MBA PIONEER PRO 2024

QUANTITATIVE APTITUDE

DPP: 4

Quadratic Equation - 2

Q1 Find the sum of all possible real values of p for which the equations $3x^2 - x - 2p = 0$ and $2x^2 - x - p = 0$ have a common root.

(A) -1
(B) 2
(C) 1
(D) 0

Q2 $x(3y^2+3xy+2) = 255 - 2y$, and $xy = 5$. If x and y are the roots of a quadratic equation, find the quadratic equation.

(A) $x^2 - 15x + 5 = 0$
(B) $2x^2 - 17x + 10 = 0$
(C) $x^2 - 9x + 5 = 0$
(D) $x^2 - 11x + 5 = 0$

Q3 Find the product of the roots of the equation $2x^2 + 17x + 81 = 2\sqrt{2x^2 + 17x + 84}$

(A) 42
(B) $\frac{81}{2}$
(C) $\frac{75}{2}$
(D) None of these

Q4 If one root of equation $x^2 - 3x + 3k^2 - 5k = 0$ is double of the other then find the sum of all possible values of k.

(A) $\frac{2}{3}$
(B) 1
(C) $\frac{5}{4}$
(D) $\frac{5}{3}$

Q5 Compare the roots of the following quadratic equations:

I. $x^2 - 22x + 117 = 0$
II. $y^2 - 29y + 204 = 0$

(A) $x \le y$
(B) $x \ge y$
(C) $x < y$
(D) Relationship between x and y can't be established.

Q6 Compare the roots of the following quadratic equations:

I. $45x^2 + 56x - 45 = 0$
II. $40y^2 + 39y - 40 = 0$

(A) $x \le y$
(B) $x \ge y$
(C) $x < y$
(D) Relationship between x and y can't be established.

Q7 Compare the roots of the following quadratic equations:

I. $14x^2 - 53x + 14 = 0$
II. $6y^2 + 38y + 60 = 0$

(A) $x \le y$
(B) $x \ge y$
(C) $x > y$
(D) Relationship between x and y can't be established.

Q8 Compare the roots of the following quadratic equations:

I. $40x^2 + 31x - 156 = 0$
II. $49y^2 - 175y + 156 = 0$

(A) $x \le y$
(B) $x \ge y$
(C) $x < y$
(D) Relationship between x and y can't be established.

Q9 Find the minimum value of $5x^2 + 10x + 9 = 0$

(A) 1 (B) 2

(C) 3 (D) 4

Q10 Compare the roots of the following quadratic equations:

I. $40x^2 - 72x - 144 = 0$

II. $5y^2 - 9y - 18 = 0$

(A) $x \leq y$ (B) $x \geq y$

(C) $x < y$ (D) $x = y$

Q11 What is the minimum value of $x^2 + x + 1 = 0$?

(A) 5

(B) 0

(C) -1

(D) $\frac{3}{4}$

Q12 The maximum value of $-x^2 + bx + 3 = 0$ can be obtained at x = 1, then what is the value of b ?

Q13 The minimum value of $ax^2 + 12x + 8 = 0$ can be obtained at x = -2, then what is the value of a?

Q14 The maximum value of $-3x^2 + bx + c$ can be obtained at x = 3. If the product of the roots for $-3x^2 + bx + c = 0$ is 6, then what is the value of b + c?

Q15 What is the absolute value of α for which the difference between the roots of the quadratic equation $x^2 - \alpha x + 15 = 0$ is 2?

Q16 Let u, v, and w are positive real numbers such that $v^2 = uw$. If the roots of the quadratic equation $ux^2 + vx + w = 0$ are real and equal, then what is the value of $(v^2+4uw)^{2022}$?

Q17 Let the roots of the quadratic equation $ax^2 + bx + c = 0$ are $\sqrt{2}$ and $-\sqrt{2}$. Then, what is the value of $\frac{3a^2 - ab + ac}{a^2}$?

Q18 The minimum value of $5x^2 + bx + c = 0$ can be obtained at x = 2. If the product of the roots for $5x^2 + bx + c = 0$ is 8, then what is the value of $-\frac{c}{b}$?

