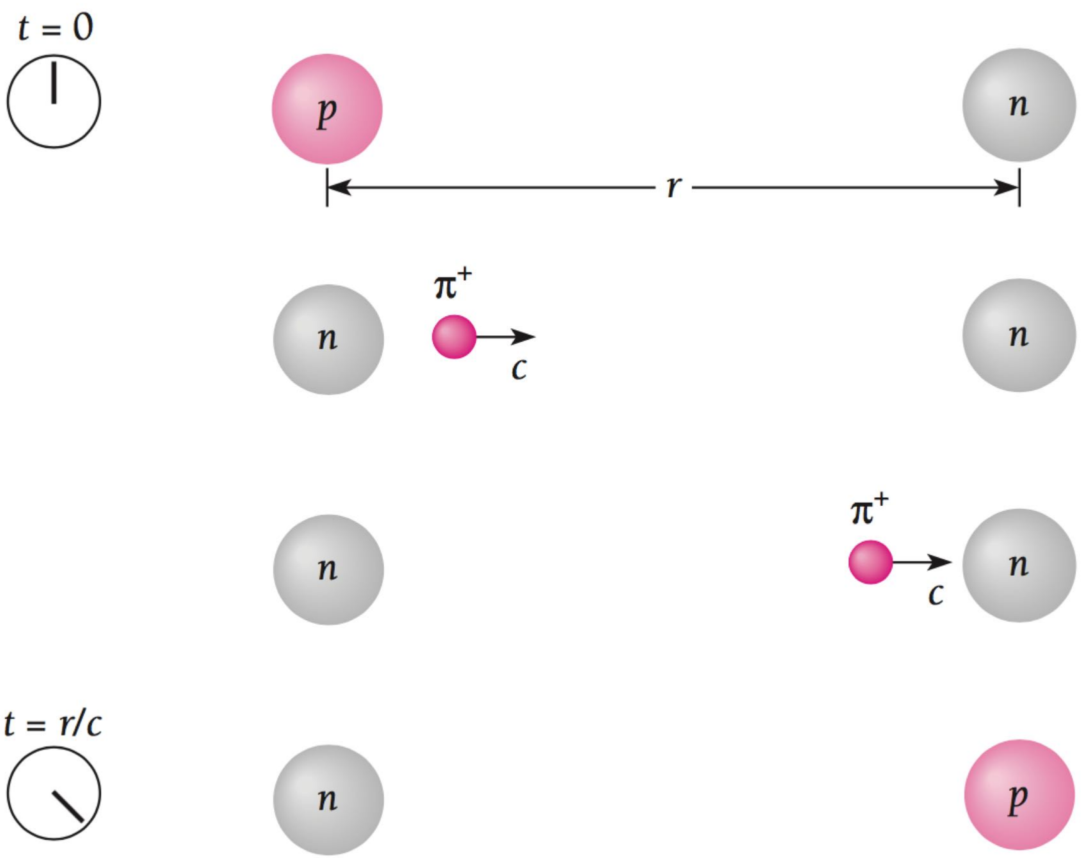
If nucleons constantly emit and absorb pions, why are neutrons & protons never found with other than their usual masses?

The uncertainty principle permits the creation, transfer and disappearance of a pion to occur without violating conservation of energy provided the sequence takes place fast enough.

According to Heisenberg's uncertainty principle in the form

$$\Delta E \, \Delta t \; \geq \frac{\hbar}{2}$$

an event in which an amount of energy ΔE is not conserved is not prohibited if Δt does not exceed $\frac{\hbar}{2\Delta E}$.



A positive pion is emitted by a proton and it becomes a neutron

The positive pion absorbed by a neutron.

