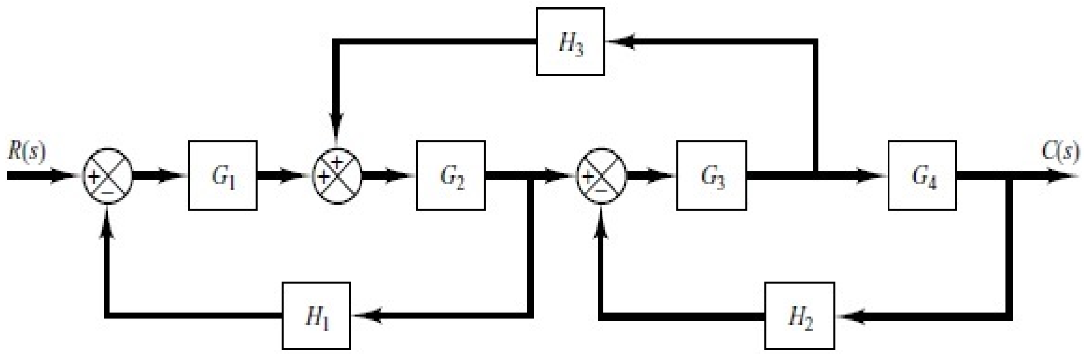
Control System Tutorial

Subject Code: EC380

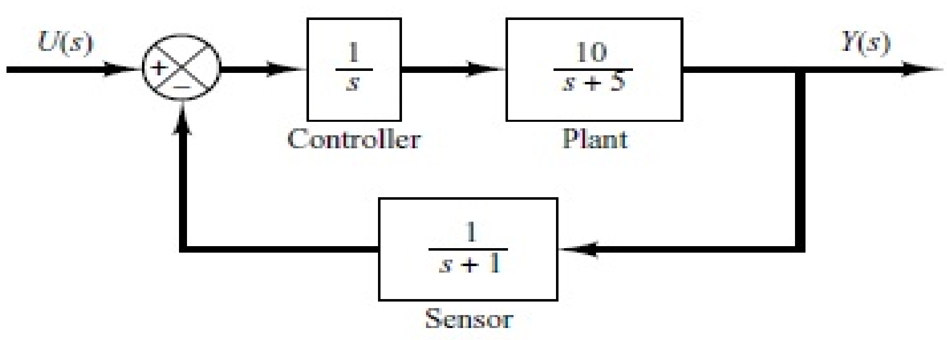
INDIAN INSTITUTE OF INFORMATION TECHNOLOGY GUWAHATI

I. Block diagram representation

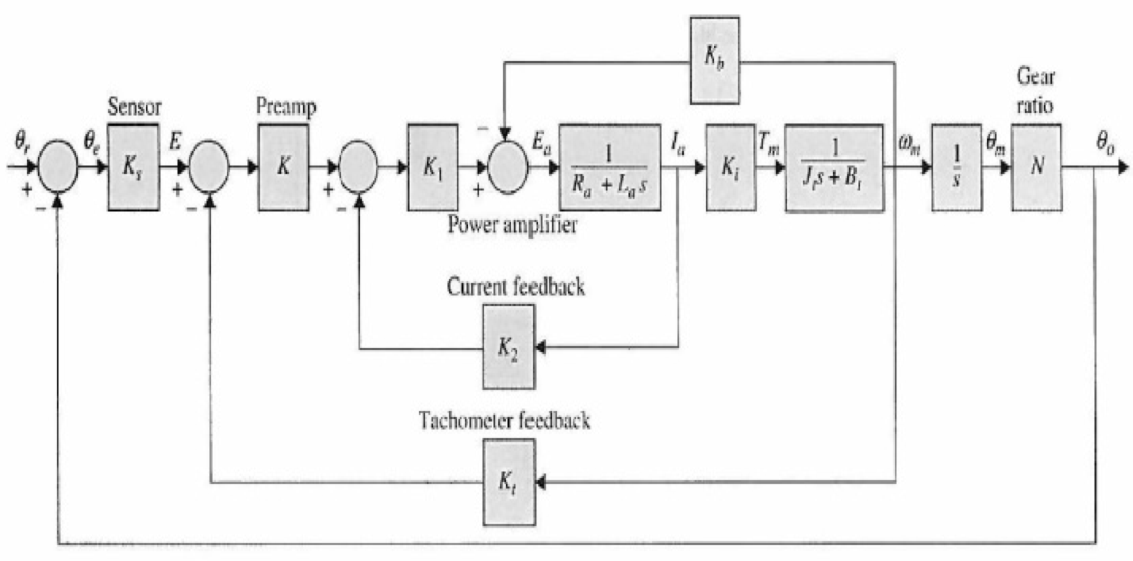
1. Simplify the block diagram shown below and obtain the closed-loop transfer function $C(s)/R(s)$



