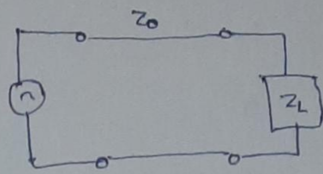
Date
09/01/2024

The Smitch Chart

$$\Gamma = \Gamma_r + j\,\Gamma_i$$



$Z_L = Z_0$ matched

$|\Gamma| = 0$

$S = 1$

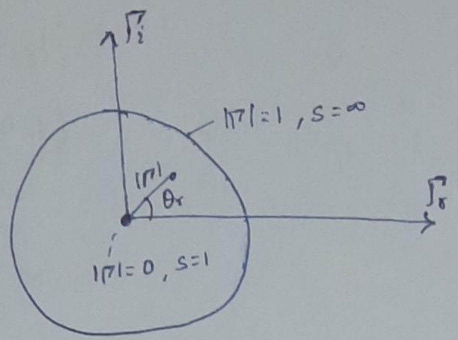
$Z_L \neq Z_0$ mismatched

$|\Gamma| \neq 0 \quad S > 1$

$\infty \ngtr S \geq 1$

$1 \leq \Gamma \leq -1$

$1 \leq |\Gamma| \leq 0$



$Z_L = R \pm jX \ \Omega$

Normalizing load

$$\frac{Z_L}{Z_0} = \frac{R}{Z_0} \pm j\frac{X}{Z_0}$$

$$z_L = r \pm jx \quad \text{(normalized load)}$$

$\bar{z}_L$ – normalized

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\bar{z}_L - 1}{\bar{z}_L + 1}$$

$$\bar{z}_L = r + jx = \frac{(1+\Gamma_r) + j\,\Gamma_i}{(1-\Gamma_r) - j\,\Gamma_i}$$

$$(x-a)^2 + (y-b)^2 = r^2$$

eq^n of circle in xy plane

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2$$

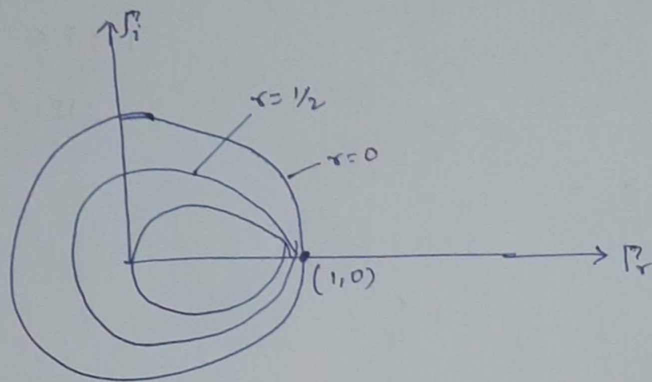
plane $\Gamma_r \Gamma_i$

centre $\left(\frac{r}{1+r}, 0\right)$

radius $= \frac{1}{1+r}$.

Equation of r-circle.

normalized resistance r	Radius $\left(\frac{1}{1+r}\right)$	Centre $\left(\frac{r}{1+r}, 0\right)$
0	1	(0,0)
1/2	2/3	(1/3, 0)
1	1/2	(1/2, 0)
∞	0	(1, 0).



$$(\Gamma_r - 1)^2 + \left[\Gamma_i - 1/x\right]^2 = \left[1/x\right]^2$$

normalized reactance (x)	Radius ($1/x$)	centre ($1, 1/x$)
0	∞	(1, ∞)
±1/2	2	(1, ±2)
±1	1	(1, ±1)

load Inductive — upper circle (+ve)

load capacitive — lower circle (-ve.

Q. $Z_L = 60 + j40\ \Omega$, $Z_0 = 50\ \Omega$ Smitch chart

Normalized Impedance

$$\bar{z}_L = \frac{Z_L}{Z_0} = \frac{60}{50} + j\frac{40}{50} = 1.2 + j0.8$$

$$= r + jx$$

$r = 1.2$

$x = 0.8$

OP = 2.85 cm

OQ = 7.95 cm

$$|\Gamma| = \frac{2.85}{7.95} = \frac{OP}{OQ}$$

$$= 0.358.$$

mark point $r = 1.2$ and $x = 0.8$ as P

extends OP points to the outer circle as Q.

$\angle\theta_\Gamma = \angle OPQ. = 55^\circ.$

Q. ~~How to~~ $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 + j40 - 50}{60 + j40 + 50}$

$$= \frac{10 + j40}{110 + j40}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

S value using Smitch chart.

and minima

→ Distance b/w two consecutive maxima

is $\frac{d_g}{2}$.

