NUMBER SYSTEM

MUST-KNOW CAT
TIPS & TRICKS

CAT

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Chapter 1: Types of Numbers

Natural Numbers: 1,2,3,4 are called natural numbers or positive integers.

Whole Numbers: 0,1,2,3 are called whole numbers. Whole numbers include "0".

Integers: -3, -2, -1, 0, 1, 2, 3 are called integers.

Negative Integers: -1, -2, -3 are called negative integers.

Positive Fractions: 2/3, 4/5, 7/8 are called positive fractions.

Negative Fractions: The numbers -6/8, - 7/19, - 12/47, ...are called negative fractions.

Rational Numbers: Any number which is a positive or negative integer or fraction, or zero is called a rational number. A rational number is one which can be expressed in the following format $\Rightarrow$ **a/b**, where b $\neq$ 0 and a & b are positive or negative integers.

Irrational Numbers: An infinite non-recurring decimal number is known as an irrational number. These numbers cannot be expressed in the form of a proper fraction a/b where b $\neq$ 0., e.g. $\sqrt{2}$, $\sqrt{5}$, π, etc.

Surds: Any root of a number, which cannot be exactly found is called a surd. Essentially, all surds are irrational numbers. e.g. $\sqrt{2}$, $\sqrt{5}$ etc.

Surds of the form $x + \sqrt{y}$, $x - \sqrt{y}$ are called binomial quadratic surds, where $x + \sqrt{y}$ and $x - \sqrt{y}$ are called conjugate surds, each being the conjugate of the other.

Even Numbers: The numbers divisible by two are called even numbers,e.g., 4, 0, 2, 16 etc.

Odd Numbers: The numbers not divisible by two are odd numbers, e.g., 7, -15, 5, 9 etc.

Prime Numbers: Those numbers, which are divisible only by themselves and 1, are called prime numbers. In other words, a number, which has only two factors 1 and itself, is called a prime number. e.g. 2, 3, 5, 7, etc.

2 is the only even prime number.

There are 25 prime numbers upto 100. These are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 & 97. These should be learnt by heart.

Co-Prime Numbers: When two numbers have their HCF as 1 they are considered to be prime to each other. e.g. 5 and 21 are prime to each other. In other words, 5 and 21 are **co-prime numbers.**

Remember: A number Z can be written as the product of two factors, which are co-prime to each other, in 2^{Y-1} ways, where Y is the number of different prime factors of Z.

E.g. Z = 120 = $2^3 \times 3^1 \times 5^1$. Now here the number of different prime factors of 120 is 3 (2, 3 & 5).

So the value of *Y* is 3. 120 can be written as the product of two numbers which are co-prime to each other as 2^{3-1}. These are 15×8, 24×5, 40×3, 120×1

To Check whether a number is prime, e.g. 113, we do not need to check all the factors below 113. The square of 10 is 100 and that of 11 is 121. Therefore, test if any of the prime numbers less than 11 is a factor of 113. And 2, 3, 5, 7, 11 are prime numbers which are not factors of 113 and hence 113 is a prime number.

Composite Number: A number, which has factors other than itself and 1, is called a composite number. e.g. 9, 14, 25....or the number which has more than two factors are called composite number. So, 4 is the first composite number.

1 is neither a composite number nor a prime number.

Consecutive Numbers: Numbers arranged in increasing order and differing by 1 is called consecutive numbers. e.g. 4, 5, 6, 7

Real Numbers: The above sets of natural numbers, integers, whole numbers, rational numbers and irrational numbers constitute the set of real numbers. Points can represent every real number on a number line.

Perfect Numbers: If the sum of all the factors of a number excluding the number itself happens to be equal to the number, then the number is called as perfect number. 6 is the first perfect number. The factors of 6 are 1, 2, 3 & 6. Leaving 6 the sum of other factors of 6 are equal to 6. The next three perfect numbers after 6 are : 28, 496 and 8128.

Fibonacci Numbers: The numbers, which follow the following series are known as Fibonacci numbers. E.g. *1,1,2,3,5,8,13,21....*

The series is obtained by adding the sum of the preceding two numbers. In general for a Fibonacci number X, $X_{i+2} = X_{i+1} + X_i$.

For a Fibonacci series,

The sum of the first n terms is $X_{n+2} - X_2$


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Chapter 2: Operations on Odd & Even Numbers

- Addition or subtraction of any two odd numbers will always result in an even number or zero.

  For example: 1 + 3 = 4; 5 - 3 = 2.

- Addition or subtraction of any two even numbers will always result in an even number or zero.

