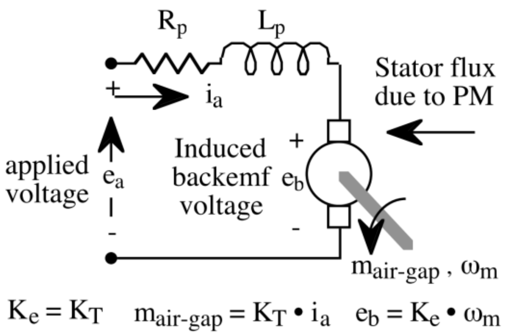
Digital Control

2021.06.17

Final exam

(Due : 2021.06.24 12pm)

Final exam is to design the digital current controller for motor drives. Considering the voltage input motor drive in Fig. 1, the corresponding discrete-time model is shown in Fig. 2. By adding the current sensor, the current regulator can be implemented. The expected controller is shown in Fig. 3. The controller input is current command i_a*(k) and output is the actual motor current, i_a(k). The EMF voltage drop is assumed as the external disturbance e_{dis}(k) for simplicity.



Jp	=	0.015×10^{-3}	Kg-m^2	polar moment of inertia
K_T	=	0.14	Nm/Amp	torque constant
K_e	=	0.14	volts/rad/sec	backemf constant
Rp	=	2.6	ohm	armature resistance
Lp	=	4.3	mH	armature inductance
T	=	0.0002	sec (5kHz)	sample period (frequency)

e_a = applied terminal voltage in volts
i_a = armature current in amperes
mag = electromagnetic air-gap torque (moment) = KT *i_a
e_{bk} = induced (back emf) voltage = Ke*Wm in volts
ω_m = load angular velocity in rad per sec

Fig. 1. DC motor with voltage input

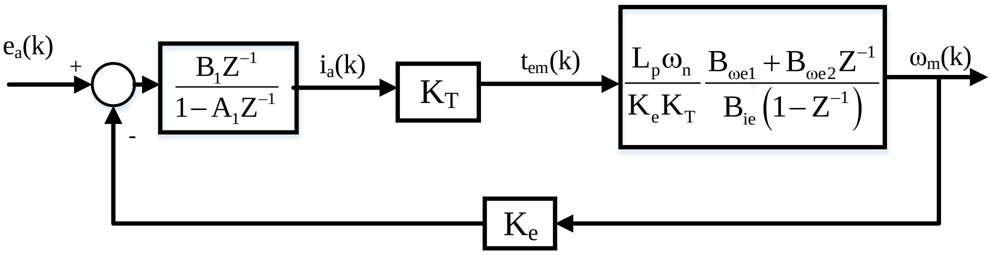


Fig. 2. The Z-domain model for the DC motor with voltage input

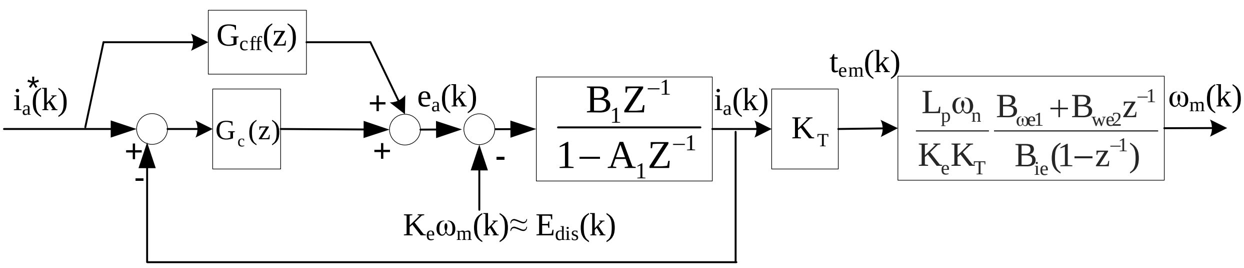


Fig. 3. The voltage manipulated DC motor with current regulation

For your convenience, some Z-domain functions from the Project#1 are given here.