(A) 5 (B) 9

(C) 4 (D) 2

Q19 The maximum value of $ax^2 + 4x + c$ can be obtained at x = 1. If the product of the roots for $ax^2 + 4x + c = 0$ is -8, then what is the value of c?

(A) 15 (B) 16

(C) 7 (D) 12

Q20 The minimum value of $ax^2 + 6x + c = 0$ can be obtained at x = 3. If the product of the roots for $ax^2 + 6x + c = 0$ is -9, then what is the value of a + c?

(A) 6 (B) 8

(C) 4 (D) 5

Q21 Let the roots of a quadratic equation $x^2 + px + q = 0$ are k and 3-k. If k is an integer, then what is the maximum integral value which q can take?

(A) 1 (B) 2

(C) 3 (D) 4

Q22 If one of the roots of the quadratic equation $(ax^2 + bx + c) = 0$ is $5 + 2\sqrt{6}$ where a, b and c are natural numbers, then find the value of p so that $(cx^2 + px + a) = 0$ whose one root is $3 + 2\sqrt{2}$.

(A) $\frac{3b}{5}$

(B) -6c

(C) 6c

(D) Both option a & b are correct

Q23 What is the maximum value of $34 + 24x^2 - 9x^4 = 0$?

Q24

If for the quadratic equation $ax^2+bx+c=0$, the sum of the squares of its roots is equal to the sum of the cubes of its roots and $b^3 + ab^2 = 6a + 9b \neq 0$, then what is the value of ac?

Q25 A quadratic equation attains a maximum of 4 at x = 1. The value of the equation at x = 0 is -1. What is the absolute value of f(x) at x = 7?

Q26 If the sum of the reciprocals of the roots of a quadratic equation is $\frac{5}{6}$ and the sum of the squares of the roots be 13, then find the equation.

(A) $25x^2 + 65x - 78 = 0$

(B) $x^2 - 5x + 6 = 0$

(C) both (a) and (b)

(D) None of the above

Q27 For the equation $x - \frac{2}{x-p} = p - \frac{2}{x-p}$, where p > 0, find the number of roots of the equation.

(A) 2 (B) 1

(C) 3 (D) 0

Q28 Find the quadratic equation whose roots are $(\alpha + \beta)^3$ and $(\alpha - \beta)^3$ if α, β are the roots of the equation $x^2 + 9mx + 8m^2 = 0$, where m is any real constant.

(A) $x^2 + 386m^2 - 250047m^6 = 0$

(B) $x^2 - 386m^2 + 25047m^6 = 0$

(C) $x^2 - 386m^2 - 250047m^6 = 0$

(D) $x^2 + 386m^2 + 25047m^6 = 0$

Q29 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$, then find the equation whose coefficient of x^2 is 1 and whose roots are α^{2022} & β^{2022}.

(A) $(x^2 + 2^{2023}x + 2^{2022}) = 0$

(B) $(x^2 + 2^{2022}) = 0$

(C) $(x^2 + 8x + 2^{2022}) = 0$

(D) $(x^2 + 4x + 2^{2022}) = 0$

Q30 Find the minimum value of $\frac{x^2+x}{x^2+x+1}$ where x is a real number.

(A) 0

(B) $-\frac{1}{3}$

(C) 1

(D) cannot be determined

Answer Key

Q1	(C)
Q2	(A)
Q3	(C)
Q4	(D)
Q5	(D)
Q6	(D)
Q7	(C)
Q8	(C)
Q9	(D)
Q10	(D)
Q11	(D)
Q12	2
Q13	3
Q14	0
Q15	8
Q16	0
Q17	1
Q18	(D)
Q19	(B)
Q20	(B)
Q21	(B)
Q22	(D)
Q23	50
Q24	3
Q25	176
Q26	(C)
Q27	(D)
Q28	(A)
Q29	(B)
Q30	(B)

Hints & Solutions

Q1 Text Solution:

