

# Linear Algebra HW9

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**1**

$$\begin{aligned} \|Q_1 Q_2 x\| &= (Q_1 Q_2 x)^T (Q_1 Q_2 x) = x^T Q_2^T Q_1^T Q_1 Q_2 x = x^T I x \\ &\Rightarrow (Q_1 Q_2)^T (Q_1 Q_2) = I \\ &\Rightarrow Q_1 Q_2 \text{ is a othogonal matrix.} \end{aligned}$$

**2**

$$\begin{aligned} q_1 &= a/\|a\| = (0.7071, 0.7071, 0)^T - <ans> \\ b' &= b - (q_1^T b)q_1 \\ q_2 &= b'/\|b'\| = (0.4082, -0.4082, 0.8165)^T - <ans> \\ c' &= c - (q_1^T c)q_1 - (q_2^T c)q_2 \\ q_3 &= c'/\|c'\| = (-0.5774, 0.5774, 0.5774)^T - <ans> \end{aligned}$$

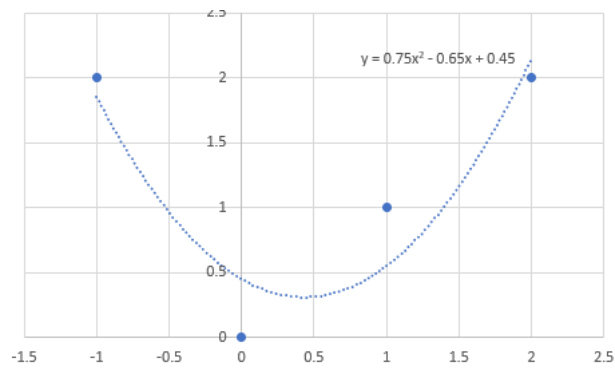
**3**

**(a)**

$$\begin{aligned} A^T A \hat{x} &= A^T b \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix} \\ \begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} &= \begin{bmatrix} 0.45 \\ -0.65 \\ 0.75 \end{bmatrix} - <ans> \end{aligned}$$

**(b)**

$$\begin{aligned} q_1 &= a/\|a\| \\ b' &= b - (q_1^T b)q_1 \\ q_2 &= b'/\|b'\| \\ c' &= c - (q_1^T c)q_1 - (q_2^T c)q_2 \\ q_3 &= c'/\|c'\| \\ A &= QR = \begin{bmatrix} 0.5 & -0.6708 & 0.5 \\ 0.5 & -0.2336 & -0.5 \\ 0.5 & 0.2236 & -0.5 \\ 0.5 & 0.6708 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2.2361 & 2.2361 \\ 0 & 0 & 2 \end{bmatrix} \\ A^T A \hat{x} &= A^T b \Rightarrow R^T Q^T Q R \hat{x} = R^T Q^T b \Rightarrow R \hat{x} = Q^T b \\ \hat{x} &= \begin{bmatrix} 0.45 \\ -0.65 \\ 0.75 \end{bmatrix} - <ans> \end{aligned}$$



4

$$b \text{ projects on } (1,0) = \frac{b \cdot (1,0)}{\|(1,0)\|} (1,0) = (1,0)$$

$$b \text{ projects on } (1,1) = \frac{b \cdot (1,1)}{\|(1,1)\|} (1,1) = \frac{3\sqrt{2}}{2} (1,1)$$

$$b = (1,2) \neq (1,0) + \frac{3\sqrt{2}}{2} (1,1)$$

5

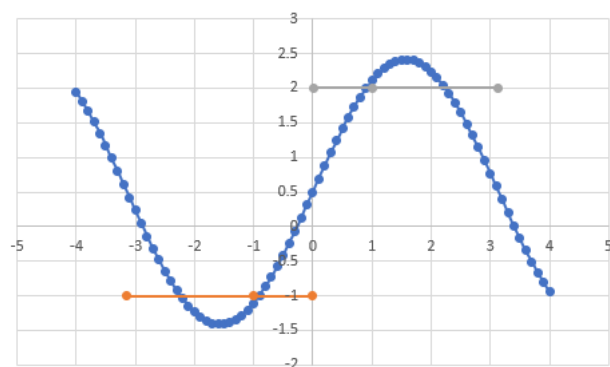
$$a_0 = \frac{\langle y, 1 \rangle}{\langle 1, 1 \rangle} = 0.5$$

$$a_1 = \frac{\langle y, \sin x \rangle}{\langle \sin x, \sin x \rangle} = 0$$

$$a_2 = \frac{\langle y, \sin 2x \rangle}{\langle \sin 2x, \sin 2x \rangle} = 0$$

$$b_1 = \frac{\langle y, \cos x \rangle}{\langle \cos x, \cos x \rangle} = \frac{6}{\pi}$$

$$b_2 = \frac{\langle y, \cos 2x \rangle}{\langle \cos 2x, \cos 2x \rangle} = 0$$