$d/2 \sim 360^\circ$

$d \sim 720^\circ$

$u = 0.6c$
$f = 2M$

$$\lambda = \frac{u}{f} = \frac{0.6c}{2M}$$

$$= \frac{0.6 \times 3 \times 10^8}{2 \times 10^6}$$

$$= 90 \text{ m}$$

$$l = 30\text{m} = \frac{30}{\lambda} \times \lambda = \frac{30}{90} \times 90$$

$$= \frac{1}{3}\lambda.$$

In degree

$$l = \lambda/3 = \frac{720^\circ}{3} = 240^\circ.$$

$$180^\circ + 60^\circ.$$

mark point ~~180°~~ on the circle

at determine r and x

$$Z_{in} = r + jx$$

$$= 0.5 + j\,0.04.$$

Q. $Z_L = 100 + j150$ $\quad Z_0 = 75\,\Omega.$

(1) Γ

(b) S

(c) The load admitance Y_L

(d) Zin at 0.4λ from the load

(e) The location of Vmax and Vmin with respect to load if the line is 0.6λ long

(f) Zin at the generator.

$$\bar{Z}_L = \frac{100}{75} + j\frac{150}{75}$$

$$= 1.33 + j\,2$$

$$|\Gamma| = \frac{OP}{O\theta} = \frac{5.2\text{ cm}}{8\text{ cm}} = 0.65$$

$O\theta = 8$ cm
$OP = 5.2$ cm

$$S = \frac{1 + 0.65}{1 - 0.65} = \frac{1.65}{0.35}$$

$$= 4.71$$

$$\Gamma = |\Gamma| \angle \theta OP$$
$$= 0.65 \angle 40^\circ.$$

$\bar{Y}_L$ – opposite dirn of $\bar{Z}_L$

$r = 0.228$
$x = 0.35$

$\bar{Y}_L = r - jx$

$$Y_L = \bar{Y}_L Y_0$$
$$= \frac{1}{75}(0.228 - j0.35).$$

(d) 0.4d from the load. — clock.

$$l = 0.4d = 0.4 \times 720^\circ$$
$$= 288^\circ.$$

Mark point by moving clockwise by

$= 280^\circ$

$\bar{Z}_{in} = 0.3 + j0.6$

$Z_{in} = 75(0.3 + j0.63)$

By formulale

$$\beta l = \frac{2\pi}{d} \times 0.4d = 0.4 \times 360^\circ = 144^\circ.$$

$$Z_{in} = Z_0 \left(\frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right)$$

$$= 75 \left(\frac{100 + j150 + j75 \tan(144^\circ)}{(75 + j100 - j150 \tan 144^\circ} \right)$$

$$= 54.41 \angle 65.25^\circ.$$

(e) $l = 0.6d$

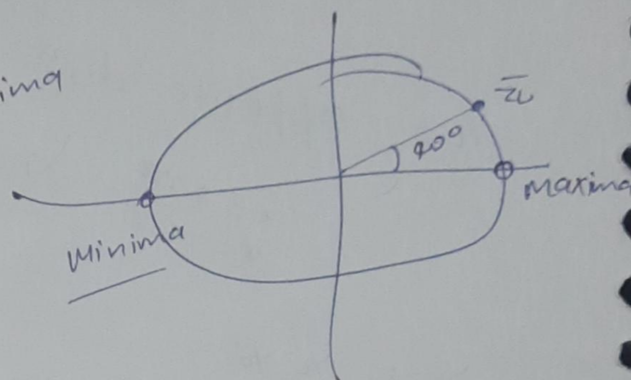
$= 0.6 (720^\circ)$ $\quad l = 0.6d$ → line length.

$= 432^\circ$

↓

$360^\circ + 72^\circ$

2 maxima → 1 minima



1st maxima $\frac{90^\circ}{720^\circ} \times d$

$= 0.05 d$

1st minima $= 0.05d + d/4$ — distance b/w two consecutive maxima & minima

$= 0.05d + 0.25d$

$= 0.3d.$

2nd maxima $= 0.3d + d/4$

$= 0.55d.$

(f) $Z_{in} = 0.6d.$

Calculate the input impedance

$r = 1.8$

$x = -2.2$

$\bar{Z}_{in} = r + jx$

$= 1.8 - j2.2.$

$Z_{in} = 75 (1.8 - j2.2).$

$$V_s(z) = V_0^+\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right]$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$I_s(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} - \Gamma e^{j\beta z}\right]$$

Short circuit : $(Z_L = 0)$

$$\Gamma = -1 \quad , \quad s = \infty.$$

$$V_s(z) = V_0^+\left[e^{-j\beta z} - e^{j\beta z}\right]$$

$$= -V_0^+ \, 2j \left[\frac{e^{j\beta z} - e^{-j\beta z}}{2j}\right]$$

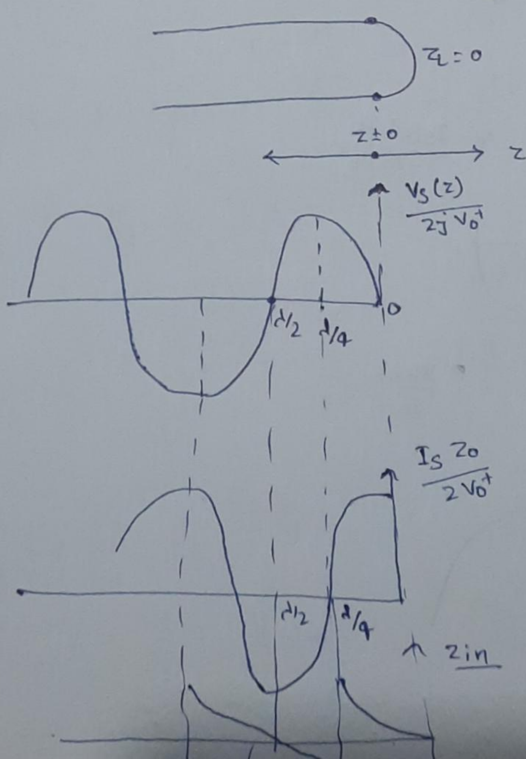
$$\boxed{V_s(z) = -V_0^+ \, 2j \sin\beta z}$$