Neutron becomes a proton after pion absorption

Calculation of pion mass

Let us assume that

- a pion travels between nucleons at a speed of $v \sim c$ (actually $v < c$)
- the emission of a pion of mass m_π represents a temporary energy discrepancy $\Delta E \sim m_\pi c^2$ (neglecting pion's kinetic energy)
- $\Delta E \Delta t \sim \hbar$

Nuclear forces have a maximum range r of about $1.7\ fm$. The time needed for the pion to travel this far is

$$\Delta t = \frac{r}{v} \sim \frac{r}{c}$$

Sub Δt,

$$\Delta E \Delta t \sim \hbar$$

$$(m_\pi c^2)\left(\frac{r}{c}\right) \sim \hbar$$

$$m_\pi \sim \frac{\hbar}{rc}$$

$$m_\pi \sim \frac{1.05 \times 10^{-34}\ J.s}{(1.7 \times 10^{-15}\ m)(3 \times 10^{8}\ m/s)} \sim 2 \times 10^{-28}\ kg$$

Thus the mass of pion is about 220 times the rest mass m_e of the electron.

A dozen years after Yukawa's proposal, particles with properties he had predicted were actually discovered. The rest mass of charged pions is $273\ m_e$ and that of neutral pions is $264\ m_e$.

The saturation of nuclear forces:

The nuclear forces are limited in range, and as a result nucleons interact strongly only with nearest neighbours. This effect is referred to as the saturation of nuclear forces.

If every particle in the nucleus is supposed to interact with every other particle, we expect

$$\begin{aligned}\text{Binding energy} &\propto \text{number of interacting pairs}\\ &\propto A(A-1)/2\end{aligned}$$

But this calculated result is in sharp contrast with the experimental result:

$$\text{Binding energy} \propto A$$

This suggests that a nuclear particle does not interact with all the other particles in the nucleus, but only with a limited number of them.

This situation is analogous to that in a liquid or solid, in which each atom is linked by chemical bonds to a number of nearest neighbours rather than to all the other atoms.

If a particular nuclear particle does not interact with all the other particles in the nucleus but only some of its neighbours, then the nuclear forces must have a **short range.**

The mass number A is approximately equal to twice the nuclear charge Z (accurate for light nuclei).

For heavier nuclei, the value of A increases more rapidly than 2Z.

∴ In light nuclei,

number of neutrons(n) = number of protons(p)

"suggests strong attractive force between n and p".

In heavier nuclei, n increases more rapidly than p.

If there is a strong specifically nuclear attractive force between a neutron & a proton, then we can suppose that there are similar forces between two neutrons and between two protons.

So, the force between two neutrons must be very nearly equal to that between two protons.

From the observed mass-to-charge ratio, there are two possibilities:

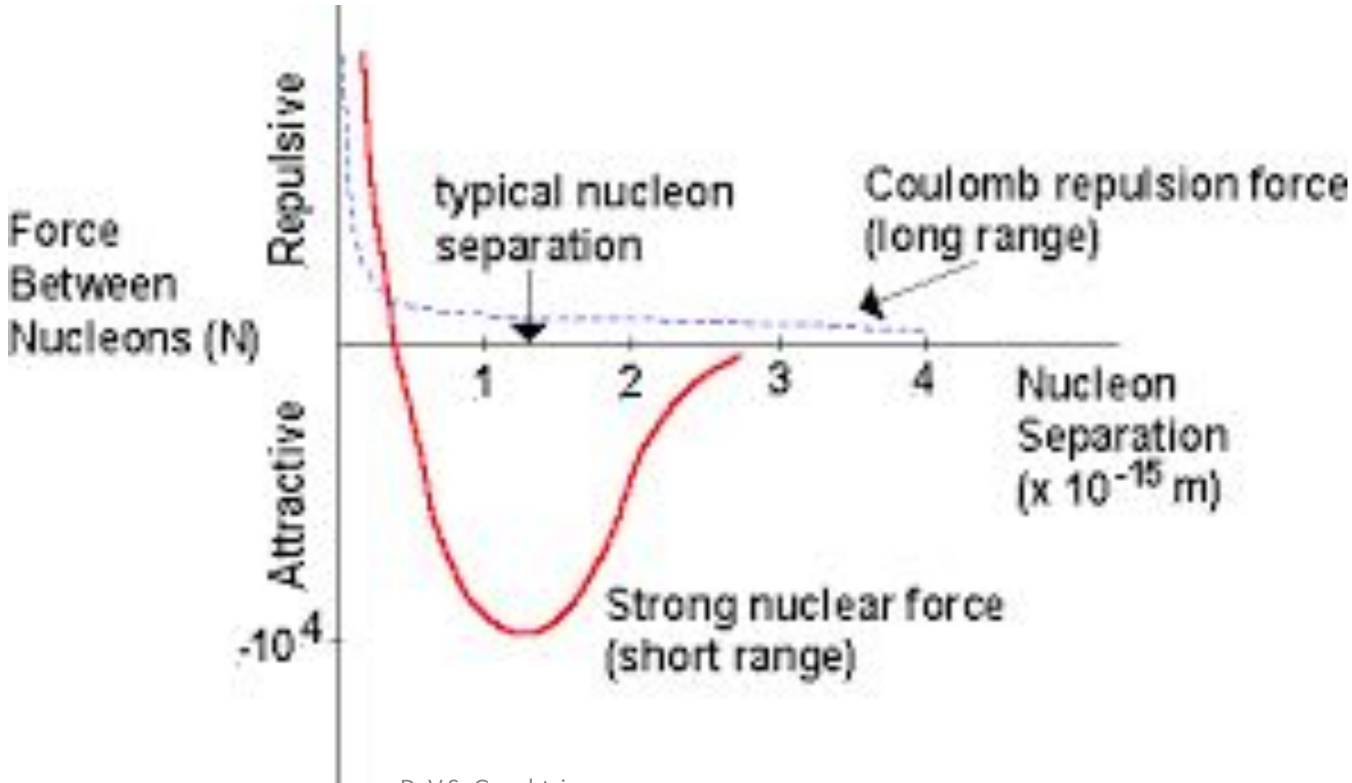
$n - p \approx n - n \approx p - p$ "charge independence of nuclear forces"

$n - p \gg n - n;\; n - p \gg p - p; n - n \approx p - p$

"charge symmetry of nuclear forces"

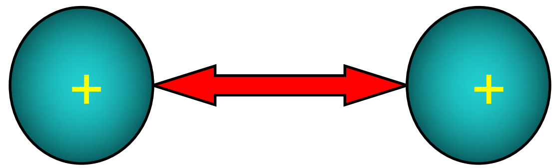
Two main forces are present between the nucleons in a nucleus

- Electrostatic Coulomb force of repulsion
- Inter-nucleon attractive forces: Strong nuclear force



Electrostatic Coulomb force of repulsion:

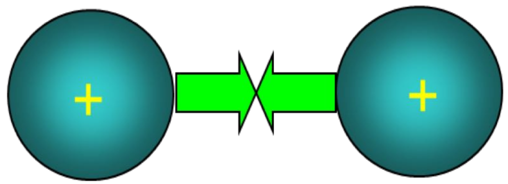
- Exists between *+ve charged protons* and extends over *long range* beyond the nuclear radius.
- A *repulsive force* present due to positive potential for distance *less than 0.5F*
- This disruptive force tends to *reduce the stability of nucleus*



Electrostatic repulsion: pushes protons apart

Inter-nucleon attractive forces: Strong nuclear force:

- Exists between nucleons which *increases nuclear stability.*
- *Attractive* in nature, when nucleons are at a distance between *0.5F-2.5F*
- A *short ranged force* and have maximum value for distances of order of about 2F.



Strong nuclear force: pulls protons together

Characteristics of Nuclear Force

- The nuclear force is the strongest of the known forces.