Let a be the common root,

Then,

$3a^2 - a - 2p = 2a^2 - a - p$

=> $a^2 = p$

Therefore,

$3a^2 - a - 2a^2 = 0$

=> $a(a-1) = 0$

Therefore, a = 0 or a = 1

i.e., p = 0 or p = 1

Sum of all possible values = 1

Q2 Text Solution:

$x(3y^2 + 3xy + 2) = 255 - 2y$

$3xy^2 + 3x^2y + 2x + 2y = 255$

$3xy(x+y)+2(x+y) = 255$

$15(x+y)+2(x+y)=255$

$17(x+y) = 255$

$x+y = \frac{255}{17} = 15$

Since x and y are roots of an equation, so, $x + y = -\frac{b}{a}$ and $xy = \frac{c}{a}$

Thus, the equation is $x^2 - 15x + 5 = 0$.

Q3 Text Solution:

Let us assume that

$\sqrt{2x^2 + 17x + 84}$

$= y$

Thus, from the question we have

$y^2 - 3 = 2y$

$\Rightarrow y^2 - 2y - 3 = 0$

$\Rightarrow (y-3)(y+1) = 0$

So, y = 3 as y cannot be equal to a negative value

Now, we have $= 3$

$2x^2 + 17x + 84 = 9$

$2x^2 + 17x + 75 = 0$

Hence, Product of the roots = $\frac{c}{a} = \frac{75}{2}$

Q4 Text Solution:

Let roots are a & 2a.

Sum of the roots : a+2a = 3

=> a = 1

Product of the roots: $(a)(2a) = 3k^2 - 5k$

or, $2a^2 = 3k^2 - 5k$

or, $2(1) = 3k^2 - 5k$ (since a = 1)

or, $3k^2 - 5k - 2 = 0$

or, $(3k + 1)(k - 2) = 0$

Either k = 2 or, $k = -\frac{1}{3}$

Hence, the required sum of the two possible values of k is

$2 + (-\frac{1}{3}) = \frac{5}{3}$

Q5 Text Solution:

The given equation is $x^2 - 22x + 117 = 0$

=> $x^2 - (13 + 9)x + 117 = 0$

=> $x^2 - 13x - 9x + 117 = 0$

=> $x(x-13) - 9(x-13) = 0$

=> $(x-9)(x-13) = 0$

=> x = 9, 13

Also, the other equation is $y^2 - 29y + 204 = 0$

=> $y^2 - (12 + 17)y + 204 = 0$

=> $y^2 - 12y - 17y + 204 = 0$

=> $y(y - 12) - 17(y - 12) = 0$

=> $(y - 12)(y - 17) = 0$

=> y = 12, 17

It can be easily seen from above that, 9 < 12, and 9 < 17, but 13 > 12.

As one of the values of x is more than one value of y and the other one is less than the other value of y, we cannot establish any relation between x & y.

So, the relation cannot be established.

Q6 Text Solution:

The given equation is $45x^2 + 56x - 45 = 0$

=> $45x^2 + (81 - 25)x - 45 = 0$

=> $45x^2 + 81x - 25x - 45 = 0$
=> $9x(5x + 9) - 5(5x + 9) = 0$
=> $(9x - 5)(5x + 9) = 0$
=> $x = \frac{5}{9}, x = -\frac{9}{5}$
Also, the other equation is $40y^2 + 39y - 40 = 0$
=> $40y^2 + (64 - 25)y - 40 = 0$
=> $40y^2 + 64y - 25y - 40 = 0$
=> $8y(5y + 8) - 5(5y + 8) = 0$
=> $(8y - 5)(5y+8) = 0$
=> $y = \frac{5}{8}, y = -\frac{8}{5}$
We can see that, $\frac{5}{9} < \frac{5}{8}$, but $\frac{5}{9} > -\frac{8}{5}$.
As one of the values of x is more than one value of y and the other one is less than the other value of y, we cannot establish any relation between x & y.
Hence, the relation cannot be established.

