Astrophysics

Books

1. Pankaj Jain, *Introduction to Astronomy and Astrophysics*, Boca Raton, CRC Press, 2015

2. Bradley W. Carroll and Dale A. Ostlie, *An introduction to modern Astrophysics*, Addison Wesley, 2007.

Astronomical scale and dimensions

Astronomical Unit (AU)

The radius and mass of the Earth:

$$R_E = 6378\ Km. \qquad M_E = 5.974 \times 10^{24}\ Kg$$

The sun is about a million times more massive with about 100 times larger radius:

$$R_s = 6.96 \times 10^5\ Km. \qquad M_S = 1.989 \times 10^{30}\ Kg$$

The Earth-Sun distance is called one Astronomical Unit (AU)

$$1\ AU = 1.496 \times 10^8\ Km$$

Nearest star : The Sun

The next nearest star : Proxima Centauri at a distance of 1.31 pc

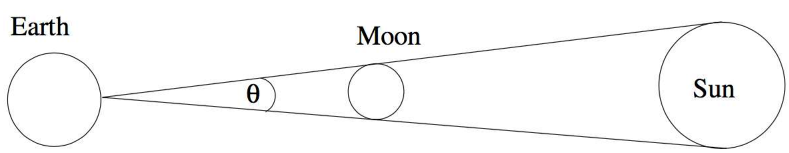
$$1\ parsec\ (pc) \approx 3\ \times 10^{13}\ Km$$

Hence, the next nearest star is roughly 200,000 times the distance to the Sun.

Another unit of distance in astronomy is a light year – distance travelled by light in 1 year.

$$1\ pc = 3.26\ light\ years$$

∴ Proxima centauri is 4.3 light years away.
The current best estimate of the age of the Universe is about 14 Giga years (4 Gpc)



The angle θ subtended by the Sun at Earth is approximately the same as that subtended by the Moon on Earth.

$$\theta = \frac{2\ \times\ 6.96\ \times\ 10^{6}}{1.496\ \times\ 10^{8}} = 0.0093\ radians$$

$$\approx 0.5\ degrees$$

Our Universe contains wide range of structures

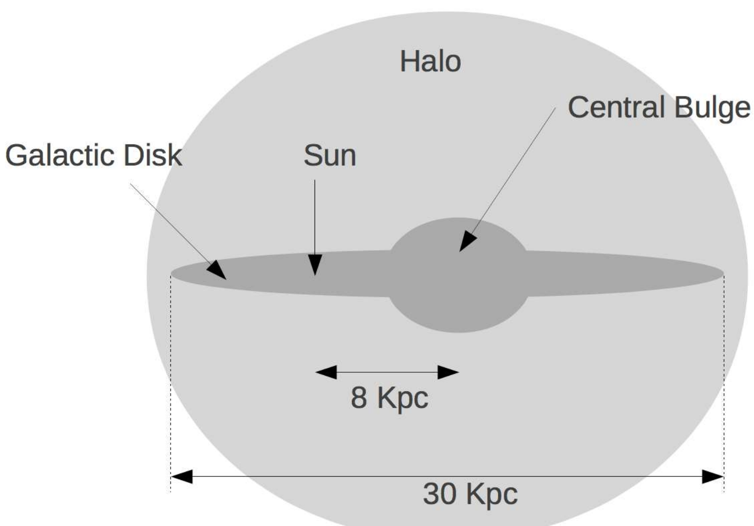
- The Universe includes *the solar system* and *the planetary systems* associated with other stars.
- The stars themselves often form clusters that are part of bigger structures called *galaxies*.
- Furthermore, the galaxies are also not found in isolation and form groups or clusters of galaxies that form larger clusters called *superclusters*.
- The superclusters are the largest structures observed.
- Universe is roughly 50 times larger than the size of the largest supercluster.

Galaxies:

They are organized structures, with a high density of matter near the center. Many galaxies show evidence of rotation about the center.

Example:
Milky way--
consists of over 200 billion stars.

Milky way Galaxy



One video on milky way

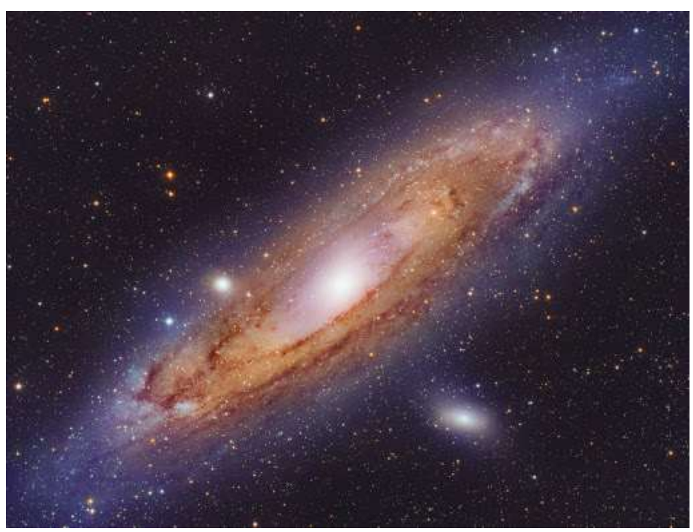
Milky way Galaxy (from Earth)

The "milky" is derived from the appearance from Earth of the galaxy – a band of light seen in the night sky formed from stars that cannot be individually distinguished by the naked eye.

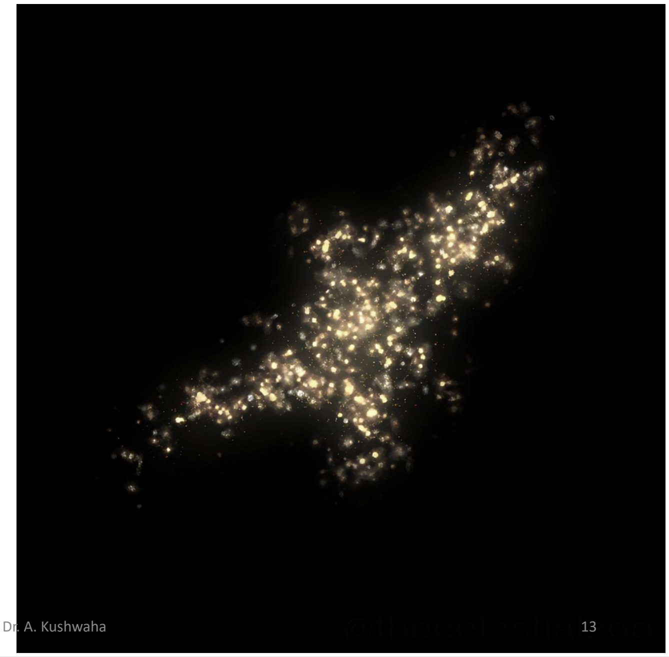
From Earth, the Milky Way appears as a band because its disk-shaped structure is viewed from within.

Andromeda Galaxy

Light from the Andromeda galaxy (located at a distance of about 1 Mpc) left this galaxy about 3 million years ago.



Superclusters



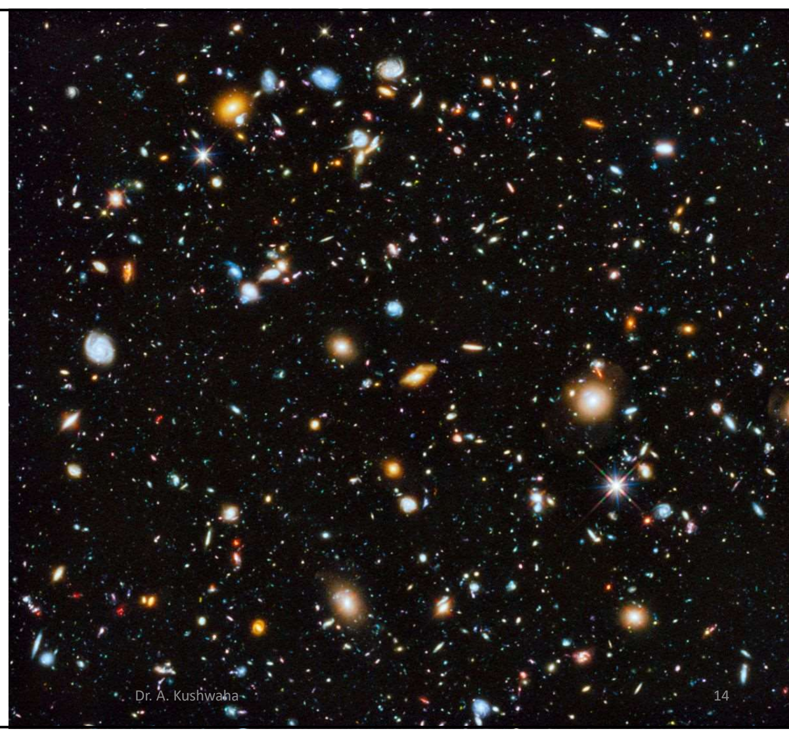
They are the largest structures seen in the universe.

Typical distance of superclusters is about 10 to 100 Mpc.

Hubble space telescope's image showing thousands of galaxies

Size of the observable universe is on the order of a few Gpc
($1\ Gpc\ = 10^9\ pc$)

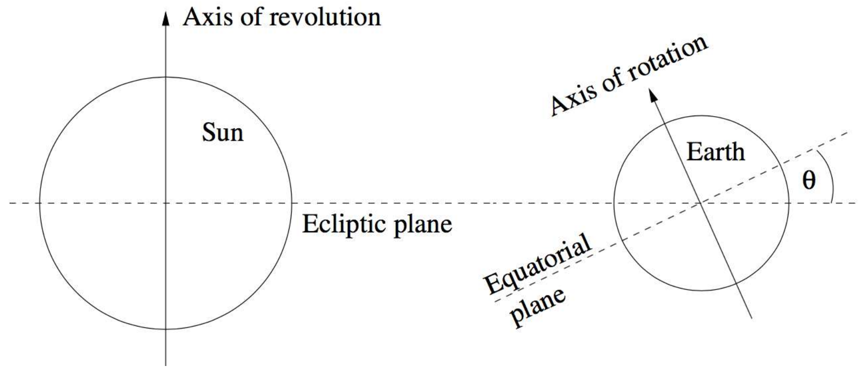
The observable universe contains on the order 100 billion galaxies. The typical distance scale of galaxy clusters range from 1 to 10 Mpc (1 Mpc = 1 million pc)



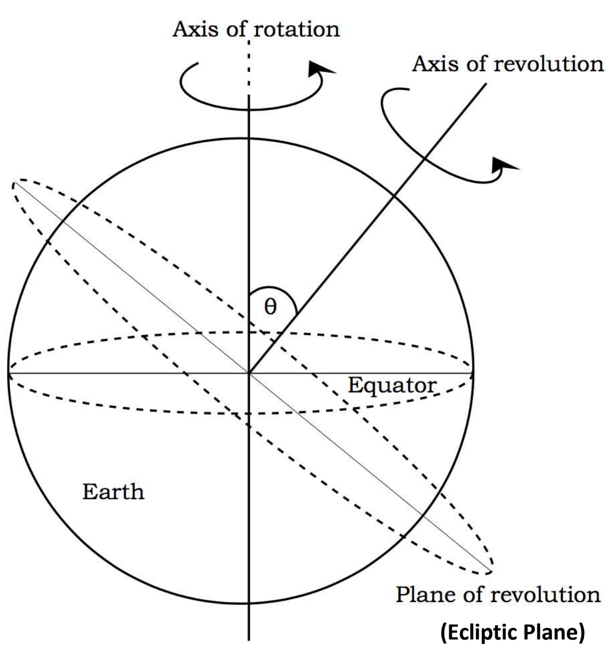
Night sky

The night sky is continuously changing due to

1. the rotation of the Earth about its axis.
2. revolution of the Earth around the Sun.



The plane of revolution of the Earth around the Sun is called the ecliptic plane.



The axis of rotation of the Earth is perpendicular to the equatorial plane.

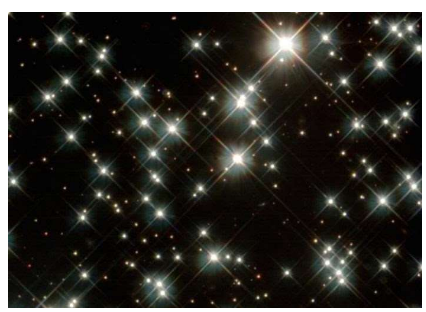
The plane of revolution of the Earth around the Sun is inclined at an angle $\theta = 23.5\,°$ with respect to the equatorial plane.

In the night sky, we can see large variety of objects:

- Stars
- planets
- shooting stars or meteors
- meteorites
- comets

Stars

- *A star* is approximately fixed in space.
- It appears to move only due to the motion of the observer on Earth.
- Its position relative to other stars or with respect to the Sun does not show much change over a period of days or even a year.
- It changes at a very slow rate due to the proper motion of stars.



In the northern hemisphere, few stars are prominent in summer:

- Vega
- Altair
- Deneb

In the northern hemisphere, few stars are prominent in winter: Capella, Betelgeuse, Sirius.

Planets

A planet moves around the Sun in an elliptical orbit.

Its position changes significantly with respect to the background fixed stars.

Meteoroids

- There also exist many interesting phenomena caused by smaller objects in the solar system called meteoroids.
- It is made up of a piece of rock, ice and metals ($10\ \mu m\ to\ 1\ m\ wide$).
- Meteoroids may enter the Earth's atmosphere at high speeds on the order of 10 to 75 Km/s.
- A small meteoroid last a very short interval and produces a short streak of light called a *meteor or a shooting star*.

