# Linear Algebra HW7

## 1

## 2

### (a)

$$T=\begin{bmatrix}0&1\\1&0\end{bmatrix}$$
$$T(\vec{0})=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$
$$T(cv)=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}cv_1\\cv_2\end{bmatrix}=\begin{bmatrix}cv_2\\cv_1\end{bmatrix}=cT(v)$$
$$T(u+v)=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}u_1+v_1\\u_2+v_2\end{bmatrix}=\begin{bmatrix}u_2\\u_1\end{bmatrix}+\begin{bmatrix}v_2\\v_1\end{bmatrix}=T(u)+T(v)$$
$$\Rightarrow T\quad is\quad linear.-<ans>$$
$$Range\quad is\quad c_1\begin{bmatrix}0\\1\end{bmatrix}+c_2\begin{bmatrix}1\\0\end{bmatrix}-<ans>$$
$$Kernel\quad is\quad Tv=0,\quad v=\begin{bmatrix}0\\0\end{bmatrix}-<ans>$$

### (b)

$$T=\begin{bmatrix}1&0\\1&0\end{bmatrix}$$
$$T(\vec{0})=\begin{bmatrix}1&0\\1&0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$
$$T(cv)=\begin{bmatrix}1&0\\1&0\end{bmatrix}\begin{bmatrix}cv_1\\cv_2\end{bmatrix}=\begin{bmatrix}cv_1\\cv_1\end{bmatrix}=cT(v)$$
$$T(u+v)=\begin{bmatrix}1&0\\1&0\end{bmatrix}\begin{bmatrix}u_1+v_1\\u_2+v_2\end{bmatrix}=\begin{bmatrix}u_1\\u_1\end{bmatrix}+\begin{bmatrix}v_1\\v_1\end{bmatrix}=T(u)+T(v)$$
$$\Rightarrow T\quad is\quad linear.-<ans>$$
$$Range\quad is\quad c_1\begin{bmatrix}1\\1\end{bmatrix}<ans>$$
$$Kernel\quad is\quad Tv=0,\quad v=c_1\begin{bmatrix}0\\1\end{bmatrix}-<ans>$$

### (c)

$$T=\begin{bmatrix}0&0\\1&0\end{bmatrix}$$
$$T(\vec{0})=\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$
$$T(cv)=\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}cv_1\\cv_2\end{bmatrix}=\begin{bmatrix}0\\cv_1\end{bmatrix}=cT(v)$$
$$T(u+v)=\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}u_1+v_1\\u_2+v_2\end{bmatrix}=\begin{bmatrix}0\\u_1\end{bmatrix}+\begin{bmatrix}0\\v_1\end{bmatrix}=T(u)+T(v)$$
$$\Rightarrow T\quad is\quad linear.-<ans>$$
$$Range\quad is\quad c_1\begin{bmatrix}0\\1\end{bmatrix}<ans>$$
$$Kernel\quad is\quad Tv=0,\quad v=c_1\begin{bmatrix}0\\1\end{bmatrix}-<ans>$$

### (d)

$$T(\vec{0})=(0,1)$$
$$T\quad is\quad not\quad linear.-<ans>$$

# 3

## (a)

$$\frac{d^2}{dt^2}(0) = 0$$
$$\frac{d^2}{dt^2}(cA) = \frac{d^2}{dt^2}(ca_0 + ca_1t + ca_2t^2 + ca_3t^3) = c(2a_2 + 6a_3t) = c\frac{d^2}{dt^2}(A)$$
$$\frac{d^2}{dt^2}(A+B) = \frac{d^2}{dt^2}(a_0 + a_1t + a_2t^2 + a_3t^3 + b_0 + b_1t + b_2t^2 + b_3t^3) = (2a_2 + 6a_3t) + (2b_2 + 6b_3t) =$$
$$The \quad transformation \quad is \quad linear.$$