  For example: 2 + 4 = 6; 12 - 4 = 8.

- Addition or subtraction of an odd number from an even number will result in an odd number.

  For example: 4 + 3 = 7; 10 - 3 = 7.

- Addition or subtraction of an even number from an odd number will result in an odd number.

  For example: 0. 3 + 4 = 7; 5 - 2 = 3.

- Multiplication of two odd numbers will result in an odd number. For example: $3 \times 3 = 9$.
- Multiplication of two even numbers will result in an even number. For example: $2 \times 4 = 8$.
- Multiplication of an odd number by an even number or vice versa will result in an even number.

  For example: $3 \times 2 = 6$.

- An odd number is raised to an odd or an even power is always odd.
- An even number is raised to an odd or an even power is always even.
- The standard form of writing a number is $m \times 10^n$ where m lies between 1 and 10 and n is an integer. e.g. $0.89713 \Rightarrow 8.9713/10^1 . \Rightarrow 8.9713 \times 10^{-1}$.
- If n is odd. $n(n^2 – 1)$ is divisible by 24. Take $n = 5 \Rightarrow 5(5^2 – 1) = 120$, which is divisible by 24.
- If n is odd prime number except 3, then $n^2 – 1$ is divisible by 24.
- If n is odd. $2^n + 1$ is divisible by 3.
- If n is even. $2^n – 1$ is divisible by 3.
- If n is odd. $2^{2n} + 1$ is divisible by 5.
- If n is even. $2^{2n} – 1$ is divisible by 5.
- If n is odd. $5^{2n} + 1$ is divisible by 13.
- If n is even. $5^{2n} – 1$ is divisible by 13

Chapter 3: Tests of Divisibility

1. **By 2** - A number is divisible by 2 when its unit's place is 0 or divisible by 2., e.g. 120, 138.
2. **By 3** - 19272 is divisible by 3 when the sum of the digits of 19272 = 21 is divisible by 3.
3. **By 4** - A number is divisible by 4 when the last two digits of the number are 0s or are divisible by 4. As 100 is divisible by 4, it is sufficient if the divisibility test is restricted to the last two digits. e.g. 145896, 128, 18400
4. **By 5** - A number is divisible by 5 if its unit's digit is 5 or 0., e.g. 895, 100
5. **By 6** - A number is divisible by 6 if it is divisible by both 2 and by 3 i.e. the number should be an even number and the sum of its digits should be divisible by 3.
6. **By 8** - A number is divisible by 8 if the last three digits of the number are 0s or are divisible by 8. As 1000 is divisible by 8, it is sufficient if the divisibility test is restricted to the last three digits, e.g. 135128, 45000

7. **By 9** - A number is divisible by 9 if the sum of its digits is divisible by 9. e.g. 810, 92754

8. **By 11** - A number is divisible by 11 if the difference between the sum of the digits at odd places and the sum of the digits at even places of the number is either 0 or a multiple of 11.

   e.g. 121, 65967. In the first case 1+1 - 2 = 0. In the second case 6+9+7 = 22 and 5+6 = 11 and the difference is 11. Therefore, both these numbers are divisible by 11.

9. **By 12** - A number is divisible by 12 if it is divisible by both 3 and by 4. i.e., the sum of the digits should be divisible by 3 and the last two digits should be divisible by 4. e.g. 144, 8136

10. **By 15 –** A number is divisible by 15 if it is divisible by both 5 and 3.

11. **By 25** – 2358975 is divisible by 25 if the last two digits of 2358975 are divisible by 25 or if the last two digits are 0s.

12. **By 75** - A number is divisible by 75 if it is divisible by both 3 and 25 i.e. the sum of the digits should be divisible by 3 and the last two digits should be divisible by 25.

13. **By 125 -** A number is divisible by 125 if its last three right-hand digits are divisible by 125 or if the last three digits are 0s. e.g. 1254375, 12000

14. The number of factors of a number can be found by knowing how many prime factors it has.

The number of factors of a number, say 48, can be found by knowing how many prime factors it has. 48 has four 2s and one 3.

$(2^4 \times 3^1)$

So 48 has $\Rightarrow (4 + 1)(1 + 1) = 10$.

Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24 & 48.

If $J = p^x \times q^y \times r^z$, (p, q & r are prime) then J has

$(x + 1)(y + 1)(z + 1)$ factors.

15. **By 7** – A number is divisible by 7, if the number of tens added to five times the number of units is divisible by 7. e.g. if you check 259, number of tens = 25 and 5 times units digit $5 \times 9 = 45$ now $25 + 45 = 70$. As 70 is divisible by 7 so this number is divisible by 7.