Bright, basketball- sized Meteor in Ontario (2014)

Image credit: University of western ontario

- On some dates one might see a meteor shower, containing a large number of meteors, which appear to diverge from a small region called the *radiant*.
- If the mass of the meteoroid exceeds 1 Kg, then it might survive until it reaches ground level. The resulting fragment is called a *meteorite*.

Meteor shower

4/22/2021 Dr. A. Kushwaha 25

Comets

- They consist of a nucleus and a tail.
- The nuclei are composed of ice, rock and dust.
- They have diameters ranging from a few hundred meters to tens of kilometers.
- Most of the comets are too dim to be seen by the naked eye.
- On average, one might be bright enough to be observable without a telescope every 10 years.

4/22/2021 Dr. A. Kushwaha 26

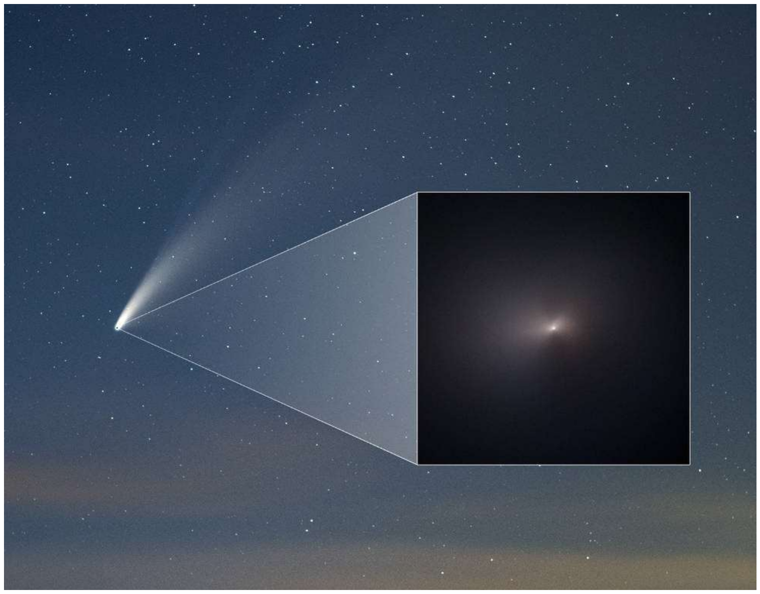


Image of Comet **NEOWISE.**
Inset: close-up of the comet after its pass by the sun.

Constellations

- The stars are grouped into different constellations.
- Historically, these groupings were made on the basis of some figure, these groups of stars appear to resemble.
- The choice of these figures was made entirely on the basis of human imagination.

- Constellations played an important role in ancient times because they were very useful for navigation.
- A person trained in reading the night sky could use them to identify different directions.
- There are a large number of mythological stories associated with stars and constellations.
- Instead of just viewing stars, one now sees gods, goddesses, saints, warriors, nymphs, etc. in the night sky. It is filled with drama, action, and poetry.

- The entire sky was divided into 88 constellations.
- The constellation boundaries were made precise by the International Astronomical Union (IAU) in 1928.
- The boundaries were chosen on the basis of constant right ascension and declination using the equatorial coordinate system.
- The coordinate system is updated every 50 years in order to account for the precession of the Earth's rotation axis. Hence, these boundaries do not correspond to constant right ascension and declination in the equatorial system, J2000, currently in use.

- These constellations play an extremely important role if one wants to recognize a particular star in the sky.
- A person trained in recognizing the constellations can identify a star even if he/she does not know its precise coordinates.
- Some examples of the constellations are Andromeda, Aries, Centaurus, Cygnus, Orion, etc.

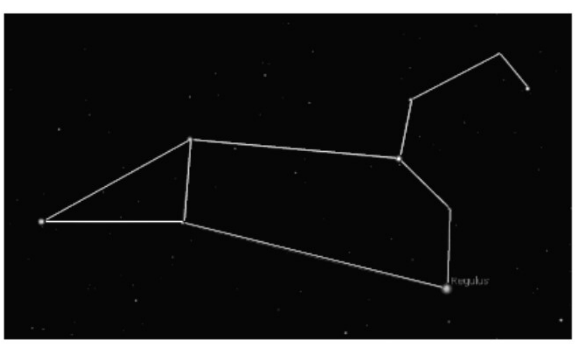
Some of the prominent constellations visible in the Northern hemisphere at different times of the year.

The Orion constellation

- The Orion constellation, named after the mythological hunter, Orion.
- The brightest stars in this constellation are Rigel and Betelgeuse.

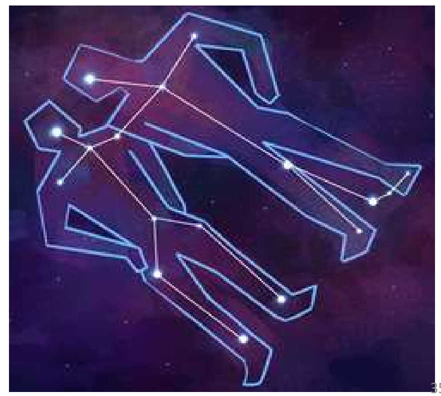
The Leo constellation

- The Leo constellation which roughly appears as a lion.
- The brightest stars in this constellation are Regulus and Beta Leonis.



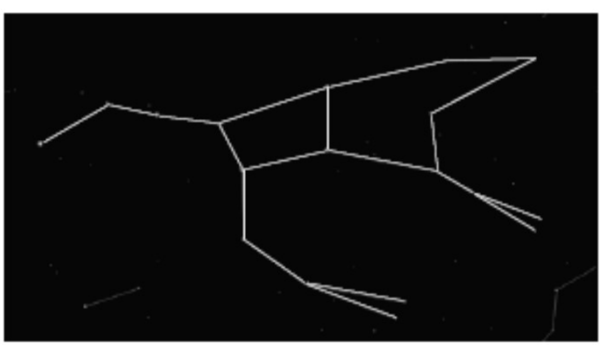
The Gemini constellation

- The Gemini constellation takes the shape of twins standing next to one another.
- Gemini is the Latin word for twins.
- The two brightest stars in this constellation are named after the twins, Pollux and Castor, in Greek mythology.

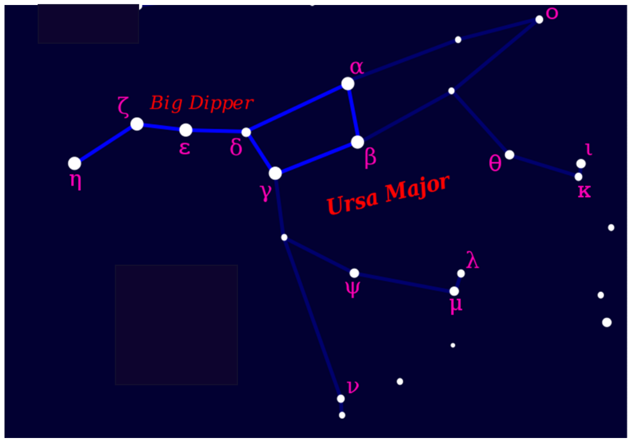


The Ursa Major constellation

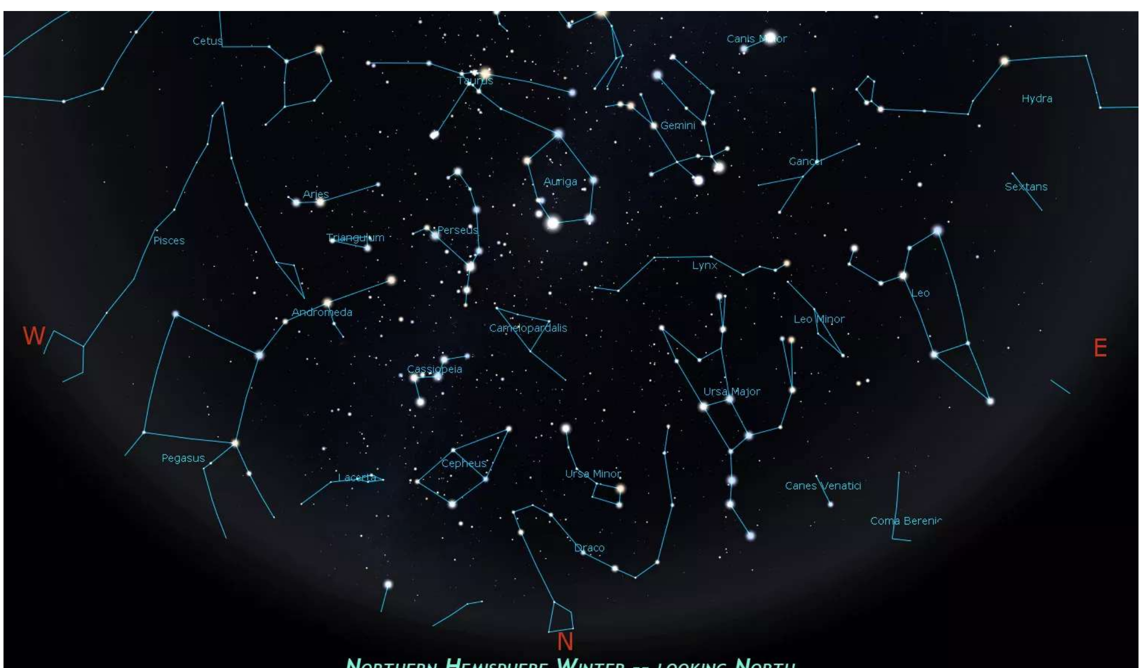
The Ursa Major constellation is also called the Great Bear.



- It contains seven bright stars seen in the upper left region of this figure.
- These are easily identifiable in the night sky and are collectively called the Big Dipper.
- These stars are also helpful in locating the North Star.

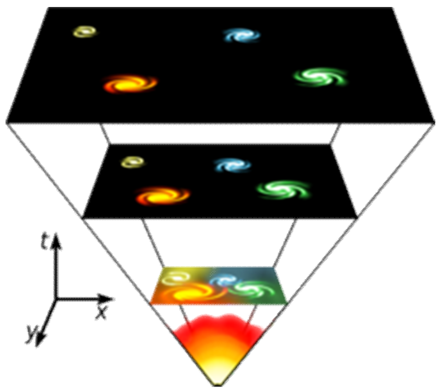


The Big Dipper (or Plough) within Ursa Major



Our Universe

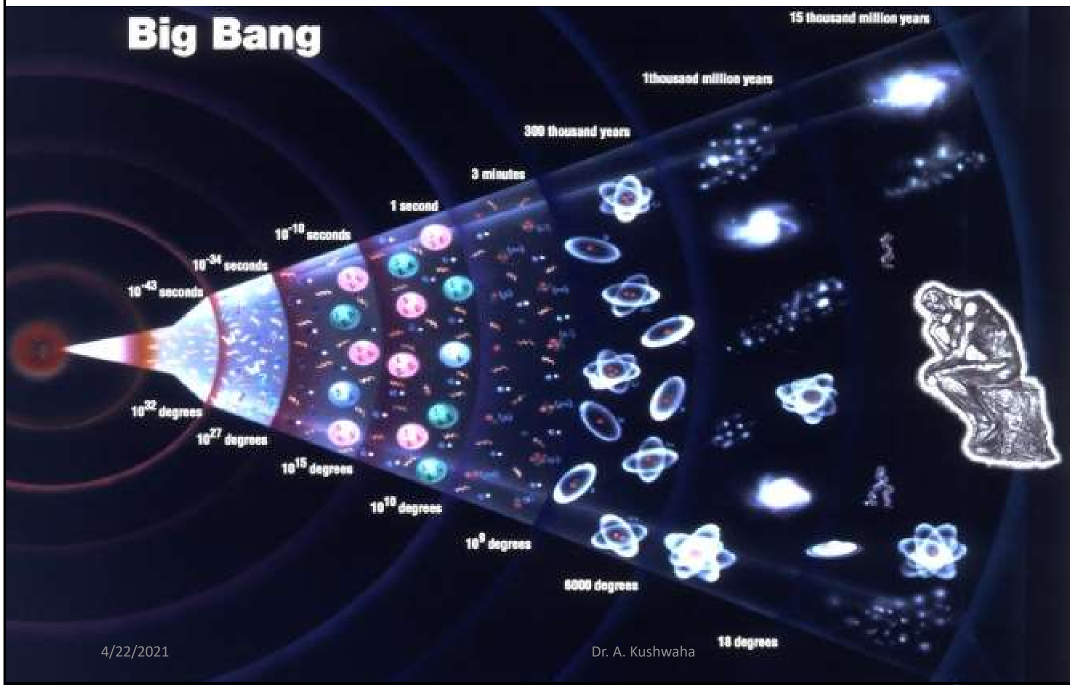
- The Universe is continuously evolving.
- The planets and the stars, which appear to us as everlasting, originated at some time in the past.
- The stars evolve with time, slowly exhaust their nuclear fuel, and eventually die.
- The galaxies also have an origin and slowly evolve into the structures we observe today.



An artist's concept illustrating the expansion of a portion of a flat universe.

Big Bang model

- The entire Universe also had an origin
- The model that appears to best describe the origin and evolution of the Universe is called the ***Big Bang model.***
- This model has several postulates.