$$\frac{I_a(z)}{E_a(z)} = \frac{B_1 z^{-1}}{1-A_1 z^{-1}} = \frac{0.0437z^{-1}}{1-0.8805z^{-1}}$$

$$\frac{\omega_m(z)}{I_a(z)} = \frac{L_p\omega_n}{K_eK_T}\frac{B_{\omega e1}+B_{\omega e2}z^{-1}}{B_{ie}(1-z^{-1})} = 16.9312\ \frac{0.0058+0.0056z^{-1}}{0.1036(1-z^{-1})}$$

$$A_1 = \frac{C_{2t}}{\omega_n^2}\quad B_1 = \frac{C_{1t}}{Lp\ \omega_n^2}\quad B_{\omega e1} = C_{3t}\quad B_{ie} = \frac{C_{1t}}{\omega_n}\quad B_{\omega e2} = \frac{C_{1t}^2-C_{2t}C_{3t}}{\omega_n^2}$$

$$\omega_n^2 = \frac{Ke\ Kt}{Lp\ Jp}\quad \zeta = \frac{Rp}{2\ \omega_n Lp}\quad C_1 = \frac{\omega_n}{\sqrt{1-\zeta^2}}\quad C_2 = -\zeta\omega_n T\quad C_3 = \sqrt{1-\zeta^2}\omega_n T\quad \varphi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\quad C_{1t} = C_1 e^{C2}\sin(C_3)$$

$$C_{2t} = -\frac{\omega_n^2}{\sqrt{1-\zeta^2}}\ e^{C2}\sin(C_3-\varphi)\quad C_{3t} = 1-\frac{1}{\sqrt{1-\zeta^2}}\ e^{C2}\sin(C_3+\varphi)$$

You are asked to：

1.(20%)PI control: Design a current controller $C_c(z) = C_{LP}(z)$ with zero steady state error, no overshoot, and 100Hz bandwidth ($s = -100\times2\pi$ and $z=e^{ST}$). You are asked to use **direct Z-domain design method** to let the whole system ($\frac{I_a(z)}{I_a{*}(z)}$)behave as a low pass filter. Show the controller $C_{LP}(z)$ transfer function with the numerical value.

2.(20%)Deadbeat control: You are asked to design a feedback control $C_c(z)=C_{db}(z)$ with the fastest dynamic response. **The deadbeat control is defined by $G_{dr}(z) = \frac{I_a(z)}{I_a{*}(z)} = z^{-1}$.** Show the controller $C_{db}(z)$ transfer function with the numerical value.

3.(10%)One step delay: Based on Simulink, apply a 1A/100Hz sine wave current command. Overlay two figures, $i_a{*}(k)$&$i_a(k)$ vs. time, and $i_{err}(k)= i_a{*}(k)-i_a(k)$ vs. time with deadbeat $C_{db}(z)$ control. Explain the one step delay issue on $i_a(k)$. **Propose your own controller to solve this delay. Otherwise, explain the reason why the one-step delay cannot resolve.**

4. (20%)Command tracking: Design a current feedforward controller $C_{cff}(z)$ in Fig. 3. Assuming the ideal parameter estimation, show $C_{cff}(z)$ transfer function with the numerical value. Apply a 1A/100Hz sine wave current command to the PI plus CFF control system. Overlay another two figures, $i_a{*}(k)$&$i_a(k)$ vs. time, and $i_{err}(k)= i_a{*}(k)-i_a(k)$ vs. time. Compare $i_a(k)$ response with Problem 2. With CFF, no phase delay should be observed ? **Otherwise, explain the reason why the delay still appears with CFF.**

5. (15%)Disturbance rejection: Apply a discretized 1V/100Hz sine wave voltage disturbance $e_{dis}(k)$ in Fig. 3. Explain the performance difference between PI plus CFF and deadbeat without CFF. Explain the performance difference between two different control methods.

6.(15%)Parameter sensitivity: Re-evaluate the command tracking (Problem 4) and disturbance rejection (Problem 5) on two different control methods, PI plus CFF and deadbeat without CFF. However, the parameter error is considered under $\hat{R}_p=4R_p$. Considering the resistance parameter error, summary differences between two control methods from both command tracking and disturbance rejection. **No simulation is required.** Only theory explanation with the support from key equations are expected.

Note: The design of CFF and deadbeat control are all based on the knowledge of new system dynamic (NSD). However, there is one difference between CFF and deadbeat control.