MBA Pioneer Pro 2024

Inequalities 2

Quantitative Aptitude

DPP: 7

Q1 $(x - 1)(x - 2)^2 (x - 3)^3 (x - 4)^4 < 0$, then how many integral values 4x can assume?

Q2 Arrange the numbers in ascending order.

$6^{\frac{1}{6}}, 2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 8^{\frac{1}{8}}$

(A) $6^{\frac{1}{6}} > 2^{\frac{1}{2}} > 3^{\frac{1}{3}} > 8^{\frac{1}{8}}$

(B) $2^{\frac{1}{2}} > 3^{\frac{1}{3}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$

(C) $3^{\frac{1}{3}} > 2^{\frac{1}{2}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$

(D) $2^{\frac{1}{2}} > 2^{\frac{1}{2}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$

Q3 If $(x^4 - 16x^3 + 86x^2 - 176x + 105) < 0$, then the number of integral values 3x can assume is:

Q4 If $(x - 1)^{2017}(x - 3)^{2019}(x - 5)^{2021}(x - 7)^{2023} < 0$, then how many integral values 3x can assume?

Q5 If $x^4 - 22x^3 + 159x^2 - 418x + 300 < 20$, then 3x can assume how many integral values?

Q6 If $x(x^2 - 16x + 65) > 50$ and $x^2 - 10x + 36 \leq 20$, then find the values x can assume (the value of pi is to be taken as 3.14)

(A) π (B) $\frac{\pi}{\sqrt{3}}$

(C) $\pi\sqrt{3}$ (D) 2π

Q7 If $x(x - 7)^2 < 64 - 7x$ and $x(x - 8)^2 < 50 - x$, then what can be the value of x if x > 1?

(A) π (B) $\pi\sqrt{3}$

(C) $\frac{\pi}{\sqrt{3}}$ (D) π^2

Q8 If $x(x - 7)^2 < 64 - 7x$ and $x(x - 8)^2 > 50 - x$, then what can be the value of x?

(A) π (B) $\pi\sqrt{2}$

(C) $\pi\sqrt{3}$ (D) $\frac{\pi}{\sqrt{2}}$

Q9 If a + 3b + 5c + 7d < 31 where a, b, c, d are distinct natural numbers, then $\frac{(x-a)(x-d)^d}{\left(x-b\right)\left(x-c\right)+\frac{(b+c)^d}{a}}$ can assume which of the below values:

1. $-\frac{\pi}{3}$
2. $\frac{\pi}{3}$
3. $-\pi$
4. $\frac{\pi}{4}$

(A) Only 1 and 2

(B) Only 2, 3, and 4

(C) Only 1 and 3

(D) Only 1 and 4

Q10 If a + 3b + 5c + 7d < 31, where a, b, c, d are distinct natural numbers and if $\frac{(x-a)^a(x-d)^d}{(x-b)^b(x-c)^c} < 0$ then x can assume which of the below values:

(A) $-\frac{\pi}{3}$ (B) $\frac{\pi}{3}$

(C) 2 (D) $\frac{\pi}{4}$

Q11 If $x(x - 6)^2 < 28 - 3x$, $x(x - 9)^2 < 120 - 11x$ and $(x - 4)(x - 8) < 5$, then x can assume which of the following values?

(A) $\pi\sqrt{1}$ (B) $\pi\sqrt{3}$

(C) π^2 (D) $\pi\sqrt{4}$

Q12 If $x(x - 6)^2 > 28 - 3x$, $x(x - 9)^2 < 120 - 11x$ and $(x - 4)(x - 8) < 5$, then x can assume which of the below values?

(A) $\pi\sqrt{1}$ (B) $\pi\sqrt{3}$

(C) $\pi\sqrt{4}$ (D) $\pi\sqrt{6}$

Q13 If $(x^3 - 3^4) < 9(x - 1)(2x - 9)$ and $x^2(x - 10)2 < 9(10 - x)(3x - 2)$, then x can assume which of the below values?

1. π
2. 2π
3. $\frac{\pi}{2}$
4. $\sqrt{\pi}$

(A) Only 1 and 2

(B) Only 2 and 3

(C) Only 1 and 3

(D) Only 2, 3 and 4

Q14

If $\left(x^3-3^4\right) > 9\left(x-1\right)\left(2x-9\right)$ *and*, then x
$x^2\left(x-10\right)2 < 9\left(10-x\right)\left(3x-2\right)$
can assume which of the below values?
1. π
2. 2π
3. 3π
4. π^2

(A) Only 1 and 2
(B) Only 2 and 3
(C) Only 3 and 4
(D) Only 2 and 4

Q15 If a + 3b + 5c + 7d < 31, where a, b, c, d are distinct natural numbers and if $\frac{(x-a)^a(x-d)^d}{(x-b)^b(x-c)^c} > 0$, then x can assume which of the below values:
1. $-\frac{\pi}{3}$
2. π
3. 4
4. $\frac{\pi}{4}$

(A) Only 1 and 2
(B) Only 1, 2, and 4
(C) Only 1 and 3
(D) Only 2 and 4

Q16 If
If $\frac{x^3-12x^2+39x-28}{x^3-18x^2+99x-162} < 0$, then x can assume,
then x can assume
1. π
2. $\frac{\pi}{2}$
3. $\pi\sqrt{3}$
4. 3π

(A) Only 1 and 2
(B) Only 2 and 3
(C) Only 2, 3, and 4
(D) Only 1 and 3

Q17 If $(x^3 - 3^4) < 9(x - 1)(2x - 9)$ and $x^2(x - 10)^2 > 9(10- x)(3x - 2)$, then x can assume which of the below values? (the value of pi is to be taken as 3.14)
1. π
2. $\frac{\pi}{2}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

(A) Only 1, 2 and 5
(B) Only 1 and 4
(C) Only 3 and 4
(D) Only 4

Q18 If $\sqrt{3-x} > \sqrt{x+1}$ then how many integral values 4x can assume?

Q19 If $\sqrt{20-x^2} > \sqrt{x+8}$, then how many integral values x can assume?

Q20 If $\sqrt{28-7x} > \sqrt{x(x-4)(x-8)}$, then how many values x can assume?
1. 0
2. 1
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

(A) Only 1 and 3
(B) Only 1 and 4
(C) Only 2 and 3
(D) Only 2 and 4

Q21 If $\sqrt{28-7x} < \sqrt{x(x-4)(x-8)}$, then how many values x can assume?
1. 0
2. π
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

(A) Only 1 and 3
(B) Only 1 and 4
(C) Only 2 and 3
(D) Only 2 and 4

Q22 If $\sqrt{x+9} > \sqrt{16-x}$ then how many integral values x can assume?

Q23 If $\sqrt{36-x^2} > \sqrt{x+16}$, then find which of the below values x can assume?
1. π
2. $-\pi\sqrt{2}$
3. $\pi\sqrt{2}$
4. π^2

(A) Only 2
(B) Only 1 and 2
(C) Only 2, 3 and 4
(D) Only 3

Q24 If $x^4-8x^3+14x^2+8x < 15$, then $x\sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}}$ can assume how many integral values?