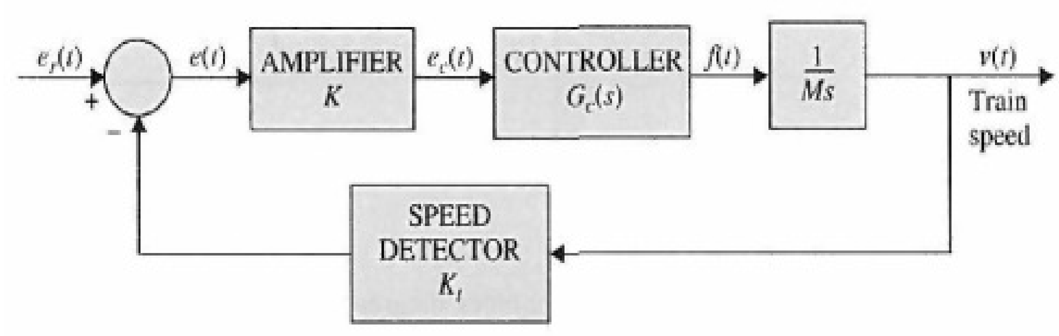
2. Obtain the closed-loop transfer function $Y(s)/U(s)$ of the following system



3. Following is the block-diagram of the position-control system of an electronic word processor. Obtain the loop transfer function $\theta_0(s)/\theta_e(s)$ (considering outer feedback path open) and closed-loop transfer function $\theta_0(s)/\theta_r(s)$.



4. The following is a block diagram of electric train control, The system parameters are given below:
$e_r(t)$=voltage representing the desired train speed, V
v_t=speed of train in ft/sec
M=$30,000 lb/sec^2$
K=amplifier gain
K_t=gain of speed indicator=$0.15V$/ft/sec



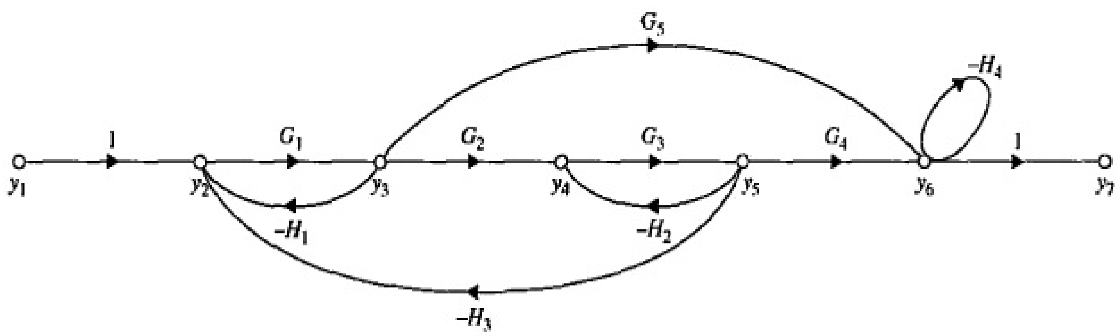
To determine the transfer function of the controller, a step function of $1V$ is applied to the input of the controller i.e., $e_c(t) = u_s(t)$. The output of the controller is measured and described by the following equation:

