Linear Algebra HW15

1

$Case: one\ \ eigenvalue\ \ is\ \ zero.$
$The\ \ ellipsoid\ \ will\ \ become\ \ infinite\ \ long\ \ cylinder\ \ with\ \ ellipse\ \ section.$
$Case: two\ \ eigenvalues\ \ are\ \ zero.$
$The\ \ ellipsoid\ \ will\ \ become\ \ two\ \ parallel\ \ planes\ \ with\ \ distance\ \ \frac{1}{\sqrt{\lambda}}$
$Case: All\ \ eigenvalues\ \ are\ \ zero.$
$0 \neq 1 \Rightarrow Not\ \ exist.$

2

$$\frac{\partial P_1}{\partial x} = x + y = 0$$
$$\frac{\partial P_1}{\partial y} = x + 2y - 3 = 0$$
$$\Rightarrow P_1\ \ at\ \ (x, y) = (-3, 3)\ \ has\ \ minimum\ \ -\frac{9}{2}$$
$$let\ \ P_2 = \frac{1}{2}\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 0 \\ 3 \end{bmatrix}$$
$$But\ \ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}\ \ has\ \ no\ \ solution. \Rightarrow minP_2\ \ doesn't\ \ exist.$$

3

$$R(x) = \frac{x_1^2 - x_1x_2 + x_2^2}{x_1^2 + x_2^2} = \frac{\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} 1 & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}{\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \frac{X^TAX}{X^TX}$$
$$By\ \ Rayleigh's\ \ quotient,\ \ \lambda(A)_{min} \leq R \leq \lambda(A)_{max}$$
$$det(A - \lambda I) = 0 \Rightarrow \lambda = \frac{1}{2}, \frac{3}{2}$$
$$\Rightarrow minR(x) = \frac{1}{2} - < ans >$$
$$let\ \ Y = \begin{bmatrix} \sqrt{2}x_1 \\ x_2 \end{bmatrix} \Rightarrow R(x) = \frac{Y^T\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{2}}{4} \\ \frac{-\sqrt{2}}{4} & 1 \end{bmatrix}Y}{Y^TY} = \frac{Y^TAY}{Y^TY}$$
$$det(A - \lambda I) = 0 \Rightarrow \lambda(A) = \frac{4-\sqrt{2}}{4}, \frac{4+\sqrt{2}}{4}$$
$$minR(x) = \frac{4-\sqrt{2}}{4} - < ans >$$

4

(a)

$$covariance\ \sigma_{ij} = \frac{\sum_{i=1}^{i=10}(x_{ki}-\bar{x}_i)(x_{kj}-\bar{x}_j)}{10-1},\ where\ \bar{x}_i = 62309.1,\ \bar{x}_j = 2927.3$$

$$Then\ covariance\ matrix\ \Sigma = \begin{bmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{bmatrix} = \frac{A^TA}{10-1}$$

$$= \frac{1}{9}\begin{bmatrix}9004582022.9 & 230180396.7\\ 230180396.7 & 12870180.1\end{bmatrix}$$

$$= \begin{bmatrix}1000509113.7 & 25575597.6\\ 25575599.6 & 1430020.01\end{bmatrix} - < ans >$$

$$correlastion\ matrix\ \rho = \begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{bmatrix},\ where\ \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i\sigma_j}$$

$$Then\ \rho = \begin{bmatrix}1 & 0.67615185\\ 0.67615185 & 1\end{bmatrix} - < ans >$$

(b)

$det(\Sigma - \lambda I) = 0,\ \lambda = 1.00116e9,\ 775734$

$for\ \lambda = 1.00116e9,\ \ (\Sigma - \lambda I)x_1 = 0 \Rightarrow x_1 = (0.99967, 0.02557)^T$

(c)

$det(\rho - \lambda I) = 0,\ \lambda = 1.67615, 0.323848$

$for\ \lambda = 1.67615,\ (\rho - \lambda I)x_2 = 0 \Rightarrow x_2 = (1,1)^T$

(d)

b. weighted index is majorly determined by sales.

c. weighted index are almost the same, because normalized process.

(e)

$$for\ \lambda = 0.32848,\ (\rho - \lambda I)x_3 \Rightarrow (\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2})^T$$

$Cov(e_2^Ty, e_1^Ty) = e_2^TB^TBe_1 = x_3^T\rho x_2 = 0.$

$where\ x_2 = sales + profit,\ \ x_3 = sales - profit$