

# Definitions of Graph-Theoretic Measures

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## 1. Transitivity

A global measure of clustering in a graph. It is defined as the ratio of three times the number of triangles in the graph to the number of connected triplets of nodes. It indicates the overall tendency of nodes to form tightly connected groups.

## 2. Top Eigenvalue

The largest eigenvalue of a graph's adjacency matrix. It reflects the graph's overall connectivity and is often related to processes such as diffusion, synchronization, and epidemic spreading on networks.

## 3. Average Closeness Centrality

The average of closeness centrality values over all nodes. Closeness centrality of a node is the inverse of the average shortest-path distance from that node to all other reachable nodes.

## 4. Average Degree Centrality

The mean of degree centrality values for all nodes. Degree centrality of a node is the number of edges connected to it, usually normalized by the maximum possible degree.

## 5. Density

The ratio of the number of edges in the graph to the maximum possible number of edges. It measures how close the graph is to being fully connected.

## 6. Algebraic Connectivity

The second-smallest eigenvalue of the graph Laplacian matrix. It measures how well-connected the graph is; higher values indicate stronger connectivity and robustness.

## **7. Matching Number**

The size of a maximum matching in the graph, where a matching is a set of edges with no shared vertices.

## **8. Size of Largest Component**

The number of nodes in the largest connected component of the graph.

## **9. Number of Nodes**

The total count of vertices present in the graph.

## **10. Graph Energy**

The sum of the absolute values of all eigenvalues of the graph's adjacency matrix. It originates from chemical graph theory and reflects structural complexity.

## **11. Wiener Index**

The sum of shortest-path distances between all pairs of nodes in the graph. It is a measure of overall compactness.

## **12. Average Shortest Path Length**

The average of the shortest-path distances between all pairs of reachable nodes in the graph.

## **13. Radius**

The minimum eccentricity among all nodes, where eccentricity of a node is the maximum distance from that node to any other node.

## **14. Diameter**

The maximum shortest-path distance between any pair of nodes in the graph.

## **15. Second Eigenvalue**

Typically refers to the second-largest eigenvalue of the adjacency matrix or the second-smallest eigenvalue of the Laplacian matrix, depending on context. It provides information about graph connectivity and expansion.

## 16. Average Betweenness Centrality

The mean of betweenness centrality values over all nodes. Betweenness centrality measures how often a node lies on shortest paths between other nodes.

## 17. Girth

The length of the shortest cycle in the graph. If the graph has no cycles, the girth is infinite or undefined.

## 18. Degeneracy

The smallest number  $k$  such that every subgraph has a node with degree at most  $k$ . It is also known as the graph's *k-core number*.

## 19. Number of Edges

The total count of edges present in the graph.

## 20. Average Clustering Coefficient

The average of local clustering coefficients across all nodes. It measures how close each node's neighbors are to forming a complete subgraph.

## 21. Max Flow

The maximum amount of flow that can be sent from a source node to a sink node in a flow network without exceeding edge capacity constraints.