- The initial compact, intensely hot fireball of matter and radiation from the big bang expanded, then cooled and underwent a series of transitions at specific temperatures.

- **From $10^{-43}s$ to 10^{-35} s:** Universe cooled from 10^{28} $to 10^{23}$ eV. (10^{-4} eV corresponds to ~ 1 K). At energies like these the strong, electromagnetic and weak interactions are merged into a single interaction.

- **At $10^{-35}s$:** the strong interaction became separated from the electroweak interaction. At this time, the universe was only about a millimetre across.

- **From 10^{-35} to $10^{-10}s$:** The universe consisted of a dense soup of elementary particles whose behaviour was controlled by the strong, electroweak and gravitational interactions.

- **At $10^{-10}s$:** the electroweak interaction became separated into the electromagnetic and weak components we observe today.

- **About 1s :** Nuclear reactions were starting to occur incorporating neutrons into the helium nuclei.

- **At 5 mins:** Nuclear synthesis stopped when the ratio of helium mass to total mass should have been between 24 and 24% -- Indeed this is the ratio in most of the universe today. Ex: In Sun's outer layers, the helium proportion is close to 28%.

- **From 5 mins to around 100,000 years after the Big Bang**:

- Matter & radiation were decoupled and the universe became transparent.
- We expect this remnant radiation to come equally strongly from all directions and to have a spectrum like that of a blackbody at 2.7 K
- Such radiation has been found in microwave measurements made from the earth and from satellites.

Three observations that strongly support Big-Bang cosmology:

- The uniform expansion of the universe
- The relative abundances of hydrogen and helium in the universe
- The cosmic background radiation

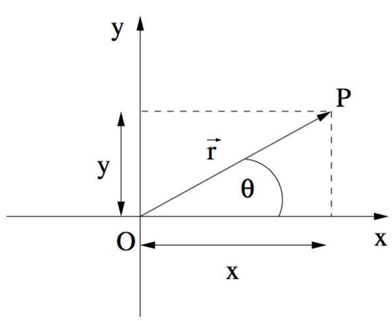
Astronomical coordinate systems

Astrometry

The branch of astronomy that deals with the positions and motions of heavenly objects.

The positions are specified in terms of their distances and angular positions on the sky.

Cartesian & Plane polar coordinates



Plane polar coordinates

$$\boxed{\begin{array}{l} r = \sqrt{x^2 + y^2} \\ \theta = tan^{-1}\dfrac{y}{x} \end{array}}$$

Cartesian coordinates

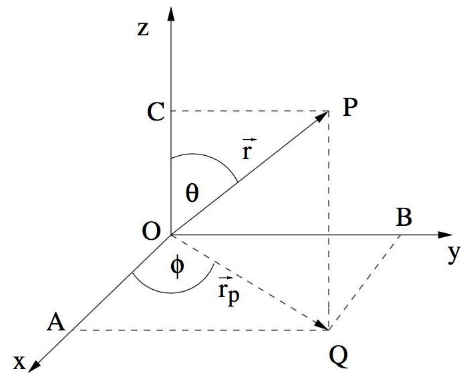
The projections of the position vector $\vec{r}$ on the $x-$ and $y-$ $axes$ are:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

For a general vector $\vec{A}$:

$$a_x = a\cos\theta\ .$$
$$a_y = a\sin\theta$$

Cartesian coordinates in 3D



The projections of the position vector $\vec{r}$ on the $x-$, $y-$, $z-$ $axes$ are:

$$x = r \sin\theta \cos\varphi$$
$$y = r \sin\theta \sin\varphi$$
$$z = r \cos\theta$$

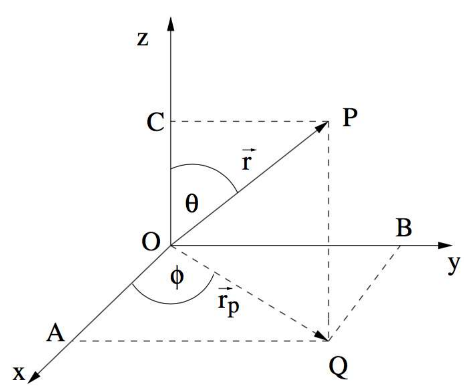
For a general vector $\vec{a}$:

$$a_x = a \sin\theta \cos\varphi \,.$$
$$a_y = a \sin\theta \sin\varphi$$
$$a_z = a \cos\theta$$

Range of values:

(x, y, z) can vary over all possible values from $-\infty$ to $+\infty$.

Spherical polar coordinates



Instead of (x, y, z) we can use (r, θ , φ)

r: radial coordinate
θ: polar coordinate
φ : azimuthal coordinate

Range of values:

$$r \geq 0$$
$$0 \leq \theta \leq \pi$$
$$0 \leq \varphi < 2\pi$$

(all the angles are given in radians)

Spherical polar coordinates in Astronomy

In astronomy, essentially spherical polar coordinates is used.

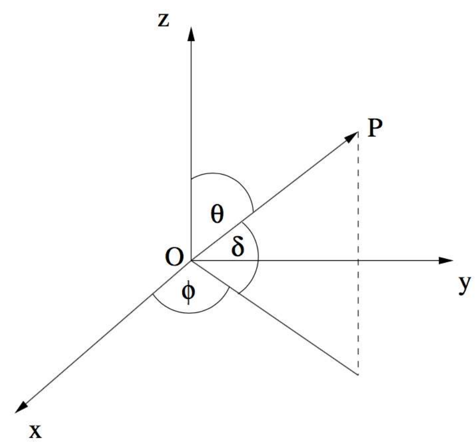
Instead of the polar coordinate θ, the **latitude**, δ, defined as

$$\delta = 90° - \theta$$

is used.

This coordinate take values in the range $-90°\ to + 90°$.

The azimuthal coordinate φ is equivalent to the **longitude.**



The angular coordinates (δ, φ) of point P

Celestial Sphere

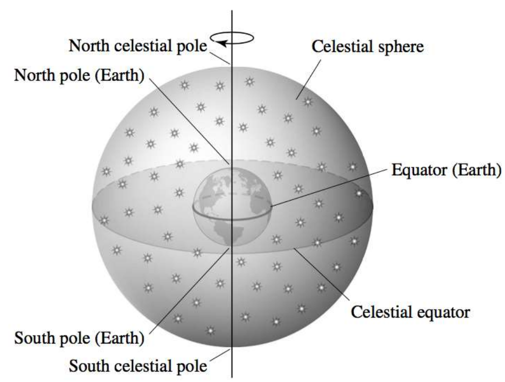
The entire surface of the Earth is mapped by longitudes & latitudes.

We can focus on the angular coordinates by projecting each object on an imaginary sphere with Earth as its center and very large (almost infinite) radius. This is called the **Celestial sphere.**

If the stars were simply attached to a celestial sphere that rotated about an axis passing through the north and south poles of Earth and intersecting the celestial sphere at the north and south celestial poles, all of the stars' known motions could be described.

The equatorial plane that is perpendicular to this axis also remains fixed. We extend this plane so that it cuts the celestial sphere. The intersection is called the equator of the celestial sphere.

The Celestial Sphere

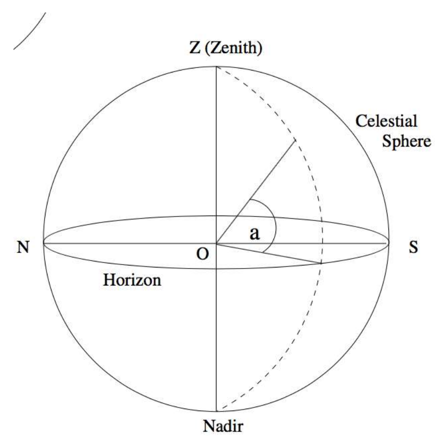


A **great circle** is the curve resulting from the intersection of a sphere with a plane Passing through the center of that sphere.

The **meridian** is a great circle – defined as passing through the observer's zenith and intersecting the horizon due north and south.

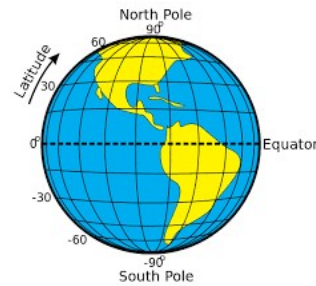
The great circle NZS is called the meridian.

All great circles passing through the zenith are called **verticals**.



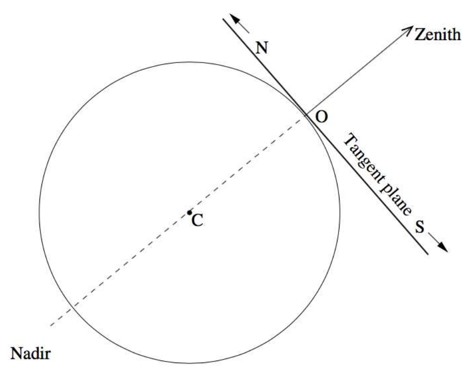
Types of Astronomical Coordinate systems

- The latitude is specified by choosing a reference plane or the equatorial plane, passing through the origin.
- This plane lies at latitude $\delta = 0$.
- The longitude is specified by choosing some reference direction.



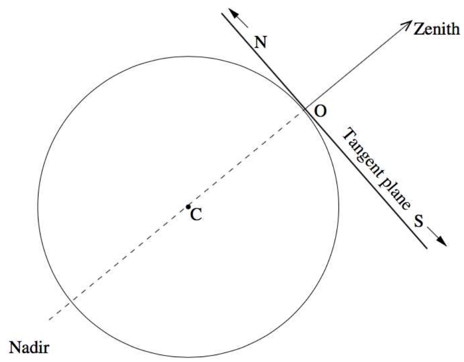
1. The altitude-azimuth (or horizon) coordinate system
2. Equatorial coordinate system
3. Ecliptic system
4. Galactic coordinate system

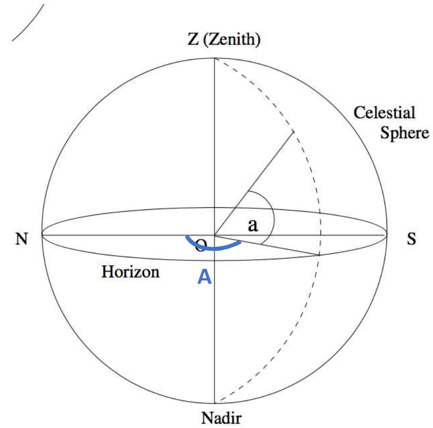
The altitude-azimuth (or horizon) coordinate System



This coordinate system is centered at the observer O, who may be located at some point on the surface of the Earth.

We consider the tangent plane at the location of the observer.





This plane cuts the celestial sphere at the horizon and is referred to as the **horizontal plane.**

The points on the horizon toward north and south are denoted as N and S respectively.

The position of any object in this coordinate system is specified by the altitude or elevation a and the azimuthal coordinate or azimuth A.

Here the horizontal plane acts as the reference plane and the verticals as the longitudes.

The altitude a is the angle along the vertical with respect to the horizontal plane. Range of a : $[-90°, 90°]$ and is analogous to the latitude ($\delta = 90° - \theta$).

The azimuth A is the angular position of the vertical passing through the object with respect to some fixed direction.

An observer chooses some convenient reference and A can be measured clockwise or counterclockwise.

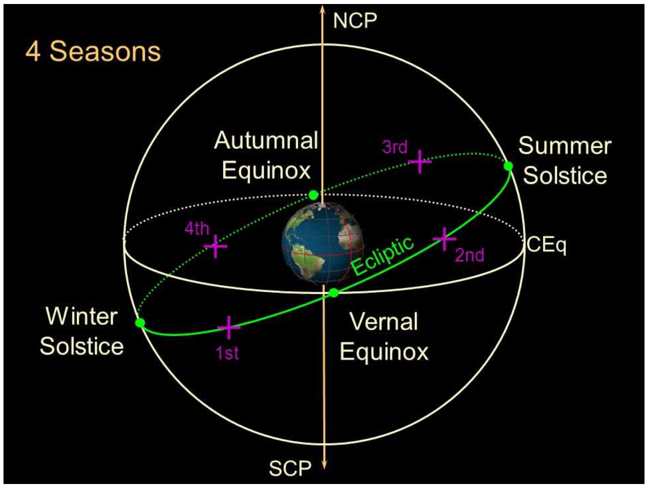
Advantage:

This coordinate is most convenient from the point of view of the observer.

Disadvantage:

Stars appear to move from east to west due to the rotation of the Earth. Hence the positions of stars change rapidly in this system and it is not convenient for use in catalogs.

Equatorial coordinate system



Seasonal climatic variations are due to the orbital motion of Earth, coupled with the approximately 23.5° tilt of its rotation axis.

As a result of the tilt, the ecliptic moves north and south of the celestial equator.

- Twice during the year, the sun crosses the celestial equator—

(i) once moving northward along the ecliptic and the point of intersection is called the **vernal equinox.** Spring begins when the center of the Sun is precisely on the vernal equinox.

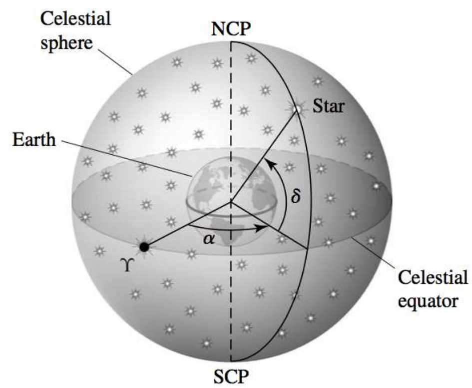
(i) Southern crossing occurs at the **autumnal equinox**, which marks the beginning of fall.

