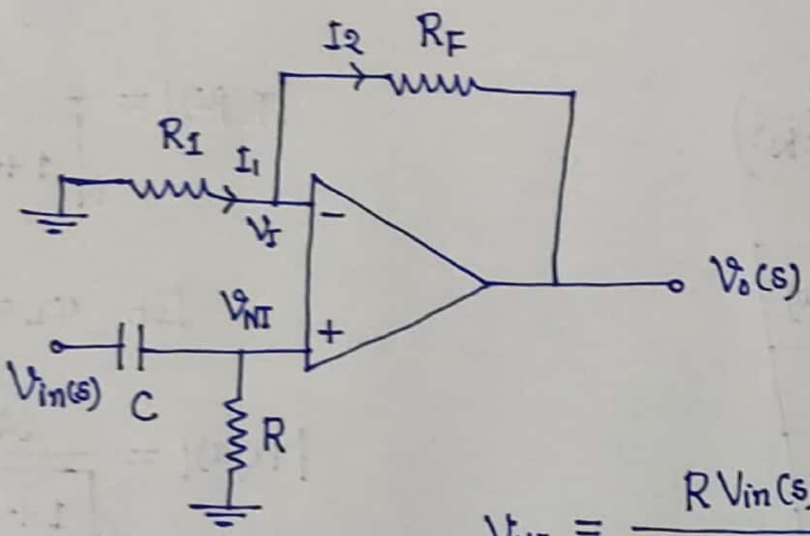
First Order Butterworth High-Pass Filter



$$V_{NI} = \frac{R\,V_{in}(s)}{R + 1/cs}$$

$$V_o(s) = \left(1 + \frac{R_F}{R_1}\right) V_I$$

$$= \left(1 + \frac{R_F}{R_1}\right) V_{NI}$$

$$= \left(1 + \frac{R_F}{R_1}\right) \cdot \frac{R\,V_{in}(s)}{R + 1/cs}$$

$$= \left(1 + \frac{R_F}{R_1}\right) \left(\frac{V_{in}(s)}{1 + \frac{1}{sCR}}\right)$$

Transfer function, $T(s) = \dfrac{V_o(s)}{V_{in}(s)} = \left(1 + \dfrac{R_F}{R_1}\right)\left(\dfrac{1}{1 + \frac{1}{sRC}}\right)$

put $s = j\omega$

let $1 + \frac{R_F}{R_1} = A$

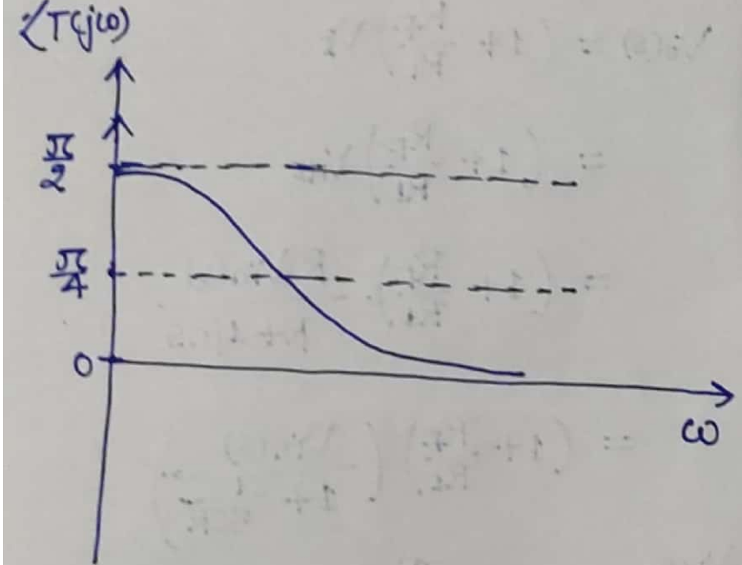
$$T(j\omega) = \frac{V_o(j\omega)}{V_{in}(j\omega)} = A \cdot \frac{1}{1 + \frac{1}{j\omega RC}}$$