OR There is a concept of seed number, which can be applied to check the divisibility of certain numbers. The process is that if the sum of the products of the digits of the number from left to right with increasing powers of the seed number is divisible by 7 or is zero. The seed number of 7 is – 2. e.g. 2863

$\Rightarrow 2(-2)^0 + 8(-2)^1 + 6(-2)^2 + 3(-2)^3$

$= 2 - 16 + 24 - 24 = -14$. As this is divisible by 7, the number is divisible by 7.

16. **By 13** – If the number of tens added to four times the number of units is divisible by 13. Then the number is divisible by 13. e.g. 4394 no. of tens = 439, number of units = $4 \times 4 = 16$

$\Rightarrow 439 + 16 = 455$, which is divisible by 13, so the number is divisible by 13

OR A number is divisible by 13, if the sum of the digits of the number from left to right with 1, 4, 3, -1, - 4, -3…. Successively is divisible by 13.

e.g. 4394 $\Rightarrow 4(1) + 3(4) + 9(3) + 4(-1) = 4 + 12 + 27 - 4 = 39$, which is divisible by 13. So the number is divisible by 13.

17. **By 17 –** A number is divisible by 17, if the number of tens added to twelve times the number of units is divisible by 17.

18. **By 19 –** A number is divisible by 19, if the number of tens added to twice the number of units is divisible by 19.

OR The seed number for 19 is 2; A number is divisible by 19 if the sum of the product of the digits of the number from left to right with increasing powers of the seed number is divisible by 19. e.g. 1083 $\Rightarrow 1(2)^0 + 0(2)^1 + 8(2)^2 + 3(2)^3$

$= 1 + 0 + 32 + 24 = 57$, which is divisible by 19, so the number is divisible by 19.

19. **By 29** – A number is divisible by 29, if the number of tens added to thrice the number of units is divisible by 29.

Chapter 4: Important Results on Numbers

1. The sum of 5 successive whole numbers is always divisible by 5.

2. The product of 3 consecutive natural numbers is divisible by 6.

3. The product of 3 consecutive natural numbers, the first of which is an even number is divisible by 24.

4. The sum of a two-digit number and a number formed by reversing its digits is divisible by 11. E.g. 28 + 82 = 110, which is divisible by 11. At the same time, the difference between those numbers will be divisible by 9. e.g. 82 – 28 = 54, which is divisible by 9.

5. $\Sigma n = \frac{n(n+1)}{2}$, Σn is the sum of first n natural numbers.

6. $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$, Σn^2 is the sum of first n perfect squares.

E.q.What is the value of P, where P = $1^2 + 2^2 + \ldots 10^2$?

You have to find the sum of first 10 perfect squares.

The above mentioned formula is to be applied.

$$\Sigma 10^2 = \frac{10 \times 11 \times 21}{6} = 385.$$

7. $\Sigma n^3 = \frac{n^2(n+1)^2}{4} = (\Sigma n)^2$, Σn^3 is the sum of first n perfect cubes.

8. $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - ... + y^{n-1})$ when n is odd. Therefore, when n is odd, $x^n + y^n$ is divisible by $x + y$. e.g. $3^3 + 2^3 = 35$ and is divisible by 5.

9. $x^n - y^n = (x + y)(x^{n-1} - x^{n-2}.y + ... y^{n-1})$ when n is even. Therefore, when n is even, $x^n - y^n$ is divisible by $x + y$.

e.g $7^2 - 3^2 = 40$, which is divisible by 10.

10. $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}.y + ... + y^{n-1})$ for both odd and even n. Therefore, $x^n - y^n$ is divisible by $x - y$.

e.g. $9^4 - 2^4 = 6545$ which is divisible by 7.

11. <u>The number of divisors of a composite number</u>: If D is a composite number in the form $D = a^p \times b^q \times c^r$, where a, b, c are primes, then the no. of divisors of D, represented by n is given by *n = (p+1)(q+1)(r +1).*

e.g. What is the total number of factors of 200?

As calculated above 200 can be written as $2^3 \times 5^2$.

The values of p & q in this case are 3 & 2 respectively.

Thus the total number of factors are (3 + 1) × (2 + 1) = 12.

So 200 has 12 factors in total.

The factors are 1 × 200, 2 × 100, 4 × 50, 5 × 40, 8 × 25, 10 × 20.

The sum of all those divisors of S_n is given by the following formula

$$S_n = \frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)}$$

e.g. What is the sum of all the factors of 200?

