

1 Basics of Numbers, Fractions and Decimals

Introduction

In order to understand quantitative skills, it is important to understand numbers as they are the basic building blocks of entire mathematics. This unit is about understanding numbers and their basic properties.

Understanding numbers

While numbers can be divided into various types, for the NMAT by GMAC™, you need to be conversant with the following types of numbers.

Natural and whole numbers

Numbers from 0, 1, 2, 3, so on are known as whole numbers.
Natural numbers do not include 0.

Rational and irrational numbers

The real numbers expressed in the form of $\frac{p}{q}$ (where $q \neq 0$) are called rational numbers. They include fractions, integers, decimals (terminating and recurring), natural numbers and whole numbers. $\sqrt[3]{27}, -4, 2.56, \frac{2}{5}, 1.\overline{33}$, etc. are the examples of rational numbers.

The real numbers that cannot be expressed in the form of $\frac{p}{q}$ (where $q \neq 0$) are called irrational numbers. They include non-terminating and non-recurring decimal numbers. $\sqrt{6}, 2\sqrt{5}, -\sqrt{8}, 1.67834569012\dots$, etc. are the examples of irrational numbers.

Integers and fractions

Numbers $-1, 0, 1, 2, \dots$, etc., which have no fractional part, are called integers. Integers include the counting numbers $(1, 2, 3, \dots)$, their negative counterparts $(-1, -2, -3, \dots)$ and 0.



Important Learning: 0 as a number is neither a negative integer nor a positive integer. Also, 0 and all positive integers are called non-negative integers.

A fraction is a quantity that represents a part of a whole. It has two parts—a numerator and a denominator. There are two types of fractions:

1. Proper fractions: Fractions of the form $\frac{A}{B}$, where A and B are integers and A is less than B, are called proper fractions. For example, $\frac{2}{3}, \frac{4}{7}, \frac{7}{11}$ and so on.
2. Improper fractions: Fractions of the form $\frac{A}{B}$, where A and B are integers and A is greater than B, are called improper fractions. For example, $\frac{5}{3}, \frac{8}{7}, \frac{11}{9}$ and so on.

Important concepts about fractions

1. If the numerator is increased while keeping the denominator constant, the fraction increases in value and vice versa.
2. If the denominator is increased while keeping the numerator constant, the fraction decreases in value and vice versa.
3. A quick method of comparing two positive fractions is to multiply the numerator of the first fraction with the denominator of the second and vice versa. If the product on the left side is larger, then the left fraction is greater and if the product on the right side is larger, then the right fraction is greater.

For example, compare $\frac{3}{7}$ and $\frac{5}{11}$. On cross-multiplying, we get $3 \times 11 = 33$ and $5 \times 7 = 35$. Since $35 > 33$, the fraction that corresponds to $\frac{5}{11}$, that is $\frac{5}{11}$, is greater.

Prime and composite numbers

A number which has exactly two different factors, that is, 1 and the number itself is a prime number (3, 11, 19, etc.) and a number having more than two different factors is a composite number (4, 12, 20, etc.).

Since the number 1 has only one factor that is 1 itself, it is neither a prime number nor a composite number.



Important Learning: 1 as a number is neither prime nor composite.

Some properties and observations on prime numbers are:

1. 2 is the smallest prime number.
2. 2 is the only even prime number; all other prime numbers are odd.
3. There are 25 prime numbers when counted from 1 to 100.
4. Every prime number, except 2 and 3, can be expressed as 1 more than a multiple of 6 or 1 less than a multiple of 6 (that is, $6N + 1$ or $6N - 1$).

Decimals

Decimals are numbers that fall in between integers. They express a part-to-whole relationship in terms of place value. For example, 1.2 is a decimal. The integers 1 and 2 are not decimals. An integer written as 1.0, however, is considered a decimal.

Digits and place value

There are 10 digits that make up all numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. For example, the three-digit number 412 consists of the digits 4, 1 and 2.

Every digit in a given number has a particular place value. The place value depends upon the digit's location relative to the decimal point.

Place value of digits in a number													
6	7	8	9	1	0	2	3	.	8	3	4		
T E N M I L L I O N S	E N D R E D T I L H O U S A U S A N D S	H U N D E R E T H O U S A N D S	T E N T H O U S A N D S	T H U N D R E N S	H U N D R E N S	T E N T H E N S	O N E S A N D S	DECIMAL	T E N T H S	H U N D R E T H S	T H O U S A N D T H S		

Calendars

The topic of calendar includes concepts such as odd days and leap year and finding the day of the week for a given date.

Concept of odd days

An ordinary year has 365 days, that is, 52 weeks and one odd day. This means, out of the 365 days in an ordinary year, 364 days will get converted into 52 weeks and one day will remain. This one day is referred to as 1 odd day.

Whenever we will have the case of certain number of days in our calculations, we will convert the days into weeks by dividing the total number of days by 7 and the remainder will be the number of odd days.

How the day of the week for the same date shifts from one year to the next

The concept of odd days means that when we proceed from one year to the next and the year is ordinary, that is, it has 365 days, the day will get shifted by one day.

As an example, if the 24th of May 2014 is a Saturday, then 24th of May 2015 will be a Sunday while 24th of May 2013 would have been a Friday and so on.

A leap year has 366 days, that is, 52 weeks and two odd days. This means that when we proceed from one year to the next and the year has the effect of leap (29th February being included), and the year has 366 days, the day would get shifted by two days.

As an example, if the 24th of May 2015 is a Sunday, then 24th of May 2016 would be a Tuesday, that is, a shift of two days.



Do not commit this common error: Normally, students make the mistake of only looking at the years (from which year to which year) to decide whether the shift will be one day or two days. This is a wrong approach. When calculating the above, we need to look at not only the years but also the date from which we are moving to the other given date. If the 29th of February falls between the two dates, there will be a shift of two days, otherwise there will be a shift of one day.

Definition of a leap year

Normally, we define a leap year by saying that a year if divisible by 4 is a leap year, and if not, then an ordinary year. This definition is unfortunately not complete. The correct definition of a leap year is as follows:

An end of the century year (the last year of a century, e.g. 1900, 2000, 2100, etc.) is a leap year only if divisible by 400.

For all the other years, check the divisibility by 4, and if the year is divisible by 4 it is said to be a leap year and will have 366 days.

This would mean that the year 1900 in spite of being divisible by 4 was not a leap year and the year 2100 will also not be a leap year.



Important Learning: Do you know why an end of the century year has to be divisible by 400 to be a leap year? This is because the revolution of the Earth around the Sun, which is normally said to be 365.25 days, which is what leads to an additional day being added in February, is actually 365.242 days.

Therefore, when we take 365.25 days in each year in our calculations, we are introducing an error of approximately 0.008 days in our calendar every year.

This error of 0.008 days does not seem very significant when viewed alone, but when its effect is taken for 400 consecutive years, it will introduce an error of 3 complete days in our calendar.

Therefore, it was decided that if the above mentioned error is to be corrected, then we need to add 3 days less in every 400 years, and so the 100th, 200th and 300th year are not leap years in spite of being divisible by 4. This error correction has also meant that after every 400 years, our calendar repeats itself for the next 400 years.

Counting odd days

1st January AD from where our calendar started was a Monday, and hence the reason for our week starting on a Monday, and Saturday and Sunday being called as weekends.

Therefore, if we are calculating from 1st January AD and after converting into weeks, whenever we have 1 odd day left, it would be a Monday. If there are 2 odd days left, then the first one would be a Monday, the second a Tuesday and so on. So, after converting into weeks,

First odd day = Monday

Second odd day = Tuesday

Third odd day = Wednesday

Fourth odd day = Thursday

Fifth odd day = Friday

Sixth odd day = Saturday

Seventh or zeroth odd day = Sunday

Concept of total odd days in 100, 200, 300 and 400 years

The total number of odd days form the basis of these calculations:

1. If we take 100 consecutive years from 1st January AD, there will be 24 leap years (remember the 100th year will not be a leap year) and 76 ordinary years.

24 leap years = $24 \times 2 = 48$ odd days, that is, 42 days being converted into 6 weeks and 6 odd days.

76 ordinary years = $76 \times 1 = 76$ odd days, that is, 70 days being converted into 10 weeks and 6 odd days.

Total = $6 + 6 = 12$ odd days, that is one week and 5 odd days.

So, 100 consecutive years from 1st January AD will give 5 odd days.

2. Similarly, 200 consecutive years from 1st January AD = 10 odd days, that is, 3 odd days.
3. 300 consecutive years will be 15 odd days, that is, 1 odd day.
4. But, 400 consecutive years = $20 + 1$, that is, 21 odd days, that is, 0 odd days. (This is because the 400th year will be a leap year and contribute 1 extra day.)

Also, any multiple of 400 consecutive years will always give 0 odd days. This is used along with the other four concepts to calculate a day if a date is given.

5. Also, 100 consecutive years will have 5 odd days, that is, the last day of 100 years will be a Friday. Similarly, the last day of the 200th, 300th and the 400th years will be Wednesday, Monday and Sunday, respectively.

Any two years will have the same calendar if they are both of the same type (that is, both ordinary or both leap) and the first days of both the years are the same.



Important Learning: The last day of a century will definitely be one day out of Friday, Wednesday, Monday or Sunday.

Continuing with the same logic, the first day of a new century will be a Saturday, Thursday, Tuesday or Monday.

Problem types based on calendars

Type 1

Example 1

What was the day on 24th May 2014?

- (A) Tuesday
- (B) Wednesday
- (C) Thursday
- (D) Friday
- (E) Saturday

Solution

In such questions, it would always depend on whether we have a reference point or not. As we do not have a reference point in this case, we will start our calculations from 1st January AD.

The first 2,000 years = 0 odd days

Next 13 years will have:

3 leap years \times 2 odd days = 6 odd days

10 ordinary years \times 1 odd day = 10 odd days = 3 odd days

For the year 2014:

January: 31 days, 3 odd days

February: 28 days, 0 odd days

March: 31 days, 3 odd days

April: 30 days, 2 odd days

May: 24 days, 24 odd days = 3 odd days

Total number of odd days for the year 2014 = $3 + 0 + 3 + 2 + 3 = 11$ odd days = 4 odd days

Total odd days = $6 + 3 + 4 = 13$ odd days = 6 odd days

Thus, 24th May 2014 will be a Saturday.

The correct answer is E.

Example 2

If 31 March 2017 is a Saturday, find the day of the week on 1 January 2014.

- (A) Wednesday
- (B) Friday
- (C) Thursday
- (D) Monday
- (E) Tuesday

Solution

The day of the week on 1 January 2014 can be determined as:

31 March 2017 is a Saturday.

31 March 2016 will be a Friday.

31 March 2015 will be a Wednesday.

31 March 2014 will be a Tuesday.

3 March 2014 (28 days before) will be a Tuesday.

28 February 2014 will be a Saturday.

31 January 2014 will be a Saturday.

3 January 2014 will be a Saturday.

So, 1 January 2014 will be a Thursday.

The correct answer is C.

2 Divisibility of Numbers and Finding Unit Digit

Now, let us look at the divisibility rules for some important numbers that will help you make quick calculations.

1. Divisibility rule for 2: A number is divisible by 2 if its last digit is 0 or even, that is, 0, 2, 4, 6 or 8. All even numbers are divisible by 2. For example, the last digit of 15646790 is 0; therefore, it is divisible by 2.
2. Divisibility rule for 3: A number is divisible by 3 if the sum of all the digits of the number is divisible by 3. For example, let us take the number 4,689. Now, the sum of the digits will be $4 + 6 + 8 + 9 = 27$; and since 27 is divisible by 3, the number 4,689 is also divisible by 3.
3. Divisibility rule for 4: A number is divisible by 4 if the number formed by its last 2 digits is divisible by 4. Let us take the example of 4,689. The number formed by the last 2 digits is 89, which is not divisible by 4. So, the given number is not divisible by 4. Also, since 89 divided by 4 gives a remainder of 1, so 4,689 when divided by 4 will also give a remainder of 1.
4. Divisibility rule for 5: A number is divisible by 5 if its last digit is 0 or 5. For example, the last digit of 567899239645 is 5; therefore, it is divisible by 5.
5. Divisibility rule for 6: A number is divisible by 6 if it is divisible by both 2 and 3. For example, the last digit of 1236 is 6. It is even; hence, it is divisible by 2. The sum of its digits ($1 + 2 + 3 + 6$) is 12, which is divisible by 3. So, the given number is divisible by 6.
6. Divisibility rule for 7: If the difference of twice of last digit of a number and the number formed by its remaining digits is either 0 or a multiple of 7, then the number is divisible by 7 (Repeat this process until we get a smaller number whose divisibility is known to us).

Let us check whether 1456 is divisible by 7 or not.

$$145 - (6 \times 2) = 133$$

$$13 - (3 \times 2) = 7$$

Therefore, 1456 is divisible by 7.

7. Divisibility rule for 8: A number is divisible by 8 if the number formed by its last three digits is divisible by 8. For example, the last three digits of 123696 are 696. 696 is divisible by 8; therefore, 123696 is divisible by 8.
8. Divisibility rule for 9: A number is divisible by 9 if the sum of all its digits is divisible by 9. For example, the sum of all the digits of 6794568 is 45 ($6 + 7 + 9 + 4 + 5 + 6 + 8$) and 45 is divisible by 9. Therefore, 6794568 is divisible by 9.
9. Divisibility rule for 10: A number is divisible by 10 if its last digit is 0. For example, the last digit of 27798870 is 0. Therefore, it is divisible by 10.
10. Divisibility rule for 11: If the difference between the sum of digits of a number at odd place and the sum of digits of that number at even place is either 0 or a multiple of 11, then that number is divisible by 11.

For example, the difference between the sum of digits of 23452 at odd place ($2 + 4 + 2 = 8$) and at even place ($3 + 5 = 8$) is 0 ($8 - 8 = 0$). Therefore, 23452 is divisible by 11.

11. For divisibility of other composite numbers: Any number is divisible by a composite number N (where $N = m \times p$) if it is divisible by both m and p. There is a condition that m and p must be co-prime.

For example, any number is divisible by 88 if it is divisible by both 8 and 11, but, not if it is divisible by both 4 and 22.

12. Divisibility rule for 12: A number is divisible by 12 if it is divisible by both 3 and 4. For example, the sum of all the digits of 7896 is 30 that is divisible by 3 and the number formed by the last two digits is 96 that is divisible by 4. So, 7896 is divisible by 12.

Example 1

If the number $2546bc$ is completely divisible by 3, find the possible values of $b + c$.

- (A) 2
- (B) 3
- (C) 5
- (D) 13
- (E) 17

Solution

For a number to be divisible by 3, the sum of all the digits should be divisible by 3.

Now, $2 + 5 + 4 + 6 + b + c$ should be divisible by 3.

$17 + b + c$ must be divisible by 3.

Therefore, $b + c$ must be a (multiple of 3) + 1.

That is, 1, 4, 7, 10, 13, 16, 19.

$b + c$ can take the mentioned nine values. 13 is the only value available in the options.

The correct answer is D.

Example 2

A number Q gives a remainder of 5 when divided by 7. Find the remainder when $2Q$ is divided by 7.

- (A) 2
- (B) 3
- (C) 5
- (D) 7
- (E) 9

Solution

$2Q$ when divided by 7 will give a remainder of 2×5 , that is, 10. This can be divided by 7 and the required remainder will be 3.

The other way of approaching this question is to pick a number that satisfies the criteria given in the question, that is, it should leave a remainder of 5 when divided by 7. Let us pick Q as 12. So, when $2Q$, that is, 24 is divided by 7, the remainder is 3, which is your answer. You can try this with any other number, such as 19, the result will remain the same $\left(\frac{38}{7} = 3\right)$.

The correct answer is B.

Word Problems on Numbers and Variables

There are a few things that need to be known before we look at various problems based on numbers and variables:

1. A two-digit number will always be written as $10x + y$ and not as xy . Similarly, a three-digit number will be written as $100x + 10y + z$. But the digits of a two-digit number will be called x and y .
2. The sum of a two-digit number and the number obtained by interchanging the digits is always divisible by 11.

$$(10x + y) + (10y + x) = 11x + 11y = 11(x + y)$$

3. The difference of a two digit number and the number obtained by interchanging the digits is always divisible by 9.
4. Any two two-digit numbers can be added to make a maximum sum of 198.

Example 3

The sum of a two-digit number and the number obtained by reversing the digits is a multiple of 88. If the difference of the digits at the ten's place and unit's place is 6, find the digit at the ten's place of the number.

- (A) 1
- (B) 3
- (C) 5
- (D) 6
- (E) 7

Solution

Let the two-digit number be $10x + y$ and so the number obtained by reversing the digits is $10y + x$.

Their sum will be $11x + 11y$, that is, $11(x + y)$.

If $11(x + y)$ is a multiple of 88, then $x + y$ is a multiple of 8, that is, $x + y$ can be either 8 or 16.

Also, $y - x = 6$

Thus, one possible result is $y = 7$ and $x = 1$. The other result is $y = 11$ and $x = 5$, which is not possible.

The correct answer is E.

VBODMAS

VBODMAS stands for Vinculum Brackets, Of, Division, Multiplication, Addition and Subtraction. This acronym helps us remember the sequence in which to carry out arithmetic operations.

Order of Operations

V – Vinculum or Bar

The bar you see on top of the values like \overline{x} must be solved first.

B – Brackets

Parts of calculation inside the brackets are always done first after the vinculum or bar.

O – Orders

Solve for orders if there is any, like powers, square roots or cube roots.

DM – Divide or multiply before addition or subtraction

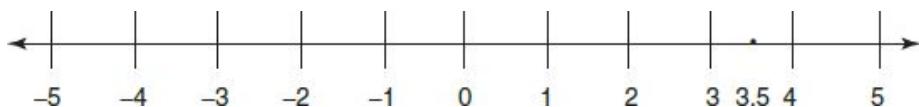
AS – Do addition and subtraction in the last (from left to right).

Note: ‘Of’ is used as a multiply operand in simplification.

While performing simplification, always ensure that the brackets are opened in the following order of priority, starting from (i)

1. Bar bracket ‘—’
2. Circular bracket ()
3. Curly bracket { }
4. Square bracket []

Number line and absolute value



Number line

A number line is a line on which all real numbers can be placed as per their value. Each point on the number line corresponds to a real number. For example, in the figure, above, the number 3.5 corresponds to a point on the number line which is halfway between 3 and 4.

The absolute value of a point is its distance from 0 on the number line. A positive number is already in the same form as that number's absolute value. For a negative number, remove the negative sign to get that number's absolute value. For example, the absolute value of -2 is 2 . The absolute value is denoted by two vertical parallel lines.



Important Learning: The absolute value of a number cannot be negative.

Largest Power of a Number in a Factorial

For Prime Numbers

Largest power of a prime number 'p' in factorial of any natural number 'N'

$$= \left[\frac{N}{p} \right] + \left[\frac{N}{p^2} \right] + \left[\frac{N}{p^3} \right] + \left[\frac{N}{p^4} \right] + \dots + \left[\frac{N}{p^n} \right], \text{ where } N \geq p^n.$$

For any non-negative integer X, [X] denotes the greatest integer less than or equal to X.

For example, [3.15] = 3

Example 4

Find the largest power of 3 in $100!$

- (A) 46
- (B) 47
- (C) 48
- (D) 49
- (E) 50

Solution

$$\text{Largest power of 3 in } 100! = \left\lfloor \frac{100}{3} \right\rfloor + \left\lfloor \frac{100}{3^2} \right\rfloor + \left\lfloor \frac{100}{3^3} \right\rfloor + \left\lfloor \frac{100}{3^4} \right\rfloor$$

Note that here we cannot take $\left\lfloor \frac{100}{3^5} \right\rfloor$ as $3^5 > 100$.

Therefore, the largest power of 3 in $100! = 33 + 11 + 3 + 1 = 48$

The correct answer is C.

For Composite Numbers

In order to find the largest power of a composite number in $N!$ follow the steps given below.

1. Factorise the given number into its prime factors.
2. Find the largest power of largest prime factor of given number in $N!$.

The largest power of largest prime factor in $N!$ is the largest power of given number in $N!$.

Example 5

Find the largest power of 30 in $50!$

- (A) 10
- (B) 11
- (C) 12
- (D) 13
- (E) 14

Solution

$$30 = 2 \times 3 \times 5$$

Since largest prime factor of 30 is 5, therefore, largest power of 5 in $50!$ is the largest power of 30 in $50!$.

$$\text{Largest power of 5 in } 50! = \left[\frac{50}{5} \right] + \left[\frac{50}{5^2} \right] = 10 + 2 = 12$$

Therefore, the largest power of 30 in $50! = 12$

The correct answer is C.

Find units digit of p^q

Unit's place of p^q depends on unit's place digit of 'p' and the divisibility of power 'q'. Consider powers of 2, as we know, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$ and so on.

The units place digit for powers of 2 repeat in an order: 2, 4, 8, 6. So, the cyclicity of number 2 is 4 (that means the pattern repeats after 4 occurrences) and the cycle pattern is 2, 4, 8, 6. From this, you can see that to find the units place digit of powers of 2, you have to divide the exponent by 4.

Shortcuts to solve problems related to units place digit of p^q

1. Case 1: If 'q' is a multiple of 4

If 'p' is an even number, that is: 2, 4, 6 or 8, then the units place digit is 6

If 'p' is an odd number, that is: 1, 3, 7 or 9, then the units place digit is 1

2. Case 2: If 'q' is not a multiple of 4

Let 'r' be the remainder when 'q' is divided by 4, then units place of p^q will be equal to units place of p^r .

Cyclicity of numbers up to 9:

Number	$\wedge 1$	$\wedge 2$	$\wedge 3$	$\wedge 4$	$\wedge 5$	Cyclicity
2	2	4	8	6	2	4
3	3	9	7	1	3	4
4	4	6	4	6	4	2
5	5	5	5	5	5	1
6	6	6	6	6	6	1
7	7	9	3	1	7	4
8	8	4	2	6	8	4
9	9	1	9	1	9	2

Let us take some examples to understand it clearly.

1. The last digit for $(163)^{11}$ is the same as that of 3^{11} , that is, $3^{4 \times 2 + 3}$ which is the same as the last digit for 3^3 , that is, last digit for 27 and so our last digit for $(163)^{11}$ is 7.
2. Last digit for $(29)^{19}$ is the same as that of 9^{19} , that is, $9^{2 \times 9 + 1}$ which is the same as the last digit for $9^1 = 9$.

3 Factors, Multiples, Least Common Multiple and Highest Common Factor

Factors and Multiples

A factor is a number that is able to completely divide a number greater than or equal to it. For example, 2 is a factor of 4 and 3 is a factor of 15, but 2 is not a factor of 15.

On the other hand, a multiple is a number that may be divided by another number a certain number of times without leaving a remainder. So, 4 is a multiple of 2, 15 is a multiple of 3, but 15 is not a multiple of 2 since dividing 15 by 2 will leave a remainder of 1.

Factor foundation rule

If a is a factor of b and b is a factor of c , then a is also a factor of c . For example, 3 is a factor of 9 and 9 is a factor of 81. Therefore, 3 is also a factor of 81.

Prime factorisation

Prime factorisation is a way to express any number as a product of prime numbers. For example, the prime factorisation of 30 is $2 \times 3 \times 5$. Prime factorisation is useful in answering questions about divisibility.

Example 1

Given that $1,176 = 2^p \times 3^q \times 7^r$, find the value of $p + q + r$.

- (A) 6
- (B) 8
- (C) 9
- (D) 10
- (E) 12

Solution

The given number can be written as below:

$$1176 = 4 \times 294 = 4 \times 3 \times 98 = 4 \times 3 \times 2 \times 49 = 2^3 \times 3^1 \times 7^2$$

Since $1176 = 2^p \times 3^q \times 7^r$, therefore, $p = 3$, $q = 1$ and $r = 2$

$$\text{Hence, } p + q + r = 3 + 1 + 2 = 6$$

The correct answer is A.

Highest Common Factor

We already have an understanding of what is a factor. The Highest Common Factor or HCF is the highest common factor among all the factors of a set of given numbers. It is also known as the Greatest Common Factor (GCF) or the Greatest Common Divisor (GCD). For example, the HCF of 24 and 40 will be 8, since 8 is the largest number that can completely divide both 24 and 40.

Least Common Multiple

The Least Common Multiple or LCM is the least common multiple of any set of given numbers. LCM refers to the smallest multiple of two (or more) integers. Multiples will be equal to or larger than the given integers. The LCM of 6 and 15 is 30, because 30 is the smallest number that both 6 and 15 go into.

Properties of HCF and LCM

Following are properties of HCF and LCM:

1. For any set of given numbers, the LCM is always a multiple of the HCF.
2. For two given numbers, product of the numbers is equal to the product of their HCF and LCM. (This property will also hold true for 3 numbers, 4 numbers and so on, provided none of them have a common factor other than 1.)
3. For a set of given fractions

$$\text{LCM of fractions} = \frac{\text{LCM of the numerator}}{\text{HCF of the denominator}}$$
$$\text{HCF of fractions} = \frac{\text{HCF of the numerator}}{\text{LCM of the denominator}}$$

Before doing this, we need to bring the fractions to the smallest or lowest form.

Example 2

What is the smallest integer which is greater than 1 and which leaves a remainder of 2 when divided by any of the integers 3, 5 and 7?

- (A) 18
- (B) 38
- (C) 105
- (D) 107
- (E) 213

Solution

You start by finding out the smallest number that is divisible by 2, 5 and 7 (which will be their LCM) and add 2 to the result.

The LCM of 3, 5 and 7 is 105, so our desired answer is $105 + 2 = 107$.

The correct answer is D.

Example 3

What minimum number must be subtracted from 247 so that the number is divisible by both 6 and 7?

- (A) 35
- (B) 36
- (C) 37
- (D) 38
- (E) 39

Solution

LCM of 6 and 7 is 42.

We need to find out a multiple of 42 closest to 247.

42×5 , that is, 210 is the multiple of 42 closest to 247 and so the value to be subtracted from 247 so that the number left is completely divisible by both 6 and 7 = $247 - 210 = 37$.

The correct answer is C.

4 Logarithm

Every positive real number N can be expressed in exponential form as

$$N = a^x \quad (1)$$

Where 'a' is also a positive real different than unity and is called the base and 'x' is called the exponent.

We can write the relation (1) in logarithmic form as

$$\log_a N = x \quad (2)$$

Hence the two relations

$$\begin{aligned} & a^x = N \\ \text{and } & \log_a N = x \end{aligned}$$

are identical where $N > 0, a > 0, a \neq 1$

Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get that number. Logarithm of zero does not exist and logarithm of (-) ve reals are not defined



Important Learning: Unless stated otherwise, we assume the base to be 10.

Important Formula

1. $\log(ab) = \log a + \log b$
2. $\log(a/b) = \log a - \log b$
3. $\log(a^n) = n \log a$
4. $\log_a b = \log b / \log a$ (to any base)

The logarithm of 1

Recall that any number raised to the power zero is 1: $a^0 = 1$.

The logarithmic form of this is

$$\log_a 1 = 0$$



Important Learning: $\log_b a = \frac{1}{\log_a b}$.

2.5 Arithmetic

2.6 What is Measured?

The arithmetic section comprises topics such as averages, profit and loss, ratio and proportion, percentages, simple and compound interest, mixtures, calendars, etc.

Basic arithmetic questions test your ability to interpret and solve problems of a mathematical nature, using such operations as addition, subtraction, division and multiplication, and in a variety of problem formats and situations.

While most of the concepts in arithmetic are quite simple, the NMAT by GMAC™ will not always test you on straightforward concepts; rather, it will mix up two or more topics, so you may see a question that combines percentages and ratio and proportion.

The arithmetic section will also test your mental math skills as you will be required to make quick calculations in your head.

2.7 Overall Test Taking Strategies

1. Read the question carefully.
2. Look at the options once before you start solving the question.
3. Remember the frequently tested squares and cubes and multiplication tables up to 20.
4. Be clear about the equivalent of common fractions as percentage and also about decimal terms.
5. Try to pick numbers and back-solve from the answer choices.

The next few sections will provide you with in-depth strategies for approaching each topic.

1 Percentages

Introduction

Percentage is an important topic in the NMAT by GMAC™, not only because you may be asked questions specifically related to percentages, but also because the knowledge of percentages helps you to easily understand and attempt problems from other areas in arithmetic.

Also, the understanding of percentages forms an important aspect of data interpretation (DI), in which questions require you to calculate percentage values, growth rates and other percentage changes.

So, what is a percentage? Any value expressed on a base of 100 or over a base of 100 is called percentage and is represented as % (cent represents the base 100).

A fraction is another way in which the value of a particular percentage can be represented. Therefore, one can say that percentages and fractions are equivalent and can be converted into one another as per the need.

For example, 25% is the same as $\frac{1}{4}$, 33.33% is the same as $\frac{1}{3}$, 50% is the same as $\frac{1}{2}$ and so on.

1. To convert a percentage into a fraction, divide the percentage by 100.

For example, 20% is same as $\frac{20}{100} = \frac{1}{5}$.

2. Similarly, to convert a fraction into a percentage, multiply the fraction by 100.

For example, $\frac{2}{5} = \frac{2}{5} \times 100 = 40\%$

Percentage equivalent of fractions

We need to know the percentage equivalent of fractions in order to enhance our understanding of percentages as a concept and to help in quick calculations.

For example, the percentage equivalent of $\frac{1}{2}$ will be $\left(\frac{1}{2}\right) \times 100 = 50\%$.

Instead of saying $\frac{1}{2}$ or half of any value, we can also say that we are calculating 50% of the given value.

Students must know the percentage equivalent of fractions up to $\frac{1}{20}$.

Let us look at some of these values:

$\frac{1}{1} = 100\%$	$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33.33\%$	$\frac{1}{4} = 25\%$	$\frac{1}{5} = 20\%$
$\frac{1}{6} = 16.67\%$	$\frac{1}{7} = 14.28\%$	$\frac{1}{8} = 12.5\%$	$\frac{1}{9} = 11.11\%$	$\frac{1}{10} = 10\%$

Percentage Change

$$1. \text{ Percentage change} = \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100$$

Percentage Increase/Decrease

2. Percentage increase = $\frac{\text{Actual increase}}{\text{Initial quantity}} \times 100$

3. Percentage decrease = $\frac{\text{Actual decrease}}{\text{Initial quantity}} \times 100$

4. If the price of a commodity increases by R%, then the reduction in consumption so as not to change the expenditure is $\left[\frac{R}{(100+R)} \times 100 \right] \%$

5. If the price of a commodity decreases by R%, then the increase in consumption so as not to change the expenditure is $\left[\frac{R}{(100-R)} \times 100 \right] \%$

Example 1

If the price of a commodity increases by 20%, then by what percentage the consumption should be reduced so as not to increase the expenditure?

- (A) 12.5%
- (B) 14.28%
- (C) 16.67%
- (D) 18.12%
- (E) 20.25%

Solution

$$\text{Required percentage} = \frac{20}{100+20} \times 100 = \frac{20}{120} \times 100 = 16.67\%$$

The correct answer is C.

Results on Population

Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

$$1. \text{ Population after } n \text{ years} = P \left(1 + \frac{R}{100} \right)^n$$

$$2. \text{ Population } n \text{ years ago} = \frac{P}{\left(1 + \frac{R}{100} \right)^n}$$

Results on Depreciation

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

1. Value of the machine after n years = $P \left(1 - \frac{R}{100}\right)^n$
2. Value of the machine n years ago = $\frac{P}{\left(1 - \frac{R}{100}\right)^n}$

Successive Percentage Change

If any quantity is increased by $x\%$, then $y\%$ and later on $z\%$, then the overall or effective percentage increase is:

$$\left[\left(\frac{100+x}{100} \right) \left(\frac{100+y}{100} \right) \left(\frac{100+z}{100} \right) - 1 \right] \times 100$$

Successive percentage change formula

When a number increases by A% and then by B%, the overall percentage increase is equal to

$$\left(A + B + \frac{AB}{100} \right) \%$$

Example 2

A number is increased by 20% and then the increased number is again increased by 10%. What is the total increment in the number?

- (A) 30%
- (B) 31%
- (C) 32%
- (D) 33%
- (E) 34%

Solution

$$\text{Required percentage} = 20 + 10 + \frac{20 \times 10}{100} = 20 + 10 + 2 = 32\%$$

The correct answer is C.

Application of successive percentage change formula

We can use successive percentage change formula to solve percentage-related problems where the product of two quantities equals the third quantity. For example,

$$\text{Length} \times \text{Breadth} = \text{Area}$$

$$\text{Price} \times \text{Quantity purchased} = \text{Expenditure}$$

Multiplication factor

To find the value of R% of a number, we multiply that number by $\frac{R}{100}$. If we want to find out 35% of a given number, we need to multiply the number by $\frac{35}{100}$ or 0.35.

To increase a number by R%, we multiply the number by $\frac{(100+R)}{100}$, and to decrease a number by R%, we multiply the number by $\frac{(100-R)}{100}$.

We need to understand that finding out R% of a number and increasing or decreasing a given number by R% are different operations.

The factor with which we multiply a number in order to (a) find the value of certain percentage of a given number, (b) increase the value of a number by a particular percentage or (c) decrease the value of a number by a particular percentage is called the multiplication factor.

For example, if we have to increase 120 by 20%, we need to multiply 120 by $\frac{(100+20)}{100}$ or $\frac{120}{100}$ or 1.2. In this case, 1.2 is the multiplication factor. The result is $120 \times 1.2 = 144$. Therefore, if we increase 120 by 20%, the final result will be 144.

Let us look at the multiplication factor for some cases:

1. To increase a number by 17%, the multiplication factor will be = $\frac{(100+17)}{100} = 1.17$.
2. To decrease a number by 11%, the multiplication factor will be = $\frac{(100-11)}{100} = 0.89$.
3. To increase a number by 34%, the multiplication factor will be = $\frac{(100+34)}{100} = 1.34$.
4. To decrease a number by 30%, the multiplication factor will be = $\frac{(100-30)}{100} = 0.7$.
5. To find 40% of a number, the multiplication factor will be 0.4.



Important Learning: If A is 20% more than B, then B will not be 20% less than A.

Base and base change

In percentages, it is very important to understand the base on which the change is happening. For example, let us take two numbers, say 40 and 50, and carry out the following operations:

1. Determine what percent of 40 is 50.

For this, we need to express 50 as a percentage of 40, that is, $\left(\frac{50}{40}\right) \times 100 = 125\%$. Therefore, 50 is 125% of the given value 40.

2. Express 40 as a percent of 50.

For this, we need to find out 40 as a percentage of the given base value, that is, 50. Therefore, $\left(\frac{40}{50}\right) \times 100 = 80\%$



Do not commit this common error: There are two things that a student must appreciate. 50% is what percentage of 40% and 50% is how much more than 40% are two different problems. In this example, we are being asked about how much more is 50% than 40%.

Many students make the mistake of saying that the required answer is 10%, which is the difference between the given values 50 and 40. The percentage affixed after the given values probably creates this confusion. What if the two values were 50 km/h and 40 km/h, that is, speed, or 50 kg and 40 kg, that is, weight?

Points to Remember

Some points to remember while resolving percentage related problems are listed as follows:

1. $A\% \text{ of } B = B\% \text{ of } A$

For example, $20\% \text{ of } 80 = 80\% \text{ of } 20 = 16$

2. If percentage increase in initial quantity is $k\%$, then the new value $= \left(\frac{k}{100} + 1\right) \times \text{Initial quantity.}$

3. If new quantity becomes k times the old quantity, then the percentage increase is $(k - 1) \times 100\%$. For example, if a quantity becomes 5 times of its initial value, then the percentage increase is 400% .

4. If A is $k\%$ more than B, then B is $\frac{k}{(100+k)} \times 100\%$ less than A.

5. If A is $k\%$ less than B, then B is $\frac{k}{(100-k)} \times 100\%$ more than A.

6. If the price of a commodity increases by $k\%$, then to keep expenditure constant, decrease in consumption is $\frac{k}{(100+k)} \times 100\%$.

7. If the price of a commodity decreases by $k\%$, then to keep expenditure constant, increase in consumption is $\frac{k}{(100-k)} \times 100\%$.

8. If increase in price is $\frac{1}{x}$ of the original price, then decrease in quantity purchased so that expenditure remains unchanged is equal to $\frac{1}{x+1}$ of the original quantity and vice versa; where x is a natural number.

For example, if price increases by 16.67% or $\frac{1}{6}$, then in order to keep the expenditure unchanged, the quantity has to be decreased by $\frac{1}{7}$ or 14.28% of its original value.

9. For the same expenditure, if increase in the quantity purchased is $\frac{1}{x}$ of the original quantity, then decrease in price is equal to $\frac{1}{x+1}$ of the original price and vice versa; where x is a natural number.

For example, if for the same expenditure, increase in quantity purchased is 20% or $\frac{1}{5}$ of the original quantity, then decrease in price is equal to 16.67% or $\frac{1}{6}$ of original price.

Example 3

In a class having 60% girls, 40% of the students qualified in a test. If 50% of the girls qualified, find the number of boys who did not qualify in the test as a percentage of the total strength of the class?

- (A) 10%
- (B) 30%
- (C) 45%
- (D) 55%
- (E) 90%

Solution

Therefore, as a percentage of the total strength of the class, 30% of the boys have not qualified. Let the total number of students in the class be 100. Then the number of girls = 60 and number of boys = 40. Total number of students who qualified = 40% of 100 = 40. Of those 40 students, 30 are girls (since 50% of the girls have qualified and 50% of 60 = 30). So, 10 boys have qualified, which means that 30 boys have not qualified.

The correct answer is B.

2 Simple and Compound Interest

Interest

Interest is an additional amount that a person gets against investment of capital. Interest earnings can be of two types: Simple Interest (SI) and Compound Interest (CI). Let us look at and understand both of them.

Simple Interest (SI)

The basic formula for simple interest is

$$SI = \frac{P \times R \times T}{100}$$

where P is the principal, R is the rate % per annum and T is the time period (in years) of investment.

Compound Interest (CI)

Compound interest is calculated on the principal amount and also on the accumulated interest of previous periods. This compounding effect can make a big difference to the total interest payable on a loan.

The following basic formula is used for calculating compound interest. The formula calculates the amount, using which the interest can be calculated, that is

$$A = P \left(1 + \frac{R}{100}\right)^n$$

where A is the amount, P is the Principal, R is the rate % applicable and n is the number of periods.

Compound interest calculation if interest is payable more than once a year

If interest is paid before completion of the year, then amount

$$A = P \left(1 + \frac{R}{100p}\right)^{np}$$

where p = number of times interest is paid in 1 year and n is number of years. So, if interest is paid semiannually, quarterly or monthly, the value of p is 2, 4 and 12 respectively.



Important Learning:

1. In the first period, SI and CI are equal. In all the other periods after the first period, the CI is greater than the SI.
2. In simple interest, the total rate of interest applicable is the sum of all the respective rates applicable.
3. In compound interest, the total rate of interest applicable is the successive effect of the respective rates given.

4. If an amount becomes N times itself in T years at SI, then
the required rate of Interest $R = \frac{(N-1)}{T} \times 100\%$
5. Difference between compound interest and simple interest
- For Two years, $CI - SI = P \left(\frac{R}{100} \right)^2$
 - For Three years, $CI - SI = P \left(\frac{R^2}{100^2} \right) \times \frac{300+R}{100}$
6. Ratio of CI and SI for two years, $\frac{CI}{SI} = \frac{200+R}{200}$

Example 1

The simple interest for 10 years is Rs. 6,000. The compound interest for 2 years is Rs. 1,400. Find the rate of interest and the principal.

- (A) 30%, 1,800
- (B) 33.33%, 1,600
- (C) 33.33%, 1,800
- (D) 35%, 1,500
- (E) 66.66%, 1,600

Solution

SI for 10 years is Rs. 6,000.

Therefore, SI for 1 year will be Rs. 600.

SI for 2 years will be Rs. 1,200.

CI for 2 years is Rs. 1,400.

Difference = Rs. 200. This is because of interest received on the first period's interest.

Therefore,

$$200 = 600 \times \frac{R}{100}$$

Therefore,

$$R = \frac{200}{6} = 33.33\%$$

Also, interest for the first period is Rs. 600, rate is 33.33% and time period is 1 year. Therefore,

$$600 = \frac{P \times 33.33 \times 1}{100} \Rightarrow P = \text{Rs. } 1,800$$

The correct answer is C.

3 Profit, Loss and Discounts

Introduction

Profit is an additional amount that a customer pays in return for buying an item that is owned by or sold by someone else.

$$\text{Profit} = \text{SP} - \text{CP}$$

where SP is the selling price, that is, the price at which the item is sold, and CP is the cost price, that is, the price at which it was originally manufactured or purchased by the seller.

When profit is expressed as a percentage of CP, it is known as profit %. Therefore,

$$\text{Profit \%} = \frac{(\text{SP} - \text{CP})}{\text{CP}} \times 100$$

Sometimes, the product is sold at a price lower than the CP. This is called loss.

Loss can be written either as $(\text{CP} - \text{SP})$, in which case it has a positive sign or $(\text{SP} - \text{CP})$, in which case it has a negative sign. So, loss = $\text{SP} - \text{CP}$.

$$\text{Loss \%} = \frac{(\text{SP} - \text{CP})}{\text{CP}} \times 100$$



Important Learning: Profit or loss % is always expressed as a percentage of the cost price.

Example 1

50 kg of a product is sold and the profit generated is equal to the cost price of 20 kg of the product. Find the profit percentage made.

- (A) 20%
- (B) 25%
- (C) 35%
- (D) 40%
- (E) 45%

Solution

As per the problem, we have:

$$\text{SP of } 50 \text{ kg} - \text{CP of } 50 \text{ kg} = \text{CP of } 20 \text{ kg}$$

$$\text{CP of } 70 \text{ kg} = \text{SP of } 50 \text{ kg}$$

$$\text{Therefore, } \frac{70 - 50}{50} \times 100 = 40\%$$

The correct answer is D.

False weights

If an item is claimed to be sold at cost price using false weights, then the overall percentage profit is given by

$$\text{Percentage profit} = \left(\frac{\text{Claimed weight of item}}{\text{Actual weight of item}} - 1 \right) \times 100$$

Example 2

A dishonest dealer claims to sell his good at cost price but uses a false weight, which reads 1000 gm for 800 gm. What is his net profit percentage?

- (A) 20%
- (B) 25%
- (C) 35%
- (D) 40%
- (E) 45%

Solution

$$\text{Required percentage} = \left(\frac{1000}{800} - 1 \right) \times 100 = \frac{200}{800} \times 100 = 25\%$$

The correct answer is B.

Discount

A discount is a reduction in the marked (or list) price of an article. “25% discount” means a reduction of 25% in the marked price of an article. For example, if the marked price of an article is Rs. 400, it is sold for Rs. 300, that is, Rs. 100 less than the marked price.

Let us define the following:

Marked Price (or List price)

The marked price (M.P.) of an article is the price at which the article is listed for sale. Since this price is written (marked) on the article, so it is called the marked price.

Net Selling Price (S.P.)

In case of discount selling, the price of the article obtained by subtracting discount from the marked price is called the net selling price or selling price (S.P.).

$$\text{Discount} = \text{Marked price} - \text{Selling price}$$

$$\text{Discount percentage} = \frac{\text{Discount}}{\text{Marked price}} \times 100$$

Successive Discounts

When a discount of a% is followed by another discount of b%, then

$$\text{Total discount} = a + b - \frac{ab}{100}$$

Important points and formulae

1. While the mark-up is always calculated as a percentage of the cost price, discount % is always calculated as a percentage of the marked price.
2. If two items are sold for Rs. x each, the first one at a profit of P% and the other at a loss of P%, then the overall loss will be
$$= \left(\frac{P^2}{100} \right) \%$$
.
3. When the CP and SP are either both increased or both decreased by the same percentage, there is no change in the existing profit % or loss %.
4. If two discounts are A% and B%, then the single equivalent discount =
$$\left(A + B - \frac{AB}{100} \right) \%$$
5. If a person wants to make a profit of A% after giving a discount of B%, then the Marked Price,
$$MP = CP \times \frac{100 + A}{100 - B}$$

4 Ratio, Proportion and Average

Introduction

Ratio is a comparison between two or more similar quantities having the same dimensions; therefore, ratio happens to be a dimensionless quantity.

For example, when we mention that the speed of two persons A and B is in the ratio 2 : 3, we do not ask whether the speed is in km/h or m/s, as the ratio is a simple comparison between two similar variables or values.

A ratio and fraction are synonymous yet different entities. When we say that $a : b$ is 2 : 3, we are talking about the ratio. Ratios are used to make comparisons, but when we need to find the individual contributions or values, fractions are required for the same.