- The most northern excursion of the Sun along the ecliptic occurs at the **summer solstice** – represents the start of summer.
- The southern most position of the sun is defined as the winter solstice.

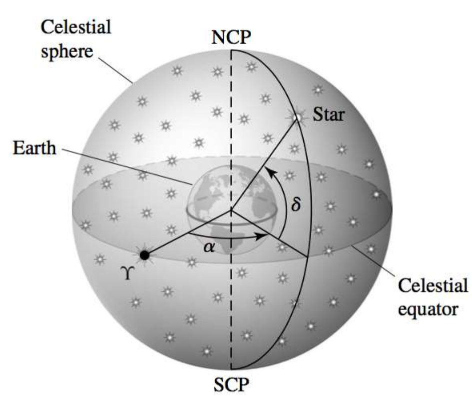
Equatorial Coordinate System

In this coordinate system, we use the fact that the axis of rotation of Earth remains approximately fixed.

Celestial equator is the reference plane to specify the latitude of the object. This coordinate is called the **declination $\boldsymbol{\delta}$ (Dec).** **Range of $\boldsymbol{\delta}$:** : $[-90°\,, 90°]$, with points on the equatorial plane having $\boldsymbol{\delta} = \mathbf{0}$.



- ❖ The azimuthal coordinate is called **right ascension $\boldsymbol{\alpha}$ (RA).**
- ❖ **Range of $\boldsymbol{\alpha}$:** $0\ to\ 360°$ -- analogous to the longitude.
- ❖ It is measured counterclockwise from vernal equinox(γ).
- ❖ It is traditionally measured in time units : hours, minutes and seconds.
- ❖ 24 hours of right ascension is equivalent to 360° or 1 hour = 15°
- ❖ Example: $\boldsymbol{\alpha} = 90°$ is equal to 6 hours, denoted as 6^h in time units.



- In this system, the positions of stars remain fixed with time, at least approximately.
- If we choose the origin at the center of the Earth, then the angular positions of stars will change slightly with time due to Earth's revolution. This shift can be eliminated by choosing the center of the Sun as the origin.
- The axis of rotation of Earth also undergoes a slow change due to the precession. Hence the equatorial system changes slowly and one has to specify the time or epoch in which the system is used.
- The system is updated every 50 years. Before 2000, the system B1950 was in use. Now the system J2000 is being used.

Sidereal time

Solar time: defined with reference to the Sun.

One solar day is the time interval after which the Sun is again at maximum elevation in the sky.

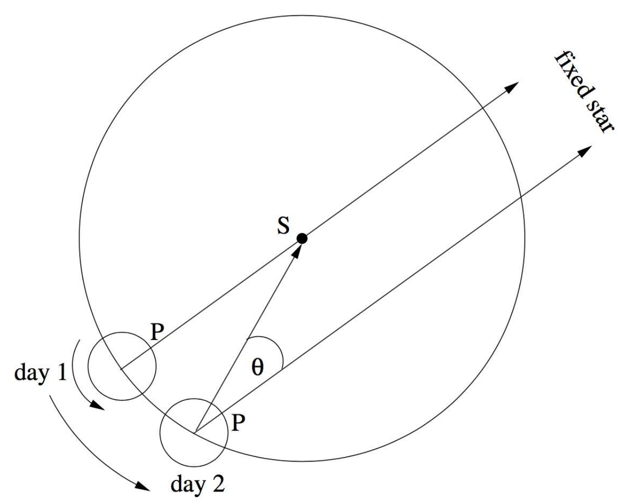
Sidereal time: defined with reference to the fixed stars.

It is based on consecutive meridian crossings of a star.

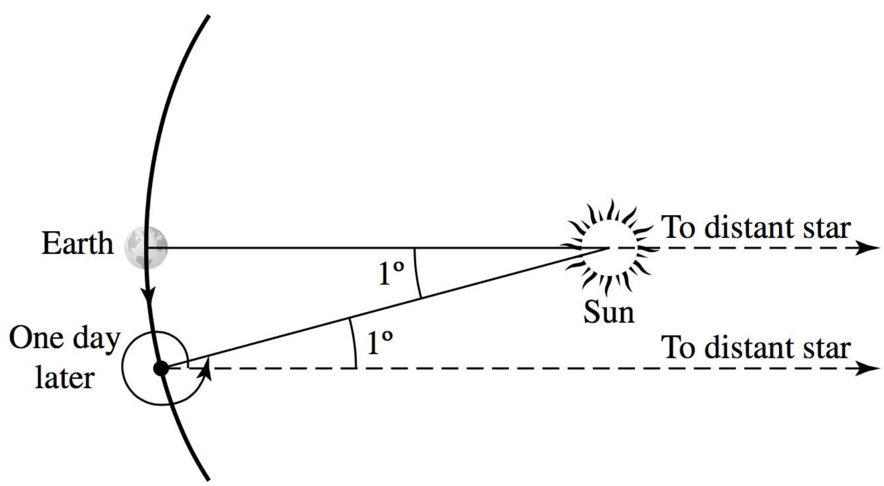
An observer P at Earth is directly below the Sun and a fixed star at noon on day 1.

After completing 1 sidereal day, P returns to its position below the fixed star.

The solar day is longer because P has to traverse an extra angle θ, before the Sun is again directly overhead.



- After 6 months, Earth is on the opposite side of the Sun.
- Now the star is overhead P at 12 midnight. The Sun comes overhead half a day later, that is , at noon.
- This implies that the number of sidereal days that have passed by now are half a day more than the solar days.
- Hence, in 1 year, the number of sidereal days completed is one more than the number of solar days.



Earth must rotate nearly 361° per solar day and only 360° per sidereal day

Earth completes one sidereal period ≈ 365.26 days
So, it moves slightly less than 1° around its orbit in 24 hours.

Thus Earth must actually rotate nearly 361° to bring the Sun to the meridian on two successive days.

It takes approximately 4 minutes for Earth to rotate the extra 1°.

Therefore, a given star rises 4 minutes earlier each night.

Let
T : time intervals corresponding to solar days
T_1: time interval corresponding to sidereal days
T_0 : the period of orbital revolution (= 1year).

The number of sidereal days in a year are **one** more than the number of solar days.

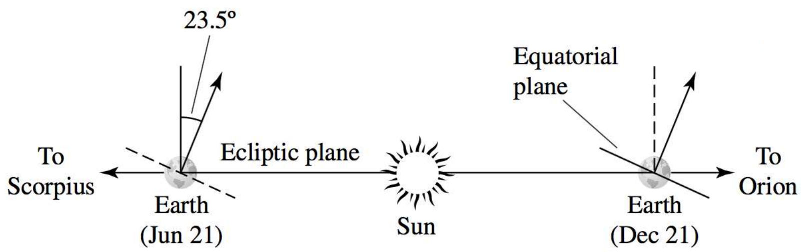
$$\frac{T_0}{T_1} - \frac{T_0}{T} = 1$$

Which gives $$\frac{1}{T_1} = \frac{1}{T} + \frac{1}{T_0}$$

The period of revolution P = 365.2563666 mean solar days (T).

Hence, $T_1 = 0.99727$ solar days.

Seasonal changes in constellations



Season	Constellation that the Sun appears to travel across
Spring	Virgo
Summer	Orion
Autumn	Aquarius
Winter	Scorpius

This seasonal change in the constellations is directly related to the fact that a given star rises approximately 4 minutes earlier each day.

Space Velocity and Proper Motion of Stars

- The stars remain approximately fixed in the sky.
- Their relative positions, with respect to one another, change only slightly with time.
- This is attributed to their intrinsic motion.
- The shift is so small that it is detectable only over a long time interval with sophisticated instruments.

Space velocity of a star

The velocity of a star with respect to the Sun is called the **space velocity.**

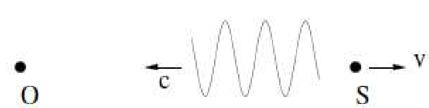
- Radial components, v_r
  - This is the velocity of the object along the line of sight.
  - This is measured by the Doppler shift and is relatively easy to measure.
- Tangential components, v_t
  - This is perpendicular to the radial direction.
  - This is the component tangential to the celestial sphere and can point in any direction on the tangent plane.

Therefore, the space velocity of a star can be written as

$$v_s = (v_r, v_t)$$

Doppler Shift

- A source S moving away from an observer O at speed v emits light of wavelength λ_0. Due to the Doppler effect, the wavelength λ observed by O is larger than λ_0.
- The wavelength seen by the observer, $\lambda = \lambda_0 + \Delta\lambda$, turns out to be larger by an amount $\lambda \approx \lambda_0(v/c)$, where c is the speed of light. This relationship is valid for $v << c$.
- Hence the wavelength becomes larger (is red shifted) if the source is moving away and smaller (is blue shifted) if it is moving toward the observer.
- This shift in wavelength is called the Doppler effect.
- In general, the source may be moving in any direction, not necessarily along the line of sight.



Let its velocity be denoted by $\vec{v}$. In this case the Doppler shift is determined by the component of $\vec{v}$ along the line of sight.

Assuming that O is at the origin of the coordinate system, this is just the radial component v_r. Hence we obtain

$$\frac{\Delta\lambda}{\lambda_0} \approx \frac{v_r}{c}$$

valid for $v_r << c$. This effect can be used to deduce the radial velocity v_r of a star.

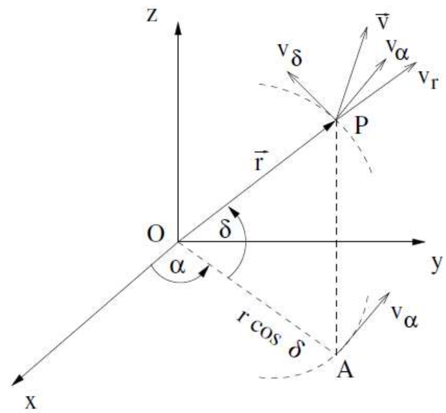
V_s: True velocity of a star, Space velocity

V_r: Radial velocity of a star, measured by Doppler shift.

V_t: Tangential velocity of a star

In astronomy, the entire sky is mapped in a grid of latitudes and longitudes.

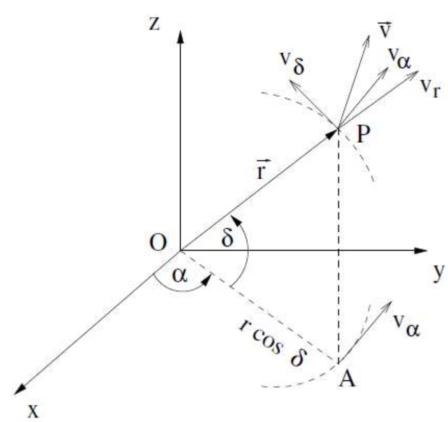
- Suppose, the position vector of an object P is $\vec{r}$. It is located at coordinates (r, δ, α).
- Its space velocity is indicated by $\vec{v}$.
- The radial component, v_r is the component in the direction of $\vec{r}$ and the tangential component, $\vec{v_t}$ lies in the plane perpendicular to $\vec{r}$.
- We can decompose $\vec{v_t}$ in terms of its components along the local latitude and longitude.



- In equatorial coordinates, these two components of $\vec{v_t}$ are denoted as v_δ and v_α, respectively.
- Hence we can write,

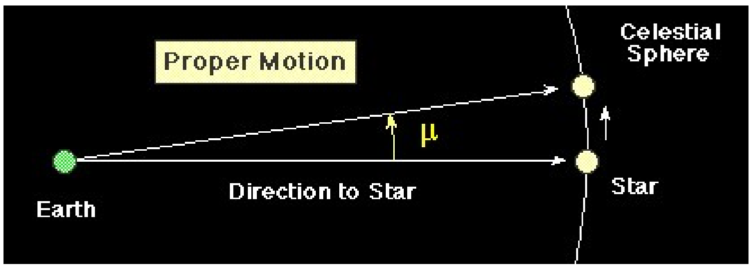
$$\vec{v_t} = (v_\delta, v_\alpha).$$

- Drop a perpendicular PA on the x−y plane.
- The component v_δ lies in the plane OPA and points in the direction of increasing δ at P.
- The component v_α is parallel to the x − y plane and points in the direction of increasing α at P.



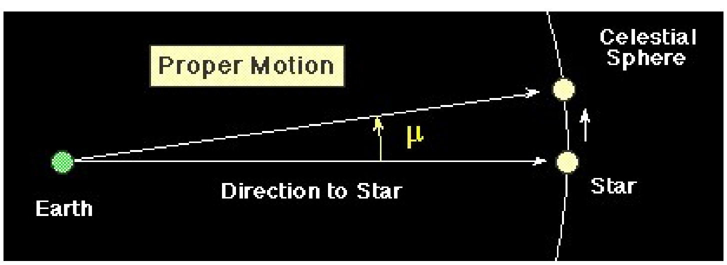
Proper Motion of Stars

- Sometime, we are interested in angular velocity of stars only, then we use proper motion of stars.
- The angular change in position of a star on the celestial sphere is called the proper motion of the star.



Proper velocity: Angular velocity of the star

- Angular velocity is the rate at which the angular position of a star changes with time and is called the proper velocity.
- It is denoted by $\vec{\mu}$.
- Proper velocity is usually give in units of seconds of arc per year.