$$f(t) = 100(1 - 0.3e^{-6t} - 0.7e^{-10t})u_s(t)$$

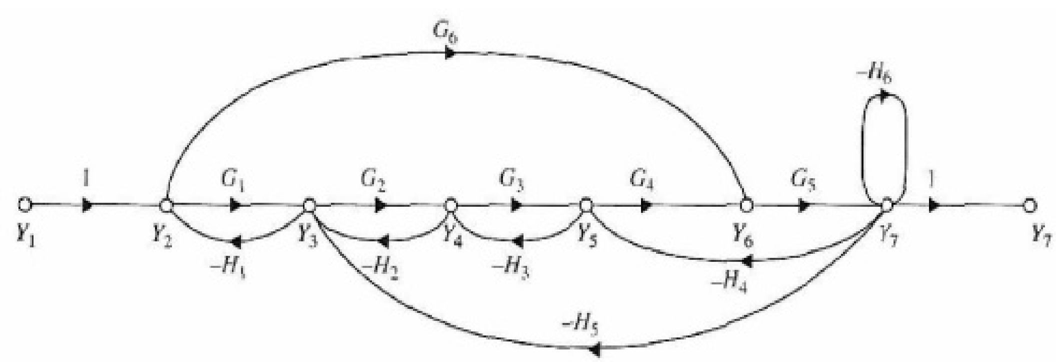
(a) Find the transfer function $G_c(s)$ of the controller.
(b) Derive the forward path transfer function $\frac{V(s)}{Es}$ of the system.
(c) Derive the closed loop transfer function $\frac{V(s)}{Ers}$ of the system.
(d) Assuming that K is set at a value so that the train will not run-away i.e. unstable, find the steady-state speed of the train in ft/sec when input is $e_r(t) = u_s(t)\ V$

II. SIGNAL FLOW GRAPH

5. Consider the following signal flow graph (SFG). Find the input-output relationship between y_7 and y_1 using gain formula