$$T(j\omega) = \frac{A}{1 - j\frac{1}{\omega RC}}$$

Phase

$$\angle T(j\omega) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

ω	$\angle T(j\omega)$
0	$\pi/2$
∞	0
ω_L	$\pi/4$

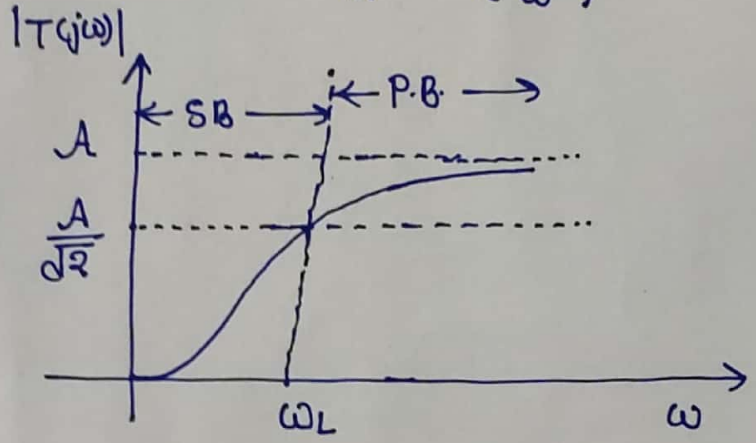


Magnitude

$$|T(j\omega)| = \frac{A}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

$$\text{Let } \omega_L = \frac{1}{RC}$$

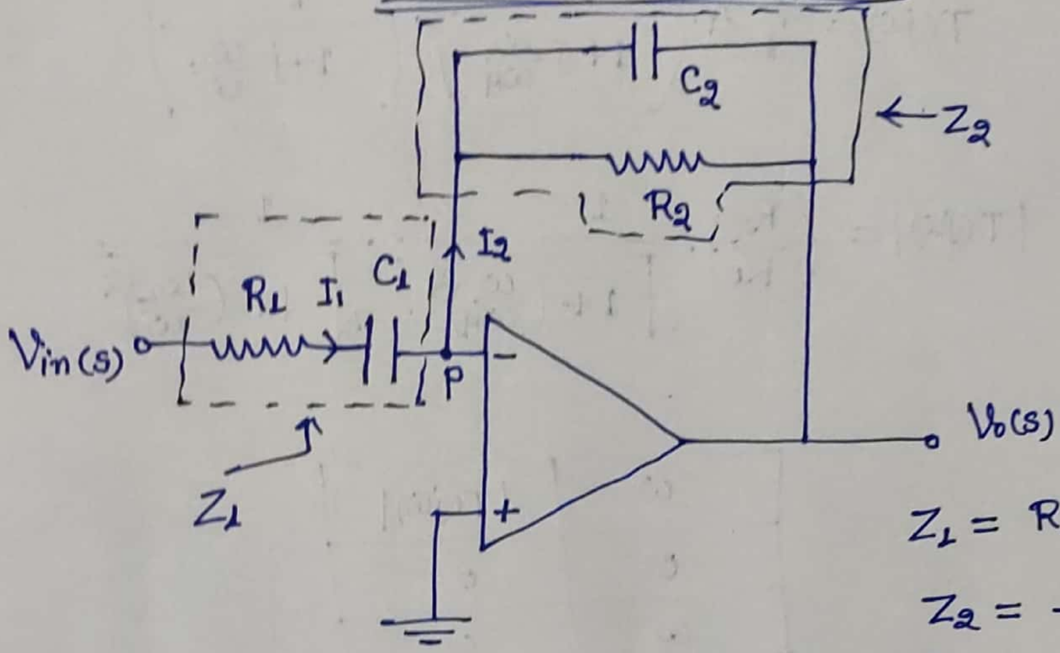
$$|T(j\omega)| = \frac{A}{\sqrt{1+\left(\frac{\omega_L}{\omega}\right)^2}}$$



