Linear Algebra and its Applications

HW#5

1. Find the dimension and a basis for the four fundamental subspaces for

$$A=\begin{bmatrix}1 & 2 & 0 & 1\\ 0 & 1 & 1 & 0\\ 1 & 2 & 0 & 1\end{bmatrix} \text{ and } U=\begin{bmatrix}1 & 2 & 0 & 1\\ 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

2. By performing the elimination to A and b so that A is reduced to a echelon form:

$$\begin{bmatrix}A & b\end{bmatrix}=\begin{bmatrix}1 & 2 & 3 & b_1\\ 4 & 5 & 6 & b_2\\ 7 & 8 & 9 & b_3\end{bmatrix}\to\begin{bmatrix}1 & 2 & 3 & b_1\\ 0 & -3 & -6 & b_2-4b_1\\ 0 & 0 & 0 & b_3-2b_2+b_1\end{bmatrix}$$

   (a) What is the basis for the null space?
   (b) What is the basis for the left-null space?
   (c) What is the basis for the row space?
   (d) What is the basis for the column space?
   (e) Show that the inner product between any vector in the left-null space and any vector in the column space is zero.

3. Find a 1 by 3 matrix whose nullspace consists of all vectors in R^3 such that $x_1+3x_3=0$. Find a 3 by 3 matrix with that same nullspace.

4. A is an m by n matrix of rank r. Suppose there are right-hand sides b for which $Ax = b$ has no solution.
   (a) What inequalities ($<$ or $\leq$) must be true between m, n, and r?
   (b) How do you know that $A^{T}y = 0$ has a nonzero solution?

5. Find the rank of A and write the matrix as $A = uv^T$:

$$A=\begin{bmatrix}1 & 0 & 0 & 3\\ 0 & 0 & 0 & 0\\ 2 & 0 & 0 & 6\end{bmatrix} \text{ and } A=\begin{bmatrix}2 & -2\\ 6 & -6\end{bmatrix}$$

6. If A is 2 by 3 and C is 3 by 2, show from its rank that there exit no C such that $CA=I$.

7. Calculate $(A^{T}A)^{-1}A^{T}$ or $A^{T}(AA^{T})^{-1}$ and find a left-inverse and/or a right-inverse (when they exist) for

$$A=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix} \quad\text{and}\quad M=\begin{bmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix} \quad\text{and}\quad T=\begin{bmatrix}a & b\\ 0 & a\end{bmatrix}$$