Q25 Let
$$\left(\sqrt{x}+\sqrt{4}\right)^2\left(\sqrt{x}+\sqrt{2}\right)^3\left(\sqrt{x}-\sqrt{2}\right)^4\left(\sqrt{x}-\sqrt{4}\right)^5 < 0$$

If $P = X^{\frac{1}{\sqrt{2\sqrt{2\sqrt{2\ldots\infty}}}}}$ is a real number, then

(A) $-\sqrt{4} < P < \sqrt{4}$

(B) $0 < P\sqrt{4}$

(C) $0 \leq P < \sqrt{2}, \sqrt{2} < P < \sqrt{4}$

(D) $\sqrt{2} < P < \sqrt{4}$

Q26 If $\left(x^2 - 17\right)^{\left(x^3 - 6x^2 + 11x - 6\right)} = 1$ and if $(x + 4)^{2021}(x - 5)^{2023} < 0$ then find the number of values x can assume.

Q27 If $x^3 - 9x^2 + 23x - 15 > 0$ and if $x^2 - 6x + 5 < 0$, then find the possible numbers of integer values 3x can assume.

Q28 If $|x^2 - 9x + 18| > x^2 - 9x + 18$, then which is true?

(A) x 3 ≤ or x ≥ 6

(B) 3 ≤ x ≤ 6

(C) 3 < x < 6

(D) None of the above

Q29 $\left(\sqrt{x\sqrt{x\sqrt{x\ldots\infty}}}\right)^{\sqrt{2+\sqrt{2+\sqrt{2+\ldots\infty}}}}\left(x\right.$So,

$\left.-3\right)\left(x^2 - x + 4\right)^{\frac{3}{2}}\left(x - 9\right)^3 < 0.$

find the number of integral values that x can assume.

Q30 If $\frac{x^3 - 12x^2 + 39x - 28}{x^3 - 18x^2 + 99x - 162} > 0$ then x can assume

1. π
2. $\frac{\pi}{2}$
3. $\pi\sqrt{2}$
4. 3π

(A) Only 1 and 2

(B) Only 2 and 3

(C) Only 1, 3, and 4

(D) Only 1 and 4

Answer Key

Q1	6
Q2	(C)
Q3	10
Q4	10
Q5	16
Q6	(A)
Q7	(B)
Q8	(B)
Q9	(D)
Q10	(B)
Q11	(D)
Q12	(D)
Q13	(D)
Q14	(C)
Q15	(B)
Q16	(B)
Q17	(D)
Q18	8
Q19	6
Q20	(B)
Q21	(C)
Q22	13
Q23	(B)
Q24	10
Q25	(C)
Q26	5
Q27	5
Q28	(C)
Q29	5
Q30	(D)

Hints & Solutions

Q1 Text Solution:

$(x - 1)(x - 2)^2(x - 3)^3(x - 4)^4 < 0$

As, $(x - 2)^2 \geq 0$ and $(x - 4)^4 \geq 0$

So, $(x - 1)(x - 3)^3 < 0$

$\Rightarrow (x - 1)(x - 3) < 0$ [Since, $(x - 3)^2 \geq 0$]

$\Rightarrow 1 < x < 3$

But, $x \neq 2$ as $x = 2$ will give $(x - 1)(x - 2)^2(x - 3)^3(x - 4)^4 = 0$

So, $1 < x < 2$ and $2 < x < 3$

$\Rightarrow 4 < 4x < 8$ and $8 < 4x < 12$

So, 4x can assume values from 5, 6, 7, 9, 10, 11

Therefore, total 6 values can be assumed by 4x.

Q2 Text Solution:

$\text{Let } a = 6^{\frac{1}{6}}$

$\Rightarrow a^6 = 6 \quad \ldots\ldots (i)$

$\text{Again, } b = 2^{\frac{1}{2}}$

$b^2 = 2$

$\Rightarrow b^6 = 2^3 = 8$

$\Rightarrow b^6 > a^6 \quad \ldots\ldots (ii)$

$\text{Also, } c = 3^{\frac{1}{3}}$

$\Rightarrow c^3 = 3$

$\Rightarrow c^6 = 3^2 = 9$

$\Rightarrow c^6 > b^6 > a^6$

$\Rightarrow c > b > a \quad \ldots\ldots (iii)$

$\text{Also, } d = 8^{\frac{1}{8}}$

$\Rightarrow d^8 = 8 = 2^3$

$\Rightarrow d^8 = b^6$

$\Rightarrow d^6 < b^6$

$\Rightarrow d < b \quad \ldots\ldots (iv)$

Again,

$d^{24} = 2^9 = 512$

$a^{24} = 6^4 > d^{24}$

$\Rightarrow a > d \quad \ldots\ldots (v)$

Hence, from (iii), (iv) and (v), we can conclude that, $c > b > a > d$

$\text{So, } 3^{\frac{1}{3}} > 2^{\frac{1}{2}} > 6^{\frac{1}{6}} > 8^{\frac{1}{8}}$

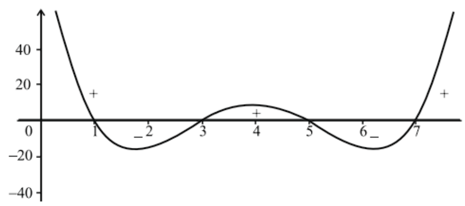
Q3 Text Solution:

$x^4 - 16x^3 + 86x^2 - 176x + 105 < 0$

$\Rightarrow (x^4 - 4x^3 + 3x^2 - 12x^3 + 48x^2 - 36x + 35x^2 - 140x + 105) < 0$

$\Rightarrow (x2 - 4x + 3)(x2 - 12x + 35) < 0$

$\Rightarrow (x - 1)(x - 3)(x - 5)(x - 7) < 0$



So, $1 < x < 3$ and $5 < x < 7$.

Thus, 3x can assume a total of 10 integer values.

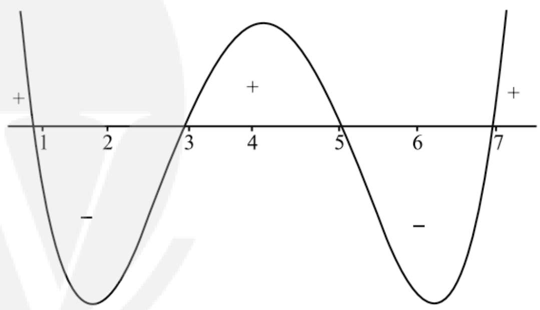
Q4 Text Solution:

As $(x - 1)^{2016} \geq 0$, $(x - 3)^{2018} \geq 0$, $(x - 5)^{2020} \geq 0$ and

$(x - 7)^{2022} \geq 0$, so

$(x - 1)^{2017}(x - 3)^{2019}(x - 5)^{2021}(x - 7)^{2023} < 0$

$\Rightarrow (x - 1)(x - 3)(x - 5)(x - 7) < 0$



So, $1 < x < 3$; $5 < x < 7$

$\Rightarrow 3 < 3x < 9$; $15 < 3x < 21$

So, 3x can assume 10 integral values.

