Linear Algebra and its Applications

HW#9

1. If Q_1 and Q_2 are orthogonal matrices, so that $Q^TQ = I$, show that Q_1Q_2 is also orthogonal.

2. Apply the Gram-Schmidt process to a=[1, 1, 0]^T, b=[1, 0, 1]^T and c=[0, 1, 1]^T and write the result in the form A=QR.

3. (a) Find the parabola: $y = C + Dt + Et^2$ fit to the following measurements by solving the normal equations:
   $y = 2$ at $t = -1$,
   $y = 0$ at $t = 0$,
   $y = 1$ at $t = 1$,
   $y = 2$ at $t = 2$.
   (b) Find your approximate solution by QR factorization and draw the observations with best-fit parabola on Excel.

4. Project the vector b=(1, 2) onto a 2-dimensional space with two basis vectors, (1, 0) and (1, 1), and show that, unlike the orthogonal basis, the sum of the two projections does not equal to b.

5. Find the Fourier coefficients a_0, a_1, b_1, a_2, b_2 to approximate a step function $y(x)$ which equals to -1 for the interval $-\pi \le x \le 0$ and equals to - for interval $0 < x \le \pi$. Use Excel to plot $y(x)$ and the Fourier series on the same figure.

6. Find the closest degree-3 polynomial function to fit the same step function in problem (4) over $-\pi \le x \le \pi$ by:
   (1) solving the normal equation
   (2) minimizing the least square
   (3) the Legendre polynomials and
   (4) Use Excel to plot the original step function and the fitted polynomial function on the same figure.