MBA PIONEER 2024

Quantitative Aptitude

DPP:03

Unit Digit Theorem

Q1 Find the unit digit of the expression :
$12 \times 16 \times 18 \times 24$
(A) 2 (B) 4
(C) 6 (D) 8

Q2

$$\text{If } a = \frac{27^{52}}{3^5} + \frac{16^{36}}{4^6}, \text{ then find the last digit of a.}$$

(A) 0 (B) 1
(C) 5 (D) 3

Q3 What is the unit digit of the result of $\left(12^{12} + 13^{13} + 14^{14} + 15^{15}\right)$?
(A) 1 (B) 2
(C) 0 (D) 7

Q4 The last digit of $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times \ldots\ldots \infty$ is :
(A) 0
(B) 2
(C) 5
(D) Can't be determined

Q5

$$\text{If } N = 17^{5^4}, \text{ then unit digit of N is :}$$

(A) 1 (B) 7
(C) 9 (D) 3

Q6 A number is of the form $2^a \times 3^b$ where $a + b = 6$. If number is less than 100 but greater than 90 , find the unit digit of number.
(A) 6
(B) 8
(C) 9
(D) Can't be determined

Q7 Find the units digit of the following expression :
9 + 99 + 999 + ... till 1012 terms.

Q8 Find the units digit of expression :

$$(196)^{143} + (174)^{199} + (205)^{218}$$

(A) 0 (B) 2
(C) 5 (D) 7

Q9 $a = 100^n - (1 + 75^m)$ where $(n, m \in N$ and $m > n)$ Find the last digit of a

Q10 If $[(-9)^m + (-4)^n] \times 2^4 = N$ such that $(m = n)$ and $m, n \in Z$.
Find the sum of all possible units digit of N.
(A) 10 (B) 12
(C) 6 (D) 0

Q11 $17^{117} = a, 15^{115} = b$ and $n^m = c$. If the units place of $(a \times b \times c)$ is 0 , then find the highest possible units digit of n (Given $n, m \in N$)
(A) 0 (B) 1
(C) 5 (D) 8

Q12 What is the sum of all the possible units digit of $(3^x + 5^y + 6^z)$ if x, y and z are natural numbers?
(A) 14 (B) 12
(C) 10 (D) 7

Q13 abcd is a 4 digit number formed by

$$3^{x} \xrightarrow{\text{units place digit}} a,\ 7^{y} \xrightarrow{\text{units placedigit}} , b,$$
$$2^{z} \xrightarrow{\text{units placedigit}} c \text{ and } 8^{t} \xrightarrow{\text{units placedigit}} d$$

(given, each of $x, y, z, t \in N$). Find the largest number abcd such that each digit is distinct.

(A) 9876 (B) 9678
(C) 9867 (D) None of these

Q14 Assume x and y is any natural number. Then, units digit of which one among the options is same as of $\mathrm{x}^{21}+\mathrm{y}^{21}$.

(A) $x^{4}+y^{4}$
(B) $x^{10}+y^{10}$
(C) $x^{17}+y^{17}$
(D) $x^{20}+y^{20}$

Q15 If $a+\frac{1}{a} \leq 2$ and $a \in N$, then find the units digit of a^{a}.

(A) 1
(B) 0
(C) 2
(D) Inadequate data

Q16 For $\left(3^{\mathrm{m}}+3^{\mathrm{n}}\right) \times\left(4^{\mathrm{o}}+4^{\mathrm{p}}\right) \times\left(5^{\mathrm{q}}+5^{\mathrm{r}}\right)$, each $\times\left(6^{\mathrm{s}}+6^{\mathrm{t}}\right)$ power is any natural number. Find the units digit of the expression.

(A) 1 (B) 0
(C) 2 (D) Either 0 or 1

Q17 Units digit of $(1 \times 2)+(3 \times 4 \times 5)+(6 \times 7 \times 8 \times 9)$ $+\ldots\ldots . .+(28 \times 29 \times 30 \times 31 \times 32 \times 33$ $\times 34 \times 35)$ is

(A) 2 (B) 4
(C) 6 (D) 8

Q18 If $\sum_{n=1}^{t} n^{2}+7=734$, then what is the units digit of t ?

(A) 1 (B) 2
(C) 3 (D) 4

Q19 Units digit of 177^{n} is $3 .$ n can be.

(A) 20 (B) 23
(C) 24 (D) 25

Q20 Find the units digit of the following $(7 \times 9 \times 11 \times 13 \ldots \times 101)^{11 \times 13 \times 15 \ldots \times 23}$

(A) 5 (B) 7
(C) 9 (D) 0

Q21 A 30 days month start with Friday. Find the unit digit of days on which 3^{rd} Sunday falls.