- Let us assume that in a small time interval t, a star traverses a small angle Δ in the sky. During this time, it moves along a distance D across the sky.
- Clearly, $D = r\Delta$, where r is the distance of the star from the Sun and Δ is in radians.
- The angular speed of the star is $\mu = \Delta/t$ and the tangential speed $v_t = D/t$. Hence we have $v_t = \mu r$.
- The proper velocity is also related to the tangential velocity by a similar relationship, $\vec{v_t} = \vec{\mu} r$.
- Let μ_δ and μ_α represent the rate of change of the angles δ and α, respectively, with respect to time, that is, $\mu_\delta = \dot{\delta}$ and $\mu_\alpha = \dot{\alpha}$.

We can also show that the velocity components can be written as

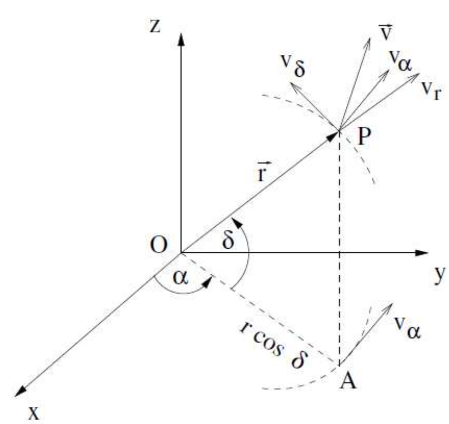
$v_\delta = r\mu_\delta$ and $v_\alpha = r\cos\delta\,\mu_\alpha$.

Hence we can write the proper velocity as $\vec{\mu}$ as

$$\vec{\mu} = (\mu_\delta, \cos\delta\,\mu_\alpha)$$

and the magnitude

$$\mu = \sqrt{\mu_\delta^2 + \cos\delta^2 \mu_\alpha^2}$$



- The largest known proper motion is that of Barnard's star, which moves across the sky at a speed of 10.3″ per year.
- Its angular velocity is equivalent to traversing the angle subtended by the diameter of the moon in less than 200 years.

Magnitude Scale

- We observe the stars and galaxies through the electromagnetic radiation they emit.
- We receive this radiation over a wide range of frequencies.
- Our modern understanding of the universe has been made possible by the quantitiative measurement of the intensity and polarization of light in every part of the electromagnetic spectrum.
- To measure the intensity (flux) of light radiated by astronomical objects, there is a technique called **Photometry.**
- In photometry, the intensity/brightness of astronomical objects can be measured by **apparent magnitude.**

Apparent magnitude

- Apparent magnitude(m) is a measure of the brightness of a star or other astronomical objects observed from Earth.
- The Greek astronomer Hipparchus was one of the first sky watchers to compile a list of the positions of 850 stars.
- He also invented a numerical scale to describe how bright each star appeared in the sky.
- He assigned an apparent magnitude (m) :

➢ m = 1 to the brightest stars in the sky
➢ m =6 for the dimmest stars visible to the naked eye.

4/22/2021 Dr. A. Kushwaha 89

- A smaller apparent magnitude means a brighter-appearing star.
- In the 19th century, it was thought that the human eye responded to the difference in the *logarithms* of the brightness of two luminous objects.
- This theory led to a scale in which a difference of 1 magnitude between two stars implies a constant *ratio* between their brightnesses.

4/22/2021 Dr. A. Kushwaha 90

The Radiation we receive from any astronomical object depends on

- Luminosity
- Distance
- Attenuation of radiation during propagation
- The area over which the radiation is received
- Sensitivity of the instrument, which is a function of the frequency of radiation.

Luminosity(L)

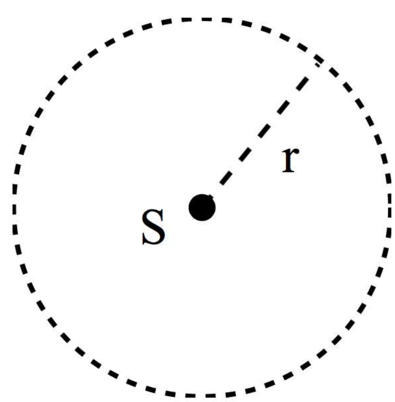
- Defined as the total amount of radiation emitted by a source per unit time.
- It is a measure of the intrinsic brightness of the source.
- Units: J/s or W.

Radiant flux or flux density F

- The brightness of a star is actually measured in terms of the radiant flux received from the star.
- Consider an isotropic source of luminosity L. If no light is absorbed during its journey, the radiant flux measured at a r is

$$F = \frac{L}{4\pi r^2}$$

- The total flux passing through the spherical surface of radius r is equal to total luminosity L.



Radiation from an isotropic source S

<u>**“inverse square law”** for light:</u>
Since L does not depend on r, $F \propto 1/r^2$

Problem

The luminosity of the Sun is $L_{\odot} = 3.839 \times 10^{26}\, W$. At a distance of $1\, AU = 1.496 \times 10^{11}\, m$, what is the radiant flux received by Earth above its absorbing atmosphere. What is its value at a distance of 10 pc ?

$$F = \frac{L}{4\pi r^2} = 1365 \text{ W } m^{-2}$$

The value of the solar flux is known as the **solar irradiance** or **solar constant.**

At a distance of 10 pc, $\mathrm{F} = 1/(2.063 \times 10^{6}\, AU)^2$
$= 3.208 \times 10^{-1}\ \ W\, m^{-2}$

Apparent magnitude

- The magnitude system is designed to specify the flux of an object with respect to the flux of some standard source used as a reference.
- The magnitude of the reference source is assigned some fixed value, such as zero.
- In astronomy, logarithmic scale is used to specify F and L of astronomical objects.

The flux density F received at Earth from a star can be expressed in terms of apparent magnitude m as:

$$m = -2.5\, log_{10} \frac{F}{F_0}, \qquad F_0: reference\ flux\ density$$

For two stars with flux densities F_1 and F_2,

$$m_1 - m_2 = -2.5\, log_{10} \frac{F_1}{F_2}$$

The star with larger m appears less bright.

If $m_1 - m_2 = 5$, then $F_2 = 100\, F_1$.

By modern definition,

- A difference of 5 magnitudes corresponds exactly to a factor of 100 In brightness.
- ∴ a difference of 1 magnitude corresponds exactly to a brightness ratio of

$$100^{1/5} \cong 2.512$$

- Thus, a first-magnitude star appears

❖ 2.512 times brighter than a second magnitude star

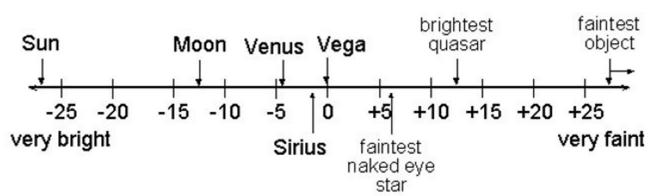
❖ $2.512^2 = 6.310$ times brighter than a third-magnitude star

❖ 100 times brighter than a sixth-magnitude star.

Hipparchus's scale has been extended in both directions:
From m = -26.83 for the Sun to approximately m=30 for the faintest object detectable.

The total range of nearly 57 magnitudes corresponds to over $100^{57/5} = (10^2)^{1\ .4} \cong 10^{23}$ for the ratio of the apparent brightness of the Sun to that of the faintest star or galaxy yet observed.

Examples of apparent magnitudes



Apparent brightnesses of some objects in the magnitude system.

- Many objects go beyond Hipparchus' original bounds of magnitude 1 to 6.
- Some very bright objects can have magnitudes of 0 or even negative numbers and very faint objects have magnitudes greater than +6.
- Brighter objects have smaller magnitudes than fainter objects.

Absolute magnitude (M)

Using the inverse square law, astronomers can assign an absolute magnitude to each star.

It is defined as the apparent magnitude of an object when it is placed at a distance of 10 pc from the observer.

It quantifies the luminosity of that object.

$$\frac{F(r)}{F(10)} = \left(\frac{10\ pc}{r}\right)^2$$

This implies that,

$$m - M = -2.5\ log_{10}\ \frac{F(r)}{F(10)}$$
$$= 5\ log_{10}\ \frac{r}{10\ pc}$$

The distance modulus

$$m - M = 5\ log_{10}\ \frac{r}{10\ pc}$$

The difference m-M is called the **distance modulus** because it is a measure of the distance of the object.

For two stars at the same distance, $\frac{F_1}{F_2} = \frac{L_1}{L_2}$

Thus the equation for absolute magnitudes becomes, $100^{(m_1 - m_2)/5} = \frac{L_2}{L_1}$

If one of the stars is Sun, $m = M_{Sun} - 2.5\ log_{10}\ \frac{L}{L_\odot}$

Hence it reveals the direct relation between a star's absolute magnitude and its luminosity.

Problem

Star A has an apparent magnitude = 5.4 and B has an apparent magnitude = 2.4. Which star is brighter and by how many times? Compare their brightness in terms of their intensities as well.

Star B is brighter than star A because it has a lower apparent magnitude.

Star B is brighter by 5.4 – 2.4 = 3 magnitudes.

In terms of intensity,

star B is $2.512^{(5.4-2.4)} = 2.512^{3.0}$
$\cong 15.8$
So, we receive almost 16 times greater energy from star B than star A.

Problem

The apparent magnitude of the Sun is $m_{Sun} = -26.83$, and its distance is $1\ AU = 4.848 \times 10^{-6}\ pc$. Calculate its absolute magnitude and distance modulus.

The absolute magnitude of the Sun is

$$M_{Sun} = m_{Sun} - 5\ log_{10}(d) + 5$$

= + 4.74

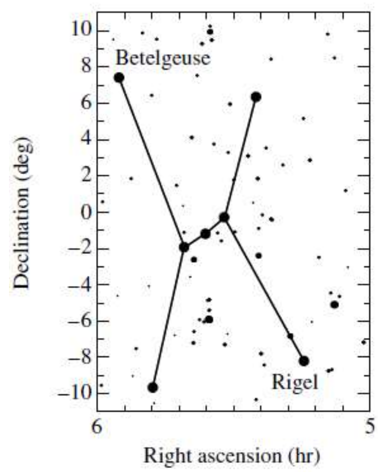
The Sun's distance modulus is : $m_{Sun} - M_{Sun}. \ =. -3\ .57$

Problem

Consider two stars A and B. Star A has an apparent magnitude m_A = +5. Calculate the apparent magnitude of Star B (m_B), if The flux of star B is twice that of Star A.

Blackbody Radiation

- If you look at the constellation of Orion on a clear winter night, you can notice strikingly different colors of red Betelgeuse (in Orion's northeast shoulder) and blue-white Rigel (in the southwest leg).
- These colors betray the difference in the surface temperatures of the two stars.
- Betelgeuse has a surface temperature of roughly 3600 K, significantly cooler than the 13,000-K surface of Rigel.



The Connection between Color and Temperature

- The connection between the color of light emitted by a hot object and its temperature was first noticed in 1792 by the English maker of fine porcelain Thomas Wedgewood.
- All of his ovens became red-hot at the same temperature, independent of their size, shape, and construction.
- Subsequent investigations by many physicists revealed that any object with a temperature above absolute zero emits light of all wavelengths with varying degrees of efficiency.

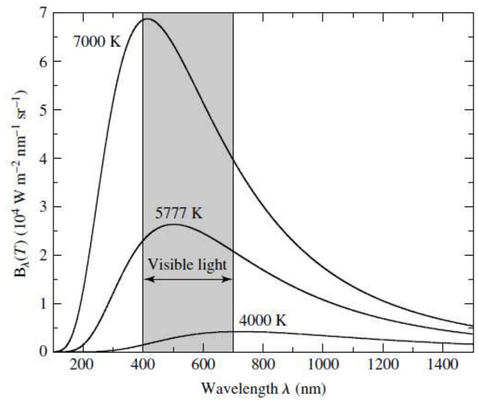
Blackbody and blackbody spectrum

- An ideal emitter is an object that absorbs all of the light energy incident upon it and reradiates this energy with the characteristic spectrum.
- Because an ideal emitter reflects no light, it is known as a **blackbody**, and the radiation it emits is called **blackbody radiation**.
- Stars and planets are blackbodies, at least to a rough first approximation.

- A blackbody of temperature T emits a **continuous spectrum** with some energy at all wavelengths and that this blackbody spectrum peaks at a wavelength λ_{max}, which becomes shorter with increasing temperature.

- The relation between λ_{max} and T is known as **Wien's displacement law.**

$$\lambda_{max}T = 0.002897755\ m\ K.$$



Blackbody spectrum [Planck function $B_\lambda(T)$]

Example:

Betelgeuse has a surface temperature of 3600 K. If we treat Betelgeuse as a blackbody, Wien's displacement law shows that its continuous spectrum peaks at a wavelength of

$$\lambda_{max} \approx \frac{0.0029 \text{ m K}}{3600 \text{ K}} = 8.05 \times 10^{-7} \text{m} = 805 \text{ nm}.$$

which is in the infrared region of the electromagnetic spectrum.

Rigel, with a surface temperature of 13,000 K, has a continuous spectrum that peaks at a wavelength of

$$\lambda_{max} \approx \frac{0.0029 \text{ m K}}{13000 \text{ K}} = 2.23 \times 10^{-7} \text{m} = 223 \text{ nm}.$$

in the ultraviolet region.