- The strong nuclear force has a very short range (Range is limited to about $10^{-15}m$)

- The magnitude of the Coulomb energy between two protons is ${e^2}/{r} \cong 0.5\ MeV$.

Hence, up to a separation of about 3 fm, the nuclear attraction between two protons is about 10-100 times stronger than the electric repulsion between them.

Despite its smalless, the coulomb repulsion becomes important for heavier nuclei because of the saturation of the attractive nuclear forces.

- The nuclear interactions between p-p, p-n and n-n appear to be identical.

- The force between nuclear particles is attractive (except for the electrostatic repulsion between two protons)

Nuclear Models

- A lot of experiential work had been done to *obtain nuclear properties, nuclear structure and nuclear forces,* but, a unique or comprehensive model could not be developed which can explain all these nuclear properties.

- A number of models have been developed which can only account for a part of the nuclear properties but it fails to explain when it is extended to the other properties/phenomenon.

    i. Liquid drop model
    ii. The Shell model
    iii. Collective model.

The theory of nuclear structure is less complete than the theory of atomic structure. In order to account for prominent aspects of nuclear properties and behavior, several models are proposed.

As a first approximation, we can think of each nucleon in a nucleus as interacting solely with its nearest neighbours.

This situation is the same as that of atoms in a solid, which ideally vibrate about fixed positions in a crystal lattice.

Drawback:

The analogy with a solid cannot be pursued because the vibrations of the nucleons about their average positions would be too great for the nucleus to be stable.

Liquid Drop Model

The properties of nuclear forces such as
(a) Saturation (b) short-range
© linear dependence of the binding energy & the volume on the number of particles in the nucleus

are analogous to the properties of the forces which hold a liquid drop together.

∴ A nucleus may be considered to be analogous to a drop of incompressible fluid of very high density ($\sim 10^{14}\ gm/cm^3$).

This analogy was proposed by George Gamow in 1929 and developed in detail by C.F. Von Weizsacker in 1935.

$$M = ZM_p + (A - Z)M_n - \frac{Binding\ energy}{c^2}$$

(M_p, M_n and M : masses of the proton, neutron and nucleus respectively)

"Semiempirical mass formula" or "semi-empirical binding energy formula"?

Weizsacker's Semi Empirical Mass Formula

For most nuclei with A >20, *the binding energy is well reproduced by a semi-empirical formula* based on the idea the nucleus can be thought of as *a liquid drop*.

Weizsacker derived a expression for mass of a nucleus, M and binding energy, B in a semi-empirical manner.

$$M_{NUCLEUS} = [Z.M_P + (A-Z)\, M_n] - B/c^2$$

where the Binding energy B is given by

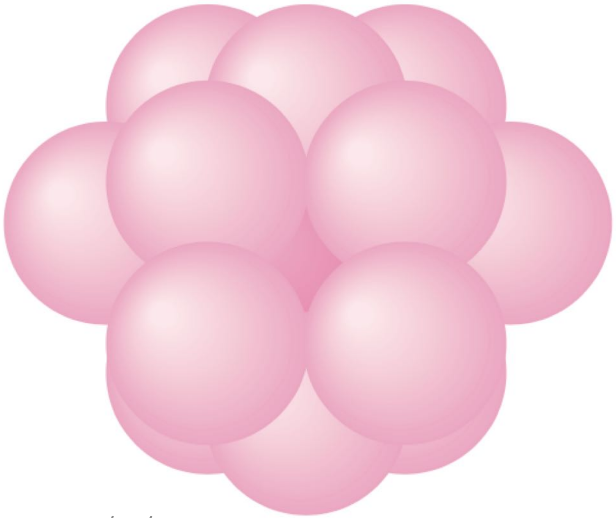
$$\boldsymbol{B = E_V + E_S + E_C + E_A + E_P}$$

= Volume energy term + Surface energy term+ Coulomb energy term+ Asymmetry energy term + Pairing energy term

Let us assume that the energy associated with each nucleon-nucleon bond = U.