$$I_s(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} + e^{j\beta z}\right]$$

$$= \frac{V_0^+}{Z_0} \times 2\cos\beta z.$$

$$\boxed{I_s(z) = \frac{2V_0^+}{Z_0}\cos\beta z}$$

$$\boxed{Z_{in} = j Z_0 \tan\beta l.}$$



Open circuit $(Z_L = \infty)$ $\quad \Gamma = 1 \quad ; \quad S = \infty$

$$V_s(z) = V_0^+ \left[e^{-j\beta z} + e^{j\beta z} \right]$$

$$= V_0^+ \, 2\cos\beta z$$

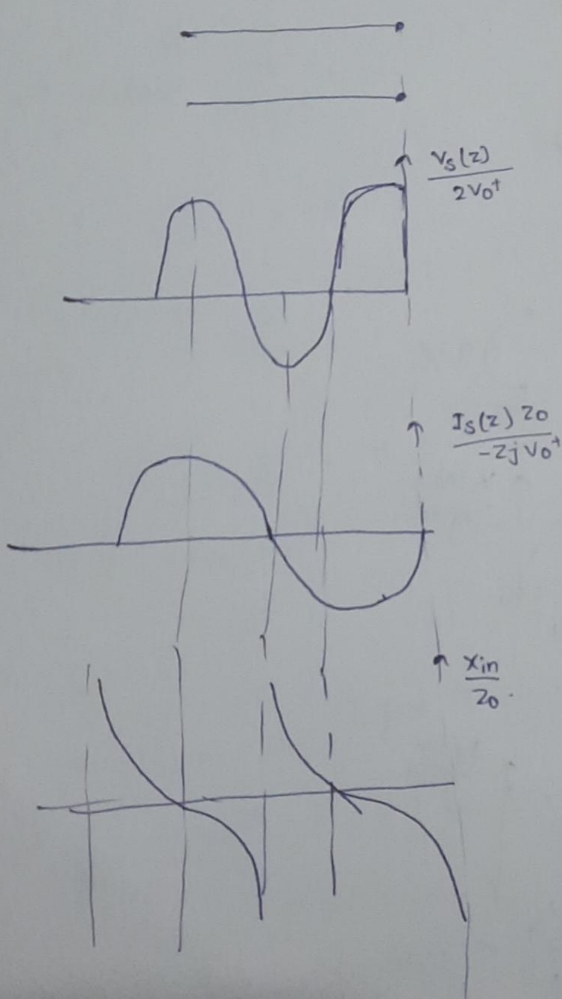
$$= 2V_0^+ \cos\beta z$$

$$I_s(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - e^{j\beta z} \right]$$

$$= \frac{V_0^+}{Z_0} \, -2j \sin\beta z$$

$$= -\frac{2V_0^+}{Z_0} \, j \, \sin\beta z.$$

$$\boxed{\hat{Z}_{in} = -jZ_0 \cot\beta l.}$$



$$\boxed{T = 1+\Gamma}$$

$$T = 1 + \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{2Z_L}{Z_L + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$

Insertion loss (IL)

$$I_L = -20 \log |T| \quad dB.$$

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Stub matching

- A stub is a short section of transmission line which is either short circuited or open circuited at one end.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$ → for impedance matching stub is used.

Q. $Z_L = 60 + j\,80\,\Omega$

$Z_0 = 50\,\Omega$

$$\bar{z}_L = \frac{60}{50} + j\frac{80}{50} = 1.2 + j\,1.6 \qquad S = \frac{OP}{OO} = \frac{4.6}{7.8}$$

step-1 mark point $\bar{z}_L$ (1)

step-2 (2) - circle

Mark the point $\bar{y}_L$ - (diagonally opposite) (3)

$$\bar{y}_L = 0.325 + j\,0.4.$$

↓ moving towards the generator.

⇓ purpose to the plane.

$\vec{Y}_{im} = 1 \pm jb.$

• $1 + j\,1.45$

$\bar{Y}_{im} = 1 \pm j\,1.45$

(2) (3) ↓ Introduce a stub.

• $1 - j\,1.45$

Extend the point $1+j1.95 \rightarrow$ (4)