Therefore, if $a : b$ is 4 : 3, we understand that for every value of 4 that a gets, b will get a value of 3, and so a gets a value of 4 for every 7 that they get together. This is known as a fraction.

Fraction of a is $\frac{4}{(4+3)}$, that is $\frac{4}{7}$, and fraction of b is $\frac{3}{(3+4)}$, that is $\frac{3}{7}$.

Important characteristics and formulae related to ratios

Some of the basic properties of ratios are listed below:

1. If both the antecedent and the consequent are multiplied or divided by the same number (except 0), the ratio will remain the same, i.e.

$$\frac{a}{b} = \frac{ka}{kb} \text{ or that, } \frac{a}{b} = \frac{a/k}{b/k}$$

2. Duplicate ratio refers to the ratio of the squares of the antecedent and the consequent. Duplicate ratio of $x : y = x^2 : y^2$.
3. Triplicate ratio refers to the ratio of the cubes of the antecedent and the consequent. Triplicate ratio of $x : y = x^3 : y^3$.
4. Sub-duplicate ratio refers to the ratio of the square roots of the antecedent and the consequent. Sub-duplicate ratio of $x : y = \sqrt{x} : \sqrt{y}$.
5. Sub-triplicate ratio refers to the ratio of the cube roots of the antecedent and the consequent. Sub-triplicate ratio of $x : y = \sqrt[3]{x} : \sqrt[3]{y}$.
6. Inverse ratio is derived by interchanging the positions of the antecedent and the consequent. Inverse ratio of $x : y = y : x$.
7. If the denominator of two ratios is same, then the ratio with larger numerator is greater than the ratio with the smaller numerator.
8. If we are given that $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = k$, then
$$k = \frac{a+c+e+g}{b+d+f+h}$$
9. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$



Important Learning: Ratio between two quantities or variables is only a comparative measure. It does not tell you anything about the actual values. For calculating actual values, you need to use fractions.

Example 1

The sum of the ages of the five members in a family is 124 years. If the ages of the children are in the ratio 3 : 4 : 5 while the combined age of their parents is 76, find the age of the youngest child.

- (A) 8
- (B) 12
- (C) 13
- (D) 14
- (E) 15

Solution

Combined age of the three children = $124 - 76 = 48$ years

Age of the youngest child will be: $\frac{3}{12} \times 48 = 12$ years

The correct answer is B.

Proportion and variation

Proportion is directly connected to ratios. Basically, a proportion is a statement that tells us that two ratios are equal. It can be written in two ways:

1. Two equal fractions, $\frac{a}{b} = \frac{c}{d}$ or
2. Using a colon, $a : b = c : d$

When two ratios are equal, the cross-multiplication of the ratios is also equal. For example,

$$\frac{2}{9} = \frac{6}{27} \Rightarrow 2 \times 27 = 6 \times 9$$

Important Characteristics and Formulae Related to Proportions

Some of the basic properties of proportions are listed below:

1. Since $\frac{a}{b} = \frac{c}{d}$, the product of extremes is equal to the product of means, i.e. $ad = bc$.
2. If a proportion is such that $a : x :: x : b$, then x is called the mean proportional or the second proportional of a and b .
3. If a proportion is such that $a : b :: b : x$, then x is called the third proportional to a and b .
4. Componendo rule: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$
5. Dividendo rule: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$
6. Componendo and Dividendo (C and D) rule: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
7. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d} = \frac{c}{d}$
8. Invertendo rule: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$
9. Alternendo rule: If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

Example 2

Three solutions having milk and water in the ratio $2 : 3$, $3 : 1$ and $4 : 5$, respectively, were mixed in the ratio $2 : 3 : 4$. Find the ratio of milk to water in the resultant mixture.

- (A) $869 : 751$
- (B) $219 : 341$
- (C) $420 : 519$
- (D) $531 : 622$
- (E) $640 : 729$

Solution

Let the solutions added be 2, 3 and 4 L, respectively.

Then, the quantity of milk in the solution is as below:

$$\begin{aligned} &= 2 \times \frac{2}{5} + 3 \times \frac{3}{4} + 4 \times \frac{4}{9} = \frac{4}{5} + \frac{9}{4} + \frac{16}{9} \\ &= \frac{144 + 405 + 320}{180} = \frac{869}{180} \end{aligned}$$

And, the quantity of water in the solution is as below:

$$= 9 - \frac{869}{180} = \frac{1620 - 869}{180} = \frac{751}{180}$$

Therefore, ratio of milk to water = 869 : 751

The correct answer is A.

Direct proportionality

Y is said to be directly proportional to X if Y increases as X increases and Y decreases as X decreases. Here, Y is called the dependent variable, while X is called the independent variable.

We can write the relation in the form $Y = KX$, where K is called the constant of proportionality.

Applications of direct proportionality

1. Distance covered is directly proportional to speed if time of travel is constant.
2. Amount of work done is directly proportional to the number of people if the number of days is constant.

Inverse proportionality

Y is said to be inversely proportional to X if Y decreases as X increases and Y increases as X decreases. We can write the relation in the form $Y = \frac{K}{X}$, where K is called the constant of proportionality.

Applications of inverse proportionality

1. Time taken is inversely proportional to speed if distance is constant.
2. Number of days is inversely proportional to the number of people if the amount of work done is constant.

Age-related problems

Problems based on ages are a simple application of the concept of ratios. In all problems of ages, we need to follow the instructions given in the problem keeping the time shift in consideration.

Important Points

1. We can take the unknown variable as the current age of the persons in the question or their age a few years earlier or a few years later. The answer will be the same, provided we keep the time shift in consideration.
2. The difference between the ages of two persons will always be the same whether the calculation is done today, a few years earlier or a few years later.
3. If the average age of a family of n members is x today, after three years, the average age of the family will be $x + 3$.

Example 3

Three years ago, the ratio of the ages of a father and a son was 6 : 1. After 3 years, the ratio will be 36 : 11. Find the present age of the son.

- (A) 3 years
- (B) 5 years
- (C) 8 years
- (D) 11 years
- (E) 17 years

Solution

Let the ages of father and son three years ago be $6x$ and x . Today their ages will be $6x + 3$ and $x + 3$ and after 3 years their ages will be $6x + 6$ and $x + 6$.

Now,

$$\begin{aligned} \frac{6x+6}{x+6} &+ \frac{36}{11} \\ \Rightarrow \frac{x+1}{x+6} &= \frac{6}{11} \\ \Rightarrow 5x &= 25 \\ \Rightarrow x &= 5 \end{aligned}$$

Present age of the son will be $x + 3$, that is, $5 + 3 = 8$ years

The correct answer is C.

Averages

An average is typically the central value of a set of numbers.

For a set of 'n' values x_1, x_2, \dots, x_n , the average is given by the following algebraic expression:

$$x_{\text{avg}} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Therefore, the formula for the average of a set of values can be expressed by the following formula:

$$\text{Average} = \frac{\text{Sum of all given values}}{\text{Number of values}}$$

Example 4

The average of five consecutive integers is 20. What is the average of the first 3 of these integers?

- (A) 15
- (B) 17
- (C) 18
- (D) 19
- (E) 21

Solution

We know that the average of consecutive integers is always the middle value. So, if the average is 20, the integers are 18, 19, 20, 21, 22.

So, the first 3 integers in this list are 18, 19, 20 whose average will again be the middle value, that is, 19.

The correct answer is D.

Properties of average

1. If each number in a set of numbers is increased by 'p', then their average is also increased by 'p'.
2. If each number in a set of numbers is decreased by 'p', then their average is also decreased by 'p'.
3. Similarly, if each number in a set of numbers is multiplied or divided by 'p', then their average also gets multiplied or divided by the same number 'p'.

Average of two different groups

Let us consider there are two groups; group 1 and group 2 with respective averages 'a' and 'b'. If the number of total items in group 1 and group 2 are 'm' and 'n' respectively, then the combined average of the two groups is given by the following expression:

$$\text{Combined average} = \frac{ma + nb}{m + n}$$

Change in average on deletion of an item

Let us consider that a value 'x' is deleted from a set of 'n' values with average 'a'. On deletion of an item the average of the remaining values may either increase or decrease which depends on the value of the deleted item. Therefore, it can be defined under two cases:

Case 1

If the average is increased by μ , then the deleted value 'x' is given by

$$x = a - (n - 1) \mu$$

Example 5

The average of a set of five values is 12. If one number is deleted, the average of the set is increased by 0.6. What is the value of the deleted number?

- (A) 8.2
- (B) 8.8
- (C) 9.2
- (D) 9.6
- (E) 10.2

Solution

Here, the original average (a) = 12

Total number of items (n) = 5

Decrease in the original average (μ) = 0.6

Therefore, the deleted value (x) = $a - (n - 1) \mu = 12 - (5 - 1) 0.6 = 9.6$

The correct answer is D.

Case 2

If the average is decreases by μ , then the deleted value 'x' is given by

$$x = a + (n - 1) \mu$$

Example 6

The average of a set of five values is 12. If one number is deleted, the average of the set is decreased by 0.6. What is the value of the deleted number?

- (A) 14.2
- (B) 14.4
- (C) 16.2
- (D) 19.6
- (E) 20.2

Solution

Here, the original average (a) = 12

Total number of items (n) = 5

Decrease in the original average (μ) = 0.6

Therefore, the deleted value (x) = $a + (n - 1) \mu = 12 + (5 - 1) 0.6 = 14.4$

The correct answer is B.

Change in average on addition of an item

Let us consider that a value 'x' is added to a set of 'n' values with the average 'a'. On addition of an item, the average may either increase or decrease which depends on the value of the added item. Therefore, it can be defined under two cases:

Case 1

If the average is increased by μ , then the added value 'x' is given by

$$x = a + (n + 1) \mu$$

Example 7

The average weight of a class of 13 students is 62.875 kg. When a new student joins the class, the average weight increases to 62.985 kg. What is the weight of the new student?

- (A) 64.415 kg
- (B) 65.825 kg
- (C) 66.545 kg
- (D) 67.215 kg
- (E) 69.615 kg

Solution

Original average (a) = 62.875 kg

Increase in average weight (μ) = $62.985 - 62.875 = 0.11$

Number of students (n) = 13

Therefore, the weight of the new students can be calculated using the formula:

$$x = a + (n + 1) \mu$$

$$x = 62.875 + (13 + 1) 0.11 = 64.415 \text{ kg}$$

The correct answer is A.

Case 2

If the average is decreased by μ , then the added value 'x' is given by

$$x = a - (n + 1) \mu$$

Example 8

Raghav has an average score of 54 in the last 15 matches. After the last match his average becomes 53. What was Raghav's score in the last match?

- (A) 30 runs
- (B) 34 runs
- (C) 38 runs
- (D) 42 runs
- (E) 46 runs

Solution

Using the formula discussed above, we get.

$$\text{Raghav's score in the last match} = 54 - (15 + 1) \cdot 1 = 38 \text{ runs}$$

The correct answer is C.

5 Time, Work and Partnership

Introduction

Time and work problems are important because there is a certain relationship between the number of persons doing the work, number of days or time taken by them to complete the work and the amount of work that is done.

The problems of time and work can primarily be divided into two types. The first type is the problem where individuals work with different efficiencies either alone or in combination to complete a task. The second type is where group efficiencies are involved. Such types of problems are also known as problems that use the chain rule.

The NMAT by GMAC™ will test you on both of these types of problems.

Problems involving individual efficiencies

In such questions, the rates at which some individuals complete a work alone is given and you are required to calculate the rate at which they can complete the work together (or vice versa). The basic formula for solving such problems is

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$$

where a and b are the time it takes the two individuals to complete a job, while working alone and c is the number of hours it takes them to complete the job working together. Let us look at an illustration to understand this concept better.

Example 1

A can do a work in 20 days. B can do the same work in 30 days. In how many days can A and B do the work together?

- (A) 8 days
- (B) 10 days
- (C) 12 days
- (D) 14 days
- (E) 16 days

Solution

Unitary method: We have been solving such problems using the unitary method.

A can do a work in 20 days. Therefore, in 1 day, A will be able to do $\frac{1}{20}$ of the work.

B can do the same work in 30 days. Therefore, in 1 day, B will be able to do $\frac{1}{30}$ of the work.

Both of them together can do $\left(\frac{1}{20} + \frac{1}{30}\right)$ of the work in 1 day = $\frac{5}{60}$ of the work in 1 day.

Therefore, the entire work will be completed in $\frac{60}{5}$ days, that is, 12 days.

The correct answer is C.

LCM method

An easier method to do such problems is to understand and use a method called the LCM method.

Let the total work be 60 units, where a unit is a simple measurement of work.

Total work = 60 units

A can to do 60 units in 20 days, that is, 3 units per day.

B can to do 60 units in 30 days, that is, 2 units per day.

Together, they are able to do $3 + 2$, that is, 5 units per day.

Therefore, they will finish the work in $\frac{60}{5}$, that is, 12 days.

With a little practice, the LCM method can be effectively used to solve a problem.



Important Learning: While using the LCM method, it is not necessary to use the LCM of the individual time taken to do the problem. One can take any convenient value and solve the problem.

Problems involving group efficiencies

We will now look at problems where people with the same efficiencies are working in groups. Let us try to understand this with the help of an example.

Example 2

A and B can do a work in 20 and 25 days, respectively. With the help of C and D, they finish the same work in 5 days. If the efficiency of C is half that of A, find the total time taken by D to finish the work alone.

(A) $\frac{100}{17}$

(B) 12

(C) $\frac{200}{17}$

(D) 15

(E) 20

Solution

Let the total work be 100 units.

A: 20 days 5 units/day

B: 25 days 4 units/day

A + B + C + D: 5 days 20 units/day

It means C and D can do 11 units per day. Since the efficiency of C is half of A, C will be able to do 2.5 units per day. It means D would be doing the remaining 8.5 units per day.

$$\text{Total time taken by D to finish the work alone} = \frac{100}{8.5} = \frac{200}{17} \text{ days.}$$

The correct answer is C.

Example 3

Three boys can do the same work as one woman. If a work is completed by 36 boys in 28 days working 9 h every day, how many women must be required to complete the same work in 7 days working 6 h every day?

- (A) 36 women
- (B) 48 women
- (C) 54 women
- (D) 66 women
- (E) 72 women

Solution

Given that 36 boys will be equivalent to 12 women.

$$12 \text{ women} \times 28 \text{ days} \times 9 \text{ h} = y \text{ women} \times 7 \text{ days} \times 6 \text{ h}$$

Therefore, $y = 72$ women

The correct answer is E.

Partnership

Partnership is defined as a legal agreement between two or more persons who agree to share profits or losses incurred by a business entity. Each person in the partnership is called a partner.

If the partnership incurs losses, then partners also share losses; and vice versa.

If $IR = \text{Investment Ratio}$

And, $TR = \text{The ratio of time periods of different partners}$,

Then, Profit Sharing Ratio (PSR) or Loss Sharing Ratio (LSR) is calculated as the product of the IR and TR.

Therefore,

$$\text{PSR or LSR} = IR \times TR$$

Important Cases Related to Partnership:

1. If there are two partners who invest I_1 and I_2 for the same period of time, then the PSR or LSR between partners 1 and 2 is calculated as:

$$\frac{\text{Profit (or loss) of Partner 1}}{\text{Profit (or loss) of Partner 2}} = \frac{I_1}{I_2}$$

2. If there are two partners who invest I_1 and I_2 amounts for time periods t_1 and t_2 , then the PSR or LSR of partners 1 and 2 is calculated as:

$$\frac{\text{Profit (or loss) of Partner 1}}{\text{Profit (or loss) of Partner 2}} = \frac{I_1 t_1}{I_2 t_2}$$

3. The share of each partner in a two-person (say Partner 1 and Partner 2) partnership, when they invest I_1 and I_2 amounts for the same duration of time, is calculated as:

$$\text{Partner 1} = \frac{I_1}{I_1 + I_2} \times P \text{ (or L)}$$

$$\text{Partner 2} = \frac{I_2}{I_1 + I_2} \times P \text{ (or L)}$$

4. The share of partners in a three-person (say Partner 1, Partner 2 and Partner 3) partnership, when they invest I_1 , I_2 and I_3 amounts for the same duration of time, is calculated as:

$$\text{Partner 1} = \frac{I_1}{I_1 + I_2 + I_3} \times P \text{ (or L)}$$

$$\text{Partner 2} = \frac{I_2}{I_1 + I_2 + I_3} \times P \text{ (or L)}$$

$$\text{Partner 3} = \frac{I_3}{I_1 + I_2 + I_3} \times P \text{ (or L)}$$

Example 4

A started a business with a capital of Rs. 5,000. Three months later, B joined with a capital of Rs. 7,000. After another 3 months, A invested Rs. 1,000 more while B withdrew Rs. 2,000. Two months later, C joined with a capital of Rs. 5,000. In what ratio should the profits get divided at the end of the year?

- (A) 22 : 4 : 17
- (B) 22 : 17 : 8
- (C) 22 : 17 : 4
- (D) 66 : 51 : 20
- (E) 66 : 17 : 20

Solution

As per the problem:

$$\text{Equivalent contribution of A} = 5,000 \times 6 + 6,000 \times 6 = 6,6000$$

$$\text{Equivalent contribution of B} = 7,000 \times 3 + 5,000 \times 6 = 51,000$$

$$\text{Equivalent contribution of C} = 5,000 \times 4 = 20,000$$

Therefore, the required ratio = 66 : 51 : 20

The correct answer is D.

2.8 Algebra and Probability

2.9 What is Measured?

The algebra and probability section will test you on topics such as equations, inequalities, sequence and series, permutation and combination and probability. You will be required to solve linear equations with one unknown and two unknowns, calculate the roots of a quadratic equation and work with the concept of absolute value.

Algebra is also tested indirectly in word problems as most of these questions will require you to make an equation and solve it.

You will also be tested on the expression of common algebraic identities such as $(a + b)^2$, $(a - b)^2$ and so on.

2.10 Overall Test Taking Strategies

1. As algebra is mostly formula-driven, you should be familiar with all the commonly used formulae.
2. If you make any change to one side of an equation or inequality, remember to make that same change to the other side as well.
3. Remember to reverse the inequality sign when changing a positive number into a negative number or vice versa.
4. A lot of the algebra questions are best tackled through the answer choices. Instead of finding the actual answer, it may be faster to back-solve from the answer choices, especially if you are able to eliminate one or two choices at first glance.

The next few sections will provide you with in-depth strategies for approaching each topic.

1 Equations and Inequalities

Equations

While arithmetic primarily deals with numbers, the basic building block of algebra is a variable which does not have a fixed value.

A variable along with a constant forms the basis of algebra. A variable by itself or a constant by itself or a variable in combination with a constant forms a term in algebra. For example, x^3 will be a term, 8 is also a term, $5x^2$ is also a term and so on.

When many terms combine together, they form an expression. For example, $x^3 - 5x^2 + 11$ is an expression in variable x. Please note that an expression is different from an equation. While an expression does not include the '=' sign, an equation will always include an '=' sign. Moreover, an expression always has a value, while an equation has roots or a solution.

Basis of classification

Algebraic expressions can be classified in the following ways:

1. Number of terms: The first basis of classification of algebraic expressions is based on the number of terms in the expression.
 - a. An expression having a single term is called a monomial, for example, $5x^2y$. Please note that the number of variables does not make any difference as long as the term is single.
 - b. An expression having two terms is called a binomial. For example, $3x + 5$.
 - c. An expression having more than two terms is called a polynomial. For example, $5x + 2y - 6$.
2. Degree of the expression: Before we understand this, we need to understand the definition of degree. Degree is defined as the highest or maximum sum of the powers of all the variables in any term of the expression.

For example,

$$3x^3 + 2x^2yz - 7y^2 + 5yz - 15x + 17$$

- a. The degree of this expression will be 4 because in the term $2x^2yz$, the power of $x = 2$, power of $y = 1$, power of $z = 1$. Hence the degree will be $2 + 1 + 1 = 4$.
- b. An expression of degree 1 is called linear.
- c. An expression of degree 2 is called quadratic.
- d. An expression of degree 3 is called cubic and so on.



Important Learning:

1. $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$
2. $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$

$$3. a^2 - b^2 = (a + b)(a - b)$$

$$4. a^2 + b^2 = \frac{(a + b)^2 + (a - b)^2}{2}$$

$$5. (a + b)^2 - (a - b)^2 = 4ab$$

$$6. (a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$7. (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$8. a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$9. a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$10. \frac{(a + b)^2 - (a - b)^2}{4} = ab$$

$$11. (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$$

$$12. \text{ If } a + b + c = 0 \text{ then } a^3 + b^3 + c^3 = 3abc$$

Linear equations

As stated earlier, an equation of degree 1 is called a linear equation. In this type of equation, all the variables are raised to the first power only (there are no squares, cubes, etc.). For example,

$$x + 10 = 25$$

In order to solve linear equations, we try to isolate the variable whose value we are trying to find by bringing it on one side of the equation and taking all other values to the other side of the equation. So, in the above equation

$$x + 10 - 10 = 25 - 10$$

$x = 25 - 10$ (isolating x by taking 10 to the other side)

$x = 15$ (the solution to the equation)

We can always verify whether we have got the correct answer by replacing the value of x (that we have arrived at) in the original equation and checking if it holds true.



Important Learning: To solve a linear equation, you just need to isolate the variable on one side and take all the numbers on the other side. Remember to reverse the signs when changing sides.

Example 1

In colony A, there are 12 houses with an average of 4 members per house, while in colony B, there are 20 houses with an average of Y members per house. If the two colonies together have an average of 3.5 members per house, find Y .

- (A) 3.2
- (B) 3.6
- (C) 4.8
- (D) 5.4
- (E) 6.2

Solution

It is given that the average number of members in the two colonies together is 3.5. Therefore,

$$12 \times 4 + 20 \times Y = 32 \times 3.5$$

$$48 + 20Y = 112$$

$$20Y = 64$$

$$Y = 3.2 \text{ members per house}$$

The correct answer is A.

Simultaneous equations

In linear equations, we were working with one variable, namely x. In simultaneous equations, we will be working with two variables, namely x and y.

Let us look at this equation

$$3x + 4y = 24$$

From this equation, can you find the values of x and y? Obviously not!

As a rule, if you want to find the numerical value for N variables, you will need N different equations. In linear equations, we are trying to find the value of one variable, so a single equation is sufficient. However, in the above equation, we are trying to find the values of two variables x and y, so we need two different equations that we will combine and solve simultaneously.



Important Learning: To find the numerical value for N variables, we need N number of equations.

Let us say we have the following two equations given to us,

$$x + 6y = 9 \quad (1)$$

$$3x + 4y = 24 \quad (2)$$

We can isolate x in Eq. (1) as

$$x = 9 - 6y$$

and we can then substitute this value of x in Eq. (2)

$$3(9 - 6y) + 4y = 24 \quad (3)$$

We can then solve Eq. (3) as a normal linear equation to get the value of y as $\frac{3}{14}$. We can then substitute this value of y in either Eq. (1) or Eq. (2) to find the value of x.

$$x + 6 \times \frac{3}{14} = 9$$

$$\Rightarrow x = \frac{54}{7}$$

Example 2

The price of two cups, seven pans and four saucers is Rs. 110 while of one cup and two saucers is Rs. 20. Find the price of three pans.

- (A) 30
- (B) 40
- (C) 60
- (D) 70
- (E) 80

Solution

As per the problem:

$$2x + 7y + 4z = 110$$

$$\text{Also, } 2x + 4z = 40$$

Therefore,

$$7y = 70 \text{ or } y = 10$$

Price of three pans will be Rs. 30.

The correct answer is A.

Example 3

Two apples and five bananas cost Rs. 17, while three apples and four bananas cost Rs. 15. What is the price of an apple?

- (A) Rs. 1
- (B) Rs. 1.50
- (C) Rs. 2
- (D) Rs. 2.50
- (E) Rs. 3

Solution

Let the price of an apple be X.

And, the price of a banana be Y.

Now, as per the question, we have:

$$2X + 5Y = 17 \quad (1)$$

$$\text{And, } 3X + 4Y = 15 \quad (2)$$

Multiply Eq. (1) by 4 and Eq. (2) by 5, we get

$$8X + 20Y = 68$$

$$15X + 20Y = 75$$

Solving, we get $X = \text{Rs. } 1$

The correct answer is A.

Quadratic Equations

An equation of the form $ax^2 + bx + c = 0$, where a , b and c are real and $a \neq 0$, is called a quadratic equation.

How to solve a quadratic equation

The following two methods are used to solve a quadratic equation:

1. Method of factorisation: This is the most popular method of solving a quadratic equation. If $ax^2 + bx + c = 0$ is the equation, we divide b into two parts such that their sum is b and product is ac.

For example, consider the equation $x^2 + 5x + 6 = 0$.

Now, 5 has to be divided into two numbers such that their product = 6 and their sum is 5. Therefore,

$$x^2 + 2x + 3x + 6 = 0$$

$$x(x+2) + 3(x+2) = 0$$

$$(x+3)(x+2) = 0$$

$$x = -3, -2$$

2. Shridharacharya's method: This is the method through which we all learnt the solution of quadratic equations for the first time. If $ax^2 + bx + c = 0$ is the equation, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $(b^2 - 4ac)$ is known as D, the discriminant of the equation, which also decides the nature of the roots of the quadratic equation, as follows:

- If $D > 0$, the roots are real and unequal.
- If $D = 0$, the roots are real and equal.
- If $D < 0$, the roots are imaginary.



Important Learning: The solutions to a quadratic equation are technically called its roots.

Properties of quadratic equations

1. If $ax^2 + bx + c = 0$ is the equation and the two roots are α and β , then

$$\alpha + \beta = \text{Sum of the roots} = -\frac{b}{a}$$
$$\alpha\beta = \text{Product of the roots} = \frac{c}{a}$$

2. If we know the roots, we can find the equation using

$$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

Example 4

If the sum of the roots of an equation is $\frac{5}{3}$ times the product of the roots, find the relation between b and c.

- (A) $b = \frac{5c}{3}$
- (B) $b = \frac{3c}{5}$
- (C) $b = -\frac{5c}{3}$
- (D) $b = -\frac{3c}{5}$
- (E) $b = \frac{2c}{5}$

Solution

For a quadratic equation,

Sum of the roots = $-\frac{b}{a}$ and product of the roots = $\frac{c}{a}$.

According to the problem,

$$-\frac{b}{a} = \frac{5}{3} \left(\frac{c}{a} \right)$$

$$-3ab = 5ac$$

$$b = -\frac{5c}{3}$$

The correct answer is C.

Note: The NMAT by GMAC™ will not test any skills beyond quadratic equations.

Inequalities

While equations tell us that two parts of an equation are equal, inequalities tell us that one part is bigger or smaller than the other.

$$2x + 3 < 7$$

Inequalities basically give us an idea of the relative size of two values.

Solution of an inequality

The value(s) of the variable(s) which makes the inequality a true statement is called its solutions. The set of all solutions of an inequality is called the solution set of the inequality. For example, $x - 1 \geq 0$, has infinite number of solutions as all real values greater than or equal to one make it a true statement. The inequality $x^2 + 1 < 0$ has no solution in R as no real value of x makes it a true statement.

How to solve an inequality

An inequality is solved, in the same way as you solve an equation, by isolating the variable on one side and simplifying it. It is just that the sign used will not be the '=' sign but the inequality sign, depending on the relation between the two parts of the inequality.

1. $>$ greater than
2. $<$ less than
3. \geq greater than or equal to
4. \leq less than or equal to

So, the solution to the inequality $2x + 3 < 7$ is $x < 2$.

However, there is one major difference between equations and inequalities that needs to be kept in mind—if the inequality is multiplied or divided by a negative number, the sign of the inequality is reversed.

For example, if the inequality $-5x > 3$ is multiplied by -1 , the resulting inequality is

$$5x < -3$$



Important Learning: If an inequality is multiplied or divided by a negative number, the sign of the inequality gets reversed.

Inequality Rules

Rule 1: Equal numbers may be added to (or subtracted from) both sides of an equation.

Rule 2: Both sides of an equation may be multiplied (or divided) by the same non-zero number.

Two Important Results

1. If $a, b \in \mathbb{R}$ and $b \neq 0$, then
 - a. $ab > 0$ or $\frac{a}{b} > 0 \Rightarrow a$ and b are of the same sign
 - b. $ab < 0$ or $\frac{a}{b} < 0 \Rightarrow a$ and b are of the opposite sign
2. If a is any positive real number, i.e., $a > 0$, then
 - a. $|x| \leq a \Leftrightarrow -a \leq x \leq a$
 $|x| \leq a \Leftrightarrow -a \leq x \leq a$
 - b. $|x| > a \Leftrightarrow x < -a$ or $x > a$
 $|x| \geq a \Leftrightarrow x \leq -a$ or $x \geq a$

Example 5

Which of the following describes all possible solutions to the inequality $|a + 4| \leq 7$?

- (A) $a < 3$
- (B) $a > -11$
- (C) $3 > a > -11$
- (D) $-11 > a > 3$
- (E) $a > 11$ or $a < -11$

Solution

Note the absolute value sign in the original inequality. This basically means that the solution could lie on either side of the number line. So, you will have to solve this inequality in two ways to get the entire range of solutions for a .

$$a - 4 < 7 \text{ or } a + 4 > -7$$

$$\text{So, } a < 3 \text{ or } a > -11$$

The correct answer is C.

2 Sequence and Series

Introduction

A series in which a particular relation exists between the terms is called a progression. There are three types of progressions: Arithmetic Progression (AP), Geometric Progression (GP) and Harmonic Progression (HP).

For the NMAT by GMAC™, you need to focus on AP and GP.

Arithmetic progression

Popularly known as AP, it is a series of terms in which the difference between a term and the next term is constant. This difference is called the common difference of the AP and is denoted by d .

Some examples of AP are

1. 2, 4, 6, 8, 10, 12, 14, ...
2. -3, -6, -9, -12, -15, -18, ...
3. 1, 2, 3, 4, 5, 6, 7, 8, ...
4. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \dots$

Properties of AP

The first term is called a , the common difference is called d and the number of terms is denoted by n . Therefore, an AP would be like $a, a + d, a + 2d, a + 3d, a + 4d$ and so on. Some important properties of AP are listed below:

1. The n th term of an AP is given by $T_n = a + (n - 1)d$, a relation between the n th term, the first term, the common difference and the number of terms.
2. If the same quantity (positive or negative) is added to each term of an AP, the series will continue to be an AP.
3. If the same quantity (positive or negative) is multiplied with or divides each term of an AP, the series will continue to be an AP.
4. In an AP, the sum of the terms equidistant from the beginning and end is a constant and is equal to the sum of the first and last terms. Let us try to understand this with the help of an example:

3, 6, 9, 12, 15, 18, 21, 24 is an AP

- a. Sum of the first and last terms = $3 + 24 = 27$
- b. Sum of the second and second last terms = $6 + 21 = 27$
- c. Sum of the third and third last terms = $9 + 18 = 27$
- d. Sum of the fourth and fourth last terms = $12 + 15 = 27$

What if the number of terms is odd?

Let us try to understand this with the help of another example:

4, 7, 10, 13, 16, 19, 22 is an AP

- a. Sum of the first and last terms = $4 + 22 = 26$
- b. Sum of the second and second last terms = $7 + 19 = 26$
- c. Sum of the third and third last terms = $10 + 16 = 26$
- d. The middle term in this case, which does not form a pair, will be half the sum of the first and last terms.

5. Sum to n terms of an AP:

$$S_n = \frac{n}{2}(a+1)$$

where l is the last term, in this case the nth term of the AP, and $l = T_n = a + (n - 1)d$ Substituting for l, we get

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

One can use either of the above formulae to find the sum to n terms of an AP.

6. Three consecutive terms in an AP will be $a - d$, a and $a + d$.
7. Four consecutive terms in an AP will be $a - 3d$, $a - d$, $a + d$ and $a + 3d$.
8. Five consecutive terms in an AP will be $a - 2d$, $a - d$, a , $a + d$ and $a + 2d$.
9. Sum of first n natural numbers = $\frac{n(n+1)}{2}$.
10. Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$.
11. Sum of cubes of first n natural numbers = $\left[\frac{n(n+1)}{2} \right]^2$.
12. If a, b and c are in an AP, then b is the arithmetic mean of the three numbers.



Important Learning: In an AP, the sum of the terms equidistant from the beginning and end is a constant and is equal to the sum of the first and the last terms.

Example 1

Which term of the sequence 3, 7, 11, 15, ... is the term 63?

- (A) 14
- (B) 15
- (C) 16
- (D) 17
- (E) 18

Solution

In the given AP

$$a_n = a + (n - 1)d = 63$$

We have $a = 3$, $d = 7 - 3 = 4$. Therefore,

$$3 + (n - 1) \times 4 = 63$$

$$(n - 1) \times 4 = 63 - 3 = 60$$

$$n - 1 = \frac{60}{4} = 15$$

$$n = 15 + 1 = 16$$

The correct answer is C.

Example 2

A man pays a rent of Rs. 50 for the first day, Rs. 100 for the second day and so on, with the rent on each day being Rs. 50 more than the rent on the previous day. What is the total rent paid for the first 10 days?

- (A) 2,750
- (B) 2,800
- (C) 3,050
- (D) 3,100
- (E) 3,350

Solution

The series is an AP with $a = 50$, $d = 50$ and $n = 10$

$$S_n = \frac{10}{2}[2 \times 50 + (10 - 1) \times 50]$$

$$S_n = 5(100 + 450)$$

$$S_n = 5(550)$$

$$S_n = \text{Rs. } 2,750$$

The correct answer is A.

Geometric Progression

GP refers to a series in which the ratio of a term to its previous term is constant. This ratio is called the common ratio of the GP and is denoted by r .

Some examples of GP are

1. 2, 4, 8, 16, 32, 64, ...
2. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \dots$
3. -2, 4, -8, 16, -32, 64, ...

Properties of GP

The first term is called a , the common ratio is r and the number of terms is denoted by n . Therefore, a GP would be like a, ar, ar^2, ar^3, ar^4 and so on.

Some important properties of a GP are listed below:

1. The n th term of a GP denoted by T_n is given by $T_n = ar^{(n-1)}$.
2. If a constant term (positive or negative) is multiplied with or divides each term of a GP, the series continues to be a GP.
3. Sum to n terms of a GP is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{\infty} = \frac{a}{1 - r}$$

4. Three consecutive terms in GP will be $\frac{a}{r}, a$ and ar .
5. Four consecutive terms in GP will be $\frac{a}{r^3}, \frac{a}{r}, ar$ and ar^3 .
6. If a, b and c are in GP, then

$$\begin{aligned}\frac{b}{a} &= \frac{c}{b} \\ b^2 &= ac \\ b &= \sqrt{ac}\end{aligned}$$

Then, b is called the geometric mean (GM) of a and c . GM of three numbers a, b and c is the cube root of the product of the three numbers.

Example 3

Find the sum to infinite terms of the series $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \dots$

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

(E) $\frac{1}{8}$

Solution

As per the problem, we have

$$a = \frac{1}{4} \text{ and } r = \frac{1}{4}$$

$$\text{Sum to infinite terms} = \frac{a}{1-r} = \frac{1/4}{(1-1/4)} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$$

The correct answer is B.

Harmonic Progression

A series of terms is said to be in Harmonic Progression (HP) if the reciprocal of the terms are in AP. As an example, if a, b and c are in HP, then $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ will be in AP. Therefore,

$$\begin{aligned}\frac{1}{b} - \frac{1}{a} &= \frac{1}{c} - \frac{1}{b} \\ \frac{2}{b} &= \frac{1}{a} + \frac{1}{c} = \frac{a+c}{ac} \\ b &= \frac{2ac}{(a+c)}\end{aligned}$$

where b is called the harmonic mean (HM) of a and c.

Example 4

If the second term of a harmonic progression is 5 and the 5th term of the same harmonic progression is 11, then find the 56th term.

- (A) $\frac{12}{99}$
- (B) $\frac{13}{99}$
- (C) $\frac{37}{99}$
- (D) $\frac{41}{99}$
- (E) $\frac{55}{99}$

Solution

The reciprocals of the HP form an arithmetic progression $a, a + d, a + 2d \dots$

$$\text{Then, } a + d = \frac{1}{5} \text{ and, } a + 4d = \frac{1}{11}$$

Solving above two equations, we get

$$a = \frac{13}{55}, d = \frac{-2}{55}$$

$$\text{Therefore, 56th term of AP} = a + 55d = \frac{13}{55} + 55 \times \frac{-2}{55} = \frac{99}{55}$$

$$\text{Hence, 56th term of HP} = \frac{55}{99}$$

The correct answer is E.

Relationship between the means of AP, GP and HP

If AM, GM and HM be the arithmetic, geometric and harmonic means between a and b, then the following results hold:

$$AM = \frac{a+b}{2} \quad (1)$$

$$GM = \sqrt{ab} \quad (2)$$

$$HM = \frac{2ab}{a+b} \quad (3)$$

Therefore, we can write:

$$AM \times HM = \frac{a+b}{2} \cdot \frac{2ab}{a+b} = ab = GM^2$$

Or

$$GM^2 = AM \times HM \quad (4)$$

Also, we have

$$\begin{aligned} AM - GM &= \frac{a+b}{2} - \sqrt{ab} = \frac{a+b+2\sqrt{ab}}{2} \\ &= \left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{2}} \right)^2 \end{aligned} \quad (5)$$

which is positive if a and b are positive; therefore, the AM of any two positive quantities is greater than their GM.

Also, from Eq. (4) we have,

$$GM^2 = AM \times HM$$

Clearly then, GM is a value that would fall between AM and HM and from Eq. (5) it is known that $AM > GM$, therefore we can conclude that $GM > HM$.

In other words, we can say that the arithmetic, geometric and harmonic means between any two +ve quantities are in descending order of magnitude.

Sum to n Terms of Special Series

1. Sum of the first n natural numbers:

$$\sum n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

2. Sum of the squares of first n natural numbers.

$$\sum n^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Sum of cubes of first n natural numbers:

$$\sum n^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

Example 5

Evaluate: $6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2$

- (A) 449
- (B) 450
- (C) 451
- (D) 452
- (E) 453

Solution

$$\text{Required Sum} = (1^2 + 2^2 + 3^2 + \dots + 11^2) - (1^2 + 2^2 + 3^2 + \dots + 5^2)$$

$$S = \frac{11(11+1)(2 \times 11+1)}{6} - \frac{5(5+1)(2 \times 5+1)}{6}$$

$$S = \frac{11 \times 12 \times 23}{6} - \frac{5 \times 6 \times 11}{6} = 451$$

The correct answer is C.

3 Permutation and Combination

Introduction

Permutation and combination are two of the most logical topics in mathematics, and their applications can be observed and verified in real-life situations.

Fundamental principle of counting

The concepts in permutation and combination are based on the fundamental principle of counting, which in turn comprises the rule of multiplication and the rule of addition. These are listed as follows:

1. Rule of multiplication: If a certain task can be done in m ways and after having done it, another can be done in n ways, then the total number of ways in which the two tasks can be done together is $m \times n$.

Thus, if there are three ways of going from A to B and four ways of going from B to C, then the number of ways of going to C from A via B = $3 \times 4 = 12$ ways.

2. Rule of addition: If a certain task can be done in m ways and another can be done in n ways, then either of the two tasks can be done in $m + n$ ways.

Thus, when tasks A and B both have to be completed, we multiply, but when either A or B has to be completed, we add.



Important Learning: Whenever the concept in the question is the same as 'and' we will use multiplication and whenever the concept is same as 'or' we will use addition.

Permutation

While a combination deals only with selection, permutation is selection and arrangement both, that is, in permutation, we are not only interested in selecting the things, but we also take into consideration the number of possible arrangements of the selected things.

The permutation of n different things taken r at a time is

$${}^n P_r = \frac{n!}{(n-r)!}$$

The concept of permutation is illustrated in the following example.

Example 1

Suppose there are four children in a group. Find the number of ways in which any two children out of the four can be arranged for a photograph.

- (A) 8 ways
- (B) 10 ways
- (C) 12 ways
- (D) 14 ways
- (E) 16 ways

Solution

We need to select two children out of the four, and then arrange them. So, we need to take the permutation of four different things, taken two at a time.

$${}^4P_2 = \frac{4!}{(4-2)!} = \frac{24}{2} = 12 \text{ ways}$$

The correct answer is C.

Permutation of N different things taken all at a time

The permutation of n different things taken all at a time is

$${}^n P_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

This is the result of selecting n things out of n given things, and then arranging them. But, if n things are being selected out of n things, then selection will be possible in only one way and so the result obtained, that is, $n!$ is entirely due to the arrangement of the selected things.

This leads to a very important result of permutation, that is, n different things can be arranged in a straight line in $n!$ ways.

For example, if four students are to be arranged in a straight line for a photograph, the total number of ways of doing so is $4! = 4 \times 3 \times 2 \times 1 = 24$ ways.

Permutation of identical things

This refers to the number of permutations of n things taken all at a time. Here, p things are alike of one kind, q of second kind, r of third kind and so on. The permutation is given by

$$\frac{n!}{p! \times q! \times r!}$$

that is, divide by the factorial of the number of identical things.

The concept is illustrated by means of the following example.

Example 2

In how many different ways can the letters of the word ARRANGE be arranged?

- (A) 1,200 ways
- (B) 1,240 ways
- (C) 1,260 ways
- (D) 1,300 ways
- (E) 1,340 ways

Solution

$$\text{Total number of ways} = \frac{7!}{2! \times 2!} = 1,260 \text{ ways}$$

The correct answer is C.

Circular permutation

The total number of ways in which n different things can be arranged in a circle = $(n - 1)!$ ways.

For example, if three people are to be seated on a circular table for dinner, the number of ways of doing so is $(3 - 1)! = 2$ ways.



Important Learning: If there are n different things, they can be arranged in a circle in $(n - 1)!$ different ways.

Distribution

Case 1

Different things to be distributed to different groups.

Example 3

In how many ways can five different balls be distributed in three different boxes?

- (A) 143 ways
- (B) 243 ways
- (C) 343 ways
- (D) 443 ways
- (E) 543 ways

Solution

The first ball can go into any of the three boxes, and therefore, the first ball can be distributed in three ways. Having done this, the second ball can be distributed in three ways and so on.

Therefore, the five balls can be distributed in $3 \times 3 \times 3 \times 3 \times 3 = 3^5$ ways or 243 ways.

The correct answer is B.

Case 2

Identical things to be distributed to different groups.

Example 4

In how many ways can five identical balls be distributed in three different boxes?

- (A) 15 ways
- (B) 18 ways
- (C) 20 ways
- (D) 21 ways
- (E) 24 ways

Solution

Five identical balls have to be distributed in three different boxes. If space has to be divided into three boxes, we can do so by using two partitions.

Now, one of the possible arrangements will be $\frac{\text{bbb}}{\frac{\text{bb}}{0}}$, that is, three balls in the first box, two balls in the second box and no balls in the third box.

Other possible arrangements can be:

1. $\frac{\text{bb}}{\frac{\text{bbb}}{0}}$
2. $\frac{\text{b}}{\frac{\text{bbb}}{\text{b}}}$
3. $\frac{\text{b}}{\frac{\text{bb}}{\text{bb}}}$
4. $\frac{\text{b}}{\frac{\text{b}}{\text{bbb}}}$ and so on

If you observe closely, we are arranging and rearranging seven things in a straight line, out of which five (balls) are identical of one type and two (partitions) are identical of another type.

Thus, required number of ways = $\frac{7!}{5! \times 2!} = 21$ ways

The correct answer is D.

Example 5

In how many different ways can 10 students of a class with roll numbers from 1 to 10 be seated in a straight line such that one of the extreme positions has a student with an odd roll number while the other extreme position has a student with an even roll number?

- (A) $48 \times 6!$
- (B) $44 \times 12!$
- (C) $50 \times 8!$
- (D) $50 \times 6!$
- (E) $52 \times 8!$

Solution

Let the first place have an odd number. This place can be filled in five ways (1, 3, 5, 7 and 9).

The other extreme position can be also filled in five ways (2, 4, 6, 8 and 10).

These two extreme positions can be interchanged also. Therefore, the two extreme positions can be first filled in

$$5 \text{ ways} \times 5 \text{ ways} \times 2 \text{ ways} = 50 \text{ ways}$$

Now, the remaining eight people can occupy eight available positions in $8!$ ways.

$$\text{Total ways} = 50 \times 8! \text{ ways}$$

The correct answer is C.

Example 6

Eight students were to be seated along two rows such that four students will be seated in each of the two rows called A and B. Two of the eight students definitely want to be seated in row A while one of them definitely wants to be seated in row B. In how many different ways can the eight students be seated?

