Linear Algebra and its Applications

HW#08

1. Find the matrix that projects every vector in R^3 onto the intersection of the planes $x_1+x_2+x_3=0$ and $x_1-x_3=0$, which is a line.

2. Suppose the values $b_1=1$ and $b_2=7$ at times $t_1=1$ and $t_2=5$ are fitted by a line $b=Dt$ through the origin. Find $\hat{D}$ by least square and sketch the observations with the best-fit line. Find $\hat{D}$ by projection and sketch the projection of b onto the column space of t.

3. Write out $E^2=\|Ax-b\|^2$ and set to zero its derivatives with respect to u and v, if

$$A=\begin{bmatrix}1 & 0\\ 0 & 1\\ 1 & 1\end{bmatrix},\ x=\begin{bmatrix}u\\ v\end{bmatrix},\ b=\begin{bmatrix}1\\ 3\\ 4\end{bmatrix}$$

Compare the resulting equations with the normal equations. Find the least squared approximate of x and the b's projection onto the column space of A.

4. If V is the subspace spanned by (1, 1, 0, 1) and (0, 0, 1, 0), find
   (a) a basis for the orthogonal complement $V^{\perp}$
   (b) the projection matrix P onto $V^{\perp}$
   (c) the vector in V closest to the vector b = (0, 1, 0, -1) in $V^{\perp}$

5. If P is the projection matrix onto a k-dimensional subspace S of the whole space $\mathbf{R}^n$, what is the column space and nullspace of P and what is its rank?

6. If u is a unit vector, show that $Q=I-2uu^T$ is a reflection transformation. Compute Q when $u^T=(1/2, 1/2, -1/2, -1/2)$ and explain what Q does to x with Qx.

7. (a) Find the bases for the null space and the row space of

$$A=\begin{bmatrix}1 & 0 & 2\\ 1 & 1 & 4\end{bmatrix}.$$

   (b) Split $x = (3, 3, 3)^T$ into a row-space component x_r and a null-space component x_n.
   (c) Find the pseudoinverse A^+ such that $A^+Ax=x_r$.
   (d) Let $Ax=(9, 21)^T$. Recover the row space component of x.
   (e) Show that the pseudoinverse found in (c) is the right inverse of A.