As per the above formula $S_n = \frac{(2^{3+1}-1)(5^{2+1}-1)}{(2-1)(5-1)}$

$\frac{15\times 124}{4} =$ 15 × 31 = 465.

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Chapter 5: Shortcuts in Multiplication & Division

1. ***To multiply by 9, 99, 999 etc.***

To multiply a number α by 9, multiply a by 10 and subtract α from the result.

Algebraically, $\alpha \times 9 = \alpha \times (10 - 1) = 10\alpha - \alpha$.

Similarly, for 99, 999 etc multiply a by 100, 1,000 respectively.

e.g. 745 × 99 = 745 × 100 - 745

= 74,500 - 745 = 73,755.

2. ***To multiply by 5 or powers of 5***

a. To multiply by 5, multiply by 10 and divide by 2.

e.g. $137 \times 5 = \frac{1,370}{2} = 685$

b. To multiply by 25, multiply by 100 and divide by 4.

e.g. $24 \times 25 = 24 \times \frac{100}{4} = \frac{2,400}{4} = 600$

c. To multiply by 125, multiply by 1,000 and then divide by 8

e.g. $48 \times 125 = 48 \times \frac{1,000}{8} = \frac{48,000}{8} = 6,000$

d. To multiply by 625, multiply by 10,000 & then divide by 16

e.g. $64 \times 625 = 64 \times \frac{10,000}{16} = 40,000$

Chapter 6: Solved Examples

Ex. 1. Find the greatest 5-digit number, which is a multiple of 23.

To solve such a question, take the greatest five-digit number, which is 99999.

Divide this number by 23 and get the remainder as 18. Simply because the remainder is 18 if you subtract 18 from the number the remaining number will be a multiple of 23. So the greatest such number will be 99999 – 18 = 99981.

Ex. 2. Find the smallest 7-digit number, which is a multiple of 19.

To solve such question take the smallest seven-digit number, which is 1000000.

Divide this number by 19 and get the remainder as 11.

Here if you subtract 11 from the number, no doubt you will get a multiple of 19. But because you have already taken the smallest seven-digit number, if you subtract anything from it you will get a six-digit number. Think it otherwise, that instead of subtracting you should add something.

Now, what should be added to 11(the remainder) so that it becomes a multiple of 19, i.e. 19 – 11 = 8.$\Rightarrow$ 8 should be added to the number, i.e. 1000000 + 8 = 1000008 is the answer.

Ex. 3. Which letter should replace the \$ in the number 2347\$98, so that it becomes a multiple of 9.

As you know that if the sum of all the digits is divisible by 9, then the number is divisible by 9.

Now sum of the given digits is 2 + 3 + 4 + 7 + 9 + 8 = 33 + \$.

Now think the next multiple of 9 after 33, i.e. 36. This means you add 3 to this. The value of \$ is 3

Ex. 4. In a party there are 25 persons are present. If each of them shakes hand with all the other persons. In total how many handshakes will take place?

This question you can solve with the help of combinations; otherwise, you can apply other logic. Out of 25 persons, the first person will shake hand with 24 persons.

The second person will shake hand with 23 persons (because he has already shaken hand with the first person). The third person with 22 persons and so on. The second last person shakes hand with only one person. And last will shake hand with none (because he has already shaken hand with all persons). Net you have to add all the natural numbers from 1 to 24, i.e.Σ 24. $\Sigma 24 = 24 \times 25/2 =$ 300 handshakes.

Ex. 5. Find the prime factors of 1500.

The prime factor of 1500 are $2 \times 2 \times 3 \times 5 \times 5 \times 5$. So the answer is $2^2 \times 3^1 \times 5^3$. So it has three different prime factors, i.e. 2, 3 & 5.

Ex. 6. What will be the number of zeroes at the end of the product of the first 100 natural numbers?

In this kind of a question, you need to find greatest power of 5, which can divide the product of the first 100 natural numbers $\because$ a multiple of 5 multiplied by any even number, gives you a zero. Now divide 100 by 5 and take 20 as a quotient. Then divide 20 (the quotient) by 5 and get the new quotient 4, which further cannot be divided by 5. The sum of all such quotient gives you the greatest power of 5, which can divide that number. The sum is 24 and this is the number of zeroes at the end of the product of the first 100 natural numbers.