- (A) 5,760
- (B) 5,960
- (C) 6,500
- (D) 6,760
- (E) 7,160

Solution

The two students who want to be in row A can be seated in $4 \times 3 = 12$ ways

The student who wants to be in row B can be seated in four ways

The remaining five people can be seated in $5!$ ways = 120 ways

Therefore, total number of ways = $12 \times 4 \times 120 = 5,760$ ways

The correct answer is A.

Combination

Combination means selection only, that is, in combination, we are only interested in the selection of things and not in their arrangement.

In general, the number of combinations of n different things taken r at a time is given by nC_r , where

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

where $n!$ (read as n factorial) = $n \times (n - 1) \times (n - 2) \times \dots \times 4 \times 3 \times 2 \times 1$

Thus,

$$0! = 1 \text{ (by definition)}$$

$$1! = 1$$

$$2! = 2 \times 1$$

$$3! = 3 \times 2 \times 1 \text{ and so on}$$

Let us try to understand the concept of combination by taking an example.

Example 7

There are four fruits, an apple, a mango, a banana and an orange, and we need to select any two fruits out of these four fruits. In how many ways can we do this?

- (A) 4 ways
- (B) 5 ways
- (C) 6 ways
- (D) 7 ways
- (E) 8 ways

Solution

Number of ways of selecting two fruits from four fruits is 4C_2 ,
that is,

$$\frac{4!}{2! \times (4-2)!} = \frac{24}{4} = 6$$

That is, six different ways, which will be AM, AB, AO, MB, MO
and BO.

The correct answer is C.

Rules of combination

1. $nC_0 = 1$ way (there is only one way to select 0 things out of n different things).
2. $nC_n = 1$ way (there is only one way to select n things out of n different things).
3. $nC_1 = n$ ways (there are n ways to select one thing out of n different things).
4. $nC_r = nC_{n-r}$ (the number of ways of selecting r things out of n different things is the same as identifying those $(n - r)$ things that will not be selected).

Combination of N different things taken 0 or some or all at a time

The number of combinations of n different things taken 0 or some or all at a time is:

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$

The above expression is the binomial expansion of $(1 + 1)^n = 2^n$

Therefore,

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n$$

Example 8

Eleven players are to be selected for a match out of an available list of 14 players. In how many ways can this be done such that the best two identified players are always selected?

- (A) ${}^{14}C_{11}$
- (B) ${}^{12}P_9$
- (C) ${}^{12}C_9$
- (D) ${}^{12}C_9 \times 2!$
- (E) ${}^{12}C_{11}$

Solution

If two people have to be definitely selected, then nine people have to be selected of the remaining 12 people = $^{12}C_9$.

The correct answer is C.

4 Probability

Introduction

Probability is defined as the chance of happening of an event and is a measure of the likelihood that an event will occur. It is used to quantify an attitude of mind towards some proposition of whose truth we are not certain. The certainty we adopt can be described in terms of a numerical measure and this number, between 0 and 1 (where 0 indicates impossibility and 1 indicates certainty), is called probability.

Thus, the higher the probability of an event, the more certain we are that the event will occur. A simple example would be the toss of a fair coin. As the two outcomes are deemed equiprobable, the probability of ‘heads’ equals the probability of ‘tails’, and each probability is $\frac{1}{2}$ or equivalently a 50% chance of either ‘heads’ or ‘tails’.

In many ways, the concept of probability can be said to be an extension of the concepts of permutation and combination.

In examples and problems where the event is described, the basis of solving the problem happens to be the classical definition of probability, which says

$$\text{Probability of an event } E, \text{ denoted by } P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

where the favourable number of outcomes or cases is as defined in the problem under the given set of conditions and the total number of outcomes or cases is the number of ways of doing the task without any condition being applicable.

Example 1

What is the probability that the month of May will have five Tuesdays?

(A) $\frac{2}{3}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{3}{7}$

(E) $\frac{3}{8}$

Solution

The month of May will have 31 days which would get converted into 28 days (4 weeks) and 3 odd days.

Therefore, each of the 7 days will definitely appear 4 times.
The remaining 3 days can be

1. Mon, Tue, Wed
2. Tue, Wed, Thu
3. Wed, Thu, Fri
4. Thu, Fri, Sat
5. Fri, Sat, Sun
6. Sat, Sun, Mon
7. Sun, Mon, Tue

Total number of cases = 7.

Number of favourable cases = 3 (There are the three cases in which Tuesday appears.)

$$\text{Required probability} = \frac{3}{7}.$$

The correct answer is D.

Elements of probability

1. Scope: Probability is always defined for the future.
2. Random experiment: A random experiment is an experiment, trial or observation that can be repeated numerous times under the same conditions. The outcome of an individual random experiment must be independent and identically distributed. It must in no way be affected by any previous outcome and cannot be predicted with certainty.
3. Sample space: The total number of ways in which an event can happen is called the sample space of the event.
 - Sample space for toss of coins:
 - (a) Tossing a coin: two elements (H or T)
 - (b) Tossing two coins: two elements × two elements = four elements (HT, TH, TT, HH)
 - (c) Tossing three coins: two elements × two elements × two elements = eight elements
$$(\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \text{TTT}, \text{TTH}, \text{THT}, \text{HTT})$$
 - Sample space for throwing one or more dice:
 - (a) Throw a dice: six elements (1, 2, 3, 4, 5, 6)
 - (b) Throw two dice: 36 elements [(1, 1), (1, 2), (1, 3)...(1, 6), (2, 1), (2, 2)...(2, 6) and so on till (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)]
4. Range of probability of any event: As the number of favourable outcomes cannot be greater than the total number of outcomes, therefore $0 \leq P(E) \leq 1$, that is, probability of any event will always lie between 0 and 1.



Important Learning: The range of probability of any event will always lie between 0 and 1, both inclusive.

5. Any event will either happen or not happen: $P(E) + P(\sim E) = 1$, that is, sum of the probability of happening of an event and the probability of its not happening = 1.
6. Odds for and against an event: It is not necessary that a problem will always provide probabilities. There are examples where odds in favour of an event or odds against an event are provided.

$$\text{Odds in favour of an event} = \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

$$\text{Odds against an event} = \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

7. Mutually exclusive events: Any two events are said to be mutually exclusive events if they cannot occur together.

For two mutually exclusive events A and B

$$P(A \text{ or } B) = P(A) + P(B)$$

and if the events are not mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

In these expressions, $P(A \text{ or } B)$ represents the probability of either event A happening or event B happening.

8. Independent events: Any two events where the probability of happening of one event is not dependent on the other are said to be independent events.

For two independent events A and B,

$$P(A \text{ and } B) = P(A) \times P(B)$$

The above result will be true for any number of independent events.

For three independent events A, B and C

$$P(A, B \text{ and } C) = P(A) \times P(B) \times P(C)$$

9. Pack of cards: Problems related to a pack of cards are frequently asked as a part of questions on probability. In case of a pack of cards, remember the following:
- A pack has 52 cards (26 black and 26 red), which get divided into 13 cards each of four different types or variety.
 - These 13 cards are (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q and K) and the four types are Hearts, Diamond, Spade and Club. Technically, these four types are called suits. So, in a normal pack of cards, there will be four aces, four ones, four twos and so on, thus, making a total of 52 cards.
 - King, Queen and Jack are also popularly known as face cards. A pack of cards will, therefore, have 12 face cards.

Example 2

What is the probability that a card drawn at random from a pack of cards is either black or a jack?

(A) $\frac{17}{52}$

(B) $\frac{7}{13}$

(C) $\frac{27}{52}$

(D) $\frac{3}{13}$

(E) $\frac{5}{9}$

Solution

There are a total of $(13 + 13) = 26$ black cards and 4 jacks in a pack of cards. But remember that two of these jacks are also black cards, so we will have to account for this overlap.

Therefore, number of favourable cards = $13 + 13 + 4 - 2 = 28$.

Total cards = 52.

$$\text{Required probability} = \frac{28}{52} = \frac{7}{13}.$$

The correct answer is B.

10. AND rule: Suppose the probability of event A is $P(A)$ and that of event B is $P(B)$. Now, if these two events are to occur together, i.e. if we want to calculate the probability of occurrence of event A and that of event B, we need to multiply the individual probabilities of A and B.

Therefore, the probability of occurrence of A and B will be $P(A) \times P(B)$.

11. OR rule: When the probabilities of two events are given and we need to calculate the probability that at least one of them occurs, we need to add the individual probabilities of the two events. Therefore, if the probability of event A is $P(A)$ and that of event B is $P(B)$, then the probability that at least one of the two events occurs is $P(A) + P(B) - P(A \text{ and } B)$.
12. Conditional Probability: When A and B are dependent events, then $P(A/B)$ denotes probability of happening of event A when B has already occurred. Similarly, $P(B/A)$ denotes probability of happening of B when A has already occurred.

Example 3

Two students are selected from a class of 5 girls and 12 boys. Find the probability that a particular pair of girl and boy is selected.

(A) $\frac{1}{136}$

(B) $\frac{1}{63}$

(C) $\frac{1}{51}$

(D) $\frac{10}{21}$

(E) $\frac{13}{61}$

Solution

$$\text{Total number of possibilities} = {}^{17}C_2 = \frac{17 \times 16}{1 \times 2} = 17 \times 8 = 136$$

Number of favourable cases = 1

$$\text{Therefore, the required probability} = \frac{1}{136}.$$

The correct answer is A.

Example 4

A bag contains five red and nine black balls, while another has three red and five black balls. A ball is first selected at random from bag 1 and transferred to bag 2, and then a ball is drawn from bag 2. What is the probability that the ball drawn is red?

(A) $\frac{25}{103}$

(B) $\frac{47}{126}$

(C) $\frac{33}{97}$

(D) $\frac{47}{97}$

(E) $\frac{61}{131}$

Solution

We need to consider both the cases here.

Red ball is transferred, and then red ball is drawn.

Black ball is transferred, and then red ball is drawn.

$$= \frac{5}{14} \times \frac{4}{9} + \frac{9}{14} \times \frac{3}{9} = \frac{20}{126} + \frac{27}{126} = \frac{47}{126}$$

The correct answer is B.

Example 5

Three cards are drawn from a pack of cards at random. Find the probability that they consist of both colours.

(A) $\frac{^{26}C_1 \times ^{26}C_2}{^{52}C_3}$

(B) $\frac{2 \times ^{26}C_1 \times ^{26}C_2}{^{52}C_3}$

(C) $\frac{^{26}C_3}{^{52}C_3}$

(D) $\frac{2 \times ^{26}C_1 \times ^{26}C_2}{^{52}C_3}$

(E) None of these

Solution

Number of favourable cases = 2 red and 1 black or 1 red and 2 black

$$= {}^{26}C_2 \times {}^{26}C_1 + {}^{26}C_1 \times {}^{26}C_2 = 2 \times {}^{26}C_2 \times {}^{26}C_1$$

$$\text{Total cases} = {}^{52}C_3$$

$$\text{Therefore, required probability} = \frac{2 \times {}^{26}C_2 \times {}^{26}C_1}{{}^{52}C_3}$$

The correct answer is B.

Bayes' Theorem

Bayes' theorem is a direct application of conditional probabilities. This theorem is used to find the conditional probability of an event $P(A/B)$, say, when the “reverse” conditional probability $P(B/A)$ is known.

Let A_1, A_2, \dots, A_n be a set of mutually exclusive events that together form the sample space S . Let B be any event from the same sample space, such that $P(B) > 0$. Then,

$$P(A_k / B) = \frac{P(A_k \cap B)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \dots + P(A_n)P(B/A_n)}$$

We know that $P(A_k \cap B) = P(A_k)P(B/A_k)$, so, Baye's theorem can also be expressed as

$$P(A_k / B) = \frac{P(A_k)P(B/A_k)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \dots + P(A_n)P(B/A_n)}$$

Example 6

Three companies P, Q and R supply 20%, 30% and 50% of the books to a college. Past experience shows that 2%, 4% and 5% of the books produced by these companies are defective. If a book was found to be defective, what is the probability that the book was supplied by company Q?

- (A) $\frac{8}{77}$
- (B) $\frac{10}{77}$
- (C) $\frac{12}{77}$
- (D) $\frac{17}{77}$
- (E) $\frac{19}{77}$

Solution

Let us consider that P, Q and R be the events that books are supplied by companies P, Q and R respectively.

Suppose D be the event that books are defective.

According to the question,

$$P(P) = 0.2, P(Q) = 0.3 \text{ and } P(R) = 0.5$$

$$\text{Also, } P(D/P) = 0.02, P(D/Q) = 0.04, P(D/R) = 0.05$$

$$\begin{aligned}\text{Required probability, } P(Q / D) &= \frac{P(Q)P(D / Q)}{P(P)P(D / P) + P(Q)P(D / Q) + P(R)} \\ &= \frac{0.3 \times 0.04}{0.2 \times 0.02 + 0.3 \times 0.04 + 0.5 \times 0.05} \\ &= \frac{0.012}{0.04 + 0.012 + 0.025} = \frac{0.012}{0.077} = \frac{12}{77}\end{aligned}$$

The correct answer is C.

2.11 DI-Caselets and Tables

2.12 What is Measured?

The DI-Caselets and Tables section consists of caselets and tables that you will need to analyse and apply while answering the questions.

Your mental math skills get tested indirectly as you will need to make a lot of quick calculations especially using concepts of percentages, ratios, etc.

Your estimation skills also get tested as, in a lot of the questions, you will be able to arrive at the answer just by intelligent estimation and elimination of wrong answer choices.

2.13 Overall Test Taking Strategies

1. Scan the data once quickly but do not pay too much attention to every small detail as you can always do so later, based on the questions that you get.
2. The figures are all drawn to scale so you can estimate the answer, especially if the answer choices are very far apart.
3. Do not confuse percentages with actual numbers.

The next few pages will provide you with in-depth strategies for approaching this topic.

Introduction

Data Interpretation is an important area in testing the aptitude of a candidate. The primary objective in Data Interpretation (henceforth referred to as DI) is to:

1. Assess the capability of data assimilation, that is, to understand a data in its given form.
2. Carry out calculations based on the given data.
3. Take effective business decisions based on the calculations.

The data could be in the form of a caselet, bar graph, line graph, pie chart, histogram, frequency polygon, etc.

Concepts in data interpretation

Growth and growth rate

Growth in essence is the change in the value of a variable from one point in time to another point in time. Although it is generally associated with an increase in the value of a variable, mathematically, the value of growth can be negative.

Since, growth and growth rate are associated also with the change in a variable with respect to time, both of them are usually calculated on the immediate previous value until and unless mentioned otherwise. While growth is the change in the absolute value of a variable, growth rate is the percentage change in the value of a variable. In order to understand the concept of growth and growth rate, let us look at an example:

Example 1

The following table gives the value of the total sales of AR Associates across four years. The sales value is in Rs. ('000).

Year	2012	2013	2014	2015
Sale	150	200	240	300

- (A) Find the growth in the sales of AR Associates in the year 2014.
- (B) Find the growth rate of sales of AR Associates in the year 2015.
- (C) Find the growth rate of sales of AR Associates in the period 2012–2015.
- (D) Find the growth rate of sales of AR Associates in the year 2012.
- (E) If the growth rate of sales of AR Associates in the year 2012 is 25%, find the value of the sales of AR Associates in the year 2011.

Solution

- (A) The growth in the sales of AR Associates in the year 2014

$$240,000 - 200,000 = \text{Rs. } 40,000$$

- (B) Growth rate of sales in the year 2015

$$\frac{(300,000 - 240,000)}{240,000} \times 100 = \frac{6}{24} \times 100 = 25\%$$

- (C) Growth rate of sales in the period 2012–2015

$$\frac{(300,000 - 150,000)}{150,000} \times 100 = 100\%$$

- (D) If we want to find out the growth rate of sales in the year 2012, we need to find the value of sales in the immediately preceding year, that is, 2011.

Since this data is not provided, the answer to the question cannot be determined.

- (E) Let y be the sales of AR Associates in the year 2011.
Therefore,

$$\begin{aligned}y \times \frac{125}{100} &= 150,000 \\y \times \frac{5}{4} &= 150,000 \\y &= \text{Rs. } 120,000\end{aligned}$$

Percentage change in growth rate

Many of us do not differentiate between the calculation of percentage change in growth and growth rate. Actually, these are two different concepts.

Percentage change in growth rate gives the percentage change between two values that are expressed in percentage terms, that is, percentage change in growth rate gives the percentage change in two values that are growth rates.

To understand the concept better, consider Example 1 again.

Example 2

Find the percentage change in the growth rate of AR associates in the year 2014.

- (A) Decrease of 20%
- (B) Increase of 20%
- (C) Decrease of 40%
- (D) Increase of 40%
- (E) Increase of 50%

Solution

In order to answer this question, we would need the value of the growth rate of sales of AR Associates in the year 2013 and the year 2014.

Growth rate of sales of AR Associates in the year 2013

$$\frac{(200,000 - 150,000)}{150,000} \times 100 = \frac{100}{3} = 33.33\%$$

Growth rate of sales of AR Associates in the year 2014

$$\frac{(240,000 - 200,000)}{200,000} \times 100 = \frac{100}{5} = 20\%$$

Therefore, percentage change in growth rate

$$\frac{(20 - 33.33)}{33.33} \times 100 = -40\%$$

That is, a decrease of 40%.

The correct answer is C.

Percentage points

Percentage point is the difference between two values that are expressed in percentage terms. Consider the following example.

Example 3

A girl got 55% marks in the fifth semester and 57.5% marks in the sixth semester. By how many percentage points are the marks in the sixth semester more than the marks obtained in the fifth semester?

- (A) 1.5 percentage points.
- (B) 2.5 percentage points.
- (C) 3.5 percentage points.
- (D) 4.5 percentage points.
- (E) 5.5 percentage points.

Solution

Required value = 57.5% – 55% = 2.5 percentage points.

The correct answer is B.

Market share

Market share is the total percentage of the market serviced by a company, product or brand. Market share can be calculated either on volume basis (number of units sold) or value basis (revenue or sales or turnover). Also average product price is equal to the total market value divided by the total market volume. Consider the following example.

Example 4

In a market there are three products being sold. The price along with the total sale in units is given for each of the three products.

Product	Price/Unit	Sale in Numbers	Sale in Rs.
P	Rs. 5	10,000	50,000
Q	Rs. 4	15,000	60,000
R	Rs. 4.5	20,000	90,000

- (A) Find the market share of product P on volume basis.
- (B) Find the market share of product Q on value basis.
- (C) Find the average market price.

Solution

(A) Total market volume = 45,000 units.

Total sale of product P = 10,000 units.

$$\text{Market share of P on volume basis} = \frac{10,000}{45,000} \times 100 = \frac{200}{9} = 22.22\%$$

(B) Total market value = Rs. 50,000 + Rs. 60,000 + Rs. 90,000 = Rs. 200,000

Sales of product Q = Rs. 60,000

$$\text{Market share of product Q on value basis} = \frac{60,000}{200,000} \times 100 = 30\%$$

(C) Average market price is given by

$$\frac{\text{Total market value}}{\text{Total market volume}} = \frac{200,000}{45,000} = \frac{200}{45} = \frac{40}{9} = \text{Rs. } 4.44/\text{unit}$$

1 DI-Caselets

A DI-Caselet is a set of information that is given in the paragraph form. In a caselet, no graphs or tables are given. You need to read the given information and organise the given data in tabular or pictorial form to solve the questions.

Data for Examples 1–3: A person was looking at the performance data of four companies namely A, B, C and D for the year 2014. He observed that the sale of company A for the year 2014 was twice the expenses for company D in the same year. The profit for company C in the year 2014 was 25% while its sale was Rs. 300 crore. The ratio of the sales of company B and the expenses of company D was 4 : 5. Also, the expenses of company B were half of the total expenses of company D in the same year.

It was also observed that the sale of company D was Rs. 250 crore while its profit for the year was Rs. 150 crore. Also, the expenses of company A in the year 2014 were such that its profit was 50%.

(For the questions based on above data, consider, Profit = Sale – Expenses)

Example 1

What is the profit percentage of company B in the year 2014?

- (A) 35%
- (B) 40%
- (C) 50%
- (D) 55%
- (E) 60%

Example 2

Find the value of profit of company A in the year 2014.

- (A) Rs. 33.33 crore
- (B) Rs. 45 crore
- (C) Rs. 50 crore
- (D) Rs. 55 crore
- (E) Rs. 66.66 crore

Example 3

Which company had the highest profit percentage in the year 2014?

- (A) A
- (B) B
- (C) C
- (D) D
- (E) Both A and D

General Solution for Examples 1–3:

For company C:

Sales of company C = Rs. 300 crore and profit = 25%.

$$\text{Therefore, Expenses} \times \left(\frac{125}{100} \right) = 300$$

Expenses = Rs. 240 crore.

For company D:

Sales = Rs. 250 crore

Profit = Rs. 150 crore

Expenses = $250 - 150 = \text{Rs. } 100$ crore.

For company A:

Sales of company A in 2014 = $2 \times 100 = \text{Rs. } 200$ crore.

Profit of company A = 50%

$$\text{Therefore, Expenses} \times \left(\frac{150}{100} \right) = 200$$

Expenses = Rs. $\frac{400}{3}$ = Rs. 133,33 crore.

For company B:

Ratio of the sales of company B to the expenses of company D = 4 : 5

$$\text{Therefore, sales of company B} \times \frac{B}{100} = \frac{4}{5}$$

Sales of company B in 2014 = Rs. 80 crore

Expenses of company B in 2014 = Rs. 50 crore

Therefore, profit of company B in 2014 = Rs. 30 crore

Thus overall performance of companies can be tabulated as:

Company	Sales (in crore)	Expenses (in crore)	Profit (in crore)	Profit (%)
A	200	133.33	66.66	50
B	80	50	30	60
C	300	240	60	25
D	250	100	150	150

Now, all the questions can be solved easily:

1. (E)
2. (E)
3. (D)

2 DI-Tables

A DI-Table is a set of data arranged in rows and columns. It is one of the most common ways of putting information across to people. A table consists of several boxes with information inside. The first row and the first column are generally used to denote the titles.

Data for Examples 1–3: The following table gives the sales of four companies in lakhs across four years from 2004 to 2007. Read the data carefully and answer the questions that follow.

Company	2004	2005	2006	2007
A	120	130	145	160
B	150	166	160	190
C	170	185	190	200
D	110	125	140	150

Example 1

By what percentage are the sales of company B in the year 2005 more than that of company A in the year 2004?

- (A) 20%
- (B) 33.33%
- (C) 38.33%
- (D) 40%
- (E) 46%

Solution

$$\text{Required value} = \left[\frac{166 - 120}{120} \right] \times 100 = 46 \times \frac{5}{6} = \frac{230}{6} = 38.33\%$$

The correct answer is C.

Example 2

By how much value is the average sale of company C more than the average sale of company D during the period?

- (A) 35 lakhs
- (B) 38 lakhs
- (C) 45 lakhs
- (D) 55 lakhs
- (E) None of these

Solution

$$\text{Average sales of company C} = 170 + 185 + 190 + 200 = \frac{745}{4}$$

$$\text{Average sales of company D} = 110 + 125 + 140 + 150 = \frac{525}{4}$$

$$\text{Difference} = \frac{745}{4} - \frac{525}{4} = \frac{220}{4} = 55 \text{ lakh}$$

The correct answer is D.

Example 3

If the four companies account for a 40% market share by value in 2007, what was the total sale of the market?

- (A) 14 crore
- (B) 16 crore
- (C) 17.5 crore
- (D) 18 crore
- (E) 20 crore

Solution

Total sales of the four companies in 2007 = $160 + 190 + 200 + 150 = \text{Rs. } 700 \text{ lakh}$

This is 40% of the total sales. Therefore,

$$\text{Total sales} = 700 \times 2.5 \text{ lakh} = \text{Rs. } 17.5 \text{ crores}$$

The correct answer is C.

Data for Examples 4–6: Two thousand students applied for admission to various programmes at AU University. Out of the total applicants, 20% did not take the admission test. The following table gives the cumulative frequency in percentage of the mark range received by those students who appeared for the admission test.

Marks	Cumulative Frequency (%)
≤ 10 marks	15
≤ 20 marks	25
≤ 30 marks	40
≤ 40 marks	60
≤ 50 marks	85
≤ 60 marks	100

Example 4

What is the number of students who received marks in the range of 21–30 in the admission test?

- (A) 240
- (B) 400
- (C) 500
- (D) 640
- (E) None of these

Solution

As per the problem, 2,000 students applied out of which 20% did not appear for the test.

Number of students who appeared = 80% of 2,000 = 1,600

Percentage of students in the range of 21–30 marks = 40 – 25 = 15%

Therefore, 15% of 1,600 = 240 students

The correct answer is A.

Example 5

If more than 40 marks are required to qualify for the next round, find the difference between the number of students who qualified for the next round and those who failed to qualify for the next round.

- (A) 160
- (B) 240
- (C) 320
- (D) 380
- (E) 420

Solution

Percentage of students who qualified for the next round =
40%

Percentage of students who did not qualify for the next round
= 60%

Difference between the two = 20% of 1600 = 320 students

The correct answer is C.

Example 6

By what percentage is the number of students in the range of 41–50 marks more than those in the range of 0–10 marks?

- (A) 10%
- (B) 20%
- (C) 33.33%
- (D) 55.55%
- (E) 66.66%

Solution

Percentage of students in the range of 41–50 marks = 25%

Percentage of students in the range of 0–10 marks = 15%

Therefore,

$$\text{Required percentage} = \left(\frac{25 - 15}{15} \right) \times 100 = 66.66\%$$

The correct answer is E.



Important Learning: The knowledge of fractional equivalents of various percentages, such as 12.5% is the same as $\frac{1}{8}$, will be very important while attempting data interpretation questions.

Data for Examples 7–9: Go through the following information and solve the questions based on them.

Current structure	
Equity	28%
Debt	22%
Cash	4%
Real Estate	46%

Proposed structure	
Equity	70%
Debt	25%
Cash	5%

Example 7

If the real estate property of Alok is currently valued at Rs. 69 lakh, what will be the difference in lakhs between equity and debt in the proposed structure?

- (A) Rs. 150 lakh
- (B) Rs. 9 lakh
- (C) Rs. 82.5 lakh
- (D) Rs. 67.5 lakh
- (E) Rs. 70.5 lakh

Solution

Given that:

46% of the total = Rs. 69 lakh

Total = Rs. 150 lakh

Difference between equity and debt = $70 - 25 = 45\%$ of the total = $0.45 \times 150 = 4.5 \times 15$

= Rs. 67.5 lakh

The correct answer is D.

Example 8

If the difference between the contribution of equity to that of all the other components put together in the proposed structure is Rs. 16 lakh, find the value of cash in the existing structure.

- (A) Rs. 1.6 lakh
- (B) Rs. 2 lakh
- (C) Rs. 2.6 lakh
- (D) Rs. 4 lakh
- (E) Cannot be determined

Solution

Given that:

40% of the total = Rs. 16 lakh

Total = Rs. 40 lakh

Cash in the existing structure = 4% of 40 lakh = Rs. 1.6 lakh

The correct answer is A.

Example 9

By what percentage is the contribution of equity in the proposed structure more as compared to the contribution of equity in the existing structure?

- (A) 42%
- (B) 100%
- (C) 120%
- (D) 150%
- (E) 200%

Solution

$$\text{The required value} = \frac{70 - 28}{28} \times 100 = \frac{42}{28} \times 100 = 150\%$$

The correct answer is D.

2.14 DI-Graphs and Charts

2.15 What is Measured?

The DI-Graphs and Charts section consists of charts and pie diagrams that you will need to analyse and apply while answering the questions.

Your mental math skills get tested indirectly as you will need to make a lot of quick calculations especially using concepts of percentages, ratios, etc.

Your estimation skills also get tested as, in a lot of the questions, you will be able to arrive at the answer just by intelligent estimation and elimination of wrong answer choices.

2.16 Overall Test Taking Strategies

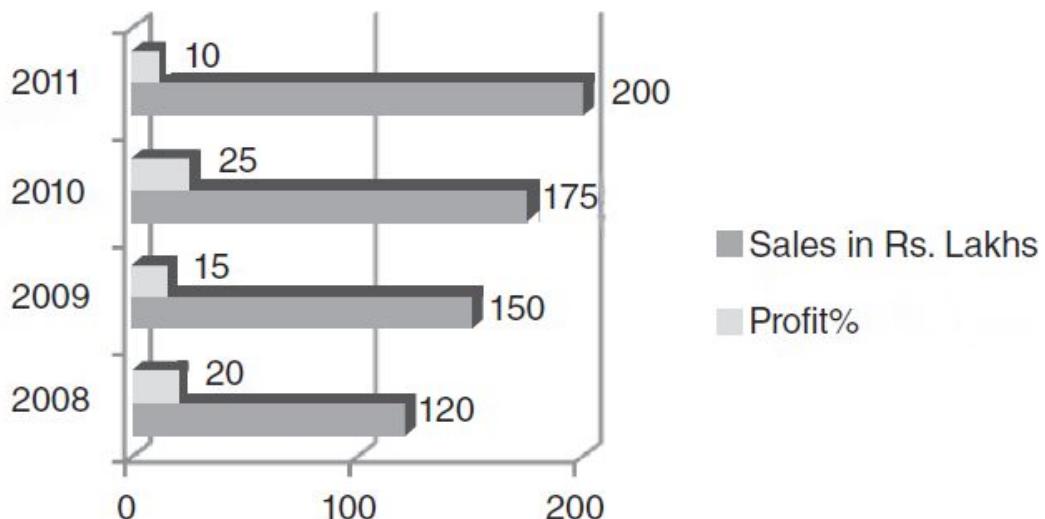
1. Scan the data once quickly but do not pay too much attention to every small detail as you can always do so later, based on the questions that you get.
2. The figures are all drawn to scale so you can estimate the answer, especially if the answer choices are very far apart.
3. Do not confuse percentages with actual numbers.

The next few pages will provide you with in-depth strategies for approaching this topic.

1 Bar Graph

A bar graph is a chart whose main purpose is to compare two or more categories. This comparison is done on the basis of quantitative value associated with each category. Horizontal or vertical bars are used to show comparisons among categories.

Data for Examples 1–3: The given bar graph shows the total sales value (in Rs. lakh) and the profit percentage for a company ABC Ltd. for 4 years from 2008 to 2011. Answer the questions based on the following data.



Example 1

If the total sales is equivalent to the selling price, what is the value of profit for ABC Ltd. for the year 2010?

- (A) Rs. 35 lakh
- (B) Rs. 38.75 lakh
- (C) Rs. 40 lakh
- (D) Rs. 45 lakh
- (E) None of these

Solution

We need to find out the profit for the year 2010. Therefore,

$$\text{CP} \times \frac{125}{100} = 175 \Rightarrow \text{CP} \times \frac{5}{4} = 175 \Rightarrow \text{CP} = 4 \times 35 = 140$$
$$\text{Profit} = 175 - 140 = \text{Rs. } 35 \text{ lakh}$$

The correct answer is A.

Example 2

Which year has shown the highest percentage change in profit %?

- (A) 2008
- (B) 2009
- (C) 2010
- (D) 2011
- (E) Cannot be determined

Solution

Percentage change in profits for 2008 cannot be calculated since we do not know the profits for 2007.

$$\text{Percentage change in profits for 2009} = \frac{(20-15)}{20} \times 100 = 25\%$$

$$\begin{aligned}\text{Percentage change in profits for 2010} &= \\ \frac{(25-15)}{15} \times 100 &= 66\% \text{ approx}\end{aligned}$$

$$\text{Percentage change in profits for 2011} = \frac{(25-10)}{25} \times 100 = 60\%$$

Thus, the maximum change has been for the year 2010.

The correct answer is C.

Example 3

By what percentage is the sales in 2011 more or less than the cost in 2008?

- (A) 50%
- (B) 100%
- (C) 200%
- (D) 250%
- (E) 300%

Solution

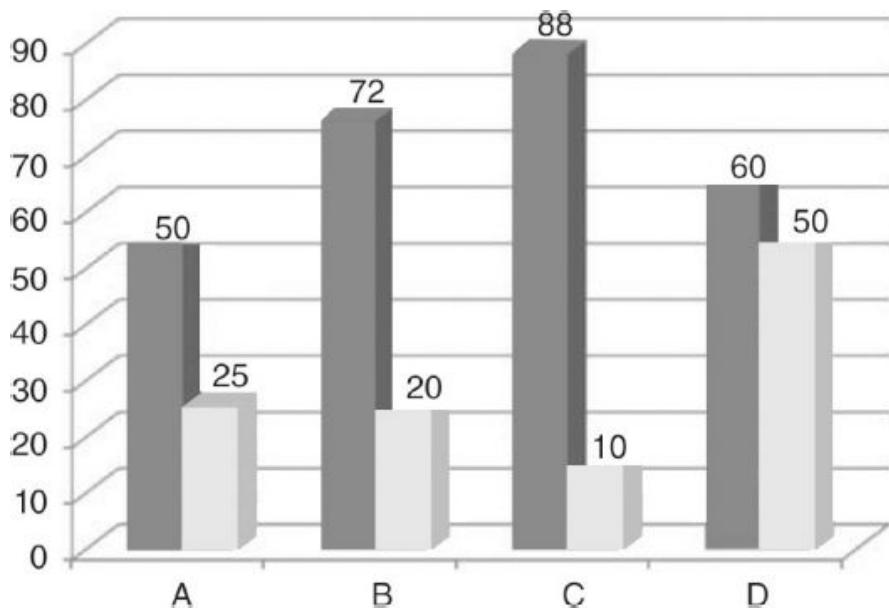
Cost in 2008 = Rs. 100 lakhs

Sales in 2011 = Rs. 200 lakhs

$$\text{Percentage change} = \left(\frac{200 - 100}{100} \right) \times 100 = 100\%$$

The correct answer is B.

Data for Examples 4–9: The following bar graph depicts the details of the number of students in four different sections of grade 1 in Ann Mary School. While the first bar represents the number of students in that section in the current year, the second bar represents the percentage change in the number of students in that section from the previous year.



Further, it was noted that no two sections had the same number of students in any of the two years. The number of sections that saw an increase in the number of students was the same as the number of sections where the total strength decreased.

Example 4

What was the total strength of Section B in the previous year?

- (A) 60
- (B) 90
- (C) 80
- (D) 70
- (E) Cannot be determined

Example 5

In the previous year, what was the ratio of the number of students in Section C to those in Section D?

- (A) 1 : 3
- (B) 3 : 1
- (C) 2 : 3
- (D) 3 : 5
- (E) None of these

Example 6

Which among the following sections saw the least change in the number of students from the previous year?

- (A) A
- (B) B
- (C) C
- (D) D
- (E) A and C

Example 7

If 50% of the total decrease is attributed to students shifting to other schools while the entire increase is due to new admissions, what is the difference between the number of students shifting to other schools and new admissions?

- (A) 18
- (B) 21
- (C) 26
- (D) 39
- (E) 60

Example 8

Which two sections saw a decrease in the number of students in the current year as compared to the previous year?

- (A) B and C
- (B) A and D
- (C) B and D
- (D) A and C
- (E) A and B

Example 9

Find the number of students in the previous year in Section D.

- (A) 40
- (B) 120
- (C) 60
- (D) 80
- (E) Cannot be determined

General Solution for Examples 4–9:

Let y be the variable which represents the number of students in each of the four sections in the previous year. Section C has 88 students and there is a percent change of 10% from the previous year. The only possibility is

$$y \times \frac{110}{100} = 88 \Rightarrow y = 80 \text{ students}$$

For section A, the change has to be an increase of 25%.

Therefore,

$$y \times \frac{125}{100} = 50 \Rightarrow y = 40 \text{ students}$$

There will definitely be a decrease in the strength of the other two sections B and D.

For section B, we have

$$y \times \frac{80}{100} = 72 \Rightarrow y = 90 \text{ students}$$

For section D, we have

$$y \times \frac{50}{100} = 60 \Rightarrow y = 120 \text{ students}$$

Now, all the questions can be solved easily:

4. (B)

5. (C)

6. (C)

7. (B) Decrease in section B = $90 - 72 = 18$ students

Decrease in section D = $120 - 60 = 60$ students

Total decrease = 78 students

50 percent of this is due to students shifting to other schools = 39 students

New admissions = $8 + 10 = 18$ new admissions

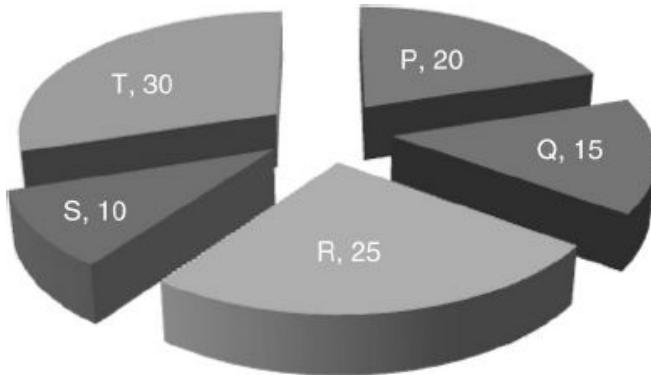
$$\text{Difference} = 39 - 18 = 21$$

8. (C)
9. (B)

2 Pie Chart

It is a type of graph in which we have a circle, which is divided into sectors each of which represents a proportion of the whole. We can also say a pie is divided into slices where length or area of each slice or sector represents the proportion of the total quantity it represents.

Data for Examples 1–3: The given pie chart shows the percentage market share by volume of five companies P, Q, R, S and T in the television market in the year 2011. The total sale volume-wise in the market in the year 2011 is 400,000 TV sets.



Example 1

If S is the others category and company X has a share of 30% in this category, find the number of TV sets sold by company X in the year 2011.

- (A) 10,000
- (B) 12,000
- (C) 15,000
- (D) 18,000
- (E) 20,000

Solution

Total sale of the others category = 10% of 400,000 = 40,000

Sales of company X = 30% of 40,000 = 12,000

The correct answer is B.

Example 2

If the total sale of two products P2 and P3 of company P is 30,000, find the percentage contribution of products P2 and P3 to the total sales unit-wise of company P in the year 2011.

- (A) 37.5%
- (B) 40%
- (C) 50%
- (D) 60%
- (E) 75%

Solution

P2 and P3 have together sold 30,000 units.

Company P has a total sale of 20% of 400,000 = 80,000

Therefore, contribution of P2 and P3

$$= \frac{30,000}{80,000} \times 100 = 3 \times \frac{100}{8} = 3 \times 12.5 = 37.5\%$$

The correct answer is A.

Example 3

By what percentage is the market share of R more than the market share of P?

- (A) 20%
- (B) 25%
- (C) 30%
- (D) 35%
- (E) 40%

Solution

The percentage by which the market share of R is more than that of P

$$= \left(\frac{25 - 20}{20} \right) \times 100 = 25\%$$

The correct answer is B.

2.17 Data Sufficiency

2.18 What is Measured?

Data sufficiency questions measure your ability to analyse data and decide whether it is enough to answer the given question. The actual answer to the question is irrelevant for our purpose.

A lot of the data sufficiency questions will test you on arithmetic concepts such as prime numbers, odd and even numbers, negative exponents and fractions, etc.

2.19 Overall Test Taking Strategies

1. Do not try to solve every statement. You only need to determine whether the given statement is sufficient to answer the given question.
2. Consider each statement separately, especially when you are looking at the second statement.
3. Read the question carefully. If the question asks you whether you can find the value of y and you realise that you cannot do so, you still have a definite answer, that is, no.
4. Remember the answer choices—(A), (B), (C), (D), (E)—and what each of them stands for.

The next few pages will provide you with in-depth strategies for approaching the topic.

1 Data Sufficiency

Introduction

The primary objective of data sufficiency questions is to find out whether the given data is sufficient to answer the question asked. In order to understand data sufficiency, we will need to look at the following aspects:

1. The structure of a data sufficiency problem.
2. The answer choices of a data sufficiency problem.
3. How to approach a data sufficiency problem.
4. The common errors in a data sufficiency problem.

Structure of a data sufficiency problem

In a data sufficiency problem, a question is asked which needs to be answered based on the given data. The given data is usually in the form of two statements called (1) and (2). The question asked needs to be answered based on the two given statements. For example:

- Question: What is the speed of the train?
- (1) The train covers 50 km in 30 minutes.
 - (2) The length of the train is 200 m.

Answer choices

The answer choices given in a data sufficiency problem are as follows:

- (A) Statement (1) ALONE is sufficient, but Statement (2) ALONE is not sufficient.
- (B) Statement (2) ALONE is sufficient, but Statement (1) ALONE is not sufficient.
- (C) BOTH statements (1) and (2) TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Note: Although the options will be the same in most of the cases, it is not necessary that they will always be the same. It is suggested that a student should always read the instructions about the options while answering the question.

Approach to a data sufficiency problem

The following steps are to be followed while solving a data sufficiency problem:

1. Check whether the question can be answered using statement 1 alone without using the data given in statement 2.
2. Check whether the question can be answered using statement 2 alone without using the data given in statement 1.
3. If neither of the two given statements can answer the question alone, combine the data given in the two statements to answer the question asked. In this case, the answer option of the question will be option (C).
4. If the question asked cannot be answered by using the given data, the answer option will be option (E).

Types of questions generally asked

Questions on data sufficiency are asked on the following three topics:

1. Number Properties Based
2. Arithmetic Based
3. Algebra and Probability Based

Common errors in data sufficiency problems

Some commonly made errors are listed as follows:

1. While checking whether the question asked can be answered by using any one of the two statements alone, you pick up data from another statement by mistake.
2. You need to combine the two statements only when you are sure that either of the two statements cannot answer the question alone.
3. If you get more than one answer, it is not acceptable in a data sufficiency problem. For example, if $x^2 = 49$, then x will have two values of +7 and -7. In a situation like this, we would say that the question cannot be answered.
4. Even a definite 'No' is an answer to a data sufficiency problem. For example, if the question asked is: 'Is 1 an example of a prime number?', then it has a definite answer which is 'No'.

These common errors in approach to data sufficiency problems are illustrated through the following examples.

Example 1

What is the value of x?

$$(1) \ x^2 - 5x + 6 = 0$$

Solution

If we solve for x , the quadratic equation will give the values as $x = 2$ or $x = 3$.

So the given problem has more than one answer. This is not acceptable in data sufficiency. We always need a unique solution. It is here that data sufficiency is different from quantitative skills.

Directions for Examples 2–10: Each data sufficiency problem consists of a question and two statements, labelled (1) and (2), which contain certain data. Using these data and your knowledge of mathematics and everyday facts (such as the number of days in July or the meaning of the word counterclockwise), decide whether the data given are sufficient for answering the question and then indicate one of the following answer choices:

- (A) Statement (1) ALONE is sufficient, but statement (2) ALONE is not sufficient.
- (B) Statement (2) ALONE is sufficient, but statement (1) ALONE is not sufficient.
- (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
- (D) EACH statement ALONE is sufficient.
- (E) Statements (1) and (2) TOGETHER are NOT sufficient.

Example 2

Can I fill up the tank of 100 L capacity completely?
(Measurements are accurate and there should be no overflow.)

- (1) I have a bucket which can exactly measure 5 L.
- (2) I have a mug which can exactly measure 3 L.

Solution

Many students make the mistake of thinking that only statement I alone can answer the question asked because the important aspect for them is that the tank should be filled whereas the important aspect is whether we can answer the question uniquely about the tank getting filled.

Therefore, in the above example, using statement I alone, the answer is yes, the tank can be filled completely and so statement 1 alone is able to answer the question asked.

Similarly, using statement 2 alone, the answer is a definite no, the tank cannot be filled completely and so statement 2 alone is also able to answer the question asked.

So, as per the standard answer options of data sufficiency, the correct answer to the question is option (D).

The correct answer is D.

Let us now look at some more illustrative examples to understand the options better.

Example 3

What is the value of x ?

(1) $x^3 = -27$

(2) $x^2 = 9$

Solution

If we use the first statement alone, we will get a unique value of $x = -3$. Therefore, the question can be answered by using statement 1 alone.

If we use statement 2 alone, we will get two values of x as 3 and -3 . As data sufficiency requires a unique answer, the question cannot be answered by using statement 2 alone.

The correct answer is A.



Important Learning: Even though one of the two answers to the question using statement 2 is -3 which is obtained from statement 1 also, the question cannot be answered by using statement 2 alone.

Example 4

On which day was Naveen born?

- (1) On 25th November, Naveen celebrated his ninth birthday.
- (2) Naveen was born on a Friday.