Q5 Text Solution:

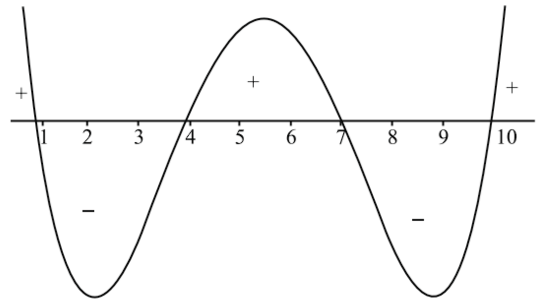
$x^4 - 22x^3 + 159x^2 - 418x + 300 < 20$

$\Rightarrow x^4 - 22x^3 + 159x^2 - 418x + 280 < 0$

$\Rightarrow (x4 - 17x^3 + 70x^2 - 5x^3 + 85x^2 - 350x + 4x^2 - 68x + 280) < 0$

$\Rightarrow (x^2 - 5x + 4)(x^2 - 17x + 70) < 0$

$\Rightarrow (x - 1)(x - 4)(x - 7)(x - 10) < 0$



So, $1 < x < 4$ and $7 < x < 10$

$\Rightarrow 3 < 3x < 12$ and $21 < 3x < 30$

3x can assume 16 integral values.

Q6 Text Solution:

Given that,

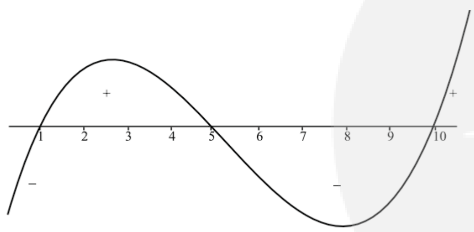
$x(x^2 - 16x + 65) > 50$

$\Rightarrow x^3 - 16x^2 + 65x - 50 > 0$

$\Rightarrow x^3 - 6x^2 + 5x - 10x^2 + 60x - 50 > 0$

$\Rightarrow (x^2 - 6x + 5)(x - 10) > 0$

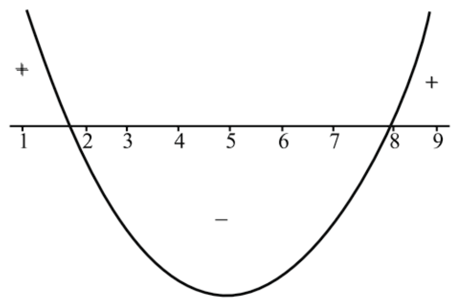
$\Rightarrow (x - 1)(x - 5)(x - 10) > 0$



Again, $x^2 - 10x + 36 \le 20$

$\Rightarrow x^2 - 10x + 16 \le 0$

$\Rightarrow (x - 2)(x - 8) \le 0$



Combining the two conditions, we can say that

$2 \le x < 5$

Hence, the only value of x satisfying this range is π.

Q7 Text Solution:

Given that, $x(x - 7)^2 < 64 - 7x$

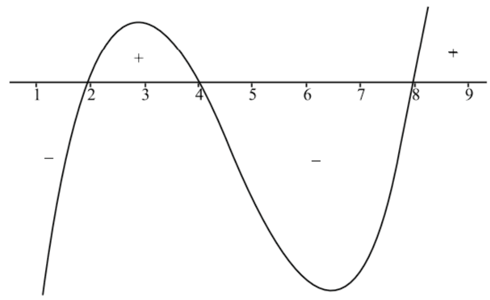
$\Rightarrow x^3 - 14x^2 + 49x < 64 - 7x$

$\Rightarrow x^3 - 14x^2 + 56x < 64$

$\Rightarrow x^3 - 6x^2 + 8x - 8x^2 + 48x - 64 < 0$

$\Rightarrow (x^2 - 6x + 8)(x - 8) < 0$

$\Rightarrow (x - 2)(x - 4)(x - 8) < 0$



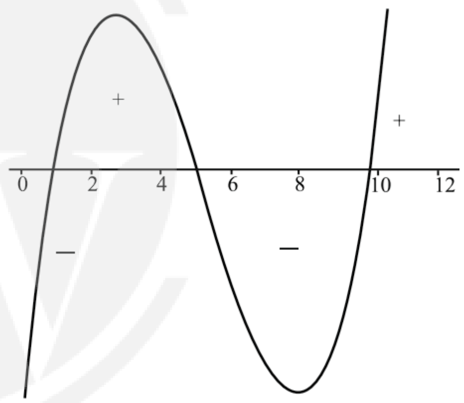
Again, $x(x - 8)^2 < 50 - x$

$\Rightarrow x^3 - 16x^2 + 65x < 50$

$\Rightarrow x^3 - 6x^2 + 5x - 10x^2 + 60x < 50$

$\Rightarrow (x^2 - 6x + 5)(x - 10) < 0$

$\Rightarrow (x - 1)(x - 5)(x - 10) < 0$



Hence, combining these two conditions, we get

$5 < x < 8$

In this range, the only x that can be satisfied is $\pi\sqrt{3}$.

Q8 Text Solution:

The given inequation is

$x(x - 7)^2 < 64 - 7x$

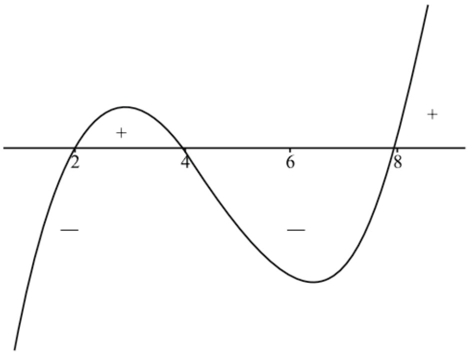
$\Rightarrow x^3 - 14x^2 + 49x < 64 - 7x$

$\Rightarrow x^3 - 14x^2 + 56x < 64$

$\Rightarrow x^3 - 6x^2 + 8x - 8x^2 + 48x - 64 < 0$

$\Rightarrow (x^2 - 6x + 8)(x - 8) < 0$

$\Rightarrow (x - 2)(x - 4)(x - 8) < 0$



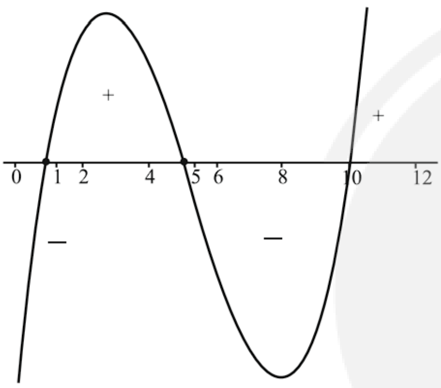
Again, $x(x-8)^2 > 50 - x$

$\Rightarrow x^3 - 16x^2 + 65x > 50$

$\Rightarrow x^3 - 6x^2 + 5x - 10x^2 + 60x > 50$

$\Rightarrow (x^2 - 6x + 5)(x - 10) > 0$

$\Rightarrow (x - 1)(x - 5)(x - 10) > 0$



Combining these two conditions, we get $4 < x < 5$.