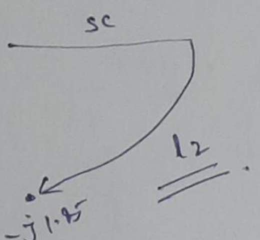
(3) - $\bar{Y}_L$

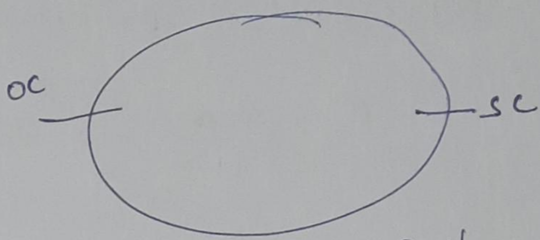
mark the point $+j1.95$

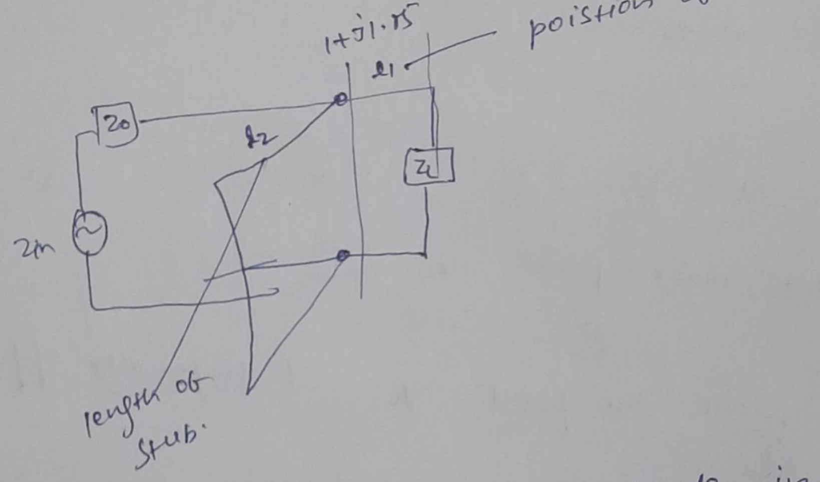
mark the point $-j1.95$

Extent point (4) outside the circle
Extent point (3) " " circle
$\bar{Y}_L$

} Mark the length as l_1.

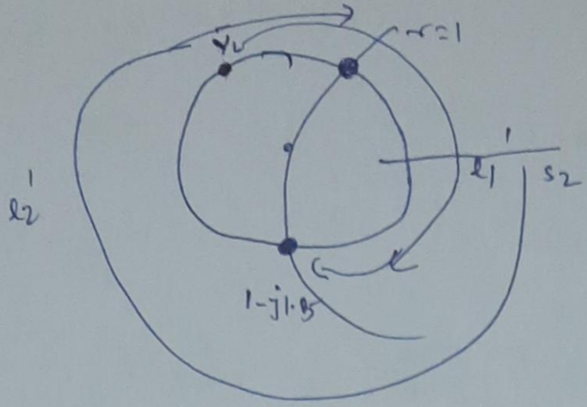


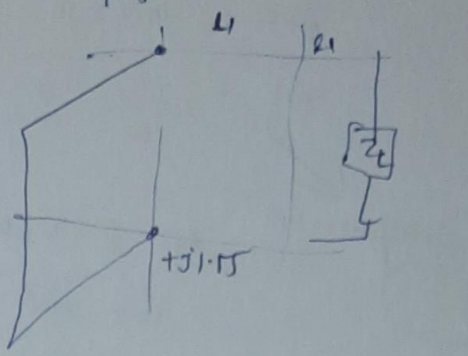




To measure l_1 and l_2 - use outer scale in wavelength.

NPTEL Prof. Girish Kumar IIT B





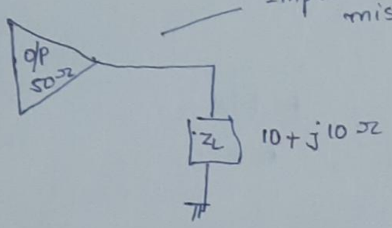
Input Impedance and VSWR

$$P_r = |\Gamma|^2$$

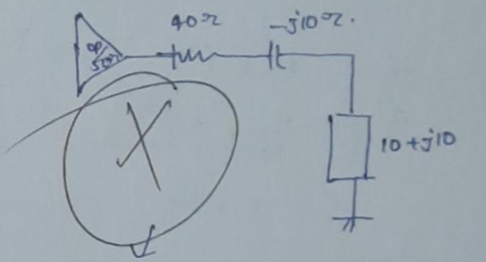
$$\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0}$$

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Lossy Impedance Matching



→



Bad Solution as resistor will consume power.

Smith chart A/Q.

Q. $Z_L = 10 + j10\ \Omega$ $\quad Z_0 = 50\ \Omega$

1. Locate Z_L on Smith Chart

$$\bar{Z_L} = \frac{Z_L}{Z_0} = \frac{10}{50} + j\frac{10}{50} = 0.2 + j0.2$$

① to ②

$$j(x_1) - (+jx_2)$$

$$j0.4 - j0.2 = j0.2. \quad \bar{X_L} \text{ — change in } z$$

Inductive Reactance.

Path ↑ change in Susceptance

$$= (-jb_0) - (-jb_1)$$

$$= -j0 - (-j2) = j2 \quad \bar{Y_L}$$

capacitive Susceptance — add capacitor in parallel.

$j\omega L = j0.2 \times 50 = j10 \Omega$

$j\omega C = \frac{j2}{50} = j0.04 \, ℧$

Susceptance

$\frac{1}{\cdot}$

At $f = 2\text{GHz}$.

$\omega = 2\pi f$

$j2\pi f L = j10$

$L = \frac{10}{2\pi f}$

$= 0.8 \text{ nH}$

$2\pi f C = 0.04$

$C = \frac{0.04}{2\pi f}$

$C = 3.2\text{pF}$.