Chapter 7: Vedic Math

Squaring a number

Part – I:

While squaring a number, you need a base. All those numbers can be taken as bases, which have a 1 and the rest number of zeroes with them (i.e. the complete round numbers like 100, 1000, 10000 etc.). The square of a number will have two parts, the left part and the right part. There is no limit for the left side, but the right side must have as many digits as the number of zeroes in the base i.e. if 100 is taken as base there should be 2 zeroes on the right side and if 1000 is taken as base then the number of digits on RHS should be 3.

Now take a number 92. The nearest complete base is 100. The difference between the base and the number given is 8. The square of this difference is 64, which will become the right side. Because it is already having two digits, so it would be simply placed on the right side. Now the difference of 8 is subtracted from the number given i.e. 92 – 8 = 84 and it will become the left side. Therefore the square of 92 is 8464.

Take another number say 94, which is 6 less than the base. While squaring 94, the right side will be $(6)^2$ i.e. 36.

And the left side would be the number given – difference i.e. 94 – 6 = 88. So the square of 94 is 8836. *If the square of the difference is having lesser digits then required, then in order to have the needed number of digits on the right side, 0's can be put with the square*. e.g. if you square a number like 97, difference is 3. The right side in this case would become 09, because 9 is a single digit number and you'll have to put a '0' before it to make it a two-digit right hand side. The left side would be 97 – 3 = 94. The square is 9409.

In case, the number of digits is more than needed, then the extra digits are carried to the left side. e.g. take 86. The difference is 14 and the square of the difference is 196, which is a 3-digit number, so the 3^{rd} extra digit 1 would be carried to the left side. The left side is 86 – 14 = 72 + 1 (carried over) = 73.

So the square of the number is

$$\begin{array}{rr} & 7\,2\,-\,- \\ + & \underline{-\;1\,9\,6} \\ & 7\;\;3\,9\,6 \end{array}$$

You can practice the following squares to get expertise on squaring.

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85 _____ 79 _____ 91 _____87 _____97_____77_____

92 _____96 _____ 99 _____ 91 _____ 76_____82 _______

81 _____88 _____89 _____

Part – II:

If the number to be squared is greater than the base, then there is only one difference in approach i.e. the difference between the number and the base is to be added in the number instead of subtracting. Take a number 107. The difference is 7. The right side will have square of difference i.e. $(7)^2 = 49$. And the left side will be 107 + 7 = 114, because the number is greater than the base. So the square is 11449. Similarly even in this case, if the number of digits on the right side less than the required number, then you can write ‘0’s with it to get the right side. The square of 103 would be: the difference is 3, its square is 9, which is a single digit number, so a 0 would be written with it i.e. 09. Then the left side is 103 + 3 = 106. The square becomes 10609.

In case the square of the difference is a 3-digit number, then the third digit would be carried to the left side. Consider one number say 118. The difference is 18 $\Rightarrow (18)^2 \Rightarrow$ 324. Out of this 3-digit number the third digit 3 would be taken to the left side. The left side would become 118 + 18 + 3 (Carried) = 139 and the square would be

```
   1 3 6 - -
 + - - 3 2 4
 -----------
   1 3 9 2 4
```

Square the following numbers:

112 _____ 108 _____ 109 _____ 121_____

124 _____113 _____ 107 _____ 102_____

123 _____105 _____101 _____ 114 _____

Multiplying numbers

When a 2-digit number is to be multiplied with a two-digit number the following process would be applied. If there were two numbers AB and CD then their product would be calculated as under.

```
  AB
  CD
 ----
```

***Step 1*:** BD (Write only the unit's digit and carry the rest to the next step).

Step 2: AD + BC + Carry over (Cross multiply and Add, write a single digit and carry the rest to the next step).

Step 3: AC + Carry over (Write the complete number because this is the last step).

Example 1:

$$\begin{array}{r} 29 \\ \underline{53} \end{array}$$

Step 1: $9 \times 3 = 27$ (Write 7 and 2 is carried over to the next step).

Step 2: $2 \times 3 + 9 \times 5 + 2$ (Carried Over) $= 53$ (Write 3 and 5 is carried over to the next step)

Step 3: $2 \times 5 + 5$ (Carried Over) $= 15$ (Write 15 because this is the last step)

Therefore 1537 is the answer

Example 2:

$$\begin{array}{r} 37 \\ \underline{73} \end{array}$$

Step 1: $7 \times 3 = 21$ (Write 1 and 2 is carried over to the next step)

Step 2: $3 \times 3 + 7 \times 7 + 2$ (Carried Over) $= 60$ (Write 0 and 6 is carried over to the next step)

Step 3: $3 \times 7 + 6$ (Carried Over) $= 27$ (Write 27 because this is the last step)

Therefore 2701 is the answer.