Q7 Text Solution:

The given equation is $14x^2 - 53x + 14 = 0$
=> $14x^2 - (4 + 49)x + 14 = 0$
=> $14x^2 - 4x - 49x + 14 = 0$
=> $2x(7x - 2) - 7(7x - 2) = 0$
=> $(2x - 7)(7x - 2) = 0$
=> $x = \frac{7}{2}, \frac{2}{7}$
Also, the other equation is $6y^2 + 38y + 60 = 0$
=> $6y^2 + y(20 + 18) + 60 = 0$
=> $6y^2 + 20y + 18y + 60 = 0$
=> $2y(3y + 10) + 6(3y + 10) = 0$
=> $(2y + 6)(3y + 10) = 0$
=> $y = -3, -\frac{10}{3}$
Hence, x > y.

Q8 Text Solution:

The given equation is $40x^2 + 31x - 156 = 0$
=> $40x^2 + (96 - 65)x - 156 = 0$
=> $40x^2 + 96x - 65x - 156 = 0$
=> $8x(5x + 12) - 13(5x + 12) = 0$
=> $(8x - 13)(5x + 12) = 0$
=> $x = (\frac{13}{8}, -\frac{12}{5})$
Also, the other equation is $49y^2 - 175y + 156 = 0$
=> $49y^2 - (91 + 84)y + 156 = 0$
=> $49y^2 - 91y - 84y + 156 = 0$
=> $7y(7y - 13) - 12(7y - 13) = 0$
=> $(7y - 12)(7y - 13) = 0$
=> $y = \frac{12}{7}, \frac{13}{7}$
Hence, x < y.

Q9 Text Solution:

We know that, any quadratic expression $ax^2 + bx + c$ has its maximum/minimum value at $x = -\frac{b}{2a}$.
Therefore, the minimum value of $5x^2 + 10x + 9$ can be obtained at $x = -\frac{10}{2\times5} = -1$
Thus, the required minimum value is $5(-1)^2 + 10 \times (-1) + 9 = 4$

Q10 Text Solution:

The given equation is $40x^2 - 72x - 144 = 0$
=> $8(5x^2 - 9x - 18) = 0$
=> $5x^2 - 9x - 18 = 0$
Also, the other equation is $5y^2 - 9y - 18 = 0$
Only the algebraic expression is different.
Therefore, the roots of the two quadratic equations are equal.

Q11 Text Solution:

$$x^2 + x + 1 = 0$$
$$x^2 + x + \frac{1}{4} + 1 - \frac{1}{4} = 0$$
$$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = 0$$

For $x = -\frac{1}{2}$, the given expression will have the minimum value.
Hence, the minimum value will be $\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + 1 = \frac{3}{4}$
Hence, option (D) is the correct answer.

Q12 Text Solution:

We know that, any quadratic expression $ax^2 + bx + c = 0$ has its maximum/minimum value at x

$= -\frac{b}{2a}$.

Therefore, the maximum value of $-x^2 + bx + 3$ can be obtained at

$x = -\frac{b}{2\times(-1)} = \frac{b}{2}$

Now, given that, at x = 1, the maximum value of $-x^2 + bx + 3 = 0$ can be obtained.

Therefore, $\frac{b}{2} = 1$

=> b = 2

Q13 Text Solution:

We know that, any quadratic expression $ax^2 + bx + c$ has its maximum/minimum value at $x = -\frac{b}{2a}$.

Therefore, the minimum value of $ax^2 + 12x + 8$ can be obtained at

$x = -\frac{12}{2a} = -\frac{6}{a}$

Now, given that, at x = -2, the minimum value of $ax^2 + 12x + 8$ can be obtained.

So,

$-\frac{6}{a} = -2$

=> a = 3.

Q14 Text Solution:

We know that, any quadratic expression $ax^2 + bx + c = 0$ has its maximum/minimum value at $x = -\frac{b}{2a}$.

Therefore, the maximum value of $-3x^2 + bx + c$ can be obtained at

$x = -\frac{b}{2(-3)} = \frac{b}{6}$

Now, it is given that, at x = 3, the maximum value of $-3x^2 + bx + c$ can be obtained.

So, $\frac{b}{6} = 3$

=> b = 18

Again, since the product of the roots for $-3x^2 + bx + c = 0$ is 6, so

$\frac{c}{(-3)} = 6$

c = -18

Hence, the value of b + c = 18 - 18 = 0

Q15 Text Solution:

Let the roots of the equation $x^2 - \alpha x + 15 = 0$ are m, n.