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Matching Network

$z_L = R_L + jX_L$

$\bar{z}_L = \frac{R_L}{Z_0} + j\frac{X_L}{Z_0}$

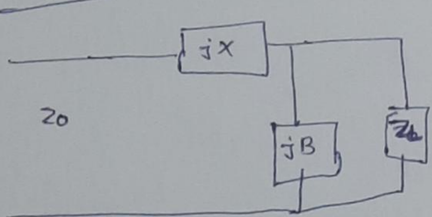
$= r + jx$

$r > 1 , \quad R_L > Z_0$

$r < 1 , \quad R_L < Z_0$

case-1

$R > Z_0$



jX - Series reactance

jB - Shunt Susceptance.

for impedance matching, we must have

$$Z_0 = jX + \frac{1}{jB + \frac{1}{R_L + jX_L}}$$

$$= jX + \frac{R_L + jX_L}{jBR_L - BX_L + 1}$$

$Y_{eq} = Y_1 + Y_2$

$= jB + 1/Z_L$

$Z_{eq} = \frac{1}{Y_{eq}}$

Equating real and imaginary

$$B = \frac{X_L \pm \sqrt{X_L^2 + (R_L^2 + X_L^2)\left(\frac{R_L}{Z_0} - 1\right)}}{(R_L^2 + X_L^2)}$$

$$X = \frac{Z_0 B R_L - X_L}{1 - B X_L}$$

Q. $Z_L = 100 - j50\ \Omega$ $\quad Z_0 = 50\ \Omega$. $\quad f = 500\ MHz$

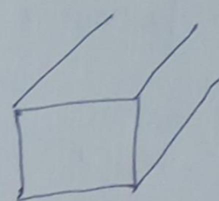
Soln:

$R_L = 100$ $\quad Z_0 = 50\ \Omega$

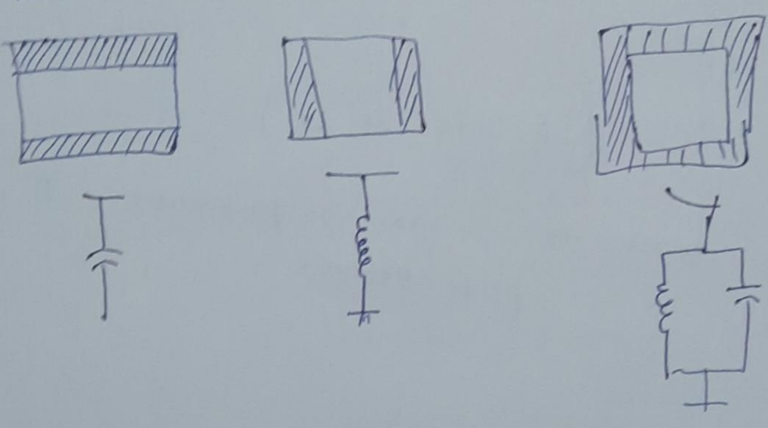
$X_L = -50$

$B =$

Slotted line



Iris



23/1/24

Waveguides

- High pass filter
- passive microwave device
- cut-off frequency (below operating freq., no-propogation)
- Power loss occurs in the walls

General solutions for TEM, TE and TM waves (Ref. David H Pozar).

$$\bar{E}(x,y,z) = [\bar{e}(x,y) + \hat{z}\, e_z(x,y)]\, e^{-j\beta z}$$

$$\bar{H}(x,y,z) = [\bar{h}(x,y) + \hat{z}\, h_z(x,y)]\, e^{-j\beta z}$$

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$

$$\nabla \times \bar{H} = j\omega\epsilon \bar{E}$$

TEM $E_z = H_z = 0$ (E & M is tr)

TE to z $\longrightarrow$ w.r.t to direction of propogation, $\vec{E}$ is perpendicular.

Both $\vec{E}$ and $\vec{H}$ are perpendicular in the dirn of propagation.

TEM ($E_z = H_z = 0$)

TE ($E_z = 0$)

TM ($H_z = 0$)

HE (hybrid modes in which all compomonents.

$K = \frac{2\pi}{\lambda}$ $\qquad K_c = \frac{2\pi}{\lambda_c}$

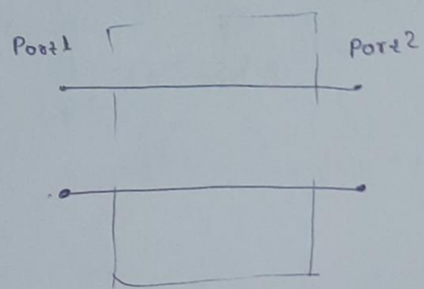
↓ for a particular frequency

$$\left.\begin{aligned} \nabla^2 E + K^2 \vec{E} &= 0 \\ \nabla^2 H + K^2 \vec{H} &= 0 \end{aligned}\right\} \text{Halmon's equation.}$$

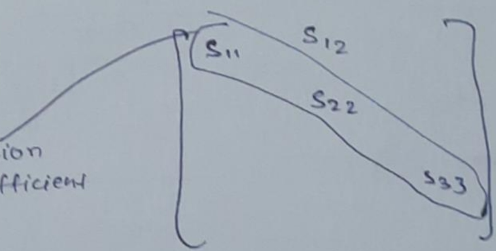
path length '↓' as frequency '↓'.

<u>wave propagation</u>

↳ At lower freq.