As each bond energy U is shared by two nucleons, each has a binding energy = $\frac{1}{2}U$.

In a tightly packed assembly of identical spheres, each interior sphere is in contact with 12 others.



∴ Binding energy of each interior nucleon = $(12)(\frac{1}{2}U) = 6\text{U}$

If all A nucleons in a nucleus were in its interior,

The total binding energy of the nucleus :

$$E_v = 6\,AU$$

Rewriting, $E_v = a_1 A.$ **"Volume energy"**

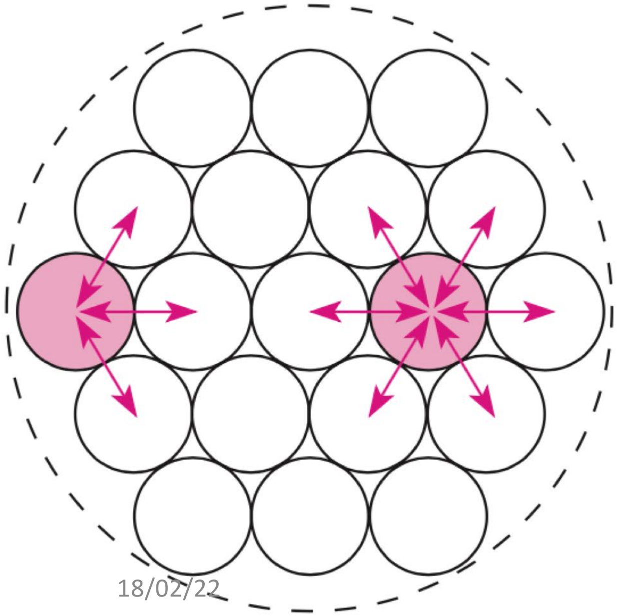
Nucleons on the surface have fewer than 12 neighbours.

The surface area of a nucleus of radius R $= 4\,\pi R^2$

$$= 4\,\pi R_0^2 A^{2/3}$$

$\therefore$ the number of nucleons with fewer than the maximum number of bonds $\propto A^{2/3}$

This reduces the total binding energy by



$$E_s = -a_2 A^{2/3}$$

"Surface energy"

The potential energy of a pair of protons r apart , due to electric repulsion :

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$$

Since there are $Z\,(Z-1)/2$ pairs of protons,

$$E_c = -\frac{Z(Z-1)}{2}\ V$$

$$= \frac{Z(Z-1)e^2}{8\pi\epsilon_0}\left(\frac{1}{r}\right)_{av}$$

(where $\left(\frac{1}{r}\right)_{av}$: $\frac{1}{r}$ averaged over all proton pairs)