(A) 1 (B) 3
(C) 7 (D) 9

Q22 Find the units digit of $\left[\left(173^{15}\right)^{72}\right]^{41}$

(A) 1 (B) 3
(C) 5 (D) 7

Q23 Units digit of a number is 2.5 times than its tens place number. Reciprocal of this number is less than original number by 24.96. Find the units place digit of the number.

(A) 10 (B) 5
(C) 15 (D) 9

Q24 Find the units place number of $\frac{(2197)^{169}}{(169)^{39}}$.

(A) 0 (B) 1
(C) 2 (D) 3

Q25 Units place of $1737^{4\mathrm{m}+3}(\mathrm{m} \in \mathrm{N})$ is same as that of

(A) 123^{75} (B) 337^{128}
(C) 413^{145} (D) None of these

Q26 Find the unit digit for $34^{34^{34^{34}}}$

Q27

If X = The product of factorials of first 500 odd natural numbers, find the unit digit of X.

Q28 Find the unit digit for the following expression:
$111^{222} \times 222^{333} \times 333^{444} \times 444^{555} \times 555^{666}$

(A) 4 (B) 2
(C) 1 (D) 0

Q29 Find the number of positive integer values of 'n' such that n < 10 for which the the units digit of $(n)^{n!}$ is maximum possible.

Q30 Find the units digit of $7^{1002} - 4^{908}$

(A) 2 (B) 3
(C) 4 (D) 5

Answer Key

Q1	(B)
Q2	(D)
Q3	(C)
Q4	(A)
Q5	(B)
Q6	(A)
Q7	8
Q8	(C)
Q9	4
Q10	(A)
Q11	(D)
Q12	(A)
Q13	(D)
Q14	(C)
Q15	(A)
Q16	(B)
Q17	(C)
Q18	(B)
Q19	(B)
Q20	(A)
Q21	(C)
Q22	(A)
Q23	(B)
Q24	(D)
Q25	(C)
Q26	6
Q27	0
Q28	(D)
Q29	1
Q30	(B)

Hints & Solutions

Q1 Text Solution:

Taking unit digit of each number, we get $2 \times 6 \times 8 \times 4$

$$\rightarrow 4$$

So, unit digit of the given expression $= 4$ Ans. b

Q2 Text Solution:

$$\begin{aligned} a &= \frac{27^{52}}{3^5} + \frac{16^{36}}{4^6} \\ &= \frac{(3)^{3\times52}}{3^5} + \frac{16^{36}}{16^3} = 3^{(156-5)} + 16^{(36-3)} \\ &= 3^{151} + 16^{33} \end{aligned}$$

Unit digit of $3^{151} = 7 \quad \left(3^{4m+3} \text{ type }\right)$
Unit digit of $16^{33} = 6 \quad \left(6^{\text{Any power}} = 6\right)$
So, $6 + 7 \rightarrow 3$ is the last digit of the given expression.
Ans. d

Q3 Text Solution:

Unit digit of $12^{12} = 6$
Unit digit of $13^{13} = 3$
Unit digit of $14^{14} = 6$
Unit digit of $15^{15} = 5$
So, unit digit of the expression $= (6 + 3 + 6 + 5)$

$$\rightarrow 0$$

Ans. c

Q4 Text Solution:

The given expression contains 2 and 5 . This implies, $(2 \times 5) = 0.$ And any number multiplied by 0 gives zero. So, unit digit of the given expression is 0 . Ans, a

Q5 Text Solution:

$$\begin{aligned} \mathrm{N} &= 17^{5^4} \\ &= 17^{625} \end{aligned}$$

Unit digit of $17^{625} = 7 \left(17^{4\,\mathrm{m}+1} \text{ type }\right)$
Ans. b

Q6 Text Solution:

$$\begin{aligned} N(\text{ say }) &= 2^a \times 3^b \\ \text{Also, } a + b &= 6 \end{aligned}$$

At $a = 5, b = 1$,
We get $N = 96$ (Satisfy)
Unit digit $= 6$
Ans. a

Q7 Text Solution:

As we can see,
Unit digit of each number is 9
There are 1012 such number
So, unit digit of (9 × 1012) is
= 8
Ans. 8

Q8 Text Solution:

$$\begin{aligned} &(196)^{143} + (174)^{199} + (205)^{218} \\ &\xrightarrow{\text{Unit's place}} \quad 6 + 4 + 5 \\ &(2\mathrm{m}+1 \text{ type}) \\ &\xrightarrow{\text{Units'splace}} \quad 5 \end{aligned}$$

Ans. c

Q9 Text Solution:

Topic - Number System

Given, n and $\mathrm{m} \in \mathrm{N}$ and $\mathrm{n} < \mathrm{m}$.