Then, m + n = α

and m × n = 15

Now, $(m+n)^2=(m-n)^2+4mn$

$= 2^2+4\times15$

= 4+60

= 64

Therefore, m+ n = ± 8

i.e., α = m+ n = ± 8

i.e. the absolute value of α is 8.

Note : Absolute Value means positive value.

Q16 Text Solution:

Given that-

$v^2 = uw$ ...... (i)

Now, the roots of the quadratic equation $ux^2 + vx + w = 0$ will be real and equal if

$v^2 - 4uw = 0$

uw - 4uw = 0 [By (i)]

uw = 0

$\Rightarrow v^2 = 0$

Therefore, $(v^2 + 4uw)^{2022} = (0 + 0)^{2022} = 0$

Q17 Text Solution:

Given, the roots are √2 and -√2

Now, since the roots of the quadratic equation $ax^2+bx+c=0$, so

$-\frac{b}{a}$ = Sum of the roots

$-\frac{b}{a}$ = + (-)

$-\frac{b}{a} = 0$

Also,

$\frac{c}{a}$ = Product of the roots

= ×(-)

= - 2

Therefore,

$\frac{3a^2-ab+ac}{a^2} = 3 - \frac{b}{a} + \frac{c}{a}$

= 3 - 0 - 2

= 1

Q18 Text Solution:

We know that, any quadratic expression ax^2 + bx + c has its maximum/minimum value at x = $-\frac{b}{2a}$.
Therefore, the minimum value of 5x^2 + bx + c can be obtained at x = - $\frac{b}{2\times5}$ = - $\frac{b}{10}$
Now, it is given that, at x = 2, the minimum value of 5x^2 + bx + c can be obtained.
So, - $\frac{b}{10}$ = 2
=> b = -20
Again, since the product of the roots for 5x^2 + bx + c = 0 is 8, so
$\frac{c}{5}$ = 8
=> c = 40
Now, - $\frac{c}{b}$ = $-\frac{40}{(-20)}$ = 2

Q19 Text Solution:

We know that, any quadratic expression ax^2 + bx + c has its maximum/minimum value at x = $-\frac{b}{2a}$.
Therefore, the maximum value of ax^2 + 4x + c can be obtained at x = $-\frac{4}{2\times a}$ = - $\frac{2}{a}$
So, - $\frac{2}{a}$ = 1
=> a = -2
Again, since the product of the roots for ax^2 + 4x + c = 0 is -8, so
$\frac{c}{a}$ = -8
c = -8a = - 8(-2) = 16.

Q20 Text Solution:

We know that, any quadratic expression ax^2 + bx + c = 0 has its maximum/minimum value at x = $-\frac{b}{2a}$.
Therefore, the minimum value of ax^2 + 6x + c can be obtained at
x = - $\frac{6}{2\times a}$ = - $\frac{3}{a}$
So, - $\frac{3}{a}$ = 3
=> 3a = -3
=> a = -1
Now, since the product of the roots for ax^2 + 6x + c = 0 is -9, so
$\frac{c}{a}$ = -9
=> c = -9a = -9(-1) = 9
So, a + c = -1 + 9 = 8.

Q21 Text Solution:

Given, the quadratic equation x^2 + px +q = 0.
Sum of the roots = -p = k+ 3-k
=> p = -3
As the equation has two distinct real roots, we get D > 0;
Thus, p^2 – 4q >0
=> (-3)^2 -4q > 0
=> 9 – 4q >0
=> q < $\frac{9}{4}$
=> q < 2.25
Thus, the maximum integral value which q can take is 2.