— two port model



reflection coefficient

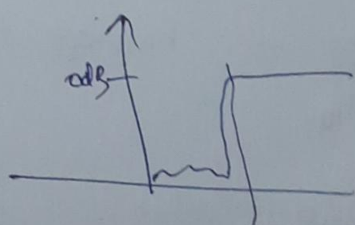
S_{ii}

S_{ij} (Transmission coefficient)

j - input port

i - output port.

S_{12} — Input at port 2 and output at port 1.



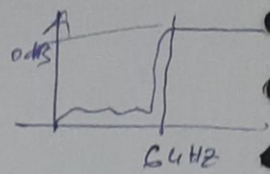
fc - cut-off frequency

$TE_{10} = 6$ GHz — lowest freq for TE_{10} mode.

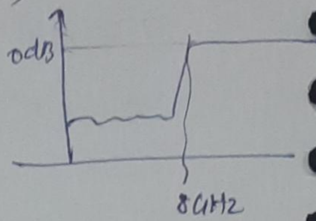
$TE_{20} = 8$ GHz

$TE_{01} = 11$ GHz

S_{12} (1)

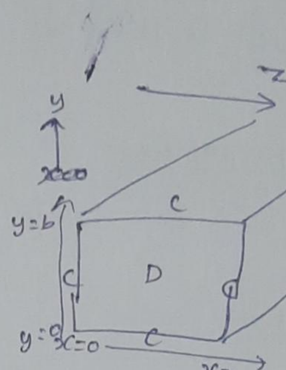


S_{12} (2)



$$\boxed{\begin{array}{l} f > f_c - PM \\ f < f_c - NPM \end{array}}$$

0dB - good



TM Modes

$H_z = 0$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) e_z(x,y) = 0$$

$$k_c^2 = k^2 - \beta^2$$

ϵ_1 / ϵ_2

$E_{z1} = E_{z2}$

$D_{n1} - D_{n2} = \rho_s$

ϵ_1 / cond.

$E_{z1} = 0$

$D_{n1} = D_{n2}$.

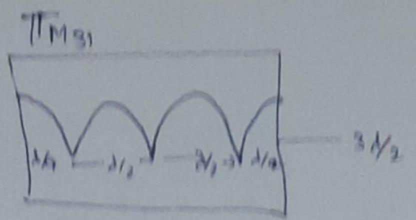
TM modes

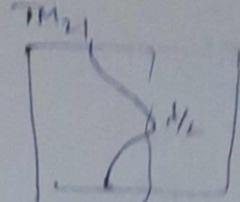
$$\left.\begin{array}{l} m = 1, 2, 3 \ldots \\ n = 1, 2, 3 \ldots \end{array}\right\} \text{(not zero)}$$

TM_{01} or TM_{10} — none of the field will exits.

TE mode $m = 0, 1, 2, 3 \ldots$

$n = 0, 1, 2, 3 \ldots$





Q. A rectangular wave guide ($a = 2.5$ cm, $b = 1$ cm) is to operate below 15.1 GHz. How many TE and TM modes can the waveguide transmit if the waveguide is filled with a medium characterized by $\sigma = 0$, $\epsilon = 4\epsilon_0$, $\mu_r = 1$. Calculate the cut-off frequencies of the mode.

Soln:-

$$\epsilon = \epsilon_r \epsilon_0$$

$$\mu = \mu_r \mu_0$$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u' = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0 4\epsilon_0}} = \frac{c}{2}$$

$$f_c = \frac{1}{2} \times \frac{c}{2} \times \frac{1}{a}\sqrt{m^2 + \frac{a^2}{b^2} \times n^2}$$

$$= \frac{3 \times 10^8}{4} \times \frac{1}{2.5 \times 10^{-2}}\sqrt{m^2 + (2.5)^2 n^2}$$

$$= 3/_{10} \times 10^9 \sqrt{m^2 + 6.25\, n^2}.$$

TE_{01} $\quad f_{c_{01}} = 3/_{10} \times 10^9 \sqrt{(6.25)\,1}$

$$= 3/_{10} \times 10^9 \times \frac{25}{10} = 7.5 \text{ GHz}.$$

TE_{02} $\quad f_{c_{02}} = 15$ GHz

TE_{03} $\quad f_{c_{03}} = 22.5$ GHz $\longrightarrow$ not possible

TE_{11} TM_{11}

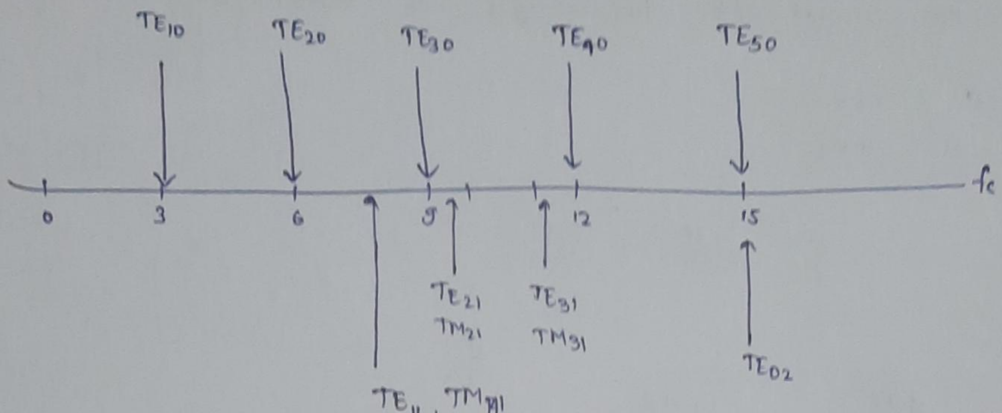
TE_{21} TM_{21}

TE_{31} TM_{31}

TE_{41} TM_{41}

TE_{12} TM_{12} $\boxed{f_{c_{12}} = 15.3 \text{ GHz}}$ X.