In this range, the only value of x will be $\pi\sqrt{2}$.

Q9 Text Solution:

$a + 3b + 5c + 7d \le 30$ as a, b, c, d are distinct natural numbers.

Minimum value of the expression will be obtained if the lower number is multiplied with the higher coefficients.

So, Min $(a + 3b + 5c + 7d) = 4 + 9 + 10 + 7 = 30$ which satisfy the equation.

Thus, a = 4, b = 3, c = 2, and d = 1

Now, we have

$$\frac{(x-4)(x-1)}{(x-3)(x-2)+\frac{(3+2^1}{4}}$$

$$= \frac{(x-4)(x-1)}{\left(x-3\right)\left(x-2\right)+\frac{5}{4}}$$

$$= \frac{x^2-5x+4}{x^2-5x+6+\frac{5}{4}}$$

$$= \frac{x^2-5x+4}{\left(x-\frac{5}{2}\right)^2+1}$$

$$= 1 - \frac{\frac{13}{4}}{\left(x-\frac{5}{2}\right)^2+1}$$

Minimum value of the expression can be obtained at $x = \frac{5}{2}$, 2 = where $\left(x - \frac{5}{2}\right)^2 + 1$is minimum.

So, Min $\left\{\frac{(x-4)(x-1)}{\left(x-3\right)\left(x-2\right)+\frac{5}{4}}\right\} = -\frac{9}{4}$

Also, the maximum value of the expression will be obtained $\left(x - \frac{5}{2}\right)^2 + 1$ if is maximized.

Thus, maximum value of the expression will be very close to 1 (but less than 1) as x assumes a very high value.

$$\text{So, } -\frac{9}{4} \le \frac{(x-1)(x-4)}{\left(x-3\right)\left(x-2\right)+\frac{5}{4}} < 1$$

Q10 Text Solution:

$a + 3b + 5c + 7d \le 30$ as a, b, c, d are distinct natural numbers. Then, the minimum value of the expression will be obtained if the lower number is multiplied with the higher coefficients.

So, Min $(a + 3b + 5c + 7d) = 4 + 9 + 10 + 7 = 30$ which satisfies the equation.

Thus, a = 4, b = 3, c = 2, and d = 1

$$\text{Therefore, } \frac{(x-4)^4(x-1)}{(x-3)^3(x-2)^2} < 0$$

$\Rightarrow \frac{x-1}{x-3} < 0$ [Since, the even powered expressions are non−negative]

$$\Rightarrow \frac{(x-1)(x-3)}{(x-3)^2} < 0 \left[x \neq 2, 3\right]$$

$$\Rightarrow \left(x - 1\right)\left(x - 3\right) < 0$$

$$\Rightarrow 1 < x < 3 \text{ except } x = 2 \text{ and } 3.$$

Q11 Text Solution:

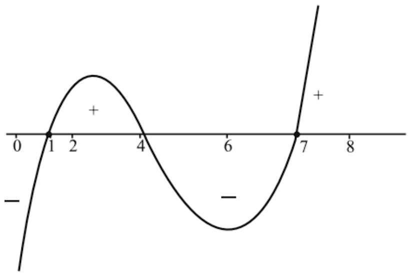
$x(x - 6)^2 < 28 - 3x$

$\Rightarrow x^3 - 12x^2 + 39x - 28 < 0$

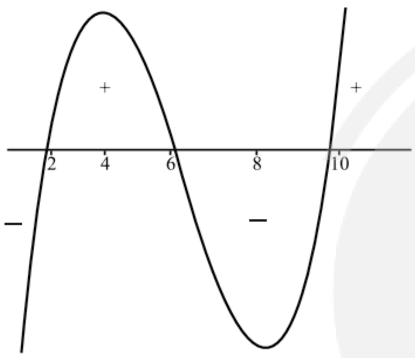
$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x - 28 < 0$

$\Rightarrow (x^2 - 5x + 4)(x - 7) < 0$

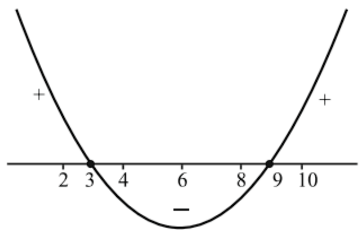
$\Rightarrow (x - 1)(x - 4)(x - 7) < 0$



$x(x-9)^2 < 120 - 11x$
$\Rightarrow x^3 - 18x^2 + 81x < 120 - 11x$
$\Rightarrow x^3 - 18x^2 + 92x < 120$
$\Rightarrow x^3 - 8x^2 + 12x - 10x^2 + 80x < 120$
$\Rightarrow (x^2 - 8x + 12)(x - 10)$ S< 0
$\Rightarrow (x-2)(x-6)(x-10) < 0$



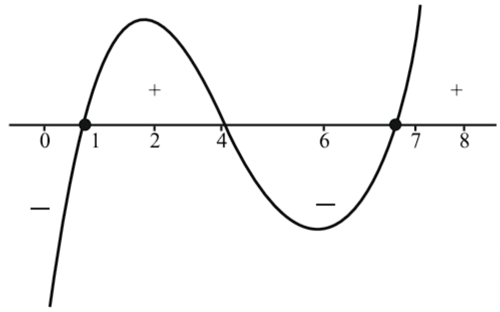
$(x-4)(x-8) < 5$
$\Rightarrow x^2 - 12x + 27 < 0$
$\Rightarrow (x-3)(x-9) < 0$



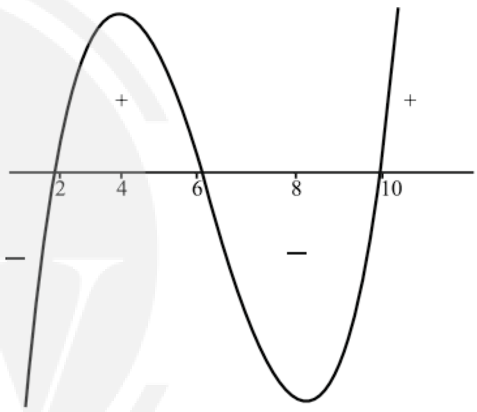
Combining three conditions, we get $6 < x < 7$.
Hence, the only value that satisfy this range is $\pi\sqrt{4}$.