6

(a)

$$\begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = y$$

$$\begin{bmatrix} (1,1) & (1,x) & (1,x^2) & (1,x^3) \\ (x,1) & (x,x) & (x,x^2) & (x,x^3) \\ (x^2,1) & (x^2,x) & (x^2,x^2) & (x^2,x^3) \\ (x^3,1) & (x^3,x) & (x^3,x^2) & (x^3,x^3) \end{bmatrix} \begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} (y,1) \\ (y,x) \\ (y,x^2) \\ (y,x^3) \end{bmatrix}$$

$$\begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.3429 \\ 0 \\ -0.1058 \end{bmatrix} - < ans >$$

**(b)**

$$E^2 = \int_{-\pi}^{\pi} [y - (C + Dx + Ex^2 + Fx^3)]^2 dx$$

$$\frac{\partial E^2}{\partial C} = \frac{4E\pi^3}{3} + \pi(4C - 2)$$

$$\frac{\partial E^2}{\partial D} = \frac{4D\pi^3}{3} + \frac{4F\pi^5}{5} - 3\pi^2$$

$$\frac{\partial E^2}{\partial E} = \pi^3 \left( \frac{4C}{3} - \frac{2}{3} \right) + \frac{4E\pi^5}{5}$$

$$\frac{\partial E^2}{\partial F} = -\frac{3}{2}\pi^4 + \frac{4D\pi^5}{5} + \frac{4F\pi^7}{7}$$

$$\begin{bmatrix} 4\pi & 0 & \frac{4\pi^3}{3} & 0 \\ 0 & \frac{4\pi^3}{3} & 0 & \frac{4\pi^5}{5} \\ \frac{4\pi^3}{3} & 0 & \frac{4\pi^5}{5} & 0 \\ 0 & \frac{4\pi^5}{5} & 0 & \frac{4\pi^7}{7} \end{bmatrix} \begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 2\pi \\ 3\pi^2 \\ \frac{2}{3}\pi^3 \\ \frac{3}{2}\pi^4 \end{bmatrix}$$

$$\begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.3429 \\ 0 \\ -0.1058 \end{bmatrix} - < ans >$$

**(c)**

$$q_1 = 1$$

$$q_2 = x - \frac{(q_1, x)}{(q_1, q_1)} q_1 = x$$

$$q_3 = x^2 - \frac{(q_1, x^2)}{(q_1, q_1)} q_1 - \frac{(q_2, x^2)}{(q_2, q_2)} q_2 = x^2 - \frac{\pi^2}{3}$$

$$q_4 = x^3 - \frac{(q_1, x^3)}{(q_1, q_1)} q_1 - \frac{(q_2, x^3)}{(q_2, q_2)} q_2 - \frac{(q_3, x^3)}{(q_3, q_3)} q_3 = x^3 - \frac{3\pi^2}{5} x$$

$$\text{Let } y = Cq_1 + Dq_2 + Eq_3 + Fq_4$$

$$\begin{bmatrix} (q_1, q_1) & (q_1, q_2) & (q_1, q_3) & (q_1, q_4) \\ (q_2, q_1) & (q_2, q_2) & (q_2, q_3) & (q_2, q_4) \\ (q_3, q_1) & (q_3, q_2) & (q_3, q_3) & (q_3, q_4) \\ (q_4, q_1) & (q_4, q_2) & (q_4, q_3) & (q_4, q_4) \end{bmatrix} \begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} (y, q_1) \\ (y, q_2) \\ (y, q_3) \\ (y, q_4) \end{bmatrix}$$

$$\begin{bmatrix} C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.7162 \\ 0 \\ -0.1058 \end{bmatrix} - < ans >$$

(d)

