

Linear Algebra and its Applications
2021 Fall HW#3

1.
 - (a) Show that A (square matrix with no row exchanges) and A^T share the same pivots.
 - (b) If A is invertible and symmetric, what is the transpose of A^{-1} ?
 - (c) Suppose R is rectangular (m by n) and A is symmetric (m by m), show that $R^T A R$ is symmetric? What shape (? by ?) is this matrix?
2. If A is invertible, which properties of A remain true for A^{-1}
 - (a) A is triangular. (b) A is symmetric. (c) A is tri-diagonal.. (d) All entries are whole numbers. (e) All entries are fractions (including numbers like $3/1$).
3. Mr. Chen travels with his motorcycle. It is known that Mr. Chen's motorcycle accelerates and decelerates according to $f(t)=\sin(4\pi t)$ from time $t=0$ to $t=1$. Let the time interval h be 0.2 and both the initial speed and the final speed be zero. Find the distance $D(jh)$, $j=1, 2, \dots, 5$, Mr. Chen travels over times $t=0.2, 0.4, 0.6, 0.8, 1.0$ by formulating and solving an $Ax=b$ problem. Plot your solution $D(jh)$ vs. j .
4. Compare the pivots produced by eliminations with and without partial pivoting for $A=\begin{bmatrix} .0001 & 0 \\ 1 & 10000 \end{bmatrix}$.
Based on the finding above, try to rescale the matrix before elimination to produce a better set of pivots. (Note: scaling an equation by multiplying both the left-hand side and the right-hand side by a scalar will not change the solution of $Ax=b$.)
5. Show which of the following subsets of \mathbb{R}^3 are actually subspaces.
 - (a) The plane of vectors (b_1, b_2, b_3) with first component $b_1=0$.
 - (b) The plane of vectors b with $b_1=1$.
 - (c) The vectors b with $b_2 b_3=0$
 - (d) All combinations of two given vectors $(1, 1, 0)$ and $(2, 0, 1)$.
 - (e) The plane of vectors (b_1, b_2, b_3) that satisfy $b_1-b_2+3b_3=0$
6. True or false (with an explanation if true and a counterexample if false)?
 - (f) The vectors b , that are not in the column space of A , form a vector subspace.
 - (g) If the column space of A contains only the zero vector, then A is the zero matrix.

- (h) The column space of $2A$ equals the column space of A .
- (i) The column space of $A-I$ equals the column space of A .
- (j) \mathbb{R}^2 isn't a subspace of \mathbb{R}^3 .

7. Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 4 & 5 & 1 & 8 \\ 2 & 1 & 1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- (a) Perform the Gaussian elimination to the A and b . (A becomes U in Echelon form).
- (b) Given (a), under what conditions on b_1 and b_2 (if any) does $Ax=b$ have a solution?
- (c) Find the nullspace of U .