Q12 Text Solution:

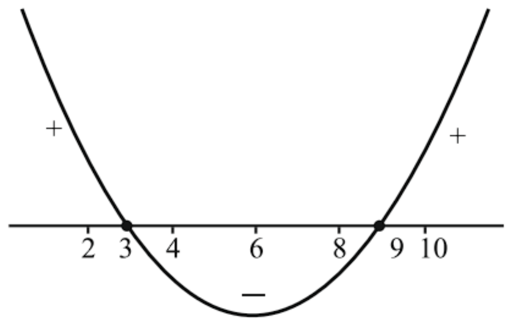
$x(x-6)^2 > 28 - 3x$
$\Rightarrow x^3 - 12x^2 + 39x - 28 > 0$
$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x > 28$
$\Rightarrow (x^2 - 5x + 4)(x - 7) > 0$
$\Rightarrow (x-1)(x-4)(x-7) > 0$



$x(x-9)^2 < 120 - 11x$
$\Rightarrow x^3 - 18x^2 + 81x < 120 - 11x$
$\Rightarrow x^3 - 18x^2 + 9^2x < 120$
$\Rightarrow x^3 - 8x^2 + 12x - 10x^2 + 80x < 120$
$\Rightarrow (x^2 - 8x + 12)(x - 10) < 0$
$\Rightarrow (x-2)(x-6)(x-10) < 0$



$(x-4)(x-8) < 5$
$\Rightarrow x^2 - 12x + 27 < 0$
$\Rightarrow (x-3)(x-9) < 0$



Combining three conditions, we get $7 < x < 9$.
Hence, the only value that satisfy this range is $\pi\sqrt{6}$.

Q13 Text Solution:

Given that $(x^3 - 3^4) < 9(x - 1)(2x - 9)$

$$\Rightarrow \left(\frac{x^3}{9} - 9\right) < 2x^2 - 11x + 9$$

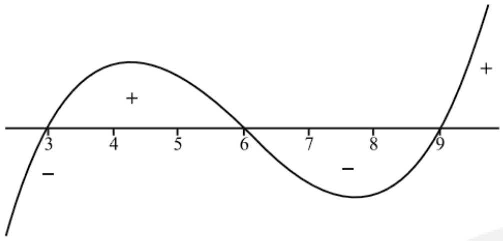
$$\Rightarrow \frac{x^3}{9} < 2x^2 - 11x + 18$$

$$\Rightarrow x^3 - 18x^2 + 99x - 162 < 0$$

$$\Rightarrow x^3 - 9x^2 + 18x - 9x^2 + 81x - 162 < 0$$

$$\Rightarrow (x^2 - 9x + 18)\left(x - 9\right) < 0$$

$$\Rightarrow \left(x - 3\right)\left(x - 6\right)\left(x - 9\right) < 0$$



$$\text{Again, } x^2\left(x - 10\right)^2 < 9\left(10 - x\right)\left(3x - 2\right)$$

$$\Rightarrow x^2\left(x - 10\right)^2 + 9\left(x - 10\right)\left(3x - 2\right) < 0$$

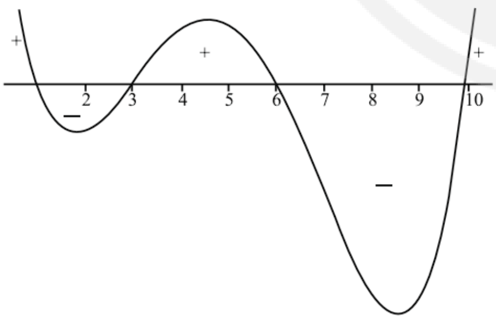
$$\Rightarrow x^2\left(x - 10\right)^2 + 9\left(3x^2 - 30x - 2x + 20\right)$$
$$< 0$$

$$\Rightarrow x^4 - 20x^3 + 127x^2 - 288x + 180 < 0$$

$$\Rightarrow x^4 - 4x^3 + 3x^2 - 16x^3 + 64x^2 - 48x$$
$$+ 60x^2 - 240x + 180 < 0$$

$$\Rightarrow \left(x^2 - 4x + 3\right)\left(x^2 - 16x + 60\right) < 0$$

$$\Rightarrow \left(x - 1\right)\left(x - 3\right)\left(x - 6\right)\left(x - 10\right) < 0$$



Hence, combining these two conditions, we get $6 < x < 9$ and $1 < x < 3$.

Q14 Text Solution:

Given that (x^3 – 3^4) > 9(x – 1)(2x – 9)

$$\Rightarrow \left(\frac{x^3}{9} - 9\right) > 2x^2 - 11x + 9$$

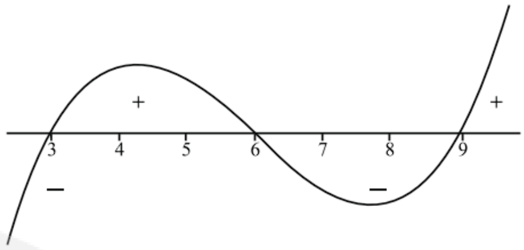
$$\Rightarrow \frac{x^3}{9} > 2x^2 - 11x + 18$$

$$\Rightarrow x^3 - 18x^2 + 99x - 162 > 0$$

$$\Rightarrow x^3 - 9x^2 + 18x - 9x^2 + 81x > 162$$

$$\Rightarrow \left(x^2 - 9x + 18\right)\left(x - 9\right) > 0$$

$$\Rightarrow \left(x - 3\right)\left(x - 6\right)\left(x - 9\right) > 0$$



$$\text{Again, } x^2\left(x - 10\right)^2 < 9\left(10 - x\right)\left(3x - 2\right)$$

$$\Rightarrow x^2\left(x - 10\right)^2 + 9\left(x - 10\right)\left(3x - 2\right) < 0$$

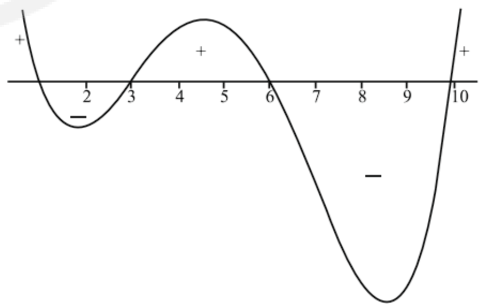
$$\Rightarrow x^2\left(x - 10\right)^2 + 9\left(3x^2 - 30x - 2x + 20\right)$$
$$< 0$$

$$\Rightarrow x^4 - 20x^3 + 127x^2 - 288x + 180 < 0$$

$$\Rightarrow x^4 - 4x^3 + 3x^2 - 16x^3 + 64x^2 - 48x$$
$$+ 60x^2 - 240x < -180$$

$$\Rightarrow \left(x^2 - 4x + 3\right)\left(x^2 - 16x + 60\right) < 0$$

$$\Rightarrow \left(x - 1\right)\left(x - 3\right)\left(x - 6\right)\left(x - 10\right) < 0$$



Hence, combining these two conditions, we get $9 < x < 10$.

Q15 Text Solution:

$a + 3b + 5c + 7d \leq 30$ as a, b, c, d are distinct natural numbers. Then, the minimum value of the expression will be obtained if the lower number is multiplied with the higher coefficients.

So, Min (a + 3b+ 5c + 7d) = 4 + 9 + 10 + 7 = 30 which satisfies the equation.