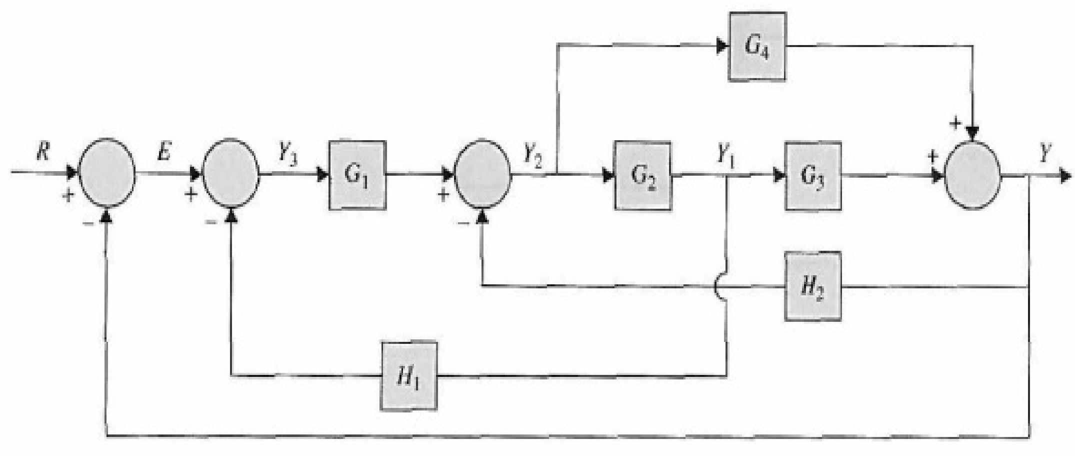


6. Find the transfer function Y_7/Y_1 of the following SFG

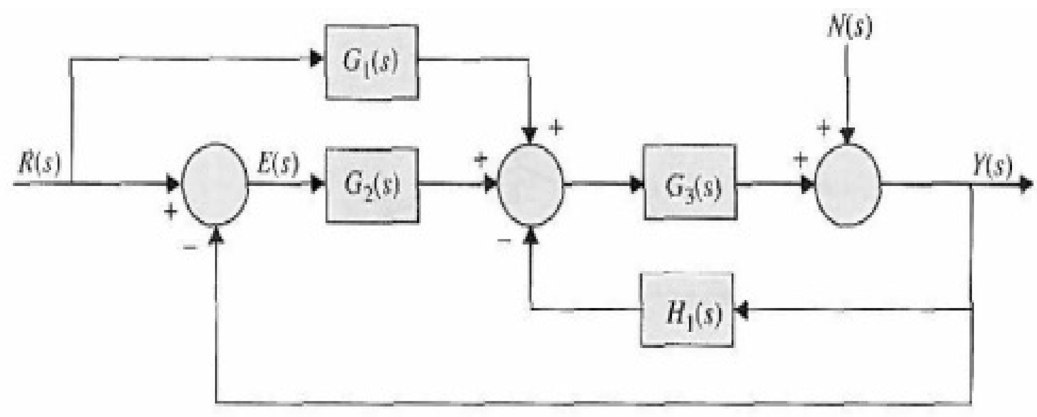


III. Signal flow graph to block diagram representation

7. Obtain the equivalent SFG of the following block diagram representation and hence find the closed-loop transfer-function $Y(s)/R(s)$ of the system.



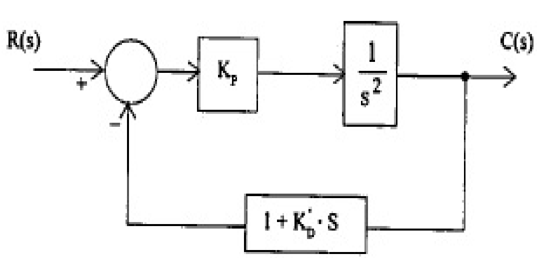
8. Find closed-loop transfer function $Y(s)/R(s)$ of the following by applying gain formula to the equivalent SFG.

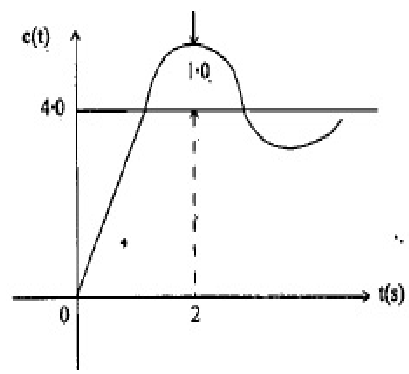


IV. Time-domain analysis

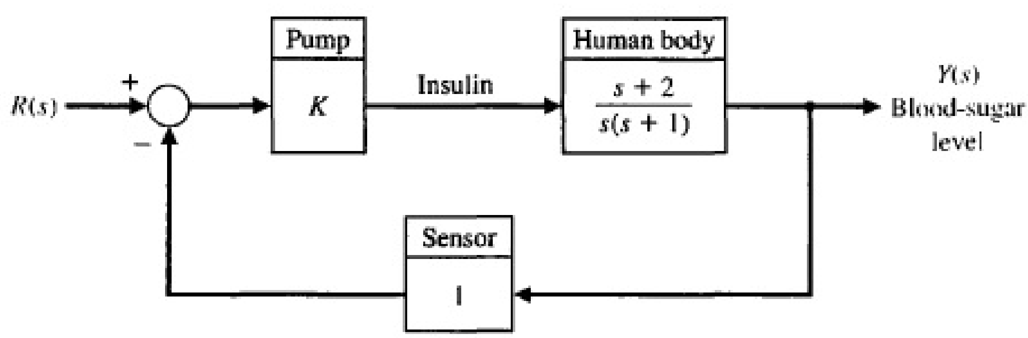
9. The open-loop transfer function of a unity feedback system is given by $G(s) = \frac{5}{s(s+1)}$. Find the rise time, percentage overshoot, time of peak overshoot and settling time (2% criterion) for a unit step input.