Try Multiplying 23 and 32 and see if the answer is 736.

Try Multiplying 28 and 82 and see if the answer is 2296.

Now we will try multiplying a three-digit number by a three-digit number. Because there are six digits, the total number of steps would be 5.

3 Digit by 3 Digit Multiplication

$$\begin{array}{r} ABC \\ \underline{DEF} \end{array}$$

Step 1: CF (Write only the unit's digit and carry the rest to the next step)

Step 2: BF + CE + Carried Over (Write only the unit's digit and carry the rest to the next step)

Step 3: AF + CD + BE + Carried Over (Write only the unit's digit and carry the rest to the next step)

Step 4: AE + BD + Carried Over (Write only the unit's digit and carry the rest to the next step)

Step 5: AD + Carried Over

(Write the complete number because this is the last step)

Example 3:

$$\begin{array}{r} 123 \\ 456 \\ \hline \end{array}$$

Step 1: $3 \times 6 = 18$ (Write 8 and 1 is carried over to the next step)

Step 2: $2 \times 6 + 3 \times 5 + 1$ (Carried Over) $= 28$ (Write 8 and 2 is carried over to the next step)

Step 3: $1 \times 6 + 3 \times 4 + 2 \times 5 + 2$ (Carried Over) $= 30$ (Write 0 and 3 is carried over to the next step)

Step 4: 1 × 5 + 2 × 4 + 3 (Carried Over) = 16 (Write 6 and 1 is carried over to the next step)

Step 5: 1 × 4 + 1 (Carried Over) = 5 (Write 5 because this is the last step). Therefore 56088 is the answer.

Example 4:

$$\begin{array}{r} 243 \\ 172 \\ \hline \end{array}$$

Step 1: 3 × 2 = 6 (Write 6 which is the single digit number)

Step 2: 4 × 2 + 7 × 3 = 29 (Write 9 and 2 is carried over to the next step)

Step 3: 2 × 2 + 1 × 3 + 4 × 7 + 2 (Carried Over) = 37 (Write 7 and 3 is carried over to the next step)

Step 4: 2 × 7 + 4 × 1 + 3 (Carried Over) = 21 (Write 1 and 2 is carried over to the next step)

Step 5: 2 × 1 + 2 (Carried Over) = 4 (Write 4 as this is the last step). Therefore 41796 is the answer.

Here are some problems for practice:

112	128	237	378
171	325	525	978
276	399	657	876
923	546	763	453
497	179	929	868
129	598	135	963

Chapter 8: Tables, Squares and Cubes to be remembered

As you have decided to improve your quantitative skills, but keep in mind you cannot be good at Math unless you are good at calculations. Take this as the starting point and make it the most important part of your preparation.

Tables:

T×1	*T×2*	*T×3*	*T×4*	*T×5*	*T×6*	*T×7*	*T×8*	*T×9*	*T×10*
12	24	36	48	60	72	84	96	108	120
13	26	39	52	65	78	91	104	117	130
14	28	42	56	70	84	98	112	126	140
15	30	45	60	75	90	105	120	135	150
16	32	48	64	80	96	112	128	144	160
17	34	51	68	85	102	119	136	153	170
18	36	54	72	90	108	126	144	162	180
19	38	57	76	95	114	133	152	171	190
21	42	63	84	105	126	147	168	189	210
23	46	69	92	115	138	161	184	207	230
24	48	72	96	120	144	168	192	216	240
27	54	81	108	135	162	189	216	243	270
29	58	87	116	145	174	203	232	261	290
37	74	111	148	185	222	259	296	333	370

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Squares:

Z	1	2	3	4	5	6	7	8	9
Z^2	1	4	9	16	25	36	49	64	81
Z	10	11	12	13	14	15	16	17	18
Z^2	100	121	144	169	196	225	256	289	324
Z	19	20	21	22	23	24	25	26	27
Z^2	361	400	441	484	529	576	625	676	729
Z	28	29	30	31	32	33	34	35	
Z^2	784	841	900	961	1024	1089	1156	1225	

Cubes:

Y	1	2	3	4	5	6	7	8
Y^3	1	8	27	64	125	216	343	512
Y	9	10	11	12	13	14	15	16
Y^3	729	1000	1331	1728	2197	2744	3375	4096
Y	17	18	19	20	21	22	23	
Y^3	4913	5832	6859	8000	9261	10648	12167	

Chapter 9: Unit Digit of a Number

The concept that revolves around finding the unit digit of a number uses the basics of the number system. Learning this concept means you have strengthened your last digit concepts.