The Stefan–Boltzmann Equation

As the temperature of a blackbody increases, it emits more energy per second at all wavelengths.

Austrian physicist Josef **Stefan** (1835–1893) in 1879, showed that the luminosity, L, of a blackbody of area A and temperature T (in kelvins) is given by

$$L = A\sigma T^4$$

Ludwig Boltzmann (1844–1906), derived this equation, now called the **Stefan–Boltzmann equation**, using the laws of thermodynamics and Maxwell's formula for radiation pressure. The Stefan–Boltzmann constant, σ, has the value

$$\sigma = 5.670400 \times 10^{-8} W m^{-2} K^{-4}$$

Effective temperature T_e

For a spherical star of radius R and surface area $A = 4\pi R^2$, the Stefan–Boltzmann equation takes the form

$$L = 4\pi R^2 \sigma T_e^4$$

Since stars are not perfect blackbodies, we use this equation to *define* the **effective temperature T_e** of a star's surface. Combining this with the inverse square law, at the surface of the star (r = R), the *surface flux* is

$$F_e = \sigma T_e^4$$

The effective temperature T_e is defined as the temperature of a blackbody that radiates the same total flux as the star.

Example:

The luminosity of the Sun is $L_\odot = 3.839 \times 10^{26}$ W and its radius is $R_\odot = 6.95508 \times 10^{8}$ m.

The effective temperature of the Sun's surface,

$$T_\odot = \left(\frac{L_\odot}{4\pi R_\odot{}^2}\right)^{\frac{1}{2}} = 5770\ K$$

The radiant flux at the solar surface is

$$F_{surf} = \sigma T_\odot^4 = 6.316 \times 10^7\ Wm^2.$$

According to Wien's displacement law, the Sun's continuous spectrum peaks at a wavelength of

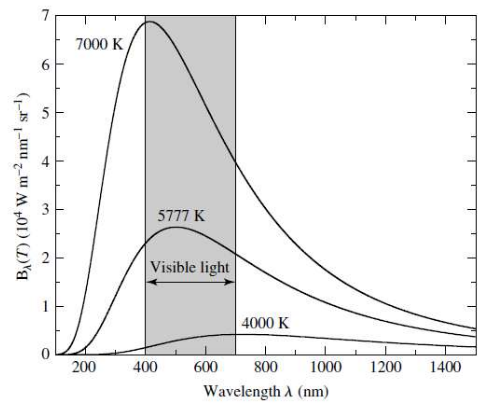
$$\lambda_{max} \approx \frac{0.0029 \text{ m K}}{5777 \text{ K}} = 5.016 \times 10^{-7}\text{m} = 501.6 \text{ nm}.$$

This wavelength falls in the *green* region (491 nm $< \lambda <$ 575 nm) of the spectrum of visible light. However, the Sun emits a continuum of wavelengths both shorter and longer than λ_{max}, and the human eye perceives the Sun's color as yellow.

Because the Sun emits most of its energy at visible wavelengths, and because Earth's atmosphere is transparent at these wavelengths, the evolutionary process of natural selection has produced a human eye sensitive to this wavelength region of the electromagnetic spectrum.

Rayleigh–Jeans law.

One of the problems haunting physicists at the end of the nineteenth century was their inability to derive from fundamental physical principles the blackbody radiation curve.



Lord Rayleigh had attempted to arrive at the expression by applying Maxwell's equations of classical electromagnetic theory together with the results from thermal physics.

The result of Rayleigh's derivation gave

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}$$ (valid only if λ is long)

which agrees well with the long-wavelength tail of the blackbody radiation curve. But, it diverges at short wavelength. This divergence is called as the ultraviolet catastrophe.

Rayleigh–Jeans law.

Lord Rayleigh had attempted to arrive at the expression by applying Maxwell's equations of classical electromagnetic theory together with the results from thermal physics.

His strategy was to consider a cavity of temperature T filled with blackbody radiation. This may be thought of as a hot oven filled with standing waves of electromagnetic radiation.

If L is the distance between the oven's walls, then the permitted wavelengths of the radiation are λ = 2L,L, 2L/3, 2L/4, 2L/5, . . . , extending forever to increasingly shorter wavelengths.

According to classical physics, each of these wavelengths should receive an amount of energy equal to kT , where k = 1.3806503 × 10^{-23} J K^{-1} is Boltzmann's constant, familiar from the ideal gas law PV = N kT . The result of Rayleigh's derivation gave

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}$$ (valid only if λ is long)

which agrees well with the long-wavelength tail of the blackbody radiation curve. But, it diverges at short wavelength. This divergence is called as the ultraviolet catastrophe.

Wien was also working on developing the correct mathematical expression for the blackbody radiation curve.

Guided by the Stefan–Boltzmann law and classical thermal physics,Wien was able to develop an empirical law that described the curve at short wavelengths but failed at longer wavelengths:

$$B_\lambda(T) \simeq a\lambda^{-5}e^{-b/\lambda T} \qquad \text{(valid only if } \lambda \text{ is short)}$$

where a and b were constants chosen to provide the best fit to the experimental data.

Planck's Function for the Blackbody Radiation Curve

Max Planck had discovered that a modification of Wien's expression could be made to fit the blackbody spectra while simultaneously replicating the long-wavelength success of the Rayleigh–Jeans law and avoiding the ultraviolet catastrophe:

$$B_\lambda(T) \simeq \frac{a/\lambda^5}{e^{b/\lambda T} - 1}$$

In order to determine the constants a and b, Planck assumed that a standing electromagnetic wave of wavelength λ and frequency $\nu = c/\lambda$ could not acquire just any arbitrary amount of energy. Instead, the wave could have only specific allowed energy values that were integral multiples of a minimum wave energy. This minimum energy, a **quantum** of energy, is given by hν or hc/λ, where h is a constant. Thus the energy of an electromagnetic wave is nhν or nhc/λ, where n (an integer) is the number of quanta in the wave.

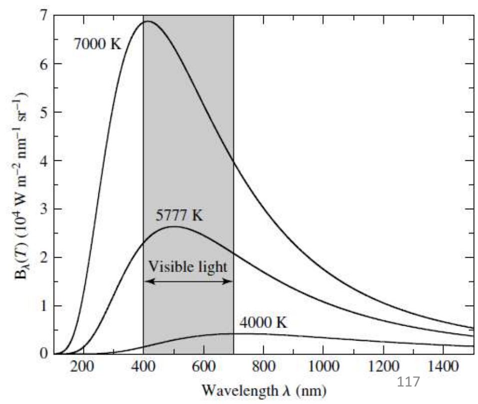
Planck gave a function known as **Planck function**, agreed wonderfully with experiment,

$$B_\lambda(T) \simeq \frac{2hc^2/\lambda^5}{e^{hc/\lambda} \quad -1}$$

where, h is Planck's constant and value of h = 6.62606876 × 10^{-34} J s.

Planck function has the form with frequency intervals $d\nu$

$$B_\nu(T) \simeq \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$



Bolometric magnitudes

- The apparent and absolute magnitudes, measured over all wavelengths of light emitted by a star, are known as **bolometric magnitudes** and are denoted by m_{bol} and M_{bol}, respectively.

- The *color* of a star may be precisely determined by using filters that transmit the star's light
- only within certain narrow wavelength bands.

- In the standard UBV system, a star's apparent magnitude is measured through three filters and is designated by three capital letters:

  - U, the star's *ultraviolet* magnitude, is measured through a filter centered at 365 nm with an effective bandwidth of 68 nm.
  - B, the star's *blue* magnitude, is measured through a filter centered at 440 nm with an effective bandwidth of 98 nm.
  - V , the star's *visual* magnitude, is measured through a filter centered at 550 nm with an effective bandwidth of 89 nm.

Color Index

A star's absolute color magnitudes M_U ,M_B, and M_V may be determined if its distance d is known.

A star's U − B **color index** is the difference between its ultraviolet and blue magnitudes, and a star's B − V color index is the difference between its blue and visual magnitudes:

$$U - B = M_U - M_B$$

and

$$B - V = M_B - M_V$$

Stellar magnitudes *decrease* with increasing brightness; consequently, a star with a smaller B − V color index is *bluer* than a star with a larger value of B − V .

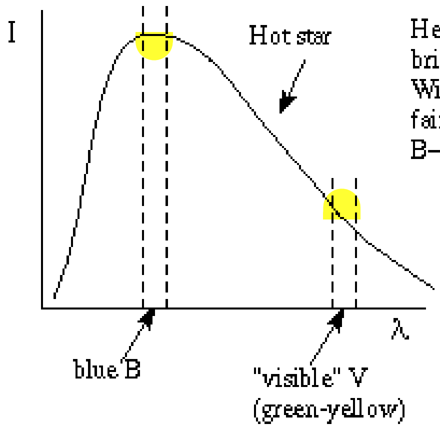
The difference between a star's bolometric magnitude and its visual magnitude is its **bolometric correction** BC:

$$BC = m_{bol} - V = M_{bol} - M_V$$

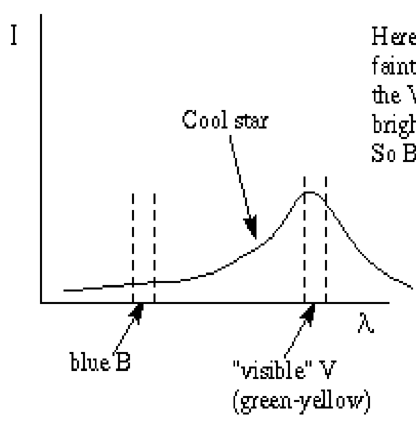
Example: B-V color index

- A hot star has a B-V color index close to 0 or negative,
- While a cool star has a B-V color index close to 2.0.
- Other stars are somewhere in between.

Hot stars appear bluer than cooler stars. Cooler stars are redder than hotter stars.



Here with the Blue (B) filter, the star will appear brighter —> smaller, more negative magnitude. With the Visible (V) filter, the star will appear fainter —> larger, more positive magnitude. So B–V = small # – large # = negative number.



Here with the Blue (B) filter, the star will appear fainter —> larger, more positive magnitude. With the Visible (V) filter, the star will appear brighter —> smaller, more negative magnitude. So B–V = large # – small # = positive number.

Example

Sirius, the brightest-appearing star in the sky, has U, B and V apparent magnitudes of U = -1.47, B = -1.43 and V = -1.44.

Thus for Sirius, $U - B = -1.47 - (-1.43) = -0.04$

and $B - V = -1.43 - (-1.44) = 0.01$

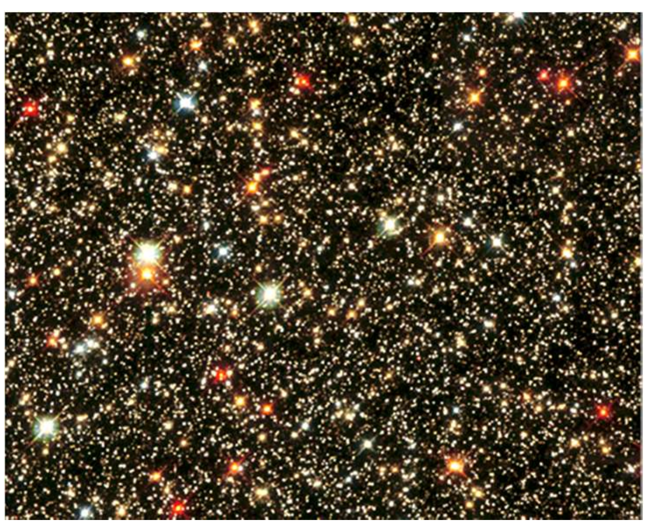
Sirius is brightest at ultraviolet wavelengths, as expected for a star with an effective temperature of $T_e = 9970\ K$. For this surface temperature,

$$\lambda_{max} = \frac{0.0029\ mK}{9970\ M} = 291\ nm$$

which is in the ultraviolet portion of the electromagnetic spectrum. The bolometric correction for Sirius is BC = -0.09, so its apparent bolometric magnitude is

$$m_{bol} = V + BC = -1.44 + (-0.09) = -1.53$$

Enhanced color picture of the sky



The color differences among the stars

Color Temperature

The color temperature of a star can be extracted by measuring its apparent magnitude corresponding to two different filters (B-V).

Let $F_E(\nu_1)$ and $F_E(\nu_1)$be the flux densities of a star observed at Earth at two different frequencies, ν_1 and ν_2, respectively. The ratio of the flux densities, assuming that the radiation emitted follows the blackbody spectrum, is given by

$$\frac{F_E(\nu_1)}{F_E(\nu_2)} = \frac{B_{\nu_1}(T)}{B_{\nu_2}(T)}$$

where $B_{\nu_1}(T)$ is the intensity of a blackbody at temperature T.

This relation provides an estimate of temperature, given a measurement of $F_E(\nu_1)$ and $F_E(\nu_2)$.

In practice, one measures the difference in magnitudes corresponding to two broad band filters, for example B − V. The difference in magnitudes at two different frequencies can be expressed as

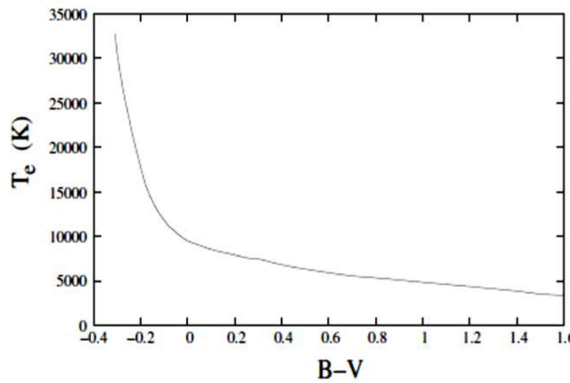
$$m_1 - m_2 = -2.5\ log_{10} \frac{F_E(\nu_1)}{F_E(\nu_2)}$$

In the case of a broad band filter we require the integrated flux density corresponding to that filter. Hence the flux densities are equal to the blackbody intensity and S_ν the sensitivity function of the filter.