Thus, a = 4, b = 3, c = 2, and d = 1

Therefore, $\frac{(x-4)^4(x-1)}{(x-3)^3(x-2)^2} > 0$

$\Rightarrow \frac{x-1}{x-3} > 0 \left[\text{Since, the even powered expressions are non-negative}\right]$

$\Rightarrow \frac{(x-1)(x-3)}{(x-3)^2} > 0 \left[x \neq 2, 3\right]$

$\Rightarrow \left(x-1\right)\left(x-3\right) > 0$

$\Rightarrow x < 1 \text{ and } x > 3 \left[x \neq 4 \text{ and } 1 \text{ as it'll make the value } 0\right]$

Q16 Text Solution:

The expression $x^3 - 18x^2 + 99x - 162$ can be written as

$x^3 - 18x2 + 99x - 162$

$= x^3 - 9x^2 + 18x - 9x^2 + 81x - 162$

$= (x^2 - 9x + 18)(x - 9)$

$= (x - 3)(x - 6)(x - 9)$

Also, $x^3 - 12x^2 + 39x - 28$

$= x^3 - 5x^2 + 4x - 7x^2 + 35x - 28$

$= (x^2 - 5x + 4)(x - 7)$

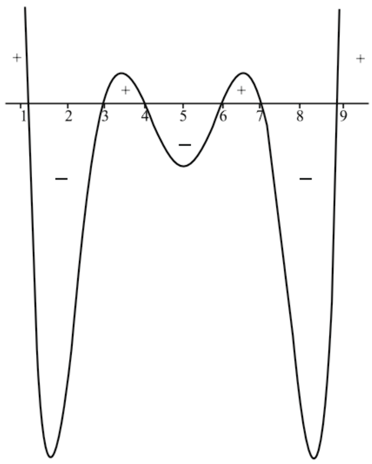
$= (x - 1)(x - 4)(x - 7)$

Now,

$\frac{x^3-12x^2+39x-28}{x^3-18x^2+99x-162} < 0$

$\Rightarrow \frac{(x-1)(x-4)(x-7)}{(x-3)(x-6)(x-9)} < 0$

$\Rightarrow \frac{(x-1)(x-3)(x-4)(x-6)(x-7)(x-9)}{(x-3)^2(x-6)^2(x-9)^2} < 0$



Q17 Text Solution:

Given that $\left(x^3 - 3^4\right) < 9\left(x-1\right)\left(2x-9\right)$

$\Rightarrow \left(\frac{x^3}{9} - 9\right) < 2x^2 - 11x + 9$

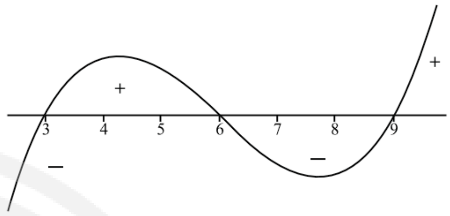
$\Rightarrow \frac{x^3}{9} < 2x^2 - 11x + 18$

$\Rightarrow x^3 - 18x^2 + 99x - 162 < 0$

$\Rightarrow x^3 - 9x^2 + 18x - 9x^2 + 81x < 162$

$\Rightarrow \left(x^2 - 9x + 18\right)\left(x-9\right) < 0$

$\Rightarrow \left(x-3\right)\left(x-6\right)\left(x-9\right) < 0$



Again, $x^2\left(x = 10\right)^2 > 9\left(10 = x\right)\left(3x-2\right)$

$\Rightarrow x^2\left(x-10\right)^2 + 9\left(x-10\right)\left(3x-2\right) > 0$

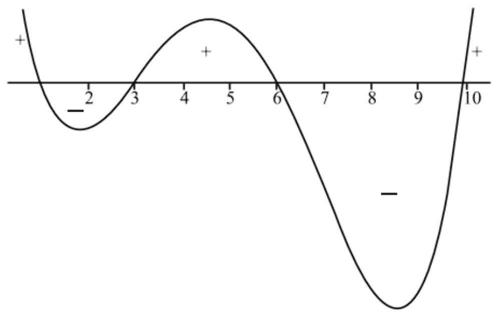
$\Rightarrow x^2\left(x-10\right)^2 + 9\left(3x^2 - 30x - 2x + 20\right) > 0$

$\Rightarrow x^4 - 20x^3 + 127x^2 - 288x + 180 > 0$

$\Rightarrow x^4 - 4x^3 + 3x^2 - 16x^3 + 64x^2 - 48x + 60x^2 - 240x > -180$

$\Rightarrow \left(x^2 - 4x + 3\right)\left(x^2 - 16x + 60\right) > 0$

$\Rightarrow \left(x-1\right)\left(x-3\right)\left(x-6\right)\left(x-10\right) > 0$



Hence, combining these two conditions, we get x < 1.

Q18 Text Solution:

$\sqrt{3-x} > \sqrt{x+1}$

As square of any non−negative number ≥ 0

3 − x > x + 1

$\Rightarrow 2 > 2x$

$\Rightarrow x < 1$

Also, $\sqrt{x+1}$ is a real number, so $x + 1 \geq 0$

$\Rightarrow x \geq -1$

So, $-1 \leq x < 1$

$\Rightarrow -4 \leq 4x < 4$

Thus, 4x can assume 8 integral values.

Q19 Text Solution:

$$\sqrt{20-x^2} > \sqrt{x+8}$$

$$\Rightarrow 20 - x^2 > x + 8$$

$$\Rightarrow 0 > x^2 + x - 12$$

$$\Rightarrow \left(x+4\right)\left(x-3\right) < 0$$

$$\Rightarrow -4 < x < 3$$

So, x can assume 6 integral values.

Q20 Text Solution:

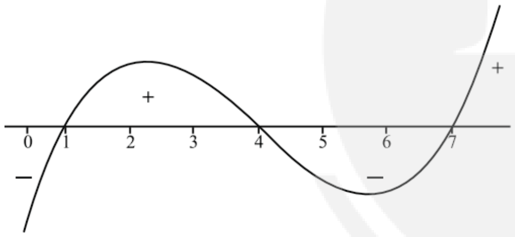
Given that, $\sqrt{28-7x} > \sqrt{x}\left(x-4\right)\left(x-8\right)$

$$\Rightarrow x^3 - 12x^2 + 39x - 28 < 0$$

$$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x - 28 < 0$$

$$\Rightarrow \left(x^2-5x+4\right)\left(x-7\right) < 0$$

$$\Rightarrow \left(x-1\right)\left(x-4\right)\left(x-7\right) < 0$$



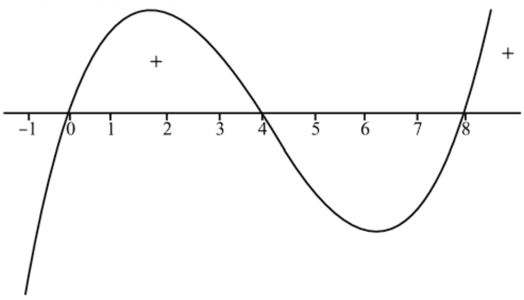
As, $\sqrt{28-7x}$ is a real number, so

$28 - 7x \geq 0$

$\Rightarrow x \leq 4$

As, $\sqrt{x(x-4)(x-8)}$ is a real number, so

$x(x - 8)(x - 4) \geq 0$



$\Rightarrow 0 \leq x \leq 4$ and $x \geq 8$.

