Linear Algebra HW6

1

(a)

$$\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0
\end{bmatrix}$$

(b)

$$aftet \quad Gaussian \quad elimination:$$
$$\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$
\begin{aligned}
&the \quad basis \quad for \quad the \quad row \quad space \quad are:\\
&(-1,1,0,0,0,0,0,0,0,0,0),(0,-1,1,0,0,0,0,0,0,0,0)\\
&(0,0,0,-1,1,0,0,0,0,0,0),(0,0,0,0,-1,1,0,0,0,0,0)\\
&(0,0,0,0,0,1,-1,0,0,0,0),(0,0,0,0,0,0,-1,1,0,0,0)\\
&(0,0,0,0,0,0,0,-1,1,0,0),(0,0,0,0,0,0,0,0,-1,1,0)\\
&the \quad basis \quad for \quad the \quad column \quad space \quad are:\\
&(-1,0,1,0,0,0,0,0,0,0,0)^T,(1,-1,0,0,0,0,0,0,0,0,0)^T\\
&(0,0,0,-1,0,1,1,0,0,0,0)^T,(0,0,0,1,-1,0,0,0,0,0,0)^T\\
&(0,0,0,0,1,-1,0,0,0,0,0)^T,(0,0,0,0,0,0,-1,-1,0,0,1)^T\\
&(0,0,0,0,0,0,0,1,-1,0,0)^T,(0,0,0,0,0,0,0,0,1,-1,0)^T\\
&AX=0: the \quad basis \quad for \quad the \quad null \quad space \quad are:\\
&(1,1,1,0,0,0,0,0,0,0,0)^T,(0,0,0,1,1,1,1,1,1,1,0)^T\\
&(0,0,0,0,0,0,0,0,0,0,1)^T\\
&by \quad the \quad loops \quad in \quad the \quad graph,\\
&the \quad basis \quad for \quad the \quad left-null \quad space \quad are:\\
&(1,1,1,0,0,0,0,0,0,0,0),(0,0,0,1,1,1,0,0,0,0,0)\\
&(0,0,0,0,0,0,0,1,1,1,1)
\end{aligned}
$$

(c)

$$null \quad space \quad means \quad that \quad the \quad elements \quad are \quad connected.$$

$$left-null \quad space \quad means \quad that \quad the \quad loops \quad in \quad the \quad graph.$$

2

(a)

$$
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0\\
0 & -1 & 1 & 0 & 0\\
0 & 0 & 1 & -1 & 0\\
0 & 0 & 0 & -1 & 1\\
1 & 0 & 0 & 0 & -1\\
1 & 0 & -1 & 0 & 0\\
0 & 0 & -1 & 0 & 1\\
0 & -1 & 0 & 0 & 1\\
0 & -1 & 0 & 1 & 0\\
-1 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
NTU\\ NTHU\\ NCTU\\ NCCU\\ NCKU
\end{bmatrix}
=
\begin{bmatrix}
1\\1\\3\\2\\3\\3\\1\\2\\4\\1
\end{bmatrix}
$$

(b)

$$Rank(A) = 4$$

$$dimensions \quad for \quad row \quad space \quad is \quad 4$$

$$dimensions \quad for \quad column \quad space \quad is \quad 4$$

$$dimensions \quad for \quad null \quad space \quad is \quad 10 - 4 = 6$$

$$dimensions \quad for \quad left - null \quad space \quad is \quad 5 - 4 = 1$$

(c)

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} NTU \\ NTHU \\ NCTU \\ NCCU \\ NCKU \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \\ b_9 \\ b_{10} \end{bmatrix}$$

$$For \quad the \quad problem \quad to \quad be \quad solvable, \quad the \quad score \quad diffenence$$

$$in \quad a \quad loopmust \quad add \quad to \quad 0.$$

$$That \quad is, \quad b_1 + b_2 - b_3 + b_4 + b_5 = 0$$

$$b_1 + b_2 + b_6 = 0$$

$$b_3 - b_4 + b_7 = 0$$

$$b_3 - b_2 - b_4 + b_8 = 0$$

$$b_3 - b_2 + b_9 = 0$$

$$b_3 - b_2 - b_1 - b_4 + b_{10} = 0$$

3

(a)

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

(b)

$$After \quad Gaussian \quad elimination, \quad A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad Rank(A) = 3$$

$$dimensions \quad for \quad the \quad row \quad space : 3$$

$$basis \quad for \quad the \quad row \quad space : (-1,1,0,0), (0,-1,1,0), (0,0,-1,1)$$

$$dimensions \quad for \quad the \quad column \quad space : 3$$

$$basis \quad for \quad the \quad column \quad space : (-1,1,0,0), (0,-1,1,0), (0,0,-1,1)$$

$$dimensions \quad for \quad the \quad null \quad space : 4 - 3 = 1$$

$$basis \quad for \quad the \quad null \quad space : (1,1,1,1)$$

$$dimensions \quad for \quad the \quad left-null \quad space : 6 - 3 = 3$$

$$from \quad graph, \quad basis \quad for \quad the \quad left-null \quad space :$$

$$(-1,0,0,-1,1,0), (0,0,-1,1,0,-1), (0,-1,0,0,1,-1)$$

(c)

$$from \quad left-null \quad space, \quad Kirchhoff's \quad Voltage \quad Law :$$

$$y_5 - y_4 - y_1 = 0, \quad y_4 - y_6 - y_3 = 0, \quad y_5 - y_6 - y_2 = 0$$

(d)

$$A^T y = f \Rightarrow$$

$$\begin{bmatrix} -1 & -1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} y = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

$$after \quad Gaussian \quad elimination,$$

$$\begin{bmatrix} -1 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} y = \begin{bmatrix} f_1 \\ f_1 + f_2 \\ f_1 + f_2 + f_3 \\ f_1 + f_2 + f_3 + f_4 \end{bmatrix}$$

$$Kirchhoff'CurrentLaw : f_1 + f_2 + f_3 + f_4 = 0 - < ans >$$