The concept of the unit digit can be learned by figuring out the unit digits of all the single digit numbers from 0 - 9 when raised to certain powers. The first learning in that for you will be that these numbers can be broadly classified into three categories for this purpose:

Digits 0, 1, 5 & 6:

When we observe the behaviour of these digits, they all have the same unit's digit as the number itself when raised to any power, i.e. $0^n = 0$, $1^n = 1$, $5^n = 5$, $6^n = 6$. So, it becomes simple to understand this logic.

e.g. Finding the Unit digit of following numbers:

$185^{563} = 5$; $271^{6987} = 1$; $156^{25369} = 6$; $190^{654789321} = 0$.

Digits 4 & 9:

Both these numbers are perfect squares and also have the same behaviour with respect to their unit digits, i.e. they have a cyclicity of only two different digits as their unit's digit.

Have a look at how the powers of 4 operate:

$4^1 = 4$, $4^2 = 1\underline{6}$, $4^3 = 6\underline{4}$ and so on

Hence, the power cycle of 4 contains only 2 numbers 4 & 6, which appear in case of odd and even powers respectively.

Likewise $9^1 = 9$, $9^2 = 81$, $9^3 = 729$ and so on.

Hence, the power cycle of 9 also contains only 2 numbers 9 & 1, which appear in case of odd and even powers respectively.

So broadly these can be remembered in even and odd only, i.e. $4^{odd} = 4$ and $4^{even} = 6$ and likewise $9^{odd} = 9$ and $9^{even} = 1$.

e.g. Finding the Unit digit of following numbers:

$189^{562589743} = 9$ (since power is odd); $279^{698745832} = 1$(since power is even);

$154^{258741369} = 4$ (since power is odd); $194^{65478932} = 6$ (since power is even)

<u>Digits 2, 3, 7 & 8:</u>

These numbers have a power cycle of 4 different numbers.

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$ & $2^4 = 16$ and after that it starts repeating.

So, the cyclicity of 2 has 4 different numbers 2, 4, 8, 6.

$3^1 = 3$, $3^2 = 9$, $3^3 = 27$ & $3^4 = 81$ and after that it starts repeating.

So, the cyclicity of 3 has 4 different numbers 3, 9, 7, 1. 7 and 8 follow similar logic.

So these four digits, i.e. 2, 3, 7 and 8 have a unit digit cyclicity of four steps.

To summarize, we can say that since the power cycle of these numbers has 4 different digits, we can divide the power by 4, find the remaining power and calculate the unit's digit using that.

e.g. Find the Unit digit of 287^{562581}

The first observation for this question: the unit digit involved is 7, which has a four-step cycle. You need to divide the power by 4 and obtain the remaining power. Doing so, you get the result as 1. Now the last step is to find the unit's digit in this power of the base, i.e. 7^1 has the unit's digit as 7, which will become the answer.

The above set of examples explain how you can use the concept of cyclicity to obtain the unit digit of numbers. In case you understood the above examples, you should be easily able to obtain the unit digit of the numbers given above, and in fact, you should be able to extend this learning to as many examples as you want.

Chapter 10: Digital Root

One of the short cuts which you must learn in number system is that of digital roots. This amazing concept of Digital Roots will help you solve numerous questions on remainders, divisibility and sum of digits. Keep in mind that from now onwards in the article, the term ‘digital root’ will be referred to as DR.

DR (also known as repeated digital sum/seed number) of a number is the number obtained by adding all the digits, then adding the digits of that number, and then continuing until a single-digit number is obtained.

First try to understand this with the help of the following Examples:

e.g. DR (48512) = DR (4+8+5+1+2) = DR (20) = DR (2+0) = 2.

e.g. DR (9873429222) = DR (9+8+7+3+4+2+9+2+2+2) = DR (48) = DR (4+8) = DR (12) = DR (1+2) **=** 3.

Divisibility Rule of 9:

What is the remainder when 48512 is divided by 9?

In order to check for the divisibility of a number by 9, we add all the digits and find the digital root. The digital root of the number is the remainder of these with 9.

Here the approach we follow is that we determine the digital root (DR) = 4 + 8 + 5 + 1 + 2 = 20 = 2 + 0 = 2. So, the digital root is 2, remainder when the number 48512 is divided by 9 is 2.

Ex.1: What is remainder when 9873429222 is divided by 9?

Sol: Again the divisibility rule of 9 will be applied.