11 TE modes 4 TM modes.



TE_{10} – dominant mode of rectangular waveguide.

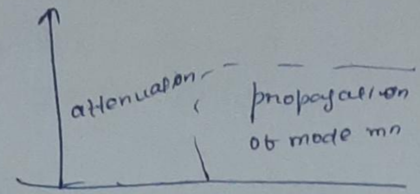
TM_{11} – dominant TM mode of rectangular waveguide

lowest frequency

Degenerate mode – Same cut-off frequency.

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$k > k_c$: β is real



Basic Quiz

6th feb — Transmission line Theory, Smith Chart.

Question

- No. of maxima & min

Q. 10.8
Ref. (Chen)

Surface current density

$$\vec{J_s} = \hat{a}_n \times \vec{H}$$

TE_{10} mode :

↳ write eq^n

$$P_{10} = \frac{1}{2} Re \int_{x=0}^{a} \int_{y=0}^{b} \vec{E} \times \vec{H}^* \, \hat{a}_z \, dy \, dx.$$

$$P_{10} = \frac{\omega \mu a^3}{2\pi^2} Re(\beta) |A_{10}|^2 \int_{x=0}^{a} \int_{y=0}^{b} \sin^2 \frac{\pi x}{a} \, dx \, dy.$$

$$\vec{E} = E_y \hat{a}_y$$

$$\vec{H} = H_x \hat{a}_x + H_z \hat{a}_z$$

TE_{10}

$$\vec{E} \times \vec{H} = -E_y H_x^* \hat{a}_z + E_y H_z^* \hat{a}_x.$$

Attenuation due to dielectric
" " " conductor

$$\alpha = \alpha_d + \alpha_c.$$

$$\gamma = \alpha_D + j\beta = \sqrt{k_c^2 - k^2}$$

$$\boxed{\alpha_D = \frac{k^2 \tan\delta}{2\beta}}$$ — TE or TM mode

$$\boxed{\alpha_D = \frac{k \tan\delta}{2} \text{ Np/m}}$$ (TEM modes).

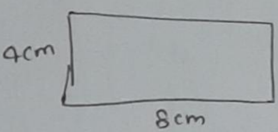
- Transmission line
- Smitch chart
- Matching Network
- Rectangular waveguide
- Rectangular Cavity
- S- parameters.
  power divider

Mid-sem

Quiz!
- Transmission line
- Smitch Chart

Group - 2
14th feb
Mid sem lab
↳ CST lab.

Q.



Operates at TE_{10} modes.
calculate
f_c, v_g,

(a) $f_c = \dfrac{c}{2a} = \dfrac{3\times10^8}{2\times 8\times10^{-2}} = 1.875$ GHz.

$v_g = \dfrac{v_p}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{3\times10^8}{\sqrt{1-\left(\frac{1.87}{3.75}\right)^2}} = 3.48\times10^8$ m/s

phase velocity

$\lambda_g = \dfrac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{c/f}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$.

Q2.

$\lambda_0 = \dfrac{c}{f_0}$

Now, $\dfrac{1}{\lambda_0^2} = \dfrac{1}{\lambda_g^2} + \dfrac{1}{\lambda_c^2}$

$\lambda_c = 2a$
$= 6.028$ cm

$\lambda_g = 4$cm given

$a = \lambda_c/2$.

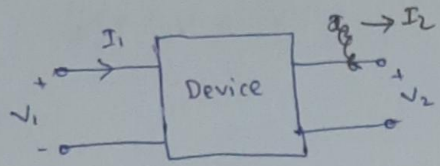
e.g. 3.1 Pozer

$R_s \; \alpha_c = R_s$

Ex 10.9

30/01/2024

Microwave Network



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}.$$

Output open-ckt

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

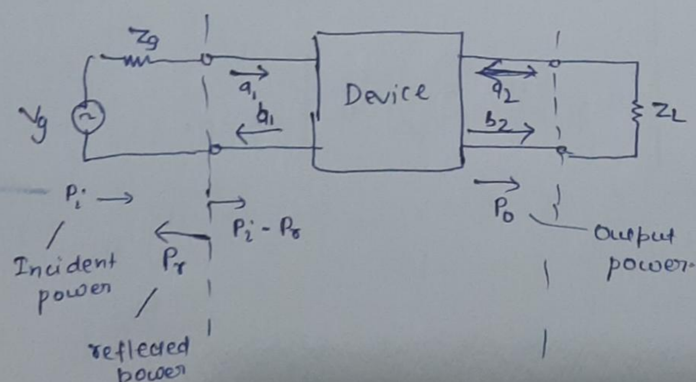
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}.$$ h-parameter

