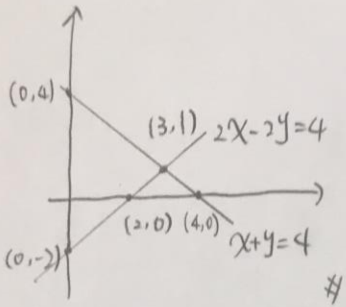
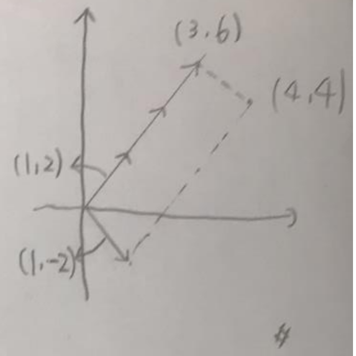
1. row picture



#

column picture:

$$\begin{bmatrix}1\\2\end{bmatrix}x+\begin{bmatrix}1\\-2\end{bmatrix}y=\begin{bmatrix}4\\4\end{bmatrix}$$



#

2. 整理成 $Ax=B$ 之形式

$$\underbrace{\begin{bmatrix}1&1&1&1\\1&0&1&1\\1&0&1&0\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}u\\v\\w\\z\end{bmatrix}}_{x}=\underbrace{\begin{bmatrix}6\\4\\2\end{bmatrix}}_{B}$$

因為Rank(A)=3,而變數有4個,4-3=1,因此為一條直線 #

If $u=-1$ included, then

$$\begin{bmatrix}1&1&1&1\\1&0&1&1\\1&0&1&0\\1&0&0&0\end{bmatrix}\begin{bmatrix}u\\v\\w\\z\end{bmatrix}=\begin{bmatrix}6\\4\\2\\-1\end{bmatrix}$$

we can solve $(u,v,w,z)=(-1,2,3,2)$, the intersection is a point. #

3.

$$u+v+w=2 \quad -(1)$$
$$u+2v+3w=1 \quad -(2)$$
$$v+2w=0 \quad -(3)$$

(a)

$\underline{(1)+(-1)\times(2)+(3) \Rightarrow 0=1}$ #

(b)

$$\begin{bmatrix}1\\1\\0\end{bmatrix}u+\begin{bmatrix}1\\2\\1\end{bmatrix}v+\begin{bmatrix}1\\3\\2\end{bmatrix}w=\begin{bmatrix}2\\1\\0\end{bmatrix}$$

we can let $\underline{\begin{bmatrix}1\\1\\0\end{bmatrix}+\begin{bmatrix}1\\3\\2\end{bmatrix}=\begin{bmatrix}2\\4\\2\end{bmatrix}=2\times\begin{bmatrix}1\\2\\1\end{bmatrix}}$ #

from the result above, we choose $(u,v,w)=(-1,2,-1) \neq (2,1,0)$

Hence, there is no solution. #

If $b=\begin{bmatrix}0\\0\\0\end{bmatrix}$, then

$$\begin{bmatrix}1&1&1\\1&2&3\\0&1&2\end{bmatrix}\begin{bmatrix}u\\v\\w\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}, \text{ by}$$

$$\Rightarrow \begin{bmatrix}1&1&1\\0&1&2\\0&0&0\end{bmatrix}\begin{bmatrix}u\\v\\w\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix},$$

有無限多解，

如果 $2v=-w=-u$ #

4、在斜率相同時，即 $\frac{y_2-y_1}{1-0}=\frac{y_3-y_2}{2-1}=\frac{y_3-y_1}{2-0}$

$\Rightarrow 2(y_2-y_1)=2(y_3-y_2)=(y_3-y_1)$

令 $y_1=K$，可得 $y_2=2K$，$y_3=3K$

當 $y_1=k$，$y_2=2k$，$y_3=3k$，$k=$ 任意常數 #

5、 $\begin{bmatrix}1\\0\\0\end{bmatrix}u+\begin{bmatrix}-1\\1\\0\end{bmatrix}v+\begin{bmatrix}-1\\1\\1\end{bmatrix}w=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

Let $w=b_3$, then $\begin{cases}v=b_2-b_3 & \because v+w=b_2\\ u=b_1+b_2 & \because u-v-w=b_1\end{cases}$

then the columns combination is

$\begin{bmatrix}1\\0\\0\end{bmatrix}(b_1+b_2)+\begin{bmatrix}-1\\1\\0\end{bmatrix}(b_2-b_3)+\begin{bmatrix}-1\\1\\1\end{bmatrix}(b_3)=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$ #

6、 $\begin{cases}t=0 & -① \\ z=0 & -② \\ x+y+z+t=1 & -③\end{cases}$ 由①，②式代入③，可得

$x+y=1$，令 $x=k$，$y=1-k$，k 為任意常數

$\begin{cases}k=0, & (x,y,z,t)=(0,1,0,0)\\ k=1, & (x,y,z,t)=(1,0,0,0)\end{cases}$ #