$9 + 8 + 7 + 3 + 4 + 2 + 9 + 2 + 2 + 2 = 48 = 4 + 8 = 12 = 1 + 2 = 3$

So, remainder is 3 in this case.

It can be concluded from the above examples that whether we take DR of any number say N or remainder when N is divided by 9, the answers are same. Actually both these concepts are same. To review the same, it can be stated that if a number has a DR 1, it will leave a remainder 1 when divided by 9. If it has a DR 2, the remainder will be 2 and so on, till the last point where if it has a DR 9, the remainder will be 0, when divided by 9.

Ex.2: What is the digital root of 26! (factorial)?

Sol: Since 26! has the product of all the digits from 1 till 26, it will be divisible by 9. Or in other words, it will have a digital root equal to 9.

Ex. 3: How many natural number < 500 exist whose seed number is 9?

Sol: Digital root or seed number means remainder is 0 when number is divided by 9. So, question is *"How many natural numbers < 500 divisible by 9"* and it can be calculated that there are 500/9 = 55 such numbers as it is starting from 1 only. Thus, 55 will be the answer.

Ex.4: In first 100 natural numbers, how many numbers have DR 1?

Sol: From 1- 9, there is only 1 number whose DR is 1 ; from 10-18, there is only 1 number whose DR is 1; from 19-27, there is only 1 number whose DR is 1 and so on.

Now in order to answer the question, instead of counting all the numbers, you should count "how many blocks of 9 numbers (1-9, 10-18, 19-27,……, 91-99) can be made out of first 100 natural numbers". There are 100/9=11 such blocks. From each block, we can find only and exactly one number with DR 1. So, we will get 11 numbers corresponding to 11 blocks, but 11 will not be answer, because besides these 11 blocks, there is one extra number 100, whose DR is 1 + 0 + 0 = 1. So, there are 11 + 1 = 12 such numbers in total.

Ex.5: What is DR of 123456……181920 (from 1 to 20 written side by side)?

Sol: One way is to count the frequency of each digit appearing till 20. Then calculate sum of digit. But that will be a long process. Instead of doing that, now if you take sum of numbers only, you will get the same answer. 12345…….20 = 1 + 2 + 3 + 4 + 5 +…..+ 19 + 20 = $(20 \times 21)/2 = 10 \times 21 = (1 + 0) \times (2 + 1) = 1 \times 3 = 3$. Thus, 3 will be the answer.

Ex.6: What is remainder when 11121314 is divided by 9?

Sol: Sum of digits = 1 + 1 + 1 + 2 + 1 + 3 + 1 + 4 = 14 = 1 + 4 = 5. Now in this case, if instead of taking sum of digits of numbers, if you take sum of numbers, you will get the same result. E.g. 11 + 12 + 13 + 14 = 50 = 5 + 0 = 5 is the answer. Here it can be concluded that when DR is asked, or remainder by 9 is asked, you can follow either of the two approaches (sum of digits or sum of numbers) and you will get the same answer.

NOTE: DR of a perfect square is always 1, 4, 7, 9.

Ex.7: How many 5-digit perfect squares can be made by using all the digits 1, 2, 3, 4, 5?

Sol: Since DR of any number by using digits 1, 2, 3, 4, 5 is 1 + 2 + 3 + 4 + 5 = 15 =1 + 5 = 6. So, no perfect square can be made.

Note: Digital root also can be used to check multiplication, addition, subtraction.

Ex. 8: What is 14723 × 58469?

(i) 860839087 (ii) 850839087

(iii) 860729087 (iv) 860729089

Sol: Just apply the following trick, which will give you the correct answer in most of the cases. Now DR of 14723 × 58469 = (1 + 4 + 7 + 2 + 3) × (5 + 8 + 4 + 6 + 9) = (17) × (32) = (1 + 7) × (3 + 2) = 8 × 5 = 40 = 4 + 0 = 4. Now check DR for all the options.

i) 8 + 6 + 0 + 8 + 3 + 9 + 0 + 8 + 7 = 49 = 13 = 4 (It can be answer as DR is same)

ii) 8 + 5 + 0 + 8 + 3 + 9 + 0 + 8 + 7 = 48 = 12 = 3

iii) 8 + 6 + 0 + 7 + 2 + 9 + 0 + 8 + 7 = 11 = 2

iv) 8 + 6 + 0 + 7 + 2 + 9 + 0 + 8 + 9 = 10 = 4 (It can be answer as DR is same).

Now between 1 and 4, the fourth option can be eliminated as the last digit of the product of 14723 × 58469 should be 7 only. Hence answer will be 1st option only.