Solution

The question is asking about the day when Naveen was born and not the date. From statement 1 we get the date but not the day.

Using statement 2 alone, we can conclude that Naveen was born on a Friday.

The question can be answered by using statement 2 alone but not by using statement 1 alone.

The correct answer is B.

Example 5

What is the speed of the train?

- (1) In an hour, the train covers a distance of 50 km after stopping for 15 min.
- (2) The train is 150 m long and crosses a man moving in the same direction at 5 km/h in 10 s.

Solution

We can find the speed of the train using statement 1 alone. Do remember, the question does not want us to find the value of the speed of the train.

What is required to be known is that distance covered and time taken can be used to find out the speed of a moving body.

Similarly, using statement 2 alone, the question can be answered.

The correct answer is D.

Example 6

What is the profit earned in rupees?

- (1) The marked price is Rs. 400 and it is sold at a discount of 30%.
- (2) The marked price is 50% more than the cost price.

Solution

In order to answer the question, we will require the values of the cost price and the selling price.

While statement 1 can give us the selling price, we do not know the cost price.

Also, statement 2 can give us the cost price but we do not know the selling price.

Therefore, either of the two statements is not sufficient to answer the question alone.

Both the statements together can answer the question asked.

The correct answer is C.

Example 7

What is the ratio of the prices of A and B?

- (1) Three years ago, their prices were in the ratio 4 : 5.
- (2) In the last 3 years, the price of both A and B has increased by Rs. 5000.

Solution

When we look at a question like this, many of us would be tempted to mark option (C) as the answer.

The question cannot be answered by using either of the two statements alone, and therefore, we will try and answer the question by combining the two statements.

Let their prices be $4x$ and $5x$, respectively.

Also, their prices have increased by Rs. 5000 each.

The ratio of their prices will be $(4x + 5000) : (5x + 5000)$, which will not give any definite value of their prices.

The correct answer is E.

Example 8

In how much time will the work be completed if 15 men work for 8 h every day?

- (1) 20 women working for 7 h can do the work in 12 days.
- (2) 8 men working for 15 h can do the work in 18 days.

Solution

The question cannot be answered by using statement 1 alone because we do not know the efficiency comparison of a man and a woman.

Using statement 2 alone and equating the total work to be done in the section Time and Work, we have

$$8 \times 15 \times 18 = 15 \times 8 \times y$$

Solving, we get $y = 18$ days

The correct answer is B.



Important Learning: Do not make the mistake of picking up information from Statement 1 while checking Statement 2 and vice versa.

Note: In data sufficiency problems that ask for the value of a quantity, the data given in the statements are sufficient only when it is possible to determine exactly one numerical value for quantity.

Example 9

Find the value of the number if it is less than 100.

- (1) The number is a perfect square.
- (2) It is a multiple of 16.

Solution

Using Statement 1 alone, the question cannot be answered as there can be a number of possibilities, such as 1, 4, 9, 16, 25, 36 and so on.

Using Statement 2 alone, there will again be a number of possibilities, such as 16, 32, 48, 64, etc.

However if we combine both the statements, there is only one possibility = 64.

Thus, C is the answer.

The correct answer is C.

Example 10

Find the speed of the stream.

- (1) A log of wood travels 4 km in 3 h.
- (2) Speed of a boat while moving downstream is 7 km/h.

Solution

A log of wood does not have any power of its own so it moves at the same rate as the speed of the stream. Thus, Statement 1 is sufficient to answer the question.

The speed of a boat does not tell us anything about the speed of the stream. So Statement 2 is not sufficient.

The correct answer is A.

3.0 Quantitative Skills Practice

3.1 Practice Questions

Solve the problems and indicate the best of the answer choices given.

Numbers: All numbers used are real numbers.

Figures: A figure accompanying a problem-solving question is intended to provide useful information for solving the problem. Figures drawn as accurately as possible. Exceptions be clearly noted. Lines shown as straight are straight, and lines that appear jagged are also straight. The positions of points, angles, regions, etc., exist in the order shown, and all measures are greater than zero. All figures lie in a plane unless otherwise indicated.

1 Number Properties

1. Which of the following is equal to $\left(\frac{\sqrt{12}}{5}\right)\left(\frac{\sqrt{60}}{2^4}\right)\left(\frac{\sqrt{45}}{3^2}\right)$
(A) $\frac{1}{12}$
(B) $\frac{1}{6}$
(C) $\frac{1}{4}$
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$
2. Express $7.\overline{583}$ as a fraction:
(A) $\frac{91}{12}$
(B) $\frac{44}{6}$
(C) $\frac{99}{14}$
(D) $\frac{22}{3}$
(E) $\frac{148}{21}$
3. Prakash bought a bag of 15 magic pencils for Rs. 60. One-third of the pencils cost Rs. 2 each and the rest cost Rs. 5 each. If there was a hole in the bag and all of the more expensive pencils fell out, the lost pencils represented approximately what percentage of the money Prakash paid for all the pencils?
(A) 7%
(B) 13%
(C) 67%
(D) 83%
(E) 88%

4. Company H distributed Rs. 4,000 and 180 pens evenly among its employees, with each employee getting an equal integer number of Rupees and an equal integer number of pens. What is the highest number of employees that could work for Company H?
- (A) 9
(B) 10
(C) 20
(D) 40
(E) 180
5. If t is divisible by 12, what is the least possible integer value of a for which $\frac{t^2}{2^a}$ might not be an integer?
- (A) 2
(B) 3
(C) 5
(D) 6
(E) 40
6. If $5^{k+1} = 2,000$, what is $5^k + 1$?
- (A) 399
(B) 401
(C) 1,996
(D) 2,000
(E) 2,001
7. Which of the following is equal to $(\sqrt[2]{x})(\sqrt[3]{x})$?
- (A) $\sqrt[5]{x}$
(B) $\sqrt[6]{x}$
(C) $\sqrt[3]{x^2}$
(D) $\sqrt[5]{x^6}$
(E) $\sqrt[6]{x^5}$

- 8.** If $125^{14}48^8$ is written as an integer, how many consecutive zeroes will that integer have at the end?
- (A) 22
(B) 32
(C) 42
(D) 50
(E) 112
- 9.** If n is the smallest of three consecutive positive integers, which of the following must be true?
- (A) n is divisible by 3
(B) n is even
(C) n is odd
(D) $(n)(n + 2)$ is even
(E) $n(n + 1)(n + 2)$ is divisible by 3
- 10.** If $\frac{17}{2^{10} \times 5^{13}}$ is expressed as a terminating decimal, how many zeroes are located to the right of the decimal point before the first non-zero digit?
- (A) 10
(B) 12
(C) 13
(D) 15
(E) 17
- 11.** If $25^54^6 = 10^x + a$, and x is an integer, what could be the minimum positive value of a ?
- (A) 0
(B) 30,000
(C) 30,000,000
(D) 10,000,000,000
(E) 30,000,000,000

- 12.** What is the unit digit of 7^{86} ?
- (A) 0
(B) 1
(C) 3
(D) 7
(E) 9
- 13.** What is the difference between the sum of all even positive integers between 1 and 100 (inclusive) and the sum of all odd positive integers between 100 and 150?
- (A) -575
(B) -475
(C) 225
(D) 475
(E) 575
- 14.** When 5, 8 and 12 divide a multiple of 13, they leave remainders of 3, 6 and 10, respectively. What is the least such number? (*Real NMAT Question*)
- (A) 169
(B) 478
(C) 598
(D) 1,298
(E) 1,602
- 15.** The sum of the last digits of the numbers of the form 2^{2n+1} , for $n = 0, 1, 2, 3$ and 4 , when divided by 7 gives a remainder
- (A) 1
(B) 2
(C) 3
(D) 4
(E) 5

- 16.** If ‘a’ and ‘b’ are prime numbers, then what is the H.C.F. of the numbers $(a^2 + b^2)$, $(a + b + 1)$ and $(a^2 + b^2 - 1)$?
- (A) 1
(B) 2
(C) 3
(D) 4
(E) Cannot be determined
- 17.** A three digit number is such that its hundredth digit is equal to the product of the other two digits which are prime numbers. Also, the difference between the number and its reverse is 297. Then, what is the ten’s digit of the number?
- (A) 2
(B) 3
(C) 5
(D) 6
(E) 7
- 18.** When a two digit number is divided by the sum of its digits, the quotient is 4. If the digits are reversed, the new number is 6 less than twice the original number. The number is
- (A) 12
(B) 21
(C) 24
(D) 42
(E) Both (C) and (D)
- 19.** A five-digit number is formed using the digits 1, 3, 5, 7 and 9 without repetition. What is the sum of all such possible numbers?
- (A) 6666600
(B) 6666660
(C) 6666666
(D) 6666000

- (E) None of these
20. A positive integer 'A' is a multiple of 180 and it has 40 factors. If 'A' is less than 3,000, then the value of $\frac{A}{40}$ is
- (A) 54
(B) 60
(C) 240
(D) 270
(E) Cannot be determined
21. If the number 5237ab is completely divisible by 3, find the possible values of a + b.
- (A) 2
(B) 5
(C) 8
(D) 15
(E) 16
22. A number A gives a remainder of 7 when divided by 9. Find the remainder when 2A is divided by 9.
- (A) 1
(B) 2
(C) 5
(D) 8
(E) 14
23. At a nature trail camp, one-fifth of the total members went rock climbing; twice the square root of the total members went hiking up a mountain trail. The remaining 10 were exploring in caves. How many members went hiking? (**Real NMAT Question**)
- (A) 5
(B) 10
(C) 15

(D) 20

(E) 25

- 24.** What is the highest power of 2 in the expression $(2^{10} - 1)!$? (*Real NMAT Question*)

(A) $2^5 - 1$

(B) $2^7 - 6$

(C) $2^8 - 9$

(D) $2^9 - 10$

(E) $2^{10} - 11$

- 25.** During the morning assembly in a school, all the students can stand in rows so that each row has 8, 10 or 12 students. Which of the following could be the least number of students in the school? (*Real NMAT Question*)

(A) 20

(B) 30

(C) 60

(D) 100

(E) 120

- 26.** What is the value of the following expression? $2^{\log_2 \log_2 2^{10}}$ (*Real NMAT Question*)

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) 4

- 27.** Which of the following numbers is divisible by 9? (*Real NMAT Question*)

(A) 1,203

(B) 2,256

- (C) 42,651
 - (D) 71,623
 - (E) 92,423
- 28.** If August 15, 1947 was a Friday, then, what was the day on January 26, 1950?
- (A) Thursday
 - (B) Friday
 - (C) Saturday
 - (D) Sunday
 - (E) Monday
- 29.** What was the day on April 20, 1984?
- (A) Thursday
 - (B) Friday
 - (C) Saturday
 - (D) Sunday
 - (E) Monday
- 30.** If in a certain year, the month of January had exactly 4 Wednesdays and 4 Sundays, then January 1 of that year was a
- (A) Saturday
 - (B) Monday
 - (C) Wednesday
 - (D) Friday
 - (E) Thursday
- 31.** Puja born in 1900s realised that in 1980 his age was the square root of the year of her birth. When was Puja born?
- (A) 1929
 - (B) 1936
 - (C) 1940
 - (D) 1946

(E) 1949

32. If 09.12.2001 happens to be a Sunday, then 09.12.1971 would have been a
- (A) Saturday
 - (B) Monday
 - (C) Wednesday
 - (D) Friday
 - (E) Thursday
33. If $\log_{10} 2 = 0.3010$, what is the value of $\log_5 256$? (*Real NMAT Question*)
- (A) 3.11
 - (B) 3.26
 - (C) 3.44
 - (D) 3.67
 - (E) 3.82
34. Find the value of 'x' if $625^{\log_{36} 6} + 12^{\log_7 49} = 11^{\log_x 169}$
- (A) 10
 - (B) 11
 - (C) 13
 - (D) 17
 - (E) 19
35. If $\log_{30} 3 = x$ and $\log_{30} 5 = y$, then find the value of $\log_8 30$.
- (A) $3(1 - x - y)$
 - (B) $\frac{1}{3(1 - x - y)}$
 - (C) $\frac{3}{(1 - x - y)}$
 - (D) $\frac{1 - x - y}{3}$
 - (E) None of these

36. If $a^p = b^q = c^r = d^s$, then find the value of $\log_a(bcd)$.

(A) $p\left(\frac{1}{q} + \frac{1}{r} + \frac{1}{s}\right)$

(B) 1

(C) $\frac{1}{q} + \frac{1}{r} + \frac{1}{s}$

(D) $\frac{p}{\left(\frac{1}{q} + \frac{1}{r} + \frac{1}{s}\right)}$

(E) $\frac{\left(\frac{1}{q} + \frac{1}{r} + \frac{1}{s}\right)}{p}$

37. If $\log_y x = 10$, then find the value of $\log_{x^3} y^6$

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) 5

(D) 6

(E) Both $\frac{1}{6}$ and $\frac{1}{5}$

38. If $P = 625^{\frac{2}{\log_3 25}} + 25^{\log_{125} 27} + 5^{\frac{5}{\log_5 125}}$, then find the value of 'P'.

(A) 90

(B) 92

(C) 122

(D) 136

(E) 154

2 Arithmetic

1. The number that is 50% greater than 80 is what percent less than the number that is 25% less than 200?
 - (A) 5%
 - (B) 10%
 - (C) 15%
 - (D) 20%
 - (E) 25%
2. Akash spends 50% of his income on rent, utilities, and insurance, and 20% on food. If he spends 30% of the remainder on video games and has no other expenditure, what percent of his income is left after all the expenditure?
 - (A) 0%
 - (B) 9%
 - (C) 20%
 - (D) 21%
 - (E) 30%
3. In a class of 40 students, exactly 90% had lower marks than Varun's marks. 60 new students join Varun's class. If Varun's marks were higher than those of 80% of the new arrivals, what percent of the combined class now had higher marks than Varun's marks?
 - (A) 86%
 - (B) 85%
 - (C) 16%
 - (D) 15%
 - (E) 14%
4. Machines X and Y pack books continuously, each working at a constant rate, but Machine Y works 50% faster than Machine X. If Machine Y packs 48,000 more books in a 24-hour period than

Machine X does, what is Machine X's packing rate in books per hour?

- (A) 4,000
 - (B) 6,000
 - (C) 8,000
 - (D) 12,000
 - (E) 16,000
5. An apple vendor sells 50% of his stock everyday. 10% of his stock gets spoiled overnight. After three nights he calculates that a total of 3,305 apples have spoiled. How many apples did he start out with on the first day? (Assume that he has purchased apples only on the first day.) **(Real NMAT Question)**
- (A) 25,000
 - (B) 30,000
 - (C) 35,000
 - (D) 40,000
 - (E) 45,000
6. Ankur bought 5 Pizzas, 7 Samosas and 4 ice-creams. Sanjeev bought 6 Pizzas, 14 Samosas and 8 Ice creams for an amount which was 50% more than what Ankur paid. What percentage of the total amount spent by Ankur was spent on the Pizzas?
- (A) 37.5%
 - (B) 45%
 - (C) 50%
 - (D) 56.5%
 - (E) 62.5%
7. By selling the burger at Rs. 260 per piece, Sameer gains 30%. Find the cost price of the burger per piece?
- (A) Rs. 150
 - (B) Rs. 200
 - (C) Rs. 250

- (D) Rs. 300
(E) Rs. 350
8. Ghosh Babu, a trader, marked up his goods 30% over the cost price and then he gave the discount of 5%. What was the profit percentage of Ghosh Babu in the whole transaction?
- (A) 19.5%
(B) 21.5%
(C) 23.5%
(D) 25.5%
(E) None of these
9. The value of $(p\%q + q\%p)$ is:
- (A) $p\% \text{ of } q$
(B) $q\% \text{ of } p$
(C) $2\% \text{ of } pq$
(D) $pq\% \text{ of } 3$
(E) None of these
10. 'p' is five times as large as 'q'. By what percent is q less than p?
- (A) $16\frac{2}{3}\%$
(B) 37.5%
(C) 60%
(D) 80%
(E) 90%
11. A 735 gm sample of a 16% (by weight) solution of iodine in alcohol is kept for three days. Some of the alcohol gets evaporated and the concentration of the solution becomes 20% (by weight). What amount of alcohol gets evaporated?
- (A) 135 gm
(B) 140 gm
(C) 147 gm

- (D) 150 gm
(E) 215 gm
- 12.** The interest rate, compounded annually, that would bring a principal of Rs. 1,200 to a final value of Rs. 1,650 in 2 years is approximately:
(A) 17%
(B) 18%
(C) 19%
(D) 20%
(E) 21%
- 13.** An amount becomes 3 times in 6 years on a certain rate of simple interest. In how many years it will become 24 times?
(A) 48 Years
(B) 50 Years
(C) 69 Years
(D) 70 Years
(E) 72 Years
- 14.** What is the value of the rate of interest if the difference between the compound interests of the first and the second year is 4 times that of the principal?
(A) 50%
(B) 100%
(C) 150%
(D) 200%
(E) 400%
- 15.** Some amount was divided into two equal parts. The first part was invested at 10% per annum at simple interest for 4 years. The second part was invested at 10% per annum at compound interest for 3 years. If the difference in the interests earned from the two investments is Rs. 1000, find the approximate value of the total initial amount.

- (A) Rs. 28,485
 - (B) Rs. 28,985
 - (C) Rs. 29,485
 - (D) Rs. 29,985
 - (E) Rs. 30,485
- 16.** The value of a car depreciates at the rate of 10% per annum. If its present value is Rs. 121,500, then what was the value of the car two years ago?
- (A) Rs. 100,000
 - (B) Rs. 150,000
 - (C) Rs. 200,000
 - (D) Rs. 250,000
 - (E) Rs. 300,000
- 17.** Ram lent Rs. 800 to a friend for 2 years and one-fourth of this amount to another friend for 3 years. He received Rs. 275 in total as simple interest. What was the rate of interest?
- (A) 10.5%
 - (B) 12.5%
 - (C) 15.5%
 - (D) 17.5%
 - (E) 19.5%
- 18.** At what interest rate per annum will a sum of money double itself in 8 years?
- (A) $12\frac{1}{2}\%$
 - (B) 13%
 - (C) 15%
 - (D) 17%
 - (E) 19%

- 19.** Parikshit invests Rs. 1,546 in BNP bank at a certain rate of compound interest per annum. At the end of 8 years, he finds that his money has doubled. What approximately is the rate of interest BNP bank paid him?
- (A) 9%
(B) 12%
(C) 15%
(D) 16%
(E) 18%
- 20.** Giri divided his property between his children Suma and Dev. Suma invested her share at 10% per annum simple interest and Dev invested his share compounded at 8% per annum. At the end of 2 years, the interest received by Suma is Rs. 13,360 more than the interest received by Dev. What was Suma's share if the total amount divided was Rs. 2,50,000? (*Real NMAT Question*)
- (A) Rs. 50,000
(B) Rs. 63,360
(C) Rs. 1,13,360
(D) Rs. 1,50,000
(E) Rs. 1,63,360
- 21.** Which of the following is the interest rate if the difference between the compound interest and the simple interest on Rs. 12,500 for 3 years is Rs. 561.60? (*Real NMAT Question*)
- (A) 10%
(B) 11%
(C) 12%
(D) 13%
(E) 15%
- 22.** Arvind sells clothes at a roadside market for which he pays Rs. 150 per day to rent a table plus Rs. 10 per hour to his salesman. He sells an average of Rs. 78 worth of clothes per hour. Assuming no other

costs, which of the functions below best represents profit per day P in terms of hours h that Arvind works for?

- (A) $P(h) = 238 - 10h$
 - (B) $P(h) = 72 - 10h$
 - (C) $P(h) = 68h - 150$
 - (D) $P(h) = 78h - 160$
 - (E) $P(h) = -160h + 78$
- 23.** A batch of clips costs Rs. $(p + 15)$ for a company to produce and each batch sells for Rs. $p(9 - p)$. For which of the following values of p does the company make a profit?
- (A) 3
 - (B) 4
 - (C) 5
 - (D) 6
 - (E) 7
- 24.** Priya deposited Rs. 20,000 on 1st January, 2014 to open a savings account. She withdrew Rs. 1,000 on the 10th of every month. She closed her account on 6th June, 2014. If the bank pays interest at 4% p.a, then approximately how much interest did she receive on closing the account? (*Real NMAT Question*)
- (A) Rs. 192
 - (B) Rs. 283
 - (C) Rs. 296
 - (D) Rs. 384
 - (E) Rs. 420
- 25.** A teacher distributed 50 worksheets among the students of three sections in a class. The worksheets were distributed among 6 students of Section A, 12 students of Section B, and 17 students of Section C. The number of worksheets received by 2 students of Section A was equal to the number of worksheets received by 5 students of Section C. The number of worksheets received by 2 students of Section B was equal to the number of worksheets

received by 3 students of Section C. How many worksheets did each student of Section C receive?

Note: Assume that each student of a particular section received the same number of sheets. (*Real NMAT Question*)

- (A) 1
 - (B) 2
 - (C) 3
 - (D) 4
 - (E) 5
- 26.** A shopkeeper claims a loss of 4% on his goods, but uses weight equal to 840 gm instead of 1 kg. The shopkeeper actually makes a:
- (A) $11\frac{1}{7}\%$ gain
 - (B) $14\frac{2}{7}\%$ gain
 - (C) 4% loss
 - (D) 4% gain
 - (E) 2% loss
- 27.** A product priced at Rs. 1,000 would earn a shopkeeper a profit of 15%. Find the profit percentage earned by him if he decides to sell the product at a discount of Rs. 50 during the festivals.
- (A) 3.34%
 - (B) 9.15%
 - (C) 9.25%
 - (D) 9.30%
 - (E) 9.50%
- 28.** A container contains milk and water in the ratio of 6 : 5. Both milk and water are increased by the same quantity. Which of the following can be the ratio of milk and water in the new mixture? (*Real NMAT Question*)
- (A) 15 : 9

- (B) 7 : 5
- (C) 19 : 15
- (D) 13 : 10
- (E) 11 : 10

- 29.** The capital of a company, Estyle, is made of 75,000 preferred shares with a dividend of 15% and 20,000 common shares, with the par value of each type of share as Rs. 10. The total profit of Estyle was Rs. 2,40,000 of which Rs. 40,000 was kept in a reserve fund. The remaining profit was distributed to the shareholders. What would be the difference in the dividend percentage given to the common shareholders if the amount kept away in the reserve fund was reduced to Rs. 25,000? (*Real NMAT Question*)
- (A) 5.75%
 - (B) 7.5%
 - (C) 10%
 - (D) 12.75%
 - (E) 15%
- 30.** If the compound ratio of $8 : \frac{5}{3}$ and the inverse of $\frac{3}{5} : \frac{1}{3}$ is $15 : x$, then x is (*Real NMAT Question*)
- (A) 20
 - (B) 40
 - (C) 60
 - (D) 80
 - (E) None of the above
- 31.** Pipe A can fill a tank in 5 hours, while Pipe B can empty the tank in 6 hours. If both the pipes are opened simultaneously, how much time will it take to fill the tank completely? (*Real NMAT Question*)
- (A) 11 hr
 - (B) 30 hr
 - (C) 45 hr

(D) $\frac{11}{30}$ hr

(E) $\frac{8}{11}$ hr

32. A alone can complete a task in 10 days. B can complete the same task in 15 days. If A and B work together, how much time will it take to complete the same task? (*Real NMAT Question*)

(A) 3 days

(B) 5 days

(C) 6 days

(D) 9 days

(E) 10 days

33. A water tank had 3 taps. The first tap could fill the tank in 10 minutes and the second tap could fill the tank in 15 minutes. When all the 3 taps were opened simultaneously, the tank was filled in 22 minutes. In how many minutes did the third tap fill or empty the tank? (*Real NMAT Question*)

(A) 6.50

(B) 8.25

(C) 9.27

(D) 15.67

(E) 18.50

34. A chemist is mixing a solution of ink and water. She currently has 30 litres of mixture solution, of which 10 litres are ink. How many litres of ink should the chemist add to her current mixture to attain a 50 : 50 mixture of ink and water if no additional water is added?

(A) 2.5

(B) 5

(C) 10

(D) 15

(E) 20

- 35.** A full glass of lemonade is a mixture of 20% lime juice and 80% soda. The contents of the glass are poured into a pitcher that is 200% bigger than the glass. The remainder of the pitcher is filled with 16 litres of water. What was the original volume of lime juice in the mixture?
- (A) 1.6 litres
(B) 3.2 litres
(C) 4.8 litres
(D) 6.4 litres
(E) 8 litres
- 36.** In a college dramatics team, the ratio of boys to girls is 6 : 7. If there are 2 more girls than boys in the team, how many boys are in the team?
- (A) 12
(B) 18
(C) 24
(D) 30
(E) 36
- 37.** X cornflake is 55% fibre and Y cornflake is 70% fibre. Sharad combines a certain amount of the two cereals in a single bowl, creating a mixed cereal that is 65% fibre. If the bowl contains 120 grams of cereal, how much of the cereal, in grams, is X?
- (A) 30
(B) 40
(C) 60
(D) 80
(E) 90
- 38.** A pump can be used for filling as well as emptying a tank. The capacity of the tank is $1,500 \text{ m}^3$. The emptying capacity of the tank is 10 m^3 per minute higher than its filling capacity and the pump needs

5 minutes less to empty the tank than it needs to fill it. What is the emptying capacity of the tank? (*Real NMAT Question*)

- (A) 10 m^3 per minute
 - (B) 30 m^3 per minute
 - (C) 40 m^3 per minute
 - (D) 50 m^3 per minute
 - (E) 60 m^3 per minute
- 39.** In what ratio, solution X which contains 50% milk and solution Y which contains 30% milk be mixed so that the obtained solution contains 45% milk?
- (A) 1 : 3
 - (B) 2 : 3
 - (C) 3 : 2
 - (D) 3 : 5
 - (E) 3 : 1
- 40.** A tank has a solution consisting of milk and water in equal proportion. This solution is transferred into a vessel having 100 ml pure water at the rate of 10 ml per second. In how much time from the start of the transfer, will the ratio of milk to water in the vessel be 1 : 3?
- (A) 5 seconds
 - (B) 10 seconds
 - (C) 15 seconds
 - (D) 20 seconds
 - (E) 25 seconds
- 41.** The sum of the ages of the six members in a family is 130 years. If the age of the children is in the ratio 1:2:6:7 while the combined age of their parents is 82 years, find the age of the eldest child.
- (A) 7 years
 - (B) 14 years

- (C) 21 years
 - (D) 28 years
 - (E) 35 years
- 42.** The population of a country increased at the rate of 6% per year. If the present population of the country is 23,452 million, what was the approximate population of the country 3 years ago? (*Real NMAT Question*)
- (A) 18,765 million
 - (B) 18,967 million
 - (C) 19,691 million
 - (D) 20,872 million
 - (E) 21,432 million
- 43.** If the average of a, b, c, 5, and 6 is 6, what is the average of a, b, c, and 13?
- (A) 8
 - (B) 8.5
 - (C) 9
 - (D) 9.5
 - (E) 10.5
- 44.** Average weight of 37 students is 42 kg. When their teacher joined them, their average weight increased by 2 kg. What is the weight of the teacher?
- (A) 112 kg
 - (B) 114 kg
 - (C) 116 kg
 - (D) 118 kg
 - (E) 120 kg
- 45.** Two persons of average age 40 years leave a group and hence the average age of the remaining group increases from 50 to 52 years. Find the number of persons originally in the group.

- (A) 5
- (B) 7
- (C) 8
- (D) 10
- (E) 12

46. If the average weight of 8th, 9th and 10th class is in the ratio of a:b:c and the number of students in class 8th, 9th and 10th is in the ratio of x:y:z, then the average weight of all the three classes considered together is

- (A) $\frac{a+b+c}{x+y+z}$
- (B) $\frac{\frac{a}{x} + \frac{b}{y} + \frac{c}{z}}{x+y+z}$
- (C) $\frac{ax+by+cz}{x+y+z}$
- (D) $\frac{ax+by+cz}{a+b+c}$
- (E) Cannot be determined

47. The average age of a husband and wife who were married 5 years ago, was 25 years then. The average age of the family including the husband, the wife and two children who were born during the interval is 16 years now. How old are the children now? [All of them have different ages with integral values.]

- (A) 1 year, 3 years
- (B) 4 years, 1 year
- (C) 2 years, 3 years
- (D) 2 years, 2 years
- (E) None of these

48. The time it takes to construct a hut is inversely proportional to the number of workers doing the work. If it takes 40 workers giving 3 hours each to do the job, how long will it take for 140 workers to do the job, to the nearest minute?

- (A) 51 minutes

- (B) 52 minutes
- (C) 53 minutes
- (D) 54 minutes
- (E) 55 minutes

- 49.** A ski resort has enough wood to keep 20 rooms heated for 14 days. If the resort decides to save wood by turning off the heat in 5 unoccupied rooms, and each room requires the same amount of wood to heat it, how many extra FULL days will the wood supply last?
- (A) 3
 - (B) 4
 - (C) 5
 - (D) 18
 - (E) 19
- 50.** Working alone at their respective constant rates, Ajay can complete a certain job in 4 hours, while Firoz can do the same job in 3 hours. Ajay and Firoz worked together on the job and completed it in 2 hours, but while Ajay worked this entire time, Firoz worked for some of the time and took 3 breaks of equal length. How many minutes long were each of Firoz' breaks?
- (A) 5 minutes
 - (B) 10 minutes
 - (C) 15 minutes
 - (D) 20 minutes
 - (E) 25 minutes
- 51.** A machine can manufacture 20 pens per hour, and exactly 10 such pens fit into every box. Mahesh packs pens in boxes at a constant rate of 3 boxes per hour. If the machine ran for 2 hours and was then turned off before Mahesh started packing the pens in boxes, how many minutes would it take Mahesh to pack all the pens that the machine had made?
- (A) 40 minutes

- (B) 45 minutes
- (C) 80 minutes
- (D) 160 minutes
- (E) 800 minutes

- 52.** Two taps can separately fill a tank in 4 minutes and 5 minutes respectively. Due to a small hole at the bottom of the tank, the two taps together take 30 seconds more time to fill the tank. The hole can empty the completely filled tank in
- (A) $\frac{980}{71}$ minutes
 - (B) $\frac{980}{81}$ minutes
 - (C) $\frac{980}{91}$ minutes
 - (D) $\frac{980}{61}$ minutes
 - (E) $\frac{981}{51}$ minutes
- 53.** Three taps P, Q and R when filling together can fill a cistern in 3 hours. After 1 hour tap P is closed and the cistern is filled in 4 more hours. Find the time in which tap P alone can fill the cistern?
- (A) 3 hours
 - (B) 4 hours
 - (C) 5 hours
 - (D) 6 hours
 - (E) 7 hours
- 54.** Pipes P and Q can fill a tank in 12 minutes and 16 minutes respectively. Both are kept open for X minute(s) and then Q is closed and P fills the rest of the tank in 5 minutes. The time X after which Q was closed is
- (A) 2 minutes
 - (B) 3 minutes
 - (C) 4 minutes

- (D) 6 minutes
(E) 7 minutes
55. Pipe X pours a mixture of acid and water, and pipe Y pours pure water into a bucket. After 1 hour, the bucket got filled and the concentration of acid in the bucket was noted to be 8%. If pipe Y was closed after 30 minutes and pipe X continued to pour the mixture, concentration of acid in the bucket after 1 hour would have been 10%. What is the ratio of acid to the water in the mixture coming out of pipe X?
- (A) 13 : 2
(B) 2 : 15
(C) 3 : 20
(D) 1 : 5
(E) 2 : 13
56. Two taps P and Q can fill a cistern in 12 minutes and 18 minutes respectively. If both the taps are opened together, how long it take to fill the cistern?
- (A) $\frac{5}{36}$ minutes
(B) $5\frac{1}{5}$ minutes
(C) $6\frac{1}{5}$ minutes
(D) $7\frac{1}{5}$ minutes
(E) None of these
57. There are 12 workers who have been recruited to dig a 20 km long tunnel. It takes one worker to dig 250 m of tunnel in a week. How many more workers are needed to complete the work in 2 weeks?
- (A) 12
(B) 18
(C) 20
(D) 24

(E) 28

58. If 33 unskilled workers can do a work in 15 days of 12 hours each, how many skilled workers can do 50% more work in 11 days of 9 hours each? (Assume that it takes 2 skilled workers to do the work of 5 unskilled workers.)
- (A) 36
(B) 42
(C) 64
(D) 90
(E) 100
59. Ajay finishes a work in certain number of days. He got two assistants who work $\frac{3}{4}$ as fast as him. If all three work together, then in what fraction of time would they finish the job as compared to Ajay working alone?
- (A) $\frac{5}{3}$
(B) $\frac{3}{5}$
(C) $\frac{2}{3}$
(D) $\frac{2}{5}$
(E) $\frac{3}{2}$
60. A and B can do a piece of work in $22\frac{2}{9}$ days, B and C can do it in $16\frac{2}{3}$ days, A and C can do the work in $15\frac{5}{13}$ days. Arrange the three in decreasing order of productivity.
- (A) C > A > B
(B) B > C > A
(C) A > C > B
(D) C > B > A
(E) B > A > C

- 61.** Tina, Ishan, Abhishek and Fatima jointly started a business and invested a total of Rs. 80. If Tina's share increases by Rs. 3, Ishan's share increases by one-third of his share, Abhishek's share decreases by 20% and Fatima's share decreases by Rs. 4, all of them would have equal amounts of money. What is Fatima's original share?
- (A) Rs. 20.25
(B) Rs. 23.50
(C) Rs. 23.75
(D) Rs. 24.25
(E) Rs. 24.75
- 62.** Mukesh, Manish, Lalu and Jaggi bought a MOKIA mobile for £60. Mukesh paid one-half of the sum of the amounts paid by the other persons. Manish paid one-third of the sum of the amounts paid by the other persons. Lalu paid one-fourth of the sum of the amounts paid by the other persons. How much did Jaggi have to pay?
- (A) £ 13
(B) £ 15
(C) £ 17
(D) £ 23
(E) None of these
- 63.** Yogesh and Mohan, two business partners, invest Rs. 21,000 and Rs. 17,500 respectively in their garment business and at the end of the year both of them make a profit of Rs. 26,400. Find their individual shares in the profit.
- (A) Rs. 14,400 and Rs. 12,000
(B) Rs. 12,000 and Rs. 14,400
(C) Rs. 14,000 and Rs. 12,400
(D) Rs. 14,200 and Rs. 12,200
(E) None of these

3 Algebra and Probability

1. If $3x^3 - 7 = 185$, what is $x^2 - x$?
 - (A) -4
 - (B) 8
 - (C) 12
 - (D) 16
 - (E) 27
2. If the arithmetic mean of two numbers is three times their geometric mean, then the ratio of the numbers can be given by: (**Real NMAT Question**)
 - (A) $(17 - 12\sqrt{2}) : 2$
 - (B) $(17 + 12\sqrt{2}) : 1$
 - (C) $(34 - 24\sqrt{2}) : 1$
 - (D) $(17 + 12\sqrt{2}) : 2$
 - (E) $(33 + 24\sqrt{2}) : 2$
3. If the roots of the equation $ax^2 + bx + c = 0$ are reciprocal of the roots of the equation $px^2 + qx + r = 0$, then which of the following represents relation(s) between a, b, c, p, q and r? (**Real NMAT Question**)
 - (A) $a = \frac{1}{p}$
 - (B) $b = \frac{1}{q}$
 - (C) $c = \frac{1}{r}$
 - (D) $a = p, c = r$ and $b = 1$
 - (E) $a = r, c = p$ and $b = q$
4. If a, b and c are the three positive integers in geometric progression, then the roots of the equation $ax^2 + 4bx + 2c = 0$ are

- (A) Imaginary
(B) Equal
(C) Rational
(D) Real
(E) Irrational
5. If p and q are roots of $x^2 + 7x + 12 = 0$, then the equation whose roots are $(p + q)^2$ and $(p - q)^2$ is
- (A) $x^2 - 50x + 49 = 0$
(B) $x^2 + 50x - 49 = 0$
(C) $x^2 - 10x + 3 = 0$
(D) $x^2 - 10x + 4 = 0$
(E) $x^2 - 50x - 49 = 0$
6. Which of the following could be the quadratic equation for which one root is $\frac{1}{2}$ times the other root and the difference between the roots is 1?
- (A) $x^2 + 3x + 3 = 0$
(B) $x^2 + 4x + 3 = 0$
(C) $x^2 - 5x + 6 = 0$
(D) $x^2 + x - 6 = 0$
(E) $x^2 - 3x - 3 = 0$
7. If p and q are the roots of the equation $ax^2 + bx + c = 0$, then what is the value of $p^4 - q^4$? (**Real NMAT Question**)
- (A) $a^4 - b^4 + c^4$
(B) $a^4 + b^4 - c^4$
(C) $\pm \frac{b}{a^4} (b^2 - ac)$
(D) $\pm \frac{b}{a^4} (b^2 - 2ac)$
(E) $\pm \frac{b}{a^4} (b^2 - 2ac) \sqrt{b^2 - 4ac}$

8. If $2(x - 1)^3 + 3 \leq 19$, then the value of x must be:

- (A) greater than or equal to 3
- (B) less than or equal to 3
- (C) greater than or equal to -3
- (D) less than or equal to -3
- (E) less than -3 or greater than 3

9. If $|3a + 7| \geq 2a + 12$, then

- (A) $a \leq -\frac{19}{5}$
- (B) $a \geq -\frac{19}{5}$
- (C) $a \geq 5$
- (D) $a \leq -\frac{19}{5}$ or $a \geq 5$
- (E) $-\frac{19}{5} \leq a \leq 5$

10. If x is an integer, $\sqrt{192} < x\sqrt{12}$ and $\frac{x}{\sqrt{12}} < \sqrt{12}$ which of the following can be the value of x ?

- (A) 2
- (B) 3
- (C) 4
- (D) 5
- (E) 12

11. If $\left|\frac{a}{b}\right|$ and $\left|\frac{x}{y}\right|$ are reciprocals, then which of the following must be true?

- (A) $ab < 0$
- (B) $\frac{a}{b} \left(\frac{x}{y} \right) < -1$
- (C) $\frac{a}{b} < 1$
- (D) $\frac{a}{b} = -\frac{y}{x}$
- (E) $\frac{y}{x} > \frac{a}{b}$

- 12.** It costs a certain chair manufacturing unit Rs. 11,000 to operate for one month, plus Rs. 300 for each chair produced during the month. Each of the chairs sells for a retail price of Rs. 700. What is the minimum number of chairs that the manufacturing unit must sell in one month to make a profit?
- (A) 26
(B) 27
(C) 28
(D) 29
(E) 30
- 13.** Which of the following describes all possible solutions to the inequality $|p + 5| < 9$?
- (A) $p < 4$
(B) $p > -14$
(C) $4 > p > -14$
(D) $-11 > p > 3$
(E) $p > 4$ or $p < -14$
- 14.** If $x^2 - 6x + 9 = 0$, what is the value of x^3 ? (*Real NMAT Question*)
- (A) -27
(B) -9
(C) 6
(D) 9
(E) 27
- 15.** Manish has 60 marbles that he wants to divide among himself and his 12 friends. The marbles don't necessarily have to be divided equally. If Manish wants to have more marbles than any of his friends, what is the least number of marbles he can have?
- (A) 5
(B) 6
(C) 7

(D) 8

(E) 12

- 16.** If $a^2 - b^2 = 0$ and $ab \neq 0$ which of the following must be true? Indicate all such statements.

1. $a = b$

2. $|a| = |b|$

3. $\frac{a^2}{b^2} = 1$

(A) 1 only

(B) 2 only

(C) 3 only

(D) 1 and 2 only

(E) 2 and 3 only

- 17.** If $(x - y) = \sqrt{20}$ and $(x + y) = \sqrt{12}$ what is the value of $x^2 - y^2$?

(A) $2\sqrt{15}$

(B) $4\sqrt{15}$

(C) $3\sqrt{20}$

(D) $6\sqrt{12}$

(E) $2\sqrt{21}$

- 18.** If $ab \neq 0$, $\frac{a^8 - b^8}{(a^4 + b^4)(a^2 + b^2)} =$

(A) 1

(B) $a - b$

(C) $(a + b)(a - b)$

(D) $(a^2 + b^2)(a^2 - b^2)$

(E) $\frac{a-b}{a+b} 2\sqrt{2}$

- 19.** Which of the following is equal to $(a - 2)^2 + (a - 1)^2 + a^2 + (a + 1)^2 + (a + 2)^2$?

- (A) $5a^2$
(B) $5a^2 + 10$
(C) $a^2 + 10$
(D) $5a^2 + 6a + 10$
(E) $5a^2 - 6a + 10$
- 20.** a is inversely proportional to b. Also, it is given that a = 24 when b = 2. What is the value of b when a = 6?
- (A) -2
(B) -1
(C) 2
(D) 4
(E) 8
- 21.** Three people sit down to eat 14 pieces of cake. If two of the people eat the same number of pieces, and the third person eats two more pieces than each of the other two, how many pieces are eaten by the third person?
- (A) 3
(B) 4
(C) 5
(D) 6
(E) 7
- 22.** There are a number of beads of three different colours: red, blue and yellow, and each colour has a different value. If the value of a red bead plus a blue bead is 4.25, the value of a blue bead plus a yellow bead is 2.75, and the value of a red bead plus a blue bead plus a yellow bead is 4.5, what is the value of a red bead plus a yellow bead?
- (A) 0.25
(B) 2
(C) 2.25

- (D) 2.75
(E) 3
- 23.** National Cricket Academy offers two different pricing packages for cricket coaching. Under the ‘Regular’ pricing plan, classes can be bought for a flat rate of Rs. 80 per hour. Under the ‘Exclusive’ pricing plan, after paying an initial fee of Rs. 495, classes can be availed for a rate of Rs. 15 per hour. If Karan buys the ‘Exclusive’ pricing plan, how many classes does he need to take in order to have spent exactly 40% less than he would have under the ‘Regular’ plan?
- (A) 10
(B) 12
(C) 15
(D) 18
(E) 20
- 24.** A student took a test in which 3 marks were given for each correct answer and 0.5 marks were deducted for an incorrect answer. If the test had 25 questions and the student attempted all the questions and got 40 marks in total, what is the difference between the number of correct and incorrect answers?
- (A) 5
(B) 10
(C) 12
(D) 15
(E) 18
- 25.** A group of friends contributed to the cost of a party where each person had to contribute the same integer amount. Since three people did not participate, the remaining people had to pay Rs. 10 more. If the total amount contributed is the minimum value possible, what would be the per person contribution had 10 people contributed to the party?
- (A) Rs. 2
(B) Rs. 5

- (C) Rs. 6
 (D) Rs. 8
 (E) Rs. 10

26. If $p + q + r = 0$, where $p \neq q \neq r$, then $\frac{p^2}{2p^2 + qr} + \frac{q^2}{2q^2 + pr} + \frac{r^2}{2r^2 + pq} =$

- (A) 0
 (B) 1
 (C) -1
 (D) pqr
 (E) $p + q + r$

27. Out of a group of swans, seven times half of the square root of the number of swans were seen going away from a bank of a river and only one pair remained in the water. How many swans were there in the group?

- (A) 9
 (B) 16
 (C) 25
 (D) 36
 (E) 49

28. If $\frac{1}{p} + \frac{1}{q} = m$ and $pq = \frac{1}{n}$, find $\frac{1}{p^2} + \frac{1}{q^2}$

- (A) $\frac{1}{m^2} - \frac{2}{n}$
 (B) $m^2 - \frac{2}{n}$
 (C) $m^2 - 2n$
 (D) $\frac{1}{m^2} - 2n$
 (E) $\frac{1}{m^2} + 2n$

29. Let $f(x + 2) + f(5x + 6) = 2x - 1$ for all real x . Find the value of $f(1)$.

- (A) -2

- (B) -1
(C) $\frac{-5}{2}$
(D) $\frac{-3}{2}$
(E) None of these
- 30.** For what value of K, the given set of equations would have no solution?

$$4x - Ky = -7 \text{ and } 5x + 3y = 2$$

- (A) $\frac{12}{5}$
(B) 0
(C) $\frac{-12}{5}$
(D) $\frac{-6}{5}$
(E) $\frac{6}{5}$

Directions for Questions 31 and 32: Answer the questions based on the following.

The following operations are defined for real numbers.

$A @ B = A$ if A is greater than B else $A @ B = B$

$A \% B = AB$ if $A \times B$ is positive else $A \% B = A$

Note that all other mathematical symbols have their usual meanings.