If the protons are uniformly distributed throughout a nucleus of radius R,

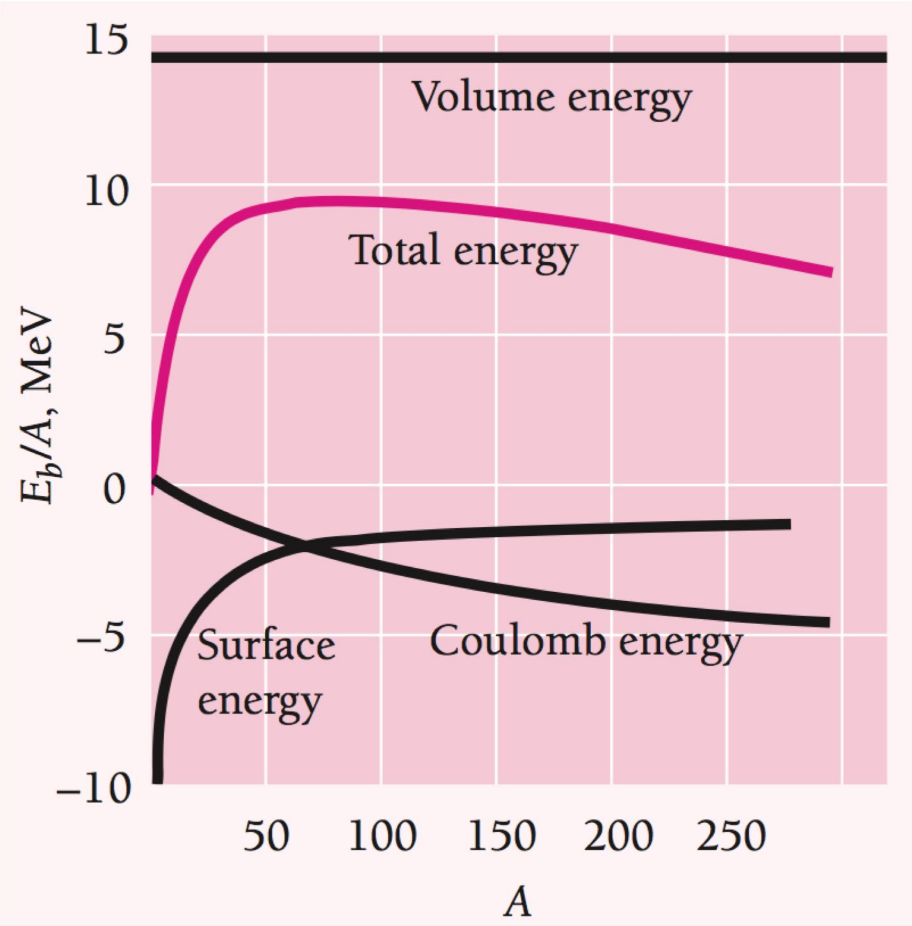
$$\left(\frac{1}{r}\right)_{av} \propto \; 1/R$$

$$\propto 1/A^{1/3}$$

$$E_c = -a_3 \frac{Z(Z-1)}{A^{1/3}}$$ **"Coulomb energy"**

The total binding energy of a nucleus is. $E_b = E_v + E_s + E_c$

$$E_b = \; a_1 \mathrm{A} - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} \ldots \ldots \ldots (1)$$

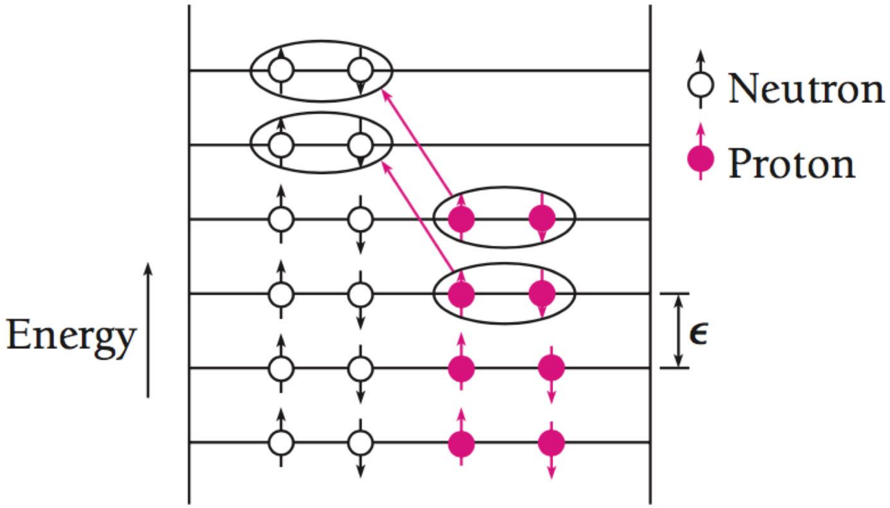


The binding energy per nucleon is

$$\frac{E_b}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{4/3}}$$

Corrections to the binding energy formula

The binding energy formula (equation 1) can be improved by including two additional effects that do not fit into the simple liquid-drop model.