ω	$\lvert T(j\omega)\rvert$
0	0
ω_L	$A/\sqrt{2}$
∞	A

Scanned with CamScanner

Active Band Pass Filter



$$Z_1 = R_1 + 1/C_1 s$$

$$Z_2 = \frac{R_2 \cdot 1/C_2 s}{R_2 + 1/C_2 s} = \frac{R_2}{sC_2R_2 + 1}$$

Apply KCL at P

$$I_1 = I_2$$

$$V_{NI} = V_I = 0$$

$$\frac{V_{in}(s) - 0}{Z_1} = \frac{0 - V_o(s)}{Z_2}$$

$$\cancel{\text{or}\ \frac{V_o(s)}{V_{in}(s)} = -\frac{Z_1}{Z_2} = -\frac{R_1\left(1+\frac{1}{SR_1C_1}\right)}{\frac{R_2}{(1+SC_2R_2)}}}$$

$$\frac{V_o(s)}{V_{in}(s)} = -\frac{Z_2}{Z_1}$$

$$= -\frac{R_2}{(1+sR_2C_2)\cdot\left(R_1 + \frac{1}{SC_1}\right)}$$

$$T(s) = -\left(\frac{R_2}{R_1}\right)\cdot\frac{1}{(1+SR_2C_2)\left(1+\frac{1}{SC_1R_1}\right)}$$

put $s = j\omega$ and let $\frac{R_2}{R_1} = A$

$$T(j\omega) = -\left(\frac{R_2}{R_1}\right)\cdot\frac{1}{(1+j\omega C_2R_2)\left(1+\frac{1}{j\omega R_1C_1}\right)}$$

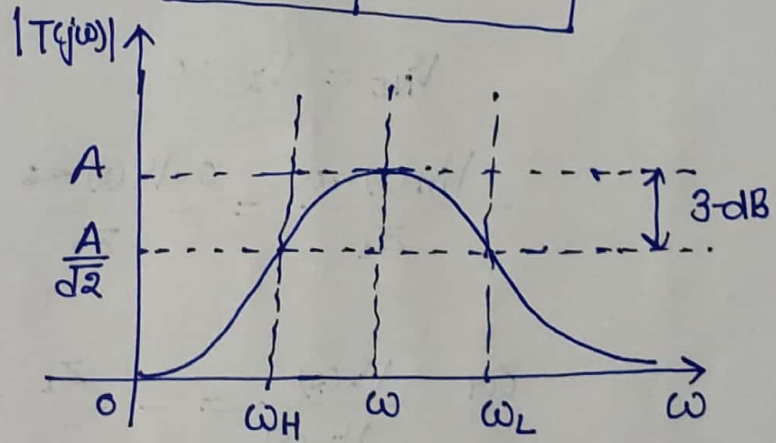
Let $\omega_H = \frac{1}{R_2C_2}$

$\omega_L = \frac{1}{R_1C_1}$

$$T(j\omega) = A \cdot \left(\frac{1}{1 + j\frac{\omega}{\omega_H}} \right) \cdot \left(\frac{1}{1 - j\frac{\omega_L}{\omega}} \right)$$

$$|T(j\omega)| = \left(\frac{R_2}{R_1} \right) \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_H}\right)^2}} \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega_L}{\omega}\right)^2}}$$

ω	$\|T(j\omega)\|$
0	0
∞	0
ω_H	$A/\sqrt{2}$
ω_L	$A/\sqrt{2}$



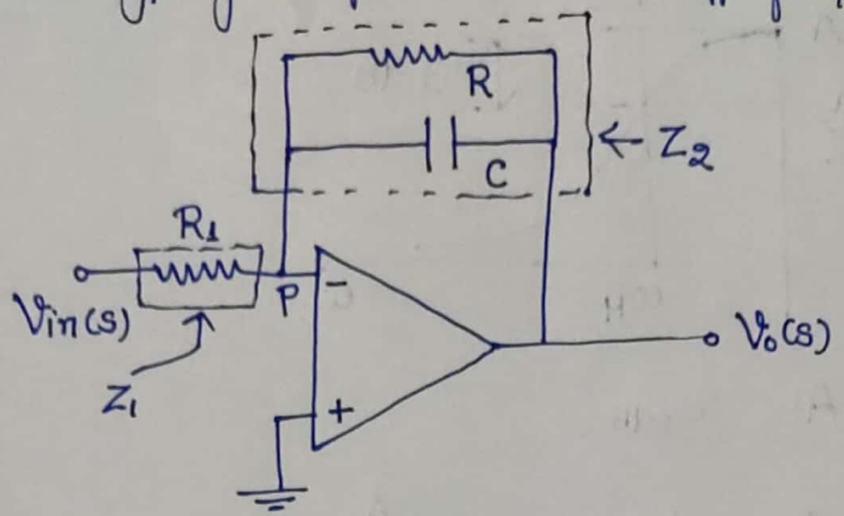
ω_H: Higher 3-dB frequency

ω_L: Lower 3-dB frequency

Bandwidth: $[\omega_H, \omega_L]$

Stopband: $\ell \cdot [0, \omega_H) \cup (\omega_L, \infty)$

Q.1: Determine type of the filter and cut-off frequency ?