Q22 Text Solution:

As a, b, c are natural numbers in the quadratic equation $\left(ax^2+bx+\text{c}\right)=0$ the other root of the equation will be $5-2\sqrt{6}$.
So,

$$-\frac{b}{a}=5+2\sqrt{6}+5-2\sqrt{6}=10$$
$$\Rightarrow b=-10a$$
$$\frac{c}{a}=(5+2\sqrt{6})(5-2\sqrt{6})=1$$
$$\Rightarrow c=a.$$

$cx^2+px+a=0$ has one root as $(3+2\sqrt{2})$ and the other root as β (lets us assume)
So,

$$\beta(3+2\sqrt{2})=\frac{a}{c}=1$$
$$\Rightarrow \beta=3-2\sqrt{2}$$

So, the sum of the roots are $(3+2\sqrt{2})+(3-2\sqrt{2})$

$$6=-\frac{p}{a}$$

$$\Rightarrow p=-6a=\frac{3b}{5}=$$

$$-6c \text{ (By using 1 and 2)}$$

Thus, both option a \& b are correct.

Q23 Text Solution:

The given expression is

$34 + 24x^2 - 9x^4$

Let assume $x^2 = y$

=> $34 + 24y - 9y^2$

=> $34 + 24y - (3y)^2$

=> $34 + 2 \times 4 \times 3y - (3y)^2$

=> $34 + 16 - 16 + 2 \times 4 \times 3y - (3y)^2$

=> $34 + 16 - [16 - 2 \times 4 \times 3y + (3y)^2]$

=> $50 - (4 - 3y)^2$

$\Rightarrow$ $50 - (4 - 3y)^2$ to be maximum $(4-3y)^2$ should be equal to zero.

Hence, the maximum value will be 50.

Q24 Text Solution:

Let p and q be the roots of the equation $ax^2 + bx + c = 0$

$p + q = -\frac{b}{a}$ and $pq = \frac{c}{a}$

Given $p^2 + q^2 = p^3 + q^3$

=> $(p + q)^2 - 2pq = (p + q)^3 - 3pq(p + q)$

=> $(-\frac{b}{a})^2 - 2(\frac{c}{a}) = (-\frac{b}{a})^3 - 3(\frac{c}{a})(-\frac{b}{a})$

=> $\frac{b^2-2ac}{a^2} = \frac{-b^3+3abc}{a^3}$

=> $ab^2 - 2a^2c = -b^3 + 3abc$

=> $b^3 + ab^2 = ac(2a + 3b)$ .... (i)

but, given that

$b^3 + ab^2 = 6a + 9b \neq 0$

$b^3 + ab^2 = 3(2a+3b)$ ..... (ii)

Using (i) and (ii),

=> $ac = 3$

Q25 Text Solution:

Let, $f(x) = ax^2 + bx + c$

At x = 1, f(1) = a + b + c = 4.............(1)

At x = 0, f(0) = c = - 1

The maximum of a quadratic expression is attained at

$x = -\frac{b}{2a}$

Therefore, $-\frac{b}{2a} = 1$ (since maximum is attained at x = 1)

and we know that a + b = 5 (since, c = - 1)

2a + b = 0 and a + b = 5

a = -5 and b = 10

Thus, $f(x) = -5x^2 + 10x - 1$

f(7) = -176

Required absolute value = 176

Q26 Text Solution:

Let the equation be $ax^2 + bx + c = 0$, and the two roots be 'p' and 'q'.

So,

And $p^2 + q^2 = 13$ .... (2)

From (1), we get

Or 6(p + q) = 5pq ....(3)

From (2), we get $(p + q)^2 - 2pq = 13$ ...(4)

Assuming (p + q) as 'A' and pq as 'B', we get the following:

From (3), 6A = 5B ...(5)

And from (4), $A^2 - 2B = 13$ ...(6)

From (5), we get, B =...(7)

Putting this in (6), we get

Or $5A^2 - 12A - 65 = 0$

So, A =

Putting this value in (7), we will get B =

So, we get two cases:

Case	Sum of roots	Product of the roots	Equation
1^{st}	$-\frac{13}{5}$	$-\frac{78}{25}$	$25x^2 + 65x - 78 = 0$
2^{nd}	5	6	$x^2 - 5x + 6 = 0$

So, there are two possibilities.

Hence, option (C) is the correct answer.

