Linear Algebra HW8

1

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - x_3 = 0$$
$$\Rightarrow x_1 = -2x_2 = x_3, \quad let \quad a = (1, -2, 1)^T$$
$$then \quad projection \quad matrix \quad P = \frac{aa^T}{a^Ta} = \begin{bmatrix} \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \\ \frac{-1}{3} & \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \end{bmatrix} - < ans >$$

2

$$by \quad least \quad square:$$
$$b = Dt \Rightarrow 1 = d_1, \quad 7 = 5d_2$$
$$let \quad E = (1 - \hat{D})^2 + (7 - 5\hat{D})^2, \quad and \quad \frac{dE^2}{d\hat{D}} = 0$$
$$\Rightarrow 18 - 13\hat{D} = 0, \quad \hat{D} = \frac{18}{13} - < ans >$$
$$by \quad projection:$$
$$A^TAx = A^Tb \Rightarrow x = (A^TA)^{-1}A^Tb$$
$$\hat{D} = (t^Tt)^{-1}t^Tb = (\begin{bmatrix} 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix})^{-1} \begin{bmatrix} 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 7 \end{bmatrix} = \frac{18}{13} - < ans >$$

3

$$E^2 = |Ax - b|^2$$
$$\frac{dE^2}{dx} = 0 = A^T(Ax - b) \Rightarrow x = (A^TA)^{-1}A^Tb = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$
$$Ax = b \Rightarrow x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$
$$projection \quad of \quad b \quad onto \quad the \quad column \quad space: A(A^TA)^{-1}A^Tb$$
$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

4

(a)

$$VV^{\perp} = 0 \Rightarrow V^{\perp} \quad is \quad in \quad null \quad space.$$

$$solve \quad VV^{\perp} = 0, \quad V^{\perp} = c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$the \quad basis \quad are: \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - < ans >$$

(b)

$$projection \quad matrix \quad P = V(V^TV)^{-1}V^T, \quad where \quad V = \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P = V(V^TV)^{-1}V^T = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} & 0 & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} & 0 & \frac{-1}{3} \\ 0 & 0 & 0 & 0 \\ \frac{-1}{3} & \frac{-1}{3} & 0 & \frac{2}{3} \end{bmatrix}$$

(c)

$$P = A^T(AA^T)^{-1}A, \quad where \quad A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$Pb = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - < ans >$$

5

$$\vec{x} \in \mathbb{R}^n, \quad P\vec{x} \subset S \Rightarrow col(P) \subset S$$
$$\vec{b} \in S, \quad P\vec{b} = \vec{b} \Rightarrow S \subset col(P)$$
$$\Rightarrow col(P) = S - < ans >$$
$$rank \quad of \quad P = rank \quad of \quad S = k - < ans >$$
$$dimension \quad of \quad null \quad space: n - k - < ans >$$

6

$$Q = I - 2uu^T = \begin{bmatrix} 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix}$$

Qx是x對u鏡射再做反向

7

(a)

$$After \quad Guassian \quad elimination \quad A \quad becomes: \begin{bmatrix}1 & 0 & 2\\0 & 1 & 2\end{bmatrix}$$
$$bases \quad of \quad row \quad space: (1,0,2),(0,1,2) - <ans>$$
$$bases \quad of \quad null \quad space: (-2,-2,1) - <ans>$$

(b)

$$x = x_r + x_n$$
$$\begin{bmatrix}3\\3\\3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\\2 & 2\end{bmatrix} x_r + \begin{bmatrix}-2\\-2\\1\end{bmatrix} x_n \Rightarrow \begin{bmatrix}1 & 0 & -2\\0 & 1 & -2\\2 & 2 & 1\end{bmatrix} \begin{bmatrix}x_{r_1}\\x_{r_2}\\x_n\end{bmatrix} = \begin{bmatrix}3\\3\\3\end{bmatrix}$$
$$x_r = \begin{bmatrix}1\\1\end{bmatrix}, x_n = -1 - <ans>$$

(c)

$$A^+(Ax) = x_r$$
$$matrix \quad projects \quad to \quad row \quad space \quad P = A^T(AA^T)^{-1}A$$
$$A^+ = A^T(AA^T)^{-1} = \begin{bmatrix}1 & \frac{-4}{9}\\-1 & \frac{5}{9}\\0 & \frac{2}{9}\end{bmatrix} - <ans>$$

(d)

$$Ax = \begin{bmatrix}9\\21\end{bmatrix}$$
$$x = \begin{bmatrix}9\\12\\0\end{bmatrix} + c\begin{bmatrix}-2\\-2\\1\end{bmatrix}$$
$$A^+A\begin{bmatrix}9\\12\\0\end{bmatrix} = \begin{bmatrix}\frac{-1}{3}\\\frac{8}{3}\\\frac{14}{3}\end{bmatrix} \subset Row \quad space \quad of \quad A$$

(e)

$$AA^+ = \begin{bmatrix}1 & 0 & 2\\1 & 1 & 4\end{bmatrix}\begin{bmatrix}1 & \frac{-4}{9}\\-1 & \frac{5}{9}\\0 & \frac{2}{9}\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$