The temperature extracted by this procedure is called the color temperature, Tc.

Hence the color indices, such as B − V , provide a measure of the color temperature of a star.

- If a star behaves as a perfect blackbody, then the color temperature would be the same as the effective temperature.
- In general, stars are not ideal blackbodies and these temperatures differ from one another.
- The color temperature is easiest to extract because it only requires measurement of apparent magnitudes.
- Hence the early estimates of temperature, made toward the beginning of the twentieth century, used this measure.
- The effective temperature can be deduced by spectroscopic measurements.

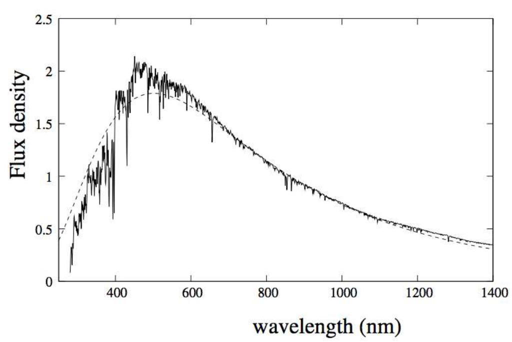


The relationship between the effective temperature and the color index, B − V

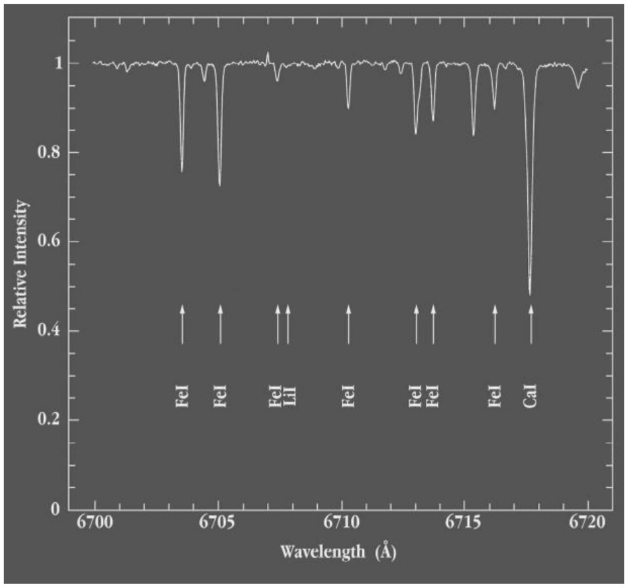
Stellar spectra & Classification

- The properties of stars show wide variation in luminosity, size, mass and spectrum.
- For most of the stars, we are only able to observe their flux density at different frequencies.
- Their spectral lines can be measured very accurately.
- Using flux & spectral measurements, it is possible to deduce other properties, such as their mass, radius, chemical composition, surface temperature etc.,
- The solar spectrum was first observed by Fraunhofer in 1814. He observed a large number of absorption lines in the Sun's spectrum.

Solar spectrum



- The stellar radiation approximately follows the blackbody distribution.
- The smooth dashed line represents a blackbody spectrum corresponding to a temperature of 5778K.
- Superimposed on the blackbody continuum there are a large number of dips, which are absorption lines produced in stellar atmospheres.



- Solar spectra showing absorption lines in a narrow range of frequencies.
- The elements that lead to these lines are also indicated.

- The first photograph of stellar spectra was obtained by Henry Draper. He photographed the spectral lines in star Vega in 1872.
- The first comprehensive survey and classification of stars, based on their spectral lines, was done at Harvard by E.C. Pickering and his two assistants, W.P. Fleming and A. J. Cannon.
- Cannon classified the spectra of about 225,000 stars in 1901 and is called the Harvard classification scheme.
- The results were published in the form of the Henry Draper Catalog between 1918 and 1924.

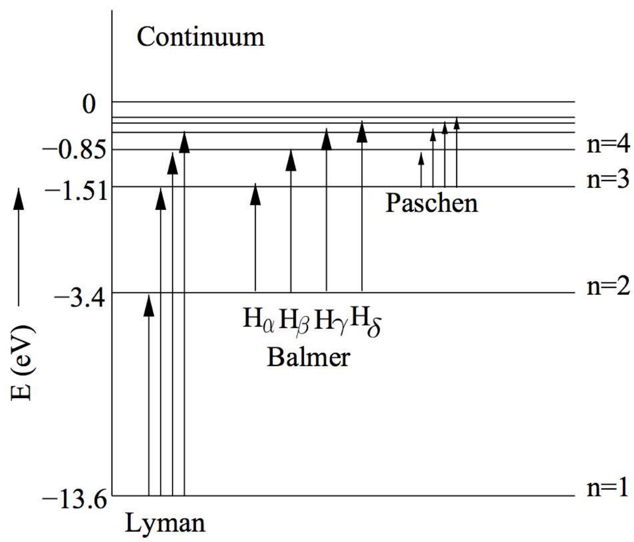
Harvard Classification of Stellar Spectra

- In the Harvard classification, the stellar spectra are classified into the following categories:

O B A F G K M

- Each of these classes is divided into subclasses by adding a numeral ranging from 0 to 9 in front of each of the letters.
- Example: the spectral type A is subdivided into A0, A1, …….A9.
- The stars are classified based on the strength of the different spectral lines at visible wavelengths.
- Ex: Hydrogen Balmer lines, lines of other elements such as He, Ca, Fe as well as molecules such as TiO.

Spectral lines of the hydrogen atom



- Only the Balmer series (n = 2 to higher n states) lies in the visible part of the spectrum.
- ∴ the spectral lines of hydrogen would contribute only if the hydrogen atom has a significant probability of being in the second excited state.

- The physical basis of the Harvard spectral classification scheme remained obscure.
- Vega (spectral type A0) displays very strong hydrogen absorption lines, much stronger than the faint lines observed for the Sun (spectral type G2).
- On the other hand, the Sun's calcium absorption lines are much more intense than those of Vega.

Is this a result of a variation in the composition of the two stars?

Or are the different surface temperatures of Vega ($T_e = 9500K$) and the Sun ($T_e = 5777\ K$) responsible for the relative strengths of the absorption lines?

- ❖ The difference between the spectra of stars with different temperatures are due to

(i) Electrons occupying different atomic orbitals in the atmospheres of these stars.

(ii) The atom can be in any one of various stages of ionizations and has a unique set of orbitals at each stage.

- ❖ We indicate the state of ionization of the different atoms by a roman numeral following the symbol for the atom.

He I	Neutral (non ionized) Helium atom
HeII	Singly ionized helium atom
OIII	Doubly ionized oxygen atom

- The proper framework for the interpretation of stellar spectra requires Boltzmann distribution and the Saha ionization equation.

  The ratio of the number of atoms N_b with energy E_b to the number of atoms N_a with energy E_a in *different states of excitation* is given by

$$\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}$$ **"Boltzmann Equation"**

  (g: degeneracy of the energy level)

- The Saha ionization equation was developed by the Indian astrophysicist Meghnad Saha in 1920.

- It allows one to compute the relative abundance of atoms in different stages of ionization.

Saha ionization equation

Let χ_i be the ionization energy needed to remove an electron from an atom or ion in the ground state, thus taking it from ionization stage i to stage $(i+1)$.

The ratio of the number of atoms in ionization stage (i+1) to the number of atoms in stage i is :

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$ **"Saha Equation"**

Where,
Z: partition function
n_e: number density of free electrons
Factor 2: reflects the two possible spins of the free electron with $m_s = \pm 1/2$.

- Using Saha ionization equation, astrophysicists were able to establish that the Harvard classification corresponds to decreasing temperature.
- Hence O type stars have the highest surface temperature while the M type stars are the coolest. Within a class, the numerical sequence 0 to 9 represents decreasing temperature.
- Cecilia Payne performed a detailed analysis and found that the most common elements in stellar atmospheres are hydrogen and helium.
- Elements such as oxygen, silicon, iron and calcium were found to be more abundant than hydrogen in the Earth's crust while He is very rare.

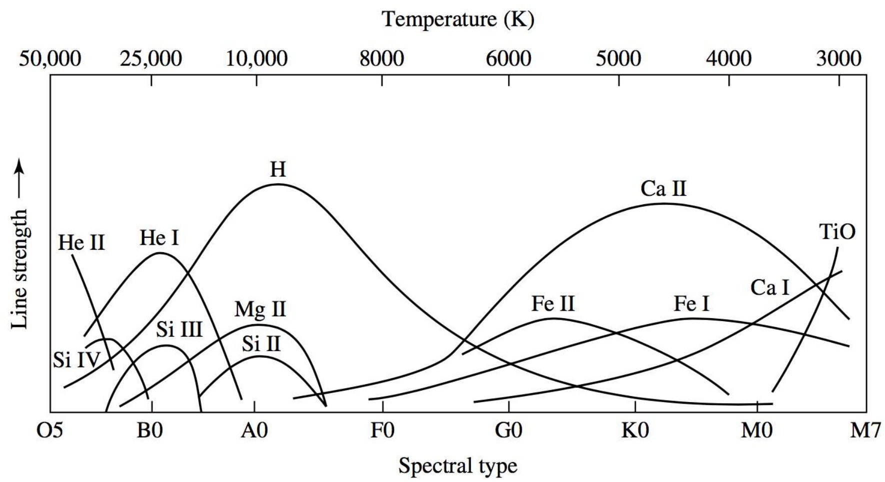
Hence relative abundance of elements in stars is very different from that of the Earth.

Type of stars	O	B	A
color	blue	blue	white
Surface temperature	40,000K to 20,000K (very hot)	20,000K to 10,000K (very hot)	10,000K TO 7,500K
Absorption lines	❖ Strong arising from ions such as $\boldsymbol{HeII, CIII, NIII, OIII, SiIV\ and\ SiV}$, as well as from the neutral Helium atom HeI.	❖ Spectra show neutral heloium lines, which are strongest at B2. ❖ Lines from ions such as $\boldsymbol{OII, SiII\ and\ MgII}$ can be seen	❖ Lines of ions such as $\boldsymbol{MgII\ and\ SiII}$ are seen. ❖ CaII lines are weak.
	❖ A few emission lines can also be seen. ❖ $\boldsymbol{HI}$ Balmer lines are visible but weak.	❖ The $\boldsymbol{HI}$ Balmer lines are relatively strong.	❖ Very strong hydrogen HI lines.
Example (visible through naked eye)	Meissa in the constellation Orion	Rigel, the brightest star in Orion constellation.	Sirius in the constellation Canis Major. (Brightest star)

Type of stars	F	G	K	M
color	Yellow-white	yellow	Orange	Cool red stars
Surface temperature	7,500K to 6,000K	6,000K to 4,500K	4,500K to 3,600K	< 3,600 K
Absorption lines	❖ HI lines are getting weaker but still relatively strong. ❖ $CaII$ and $FeII$ lines start getting stronger ❖ Neutral metal lines (FeI and CaI) also become visible.	❖ HI lines are weak. ❖ $CaII$ & $FeII$ lines continue to become stronger. ❖ The strength of FeI and CaI lines increases from G0 to G9.	❖ HI lines are very weak. ❖ $CaII$ lines are very strong. ❖ Neutral metal lines FeI and SiI are visible. ❖ The molecular bands of TiO become visible by K5.	❖ HI lines are absent. ❖ CaI lines are very strong. ❖ Spectra are dominated by TiO bands and neutral metal (FeI) Lines.
Example (visible through naked eye)	❖ Canopus in the constellation Carina. ❖ (II brightest star)	❖ Sun (G2 type star) ❖ Alpha Centauri A in the constellation Centaurus.	Aldebaran in the constellation Taurus.	Betelgeuse in the constellation Orion.

Types of stars	L	T
Color Temperature	❖ Dark red (very cool) ❖ 1300K to 2500 K	Infrared (coolest) < 1300 K
	❖ Stronger in infrared than visible. ❖ Strong molecular absorption bands of metal hydrides, water, CO and alkali metals ❖ TiO and VO are weakening.	❖ Strong methane bands but weakening CO bands

The dependence of spectral line strengths on temperature



As the temperature changes, a smooth variation from one spectral type to the next occurs, indicating that there are only minor differences in the composition of stars, as inferred from their spectra.

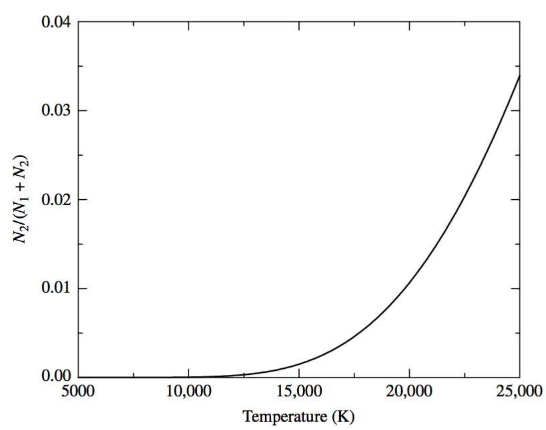
From Harvard classification, it is clear that the strength of Balmer lines diminishes at very high temperatures.