Unit digit of 75^{m} will always be 5

So, $(1+75^{\mathrm{m}}) \xrightarrow{\text{unitdigit}} 6$

So, $100^{\mathrm{n}} - 6 \xrightarrow{\text{unit'sdigit}} 4$

Q10 Text Solution:

Given, $\mathrm{m}=\mathrm{n}$

Let's take m to be an even number

Then, $(-9)^{\mathrm{m}} \xrightarrow{\text{units digit}} 1$

and $(-4)^{\mathrm{n}} \xrightarrow{\text{units digit}} 6$

Now $(-9)^{\mathrm{m}}+(-4)^{\mathrm{n}} \xrightarrow{\text{units digit}} 7$

So, $7 \times 2^4 \xrightarrow{\text{unitsdigit}} 2$

If we take n to be an odd number

Then, $(-9)^{\mathrm{m}} \xrightarrow{\text{unitsdigit}} 9$ (negative number)

and $(-4)^{\mathrm{n}} \xrightarrow{\text{unitsdigit}} 4$ (negative number)

Now, $(-9)^m+(-4)^n$ will give unit digit as 3

So, 3×2^4 will give the unit digit as 8

Hence Sum of unit digit of all possible N = (8+2)

= 10

Option A

Q11 Text Solution:

Unit digits of $17^{117}=7\left(17^{4\,\mathrm{m}+1}\right.$ type)

Unit digits of $15^{115}=5$

Now, units digit of (7×5) is 5 .

To get 0 as unit digits of $(a \times b \times c)$

We can multiply $(\mathrm{a} \times \mathrm{b})$ by any even number.

And highest units place of n can be 8 .

Ans, d

Q12 Text Solution:

Given, $\left(3^x+5^y+6^z\right)$

Units digit of 6^z is always 6

Units digit of 5^{y} is always 5

Units digit of 3^x can be $3, 9, 7$ or 1 .

So, Case $-1 \rightarrow (3+5+6) \rightarrow 4$

Case $-2 \rightarrow \quad (9+5+6) \rightarrow 0$

Case $-3 \rightarrow \quad (7+5+6) \rightarrow 8$

and Case $-4 \rightarrow (1+5+6) \rightarrow 2$

Sum of all units digit $=(4+0+8+2)=14$

Ans. a

Q13 Text Solution:

$3^{\mathrm{x}} \xrightarrow{\text{unitsplace}} 9$ (highest possible number)

or $\mathrm{a}=9$

$7^{\mathrm{y}} \xrightarrow{\text{unitsplace}} 7$ (second highest)

As we can't use 9 now

or $b=7$

$2^{\mathrm{z}} \xrightarrow{\text{unitsplace}} 8$ (highest possible number)

or $\mathrm{c}=8$

$8^{\mathrm{t}} \xrightarrow{\text{units place}} 6$ (second highest)

So, abcd $=9786$

Ans. d

Q14 Text Solution:

If we use any digit (1 to 9) such that it is raised to the power of the type $(4\,\mathrm{m}+1,\ \mathrm{m} \in \mathrm{N})$.

It will always end in the same digit or $\mathrm{x}^5, \mathrm{x}^9, \mathrm{x}^{13}, \mathrm{x}^{17}, \mathrm{x}^{21}$ and $\mathrm{y}^5, \mathrm{y}^9, \mathrm{y}^{13}, \mathrm{y}^{17}, \mathrm{y}^{21}$ So, units digit of $\left(x^{21}+y^{21}\right)$ is same as the units digit of $\left(x^{17}+y^{17}\right)$ Ans. c

Q15 Text Solution:

Given, $a \in N$

and $\mathrm{a}+\frac{1}{\mathrm{a}} \leq 2$

This can only be possible when $\mathrm{a}=1$.