$R_1 = 2K\Omega$

$R = 10K\Omega$

$C = 1nF$

Sol:

$$Z_2 = R \| 1/cs \qquad\qquad Z_1 = R_1$$

$$= \frac{R \cdot 1/cs}{R + 1/cs}$$

$\cancel{\frac{V_o(s)}{V_{in}}}$ Apply KCL at P

$$\frac{V_o(s)}{V_{in}(s)} = -\frac{Z_2}{Z_1}$$

$$= \frac{-\left(\frac{R}{1+sRC}\right)}{R_1}$$

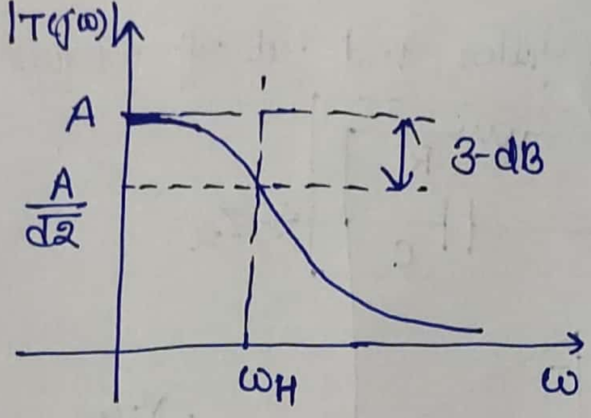
$$T(s) = -\frac{R}{R_1} \cdot \frac{1}{1+sRC}$$

$$T(s) = \frac{V_o(s)}{V_{in}(s)} = -\frac{R}{R_1} \cdot \frac{1}{1+sRC}$$

Let $A = \frac{R}{R_1}$; put $s = j\omega$

$$T(j\omega) = -A \cdot \frac{1}{1+j\omega RC}$$

$$\boxed{|T(j\omega)| = \frac{A}{\sqrt{1+\omega^2R^2C^2}}}$$



At $\omega_{3\text{-dB}}$

$$|T(j\omega)|_{\omega=\omega_{3dB}} = \frac{A}{\sqrt{2}}$$

$$\cancel{\frac{A}{\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}} = \frac{A}{\sqrt{2}}} \qquad \frac{A}{\sqrt{1+\omega_{3dB}^2 R^2C^2}} = \frac{A}{\sqrt{2}}$$

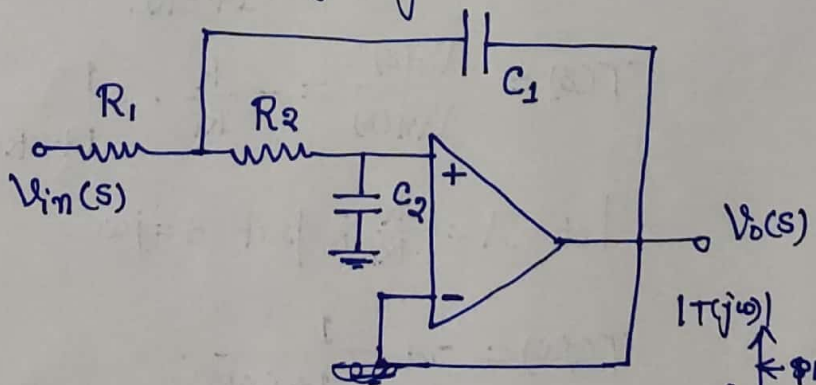
or $\omega_{3dB}^2 R^2C^2 = 2-1$

or $\boxed{\omega_{3dB} = \frac{1}{RC}}$

$$= \frac{1}{10\times10^3\times10^{-9}}$$

$$= 10^5 \text{ rad./sec.}$$

Q. Determine the type of the filter?



At $\omega \to \infty$

$C \to \infty$

$1/C_1S \to 0$

$1/C_2S \to 0$

At $\omega \to 0$