Let us assume that the uppermost neutron & proton energy levels, which the exclusion principle limits to two particles each, have the same spacing ϵ.

In order to produce a neutron excess of, say, $N - Z = 8$ without changing A,

$\frac{1}{2}(N - Z) = 4\, neutrons$ would have to replace protons in the original nucleus in which $N = Z$.

The new neutrons would occupy levels higher in energy by $2\epsilon = 4\epsilon/2$ than those of the protons they replace.

In the general case of $\frac{1}{2}(N - Z)$ new neutrons, each must be raised in energy by $\frac{1}{2}(N - Z)\,\epsilon/2$.

The total work needed is

$$\Delta E = (number\ of\ new\ neutrons)\left(\frac{energy\ increase}{new\ neutron}\right)$$

$$= \left[\frac{1}{2}(N - Z)\right]\left[\frac{1}{2}(N - Z)\,\epsilon/2\right] = \frac{\epsilon}{8}(N - Z)^2$$

$$Because\, N = A - Z, (N - Z)^2 = (A - 2Z)^2$$

$$\Delta E = \frac{\epsilon}{8}(A - 2Z)^2$$

The greater the number of nucleons in a nucleus, the smaller is the energy level spacing ϵ, with $\epsilon \propto 1/A$

$\therefore$ the asymmetry energy E_a due to the difference between N and Z can be expressed as,

$$E_a = -\Delta E = -a_4 \frac{(A-2Z)^2}{A}$$

"Asymmetry energy"

A	Z	N	E_δ
even	even	even	$+\delta/2A$
odd	even	odd	0
odd	odd	even	0
even	odd	odd	$-\delta/2A$

The last correction term arises from the tendency of proton pairs and neutron pairs to occur.

Even-even nuclei are the most stable and hence have higher binding energies. Thus nuclei such as, ${}_{2}^{4}He$, ${}_{6}^{12}C$ *and* ${}_{8}^{16}O$ appear as peaks on the empirical curve of binding energy per nucleon.

Odd-odd nuclei have both unpaired protons and neutrons --- have relatively low binding energies.

The pairing energy E_p is positive for even-even nuclei, 0 for odd-even and even-odd nuclei and negative for odd-odd nuclei. It seems to vary with A as $A^{-3/4}$.

$$E_p = (\pm\,, 0)\,\frac{a_5}{A^{3/4}}$$ **"Pairing energy"**

The final expression for the binding energy of a nucleus, first obtained by C.F. Von Weizsacker in 1935 is, of atomic number Z and mass number A:

$$E_b = a_1 \mathrm{A} - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} (\pm\,, 0) \frac{a_5}{A^{3/4}} \;\ldots\ldots.(2)$$

"Semiempirical binding energy formula"

A set of coefficients that give a good fit with the data is as follows:

$$a_1 = 14.1\ MeV; a_2 = 13.0\ MeV; a_3 = 0.595\ MeV$$

$$a_4 = 19.0\ MeV; a_5 = 33.5\ MeV$$

Equation (2) agrees better with observed binding energies than does Equation (1), which suggests that liquid-drop model, though a good approximation, is not the last word on the subject.

Equation (2) can also be rewritten to give the nuclear (or atomic) mass, since the mass and binding energy are related by the equation:

$$M = ZM_p + (A-Z)M_n - \frac{Binding\ energy}{c^2}$$

(M_p, M_n and M : masses of the proton, neutron and nucleus respectively)

The **“Semiempirical mass formula”** is given by:

$$M = ZM_p + (A-Z)M_n - \frac{a_1}{c^2}\mathrm{A} + \frac{a_2}{c^2}A^{2/3} + \frac{a_3}{c^2}\frac{Z(Z-1)}{A^{1/3}} + \frac{a_4}{c^2}\frac{(A-2Z)^2}{A}\ (\pm\ ,0)\frac{a_5}{c^2 A^{3/4}}$$