Combining these two conditions, we get

$0 \leq x < 1$

Q21 Text Solution:

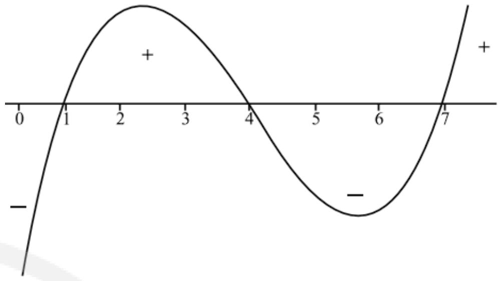
Given that, $\sqrt{28-7x} < \sqrt{x(x-4)(x-8)}$

$$\Rightarrow x^3 - 12x^2 + 39x - 28 > 0$$

$$\Rightarrow x^3 - 5x^2 + 4x - 7x^2 + 35x - 28 > 0$$

$$\Rightarrow \left(x^2-5x+4\right)\left(x-7\right) > 0$$

$$\Rightarrow \left(x-1\right)\left(x-4\right)\left(x-7\right) > 0$$

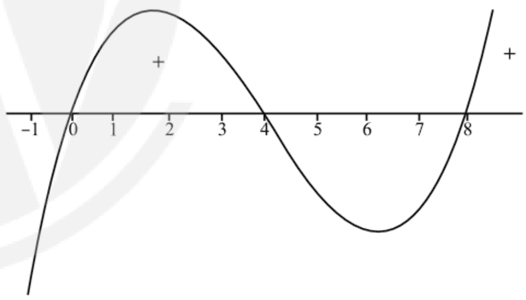


As, $\sqrt{28-7x}$ a real number, so

$\Rightarrow 28 - 7x \geq 0$

$\Rightarrow x \leq 4$

As, $\sqrt{x(x-4)(x-8)}$ is a real number, so $x(x - 8)(x - 4) \geq 0$



$\Rightarrow 0 \leq x \leq 4$ and $x \geq 8$.

Combining these two conditions, we get

$1 < x < 4$.

Q22 Text Solution:

Given that,

$$\sqrt{x+9} > \sqrt{16-x}$$

$x + 9 > 16 - x$

$\Rightarrow x > 3.5$

Also, $16 - x \geq 0$

$\Rightarrow x \leq 16$

So, $3.5 < x \leq 16$

Therefore, x can assume 13 integral values.

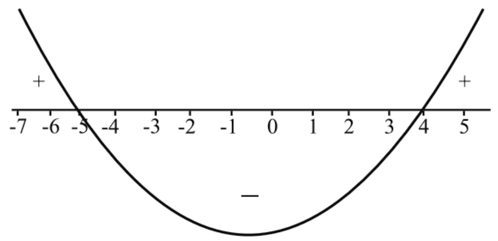
Q23 Text Solution:

$\sqrt{36-x^2} > \sqrt{x+16}$

$\Rightarrow\ 36 - x^2 > x + 16$

$\Rightarrow\ x^2 + x - 20 < 0$

$\Rightarrow\ (x-4)(x+5) < 0$



Also, $36 - x^2 \geq 0$

$\Rightarrow -6 \leq x \leq 6$

And,

$x + 16 \geq 0$

$\Rightarrow x \geq -16$

Combining all the conditions, we get

$-5 < x < 4$

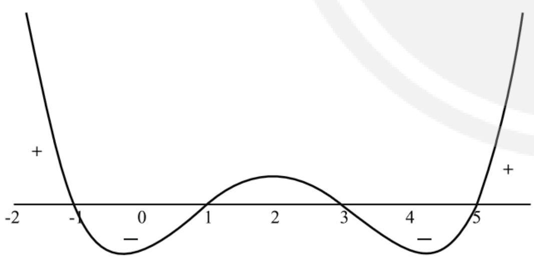
Q24 Text Solution:

Given that, $x^4 - 8x^3 + 14x^2 + 8x < 15$

$\Rightarrow x^4 - 8x^3 + 15x^2 - x^2 + 8x - 15 < 0$

$\Rightarrow (x^2-1)(x^2-8x+15) < 0$

$\Rightarrow (x+1)(x-1)(x-3)(x-5) < 0$



Again, let $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}} = y$

Then, $y^2 = 6 + y$

$y^2 - y - 6 = 0$

$\Rightarrow (y-3)(y+2) = 0$

$\Rightarrow y = 3$ [Since, $y > 0$]

So, 3x can assume

$-3 < 3x < 3;\ 9 < 3x < 15$

So, 3x can assume 10 values.

Q25 Text Solution:

Let $k = \sqrt{2\sqrt{2\sqrt{2\ldots\infty}}}$

$\Rightarrow k^2 = 2k$

$\Rightarrow k = 2$ (Since, $k \neq 0$)

So, $P = x^{\frac{1}{2}}$ is a real number.

Also, $\sqrt{x} \geq 0 \Rightarrow P \geq 0$

So, $\left(P+\sqrt{4}\right)^2\left(P+\sqrt{2}\right)^3\left(P-\sqrt{2}\right)^4\left(P-\sqrt{4}\right)^5 < 0$

$\Rightarrow \left(P+\sqrt{2}\right)\left(P-\sqrt{4}\right) < 0$

$\left[\text{Since, } \left(P+\sqrt{4}\right)^2 \geq 0, \left(P+\sqrt{2}\right)^2 \geq 0; \left(P-\sqrt{4}\right)^4 \geq 0, \left(P-\sqrt{2}\right)^4 \geq 0\right.$

$\Rightarrow -\sqrt{2} < P < 2$

As, $P \geq 0$, so $0 \leq P < 2$, but $P \neq \sqrt{2}$ as $\left(P-\sqrt{2}\right)^4 = 0$ and the expression will become 0.

So, $0 \leq P < \sqrt{2};\ \sqrt{2} < P < 2$

Q26 Text Solution:

$x^3 - 6x^2 + 11x - 6$

$= \left(x^3 - 3x^2 + 2x - 3x^2 + 9x - 6\right)$

$= \left(x^2 - 3x + 2\right)(x-3)$

$= (x-1)(x-2)(x-3)$

Now, $\left(x^2-17\right)^{\left(x^3-6x^2+11x-6\right)} = 1$ implies either,

$x^3 - 6x^2 + 11x - 6 = 0$

$\Rightarrow (x-1(x-2)(x-3) = 0$

$\Rightarrow x = 1, 2, 3$

or, $x^2 - 17 = 1$

$\Rightarrow x = \pm 3\sqrt{2}$

or, $x^2 - 17 = -1$

$\Rightarrow x = \pm 4$

Therefore,

$(x+4)^{2021}(x-5)^{2023} < 0$

As, $(x+4)^{2020} \geq 0$ and $(x-5)^{2022} \geq 0$

So, $(x+4)(x-5) < 0$

$\Rightarrow -4 < x < 5$

So, the values of x in this range are 1, 2, 3, 4, 3 $\sqrt{2}$Hence, x can assume 5 values.