31. $[(-4)@(-5)]\%2$

- (A) -8
(B) -10
(C) -5
(D) -4
(E) -7

32. $\frac{1@-1}{(-K)@(-K)} \% K, K \neq 0, K \neq 0$

- (A) K^2

- (B) $\frac{1}{K}$
- (C) $\frac{-1}{K}$
- (D) 1
- (E) Cannot be determined

33. If the sum of the roots of an quadratic equation is $\frac{5}{4}$ times the product of the roots, find the relation between b and c .

- (A) $b = \frac{5}{4}c$
- (B) $b = \frac{4}{5}c$
- (C) $b = -\frac{5}{4}c$
- (D) $b = -\frac{4}{5}c$
- (E) $b = -\frac{2}{5}c$

34. Three numbers are in geometric progression such that the product of them is 27 and the sum of the products taken in pairs is 91. What is the third number in the progression? (*Real NMAT Question*)

- (A) $\frac{1}{3}$
- (B) 3
- (C) 9
- (D) 27
- (E) $\frac{1}{3}$ or 27

35. Manoj plans to work at a coffee shop during his summer holidays. He will be paid as per the following schedule: at the end of the first week, he will receive Rs. 1,000. At the end of each subsequent week, he will receive Rs. 1,000, plus an additional amount equal to the sum of all payments he has received in the previous weeks. How much money will Manoj be paid in total if he works for 6 weeks at this coffee shop?

- (A) Rs. 18,000

- (B) Rs. 20,000
- (C) Rs. 42,000
- (D) Rs. 63,000
- (E) Rs. 81,000

36. If the collection of a movie is Rs. 100,000 for the first day, Rs. 120,000 for the second day, Rs. 140,000 for the third day and so on, that is, the collection increases by Rs. 20,000 every day, then find the total collection for the first 10 days.

- (A) Rs. 1200,000
- (B) Rs. 1400,000
- (C) Rs. 1600,000
- (D) Rs. 1700,000
- (E) Rs. 1900,000

37. If $\log_x a$, $\log_y a$ and $\log_z a$ are in HP, then x, y and z are in (**Real NMAT Question**)

- (A) AGP
- (B) AP
- (C) GP
- (D) HP
- (E) Cannot be determined.

38. A person saves Rs. 200 more each year than in the previous year. If he started with Rs. 400 in the first year, how many years would he take to save Rs. 18,000 (excluding interest)?

- (A) 10 years
- (B) 12 years
- (C) 15 years
- (D) 18 years
- (E) None of these

39. If the second term of a geometric progression is 6 and the fifth term is 48, then what is its tenth term?

- (A) 2236
(B) 2146
(C) 1536
(D) 1246
(E) 1146
- 40.** p, q, r and s are any four positive real numbers, the minimum value of $\frac{p}{q} + \frac{q}{r} + \frac{r}{s} + \frac{s}{p}$ is
- (A) 0
(B) 1
(C) 2
(D) $2\sqrt{2}$
(E) 4
- 41.** If a, b, c and d are in GP, then $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$, and $(c^3 + d^3)^{-1}$ are in
- (A) AP
(B) GP
(C) HP
(D) AP or GP
(E) None of these
- 42.** A man pays a rent of Rs. 70 for the first day, Rs. 80 for the second day and so on, with the rent on each day being Rs. 10 more than the rent on the previous day. What is the total rent paid for the first 20 days?
- (A) Rs. 2,300
(B) Rs. 2,700
(C) Rs. 3,000
(D) Rs. 3,200
(E) Rs. 3,300

- 43.** In one day, what is the sum of the numbers on which the hour hand of a clock points each time the minute hand is on 12? (*Real NMAT Question*)
- (A) 12
(B) 78
(C) 156
(D) 160
(E) 178
- 44.** If a, b and c are in arithmetic progression, then a + b, b + c, c + a (in any order) can be in (*Real NMAT Question*)
- (A) arithmetic progression.
(B) geometric progression.
(C) harmonic progression.
(D) arithmetic or geometric progression.
(E) arithmetic or geometric or harmonic progression.
- 45.** For a cricket match team selection, 2 batsmen, 3 bowlers, and 1 wicketkeeper are to be picked. There are 23 players available to play as batsmen, 21 other players available to play as bowlers, and 9 other players available to play as wicketkeepers. If the maximum possible number of complete sets of 6 players are formed, how many of the available players will not be on a team?
- (A) 7
(B) 9
(C) 11
(D) 13
(E) 15
- 46.** How many five-digit numbers can be formed using the digits 5, 6, 7, 2, 9, 0 if no digits can be repeated?
- (A) 64
(B) 120
(C) 240

- (D) 600
(E) 720
- 47.** Five friends, Akshita, Binod, Chetan, Dravid, and Eshan are to be arranged in a line. How many such arrangements are possible if Binod is not allowed to stand next to Dravid?
- (A) 24
(B) 48
(C) 72
(D) 96
(E) 120
- 48.** On January 1, Ajit put Re 1 in his piggy bank. Every day he puts in Rs 2 more than the total amount of money already in the piggy bank. Which of the following expressions gives the total amount of money in Ajit's piggy bank at the end of January? (**Real NMAT Question**)
- (A) 2^{30}
(B) 2^{31}
(C) $3(2^{30}) - 2$
(D) $3(2^{31}) - 2$
(E) $3(2^{30})$
- 49.** Among three different boxes, 10 identical balls have to be distributed. In how many ways can this be done such that every box has at least 2 balls?
- (A) 15
(B) 16
(C) 64
(D) 81
(E) None of these
- 50.** There are 6 equally spaced points A, B, C, D, E and F marked on a circle whose radius is R. How many convex pentagons of distinctly different areas can be drawn using these points as vertices?

- (A) 6P_5
(B) 6C_5
(C) 5
(D) 1
(E) 6
- 51.** One of the management test papers comprises of 9 questions divided equally among three sections, namely section I, section II and section III. There are fifteen different questions available such that there are five questions for every section for designing the test. If no two sections bear a common question, then how many different tests can be designed?
- (A) 480
(B) 640
(C) 800
(D) 880
(E) 1,000
- 52.** A shop sells 5 different types of sweets. In how many different ways a total of 8 sweets can be purchased?
- (A) 125
(B) 495
(C) 795
(D) 840
(E) 930
- 53.** A box contains 90 balls of different colours: 13 yellow, 19 green, 27 red, 10 black, 7 brown and 14 white. Find the smallest number V such that any V balls drawn from the box will contain at least 14 balls of the same colour.
- (A) 69
(B) 70
(C) 72

(D) 76

(E) 79

- 54.** Salim has total 9 friends, 5 girls and 4 boys. In how many ways can Salim invite them for his birthday party, if there have to be exactly 3 girls in the invitees list?
- (A) 80
(B) 160
(C) 200
(D) 240
(E) 320
- 55.** There are 12 holes made in the ground. At least 3 are to be filled with a red ball and the other holes can be filled with any colour ball. In how many different ways can all the holes be filled from a box of 5 red balls and 10 mixed colour balls? (**Real NMAT Question**)
- (A) 345
(B) 425
(C) 445
(D) 465
(E) 485
- 56.** If the fourth and ninth terms of a Harmonic Progression are $\frac{1}{10}$ and $\frac{1}{25}$, then find the series. (**Real NMAT Question**)
- (A) $\frac{1}{1}, \frac{1}{4}, \frac{1}{7} \dots$
(B) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots$
(C) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7} \dots$
(D) $\frac{1}{1}, \frac{1}{3}, \frac{1}{7} \dots$
(E) None of the above

Directions for Questions 57–59: The following table represent the number of players nominated for different cricket teams.

Team	Total Player nominated	Batsmen	Bowler	All rounder
A	20	8	6	6
B	15	10	4	1
C	18	12	3	3

57. In how many ways can a team selector select 6 batsmen from team A?
- (A) 18
 - (B) 20
 - (C) 24
 - (D) 28
 - (E) 30
58. In how many ways a team selector can select 10 players in team B where 6 are bats men, 3 are bowlers and 1 is an all-rounder?
- (A) 720
 - (B) 800
 - (C) 840
 - (D) 900
 - (E) None of these
59. In how many ways a team selector can select 11 players in team C where 8 are bats men, 2 are bowlers and 1 is an all-rounder?
- (A) 4,455
 - (B) 4,545
 - (C) 4,465
 - (D) 4,475
 - (E) None of these
60. Each factor of 210 is written on a piece of paper, and all the pieces of paper are mixed up. If a piece of paper is randomly picked up from this mix, what is the probability that a multiple of 42 is written on the paper?

- (A) $\frac{1}{16}$
- (B) $\frac{5}{42}$
- (C) $\frac{1}{8}$
- (D) $\frac{3}{16}$
- (E) $\frac{1}{4}$

- 61.** As per a weather forecast, the probability of hail is $\frac{1}{6}$ for any given day next week. What is the chance that there will be hail on both Thursday and Friday?
- (A) $\frac{1}{36}$
 - (B) $\frac{1}{12}$
 - (C) $\frac{1}{6}$
 - (D) $\frac{1}{3}$
 - (E) $\frac{2}{3}$
- 62.** A classroom has 12 girls and 20 boys. $\frac{1}{4}$ of the girls in the class have cell phones. If a child is selected at random from the class, what is the probability that she is a girl who does not have a cell phone?
- (A) $\frac{3}{32}$
 - (B) $\frac{9}{32}$
 - (C) $\frac{3}{8}$
 - (D) $\frac{23}{32}$
 - (E) $\frac{29}{32}$
- 63.** A cube has sides numbered 1 through 6. If the cube is rolled three times, what is the probability that at least one of the rolls will result in a number higher than 4?

(A) $\frac{13}{19}$

(B) $\frac{11}{13}$

(C) $\frac{14}{19}$

(D) $\frac{19}{27}$

(E) $\frac{12}{31}$

- 64.** There is an 80% chance that Deeksha will skip her lunch and 25% chance that there will be a power failure. If these events are independent, what is the probability that Deeksha will skip her lunch OR that there will be a power failure?
- (A) 20%
(B) 80%
(C) 85%
(D) 95%
(E) 105%
- 65.** Bag A contains 3 white and 3 red beads. Bag B contains 6 white and 3 red beads. One of the two bags will be chosen at random, and then two beads will be drawn from that bag at random without replacement. What is the probability that the two beads drawn will be of the same colour?
- (A) $\frac{7}{20}$
(B) $\frac{9}{10}$
(C) $\frac{9}{20}$
(D) $\frac{11}{20}$
(E) $\frac{13}{20}$
- 66.** Two different unbiased dice are rolled together. What is the probability of getting a sum of more than or equal to 10 after adding the numbers shown on the tops of both the dice?

(A) $\frac{1}{12}$

(B) $\frac{1}{9}$

(C) $\frac{1}{6}$

(D) $\frac{2}{16}$

(E) $\frac{1}{4}$

- 67.** Two apples and five bananas are defective out of 10 apples and 20 bananas contained in a fruit basket. If Sanjeev takes out two fruits at random, what is the probability that either both are bananas, or both are good?

(A) $\frac{119}{435}$

(B) $\frac{338}{435}$

(C) $\frac{841}{870}$

(D) $\frac{217}{870}$

(E) None of these

- 68.** If 'M' and 'N' are two independent events and $P(M) = 0.5$ and $P(N) = 0.4$, find $P(M/N)$.

(A) 0.4

(B) 0.5

(C) 0.6

(D) 0.74

(E) 0.88

- 69.** The roll numbers of students in the class are in the range from 100 to 199 (both inclusive). If the teacher selects one student at random, what is the probability that his/her roll number is divisible by 3?

(A) $\frac{1}{5}$

(B) $\frac{32}{99}$

- (C) $\frac{33}{100}$
(D) $\frac{2}{3}$
(E) None of these

70. An integer x is chosen at random from the numbers 1 to 50. Find the probability that $x + \frac{336}{X} \leq 50$.

- (A) $\frac{7}{10}$
(B) $\frac{17}{25}$
(C) $\frac{19}{50}$
(D) $\frac{13}{50}$
(E) $\frac{3}{10}$

71. In an arithmetic series, the sum of the second term and the fifth term is 24. The sixth term is greater than the third term by 12. What is the first term of the series? (**Real NMAT Question**)

- (A) 1
(B) 2
(C) 3
(D) 4
(E) 5

4 DI-Caselets and Tables

Directions for Questions 1–4: The table below* shows the information about number of laptops (figures in 1000s) of different models produced and rejected by a company over six years.

Number of laptops of different models produced and rejected by a company over the years (figures in 1000s)

Laptop model	P		Q		R		S		T	
Year	Produced	Rejected								
2010	20	2	50	3	15	0.5	80	5	60	4
2011	35	3	45	2	20	0.55	75	4	58	4
2012	15	0.5	40	2.5	17	0.7	58	2	62	3.5
2013	25	0.25	42	2.3	25	1.5	65	3	40	1.5
2014	30	1.5	48	2.5	30	2	68	3	45	2
2015	27	1.5	41	2.1	26	1.75	72	3.5	50	2.3

*Table for Questions 1–4.

1. In case of Type Q laptop, in which year was the ratio of rejection to production the highest among the given years?
 - (A) 2010
 - (B) 2011
 - (C) 2012
 - (D) 2014
 - (E) 2015
2. In which year was the ratio of rejection to production the lowest among the given years for type T laptop?
 - (A) 2010
 - (B) 2012
 - (C) 2013
 - (D) 2014
 - (E) 2015
3. What was the difference in Type R laptops rejected between 2011 and 2012?

- (A) 150
 - (B) 200
 - (C) 250
 - (D) 2000
 - (E) 2400
4. The acceptable (not rejected) Type T laptops in 2012 were what percentage of those in 2011?
- (A) 8
 - (B) 14
 - (C) 106
 - (D) 108
 - (E) 110

Directions for Questions 5–8: Refer to the following table and answer the questions that follow:

Number of trousers produced by 5 factories over 5 months of 2016.

Month	Prisma	Shelby	Kooper	Wendy	Caret
Jan	900	850	350	1000	850
Feb	800	700	1050	1100	850
Mar	1050	800	1000	1100	950
Apr	800	850	850	1100	850
May	950	900	1050	1150	850
Total	4500	4100	4300	5450	4350

5. For which factory was the number of trousers manufactured in March the highest percentage of the total number of trousers produced by that factory during the five-month period?
- (A) Prisma
 - (B) Shelby
 - (C) Kooper
 - (D) Wendy
 - (E) Caret

6. The number of trousers manufactured by Wendy in April is what percentage of the number of trousers manufactured by Wendy in January?
- (A) 10%
(B) 91%
(C) 110%
(D) 115%
(E) 125%
7. Which of the five factories has the highest ratio of the number of trousers manufactured in April to number of trousers manufactured in February?
- (A) Prisma
(B) Shelby
(C) Kooper
(D) Wendy
(E) Caret
8. For which factory was the number of trousers manufactured in February and March together the lowest among the five factories?
- (A) Caret
(B) Wendy
(C) Kooper
(D) Shelby
(E) Prisma

Directions for Questions 9–12: Answer the questions on the basis of the information given below.

The table given below* shows the number of two-wheelers (motorised) running on the roads of a country XYZ during the period 2006 – 2010. These are the only type of two-wheelers on the roads of the country.

	2006	2007	2008	2009	2010
VTS	1,120	1,300	1,800	1,900	2,100
SULPAR	194,830	249,200	266,000	325,000	438,200
SCHAPE	35,600	42,300	43,300	49,200	57,000
RHONDA	417,500	479,200	403,400	416,700	471,000
MAHA	143,600	172,600	150,400	162,400	193,800
SICTOR	1,100	1,600	2,300	3,000	2,400
MUZUKI	114,500	137,600	121,700	125,700	158,000
BAJAZ	58,800	65,000	64,900	68,000	80,400
SLOGAN	12,400	14,400	15,000	16,500	21,000

*Table for Questions 9–12.

9. The second highest annual growth over the entire period has been experienced by which of the following two-wheelers:
 - (A) RHONDA
 - (B) SICTOR
 - (C) VTS
 - (D) SCHAPE
 - (E) BAJAZ

10. What is the difference between the total numbers of two-wheelers on the roads of the country from the year 2007 to year 2009?
 - (A) 4,900
 - (B) 4,500
 - (C) 5,100
 - (D) 5,200
 - (E) 5,400

11. What is the approximate percentage of RHONDA two-wheelers out of the total number of two-wheelers on the roads of the country in the year 2008?
 - (A) 30%
 - (B) 33.33%
 - (C) 35%
 - (D) 38%

(E) 45%

- 12.** What is the average number of two-wheelers running on the roads of the country in the year 2008?
- (A) 118,756
(B) 118,765
(C) 119,576
(D) 181,756
(E) 191,756

Directions for Questions 13–15: Read the information given below and answer the questions that follow.

The table given at the bottom* of this page depicts the marks obtained by 1,000 students in English and Computer Science in an entrance exam conducted by JET (Junior Entrance Test)

- 13.** What is the difference between the percentage of students who secured more than 60% marks in aggregate and those who secured more than 40% marks in aggregate?
- (A) 0%
(B) 27%
(C) 46%
(D) 54%
(E) 73%
- 14.** What is the total number of students securing more than 20 marks in English and 40 marks in Computer Science?
- (A) 40
(B) 70
(C) 260
(D) 840
(E) Cannot be determined

- 15.** The percentage of the number of students securing more than 60% marks in Computer Science is approximately what percent of those getting more than 40% marks in aggregate?
- (A) 20%
(B) 29%
(C) 31%
(D) 36%
(E) 42%

Directions for Questions 16–19: The following table** shows the number of people employed in public and private sector in a country in the years 2005, 2006, 2007 and 2008. All the values are in '000. (**Real NMAT Question**)

- 16.** What is the ratio of private sector employment to public sector employment in 2008?
- (A) 14 : 25
(B) 14 : 23
(C) 7 : 11
(D) 2 : 31
(E) 7 : 10
- 17.** What is the difference of yearly growth rate in employment between 2006 – 2007 and 2007 – 2008?
- (A) 0.0002
(B) 0.0003
(C) 0.0006
(D) 0.0007
(E) 0.0009
- 18.** What percentage of the total employment did the private sector contribute in 2006?
- (A) 31.9%
(B) 32.2%

- (C) 32.3%
 (D) 32.6%
 (E) 32.7%
- 19.** What percentage of the total employment did the public sector contribute in 2007?
- (A) 64.9%
 (B) 65.2%
 (C) 66.0%
 (D) 66.2%
 (E) 66.7%

Subject	Marks out of 50	>40	>30	>20	>10	>0
English	90	320	800	920	1,000	
Computer Science	40	210	660	810	1,000	
Average marks per subject	70	270	730	870	1,000	

*Table for Questions 13–15.

Year	Public Sector					Private Sector			
	Central Govt	State Govt	Quasi Govt		Local Bodies	Large Estt.	Small Estt.	Total	
			Central	State					
2005	2,938.5	7,201.9	3,284.7	2,463.6	2,117.9	7,489.1	962.7	26,458.4	
2006	2,860.0	7,300.0	3,469.0	2,440.0	211.0	7,804.0	1,001.0	26,992.0	
2007	2,800.0	7,209.9	3,447.0	2,414.0	2,132.0	8,229.0	1,046.0	27,277.0	
2008	2,739.0	7,171.0	3,389.0	2,407.0	1,968.0	8,832.0	1,043.0	27,549.0	

**Table for Questions 16–19.

Directions for Questions 20–23: The following table gives the percentage breakdown of the total marks obtained by 5 students A, B, C, D and E in 6 subjects, P, Q, R, S, T and U, of their final exams. The maximum marks in each subject are 100 and every student got an integral score. (**Real NMAT Question**)

	P	Q	R	S	T	U
A	10	14	22	14	18	22
B	12	17	20	19	15	17
C	15	10	20	15	20	20
D	15	17	19	25	14	10
E	16	18	16	18	18	14

- 20.** What can be the maximum possible total score achieved by any of these 5 students?
- (A) 450
 - (B) 500
 - (C) 550
 - (D) 580
 - (E) 600
- 21.** If the pass mark in each subject is 20 and each student passed in all subjects, then what can be the minimum total score obtained by any of the 5 students?
- (A) 120
 - (B) 150
 - (C) 175
 - (D) 200
 - (E) 250
- 22.** What can be the maximum possible total score for C?
- (A) 400
 - (B) 450
 - (C) 500
 - (D) 550
 - (E) 600
- 23.** If out of these 5 students, E scored the highest marks in subject S, then his total score in all six subjects is definitely more than how many students?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

Directions for Questions 24–27: Go through the following information and answer the questions based on it.

An experiment was conducted to study the effect of acid rain on five species of aquatic animals that were released in a lake. The original pH of the lake was 6.5. (**Real NMAT Question**)

Effect of Acid Rain on Aquatic Species

Aquatic Species	Number Released at pH 6.5	pH Tolerance of Different Aquatic Species					
		pH 6.5	pH 6.0	pH 5.5	pH 5.0	pH 4.5	pH 4.0
Trout	24	✓	✓	✓	✓	✗	✗
Bass	82	✓	✓	✓	✗	✗	✗
Perch	40	✓	✓	✓	✓	✓	✗
Frog	73	✓	✓	✓	✓	✓	✓
Clam	12	✓	✓	✗	✗	✗	✗

✓ Indicates pH levels which can be tolerated

✗ Indicates pH levels which cannot be tolerated

- 24.** Approximately how many times greater is the total number of aquatic animals that will most likely not survive when the pH changes from 6.5 to 4 than the total number of aquatic animals that will most likely not survive when the pH changes from 6.5 to 4.5?

- (A) 0.5 times
- (B) 1.3 times
- (C) 2.5 times
- (D) 1 times
- (E) 7.5 times

- 25.** The pH of the lake first decreased because of acid rain from 6.5 to 4.5, and then the pH was increased artificially from 4.5 to 5.5. If 12 aquatic animals of each species were then introduced into the lake at pH 5.5, how many species would most likely record greater than 60% change in the number of animals at pH 5.5 over pH 6.5?
- (A) 1
(B) 2
(C) 3
(D) 4
(E) 5
- 26.** Assuming that the original pH level of the lake was 6.5, at what pH level would approximately 59.3% of the original number of aquatic animals that were released in the lake most likely survive?
- (A) pH 6.0
(B) pH 5.5
(C) pH 5.0
(D) pH 4.5
(E) pH 4.0
- 27.** If the pH of the lake decreases from 6.5 to 4.5 because of acid rain, what percentage of the total number of aquatic animals that were released into the lake at pH 6.5 are expected to survive?
- (A) 17.32%
(B) 23.10%
(C) 31.60%
(D) 48.92%
(E) 52.64%

Directions for Questions 28–30: Read the information given below and answer the questions that follow.

The table below* depicts the number of students of five engineering colleges A, B, C, D and E who were placed in different companies during campus placement drives.

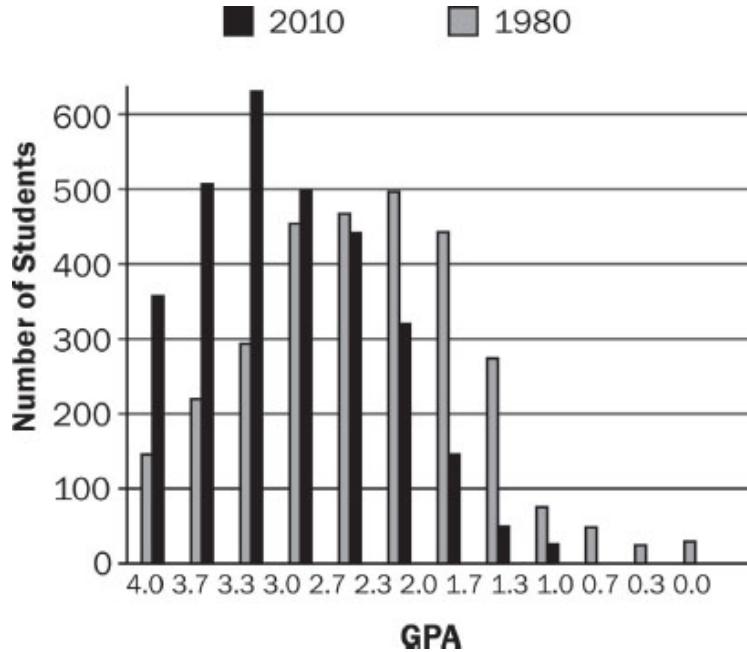
Colleges	Mechanical Engineering		Electrical Engineering		Electronics Engineering	
	Total Students	Selected	Total Students	Selected	Total Students	Selected
A	60	40	60	24	50	15
B	60	30	60	36	50	35
C	60	24	60	30	50	40
D	60	35	60	18	50	25
E	60	18	60	12	50	42

*Table for Questions 28–30.

- 28.** What is the approximate percentage of students of college C who got selected during campus placement drives?
- (A) 45%
 - (B) 50%
 - (C) 55%
 - (D) 60%
 - (E) 65%
- 29.** What is the percentage of Mechanical Engineering students from all the colleges who got selected during campus placement drives?
- (A) 40%
 - (B) 44%
 - (C) 49%
 - (D) 51%
 - (E) 53%
- 30.** What is the total number of students of Electrical Engineering from all the colleges who got selected during campus placement drives?
- (A) 100
 - (B) 110
 - (C) 120
 - (D) 130
 - (E) 136

5 DI-Graphs and Charts

Directions for Questions 1–4: Refer to the following graph and answer the questions.



Comparison of GPAs of 3000 students in 1980 and in 2010

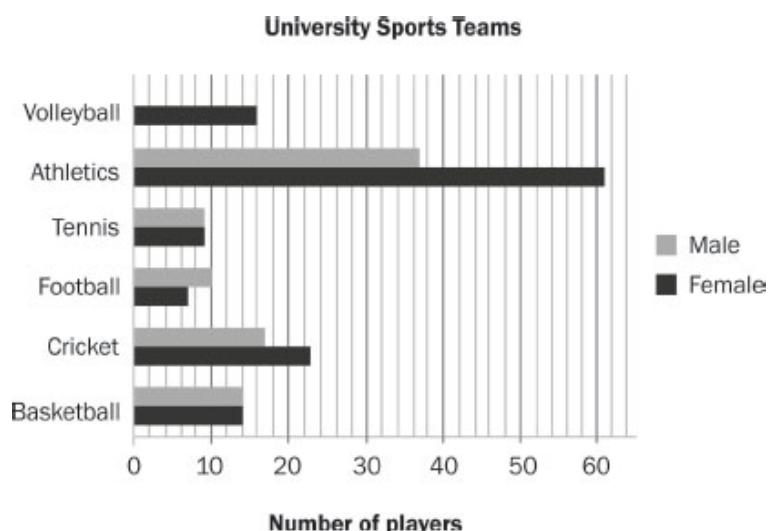
1. What was the mode for the GPA among the 3,000 students in 2010?
 - (A) 3.7
 - (B) 3.3
 - (C) 3.0
 - (D) 2.7
 - (E) 2.3

2. What was the median GPA among the 3,000 students in 1980?
 - (A) 3.7
 - (B) 3.3
 - (C) 3.0
 - (D) 2.7

(E) 2.3

3. Approximately what percentage of the students in 2010 earned at least a 3.0 GPA?
- (A) 25%
(B) 50%
(C) 67%
(D) 80%
(E) 97.5%
4. Approximately what percentage of the students in 1980 earned a GPA less than 3.0?
- (A) 33%
(B) 37.5%
(C) 50%
(D) 62.5%
(E) 75%

Directions for Questions 5–8: Go through the given graph and solve the questions based on it.



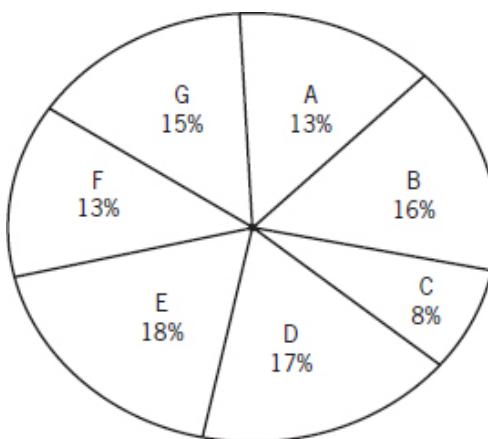
5. What is the ratio of male players to female players on the Athletics team?

- (A) 37 : 61
 - (B) 9 : 17
 - (C) 16 : 23
 - (D) 14 : 19
 - (E) 61 : 37
6. All players, except those in Athletics and Cricket teams, are a part of only one team. If there are a total of 76 male players in different university sports teams, how many male players are in both Athletics team and Cricket team?
- (A) 11
 - (B) 17
 - (C) 37
 - (D) 54
 - (E) 76
7. In which of the following university sports team(s) do male players outnumber female players?
- (A) Athletics, Tennis and Football
 - (B) Cricket
 - (C) Football and Cricket
 - (D) Football
 - (E) Tennis and Athletics
8. What is the ratio of female tennis players to male basketball players on the university sports teams?
- (A) 5 : 14
 - (B) 9 : 14
 - (C) 7 : 18
 - (D) 14 : 9
 - (E) 18 : 7

Directions for Questions 9–12: Study the following chart to answer the question given below:

Town	Percentage of Population Below Poverty Line
A	45
B	52
C	38
D	58
E	46
F	49
G	51

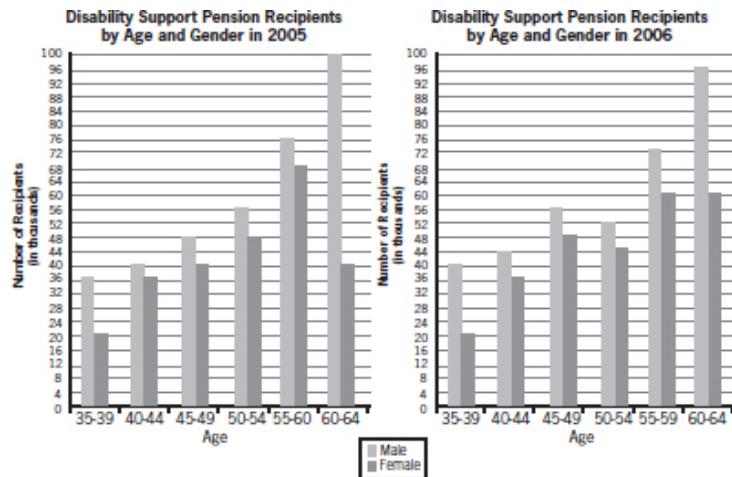
Percentage distribution of the population of seven towns of the state in 2005.



9. In 2006, the populations of Town A and Town B each increased by 10% as compared to 2005. If the population of Town A in 2005 was 5,000 and the percentage of the population living below the poverty line for all seven towns in 2006 remains the same as in 2005, which of the following is the approximate population of Town B below the poverty line in 2006?
- (A) 2,500
(B) 3,000
(C) 3,500
(D) 4,000
(E) 4,500

- 10.** In 2007, the population of Town D increased by 10% as compared to 2005 and the population of Town G reduced by 5% as compared to 2005. If the population of Town G in 2005 was 9,000, what is the total population of Towns D and G in 2007?
- (A) 19,200
(B) 19,770
(C) 19,870
(D) 19,970
(E) None of these
- 11.** If in 2005 the total population of the seven towns together was approximately 55,000, what will be the approximate population of Town F in that year below the poverty line.
- (A) 2,500
(B) 3,000
(C) 3,500
(D) 4,000
(E) 4,500
- 12.** The population of Town C is 2,000 in 2005. What will be the ratio of the population of Town C below the poverty line to that of Town E below the poverty line in that year?
- (A) 207 : 76
(B) 76 : 207
(C) 152 : 207
(D) 76 : 307
(E) 87 : 207

Directions for Questions 13–16: Go through the given graphs and solve the questions based on them. (*Real NMAT Question*)

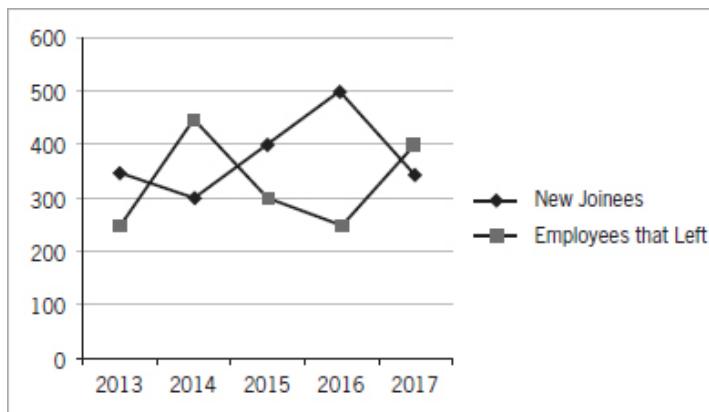


- 13.** What is the approximate percentage increase in the number of recipients in the 60–64 age group between 2005 and 2006?
- 5.5%
 - 10.2%
 - 11.4%
 - 16%
 - 45%
- 14.** In each year, there was a total of 150,000 recipients in the three age groups that are immediately before 45–49. What was the approximate percentage decrease in the number of recipients in the 30–34 age group from 2005 to 2006?
- 6%
 - 7%
 - 18%
 - 44%
 - 80%
- 15.** What is the approximate percentage decrease in the total number of recipients aged above 49 years and below 65 years from 2005 to 2006?
- 1%
 - 4.2%

- (C) 5.1%
(D) 7.8%
(E) 100%
16. The number of male recipients between the age of 35 and 69 decreased by 5% from 2005 to 2006. If there were 20,000 male recipients in the age group of 65-69 in the year 2006, how many male recipients were in the same age group in the year 2005?
- (A) 8,000
(B) 10,000
(C) 18,000
(D) 20,000
(E) 44,000

Directions for Questions 17–19: Answer the questions on the basis of the information given below.

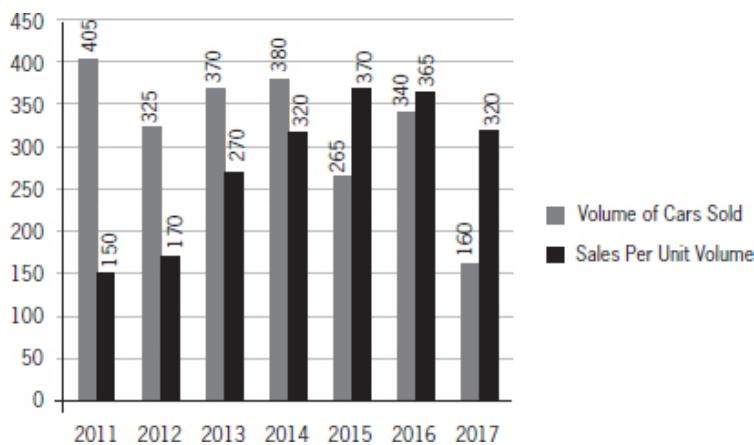
The line graph below depicts the number of employees who left the company ABC Pvt. Ltd. and the number of new joinees in that year. Also, it is known that the number of employees in the year 2012 was 2,000.



17. What was the percentage change in number of employees in the company from the year 2012 to year 2013?
- (A) 2%
(B) 3%
(C) 4%
(D) 5%

- (E) Data insufficient
18. In which of the following years was the number of employees in ABC Pvt. Ltd. the maximum?
- (A) 2014
(B) 2015
(C) 2016
(D) 2017
(E) None of these
19. In which of the following two years was the number of employees in ABC Pvt. Ltd. the same?
- (A) 2013 and 2017
(B) 2013 and 2015
(C) 2016 and 2017
(D) 2014 and 2017
(E) None of these

Directions for Questions 20–22: Answer the questions on the basis of the information given below.

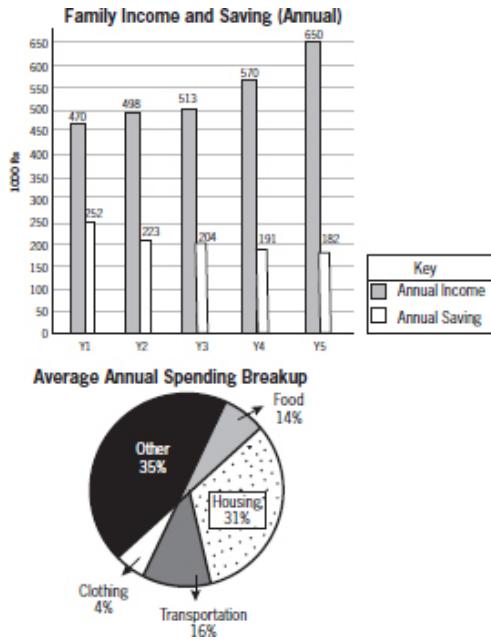


The bar chart represents the volume of cars sold in a particular year and the sales per unit volume of a particular year.

20. By what percentage is the total volume of sales in the year 2015 greater/smaller than that in 2011?

- (A) 45.6%
 - (B) 50.3%
 - (C) 54.2%
 - (D) 61.4%
 - (E) 66.67%
- 21.** What is the difference between the average of the volume of cars sold and that of the sales per unit volume for the whole period?
- (A) 40.00
 - (B) 41.57
 - (C) 43.21
 - (D) 45.12
 - (E) 50.73
- 22.** In how many of the years, the trend is such that when there is an increase in volume sold over the previous year, then there is a decrease in the sales per unit volume over the previous year and vice versa?
- (A) 1
 - (B) 2
 - (C) 3
 - (D) 4
 - (E) 5

Directions for Questions 23 to 26: Go through the given graphs and solve the questions based on them. (*Real NMAT Question*)



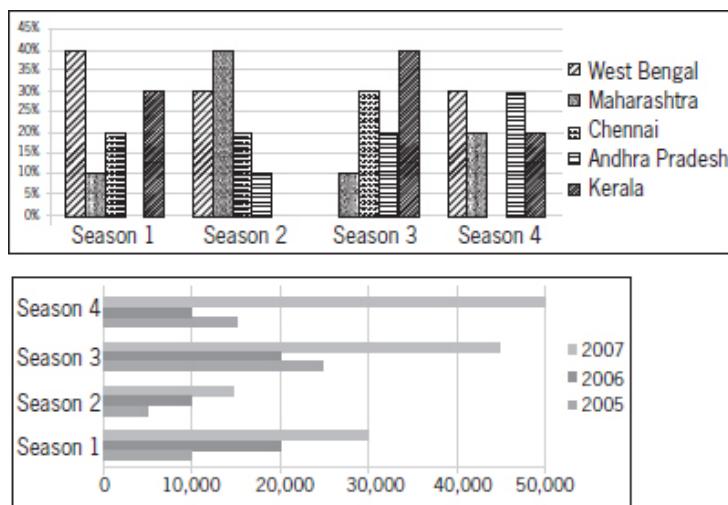
- 23.** The annual family spending in the year preceding Y1 was Rs. 150,000. Which year recorded the lowest percentage increase in the annual family spending over the preceding year?
- (A) Y1
 - (B) Y2
 - (C) Y3
 - (D) Y4
 - (E) Y5
- 24.** What was the total increase in the amount of money the family spent annually on buying food and clothing in Y2 over Y1?
- (A) Rs. 10,260
 - (B) Rs. 20,520
 - (C) Rs. 28,980
 - (D) Rs. 39,240
 - (E) Rs. 49,500
- 25.** In which year did the family spend a total of exactly Rs. 60,640 on transportation?
- (A) Y1

- (B) Y2
- (C) Y3
- (D) Y4
- (E) Y5

- 26.** On which item did the family spend between Rs. 1 lakh and Rs. 1.2 lakh annually in Y4?
- (A) Food
 - (B) Housing
 - (C) Transportation
 - (D) Clothing
 - (E) Other

Directions for Questions 27–30: Go through the following graphs and answer the questions based on them.

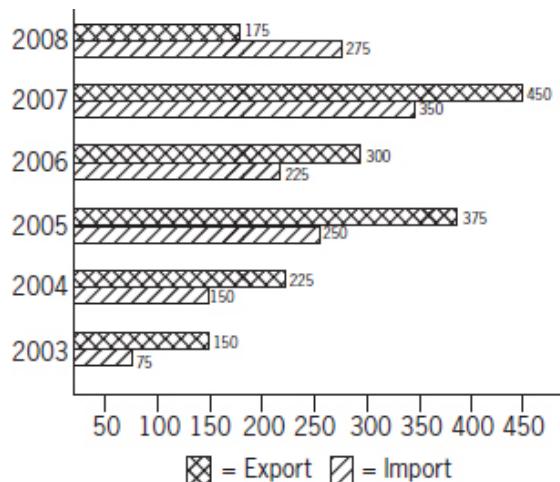
The given bar graphs represent the runs scored by the Karnataka Ranji team in domestic cricket. Bar chart 1 gives the details of the runs scored by Karnataka against each of the 5 states as a percentage of the total runs scored by Karnataka. Chart 2 gives the details of the runs scored in each of the four seasons of three different years. (**Real NMAT Question**)



Assume that the percentage break up of the runs scored against each state in each season was the same for all three years.

- 27.** Against which state did the Karnataka team score the maximum runs in the year 2005?
- (A) Kerala
 - (B) Chennai
 - (C) Maharashtra
 - (D) West Bengal
 - (E) Andhra Pradesh
- 28.** What was the percentage change in the runs scored against Chennai from 2005 to 2006?
- (A) 14.3%
 - (B) 17.2%
 - (C) 18.2%
 - (D) 20.2%
 - (E) 33.2%
- 29.** In which year and season did the runs scored against Maharashtra show the maximum change over the previous season in the year?
- (A) Season 3, year 2005
 - (B) Season 4, year 2005
 - (C) Season 2, year 2006
 - (D) Season 2, year 2007
 - (E) Season 4, year 2007
- 30.** Which of the following is the maximum difference between the runs scored against any two states in any season?
- (A) 12,000
 - (B) 13,000
 - (C) 14,500
 - (D) 15,500
 - (E) 18,000

Directions for Questions 31–34: Use the following chart, which represents the value of exports and imports (in Rs. hundred crore) of a country for a certain period, to answer the given questions. (*Real NMAT Question*)



- 31.** During which year is the percentage increase/decrease in imports from the previous year the lowest?
- (A) 2003
 - (B) 2004
 - (C) 2005
 - (D) 2006
 - (E) 2007
- 32.** What is the ratio of total imports to total exports for all the given years together?
- (A) 35 : 36
 - (B) 36 : 83
 - (C) 38 : 37
 - (D) 53 : 67
 - (E) 53 : 87
- 33.** During which period is the total value of import equal to the total value of export (in Rs. hundred crore)?
- (A) 2003–04
 - (B) 2004–07

- (C) 2005–07
 - (D) 2006–08
 - (E) 2007–08
- 34.** What percentage is the total export in the years 2003, 2006 and 2008 taken together of the total import for the same period?
- (A) 109%
 - (B) 110%
 - (C) 111%
 - (D) 112%
 - (E) 113%

6 Data Sufficiency

Directions for Questions 1 to 31: A question is followed by two statements, numbered (1) and (2). Using the information provided and general knowledge, decide whether the information given is sufficient to solve the problem.

- (A) Statement (1) ALONE is sufficient, but statement (2) ALONE is not sufficient.
 - (B) Statement (2) ALONE is sufficient, but statement (1) ALONE is not sufficient.
 - (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
 - (D) EACH statement ALONE is sufficient.
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient.
1. Three-fourths of the boys in a class are intelligent and there are 18 intelligent boys in the class. What is the number of girls in the class? **(Real NMAT Question)**
 - (1) Boys comprise two-thirds of the total number of students in the class; the rest are girls.
 - (2) The number of girls is less than the number of boys.
 2. In a group of 150 students, find the number of girls. **(Real NMAT Question)**
 - (1) Each girl was given 50 paisa, while each boy was given 25 paisa to purchase goods totaling Rs. 49.
 - (2) Girls and boys were given 30 paisa each to buy goods totaling Rs. 45.
 3. What percentage of a group of people are men with MBA degrees?
 - (1) Of all the men in the group, 25% have MBA degrees.
 - (2) Of all the women in the group, 30% have MBA degrees.

- 4.** In a classroom, one student is to be selected at random to solve a question. What is the probability that a girl will be selected?
- (1) Two-fifths of the students in the classroom are boys.
(2) 15 of the students in the classroom are boys.
- 5.** Is $x^2 + y^3$ odd? (*Real NMAT Question*)
- (1) $x^2y = 48$
(2) $xy^2 = 36$
- 6.** If a is an integer, is $a + 1$ even?
- (1) $a + 2$ is an even integer.
(2) $a - 1$ is an odd integer.
- 7.** If Udit saved Rs. 1200 of his earnings last month, how much did Udit earn last month?
- (1) Udit spent $\frac{1}{2}$ of his earnings last month on household expenses and saved $\frac{1}{4}$ of the remainder.
(2) Of his earnings last month, Udit paid twice as much in rent as he saved.
- 8.** Water is pumped into a partially filled swimming pool at a constant rate through an inlet pipe. At the same time, water is pumped out of the pool at a constant rate through an outlet pipe. At what rate, in litres per minute, is the amount of water in the pool increasing?
- (1) The amount of water initially in the pool is 500 litres.
(2) Water is pumped into the pool at a rate of 50 litres per minute and out of the pool at a rate of 20 litres every 4 minutes.
- 9.** What is the price of tea?
- (1) The price of coffee is Rs. 5 more than that of tea.
(2) The price of coffee was Rs. 5 less than the price of a cold drink, which cost three times the price of tea.
- 10.** What is the cube root of y ?