10. The plant shown below has open-loop transfer functions as $G(s) = \frac{1}{s^2}$. The plant is controlled by a forward proportional controller with gain K_P, and a rate controller in its feedback path. It is desired to obtain a response to a step input as shown below. Design the values of the gain K_P and K_D to get the desired response.





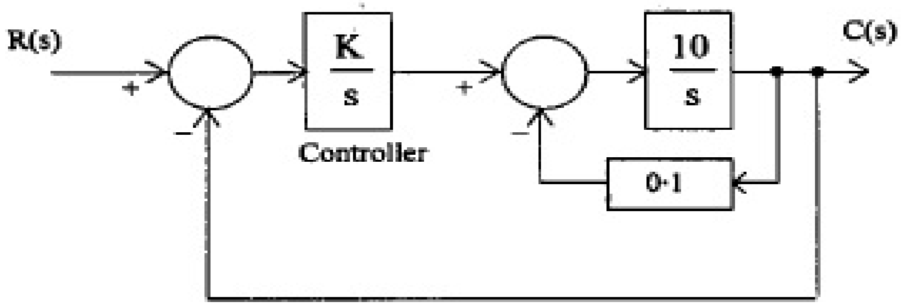
11. Automatically controlled insulin injection by means of a pump and a sensor can be very effective for better lives of diabetic persons. Such a pump and an injection system has a feedback control as shown below. Considering unity feedback, calculate the suitable gain K so that overshoot of the step-response due to the drug-injection is approximately 7%. $R(s)$ is the desired blood-sugar level and $Y(s)$ is the actual blood-sugar level.



12. Find the steady-state error for a step and ramp input in a unity feedback system with open-loop transfer function $G(s) = \frac{5(s+8)}{s(s+1)(s+4)(s+10)}$

13. For a unity feedback system, find the steady-state error for a step and ramp input when $G(s) = \frac{20}{s^2+14s+50}$

14. Consider the feedback control system shown below. The controller is an integrator with a gain of K. Find the value of K for which steady-state error to unit ramp input is less than 0.01



V. ANSWERS

$$1.\ \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_1G_2H_1+G_3G_4H_2-G_2G_3H_3+G_1G_2G_3G_4H_1H_2}$$

$$2.\ \frac{Y(s)}{U(s)} = \frac{10(s+1)}{s^3+6s^2+5s+10}$$

$$3a.\ OLTF = \frac{\theta_0(s)}{\theta_e(s)} = \frac{KK_sK_1K_iN}{s[L_aJ_ts^2+(L_aB_t+R_aJ_t+K_1K_2J_t)s+R_aB_t+K_iK_b+KK_1K_iK_t+K_1K_2B_t]}$$

$$3b.\ \frac{\theta_0(s)}{\theta_r(s)} = \frac{KK_sK_1K_iN}{[L_aJ_ts^3+(L_aB_t+R_aJ_t+K_1K_2J_t)s^2+(R_aB_t+K_iK_b+KK_1K_iK_t+K_1K_2B_t)s+KK_sK_1K_iN]}$$

$$4.a)\ G_c(s) = \frac{F(s)}{E_c(s)} = \frac{880(s+6.818)}{(s+6)(s+10)}$$

$$4.b)\ \frac{V(s)}{E(s)} = \frac{KG_c(s)}{Ms} = \frac{0.0293K(s+6.818)}{s(s+6)(s+10))}$$

$$4.c)\ \frac{Vs}{E_r(s)} = \frac{\frac{KG_c(s)}{Ms}}{1+\frac{KK_tG_c(s)}{Ms}} = \frac{0.0293K(s+6.818)}{s^3+16s^2+(0.0044K+60)s+0.03K}$$

