

Linear Algebra HW10

1

$$A = \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 1 & 2 & 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$$\det(A) = 1 * (-1) * (-2) * 10 = 20 - < ans >$$

$$B = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & \frac{3}{2} & -1 & \frac{-1}{2} \\ 0 & -1 & 2 & -1 \\ 0 & \frac{-1}{2} & -1 & \frac{3}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & \frac{3}{2} & -1 & \frac{-1}{2} \\ 0 & 0 & \frac{4}{3} & \frac{-4}{3} \\ 0 & 0 & \frac{-4}{3} & \frac{4}{3} \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & \frac{3}{2} & -1 & \frac{-1}{2} \\ 0 & 0 & \frac{4}{3} & \frac{-4}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\det(B) = 0 - < ans >$$

2

$$\det(Q^T Q) = \det(I) = 1 = \det(Q^T) \det(Q)$$

$$\therefore \det(Q^T) = \det(Q)$$

$$\therefore \det(Q^T) \det(Q) = (\det(Q))^2 = 1$$

$$\Rightarrow \det(Q) = +1 \quad \text{or} \quad -1$$

The columns of Q form a box whose volume = 1.

3

$$\text{Let } \vec{V}_1 = (0, 3, 3) - (1, 1, 1) = (-1, 2, 2)$$

$$\vec{V}_2 = (3, 0, 3) - (1, 1, 1) = (2, -1, 2)$$

$$\vec{V}_3 = (3, 3, 0) - (1, 1, 1) = (2, 2, -1)$$

$$\text{volume} = \begin{vmatrix} \vec{V}_1 \\ \vec{V}_2 \\ \vec{V}_3 \end{vmatrix} = 27 - < ans >$$

4

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

$$\text{for } a_{11} \Rightarrow 4 * 3 * 2 * 1 = 24$$

$$\text{for } a_{12}, \quad a_{13}, \quad a_{14}, \quad a_{15} \Rightarrow 3 * 3 * 2 * 1 = 18$$

$$5^5 - 24 - 4 * 18 = 3029 \quad \text{terms} - < ans >$$