- (1) The 7th root of y is 12.
(2) The 22nd root of y is 2.
- 11.** In triangle ABC, if $AB = x$ units, $BC = x + 4$ units, and $AC = y$ units, which of the three vertex angles of triangle ABC has the greatest degree measure?
(1) $y = x + 8$
(2) $x = 4$
- 12.** What percentage of Debating club members enrolled at a certain school are from India?
(1) Of the Indian students enrolled at the school, 20% are members of the Debating club.
(2) Of the non-Indian students enrolled at the school, 45% are members of the Debating club.
- 13.** If the sequence S has 150 terms, what is the 121st term of S ?
(1) The first term of S is 32.
(2) The 138th term of S is 1248, and each term of S after the first is 18 more than the preceding term.
- 14.** What is the selling price of the mixture if the ratio of the two qualities of tea mixed is 3:4? (**Real NMAT Question**)
(1) Cost price of the first quality of tea is Rs. 180 per kg.
(2) Cost price of the second quality of tea is Rs. 225 per kg.
- 15.** What is the number of trailing zeros when $P!$ is converted to base 'x'? (**Real NMAT Question**)
(1) $100 < P < 105$
(2) 'x' can be 53 or 59.
- 16.** The absolute difference between a two digit number and the number formed by reversing the digits of that number is D. What is the number?
(1) $D = 36$
(2) The sum of the digits of the number is 12.

- 17.** Who is the shortest among the five friends A, B, D, E and F?
- D is taller than F but shorter than A and F is not the shortest.
 - E is shorter than B.
- 18.** If a, b and c are digits, is $(a + b + c)$ a multiple of 9? (A digit can be one of the integers 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9.)
- The three digit number abc is a multiple of 9.
 - $[(a \times b) + c]$ is a multiple of 9.
- 19.** What is the value of X, if X is an integer?
- $X = \sqrt{9}$
 - $X^2 - 1 < 0$
- 20.** Is parallelogram ABCD a rhombus?
- The four triangles enclosed by the diagonals and the sides have equal areas.
 - A circle can be inscribed in ABCD touching all the four sides.
- 21.** Is $a = b = c = 1$?
- $a^2 + b^2 + c^2 = ab + bc + ca$
 - $a^2 + b^2 = 2c^2$
- 22.** Pipe A can fill a tank in 'a' hours and pipe B can fill the same tank in 'b' hours. If both the pipes are opened together for 2 hours, then what is the volume (in cc) of water in the tank after 2 hours?
- $a = 6$ and $b = 8$
 - Volume of the tank is 100 cc
- 23.** Four lectures Arithmetic, Biology, Chemistry and Dermatology were scheduled, one on each day on four consecutive days, but not necessarily in that order. On which day was Chemistry scheduled?
- The first lecture was scheduled on Monday, 14th January 2016 and was followed by Dermatology.
 - Arithmetic was not scheduled on 16th January 2016 and there was a gap of one day between Arithmetic and biology.

- 24.** Are the integers A and B co-primes? (*Real NMAT Question*)
- The numbers A and B are the squares of two successive even numbers.
 - Both A and B are distinct primes.
- 25.** Ramu and Somu were born into a family who had worked in the textile business for generations. While their ancestors believed that a zero clash of professional interests could be ensured only by family members taking up different segments of the textile business, the two brothers felt that if they both worked on the same segment, complementing each other but engaging in healthy competition, it would bring about brighter business prospects. Both of them stocked and sold the same merchandise and often bought textile goods from the same source, though not necessarily on the same terms and conditions.
- The two brothers bought 4 items of antique artefacts to decorate their showroom. However, on persistent requests from customers, both sold the two pieces each had kept. Whose transaction resulted in a better profit? (*Real NMAT Question*)
- Ramu sold one item at p% profit and the other at p% loss though he had bought both items at the same price.
 - Somu made q% profit on one item and on the other q% loss though he sold both items at the same price.
- 26.** A taxi charges 50 dollars when the distance is less than or equal to 300 metres and 'A' dollars for each additional 100 metres. Kamal hires a taxi from city P to city Q. Manik Lal hires a taxi from city Q to city R. If Mahesh hires a taxi from city P to city R via city Q, how much did Mahesh need to pay for the taxi? (*Real NMAT Question*)
- Kamal pays 75 dollars.
 - Manik Lal pays 95 dollars.
- 27.** The sum of the ages of three persons A, B and C is 167 years. What is the age of B? (*Real NMAT Question*)
- B is $\frac{17}{12}$ times as old as A.

- (2) C is 44 years older than A.
- 28.** Anshuman bought two articles from a sale and sold them to his friend Ankit. Did Anshuman make a profit or a loss? (*Real NMAT Question*)
- (1) Anshuman sold both the articles at the same price.
(2) Of the two articles sold, Anshuman made a profit of p% on one and incurred a loss of p% on the other.
- 29.** Sunita sold article A at x% profit. Find the price at which she bought A. (*Real NMAT Question*)
- (1) If Sunita sold A at (x - 10)% profit, her profit would have been Rs. 12 less.
(2) If Sunita sold A at (x + 20)% profit, her profit would have been Rs. 24 more.
- 30.** Find the value of $\frac{x^3}{y^2+1}$. (*Real NMAT Question*)
- (1) $\frac{x^2}{y^2} = \frac{1}{4}$
(2) $\frac{x}{y} = \frac{1}{2}$
- 31.** If Kriti paid a total of Rs. 1,350 for two dresses and sold one at 6% loss and the other at 7.5% profit, what was the price she paid for each dress? (*Real NMAT Question*)
- (1) Kriti made neither a profit nor a loss from the transaction.
(2) Kriti sold one dress for Rs. 705.

3.2 Answers and Explanations

The following discussion is intended to familiarise you with the most efficient and effective approaches to the kinds of problems common to quantitative skill questions. The particular questions in this chapter are generally representative of the kinds of problem-solving questions you will encounter on the NMAT exam. Remember that it is the problem-solving strategy that is important, not the specific answers or details of a particular question.

1 Number Properties

$$\sqrt{12} = 2\sqrt{3}$$

1. $\sqrt{60} = 2\sqrt{15}$

$$\sqrt{45} = 3\sqrt{5}$$

Thus, we get

$$\begin{aligned}\frac{2\sqrt{3}}{5} \times \frac{2\sqrt{15}}{2^4} \times \frac{3\sqrt{5}}{3^2} &= \frac{\sqrt{3}}{5} \times \frac{\sqrt{15}}{2^2} \times \frac{\sqrt{5}}{3^1} \\&= \frac{\sqrt{3}}{5} \times \frac{\sqrt{15}}{2^2} \times \frac{\sqrt{5}}{3^1} = \frac{\sqrt{3}}{5} \times \frac{\sqrt{3}\sqrt{5}}{2^2} \times \frac{\sqrt{5}}{3} \\&= \left(\frac{3 \times 5}{2^2 \times 3 \times 5} \right) = \frac{1}{4}\end{aligned}$$

The correct answer is C.

2.

$$0.\bar{3} = 0.3333 = \frac{1}{3}$$

$$\text{Thus, } 0.00\bar{3} = \frac{1}{3} \times \frac{1}{100} = \frac{1}{300}$$

$$\text{Thus, } 7.58\bar{3} = 7.58 + 0.00\bar{3}$$

$$\begin{aligned}&= \frac{758}{100} + \frac{1}{300} \\&= \frac{2275}{300} = \frac{91}{12}\end{aligned}$$

The correct answer is A.

3. $\frac{1}{3}$ of all the pencils is 5 pencils. These 5 pencils each cost Rs. 2, for a total of Rs. 10.

The remaining 10 pencils cost Rs. 5 each, for a total of Rs. 50.

If all of these more expensive pencils are lost, then the lost pencils represent $\frac{50}{60}$ of all the money paid.

$$\text{Thus, required \%} = \frac{50}{60} \times 100 = \text{approximately } 83\%$$

The correct answer is D.

4. The answer is the greatest common factor (GCF) of the two numbers.

$$\begin{aligned}4,000 &= 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \\&= 2^5 \times 5^3\end{aligned}$$

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$$

These numbers have $2 \times 2 \times 5$ in common, so 20 is the GCF.

The correct answer is C.

5. If t is divisible by 12, then t^2 must be divisible by 144 or $2 \times 2 \times 2 \times 2 \times 3 \times 3$. Therefore, t^2 can be divided evenly by 2 at least four times, so a must be at least 5.

The correct answer is C.

6. The key to solving this problem is realising that

$$5^{k+1} = 5^k 5^1$$

$$\text{Now, } 5^{k+1} = 2,000$$

$$5^k 5^1 = 2,000$$

Now divide both sides by 5:

$$5^k = 400$$

$$\text{So, } 5^k + 1 = 401$$

The correct answer is B.

7. Since a square root is the same as a $\frac{1}{2}$ exponent and a cube root is the same as a $\frac{1}{3}$ exponent.

Therefore, converting the given expression into fractional exponents, we get

$$x^{\frac{1}{2}}x^{\frac{1}{3}} = x^{\frac{1}{2} + \frac{1}{3}} = x^{\left(\frac{3+2}{6}\right)} = x^{\frac{5}{6}} = \sqrt[6]{x^5}$$

The correct answer is E.

8. In order to answer this question, you have to understand what creates zeroes at the end of a number. You will notice that zeroes are created by 10's, each of which is created by one 2 and one 5. So to answer this question, you simply need to work out how many pairs of 2's and 5's are in the given expression

$$125^{14}48^8 = (5^3)^{14} \times (2^4 \times 3)^8 = 5^{42} \times 2^{32} \times 3^8$$

Even though there are 42 powers of 5, there are only 32 powers of 2, so you can only make 32 pairs of one 5 and one 2.

Therefore, the number of consecutive zeroes at the end = 32

The correct answer is B.

9. For three consecutive integers, the possibilities are [odd, even, odd] or [even, odd, even].

Since n could be an odd or an even, option (B) and (C) are eliminated.

Option (D) is true only if n is even, but not if n is odd, so option (D) is also eliminated.

In any set of three consecutive integers, one of the integers must be divisible by 3, but not necessarily n , so option (A) is also eliminated.

For the same reason, (E) must be true, as $n(n + 1)(n + 2)$ can be thought of as "the product of any three consecutive integers." Since one of these integers must be divisible by 3, the product of those three numbers must also be divisible by 3.

The correct answer is E.

10. Decimal placement can be determined by how many times a number is multiplied or divided by 10. Multiplying moves the decimal point to the right, and dividing moves the decimal point to the left. Look for powers of 10 in the given fraction, remembering that $10 = 2 \times 5$.

$$\begin{aligned}\frac{17}{2^{10}5^{13}} &= \frac{17}{2^{10}5^{10}5^3} = \frac{17}{(2 \times 5)^{10}5^3} \\ &= \frac{17}{10^{10}5^3} = \frac{17/125}{10^{10}} = \frac{0.136}{10^{10}}\end{aligned}$$

There is no zero to the right of the decimal point before the first non-zero digit in 0.136. However, dividing by 10^{10} will move the decimal to the left 10 places, resulting in 10 zeros between the decimal and the '136' part of the number.

The correct answer is A.

- 11.** Note that $25 \times 4 = 100$, and the other side of the equation involves a power of 10. Separating out the 'pairs' of 25 and 4 on the left, we have:

$$25^54^6 = 10^x + a$$

$$4^1(4^525^5) = 10^x + a$$

$$4^1(100)^5 = 10^x + a$$

$$4(10^{10}) = 10^x + a$$

$$40,000,000,000 = [\text{a power of } 10] + a$$

To minimise 'a' while keeping it positive, maximise the power of 10 while keeping it less than 25^54^6 .

The greatest power of 10 that is less than 40,000,000,000 is 10,000,000,000, or 10^{10} .

Thus:

$$40,000,000,000 = 10,000,000,000 + a$$

$$30,000,000,000 = a$$

The correct answer is E.

- 12.** The units digits of 7 to positive integers create a repeating pattern (this works for digits other than 7 also). By multiplying 7 by itself repeatedly in the calculator, you can generate the pattern:

$$7^1 = 7$$

$$7^2 = 49$$

$$7^3 = 343$$

$$7^4 = 2,401$$

$$7^5 = 16,807$$

$$7^6 = 117,649$$

$$7^7 = 823,543$$

$$7^8 = 5,764,801$$

Pattern: 7, 9, 3, 1

To find the 86th item in a pattern of 4 repeating items, find the remainder when 86 is divided by 4, that is, 2. The second element in the pattern is 9.

The correct answer is E.

- 13.** Remember that when dealing with evenly spaced integers, the average is the middle value.

The sum of the even integers between 1 and 100 = $51 \times 50 = 2550$

The sum of the odd integers between 100 and 150 = $125 \times 25 = 3125$

The required difference = $2550 - 3125 = -575$

The correct answer is A.

- 14.** This question can easily be solved by looking at the options. Note that the number should be divisible by 13 and should leave remainders 3, 6, and 10, respectively when divided by 5, 8 and 12.

169: Incorrect. Divisible by 13 but eliminated as does not leave remainder 3 when divided by 5.

478: Incorrect. Not divisible by 13.

598: Correct. Divisible by 13 and leaves remainders 3, 6, and 10, respectively when divided by 5, 8 and 12.

1298: Incorrect. Not divisible by 13.

1602: Incorrect. Not divisible by 13.

The correct answer is C.

15. For $n = 0$, $2^{2n} + 1 = 2^0 + 1 = 2^1 = 2$

For $n = 1$, $2^{2n} + 1 = 2^2 + 1 = 2^3 = 8$

For $n = 2$, $2^{2n} + 1 = 2^4 + 1 = 2^5 = 32$

For $n = 3$, $2^{2n} + 1 = 2^6 + 1 = 2^7 = 128$

For $n = 4$, $2^{2n} + 1 = 2^8 + 1 = 2^9 = 512$

Therefore, the sum of the last digits of the above numbers = 2 + 8 + 2 + 8 + 2 = 22

When 22 is divided by 7, it gives a remainder of 1.

The correct answer is A.

16. If a and b are odd prime numbers, then

$a^2 + b^2 = \text{even number}$

$a + b + 1 = \text{odd number}$

$a^2 + b^2 - 1 = \text{odd number}$

Therefore, the H.C.F. of these numbers will be 1.

Similarly for the case $a = b = 2$, the H.C.F. will be 1.

When only one of them is equal 2, it will again give the H.C.F. as 1.

Alternatively,

$a^2 + b^2 - 1$ and $a^2 + b^2$ are two consecutive positive integers, whose H.C.F. will always be equal to 1.

The correct answer is A.

- 17.** The two digits at unit's place and ten's place can be only 2 or 3. So, the hundredth digit must be 6. So, the number is either 632 or 623. The difference between 623 and 326 is 297.

The correct answer is A.

- 18.** Such questions can be solved easily on working with the answer choices.

Checking option (C), we get

$$\frac{24}{6} = 4 \text{ and } 24 \times 2 = 48$$

Digits when reversed, number is 42.

So, the difference is $(48 - 42) = 6$

The correct answer is C.

- 19.** If we assume that any digit out of five digits is at fixed position, then the remaining four digits can be arranged in $4!$ (24) ways. So, each of the five digits will appear at each of the five places $4!$ (24) times.

So, the sum of the digits at each position is 24

$(1 + 3 + 5 + 7 + 9) = 600$ and hence the sum of all such numbers will be

$$= 600(1 + 10 + 100 + 1000 + 10000) = 6666600$$

The correct answer is A.

- 20.** $180 = 2^2 \times 3^2 \times 5^1$

We know that for a number $N = a^p \times b^q \times c^r$, the number of factors of N is calculated as:

$$n = (p + 1) \times (q + 1) \times (r + 1)$$

Now, we should try to break-up 40 as a product of 3 factors. The only possible value of A satisfying all the given conditions is as below:

$$A = 2^4 \times 3^3 \times 5^1 \text{ (As } 5 \times 4 \times 2 = 40\text{)}$$

Therefore, the required value

$$\frac{A}{40} = \frac{2^4 \times 3^3 \times 5^1}{40} = 54$$

The correct answer is A.

- 21.** For a number to be divisible by 3, the sum of all the digits should be divisible by 3.

Now, $5 + 2 + 3 + 7 + a + b$ should be divisible by 3.

$17 + a + b$ must be divisible by 3.

Therefore, $a + b$ must be a (multiple of 3) + 1.

That is, 1, 4, 7, 10, 13, 16, 19.

The correct answer is E.

- 22.** $2A$ when divided by 9 will give a remainder of 2×7 , that is, 14. This can be divided by 9 and the required remainder will be 5.

The correct answer is C.

- 23.** Let the total number of members be x^2 .

The, the number of members went for rock climbing = $\frac{x^2}{5}$

And, the number of members went for hiking = $2x$

The remaining members are = 10

$$\text{Hence, } \frac{x^2}{5} + 2x + 10 = x^2$$

Solving the equation, we get:

$$x = 5, -\frac{3}{2}$$

Since number of members cannot be negative,

So, $x = 5$

So, number of members went for hiking

$$= 2 \times 5 = 10$$

The correct answer is B.

- 24.** Highest power of 2 in the expression $(2^{10} - 1)!$

$$\begin{aligned}&= \left[\frac{2^{10}-1}{2^1} \right] + \left[\frac{2^{10}-1}{2^2} \right] + \left[\frac{2^{10}-1}{2^3} \right] + \cdots + \left[\frac{2^{10}-1}{2^9} \right] \\&= (2^9 - 1) + (2^8 - 1) + (2^7 - 1) + \cdots + (2^1 - 1) \\&= 2^1 + 2^2 + 2^3 + \cdots + 2^9 - 9(1) \\&= \frac{2(2^9 - 1)}{2 - 1} - 9 \\&= 2^{10} - 2 - 9 = 2^{10} - 11\end{aligned}$$

The correct answer is E.

- 25.** Given that all students can stand in rows so that each row has 8, 10 and 12 students.

Least number of students will be LCM of 8, 10 and 12. So, LCM of 8, 10, 12 = 120

The correct answer is E.

- 26.** Suppose $x = 2^{\log 2^{\log 2^{-x}}}$

Taking log both sides, we get

$$\log x = \log 2^{\log x}$$

$$\Rightarrow \log x = \log x \log 2$$

$$\Rightarrow \log x (1 - \log 2) = 0$$

$$\Rightarrow \log x = 0 \Rightarrow x = 1$$

The correct answer is C.

- 27.** The divisibility rule of 9 says that the sum of the digits must be divisible by 9.

So, checking the options, we get:

Option A: $1 + 2 + 0 + 3 = 6 \rightarrow$ Not divisible

Option B: $2 + 2 + 5 + 6 = 15 \rightarrow$ Not divisible

Option C: $4 + 2 + 6 + 5 + 1 = 18 \rightarrow$ Divisible

Option D: $7 + 1 + 6 + 2 + 3 = 19 \rightarrow$ Not divisible

Option E: $9 + 2 + 4 + 2 + 3 = 20 \rightarrow$ Not divisible

The correct answer is C.

- 28.** In an ordinary year, there are 52 weeks and one extra day and in a leap year there are 52 weeks and 2 extra days.

From August 15, 1947 to August 14, 1949, there is one ordinary year and one leap year, that is,

$$1 + 2 = 3 \text{ odd days}$$

So, August 15, 1949 would be a Friday + 3 days, that is, Monday.

$$\begin{aligned} \text{Number of days from August 15, 1949 to January 26, 1950} &= 16 + \\ 30 + 31 + 30 + 31 + 26 &= 164 \end{aligned}$$

Therefore, number of odd days = 3

Hence, January 26, 1950 day would be Monday + 3 days, that is, Thursday.

The correct answer is A.

- 29.** In such questions, always start from today. If today is 8 March, 2018, and it is Thursday, we can work backwards to April 20, 1984.

From 8 March, 1984 to 8 March, 2018, there are 34 years, of which there are 8 leap years and 26 normal years.

So, there would be $8 \times 2 + 26 \times 1 = 42$ odd days, which means 0 odd days.

So, 8 March, 1984 was also Thursday.

Now, from 8 March, 1984 to 20 April, 1984, number of days = 23 + 20 = 43, which means 1 odd day. So, April 20, 1984 would be Friday.

The correct answer is B.

- 30.** Since there are exactly 4 Wednesdays and 4 Sundays in the month of January having 31 days, the 1st of January that year has to be on Thursday because only then the month will end on a Saturday, with the month having 5 Thursdays, 5 Fridays and 5 Saturdays.

The correct answer is E.

- 31.** If her age is x in the year 1980, then she was born in the year $1980 - x$.

As per the question,

$$x = \sqrt{1980 - x} \text{ or } x^2 + x - 1980 = 0$$

$$\text{Or } x = \frac{-1 + \sqrt{1 + 4 \times 1980}}{2} = 44$$

So, she was born in 1936.

The correct answer is B.

Alternatively,

The year of birth has to be a perfect square. The only option is 1936.

- 32.** Total difference of years = 30 years

Leap years in these 30 years = 8

Normal years in these 30 years = 22

So, total number of odd days = $8 \times 2 + 22 = 38$

So, final odd number of days = $\frac{38}{7} = 3$ days

On 09.12.2001 is Sunday, so on 09.12.1971 there would have been Thursday.

The correct answer is E.

33. $\log_{10}2 = 0.3010$ and also $\log_{10}10 = 1$

$$\text{So, } \log_{10}(2 \times 5) = 1$$

$$\log_{10}2 + \log_{10}5 = 1 \text{ and so } \log_{10}5 = 0.6990$$

$$\begin{aligned}\log_5 256 &= \frac{\log_{10} 256}{\log_{10} 5} = \frac{\log_{10} 28}{\log_{10} 5} \\ \text{Now,} \quad &= \frac{8 \log_{10} 2}{\log_{10} 5} = 8 \times \frac{0.3010}{0.6990} = 3.44\end{aligned}$$

The correct answer is C.

34. Given $625^{\log_{36}6} + 12^{\log_7 49} = 11^{\log_x 169}$

$$\Rightarrow 625^{1/\log_{36}6} + 12^{\log_7 7} = 11^{\log_x 169}$$

$$\Rightarrow 625^{1/2} + 12^2 = 11^{\log_x 169}$$

$$\Rightarrow 25 + 144 = 11^{\log_x 169}$$

$$\Rightarrow 169 = 11^{\log_x 169}$$

$$\Rightarrow x = 11$$

The correct answer is B.

35. $\log_{30} 3 + \log_{30} 5 + \log_{30} 2 = 1$

$$[\log a + \log b + \log c = \log abc]$$

$$\Rightarrow x + y + \log_{30} 2 = 1$$

$$\Rightarrow \log_{30} 2 = 1 - x - y$$

$$\Rightarrow 3 \log_{30} 2 = 3(1 - x - y)$$

$$\Rightarrow \log_{30} 8 = 3(1 - x - y)$$

$$\Rightarrow \log_8 30 = \frac{1}{3(1 - x - y)}$$

The correct answer is B.

36. Let $a^p = b^q = c^r = d^s = k$

$$\Rightarrow a = k^{1/p}, b = k^{1/q}, c = k^{1/r} \text{ and } d = k^{1/s}$$

$$\log_a(bcd) = \log_k^{1/p}(k^{1/q} \times k^{1/r} \times k^{1/s})$$

$$= \log_k^{1/p}(k^{1/q + 1/r + 1/s})$$

$$= \frac{\left(\frac{1}{q} + \frac{1}{r} + \frac{1}{s}\right)}{\frac{1}{p}} = p\left(\frac{1}{q} + \frac{1}{r} + \frac{1}{s}\right)$$

The correct answer is A.

37. $\log_y x = 10 \Rightarrow \log_x y = \frac{1}{10}$

$$\text{So, } \log_{x^3} y^6 = \frac{6}{3} \log_x y = \frac{2}{10} = \frac{1}{5}$$

The correct answer is B.

38. Given $P = 625^{\frac{2}{\log_3 25}} + 25^{\log_{125} 27} + 5^{\frac{5}{\log_8 125}}$

$$= 625^{2 \log_{25} 3} + 25^{\frac{3 \log_5 3}{\log_5 27}} + 5^{\frac{5}{\log_8 125}}$$

$$= 625^{\frac{2 \log_5 3}{\log_5 25}} + 25^{\frac{3 \log_5 3}{\log_5 27}} + 5^{\frac{5}{\log_8 125}}$$

$$= 5^{4 \log_5 3} + 5^{2 \log_5 3} + 5^{5 \log_5 2}$$

$$= 3^4 + 3^2 + 2^5$$

$$= 81 + 9 + 32 = 122$$

The correct answer is C.

2 Arithmetic

1. The number that is 50% greater than 80 = $1.5(80) = 120$.

Similarly, the number that is 25% less than 200 = $0.75(200) = 150$.

Thus, the required percentage = $\frac{30}{150} = 20\%$

The correct answer is D.

2. The expenditure on rent, utilities, and insurance = 50%

The expenditure on food = 20%

The total expenditure on rent, utilities, insurance and food = 50% + 20% = 70%.

The income left = $100\% - 30\% = 30\%$

The expenditure on video games = 30% of 30% = $0.30 \times 0.30 = 0.09$, or 9%.

Therefore, the percentage of the income left after all of the expenditure = $30\% - 9\% = 21\%$

The correct answer is D.

3. 90% of 40 students or $0.9(40) = 36$ students had lower marks than Varun.

Of the 60 new students, 80% or $0.80(60) = 48$ students had lower marks than Varun.

Thus, $36 + 48 = 84$ students in the new, larger class have marks lower than Varun.

The new class has 100 students, 84 of whom have lower marks than Varun.

There are 16 students remaining, one of which is Varun. Since Varun has the lowest marks of this group of 16 students, there are 15

students above him.

Since the class has exactly 100 students, the required percentage = $\frac{15}{100} \times 100 = 15\%$.

The correct answer is D.

4. Machine Y's work in a 24-hour period exceeds Machine X's work by 48,000.

Let Machine X's packing rate in books per hour be a

That is to say:

$$36a - 24a = 48,000$$

$$12a = 48,000$$

$$a = 4,000$$

Machine X packs 4,000 books per hour.

The correct answer is A.

5. Suppose 'P' is the number of apples, the vendor start out with on the first day.

Following table shows the calculation of apples left at the end of each day.

Day	Stock	Sold	Spoiled overnight
1	P	$\frac{P}{2}$	$\frac{P}{20}$
2	$\frac{P}{2} - \frac{P}{20} = \frac{9P}{20}$	$\frac{9P}{40}$	$\frac{9P}{400}$
3	$\frac{9P}{20} - \frac{9P}{400} = \frac{81P}{400}$	$\frac{81P}{800}$	$\frac{81P}{8000}$

Number of apples spoiled in three nights

$$\begin{aligned}\frac{P}{20} + \frac{9P}{400} + \frac{81P}{8000} &= 3305 \\ \Rightarrow \frac{400P + 180P + 81P}{8000} &= 3305 \\ \Rightarrow \frac{661P}{8000} &= 3305 \Rightarrow P = 40,000\end{aligned}$$

Hence, the vendor started with 40,000 apples on the first day.

The correct answer is D.

6. Let 5 pizzas, 7 samosas and 4 ice-creams cost Rs. x. That is,

$$5 \text{ pizzas} + 7 \text{ samosas} + 4 \text{ ice-creams} = x \quad (1)$$

Then, 6 pizzas, 14 samosas and 8 ice-creams will cost Rs. 1.5x.
That is,

$$6 \text{ pizzas} + 14 \text{ samosas} + 8 \text{ ice creams} = 1.5x \quad (2)$$

In the first case, if Ankur has decided to buy all the items double, it would cost him Rs. 2x. That is,

$$10 \text{ pizzas} + 14 \text{ samosas} + 8 \text{ ice-creams} = 2x \quad (3)$$

Subtracting Eq. (2) from (3), we get

4 pizzas cost Rs. 0.5x.

So, 5 Pizzas will cost Rs. 0.625x.

This is the amount that Ankur have spent on Pizzas. Hence, fraction of the total amount paid

$$= 0.625 = 62.5\%$$

The correct answer is E.

7. Cost price = $260 \times \frac{100}{100 + 30}$ = Rs. 200

The correct answer is B.

8. Let cost price be Rs. 100.

Then, marked price = Rs. 130

On this a discount of 5% is given.

So, discount = 5% of 130 = Rs. 6.5

Hence, the selling price of goods

= Rs. $(130 - 6.5)$ = Rs. 123.5

So, the profit percentage = 23.5%

The correct answer is C.

- 9.** $(p \% q + q \% \text{ of } p)$

$$= \frac{pq}{100} + \frac{qp}{100} = \frac{2pq}{100}$$

$$= 2\% \text{ of } pq$$

The correct answer is C.

- 10.** Given that, $p = 5q$

Percentage by which q is less than p

$$= \frac{p - q}{p} \times 100$$

$$= \frac{5q - q}{5q} \times 100$$

$$= \frac{4q}{5q} \times 100 = 80\%$$

The correct answer is D.

- 11.** Amount of iodine in the mixture = 16% of 735 gm

Now, the amount of iodine becomes 20% of the mixture.

If amount of mixture after evaporation is x , then

20% of x = 16% of 735 or $x = 0.8 \times 735$

Amount evaporated = $735 - 0.8 \times 735 = 147$ gm

The correct answer is C.

12.

$$A = P \left[1 + \left(\frac{r}{100} \right) \right]^t$$

where A = amount; P = principle; r = rate of interest; t = time intervals

$$\Rightarrow 1650 = 1200 \left[1 + \left(\frac{r}{100} \right) \right]^2$$

$$\Rightarrow \frac{1650}{1200} = \left[1 + \left(\frac{r}{100} \right) \right]^2$$

$$\Rightarrow \sqrt{1.375} = 1 + \left(\frac{r}{100} \right)$$

$$\Rightarrow 1.172 = 1 + \left(\frac{r}{100} \right)$$

Thus, $r = 17\%$ approx.

The correct answer is A.

- 13.** Given that an amount 'A' becomes three times in 6 years which means increment of '2A' occurs in 6 years. That is, increment of 'A' occurs in 3 years.

For the amount to be 24 times, increment of '23A' is required. Therefore, it will take 69 years to become 24 times.

The correct answer is C.

- 14.** Let us consider the compound interests for the first and the second year be CI_1 and CI_2 respectively.

$$CI \text{ for the first year, } CI_1 = \frac{PR}{100}$$

$$CI \text{ for the second year, } CI_2 = \left[P \left(1 + \frac{R}{100} \right)^2 - P \right] - CI_1$$

$$= \left[P \left(1 + \frac{R}{100} \right)^2 - P \right] - \frac{PR}{100} = P \left(\frac{R}{100} \right)^2 + \frac{PR}{100}$$

Therefore, the difference between the compound interests for the first year and the second year is given by:

$$CI_2 - CI_1 = \frac{PR^2}{100^2}$$

According to the question,

$$4P = \frac{PR^2}{10000}$$

$$\Rightarrow R = 200\%$$

The correct answer is D.

Alternatively,

We know that difference between SI and CI for the first two years is $\frac{PR^2}{100^2}$. It is given that difference of CI in the second year and that in the first year is 4 times the principal. Since CI for first year is same as SI for each of the years, the above difference is also the difference between CI of the first and the second year.

$$\text{So, } \frac{PR^2}{100^2} = 4P \text{ or } r = 200\%$$

- 15.** Let the amount be $2x$.

Interest earned on amount deposited at

$$\text{Simple interest} = \frac{(4x)(10)}{100} = 0.4x$$

Interest earned on amount deposited at

$$\text{Compound interest} = x \left[\left(1 + \frac{10}{100} \right)^3 - 1 \right] = 0.331x$$

$$\text{Difference} = 0.069x = 1000$$

$$\Rightarrow x = \frac{1000}{0.069}$$

Therefore, total initial amount

$$= 2x = \frac{2000}{0.069} = \text{Rs. } 28,985$$

The correct answer is B.

- 16.** According to the question, we have

$$\begin{aligned} 121500 &= P\left(1 - \frac{r}{100}\right)^2 = P\left(1 - \frac{10}{100}\right)^2 = P(0.9)^2 \\ \Rightarrow P &= \frac{121500}{0.9^2} = 150000 \end{aligned}$$

The correct answer is B.

- 17.** Let the rate of interest be r.

According to the question,

$$\frac{800 \times 2 \times r}{100} + \frac{200 \times 3 \times r}{100} = 275$$

Solving the above equation, we get

$$r = 12.5\%$$

The correct answer is B.

- 18.** Simple interest = $\frac{PRT}{100}$

$$P = \frac{P \times R \times 8}{100}$$

$$R = 12.5\%$$

The correct answer is A.

Alternatively,

In case of simple interest, an amount doubles if the rate of interest becomes 100%.

Let us consider the rate of interest is r.

Then, the time period ' t_d ' in which total interest becomes 100% is given by as below:

$$t_d = \frac{100}{r} \Rightarrow r = \frac{100}{t_d} = \frac{100}{8} = 12.5\%$$

- 19.** An amount doubles if the rate of interest becomes 100%.

Let us consider the rate of interest compounded annually is r .

Then, we know that the time period ' t_d ' in which total interest becomes 100% is given by as below:

$$\begin{aligned} t_d &= \frac{72}{r} \\ \Rightarrow r &= \frac{72}{t_d} = \frac{72}{8} = 9\% \end{aligned}$$

The correct answer is A.

- 20.** Let Dev's share be Rs. x .

Suma's share = Rs. $(250000 - x)$

Suma invested her share at 10% per annum simple interest for 2 years

$$S.I = \frac{P \times R \times T}{100}$$

Suma's interest

$$\begin{aligned} &= \text{Rs.} \frac{(250000 - x) \times 10 \times 2}{100} \\ &= \text{Rs.} \frac{(250000 - x)}{5} \end{aligned}$$

Dev invested his share compounded at 8% per annum for 2 years

$$\begin{aligned} C.I &= P \left(1 + \frac{R}{100} \right)^T - P \\ &= x \left(1 + \frac{8}{100} \right)^2 - x = \frac{104x}{625} \end{aligned}$$

According to the question,

$$\begin{aligned} \frac{250000 - x}{5} - \frac{104x}{625} &= 13360 \\ \Rightarrow \frac{31250000 - 125x - 104x}{625} &= 13360 \\ \Rightarrow 229x &= 31250000 - (13360 \times 625) \\ \Rightarrow 229x &= 22900000 \\ \Rightarrow x &= 100000 \end{aligned}$$

Suma's share = Rs. $(250000 - x)$
 $= \text{Rs. } (250000 - 100000) = \text{Rs. } 1,50,000$

The correct answer is D.

- 21.** Principle, P = Rs. 12,500

Time, T = 3 years

Difference between CI and SI = Rs. 561.60

Difference between CI and SI for three years is

$$CI - SI = P \left(\frac{R}{100} \right)^2 \times \frac{300+R}{100}$$

Substituting the values, we get

$$\begin{aligned} 561.60 &= 12,500 \left(\frac{R}{100} \right)^2 \times \frac{300+R}{100} \\ \Rightarrow 2(300+R) &= \frac{561.60}{12,560} \times (100)^3 \\ \Rightarrow R^2(300+R) &= 44,713 \end{aligned}$$

On solving, we get $R = 12\%$

The correct answer is C.

- 22.** Arvind's per hour sale = Rs. 78

Amount paid to the salesman per hour = Rs. 10

Therefore, the profit per hour = Rs. 68

Also, the amount paid for the roadside market per day = Rs. 150

So, the formula for his daily profit will be

Profit = Revenue – Expenses

$$P(h) = 68h - 150$$

The correct answer is C.

- 23.** We know that profit equals revenue minus cost. Therefore, the company's profit is

$$\begin{aligned} p(9-p) - (p+15) &= 9p - p^2 - p - 15 \\ &= -p^2 + 8p - 15 \\ &= -(p^2 - 8p + 15) \\ &= -(p-5)(p-3) \end{aligned}$$

When $p = 5$ or $p = 3$, the profit becomes 0, so eliminate (A) and (C). For $p > 5$, the profit becomes negative (that is, the company loses money). The profit is only positive if $(p-5)$ and $(p-3)$ have opposite signs, which occurs when $3 < p < 5$.

The correct answer is B.

- 24.** Total amount deposited = Rs. 20,000

Rate of interest = 4% per annum = $\frac{1}{3}\%$ per month

Priya withdraws Rs. 1,000 on 10th of every month. So, the amount on which the interest gets calculated is reduced by Rs. 1,000 every month.

Therefore, interest earned till 6th June

$$\begin{aligned} &= \frac{20,000}{300} + \frac{19000}{300} + \frac{18000}{300} + \frac{17000}{300} + \frac{16000}{300} \\ &= 66.67 + 63.33 + 60 + 56.67 + 53.33 \\ &= \text{Rs. } 300 \end{aligned}$$

The closest option is Rs. 296.

The correct answer is C.

- 25.** Total number of worksheets = 50

Number of worksheets received by 2 students of section A was equal to the number of worksheets received by 5 students of section C. So,

Ratio of worksheets distributed to students of section A to C = 5 : 2

Similarly, ratio of worksheets distributed to students of section B to C = 3 : 2

Therefore, ratio of worksheets distributed to students of sections A, B and C = 5 : 3 : 2

The worksheets were distributed among 6 students of Section A, 12 students of Section B, and 17 students of Section C.

Let x be the ratio factor. Then,

$$6 \times 5x + 12 \times 3x + 17 \times 2x = 50$$

$$\Rightarrow 30x + 36x + 34x = 50$$

$$\Rightarrow x = 0.5$$

Hence, number of worksheets distributed to each student of Section C = $2x = 2 \times 0.5 = 1$

The correct answer is A.

- 26.** If CP is Rs. 100 for 1 kg, actual CP = Rs. 84

And, SP = Rs. 96

$$\text{So, profit} = \frac{12}{84} \times 100 = 14 \frac{2}{7} \% \text{ gain}$$

The correct answer is B.

- 27.** Profit = 15%, Discount = 5%

$$\text{So, net profit \%} = 15 - 5 + \frac{15(-5)}{100}$$

$$= 10 - 0.75 = 9.25\%.$$

The correct answer is C.

28. Ratio of milk and water = 6 : 5

Suppose quantity of milk and water initially be $6x$ and $5x$, respectively. Assume that y litres of both milk and water are added. Then,

$$\text{Ratio of milk and water in new mixture} = \frac{6x+y}{5x+y}$$

Now, looking at the options, only option E satisfies this condition by putting $x = 1$ and $y = 5$ as following:

$$\frac{6x+y}{5x+y} = \frac{6 \times 1 + 5}{5 \times 1 + 5} = \frac{11}{10} = 11:10$$

The correct answer is E.

29. Amount kept in reserve fund = Rs. 40,000

Total dividend

$$= \text{Rs. } 240,000 - \text{Rs. } 40,000 = \text{Rs. } 200,000$$

Face value of each preferred share = 10 Dividend per preferred share

$$= 10 \times \frac{15}{100} = 1.5$$

Total dividend in all preferred shares

$$= 1.5 \times 75,000 = \text{Rs. } 112,500$$

Total dividend in all common shares

$$= 200,000 - 112,500 = 87,500$$

Face value of each common share = 10

Let the dividend for common share be $x\%$.

$$\text{Dividend per common share} = 10 \times \frac{x}{100} = 0.1x$$

Total dividend in all common shares

$$= 0.1x \times 20,000 = 2,000x$$

According to the question,

$$2,000x = 87,500$$

$$\Rightarrow x = 43.75\%$$

New, total dividend in all common shares

$$= 240,000 - 112,500 - 25,000$$

$$= \text{Rs. } 102,500$$

Let the new dividend for common share be $y\%$.

Dividend per common share

$$= 10 \times \frac{y}{100} = 0.1y$$

New total dividend in all common shares

$$= 0.1y \times 20,000 = 2,000y$$

According to the question,

$$2,000y = 102,500$$

$$\Rightarrow y = 51.25\%$$

Required difference

$$= 51.25\% - 43.75\% = 7.5\%$$

The correct answer is B.

Alternatively,

Difference in money distributed as dividend to common shareholders

$$= 40,000 - 25,000 = 15,000$$

If $x\%$ and $y\%$ are the percentage dividend in the earlier and later cases, then

$$y\% \text{ of } 10 \times 20,000 - x\% \text{ of } 10 \times 20,000 = 15,000$$

$$\Rightarrow 2,000(y - x) = 15,000$$

$$\Rightarrow (y - x) = 7.5\%$$

- 30.** Simplifying the ratio, we get

$$8:5\frac{1}{3} = 8:\frac{16}{3} = 24:16 = 3:2$$

$$3\frac{1}{5}:1\frac{1}{3} = \frac{16}{5}:\frac{4}{3} = \frac{4}{5}:\frac{1}{3} = 12:5$$

Compound ratio of 3 : 2 and 5 : 12

$$= (3 \times 5) : (2 \times 12)$$

$$= 15 : 24$$

As per the question, we have

$$15 : 24 = 15 : x$$

$$\Rightarrow x = 24$$

The correct answer is E.

- 31.** Time taken by pipe A to fill the tank = 5 hours

Time taken by pipe B to empty the tank = 6 hours

Let the capacity of tank be 30 litres.

Filling capacity of pipe A = 6 litres/hour

Emptying capacity of pipe B = 5 litres/hours

So, tank filled in one hour = 1 litre

Hence, time taken to fill the tank completely = 30 hours

The correct answer is B.

- 32.** Time taken by A to complete the task = 10 days

Time taken by B to complete the task = 15 days

Let the total task be 30 units.

Task completed by A in one day = 3 units

Task completed by B in one day = 2 units

Task completed by both A and B in one day = 5 units

Hence, time taken to complete the task

$$= \frac{30}{5} = 6 \text{ days}$$

The correct answer is C.

- 33.** Time taken to fill the tank by the first tap = 10 minutes

Time taken to fill the tank by the second tap = 15 minutes

Let the capacity of tank be 150 litres.

Filling capacity of the first step = 15 litres/minute

Filling capacity of the second tap = 10 litres/minute

Suppose filling capacity of the third tap = x litres/minute

When all the 3 taps were opened simultaneously, the tank was filled in 22 minutes. So,

$$22(15 + 10 + x) = 150$$

$$\Rightarrow 550 + 22x = 150$$

$$\Rightarrow x = -\frac{400}{22} = -\frac{200}{11} \text{ litres/minutes}$$

Negative sign indicates that the third tap is emptying the tank.

Hence, time required to empty the tank by the third tap.

$$= -\frac{150}{200/11} = \frac{150}{200} \times 11 = 8.25 \text{ minutes} = 8.25 \text{ minutes}$$

The correct answer is B.

- 34.** The chemist now has 10 litres of ink in a 30-litre mixture, so she must have 20 litres of water. You want to know the amount of ink you must add in order to make this mixture a 50% solution. Since no additional water is added, the solution must finish with 20 litres of water. Therefore, she also needs a total of 20 litres of ink, or 10 more litres than the mixture currently contains.

The correct answer is C.

- 35.** 200% bigger means ‘three times as big as’ the original. If the pitcher is three times as big as the glass, then pouring the contents of the glass into the pitcher will make the pitcher $\frac{1}{3}$ full. If adding another 16 litres fills up the pitcher, the 16 litres must be equal to the remaining $\frac{2}{3}$ of the pitcher’s capacity.

Then, $\frac{1}{3}$ of the pitcher’s capacity = $\frac{16}{2} = 8$ litres.

So, the quantity of the juice mixture = 8 litres.

Therefore, the volume of lime juice in the mixture = 20% of 8 litres.

$$= (8)(0.2) = 1.6 \text{ litres}$$

The correct answer is A.

- 36.** The ratio of boys to girls is 6 : 7. If you introduce the unknown multiplier x , the number of boys is $6x$, and the number of girls is $7x$, where x is a positive integer.

According to the question,

$$7x - 6x = 2$$

$$\Rightarrow x = 2$$

Finally, substitute the value of x into the expression for the number of boys: $6x = 6(2) = 12$. There are 12 boys in the team.

The correct answer is A.