- But according to Boltzmann equation,

$$\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}$$

we expect a greater proportion of the electrons to be in the first excited state rather than in the ground state.

What causes diminishing strength of the Balmer lines at higher temperatures?

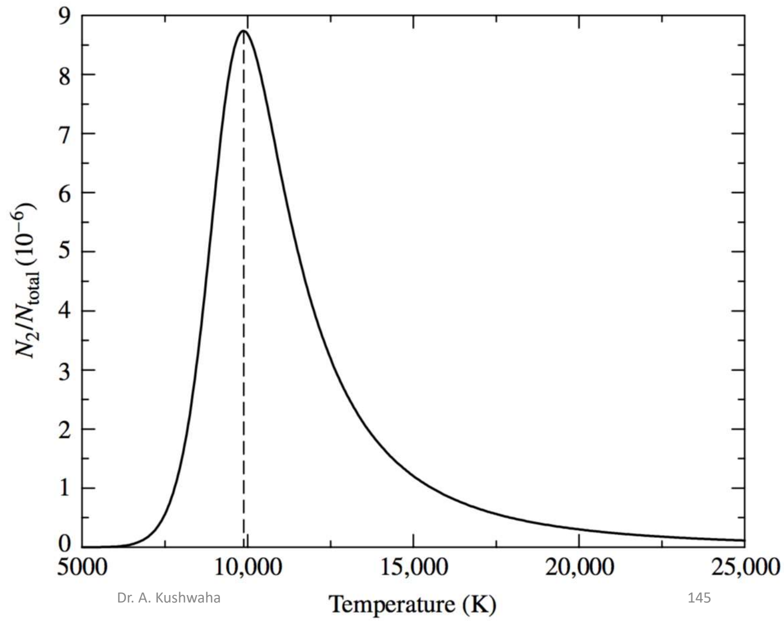


From Boltzmann equation

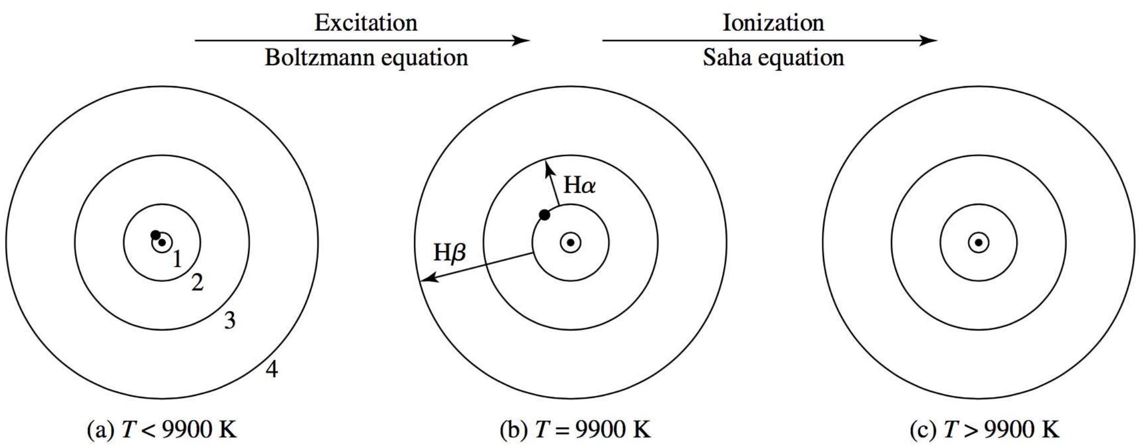
Combining the Boltzmann and Saha equations

The hydrogen gas would produce the most intense Balmer lines at a temperature of 9900 K.

The diminishing strength of the Balmer lines at higher temperatures is due to the rapid ionization of hydrogen above 10,000K



The electron's position in the hydrogen atom at different T



Electron in the ground state

Electron in the first excited state
"Balmer lines are produced"

Atom has been ionized

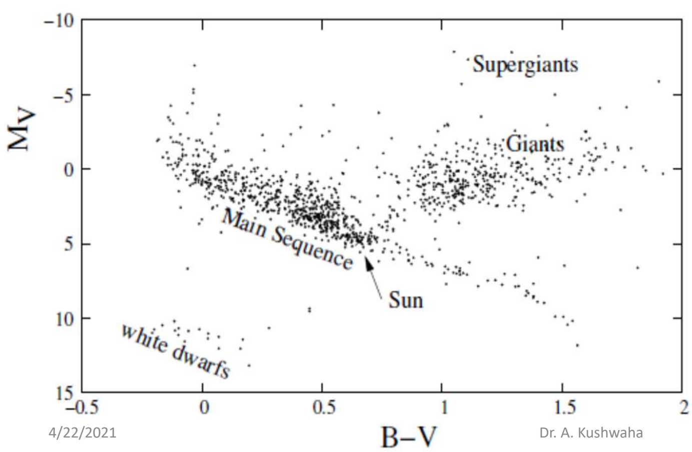
After having accumulated the temperature and absolute magnitude data for a large number of stars, astronomers tried to find out if there exists any relationship among these parameters.

For example, is it possible that the luminosities of different stars may be related to some other parameter such as the temperature, mass, or size of the star?

Hertzsprung–Russell or HR diagram

A relationship was found empirically by making a plot between the absolute visual magnitude M_v and the color index B − V .

This plot is now known as the Hertzsprung–Russell or HR diagram, after its discoverers.



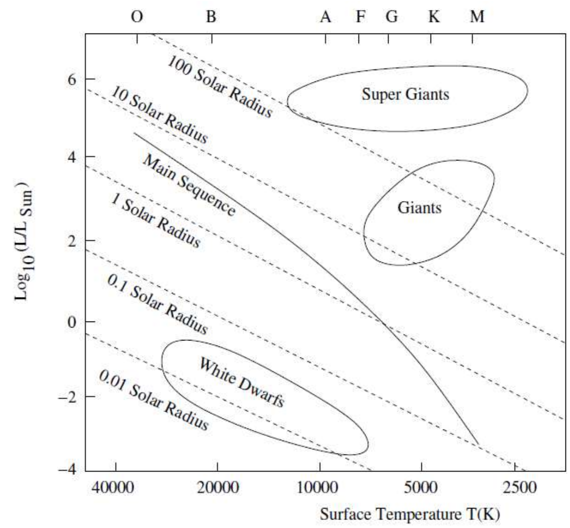
The Sun is located at B − V = 0.656 and M_V = 4.83.

The color index B − V is directly observable and is a measure of the effective surface temperature of a star.

The absolute visual magnitude is estimated from the apparent magnitude after determining the distance of the star. It provides an estimate of the luminosity L of a star.

One can, therefore, plot the HR diagram in terms of the effective surface temperature and the luminosity of stars.

The Harvard spectroscopic classification is also labeled at the top of this plot. From left to right, it goes from the very hot O type stars to the cool M type stars.



We find that most of the stars (80% to 90%) lie roughly on a narrow diagonal band on this plot. These are called the main sequence stars.

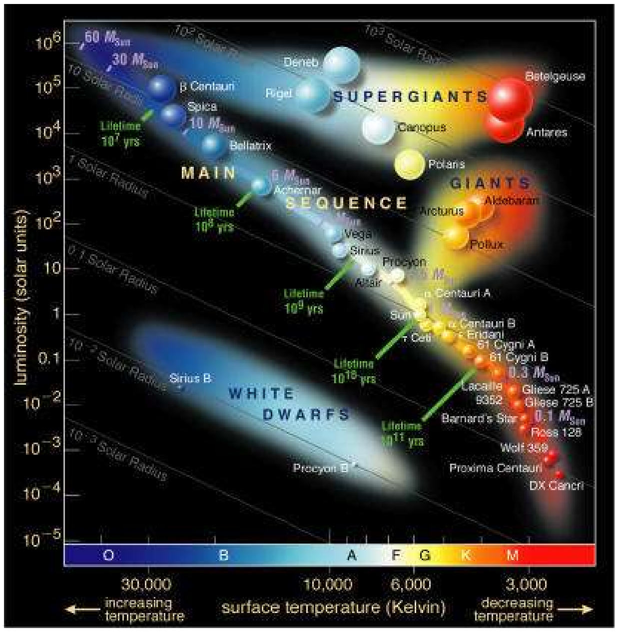
The stars in the upper left corner along this line are very hot and luminous. As we come down this line, we find that both the temperature and the luminosity decrease.

Besides the main sequence, we find stars along two horizontal branches in the upper half plane of the diagram.

The stars along the lower branch are called the red giants. If we compare these to the main sequence stars of comparable luminosity, we find that these stars have a lower temperature.

The stars along the upper branch are called the red supergiants. These stars are considerably more luminous compared to the main sequence stars and the red giants of comparable temperature.

Finally we see a branch of stars with very hot temperatures but low luminosity at the bottom of the diagram. These are called the white dwarfs.



- From Stefan-Boltzmann law, we know that: $\boldsymbol{R = \frac{1}{T_e^2}\sqrt{\frac{L}{4\pi\sigma}}}$. ……………(1)

- So, if two stars have the same temperature, then the more luminous star must be larger.

- The radius of a star can be easily determined from its position on the H-R diagram. If two stars have the same surface temperature, but one star is 100 times more luminous than the other, then from equation (1) we can find that the radius of the more luminous star is $\sqrt{100} = 10\ times$ larger.

- The main-sequence stars show some variation in their sizes, ranging from roughly 20 $R_\odot$ at the upper extreme left end of main sequence down to 0.1 $R_\odot$ at the lower end.

- The giant stars fall between roughly 10 $R_\odot$ and 100 $R_\odot$.

- Super giant stars are even larger.
- Ex: Betelgeuse contracts & expands between 700 $R_{\odot}$ and 1000 $R_{\odot}$.
- Hence the position of a star on the main sequence is governed by a single factor –**Mass.**
- O stars have masses of 60 $M_{\odot}$ and M stars have at least 0.08 $M_{\odot}$.
- From masses & radii for main-sequence stars, we can calculate the average density of the stars --- They have roughly the same density as water.
- Moving up the main sequence, more massive stars have a lower average density.

Example:

Density of Sun $\bar{\rho} = \frac{M_{\odot}}{\frac{4}{3}\pi R_{\odot}^3} = 1410 \text{ kg } m^{-3}$

Density of Sirius $\bar{\rho} = \frac{2.2\, M_{\odot}}{\frac{4}{3}\pi (1.6 R_{\odot})^3} = 0.54\, \bar{\rho}_{\odot}$. (76 % density of water)

Density of Betelgeuse

$$\bar{\rho} = \frac{10\, M_{\odot}}{\frac{4}{3}\pi (1000\, R_{\odot})^3} = 10^{-8}\, \bar{\rho}_{\odot}$$ (hundred thousand times less dense than air)

Different stages of stellar evolution on HR diagram

- The main sequence phase represents the earliest stage of evolution, where the energy of a star is provided by fusion of hydrogen into helium in the core of the star.
- After exhausting the hydrogen in its core, the star enters the giant or the supergiant phase, depending on the mass of the star.
- A star with a mass larger than roughly 10 solar masses evolves into a super giant phase, whereas lower mass stars settle into a giant phase.
- After the giant phase, a star enters its final phase of evolution, which may be a white dwarf, a neutron star, or a black hole.

- These objects are of very small size in comparison to stars in other phases.
- A neutron star or a black hole is too tiny to be represented on a HR diagram.
- A star spends much more time on the main sequence phase in comparison to the giant phase.
- Hence, most of the stars on the HR diagram are found to lie on the main sequence.
- There may be a large number of white dwarfs but most of them are too dim to be observable.

The HR Diagram: Luminosity Class

It is clear from the HR diagram that the Harvard spectral classification, which is based only on the temperature of a star, is incomplete.

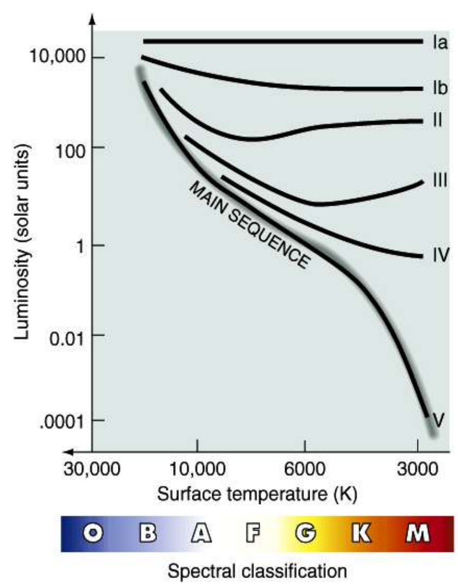
For example, a red giant and a main sequence star have the same temperature but very different luminosities.

One requires a two-dimensional classification that takes into account this difference.

This is accomplished by the Yerkes or MKK classification scheme, which identifies several different luminosity classes, labeled by Roman numerals I, II, III, etc.

The luminosity classes I, II, III, IV, and V correspond to supergiants, bright giants, normal giants, sub-giants, and main sequence stars, respectively.

For example, the symbol G2I, represents a supergiant with the same surface temperature as the Sun. The Sun is classified as G2V.



Bright Supergiants

Supergiants

Bright Giants

Giants

Sub-giants

Main-Sequence (Dwarfs)