Q27 Text Solution:

Given equation is $x - \frac{2}{x-p} = p - \frac{2}{x-p}$

Simplifying we get

$\Rightarrow \frac{x(x-p)-2}{(x-p)} = \frac{p(x-p)-2}{(x-p)}$

$\Rightarrow x^2 - px - 2 = xp - p^2 - 2$ only when $x \neq p$

(Because at x = p

denominator becomes zero which means we cannot cancel (x - p)

from both sides as it becomes 0.)

$x^2 - 2px + p^2 = 0$, but $x \neq p$

$(x - p)^2 = 0$,

$(x - p) = 0$, but $x \neq p$

Hence, no solution will exist.

Clearly, option (D) is the correct answer.

Q28 Text Solution:

Since, α, β are the roots of the equation $x^2 + 9mx + 8m^2 = 0$, so

$\alpha + \beta = -9m$ and

$\alpha\beta = 8m^2$

Now, $(\alpha + \beta)^3 = -729m^3$ and

$(\alpha - \beta)^3 = [(\alpha - \beta)^2]^{\frac{3}{2}}$

$=[(\alpha + \beta)^2 - 4\alpha\beta]^{\frac{3}{2}}$

$=[81m^2 - 32m^2]^{\frac{3}{2}}$

$=(49m^2)^{\frac{3}{2}}$

$=7 \times 49m^3$

$= 343m^3$

Thus, $(\alpha + \beta)^3 + (\alpha - \beta)^3 = -729m^3 + 343m^3 = -386m^3$

Also, $(\alpha + \beta)^3 (\alpha - \beta)^3 = (-729m^3)(343m^3)$

$= -250047m^6$

Hence, the required quadratic equation is

$x^2 + 386m^2 - 250047m^6 = 0$.

Thus, option (A) is the correct answer.

Q29 Text Solution:

Given that, $x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 4$

$= 0$

$(\alpha^4 + \beta^4) = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= -2^3$

$(\alpha^6 + \beta^6) = (\alpha^4 + \beta^4)(\alpha^2 + \beta^2) - \alpha^2\beta^2(\alpha^2 + \beta^2)$

$= 0$

$(\alpha^8 + \beta^8)$

$= (\alpha^6 + \beta^6)(\alpha^2 + \beta^2) - \alpha^2\beta^2(\alpha^4 + \beta^4)$

$= 0 - 2^2(-8)$

$= 2^5$

$(\alpha^{10} + \beta^{10})$

$= (\alpha^8 + \beta^8)(\alpha^2 + \beta^2) - \alpha^2\beta^2(\alpha^6 + \beta^6)$

$= 0.$

So, we can see that $\alpha^{(4n-2)} + \beta^{(4n-2)} = 0$ where n is any natural number.

Thus,

$(\alpha^{2022} + \beta^{2022}) = 0$ [As $2022 = 4 \times 506 - 2$]

Also,

$\alpha^{2022}\beta^{2022} = (\alpha\beta)^{2022} = (2)^{2022}$

Thus, the equation having roots α^{2022} & β^{2022} is

$K(x^2 + 2^{2022}) = 0$

Given that K = 1, we get

$(x^2 + 2^{2022}) = 0$.

Q30 Text Solution:

Given $y = \frac{x^2+x}{x^2+x+1}$

$\Rightarrow y(x^2+x+1) = (x^2+x)$

$\Rightarrow (y-1)x^2 + (y-1)x + y = 0$

The above equation is quadratic when $y \neq 1$ because if the coefficient of x^2 becomes zero then the equation can't be a quadratic equation.

Since x is real, so $D \geq 0$

$\Rightarrow (y - 1)^2 - 4(y - 1)y \geq 0$

$\Rightarrow -3y^2 + 2y + 1 \geq 0$

$\Rightarrow (3y + 1)(y - 1) \leq 0$

$\Rightarrow y \in [-\frac{1}{3}, 1]$

But, for y = 1, we have

$(x^2 + x + 1) = x^2 + x$

$\Rightarrow 1 = 0$, which is impossible.

Hence, the range of the quadratic equation is y
∈ $[-\frac{1}{3}, 1)$
Clearly, the minimum value is $-\frac{1}{3}$.

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