- 37.** Use the weighted average formula to get the ratio of X to Y:

$$\left(\frac{0.55x + 0.70y}{x + y} \right) = 0.65, \text{ where } x \text{ is the amount of}$$

X and y is the amount of Y.

$$0.55x + 0.70y = 0.65(x + y)$$

$$0.55x + 0.70y = 0.65x + 0.65y$$

$$55x + 70y = 65x + 65y$$

$$55x + 5y = 65x$$

$$5y = 10x$$

$$\text{Thus, } \frac{y}{x} = \frac{2}{1}$$

Since y and x are in a 2 to 1 ratio, $\frac{2}{3}$ of the total is y and $\frac{1}{3}$ of the total is x.

Since the total is 120 grams, X accounts for $\frac{1}{3} \times 120 = 40$ grams of the mixed cereal.

The correct answer is B.

- 38.** Let the filling capacity be $x \text{ m}^3/\text{minute}$. Then,

$$\text{Emptying capacity} = (x + 10) \text{ m}^3/\text{minutes}$$

The pump needs 5 minutes less to empty the tank than it needs to fill it. So,

$$\frac{1500}{x} - \frac{1500}{x+10} = 5$$

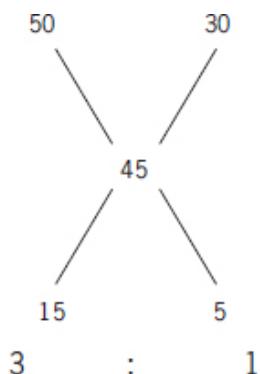
$$\Rightarrow \frac{1}{x} - \frac{1}{x+10} = \frac{1}{300}$$

$$\begin{aligned}
 &\Rightarrow \frac{x+10-x}{x(x+10)} = \frac{1}{300} \\
 &\Rightarrow x(x+10) - 3,000 = 0 \\
 &\Rightarrow x^2 + 10x - 3,000 = 0 \\
 &\Rightarrow x^2 + 60x - 50x - 3,000 = 0 \\
 &\Rightarrow x(x+60) - 50(x+60) = 0 \\
 &\Rightarrow x = 50
 \end{aligned}$$

So, emptying capacity = $x + 10 = 60 \text{ m}^3$ per minute

The correct answer is E.

- 39.** Given that solution X contains 50% milk and solution Y contains 30% milk. Using the Alligation method, we can determine the ratio of volume of the two mixtures:



Therefore, the required ratio = 3:1

The correct answer is E.

- 40.** Let the required time be 'x' seconds.

Volume of solution transferred = $10x \text{ ml}$

Volume of milk transferred = $5x \text{ ml}$

Volume of water transferred = $5x \text{ ml}$

Ratio of volume of milk to the volume of water in the solution = $5x : 5x + 100 = 1 : 3$

$\Rightarrow x = 10$ seconds

The correct answer is B.

- 41.** Combined age of the four children = $130 - 82 = 48$ years

Age of the eldest child will be

$$\frac{7}{1+2+6+7} \times 48 = \frac{7}{16} \times 48 = 21 \text{ years}$$

The correct answer is C.

- 42.** Given that the population increases at the rate of 6% per year.

Present population of country = 23,452 million

Population of country 3 years ago

$$= \frac{23,452}{\left(1 + \frac{6}{100}\right)^3} = \frac{23,452}{\left(\frac{106}{100}\right)^3} = \frac{23,452}{(1.06)^3} \approx 19,691 \text{ million}$$

The correct answer is C.

- 43.**

$$6 = \frac{a+b+c+6}{5}$$

$$\text{or, } a+b+c=19$$

It is not necessary, or possible, to determine the values of a, b, and c individually. The second average includes all three variables, so the values will be summed again anyway.

$$\text{Required average} = \left(\frac{a+b+c+13}{4} \right) = \frac{32}{4} = 8$$

The correct answer is A.

- 44.** Weight of the teacher

= Average weight + Weight contributed by increase in average

$$= 42 + 38 \times 2 = 118 \text{ kg}$$

The correct answer is D.

- 45.** Let number of men be n.

Total weight of n men initially = $50 \times n = 50n$ (1)

After 2 persons leave, then resultant weight
= $52(n - 2)$ (2)

Equations (1) and (2) can be written as

$$50n = 52(n - 2) + 80$$

$$n = 12$$

The correct answer is E.

- 46.** Let average weight of class 8th, 9th and 10th students be ka , kb and kc .

Let number of students in class 8th, 9th and 10th be jx , jy and jz .

Then average weight = $\frac{\text{Total weight of all students}}{\text{Total number of students}}$

$$\begin{aligned} &= \frac{(ka \times jx) + (kb \times jy) + (kc \times jz)}{jx + jy + jz} \\ &= \frac{jk(ax + by + cz)}{j(x + y + z)} \\ &= \frac{k(ax + by + cz)}{x + y + z} \end{aligned}$$

Here we do not have the value of k and hence it cannot be determined.

The correct answer is E.

- 47.** The average age of husband and wife now is 30 years and the average age of husband, wife and the children now is 16 years.

$$\text{Total ages of children} = (16 \times 4) - (30 \times 2)$$

$$= 4 \text{ years}$$

As all of them have different ages with integral values, the only possibility is (1, 3) years.

The correct answer is A.

- 48.** As the product of time taken to construct a hut and the number of workers doing the work is always constant, we have:

$$(3 \text{ hours}) \times (40 \text{ workers}) = (t \text{ hours}) \times (140 \text{ workers})$$

$$\begin{aligned}\Rightarrow t &= \frac{120}{140} = \frac{6}{7} \text{ hours} \\ &= \frac{6}{7} \times 60 = \text{approximately } 51 \text{ minutes.}\end{aligned}$$

The correct answer is A.

- 49.** The resort has $20(14) = 280$ of wood.

If the resort only needs to heat 15 rooms instead of 20, divide 280 by 15 to get 18.666.... You are asked for extra FULL days,

So extra FULL day will be 4.

The correct answer is B.

- 50.** Ajay and Firoz' combined rate $= \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$ So, in 2 hours, they should have completed $\frac{14}{12} = \frac{7}{6}$ of the job.

Therefore, Firoz' breaks cost them $\frac{7}{6} - 1 = \frac{1}{6}$ job worth of productivity.

Thus, Firoz' break time = the amount of time it would have taken him to do $\frac{1}{6}$ of the job = 30 minutes

Therefore, each of his 3 breaks was $\frac{30 \text{ minutes}}{3} = 10 \text{ minutes long.}$

The correct answer is B.

- 51.** First, figure out how many boxes worth of pens the machine produced in the 2 hours that it was on.

20 pens per hour is the rate, and 2 hours is the time:

Work = (20 pens per hour) \times (2 hours) = 40 pens.

Now, since there are 10 pens per box, compute the number of boxes:

$$\begin{aligned}\text{The number of boxes} &= 40 \text{ pens} \times \left(\frac{1 \text{ box}}{10 \text{ pens}} \right) \\ &= 4 \text{ boxes}\end{aligned}$$

So Mahesh must pack 4 whole boxes to accommodate all the pens that the machine had made.

Mahesh's rate is 3 boxes per hour, while the total work is 4 boxes. Rearrange and plug in:

$$\begin{aligned}\text{Time} &= \left(\frac{4}{3} \right) \text{ hours} \\ &= \frac{4}{3} \times 60 = 80 \text{ minutes}\end{aligned}$$

The correct answer is C.

52. Without the hole, the two taps will fill the tank in $\frac{1}{\frac{1}{4} + \frac{1}{5}} = \frac{20}{9}$ minutes

Let the hole empties the tank in x minutes, then

$$\begin{aligned}\frac{1}{\frac{1}{4} + \frac{1}{5} - \frac{1}{x}} &= \frac{20}{9} + \frac{1}{2} \\ \Rightarrow \frac{1}{x} &= \frac{81}{980} \\ \Rightarrow x &= \frac{980}{81} \text{ minutes}\end{aligned}$$

The correct answer is B.

53. If x , y and z are the time taken by P, Q and R respectively to fill the cistern alone, then

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{3} \quad (1)$$

Also, P works for 1 hour, whereas Q and R work for 5 hours in total. Then,

$$\frac{1}{x} + 5 \times \frac{1}{y} + 5 \times \frac{1}{z} = 1 \quad (2)$$

Multiplying Eq. (1) by 5 and subtracting Eq. (2) from it, we get

$$4 \times \frac{1}{x} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$x = 6 \text{ hours}$$

So, P alone can fill the cistern in 6 hours.

The correct answer is D.

- 54.** P works for $X + 5$ minutes whereas Q works for X minutes only. So,

$$(X + 5) \times \frac{1}{12} + X \times \frac{1}{16} = 1$$

$$\text{Or } X = 4 \text{ minutes}$$

The correct answer is C.

- 55.** In one hour,

Suppose the quantity of water from pipe Y = w_1

The quantity of water from pipe X = w_2 ,

And, acid from pipe X = a

In first case, when the bucket is full,

$$\frac{a}{w_1 + w_2 + a} = \frac{8}{100} \quad (1)$$

$$\frac{w_1 + w_2 + a}{a} = 12.5 \text{ or } \frac{w_1}{a} + \frac{w_2}{a} = 11.5$$

In second case, the bucket is not full, So,

$$\frac{a}{\frac{w_1}{2} + w_2 + a} = \frac{10}{100} \quad (2)$$

$$\frac{w_1}{2a} + \frac{w_2}{a} = 9$$

$$\Rightarrow \frac{w_2}{a} = 18 - 11.5 = 6.5 = 13 : 2$$

The ratio of acid to water in pipe X = 2 : 13

The correct answer is E.

56. Part of the cistern filled by tap P in 1 minute = $\frac{1}{12}$

Part of the cistern filled by tap Q in 1 minute = $\frac{1}{18}$

Part of the cistern filled by both the taps in 1 minute = $\frac{1}{12} + \frac{1}{18} = \frac{5}{36}$

So, the cistern can be filled in $\frac{36}{5} = 7\frac{1}{5}$ min

The correct answer is D.

57. Given that one worker can dig 250 m of tunnel in a week. It means one worker in two weeks can dig 500 m of tunnel.

So, for digging 20,000 m of tunnel in two weeks, the number of workers required is given by:

$$n = \frac{20,000}{500} = 40 \text{ workers}$$

The number of existing workers = 12

So, the number of extra workers required

$$= 40 - 12 = 28$$

The correct answer is E.

58.

Skilled workers	Days	Hours	Work
$\frac{2}{5} \times 33$	15	12	1
y	11	9	1.5

$$\text{Thus, } y = \left(\frac{2}{5}\right) \times 33 \times \left(\frac{15}{11}\right) \times \left(\frac{12}{9}\right) \times \left(\frac{3}{2}\right) = 36$$

The correct answer is A.

Alternatively,

Use the formula: $\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$.

$$\text{So, } \frac{\left(\frac{2}{5} \times 33\right) \times 15 \times 12}{1} = \frac{y \times 11 \times 9}{1.5} \text{ or } y = 36$$

59. Since each of the two assistants work $\frac{3}{4}$ as fast as Ajay, all the three put together will work

$$1 + \frac{3}{4} + \frac{3}{4} = \frac{5}{2} \text{ as fast as ajay alone can do it.}$$

Hence, if all three work together, they can finish the work in $\frac{2}{5}$ of the time taken by Ajay alone to do the work.

The correct answer is D.

Alternatively,

Let Ajay does 4 units per day. The job is 4 units of work. So assistants would do 3 units each per day. Hence, it takes them $\frac{4}{4+3+3} = \frac{2}{5}$ days which is $\frac{2}{5}$ of what Ajay takes working alone.

60. The minimum time is taken by C as he is not present in the maximum time of $22\frac{2}{9}$ days.

Similarly, the maximum time is taken by B as he is not present in the minimum time of $15\frac{5}{13}$ days.

The correct answer is A.

61. Let Tina's share be T, Ishan's share be I, Abhishek's share be A and Fatima's share be F. Then,

$$\text{Given that } T + 3 = I + \frac{1}{3} = \frac{80A}{100} = F - 4$$

$$T = F - 7 \quad (1)$$

$$I = \frac{3}{4} (F - 4) \quad (2)$$

$$A = \frac{5}{4} (F - 4) \quad (3)$$

Also given that, $T + I + F + A = 80$ (4)

Substituting the values from Eqs. (1), (2) and (3) in Eq. (4), we get:

$$\left[F + \frac{3F}{4} + \frac{5F}{4} + F \right] - 7 - 3 - 5 = 80$$

$$F = \text{Rs. } 23.75$$

The correct answer is C.

- 62.** Mukesh paid 1/2 of what others paid.

So, Mukesh paid 1/3rd of the total amount = £ 20

Similarly,

Manish paid 1/4th of the total amount = £ 15 and Lalu paid 1/5th of the total amount = £ 12.

Remaining amount of £ 60 – £ 20 – £ 15 – £ 12

= £ 13 is paid by Jaggi.

The correct answer is A.

- 63.** Since their investments in the business are for the same duration i.e. 1 year, profits will be shared in the ratio of their investments i.e. $21,000 : 17,500 = 6 : 5$.

So, Yogesh's share = $\frac{6}{11} \times 26,400 = \text{Rs. } 14,400$

Mohan's share = $\frac{5}{11} \times 26,400 = \text{Rs. } 12,000$

The correct answer is A.

3 Algebra and Probability

1. $3x^3 - 7 = 185$

$$\Rightarrow 3x^3 = 192$$

$$\Rightarrow x^3 = 192/3 = 64$$

$$\Rightarrow x = 4$$

Thus, $x^2 - x = 16 - 4 = 12$

The correct answer is C.

2. Let the two numbers be a and b.

Given that the arithmetic mean is three times their geometric mean.
So,

$$\frac{a+b}{2} = 3\sqrt{ab}$$

$$\Rightarrow \frac{a+b}{\sqrt{ab}} = 6$$

$$\Rightarrow \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = 6$$

Suppose $\sqrt{\frac{a}{b}} = t$. Then, the equation can be written as

$$t + \frac{1}{t} = 6$$

$$\Rightarrow t^2 - 6t + 1 = 0$$

$$\Rightarrow t = \frac{6 \pm \sqrt{6^2 - 4 \times 1 \times 1}}{2}$$

$$\Rightarrow t = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$$\sqrt{\frac{a}{b}} = 3 \pm 2\sqrt{2}$$

On squaring both sides, we get

$$\Rightarrow \frac{a}{b} = 9 + 8\pm 12\sqrt{2}$$

$$\Rightarrow \frac{a}{b} = 17 \pm 12\sqrt{2}$$

Therefore, the required ratio is $(17 \pm 12\sqrt{2}) : 1$.

The correct answer is B.

3. Let the roots of equation $ax^2 + bx + c = 0$ be m and n.

Then roots of equation, $px^2 + qx + r = 0$ are $1/m$ and $1/n$

$$\text{Now, } m + n = \frac{b}{a} \text{ and } mn = \frac{c}{a}$$

$$\frac{1}{m} + \frac{1}{n} = \frac{q}{p} \text{ and } \frac{1}{mn} = \frac{r}{p}$$

$$\text{Also, } \frac{1}{m} + \frac{1}{n} = \frac{(m+n)}{mn}.$$

$$\text{So, } \frac{-b}{c} = \frac{-q}{p} \text{ and also } \frac{c}{a} = \frac{p}{r}$$

By these relations we get $a = r$, $c = p$ and $b = q$.

The correct answer is E.

4. Discriminant, $D = (4b)^2 - 4a(2c)$

Since a, b and c are in GP, $b^2 = ac$

Hence, discriminant, $D = 16b^2 - 8b^2 = 8b^2 > 0$

Note that $8b^2$ cannot be zero as a, b and c are positive integers.

Hence, roots are real and distinct.

The correct answer is D.

5. The roots are -4 and -3 .

Then, $(p+q)^2 = (-7)^2 = 49$;

$$(p - q)^2 = (-4 + 3)^2 = 1$$

So, required equation is

$$x^2 - (49 + 1)x + (49)(1) = 0$$

$$\Rightarrow x^2 - 50x + 49 = 0$$

The correct answer is A.

6. Let p and q be the roots of the quadratic equation.

Given that, $p = 1.5q$ and $p - q = 1$ or $q - p = 1$

From these equations, we get

$$p = 3 \text{ and } q = 2 \text{ (or) } p = -3 \text{ and } q = -2$$

Since the quadratic equation with roots p and q is given by $(x - p)(x - q)$, it follows that equations can be $x^2 - 5x + 6 = 0$ (or) $x^2 + 5x + 6 = 0$

The correct answer is C.

7. p and q are roots of equation $ax^2 + bx + c = 0$

$$p + q = \frac{-b}{a} \text{ and } pq = \frac{c}{a}$$

We have to calculate value of $p^4 - q^4$

$$\begin{aligned} p^4 - q^4 &= (p^2 - q^2)(p^2 + q^2) \\ &= (p + q)(p - q)[(p + q)^2 - 2pq] \end{aligned}$$

To calculate the answer we have to get the value of

$$\begin{aligned}
 & (p - q) \text{ and } [(p + q)^2 - 2pq] \\
 & (p - q) = \pm \sqrt{(p + q)^2 - 4pq} \\
 & = \pm \sqrt{\left(\frac{-b}{a}\right)^2 - \frac{4c}{a}} \\
 & = \pm \frac{1}{a} \sqrt{b^2 - 4ac}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 [(p + q)^2 - 2pq] &= \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} \\
 &= \frac{b^2 - 2ac}{a^2}
 \end{aligned} \tag{2}$$

Putting values of Eq. (1) and (2), we get

$$\begin{aligned}
 p^4 - q^4 &= \left(\frac{-b}{a}\right) \left(\frac{b^2 - 2ac}{a^2} \right) \left(\pm \frac{1}{a} \sqrt{b^2 - 4ac} \right) \\
 &= \mp \frac{b}{a^4} (b^2 - 2ac) (\sqrt{b^2 - 4ac})
 \end{aligned}$$

The correct answer is E.

8. $2(x - 1)^3 + 3 \leq 19$

$$2(x - 1)^3 \leq 16$$

$$(x - 1)^3 \leq 8$$

$$x - 1 \leq 2$$

$$x \leq 3$$

The correct answer is B.

9. $(3a + 7) \geq 2a + 12$ or $-(3a + 7) \geq 2a + 12$
 $a + 7 \geq 12$ or $-3a - 7 \geq 2a + 12$
 $a \geq 5$ or $-7 \geq 5a + 12$
 or $\frac{19}{5} \geq a$

The correct answer is D.

10. $\sqrt{192} < x\sqrt{12}$,

$$\Rightarrow \frac{\sqrt{192}}{\sqrt{12}} < x$$

$$\Rightarrow \sqrt{16} < x$$

$$\Rightarrow 4 < x$$

Similarly,

$$\frac{x}{\sqrt{12}} < \sqrt{12}$$

$$\Rightarrow x < \sqrt{12}x\sqrt{12}$$

$$\Rightarrow x < 12$$

The correct answer is D.

11. If $\left(\frac{a}{b}\right)\left(\frac{x}{y}\right) < 0$, then the two fractions must have opposite signs, that is, $\left(\frac{a}{b}\right)$ must be the negative inverse of $\left|\frac{x}{y}\right|$ or $\left|\frac{a}{b}\right| = -\left(\frac{y}{x}\right)$

Thus, D is the answer. The other choices may or may not be true.

The correct answer is D.

12. Let b equal the number of chairs sold. Each chair sells for Rs. 700, so the total revenue is Rs. $700b$. The cost is equal to 11,000 plus 300 for every chair sold.

$$(700b) - (11,000 + 300b) > 0$$

$$700b - 11,000 - 300b > 0$$

$$400b - 11,000 > 0$$

$$400b > 11,000$$

$$b > 27.5$$

If b must be greater than 27.5, then the manufacturing unit needs to sell at least 28 chairs to make a profit.

The correct answer is C.

13. Note the absolute value sign in the original inequality. This basically means that the solution could lie on either side of the number line.

So, you will have to solve this inequality in two ways to get the entire range of solutions for p .

$$p + 5 < 9 \text{ or } -(p + 5) < 9$$

$$p < 4 \text{ or } p > -14$$

$$-14 < p < 4$$

The correct answer is C.

14. $x^2 - 6x + 9 = 0$

$$\Rightarrow (x - 3)^2 = 0$$

$$\Rightarrow x = 3$$

$$\text{So, } x^3 = (3)^3 = 27$$

The correct answer is E.

- 15.** The simplest way to approach this question is to divide the marbles evenly among all 13 people and then adjust this number so that Manish ends up with the most marbles.

$$\frac{60}{13} = 4.61$$

Let Manish have 5 marbles. Then each of his friends can have at the most 4 marbles, giving a total of $(12 \times 4) + 5 = 53$ marbles. We still have 7 marbles left, so Manish may not necessarily end up with the most number of marbles.

Then, let Manish have 6 marbles. Now each of his friends can have at the most 5 marbles, giving a total of $(12 \times 5) + 6 = 66$ marbles. But we only have a total of 60 marbles, so if Manish keeps 6, there is no way any of his friends can end up with a greater number of marbles. Thus, the answer is 6.

The correct answer is B.

- 16.** $a^2 - b^2 = 0$ can be rewritten as $a^2 = b^2$.

From here, it might look as though $a = b$, but this is not necessarily the case. For example, a could be 3 and b could be -3 . Algebraically, when you square root both sides of $a^2 = b^2$, you do NOT get $a = b$, but rather $|a| = |b|$. Thus, statement 1 is not necessarily true and statement 2 is true.

Statement 3 is also true:

$$a^2 - b^2 = 0$$

$$\Rightarrow a^2 = b^2$$

$$\Rightarrow \frac{a^2}{b^2} = 1$$

The correct answer is E.

- 17.** The algebraic identity $x^2 - y^2 = (x + y)(x - y)$

$$\begin{aligned}\text{Then, } x^2 - y^2 &= \sqrt{12} \times \sqrt{20} \\ &= \sqrt{240} \\ &= \sqrt{16} \times \sqrt{15} \\ &= 4\sqrt{15}\end{aligned}$$

The correct answer is B.

- 18.** We know that $x^2 - y^2 = (x + y)(x - y)$

a^8 can be written as $(a^4)^2$

a^4 can be written as $(a^2)^2$

The given expression can now be solved as follows:

$$\frac{a^8 - b^8}{(a^4 + b^4)(a^2 + b^2)} = \frac{(a^4 + b^4)(a^4 - b^4)}{(a^4 + b^4)(a^2 + b^2)} = \frac{a^4 - b^4}{a^2 + b^2}$$

Now, factor $a^4 - b^4$ according to the same pattern:

$$\frac{a^4 - b^4}{a^2 + b^2} = \frac{(a^2 + b^2)(a^2 - b^2)}{(a^2 + b^2)} = a^2 - b^2$$

$$= (a + b)(a - b)$$

The correct answer is C.

- 19.** We know that $(a - b)^2 = a^2 - 2ab + b^2$ and

$$(a + b)^2 = a^2 + 2ab + b^2$$

Then,

$$\begin{aligned} & (a - 2)^2 + (a - 1)^2 + a^2 + (a + 1)^2 + (a + 2)^2 \\ &= (a^2 - 4a + 4) + (a^2 - 2a + 1) + (a^2) + (a^2 + 2a + 1) \\ &\quad + (a^2 + 4a + 4) \\ &= (a^2 + 4) + (a^2 + 1) + (a^2) + (a^2 + 1) + (a^2 + 4) \\ &= 5a^2 + 10 \end{aligned}$$

The correct answer is B.

- 20.** The phrase “inversely proportional” means that $a = k/b$ or $ab = k$, where k is a constant.

As the product of a and b is always constant, we have:

$$24 \times 2 = 6 \times b,$$

$$\Rightarrow b = 8.$$

The correct answer is E.

- 21.** Let the number of pieces of cake eaten by each of the two people who eat the same quantity be P and the number of pieces of cake eaten by the third person be T .

According to the question, we have

$$P + P + (P + 2) = 14$$

$$\Rightarrow 3P + 2 = 14$$

$$\Rightarrow 3P = 12$$

$$\Rightarrow P = 4$$

You can use the value of P to solve for T .

Therefore, the number of pieces eaten by the third person = $4 + 2 = 6$

The correct answer is D.

- 22.** Suppose the value of the red beads, the blue beads and the yellow beads are r , b and y respectively.

From the question,

$$r + b = 4.25$$

$$b + y = 2.75$$

$$r + b + y = 4.5$$

You need to find out ' $r + y$ '

$$\begin{aligned} r + y &= 2(r + b + y) - [(r + b) + (b + y)] \\ &= 2(4.5) - [4.25 + 2.75] \\ &= 9 - 7 = 2 \end{aligned}$$

The correct answer is B.

- 23.** Let us consider,

E = price under the 'Exclusive' plan

R = price under the 'Regular' plan

x = the number of classes Karan takes

Then,

$$E = 495 + 15x$$

$$R = 80x$$

$$0.6R = E$$

Now, solving these equations by substitution:

$$0.6R = 495 + 15x$$

$$0.6(80x) = 495 + 15x$$

$$48x = 495 + 15x$$

$$33x = 495$$

$$x = 15$$

The correct answer is C.

- 24.** Let the number of correct answers be x ; so the number of incorrect answers will be $25 - x$.

According to the problem,

$$3x - 0.5(25 - x) = 40$$

$$\Rightarrow 3x - 12.5 + 0.5x = 40$$

$$\Rightarrow x = 15$$

$$\text{The number of incorrect answers} = 25 - 15 = 10$$

$$\text{The required difference} = 15 - 10 = 5$$

The correct answer is A.

- 25.** Let the number of persons be x and contribution per person be Rs. y .

Therefore, the total contribution = Rs. xy

Also, the total contribution

$$= (x - 3)(y + 10) = xy + 10x - 3y - 30$$

Since the contribution is the same, therefore:

$$xy = xy + 10x - 3y - 30$$

$$\Rightarrow 10x - 3y = 30$$

$$\Rightarrow 3y = 10x - 30$$

If $x = 3$, $y = 0$; this is not possible as the total contribution will be 0.

(Note that when $x = 3$ or 4, y will not be an integer value).

If $x = 6$, $y = 10$; this condition is possible, so the minimum total contribution possible is Rs. 60.

If 10 people had joined for the party, contribution per person = $\frac{60}{10} =$ Rs. 6

The correct answer is C.

- 26.** Assume some values of p , q and r such that $p + q + r = 0$ and find the value of the expression that is given, so suppose $p = 1$, $q = -1$ and $r = 0$.

We find that,

$$\frac{p^2}{2p^2 + qr} + \frac{q^2}{2q^2 + pr} + \frac{r^2}{2r^2 + pq} = \frac{1}{2} + \frac{1}{2} + 0 = 1$$

The correct answer is B.

- 27.** Let the number of swans be x .

So, we get: $7 \times \frac{1}{2} \times \sqrt{x} + 2 = x$

Now using options, we realise that $x = 16$ satisfies the equation.

So, number of swans is 16.

The correct answer is B.

- 28.** $\frac{1}{p^2} - \frac{1}{q^2} = \left(\frac{1}{p} + \frac{1}{q}\right)^2 - 1 \times \frac{1}{pq} = m^2 - 2n$

The correct answer is C.

- 29.** Given $f(x + 2) + f(5x + 6) = 2x - 1$

When $(x + 2) = 1$, $x = -1$, at which the value of $(5x + 6)$ is also 1.

So, putting $x = -1$ in the given equation, we get:

$$f(1) + f(1) = -3$$

$$f(1) = \frac{-3}{2}$$

The correct answer is D.

30. For no solution of the equations

$$4x - Ky = -7 \text{ and } 5x + 3y = 2$$

$$\frac{4}{5} = \frac{-K}{3} \neq \frac{-7}{2}$$

$$K = \frac{-12}{5}$$

The correct answer is C.

31. $-4 > -5$, therefore, $[(-4)@(-5)] = (-4)$

Then, $(-4) \% 2 = -4$ (Since A \times B is negative)

The correct answer is D.

32. $\frac{1 @ -1}{(-K) @ (-K)} \% K, K > 0$

$$= \frac{1}{-K} \% K$$

$$= \frac{1}{-K}$$

The correct answer is C.

33. For a quadratic equation,

$$\text{Sum of the roots} = \frac{-b}{a}$$

$$\text{And product of the roots} = \frac{c}{a}$$

According to the problem,

$$-\frac{b}{a} = \frac{5}{4} \left(\frac{c}{a} \right)$$

$$\Rightarrow -4ab = 5ac$$

$$\Rightarrow b = -\frac{5}{4}c$$

The correct answer is C.

34. Let the three numbers in GP be a , ar and ar^2 .

Given that the product of the three terms is 27. So,

$$a \times ar \times ar^2 = 27$$

$$\Rightarrow a^3r^3 = 27 \quad ar = 3$$

Also given that the sum of the products taken in pairs is 91. So,

$$(a \times ar) + (ar \times ar^2) + (ar^2 \times a) = 91$$

$$\Rightarrow a^2r + a^2r^3 + a^2r^2 = 91$$

Substituting $ar = 3$, we get

$$\Rightarrow 3a + 9r + 9 = 91$$

$$\Rightarrow 3 \times \frac{3}{r} + 9r - 82 = 0$$

$$\Rightarrow 9 + 9r^2 - 82r = 0$$

$$\Rightarrow 9r^2 - 82r + 9 = 0$$

$$\Rightarrow 9r^2 - 81r - r + 9 = 0$$

$$\Rightarrow 9r(r - 9) - 1(r - 9) = 0$$

$$\Rightarrow (9r - 1)(r - 9) = 0$$

$$\Rightarrow r = \frac{1}{9}, 9$$

When $r = \frac{1}{9}$, $a = \frac{3}{r} = \frac{3}{1/9} = 27$

$$\text{Third term} = ar^2 = 27 \times \left(\frac{1}{9}\right)^2 = \frac{1}{3}$$

When $r = 9$, $a = \frac{3}{r} = \frac{3}{9} = \frac{1}{3}$

$$\text{Third term} = ar^2 = 9 \times \frac{1}{3} = 3$$

So, the third term will be either 3 or $\frac{1}{3}$.

The correct answer is E.

- 35.** The given information can be tabulated as follows:

Week no.	Payment for the week	Cumulative payment
1	1,000	1,000
2	$1,000 + 1,000 = 2,000$	$1,000 + 2,000 = 3,000$
3	$1,000 + 3,000 = 4,000$	$3,000 + 4,000 = 7,000$
4	$1,000 + 7,000 = 8,000$	$7,000 + 8,000 = 15,000$
5	$1,000 + 15,000 = 16,000$	$15,000 + 16,000 = 31,000$
6	$1,000 + 31,000 = 32,000$	$31,000 + 32,000 = 63,000$

The correct answer is D.

- 36.** The collection figures are in AP, with the first term as 100,000 and the common difference is 20,000.

Then

$$\begin{aligned}S_{10} &= \frac{10}{2} [2a + (n - 1) \times d] \\&= 5[200,000 + (9 \times 20,000)] \\&= 5 (380,000) \\&= \text{Rs. } 1900,000\end{aligned}$$

The correct answer is E.

- 37.** Given that \log_x^a , \log_y^a and \log_z^a are in HP. Therefore,

$\frac{1}{\log^a_x}, \frac{1}{\log^a_y}$ and $\frac{1}{\log^a_z}$ will be in AP. So,

$$2 \times \frac{1}{\log^a_y} = \frac{1}{\log^a_x} + \frac{1}{\log^a_z}$$

$$\Rightarrow 2 \times \frac{\log y}{\log a} = \frac{\log x}{\log a} + \frac{\log z}{\log a}$$

$$\Rightarrow 2 \log y = \log x + \log z$$

$$\Rightarrow \log y^2 = \log(xz)$$

$$\Rightarrow y^2 = xz$$

Therefore, x, y and z are in GP.

The correct answer is C.

- 38.** Here, the person saves Rs. 400 in the first year, Rs. 600 in the second year, Rs. 800 in the third year and so on.

Hence, this forms an AP with the first term, $a = 400$ and the common difference, $d = 200$.

According to the question,

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow 18000 = \frac{n}{2} [800 + (n - 1)200]$$

$$\Rightarrow 18000 = \frac{n}{2} [600 + 200n] = 300n + 100n^2$$

$$\text{We have } n^2 + 3n - 180 = 0$$

$$\Rightarrow n = 12$$

The correct answer is B.

- 39.** Given that, $T_2 = ar = 6$ and $T_5 = ar^4 = 48$

Hence, $r = 2$ and $a = 3$

$$\text{So, } T_{10} = 3 \times 2^9 = 1,536$$

The correct answer is C.

40. $\frac{p}{q}, \frac{q}{r}, \frac{r}{s}, \frac{s}{p}$ are all positive real numbers and for positive numbers.

A.M. \geq G.M.

$$\text{So, } \frac{\frac{p}{q} + \frac{q}{r} + \frac{r}{s} + \frac{s}{p}}{4} \geq \sqrt[4]{\frac{p}{q} \times \frac{q}{r} \times \frac{r}{s} \times \frac{s}{p}}$$

$$\text{or, } \frac{p}{q} + \frac{q}{r} + \frac{r}{s} + \frac{s}{p} \geq 4$$

The correct answer is E.

41. Suppose $b = ar$, $c = ar^2$ and $d = ar^3$. Then,

$$\frac{1}{a^3 + b^3} = \frac{1}{a^3(1+r^3)}$$

$$\frac{1}{b^3 + c^3} = \frac{1}{a^3 r^3(1+r^3)}, \text{ and}$$

$$\frac{1}{c^3 + d^3} = \frac{1}{a^3 r^6(1+r^3)}$$

Clearly, $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$, and $(c^3 + d^3)^{-1}$ are also in GP with common ratio $\frac{1}{r^3}$

The correct answer is B.

42. The series is an AP with $a = 70$, $d = 10$ and $n = 20$.

$$S_n = \frac{20}{2} [2 \times 70 + (20-1)10]$$

$$S_n = 10(140 + 190)$$

$$S_n = 10(330)$$

$$S_n = 3300$$

The correct answer is E.

43. We have to calculate the sum of values of hour hand in a day when minute hand is on 12.

Also, when minute hand is on 12, hour hand point towards integral values between 1 to 12 because of exact hour completion.

In a single day, hour hand rotates two times pointing integer between 1 to 12.

Therefore, summation of required values

$$= 2 \cdot \sum_{k=1}^{12} k = 2 \cdot \frac{12 \times 13}{2} = 156$$

The correct answer is C.

- 44.** Given that a, b and c are in arithmetic progression. So, we have

$$2b = a + c \Rightarrow c = 2b - a$$

Substituting the value of 'c' in the following terms, we get

$$b + c = b + 2b - a = 3b - a$$

$$c + a = 2b - a + a = 2b$$

Therefore the terms (a + b), (b + c) and (c + a) or the terms (a + b), (3b - a) and (2b) can be in arithmetic or geometric or harmonic progression.

The correct answer is E.

- 45.** To figure out the 'limiting factor', take the number of players available for each position and figure out how many sets could be formed in each case, if there were more than enough players in all the other positions.

Batsmen: 23 players available \div 2 players needed per set = 11.5 sets = 11 complete sets.

Bowlers: 21 players available \div 3 bowlers needed per set = 7 complete sets.

Wicketkeepers: 9 players available \div 1 wicketkeeper needed per set = 9 complete sets.

Thus, only 7 complete sets can be formed, using all of the available bowlers and some of the other players. A total of $7 \times 2 = 14$ batsmen are required, leaving $23 - 14 = 9$ unused batsmen. Likewise, $7 \times 1 = 7$ wicketkeepers are required, leaving

$9 - 7 = 2$ unused wicketkeepers. In all, there are
 $9 + 2 = 11$ unused players, who will not be on any team.

The correct answer is C.

- 46.** For the first digit, there are only five options (5, 6, 7, 8, and 9) because a five-digit number must start with a non-zero integer. For the second digit, there are 5 choices again, because now zero can be used but one of the other numbers has already been used, and numbers cannot be repeated. For the third number, there are 4 choices, for the fourth there are 3 choices, and for the fifth number there are 2 choices. Thus, the total number of choices is $(5)(5)(4)(3)(2) = 600$.

The correct answer is D.

- 47.** The number of ways in which the friends can be arranged with Binod and Dravid separated is equal to the total number of ways in which the friends can be arranged minus the number of ways they can be arranged with Binod and Dravid together.

The total number of ways to arrange 5 people in a line is $5! = 120$.

To compute the number of ways to arrange the 5 friends such that Binod and Dravid are standing together, group Binod and Dravid as one person, since they must be lined up together. Then the problem becomes one of lining up 4 students, which gives $4!$ possibilities.

However, remember that there are actually two options for Binod and Dravid coming together: Binod first and then Dravid or Dravid first and then Binod. Therefore, there are $(4!)(2) = (4)(3)(2)(1)(2) = 48$ total ways in which the five friends can be lined up with Dravid and Binod standing together.

Therefore, there are $120 - 48 = 72$ arrangements where Binod will be separated from Dravid.

The correct answer is C.

- 48.** Given that Ajit put Re 1 in his piggy bank on January 1. Every day he puts in Rs 2 more than the total amount of money already in the piggy bank.

Total amount of money in Ajit's piggy bank at the end of January is

$$= 1 + (1 + 2) + \{1 + (1 + 2) + 2\} + \{\{1 + (1 + 2) + 2\} + 2\} + \dots \text{ 31 terms}$$

The nth term of this series is

$$t_n = 1 + 3 + 6 + 12 + 24 + 48 + 96 \dots$$

Required sum is

$$\begin{aligned} S_n &= 1 + \frac{3(2^{30} - 1)}{2 - 1} \\ &= 1 + 3(2^{30}) - 3 = 3(2^{30}) - 2 \end{aligned}$$

The correct answer is C.

- 49.** First distribute 2 balls in each of the boxes. So, we are left with 4 identical balls to be distributed in 3 boxes.

We know that the number of ways of distributing 'n' identical balls among 'k' different boxes = ${}^{n+k-1}C_{k-1}$. The number of ways = ${}^{4+3-1}C_{3-1} = {}^6C_2 = \frac{6!}{4! \times 2!} = 15$

The correct answer is A.

- 50.** Since the hexagon formed by joining those 6 points is regular, any pentagon formed using 5 vertices will be congruent.

Hence, only one pentagon with distinctly different area can be formed.

The correct answer is D.

- 51.** In each section 3 questions are to be selected from the five designated questions. This can be done in 5C_3 ways.

Hence, the total number of different tests

$$= {}^5C_3 \times {}^5C_3 \times {}^5C_3 = 1,000$$

The correct answer is E.

- 52.** If five different types of sweets A, B, C, D and E then the required answer will be non-negative integral solution of the equation

$$A + B + C + D + E = 8$$

Hence required answer

$$= {}^{8+5-1}C_{5-1} = {}^{12}C_4 = 495$$

The correct answer is B.

- 53.** We have 13 Y (yellow) balls, 19 G (green) balls, 27 R (red) balls, 10 B (black) balls, 7 b (brown) balls and 14 W (white) balls in all.

Let us take out maximum 13 balls of different colours, then we can

take out $(13Y) + (13G) + (13R) + (10B) + (7b) + (13W)$ balls = 69 balls.

If we take one more ball out now, we will certainly get 14 balls of either green, red or white colour. That is, if we take out 70 balls from the bag, we will get 14 balls (at least) of same colour.

The correct answer is B.

- 54.** Out of 5 girls, 3 girls can be invited in 5C_3 ways. Nothing is mentioned about the number of boys that Salim has to invite.

Salim can invite 1, 2, 3, 4, or even no boy.

Out of 4 boys, Salim can invite them in the said manner in $= {}^5C_3 \times (2)^4 = 10 \times 16 = 160$

The correct answer is B.

- 55.** Out of total 12 holes, at least 3 holes with red coloured balls can be filled in the following ways:

3 red coloured balls + 9 other coloured balls

$$= {}^5C_3 \times {}^{10}C_9 = 100$$

4 red coloured balls + 8 other coloured balls

$$= {}^5C_4 \times {}^{10}C_8 = 225$$

5 red coloured balls + 7 other coloured balls

$$= {}^5C_5 \times {}^{10}C_7 = 120$$

The sum of these three cases

$$= 100 + 225 + 120 = 445$$

The correct answer is C.

- 56.** Let a and d be the first term and the common difference of the arithmetic progression.

As per the problem, we have

$$a + 3d = 10 \quad (1)$$

$$a + 8d = 25 \quad (2)$$

Subtracting Eq. (1) from Eq. (2), we get

$$a + 8d - (a + 3d) = 25 - 10$$

$$5d = 15 \Rightarrow d = 3$$

Substituting the value of 'd' in Eq. (1), we get

$$a + 3(3) = 10 \Rightarrow a = 1$$

So, the arithmetic progression will be 1, 4, 7, 10, ...

Therefore, the required harmonic progression will be $\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \dots$

The correct answer is A.

- 57.** There are total 8 batsmen in team A and we want to select 6.

The required number of ways $= {}^8C_6 = \frac{8!}{2!6!} = 8 \times \frac{7}{2} = 28$ ways.

The correct answer is D.

- 58.** Total number of ways $= {}^{10}C_6 \times {}^4C_3 \times {}^1C_1$

$$= \frac{10 \times 9 \times 8 \times 7}{4!} \times 4 \times 1 = 840$$

The correct answer is C.

- 59.** Number of ways $= {}^{12}C_8 \times {}^3C_2 \times {}^3C_1$
 $= \frac{12 \times 11 \times 10 \times 9}{4!} \times 3 \times 3 = 4455$

The correct answer is A.

- 60.** The factors of 210 are as follows:

1 and 210

2 and 105

3 and 70

5 and 42

6 and 35

7 and 30

10 and 21

14 and 15

Out of the list of 16 factors, there are two multiples of 42 (42 and 210).

Thus, the answer is $\frac{2}{16}$ or $\frac{1}{8}$.

The correct answer is C.

- 61.** In this case, there are two independent events: hail on Thursday and hail on Friday. The question asks about the probability that there will be hail on both the days. Hence, we simply need to multiply the individual probabilities together as follows to arrive at the answer:

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

The correct answer is A.

- 62.** There are 12 girls and 20 boys in the classroom. If $\frac{1}{4}$ of the girls have cell phones, then there are $12 \times \frac{1}{4} = 3$ girls with cell phones. Therefore, there are $12 - 3 = 9$ girls who do NOT have cell phones.

Therefore, the probability of choosing a girl who does not have a cell phone is the number of girls without cell phones divided by the total number of children, which is $\frac{9}{32}$

The correct answer is B.

- 63.** The probability that at least one roll results in a number higher than 4 is equal to 1 minus the probability that all three of the rolls result in numbers 4 or lower. For one roll, there are 6 possible outcomes and 4 ways in which the outcome can be 4 or lower, so the probability is $\frac{4}{6} = \frac{2}{3}$. Thus, the probability that all three rolls result in numbers 4 or lower is $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$. This is the result that you do NOT want; subtract this from 1 to get the probability that you do want:

$$\text{Therefore, the required probability} = 1 - \left(\frac{8}{27}\right) = \frac{19}{27}$$

The correct answer is D.

- 64.** This question is an OR question, so you may be tempted to simply add the two probabilities. However, this will give you a number greater than 100%, which is NEVER possible: $0.8 + 0.25 = 1.05$. This is because this figure double-counts the cases where Deeksha skips her lunch and there is a power failure. Subtract out these cases in order to find the desired value.

In order to calculate the probability that Deeksha will skip her lunch AND that there will be a power failure, multiply the individual probabilities together:

$$0.8 \times 0.25 = 0.2$$

Then, subtract to find the required probability:

$$1.05 - 0.2 = 0.85, \text{ or } 85\%$$

The correct answer is C.

- 65.** The probability of choosing Bag A, $P(A)$, and the probability of choosing Bag B, $P(B)$, must be the same, that is, $P(A) = P(B) = \frac{1}{2}$.

If Bag A is chosen, what is the probability of a matched pair? First, compute the probability of two whites. The probability of the first white is $\frac{3}{6}$ and the probability of the second white is $\frac{2}{5}$ so the probability of a first AND second white is $\left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{1}{5}$. Similarly, the probability of two reds is $\left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{1}{5}$. If Bag A is chosen, you can obtain a match by either choosing a pair of white OR a pair of red, so you must add their probabilities to get the total chance of a pair. This gives $P(\text{Bag A Pair}) = \frac{1}{5} + \frac{1}{5} + \frac{2}{5}$.