## (b)

$$a_0 + a_1t + a_2t^2 + a_3t^3 \quad can \quad be \quad expressed \quad a_0\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + a_1\begin{bmatrix}0\\1\\0\\0\end{bmatrix} + a_2\begin{bmatrix}0\\0\\1\\0\end{bmatrix} + a_3\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$After \quad taking \quad second \quad derivative, \quad the \quad basis \quad become \quad \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\6\\0\\0\end{bmatrix}$$

$$then \quad \frac{d^2}{dt^2}A = a_0\begin{bmatrix}0\\0\\0\\0\end{bmatrix} + a_1\begin{bmatrix}0\\0\\0\\0\end{bmatrix} + a_2\begin{bmatrix}2\\0\\0\\0\end{bmatrix} + a_3\begin{bmatrix}0\\6\\0\\0\end{bmatrix} - < ans >$$

## (c)

$$From \quad the \quad result \quad in \quad (b), \quad the \quad matrix \quad can \quad be \quad expressed:$$
$$\begin{bmatrix}0 & 0 & 2 & 0\\0 & 0 & 0 & 6\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix} - < ans >$$

## (d)

$$From \quad the \quad result \quad in \quad (c), \quad the \quad matrix \quad can \quad be \quad expressed:$$
$$\begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} - < ans >$$

$$\frac{d^2}{dt^2}(4 + 3t + 2t^2 + t^3) \Rightarrow \begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}4\\3\\4\\6\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix} \Rightarrow 4 + 6t - < ans >$$

# 4

$$\begin{bmatrix}cos2\theta & sin2\theta\\sin2\theta & -cos2\theta\end{bmatrix}\begin{bmatrix}cos2\alpha & sin2\alpha\\sin2\alpha & -cos2\alpha\end{bmatrix} = \begin{bmatrix}cos(2\theta - 2\alpha) & -sin(2\theta - 2\alpha)\\sin(2\theta - 2\alpha) & cos(2\theta - 2\alpha)\end{bmatrix}$$
$$the \quad matrix \quad is \quad rotation \quad matrix \quad with \quad angle \quad 2\theta - 2\alpha.$$

# 5

$Null \quad space \quad is \quad orthogonal \quad to \quad row \quad space.$
$Left-null \quad space \quad is \quad orthogonal \quad to \quad column \quad space.$
$Hence, \quad x \quad is \quad in \quad the \quad null \quad space, \quad y \quad is \quad in \quad the \quad left-nullspace, \quad and \quad z \quad is \quad in \quad t$
$Solving \quad Ax=0 \quad can \quad get \quad null \quad space, \quad and \quad solving \quad A^Ty=0 \quad can \quad get \quad left-null \quad sp$
$x=(-2,1,0)^T - <ans>$
$y=(-1,-1,1)^T - <ans>$
$z=(1,2,1) - <ans>$

## 6

$(\Rightarrow): (x-y) \quad is \quad orthogonal \quad to \quad (x+y), \quad that \quad is \quad (x-y)^T(x+y)=0$
$(x-y)^T(x+y)=[(x_1-y_1)(x_1+y_1)+\ldots+(x_n-y_n)(x_n+y_n)]=||x||-||y||$
$(x-y)^T(x+y)=0=||x||-||y|| \Rightarrow ||x||=||y||$
$(\Leftarrow): ||x||=||y||, \quad then \quad proof \quad (x-y) \quad is \quad orthogonal \quad to \quad (x+y)$
$||x||-||y||=(x_1^2+\ldots+x_n^2)-(y_1^2+\ldots+y_n^2)=[(x_1-y_1)(x_1+y_1)+\ldots+(x_n-y_n)(x_n+y_n)]=(x-y$
$\because ||x||=||y|| \Rightarrow ||x||-||y||=0=(x-y)^T(x+y)$

## 7

$A=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\end{bmatrix}, \quad get \quad null \quad space \quad by \quad solving \quad Ax=0.$
$x=(-1,-1,1)^T \quad is \quad the \quad orthogonal \quad complement. - <ans>$

## 8