4.d) $e_r = 1V$ i.e., $E_r(s) = \frac{1}{s}$

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} sV(s) = 6.66 ft/sec$$

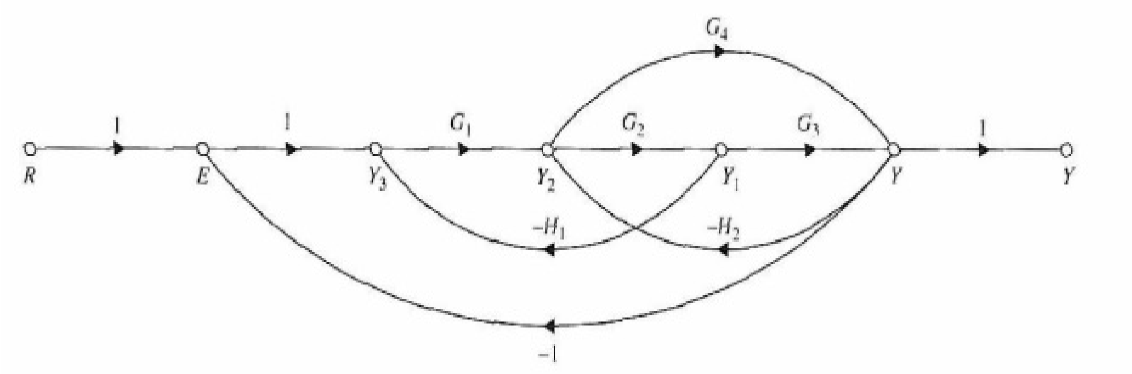
$$5.\ \frac{y_7}{y_1} = \frac{G_1G_2G_3G_4+G_1G_5(1+G_3H_2)}{\Delta}$$

where $\Delta = 1+G_1H_1+G_3H_2+G_1G_2G_3H_3+H_4+G_1G_3H_1H_2+G_1H_1H_4+G_3H_2H_4+G_1G_2G_3H_3H_4+G_1G_3H_1H_3H_4$

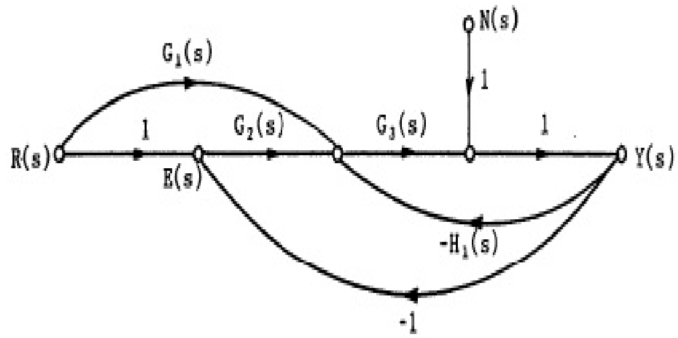
$$6.\ \frac{Y_7}{Y_1} = \frac{G_1G_2G_3G_4G_5+G_5G_6(1+G_2H_2+G_3H_3)}{\Delta}$$

where $\Delta = 1+G_1H_1+G_2H_2+G_3H_3+G_4G_5H_4+H_6+G_2G_3G_4G_5H_5-G_5G_6H_1H_5-G_5G_6H_1H_2H_3H_4+G_1G_3H_1H_3+G_1G_4G_5H_1H_4+G_1H_1H_6+G_2G_4G_5H_2H_4+G_2H_2H_6+G_3H_3H_6-G_3G_5G_6H_1H_3H_5+G_1G_3H_1H_3H_6$

7.



8.



9. $t_r = 0.83sec$, $M_p = 48.7\%$, $t_p = 1.44sec$, $t_s = 8sec$

10. $M_p = 1$ implies $\zeta = 0$, which implies $w_n = 1.57rads^{-1}$ which in turn gives $K_P = 1.25$ and $K_D = 0$

11. K=1.67

12. $K_p = \infty$ and $K_v = 1.0$ for type-1 system. So, $e_{ss} = 0$ for step-input and $e_{ss} = 1a_0$ for ramp-input where a_0 is the slope of ramp signal.

13. $e_{ss} = 0.71$ for step input and $e_{ss} = \infty$ for a ramp input

14. $K > 10$