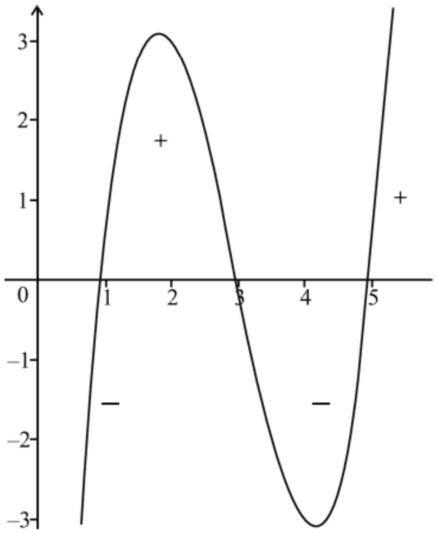
Q27 Text Solution:

$x^3 - 9x^2 + 23x - 15 > 0$

$\Rightarrow x^3 - 4x^2 + 3x - 5x^2 + 20x - 15 > 0$

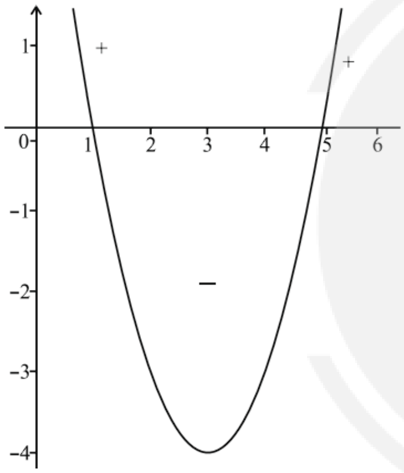
$\Rightarrow \left(x^2 - 4x + 3\right)(x-5) > 0$

$\Rightarrow (x-1)(x-3)(x-5) > 0$



Also, $x^2 - 6x + 5 < 0$

$\Rightarrow (x - 1)(x - 5) < 0$



$\Rightarrow 1 < x < 3$

So, $3 < 3x < 9$.

Hence, 3x can assume total of 5 integer values.

Q28 Text Solution:

We have the inequality

$|x^2 - 9x + 18| > x^2 - 9x + 18$

First let, $x^2 - 9x + 18 \geq 0$

$\Rightarrow (x - 3)(x - 6) \geq 0$

$\Rightarrow x \leq 3$ or $x \geq 6$.

Again, if $x^2 - 9x + 18 < 0$

$\Rightarrow (x - 3)(x - 6) < 0$

$\Rightarrow 3 < x < 6$

Therefore, if $x^2 - 9x + 18 > 0$, then $x^2 - 9x + 18 > x^2 - 9x + 18$ and there is no solution.

If $x^2 - 9x + 18 < 0$, then $-(x^2 - 9x + 18) > x^2 - 9x + 18$

$\Rightarrow -x^2 + 9x - 18 > 0$

$\Rightarrow -(x - 3)(x - 6) > 0$

$\Rightarrow 3 < x < 6$

Hence, option (c) is correct.

Q29 Text Solution:

$$\text{Let } \sqrt{x\sqrt{x\sqrt{x\ldots\infty}}} = p$$

$$\text{Then, } p^2 = xp$$

$$\Rightarrow p = x \left[\text{Since, } x \neq 0\right]$$

$$\text{Also, } \sqrt{2+\sqrt{2+\sqrt{2+\ldots\infty}}} = q$$

$$\Rightarrow q^2 = 2 + q$$

$$\Rightarrow q^2 - q - 2 = 0$$

$$\Rightarrow \left(q - 2\right)\left(q + 1\right) = 0$$

$$\Rightarrow q = 2, \text{ since } q > 0$$

$$\text{So, } \left(\sqrt{x\sqrt{x\sqrt{x\ldots\infty}}}\right)^{\sqrt{2+\sqrt{2+\sqrt{2+\ldots\infty}}}}$$

$$\left(x - 3\right)\left(x^2 - x + 4\right)^{\frac{3}{2}}\left(x - 9\right)^3 < 0$$

$$\Rightarrow x^2\left(x^2 - x + 4\right)^{\frac{3}{2}}\left(x - 3\right)\left(x - 9\right)^3 < 0$$

$$\text{Now } x^2 - x + 4 = \left(x - \tfrac{1}{2}\right)^2 + \tfrac{15}{4} > 0$$

$$\text{or } \left(x^2 - x + 4\right)^{\frac{3}{2}} > 0$$

Also $x^2 \geq 0$

So, $(x - 3)(x - 9)^3 < 0$

Because $(x - 9)^2 \geq 0$

So, $(x - 3)(x - 9) < 0$

So, $3 < x < 9$

So, x can assume integral values from 4 to 8.

So, total of 5 values can be assumed by x.

Q30 Text Solution:

The expression $x^3 - 18x^2 + 99x - 162$ can be written as $x^3 - 18x^2 + 99x - 162$

$= x^3 - 9x^2 + 18x - 9x^2 + 81x - 162$

$= (x^2 - 9x + 18)(x - 9)$

$= (x - 3)(x - 6)(x - 9)$

Also, $x^3 - 12x^2 + 39x - 28$

$= x^3 - 5x^2 + 4x - 7x^2 + 35x - 28$

$= (x^2 - 5x + 4)(x - 7)$

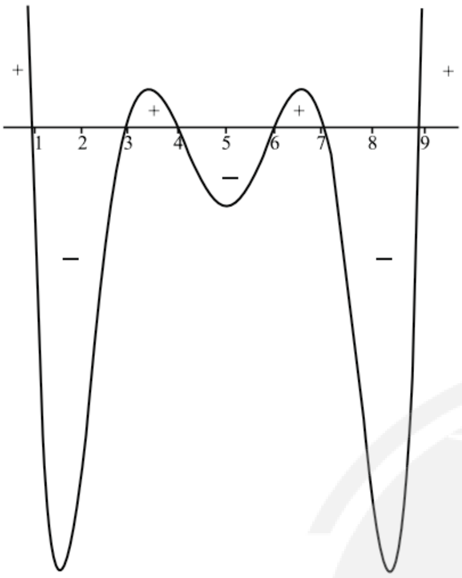
$= (x - 1)(x - 4)(x - 7)$

Now,

$$\frac{x^3-12x^2+39x-28}{x^3-18x^2+99x-162} > 0$$

$$\frac{(x-1)(x-4)(x-7)}{(x-3)(x-6)(x-9)} > 0$$

$$\Rightarrow \frac{(x-1)(x-3)(x-4)(x-6)(x-7)(x-9)}{(x-3)^2(x-6)^2(x-9)^2} > 0$$



The values of x which satisfies the inequation,

x > 9 ∪ (6,7) ∪ (3,4) ∪ x < 1