Similarly, if Bag B is chosen, the probability of a pair of white beads is $\left(\frac{6}{9}\right)\left(\frac{5}{8}\right) = \frac{5}{12}$ and the probability of a pair of red beads is $\left(\frac{3}{9}\right)\left(\frac{2}{8}\right) = \frac{1}{12}$. Therefore, the probability of a pair is $P(\text{Bag B pair})$

$= \frac{5}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$. The probability of choosing Bag A and a pair from Bag A is the product of the two events, $\left(\frac{1}{2}\right)\left(\frac{1}{5}\right) = \frac{1}{10}$.

Similarly, the probability of choosing Bag B and a pair from Bag B is $\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}$. The total probability of choosing a pair will be the probability of choosing Bag A and a pair from Bag A or choosing Bag B and a pair from Bag B, meaning you must sum up these two events. This gives: $P(\text{pair})$

$$= \frac{1}{10} + \frac{1}{8} = \frac{4}{20} + \frac{5}{20} + \frac{9}{20}.$$

The correct answer is C.

- 66.** Total possible results on rolling two dice,

$$n(S) = 6^2 = 36$$

Let A be the event that a sum of 10 or more occurs.

$$A = \{(4,6), (5,5) (5,6), (6,4), (6,5), (6,6)\}$$

$$n(A) = 6$$

$$\text{Required probability} = \frac{1}{6}$$

The correct answer is C.

- 67.** Let A be the event of getting two bananas and B be the event of getting two good fruits.

Therefore, $(A \cap B)$ will be the event of getting two good bananas.

$$P(A) = P(\text{Get 2 bananas}) = \frac{\binom{20}{2}}{\binom{30}{2}}$$

$$P(B) = P(\text{Get 2 good fruits}) = \frac{\binom{23}{2}}{\binom{30}{2}}$$

$$P(A \cap B) = \frac{\binom{15}{2}}{\binom{30}{2}}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{338}{435}$$

The correct answer is B.

- 68.** Since the events are independent, the outcome of one will not affect the other.

Given that $P(M) = 0.5$ and $P(N) = 0.4$

$P(M|N)$ means the probability of the event M given that the event N has already occurred.

So, the $P(M|N) = P(M) = 0.5$

The correct answer is B.

- 69.** Total number of roll numbers = 100

The number of rolls numbers which are multiples of 3 = 33

So, the required probability = $\frac{33}{100}$

The correct answer is C.

- 70.** Let the number be x . Then,

$$x + \frac{336}{x} \leq 50 \text{ or } (x - 8)(x - 42) \leq 0$$

$$\Rightarrow 8 \leq x \leq 42$$

So, x can be anything from 8 to 42, that is, 35 numbers.

$$\text{Required probability} = \frac{35}{50} = \frac{7}{10}$$

The correct answer is A.

- 71.** Let a and d be the first term and the common difference of the arithmetic progression.

As per the problem, we have

$$(a + d) + (a + 4d) = 24$$

$$2a + 5d = 24 \quad (1)$$

$$(a + 5d) - (a + 2d) = 12$$

$$3d = 12 \Rightarrow d = 4$$

Substituting the value of d in Eq. (1), we get

$$2a + 5(4) = 24$$

$$2a = 24 - 20 \Rightarrow a = 2$$

So, the first term is 2.

The correct answer is B.

4 DI-Caselets and Tables

1. Ratio in 2010 = $\frac{3}{50} = 0.06$

Ratio in 2011 = $\frac{2}{45} = 0.044$

Ratio in 2012 = $\frac{2.5}{40} = 0.0625$

Ratio in 2014 = $\frac{2.5}{48} = 0.052$

Ratio in 2015 = $\frac{2.1}{41} = 0.051$

Thus, the ratio was highest in 2012.

The correct answer is C.

2. Ratio in 2010 = $\frac{4}{60} = 0.066$

Ratio in 2012 = $\frac{3.5}{62} = 0.056$

Ratio in 2013 = $\frac{1.5}{40} = 0.037$

Ratio in 2014 = $\frac{2}{45} = 0.044$

Ratio in 2015 = $\frac{2.3}{50} = 0.046$

Thus, the ratio was lowest in 2013

The correct answer is C.

3. The required difference = $700 - 550 = 150$

The correct answer is A.

4. The required percentage = $\left(\frac{62 - 3.5}{58 - 4} \right) \times 100$

= approximately 108%

The correct answer is D.

5. The required percentage for Prisma

$$= \frac{1,050}{4,500} \times 100 = \text{approximately } 23.33\%$$

The required percentage for Shelby

$$= \frac{800}{4,100} \times 100 = \text{approximately } 19\%$$

The required percentage for Kooper

$$= \frac{1,000}{4,300} \times 100 = \text{approximately } 23.25\%$$

The required percentage for Wendy

$$= \frac{1,100}{5,450} \times 100 = \text{approximately } 20\%$$

The required percentage for Caret

$$= \frac{950}{4,350} \times 100 = \text{approximately } 22\%$$

The correct answer is C.

6. The required percentage $= \frac{1,100}{1,000} \times 100 = 110\%$

The correct answer is C.

7. While you can try calculating the required ratio for all the companies to get to the answer, the faster way to calculate this question is to look at the figures for the companies for April and February. Only in the case of Shelby is the April figure greater than the February figure. Thus, the highest ratio has to be that of Shelby.

The correct answer is B.

8. The number of trousers manufactured in February and March by Caret $= 850 + 950 = 1,800$

The number of trousers manufactured in February and March by Wendy = $1,150 + 1,100 = 2,200$

The number of trousers manufactured in February and March by Kooper = $1,050 + 1,050 = 2,050$

The number of trousers manufactured in February and March Shelby = $700 + 800 = 1,500$

The number of trousers manufactured in February and March by Prisma = $1,050 + 800 = 1,850$

The correct answer is D.

- 9.** Only Sulpar and Sictor have more than doubled over the entire period.

Overall growth of Sulpar

$$= \left(\frac{438,200}{194,830} - 1 \right) \times 100 = 125\%$$

Overall growth of Sictor

$$= \left(\frac{2,400}{1,100} - 1 \right) \times 100 = 118.18\%$$

Therefore, Sictor experienced the second highest annual growth rate.

The correct answer is B.

- 10.** Total number of two-wheelers on roads of the country in year 2007

$$\begin{aligned} &= 1,300 + 249,200 + 42,300 + 479,200 + 172,600 + 1,600 + 137,600 \\ &+ 65,000 + 14,400 \\ &= 1163,200 \end{aligned}$$

Total number of two-wheelers on roads of the country in year 2009

$$\begin{aligned} &= 1,900 + 325,000 + 49,200 + 416,700 + 162,400 + 3,000 + 125,700 \\ &+ 68,000 + 16,500 \\ &= 1168,400 \end{aligned}$$

Required difference = $1168,400 - 1163,200 = 5,200$

The correct answer is D.

- 11.** Total number of RHONDA two-wheelers in the year 2008 = 403,400

Total number of two-wheelers in the year 2008

$$= 1,800 + 266,000 + 43,300 + 403,400 + 150,400 + 2,300 + 121,700 \\ + 64,900 + 15,000 = 1068,800$$

So, the required percentage

$$= \frac{403,400}{1068,800} \times 100 = 37.74\% \approx 38\%$$

The correct answer is D.

- 12.** From the explanation of the previous question, we have:

Total number of two-wheelers in the year 2008

$$= 1068,800$$

Therefore, average number of two-wheelers in the year 2008

$$= \frac{1068,800}{9} = 118,756$$

The correct answer is A.

- 13.** Percentage of students securing more than 60% marks in aggregate

$$= \frac{270}{1,000} \times 100 = 27\%$$

And percentage of students securing more than 40% marks is =
 $\frac{730}{1,000} \times 100 = 73\%$

Hence, the difference = $73\% - 27\% = 46\%$

The correct answer is C.

- 14.** The number of students securing more than 20 marks in English = 800

The number of students securing 40 marks in Computer Science cannot be determined from the given information.

The correct answer is E.

- 15.** Number of students securing more than 60% in computers is 210 and number of students securing more than 40% in aggregate is 730.

$$\begin{aligned}\text{So, the required percentage} &= \frac{210}{730} \times 100 \\ &= 28.67\% = 29\%\end{aligned}$$

The correct answer is B.

- 16.** Number of people employed in private sector in 2008

$$= 8,832 + 1,043 = 9,875 \text{ thousands}$$

Number of people employed in public sector in 2008

$$= 2,739 + 7,171 + 3,387 + 2,407 + 1,938$$

$$= 17,674 \text{ thousands}$$

So, ratio of private sector employment to public sector employment in 2008

$$= \frac{9,873 \text{ thousands}}{17,674 \text{ thousands}} = 14:25$$

The correct answer is A.

- 17.** Yearly growth rate in employment in 2006–2007

$$= \frac{27,277 - 26,992}{26,992} = 0.01056$$

Yearly growth rate in employment in 2007–2008

$$= \frac{27,549 - 27,277}{27,277} = 0.00997$$

Required difference = $0.01056 - 0.00997$

$$= 0.00059 \approx 0.0006$$

The correct answer is C.

- 18.** Employment in private sector in 2006

$$= 7,804.0 + 1,001.1 = 8,805.1$$

$$\text{Required percentage} = \frac{8,805.1}{26,992} \times 100 = 32.6\%$$

The correct answer is D.

- 19.** Employment in public sector in 2007

$$= 2,800.0 + 7,209.9 + 3,447.0 + 2,414.0 + 2,132.0$$

$$= 18,002.9$$

$$\text{Required percentage} = \frac{18,002.9}{27,277} \times 100 = 66\%$$

The correct answer is C.

- 20.** Maximum marks in each subject is 100. So, the maximum total score should be such that the marks obtained by the student should not be more than 100. Let us discuss the options in decreasing order of total marks.

600: For achieving total score equal to 600, a student should not obtain more than 16.67% ($100/6$) in any subject. There is no such student so this cannot be the answer.

580: For achieving total score equal to 580, a student should not obtain more than 17.24% ($100/5.8$) in any subject. There is no such student so this cannot be the answer.

550: For achieving total score equal to 550, a student should not obtain more than 18.18% ($100/5.5$) in any subject. E is such a student so this is the correct answer.

The correct answer is C.

- 21.** Pass mark in each subject is 20 and each student passed in all subjects. Minimum total score should be such that the marks

obtained by the student should not be less than 20. Let us discuss the options in increasing order of total marks.

- 120: For achieving total score equal to 120, a student should not obtain less than 16.67% ($20/1.2$) in any subject. There is no such student so this cannot be the answer.
- 150: For achieving total score equal to 150, a student should not obtain less than 13.33% ($20/1.5$) in any subject. E is such a student. This is the correct answer.

The correct answer is B.

- 22.** The maximum total score should be such that the marks obtained by student C in any subject should not be more than 100. Let us discuss the options in decreasing order of total marks.
- 600: For achieving total score equal to 600, student C should not obtain more than 16.67% ($100/6$) in any subject. Incorrect.
- 550: For achieving total score equal to 550, student C should not obtain more than 18.18% ($100/5.5$) in any subject. Incorrect.
- 500: For achieving total score equal to 500, student C should not obtain more than 20% ($100/5$) in any subject. Correct.

The correct answer is C.

- 23.** Student E scored 18% in subject S, so students getting more than 18% in S will have lesser total marks than E. It means students B and D scored lesser total marks than the total marks scored by student E. So, E's total score in all six subjects is definitely more than 2 students.

The correct answer is C.

- 24.** Total number of aquatic animals that will not survive when pH changes from 6.5 to 4

$$= 24 + 82 + 40 + 12 = 158$$

Total number of aquatic animals that will not survive when pH changes from 6.5 to 4.5

$$= 24 + 82 + 12 = 118$$

So, required factor = $\frac{158}{118} = 1.3$ times

The correct answer is B.

- 25.** Following table shows the change in the number of species at different pH values.

Aquatic species	pH 6.5	pH 4.5	pH 5.5 (on adding 12)	% change at pH 5.5 over pH 6.5
Trout	34	0	12✓	< 60%
Bass	82	0	12✓	> 60%
Perch	40	40	52✓	< 60%
Frog	73	73	85✓	< 60%
Calm	12	0	12*	

Therefore, only Bass records greater than 60% change in the number of animals at pH 5.5 over pH 6.5.

The correct answer is A.

- 26.** Following table briefs number of aquatic animals at different pH values.

Aquatic species	pH 6.5	pH 6.0	pH 5.5	pH 5.0	pH 4.5	pH 4.0
Trout	24	24	24	24	0	0
Bass	82	82	82	0	0	0
Perch	40	40	40	40	40	0
Frog	73	73	73	73	73	73
Calm	12	12	0	0	0	0
Total	231	231	219	137	113	73

59.3% of original number of aquatic animals

$$= \frac{59.3}{100} \times 231 = 136.983 \approx 137$$

Therefore, the required answer is pH 5.0.

The correct answer is C.

- 27.** Total number of aquatic animals at pH 6.5

$$= 24 + 82 + 40 + 73 + 12 = 231$$

At pH 4.5, only Perch and Frog will survive. So,

$$\text{Number of aquatic animals at pH 4.5} = 40 + 73 = 113$$

$$\text{Required percentage} = \frac{113}{231} \times 100 = 48.92\%$$

The correct answer is D.

- 28.** Total number of students in college C

$$= 60 + 60 + 50 = 170$$

Number of students who got selected during campus placement drives

$$= 24 + 30 + 40 = 94$$

Required percentage

$$= \frac{94}{170} \times 100 = 55.29\%$$

The correct answer is C.

- 29.** Total number of Mechanical Engineering students from all the colleges

$$= 60 + 60 + 60 + 60 + 60 = 300$$

Number of students who are selected during campus placement drives

$$= 40 + 30 + 24 + 35 + 18 = 147$$

Required percentage

$$= \frac{147}{300} \times 100 = 49\%$$

The correct answer is C.

- 30.** Total number of Electrical Engineering students from all the colleges who got selected during campus placement drives

$$= 24 + 36 + 30 + 18 + 12 = 120$$

The correct answer is C.

5 DI-Graphs and Charts

1. The mode of a list of numbers is the number that occurs most frequently in the list. In the bar graph for GPA, dark gray bars represent the students in 2010, and the mode of that data set is indicated by the tallest dark gray bar. This is at grade point average of 3.3. There were 625 students with a grade point average of 3.3 in the year 2010, more students than had any other grade point average.

The correct answer is B.

2. The median is the ‘middle value’ of an ordered list of numbers, dividing the list into roughly two equal parts. For the 3,000 students in 1980, the median GPA is the average of the 1,500th and the 1,501st highest GPA. The students in 1980 are represented by the light gray bars.

150 students had a 4.0 GPA.

225 students had a 3.7 GPA.

(Total with this GPA and higher = $150 + 225 = 375$)

300 students had a 3.3 GPA.

(Total with this GPA and higher = $375 + 300 = 675$)

450 students had a 3.0 GPA.

(Total with this GPA and higher = $675 + 450 = 1,125$)

475 students had a 2.7 GPA.

(Total with this GPA and higher = $1,125 + 475 = 1,600$)

The 1,500th and 1,501st students fall between the 1,125th and 1,600th students. Thus, the 1,500th and 1,501st highest grade point averages are both 2.7.

The correct answer is D.

3. In 2010,

350 students had a 4.0 GPA.

525 students had a 3.7 GPA.

625 students had a 3.3 GPA.

500 students had a 3.0 GPA.

Thus, there were $350 + 525 + 625 + 500 = 2,000$ students who earned at least a 3.0 GPA in the year 2010, out of a total of 3,000 students. This is $\frac{2}{3}$ of the students, or about 67% of the students.

The correct answer is C.

- 4.** In 1980,

150 students had a 4.0 GPA.

225 students had a 3.7 GPA.

300 students had a 3.3 GPA.

450 students had a 3.0 GPA.

In 1980, $150 + 225 + 300 + 450 = 1,125$ students had a GPA of 3.0 or higher.

Thus, $3,000 - 1,125 = 1,875$ students earned a GPA less than 3.0.

As a percent of the class, this was $\left(\frac{1,875}{3,000}\right) \times 100 = 62.5\%$

The correct answer is D.

- 5.** Note that there are 5 vertical grid lines for every 10 players, so each vertical grid line accounts for 2 players. On the Athletics team, there are between 36 and 38 men (so it must be 37) represented by the light gray bar. On the Athletics team, there are between 60 and 62 women (so it must be 61) represented by the dark gray bar. In fractional form, the required ratio = $\frac{37}{61}$

The correct answer is A.

- 6.** Note that there are 5 vertical grid lines for every 10 players, so each vertical grid line accounts for 2 players.

Male athletes are represented by the light gray bars for each sport. Sum the male athletes in each of the separate teams.

Males in Volleyball team: 0

Males in Athletics team: 37

Males in Tennis team: 9

Males in Football team: 10

Males in Cricket team: 17

Males in Basketball team: 14

There are $0 + 37 + 9 + 10 + 17 + 14 = 87$ male players in all of the teams combined, but there are only a total of 76 male players. Thus, there must be $87 - 76 = 11$ male players who are counted twice by being on both the Athletics and Cricket teams.

The correct answer is A.

7. A sport in which male players outnumber female players will have a shorter dark gray bar than a light gray bar. This is only the case for Football, where there are 10 male players and 7 female players.

The correct answer is D.

8. There are 9 female tennis players and 14 male basketball players.

Thus, the required ratio = 9 : 14

The correct answer is B.

9. The population of B in 2005 = $5,000 \times \frac{16}{13} = 6,150$ approx.

The population of B in 2006 = $6,150 \times \frac{16}{13} = 6,750$ approx.

The population below poverty line = 52% of 6,750 = 3,500 approx.

The correct answer is C.

10. The population of Town D in 2005

$$= 9,000 \times \frac{17}{15} = 10,200$$

The population of Town D in 2007

$$= 10,200 \times \frac{110}{100} = 11,220$$

The population of Town G in 2007

$$= 9,000 \times \frac{95}{100} = 8,550$$

Therefore, the required total

$$= 11,220 + 8,550 = 19,770$$

The correct answer is B.

11. The population of Town F below the poverty line =
 $\frac{55,000}{100} \times \frac{13}{100} \times \frac{49}{100} = 3,500$

The correct answer is C.

12. The population of Town C below the poverty line

$$= 2,000 \times \frac{38}{100} = 760$$

The population of Town E below the poverty line

$$= \frac{2,000}{8} \times 18 \times \frac{46}{100} = 2,070$$

Therefore, the required ratio = $760 : 2,070$
= $76 : 207$

The correct answer is B.

Alternatively,

We do not need to know the population of any town to solve this question.

Required ratio = $(8 \times 38) : (18 \times 46) = 76 : 207$.

13. Number of recipients in the 60–64 age group in 2005

$$= 100 + 40 = 140 \text{ thousands}$$

Number of recipients in the 60–64 age group in 2006

$$= 96 + 60 = 156 \text{ thousands}$$

Required percentage increase

$$= \frac{156 - 140}{140} \times 100 = \frac{16}{140} \times 100 = 11.4\%$$

The correct answer is C.

- 14.** Number of recipients in the 35–39 and 40–44 age groups in 2005

$$= 36 + 20 + 40 + 36 = 132 \text{ thousands}$$

So, number of recipients in the 30–34 age group in 2005

$$= 150 - 132 = 18 \text{ thousands.}$$

Number of recipients in the 35–39 and 40–44 age groups in 2006

$$= 40 + 20 + 44 + 36 = 140 \text{ thousands}$$

So, number of recipients in the 30–34 age group in 2006

$$= 150 - 140 = 10 \text{ thousands}$$

$$\text{Required percentage} = \frac{18 - 10}{18} \times 100$$

$$= \frac{8}{18} \times 100 = 44.4\% \approx 44\%$$

The correct answer is D.

- 15.** Number of recipients aged above 49 years and below 65 years in 2005

$$= 56 + 48 + 76 + 68 + 100 + 40 = 388 \text{ thousands}$$

Number of recipients aged above 49 years and below 65 years in 2006

$$= 52 + 44 + 72 + 60 + 96 + 60 = 384 \text{ thousands}$$

Required percentage decrease

$$= \frac{388 - 384}{388} \times 100 = \frac{4}{388} \times 100 = 1.03\% \approx 1\%$$

The correct answer is A.

- 16.** Let the number of male recipients in the age group of 68–69 in the year 2005 be x .

Number of male recipients in the age of 35 and 69 in 2005

$$= 36 + 40 + 48 + 56 + 76 + 100 + x$$

$$= (356 + x) \text{ thousands}$$

Number of male recipients in the age of 35 and 69 in 2006

$$= 40 + 44 + 56 + 52 + 72 + 96 + 20$$

$$= 380 \text{ thousands}$$

Given that the number of male recipients between the age of 35 and 69 decreased by 5% from 2005 to 2006. So,

$$(356 + x) \times \frac{95}{100} = 380$$

$$\Rightarrow 356 + x = 400 \Rightarrow x = 44 \text{ thousands}$$

The correct answer is E.

General Solution for Questions 17–19:

Number of employees in 2013

$$= 2,000 - 250 + 350 = 2,100$$

Number of employees in 2014

$$= 2,100 - 450 + 300 = 1,950$$

Number of employees in 2015

$$= 1,950 - 300 + 400 = 2,050$$

Number of employees in 2016

$$= 2,050 - 250 + 500 = 2,300$$

Number of employees in 2017

$$= 2,300 - 400 + 350 = 2,250$$

- 17.** Number of employees in 2012 = 2,000

Change in number of employees in 2013

$$= 350 - 250 = 100$$

So, the required percentage change

$$= \frac{2,100 - 2,000}{2,000} \times 100 = 5\% = 5\%$$

The correct answer is D.

- 18.** Looking at the above calculation, it is clear that the number of employees in ABC Pvt. Ltd. was the maximum in the year 2016.

The correct answer is C.

- 19.** Looking at the above calculation, it is clear that none of the given two years has the number of employees in ABC Pvt. Ltd. the same.

The correct answer is E.

- 20.** Total volume of sales in the year 2011

$$= 405 \times 150 = 60,750$$

Total volume of sales in the year 2015

$$= 370 \times 265 = 98,050$$

Hence, the required percentage

$$= \frac{98,050 - 60,750}{60,750} \times 100 = \frac{3,730}{6,075} \times 100 = 61.4\%$$

The correct answer is D.

- 21.** Average of the volume of cars sold

$$= \frac{405 + 325 + 370 + 380 + 265 + 340 + 160}{7}$$

$$= \frac{2,245}{7} = 320.71$$

Average of the sales per unit volume

$$= \frac{150 + 170 + 270 + 320 + 370 + 365 + 320}{7}$$

$$= \frac{1,965}{7} = 280.71$$

Therefore, the required difference

$$= 320.71 - 280.71 = 40$$

The correct answer is A.

- 22.** This happens only in the duration 2011 – 2012, 2014 – 15 and 2015 – 2016. In rest of the duration the trend in both are same.

The correct answer is C.

- 23.** Family spending in Y1 = 470 – 252 = 218 thousands

Family spending in Y2 = 498 – 223 = 275 thousands

Family spending in Y3 = 513 – 204 = 309 thousands

Family spending in Y4 = 570 – 191 = 379 thousands

Family spending in Y5 = 650 – 182 = 468 thousands

Percentage increase in the annual family spending over the preceding year for

$$Y_1 = \frac{218 - 150}{150} \times 100 = \frac{68}{150} \times 100 = 45.33\%$$

$$Y_2 = \frac{275 - 218}{218} \times 100 = \frac{57}{218} \times 100 = 26.14\%$$

$$Y_3 = \frac{309 - 275}{275} \times 100 = \frac{34}{275} \times 100 = 12.36\%$$

$$Y_4 = \frac{379 - 309}{309} \times 100 = \frac{70}{309} \times 100 = 22.65\%$$

$$Y_5 = \frac{468 - 379}{379} \times 100 = \frac{89}{379} \times 100 = 23.48\%$$

Hence, Y3 is the required answer.

The correct answer is C.

- 24.** Family spending on food and clothing in Y1

$$= \frac{14+4}{100} \times (470 - 252) = \frac{18}{100} \times 218 = 39.24 \text{ thousands}$$

Family spending on food and clothing in Y2

$$= \frac{14+4}{100} \times (498 - 223) = \frac{18}{100} \times 275 = 49.5 \text{ thousands}$$

Required amount = $39.24 - 49.5 = 10.26$ thousands = Rs. 10,260

The correct answer is A.

- 25.** Amount spent on transportation in Y1

$$= \frac{16}{100} \times (470 - 252) = \frac{16}{100} \times 218 = \text{Rs. } 34,880$$

Amount spent on transportation in Y2

$$= \frac{16}{100} \times (498 - 223) = \frac{16}{100} \times 275 = \text{Rs. } 44,000$$

Amount spent on transportation in Y3

$$= \frac{16}{100} \times (513 - 204) = \frac{16}{100} \times 309 = \text{Rs. } 49,440$$

Amount spent on transportation in Y4

$$= \frac{16}{100} \times (570 - 191) = \frac{16}{100} \times 379 = \text{Rs. } 60,640$$

Amount spent on transportation in Y5

$$= \frac{16}{100} \times (650 - 182) = \frac{16}{100} \times 468 = \text{Rs. } 74,840$$

Hence, family spent Rs. 60,640 on transportation in Y4.

The correct answer is D.

- 26.** Family spending in Y4 = $570 - 191 = 179,000$

$$\text{On food} = \frac{14}{100} \times 379,000 = \text{Rs. } 53,060$$

$$\text{On housing} = \frac{31}{100} \times 379,000 = \text{Rs. } 117,490$$

$$\text{On transportation} = \frac{16}{100} \times 379,000 = \text{Rs. } 60,640$$

$$\text{On clothing} = \frac{4}{100} \times 379,000 = \text{Rs. } 15,160$$

$$\text{Other} = \frac{35}{100} \times 379,000 = \text{Rs. } 132,650$$

Hence, family spent between Rs. 1 lakh and 1.2 lakh on housing in Y4.

The correct answer is B.

- 27.** Runs scored by Karnataka against each state in 2005 is calculated in the table* below.

Total runs scored against West Bengal

$$= 4,000 + 1,500 + 4,500 = 9,000$$

Total runs scored against Maharashtra

$$= 1,000 + 2,000 + 2,500 + 3,000 = 8,500$$

Total runs scored against Chennai

$$= 2,000 + 1,000 + 7,500 = 10,500$$

Total runs scored against Andhra Pradesh

$$= 500 + 5,000 + 4,500 = 10,000$$

Total runs scored against Kerala

$$= 3,000 + 10,000 + 3,000 = 16,000$$

The correct answer is A.

- 28.** Runs scored against Chennai are calculated in the table** given at the bottom of this page.

Required percentage change

$$= \frac{12,000 - 10,500}{10,500} \times 100 = 14.28\% = 14.3\%$$

The correct answer is A.

	Season 1	Season 2	Season 3	Season 4
West Bengal	40% of 10,000 = 4,000	30% of 5,000 = 1,500		30% of 1,500 = 4,500
Maharashtra	10% of 10,000 = 1,000	40% of 5,000 = 2,000	10% of 25,000 = 2,500	20% of 15,000 = 3,000
Chennai	20% of 10,000 = 2,000	20% of 5,000 = 1,000	30% of 25,000 = 7,500	
Andhra Pradesh		10% of 5,000 = 500	20% of 25,000 = 5,000	30% of 15,000 = 4,500
Kerala	30% of 10,000 = 3,000		40% of 25,000 = 10,000	20% of 15,000 = 3,000

*Table for Question 27.

	Season 1	Season 2	Season 3	Season 4	Total
2005	20% of 10,000 = 2,000	20% of 5,000 = 1,000	30% of 25,000 = 7,500	-	10,500
2006	20% of 20,000 = 4,000	20% of 10,000 = 2,000	30% of 20,000 = 6,000	-	12,000

**Table for Question 28.

- 29.** This questions can be solved easily by looking at the options.

Option A: Season 3, Year 2005

Runs scored against Maharashtra in Season 2, Year 2005

$$= 40\% \text{ of } 5,000 = 2,000$$

Runs scored against Maharashtra in Season 3, Year 2005

$$= 10\% \text{ of } 25,000 = 2,500$$

Change over the previous season = $2,500 - 2,000 = 500$

Option B: Season 4, Year 2005

Runs scored against Maharashtra in Season 3, year 2005

$$= 10\% \text{ of } 25,000 = 2,500$$

Runs scored against Maharashtra in Season 4, year 2005

$$= 20\% \text{ of } 15,000 = 3,000$$

Change over the previous season = $3,000 - 2,500 = 500$

Option C: Season 2 Year 2006

Runs scored in Season 1, year 2006

$$= 10\% \text{ of } 20,000 = 2,000$$

Runs scored in Season 2, Year 2006

$$= 40\% \text{ of } 10,000 = 4,000$$

Change over the previous season

$$= 4,000 - 2,000 = 2,000$$

Option D: Season 2, Year 2007

Runs scored in Season 1, Year 2007

$$= 10\% \text{ of } 30,000 = 3,000$$

Runs scored in Season 2, Year 2007

$$= 40\% \text{ of } 15,000 = 6,000$$

Change over the previous season

$$= 6,000 - 3,000 = 3,000$$

Option E: Season 4, Year 2007

Runs scored in Season 3, Year 2007

$$= 10\% \text{ of } 45,000 = 4,500$$

Runs scored in Season 4, Year 2007

$$= 20\% \text{ of } 50,000 = 10,000$$

Change over the previous season

$$= 10,000 - 45,000 = 5,500$$

Hence, Season 4, Year 2007 shows the maximum change over the previous season in the year.

The correct answer is E.

- 30.** Highest run is scored against Kerala in Season 3 Year 2007 = 18,000

No run is scored against West Bengal in Season 3 Year 2007.

Hence, maximum difference between the runs scored against any two states in any season = 18,000

The correct answer is E.

- 31.** Percentage increase in imports in 2004

$$= \frac{150 - 75}{75} \times 100 = 100\%$$

Percentage increase in imports in 2005

$$= \frac{250 - 150}{150} \times 100$$

Percentage decrease in imports in 2006

$$= \frac{250 - 225}{250} \times 100 = 10\%$$

Percentage increase in imports in 2007

$$= \frac{350 - 225}{225} \times 100 = 55\%$$

Percentage decrease in imports in 2008

$$= \frac{350 - 275}{350} \times 100 = 21.42\%$$

Hence, the percentage decrease in imports is lowest in 2006.

The correct answer is D.

- 32.** Total export for all the years
= $150 + 225 + 375 + 300 + 450 + 175$
= 1,675 hundred crores
- Total imports for all the years
= $75 + 150 + 250 + 225 + 350 + 275$
= 1,325 hundred crores
- Required ratio = $\frac{1,325}{1,675} = 53:67$
- The correct answer is D.**
- 33.** Total value of imports in 2003–04
= $75 + 150 = 225$ hundred crores
- Total value of exports in 2003–04
= $150 + 225 = 375$ hundred crores
- Total value of imports in 2004–07
= $150 + 250 + 225 + 350 = 975$ hundred crore
- Total value of exports in 2004–07
= $225 + 375 + 300 + 400 = 1,350$ hundred crore
- Total value of imports in 2005–07
= $250 + 225 + 350 = 825$ hundred crore
- Total value of exports in 2005–07
= $375 + 300 + 450 = 1,125$ hundred crore
- Total value of imports in 2006–08
= $225 + 350 + 275 = 850$ hundred crore
- Total value of exports in 2006–08

$$= 300 + 450 + 175 - 925 \text{ hundred crore}$$

Total value of imports in 2007–08

$$= 350 + 275 = 625 \text{ hundred crore}$$

Total value of exports in 2007–08

$$= 450 + 175 = 625 \text{ hundred crore}$$

Hence, in 2007–08 the total value of import was equal to the total value of export.

The correct answer is E.

- 34.** Total export in the year 2003, 2006 and 2008

$$= 150 + 300 + 175 = 625 \text{ hundred crore}$$

Total import in the year 2003, 2006, 2008

$$= 75 + 225 + 275 = 575 \text{ hundred crore}$$

Required percentage

$$= \frac{625}{575} \times 100 = 108.69\% \approx 109\%$$

The correct answer is A.

6 Data Sufficiency

1. Total intelligent boys = 18

So, total boys in the class $= 18 \times \frac{4}{3} = 24$

From statement 1, the number of girls can be calculated.

From statement 2, the number of girls cannot be calculated.

Hence, statement 1 alone is sufficient, but statement 2 alone is not sufficient.

The correct answer is A.

2. Total students = 150

Let number of girls be 'x' and boys be 'y'.

$$\text{So, } x + y = 150 \quad (1)$$

From statement 1:

$$0.5x + 0.25y = 49 \quad (2)$$

'x' can be calculated using Eq. (1) and (2)

From statement 2:

$$0.3x + 0.3y = 45$$

$$x + y = 150 \quad (3)$$

Eq. (1) and (3) both are same, so x cannot be calculated.

Hence, statement 1 alone is sufficient but statement 2 alone is not sufficient.

The correct answer is A.

3. Since we don't have the total number of people given to us, or the ratio of men to women, we cannot arrive at the answer even by

combining both the statements.

The correct answer is E.

4. If $\frac{2}{5}$ of the students are boys, this means that $\frac{3}{5}$ of the students are girls. So the required probability is $\frac{3}{5}$. Statement 1 is sufficient.

Statement 2 does not help since we don't know the total number of students in the class.

The correct answer is A.

5. From statement 1:

$$x^2y = 48$$

Cannot be determined if $x^2 + y^3$ is even or odd.

So, statement 1 alone is not sufficient.

From statement 2:

$$x^2y = 36$$

Again, cannot be determined if $x^2 + y^3$ is even or odd.

So, statement 2 alone is not sufficient.

Combining both the statements, we have

$$\frac{x^2y}{xy^2} = \frac{48}{36} = \frac{4}{3}$$

$$\Rightarrow \frac{x}{y} = \frac{4}{3}$$

$$\Rightarrow x = \frac{4}{3}y$$

So, putting value of x in any of the statement will provide the values of y and then x can be calculated.

Hence, both statements together are sufficient.

The correct answer is C.

6. Statement 1: Since $a + 2$ is even, a is an even integer, and therefore $a + 1$ would be an odd integer; Statement I is Sufficient

Statement 2: Since $a - 1$ is an odd integer, a is an even integer. Therefore $a + 1$ would be an odd integer; Statement II is Sufficient.

The correct answer is D.

7. From statement 1, Let Udit's earning last month be x

$$\text{Then } \frac{1}{4} \times \frac{1}{2}x = 1200$$

By solving the above equation, we will get Udit's earnings for the last month; Sufficient. (Remember, you do not need to try to solve this equation, it will only waste your time).

Statement 2 just tells us the relation between Udit's savings and his rent, which does not tell us anything about his overall earnings. So, the statement 2 is Insufficient.

The correct answer is A.

8. From Statement 1, the original amount of water in the pool is irrelevant, since we only have to figure out the rate of increase. So, Statement 1 is Not Sufficient

From Statement 2, water is being pumped in at 50 litres per minute and pumped out at $\frac{20}{4} = 5$ litres per minute. Thus, the net rate of addition = $50 - 5 = 45$ litres per minute. So Statement 2 ALONE is sufficient.

The correct answer is B.

9. Using statement 1 alone, we can say that the price of coffee = $x+5$, where x is the price of tea.

Using statement 2 alone, we know that the price of coffee = $3x - 5$, where x is the price of tea.

Combining both the statements, we have

$$\begin{aligned}x + 5 &= 3x - 5 \\2x &= 10 \Rightarrow x = 5\end{aligned}$$

The correct answer is C.

- 10.** You don't need to actually calculate the 7th and 22nd roots of y ; doing so will only waste your time. However, the important thing to note is that you will get a unique value for y from either of the two given statements. What that value is does not matter to you. Thus, the answer is D.

The correct answer is D.

- 11.** You know that in a triangle, the largest angle lies opposite to the longest side. So all we need to figure out is which of the three sides is the longest.

From Statement 1, AC is the longest side, so it is Sufficient.

From Statement 2, we still don't have a value for y . So it is NOT Sufficient.

The correct answer is A.

- 12.** You can't do anything with either of the two statements because the total number of students is not given to you, neither is the ratio of Indian to non-Indian students given.

The correct answer is E.

- 13.** Statement 1 is not sufficient since it does not provide us with the Common difference.

Statement 2 is Sufficient because it provides us with the common difference (18) and also with one specific term.

The correct answer is B.

- 14.** The two statements simply give the cost price of the two types of tea. However, from the cost price we cannot get any idea of the selling price. Thus, both the statements together are Not Sufficient to answer the question asked.

The correct answer is E.

- 15.** Question asks to find the number of trailing zero when $P!$ is converted to base x . This number of trailing zero will depend upon the number of times x can divide successively $P!$. To find this, we should be given the values of both P and x .

From statement 1:

We are given the value of P only, so statement 1 alone is not sufficient.

From statement 2:

We are given the value of x only, so statement 2 alone is not sufficient.

On combining both the statements, we have values of both P and x , so this can be calculated.

Hence, both statements together are sufficient.

The correct answer is C.

- 16.** Let the number be xy .

Then, the number can be represented as $10x + y$ and the number formed by reversing the digits can be represented by $10y + x$.

From statement 1:

$$D = |(10x + y) - (10y + x)| = 36$$

$$\Rightarrow |9(x - y)| = 36$$

$$\Rightarrow |x - y| = 4$$

$$\Rightarrow x - y = \pm 4$$

Hence, the question cannot be answered using statement 1 alone.

From statement 2:

$$x + y = 12$$

Hence, the question cannot be answered using statement 2 alone.

Combining statements 1 and 2:

Clearly, the number could be 48 or 84 and hence, a unique number cannot be determined by combining the statements 1 and 2.

So, both the statements are not sufficient.

The correct answer is E.

- 17.** From Statement 1:

Using the first statement one can deduce that $A > D > F$ such that F is taller than at least one person, either B or E.

Hence, the question cannot be answered using statement 1 alone.

From Statement 2:

Using the second statement one can deduce that $B > E$.

Hence, the question cannot be answered using statement 2 alone.

Combining statements 1 and 2:

Combining both the statements we can deduce that F is greater than both B and E and, hence, E is the smallest.

So, the question can be answered using both the statements.

The correct answer is C.

- 18.** Divisibility rule of 9 says that a number is divisible by 9 if sum of digits is divisible by 9. So, as per statement 1, the number abc is a multiple of 9 and so sum of digits is also divisible by 9.

So, $a + b + c$ is divisible by 9.

Hence, statement 1 alone is sufficient.

Statement 2 is not sufficient since choosing $a = 0 = b$ and $c = 9$ makes it valid and $(a + b + c)$ is 9 but choosing $a = 4 = b$ and $c = 2$ also makes it valid with $(a + b + c)$ equal to 10, which is not divisible by 9.

The correct answer is A.

- 19.** Each statement independently gives the answer.

$\sqrt{9}$ gives + 3 and not -3. By convention we always take a positive root only.

The second statement gives the value of X as 0.

The correct answer is C.

- 20.** Statement 1 is true for parallelogram, rectangle as well as a rhombus.

So, statement 1 alone is not sufficient.

From statement 2, sum of AB + CD is equal to sum of AD + BC.

Since AB = CD and AD = BC (in a parallelogram), we can say that AB = BC = CD = DA, and so the parallelogram is a rhombus.

Hence, statement 2 alone is sufficient to determine the nature of ABCD.

The correct answer is B.

- 21.** Using statement 1:

$$a^2 + b^2 + c^2 = ab + bc + ca$$

$$\text{or } a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$\text{or } \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$

$$\Rightarrow a = b = c$$

Therefore, we cannot say that $a = b = c = 1$

Hence, statement 1 alone is not sufficient to answer the question.

Using statement 2

From equation $a^2 + b^2 = 2c^2$, we cannot say that a, b and c are definitely equal to 1. They could be equal to 2, 3, 4 and so on.

Hence statement 2 alone is also not sufficient to answer the question.

Combining both the statements also we cannot find the definite answer.

The correct answer is E.

- 22.** Using statement 1:

Let the total capacity of the tank be V cm³.

Fraction of the total volume of the tank filled in 2 hours when both the pipes are opened together

$$= 2\left(\frac{1}{6} + \frac{1}{8}\right)$$

$$\text{So, volume of water in the tank} = 2\left(\frac{1}{6} + \frac{1}{8}\right)V$$

Since, the value of 'V' is not known, hence statement 1 alone is not sufficient to answer the question.

Using statement 2

Obviously, this statement alone is not sufficient to answer the question.

Combining both the statement together, we get:

$$\text{Volume of water in the tank} = 2\left(\frac{1}{6} + \frac{1}{8}\right) \times 100$$

The correct answer is C.

- 23.** Using statement 1:

Nothing can be said about the schedule of Chemistry.

Using statement 2:

Nothing can be said about the schedule of Chemistry.

Combining both the statement together, we get:

14th	15th	16th	17th
Monday	Tuesday	Wednesday	Thursday
Arithmetic	Dermatology	Biology	Chemistry

Hence, by using both the statements together, we can see that Chemistry was scheduled on Thursday.

The correct answer is C.

- 24.** From statement 1:

Even numbers are those whose one common factor is 2. So, the square of an even numbers have a common factor 4.

Hence, A and B are not co-primes (co-prime number are those whose HCF is 1).

From statement 2:

HCF of two prime numbers is always 1. So, A and B are co-prime numbers.

Hence, each statement alone is sufficient to answer the question.

The correct answer is D.

- 25.** We have to determine whose transaction out of two brothers Ramu and Somu resulted in a better profit. So, we should have details of both.

From statement 1:

We have transaction detail of only Ramu but not of Somu. So, statement 1 alone is not sufficient.

From statement 2,

We have transaction detail of only Somu but not of Ramu. So, statement 2 alone is not sufficient.

On combining statements 1 and 2, we have transaction details of both Ramu and Somu and hence, profit/loss of both can be determined.

Hence, both statements together are sufficient.

The correct answer is C.

- 26.** Given that taxi charges 50 dollars for distance less or equal to 300 metres and 'A' dollar for each additional 100 metres.

Kamal travels P to Q, Manik Lal travels city Q to R and Mahesh travels city P to R via city Q. So, if we know the amount paid by Kamal and Manik Lal then we can calculate amount paid by Mahesh.

From statement 1, amount paid by Kamal only is known. So statement 1 alone is not sufficient.

From statement 2, amount paid by Manik Lal is known but not by Kamal. So statement 2 alone is not sufficient.

On combining statements 1 and 2, we know the amounts paid by both Kamal and Manik Lal, so amount paid by Mahesh can be calculated now.

Hence, both statements together are sufficient.

The correct answer is C.

- 27.** The sum of the ages of A, B and C is 167 years. So,

$$A + B + C = 167$$

From statement 1, relation between ages of B and C is

$$B = \frac{17}{12}A$$

From this expression, the age of B cannot be calculated.

From statement 2, relation between ages of A and C is

$$C = A + 44$$

From this expression also, we cannot calculate the age of B.

On combining both the statements, we can calculate the age of B.

Hence, both statements together are sufficient.

The correct answer is C.

- 28.** From statement 1, it is not clear that the selling price is greater or smaller than the cost price.

From statement 2, overall profit or loss cannot be calculated.

On combining both the statements, the question can be solved.

Hence, both statements together are sufficient.

The correct answer is C.

- 29.** From statement 1, it is clear that 10% of the price value is Rs. 12. So, cost price can be calculated using statement 1 alone.

From statement 2, it is clear that the 20% of the price value is Rs. 24. So, cost price can be calculated using statement 2 alone.

Hence, each statement alone is sufficient.

The correct answer is D.

- 30.** From Statement 1:

$$\frac{x^2}{y^2} = \frac{1}{4} \Rightarrow y^2 = 4x^2$$

The given expression becomes:

$\frac{x^3}{4x^2 + 1}$, whose unique value cannot be calculated.

So, statement 1 alone is not sufficient.

From statement 2:

$$y = 2x$$

Putting in the given expression, we get:

$\frac{x^3}{4x^2 + 1}$, whose unique value cannot be calculated.

So, statement 2 alone is also not sufficient.

Combining the two statements does not help us either.

The correct answer is E.

- 31.** Given that the cost price of two dresses is Rs. 1,350.

Loss on first dress = 6%

Profit on second dress = 7.5%

From statement 1:

As Kirti made no profit and no loss, we can assume price of first dress as Rs. x and can form the following equation:

$$x \times \left(\frac{100 - 6}{100} \right) \times 100 + (1,350 - x) \left(\frac{100 + 7.5}{100} \right) \times 100 = 1,350$$

The value of x can be calculated from this equation. So, statement 1 alone is sufficient.

From statement 2:

Selling price of one dress is given but it is not clear that this is for the first dress or the second dress. So, question cannot be solved using statement 2 alone.

Hence, statement 1 alone is sufficient but statement 2 alone is not sufficient.

The correct answer is A.