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & +B \\ C & +D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}.$$ — ABCD parameter

$\rightarrow I_2 \rightarrow +I_2$

$\leftarrow I_2 \rightarrow -I_2$

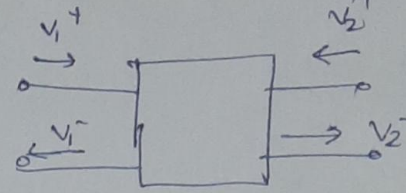
S-parameters (Scattering parameters)



a – incoming
b – outgoing.

a's represent normalized incident wave amplitude

b's " " reflected " "



$$a_1 = \frac{V_1^+}{\sqrt{Z_0}} = \frac{V_1 - V_1^-}{\sqrt{Z_0}}$$

$V_1 = V_1^+ + V_1^-$

$$b_1 = \frac{V_1^-}{\sqrt{Z_0}} = \frac{V_1 - V_1^+}{\sqrt{Z_0}}$$

$V_2 = V_2^+ + V_2^-$

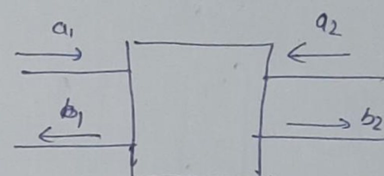
$$a_2 = \frac{V_2^+}{\sqrt{Z_0}} =$$

$$b_2 = \frac{V_2^-}{\sqrt{Z_0}}$$

Total or net power flow into any point is given by,

$$P = P_i - P_r = \frac{1}{2}\left(|a|^2 - |b|^2\right)$$

For 2- port network



$b_1 = S_{11}\, a_1 + S_{12}\, a_2$

$b_2 = S_{21}\, a_1 + S_{22}\, a_2$

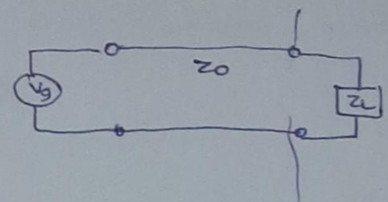
Reflection coefficient

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1 = 0}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1 = 0} \qquad S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0}.$$

represents matching of the terminal.

$\Gamma = \frac{V_{ref}}{V_{inc}}$



$\Gamma = 0$, $V_{ref} = 0$.

$Z_L \neq Z_0$

$\Gamma = 1$ — Short circuit.

$Z_L = Z_0$ (matched)

$P_{ref} = 0$

$0 \leq |\Gamma| \leq 1$

$1 \leq S < \infty$.

VSWR.

Ideally matched

reflection coefficient $= 0$ $\Rightarrow$ $S = 1$.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$ — reflection coefficient.

$S_{ii} \leq -10\,dB$ — SWR ≤ 2

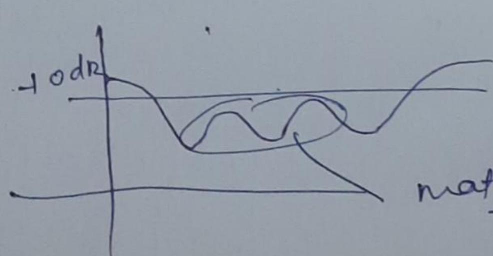
SWR $= 1$ ↳ Matched condition.

Q. specified $|\Gamma| = 1/2$ and $P_{in} = 1W$ what will the amplitude

$P_r = |\Gamma|^2 P_{in}$

$= 1/4 \times 1 = 1/4\,W$

$P_{in} - P_{ref} = 1 - 1/4 = 3/4\,W$ - power delivered to the load.



matched port.

→ To check matched port, S_{ii} must be less than -10 dB

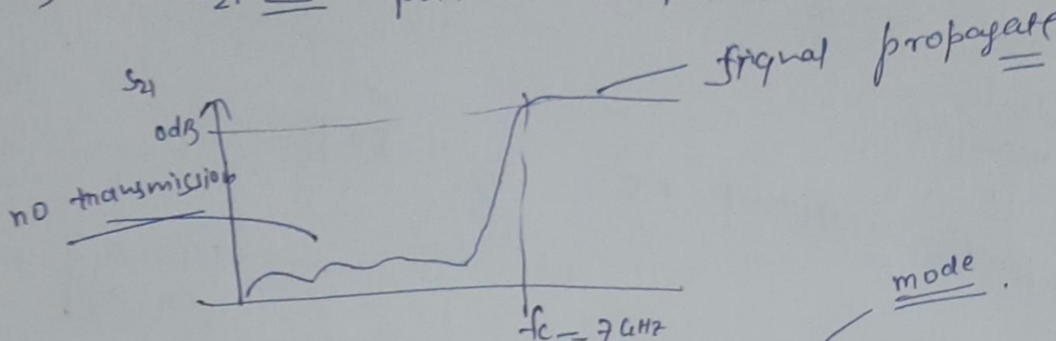
$$S_{ii} \leq -10\,dB.$$

S_{ij} – transmission co

Input port / j – port no. for incident wave

Output port / i – port no for reflected wave.

→ S_{21} = port 1 to port 2.



TE_{10} – $f_c = 7\ GHz$

TE_{01} – $f_c = 10\ GHz$

TE_{20} $f_c = 14\ GHz$