At $\mathrm{a}=1, \mathrm{a}+\frac{1}{\mathrm{a}}=2$

For any other natural number,

$$a+\frac{1}{a}>2$$

Now, 1^1

$$=1$$

So, units digit $= 1$
Ans. a

Q16 Text Solution:

Units digit of 5^q is 5 and units digit of 5^r is 5

$$\text{So, } 5+5 \xrightarrow{\text{units digit}} 0$$

And any number having 0 as a units digit, when multiplied by any other number. The units digit of resulting number is also 0 .
Ans. b

Q17 Text Solution:

Other expression according to the given expression are
Units digit of 5 term expression $= 0$,
Units digit of 6 term expression $= 0$,
Units digit of 7 term expression $= 0$.
Similarly for 8 term expression, units digit $= 0$
Units digit of $(3 \times 4 \times 5) = 0$
Units digit of $(6 \times 7 \times 8 \times 9) = 4$
Therefore, units digit of the expression is

$$\begin{aligned} &= 2+0+4+0+0+0+0 \\ &= 6 \end{aligned}$$

Ans. c

Q18 Text Solution:

$$\begin{aligned} &\sum_{n=1}^{t} n^2 + 7 = 734 \\ &\Rightarrow \frac{t \times (t+1) \times (2t+1)}{6} + 7t = 734 \end{aligned}$$

Using hit and trial, t $= 12$ satisfy the above equation So, units digit of $12 = 2$ Ans. b

Q19 Text Solution:

We know that,
Units digit of $7^{4\,\mathrm{m}}, 7^{4\,\mathrm{m}+1}, 7^{4\,\mathrm{m}+2}$ and $7^{4\,\mathrm{m}+3}$ are $1, 7, 9$ and 3 respectively.
$(\mathrm{m} \in \mathrm{N})$
So, n is of $4m+3$ type.
Option b satisfy.
Ans. b

Q20 Text Solution:

Multiplication of odd numbers give odd numbers and 15 multiplied by any odd number result in the number with 5 as units digit.

Now, $5^{\text{Any power}} \xrightarrow{\text{units digit}} 5$
Ans. a

Q21 Text Solution:

1^{st} day of month is Friday
So, 3^{rd} day will be Sunday (1^{st} Sunday)
And then, 2^{nd} Sunday will fall on 10^{th} days.
This means 3^{rd} Sunday will fall on 17^{th} days.
Ans. c

Q22 Text Solution:

Units digit of 173^{15} is 7
And 7^{72}, units digit $= 1$
This means 1^{41} gives the units digit as 1 .
Ans. a

Q23 Text Solution:

Lex x be the tens place digit of number. Then, $2.5\mathrm{x}$ is the units place digit. Now, according to the question

$$\begin{aligned} x2.5x - \frac{1}{x2.5x} &= 24.96 \\ \Rightarrow 10x + 2.5x - \frac{1}{10x+2.5x} &= 24.96 \\ \Rightarrow 12.5x - \frac{1}{12.5x} &= 24.96 \end{aligned}$$

At $x = 2$, above equation satisfy.

Thus, units place number $= (2.5 \times 2) = 5$

Ans. b

Q24 Text Solution:

$$\frac{(2197)^{169}}{(169)^{39}}$$
$$= \frac{\left(13^3\right)^{169}}{\left(13^2\right)^{39}}$$
$$= \frac{13^{3\times169}}{13^{2\times39}}$$
$$= 13^{507-78}$$
$$= 13^{429}$$
$$13^{429} \xrightarrow{\text{unitsplace}} 3 \quad \left(3^{4\,\mathrm{m}+1}\text{ type, m} \in \mathrm{N}\right)$$

Ans. d

Q25 Text Solution:

Units place of $1737^{4\,\mathrm{m}+3} = 3$

Checking option $\rightarrow$

(a) Units place of $123^{75} = 7$

(b) Units place of $337^{128} = 1$

(c) Units place of $413^{145} = 3$ (Satisfy)

Ans. c

Q26 Text Solution:

As the base is 34, the unit digit will be determined by the unit digit of 34 which is 4.

4 follows a cyclicity of 2 as far as the units digits are concerned.

$4^{odd\ number}$ always ends with 4 and $4^{even\ number}$ always ends with 6

As, $34^{34^{34}}$ is even, therefore the units digit will be 6

Q27 Text Solution:

X = 1! x 3! x 5! x 7! x 9! x 11! .... x 999!

The value of 5! = 120 i.e. it ends in 0.

Therefore the above product would contain atleast one trailing zero, therefore the unit digit of X is 0.

Q28 Text Solution:

No matter, how big the base, we will only consider its unit digit for the calculation.

Unit digit of 111^{222} will be **1**.

Unit digit of 222^{333} will be same as the unit digit of 2^{333}, 2 has a cyclicity of 4 for its units digits.

333 divided by 4, leaves a remainder of 1, therefore the unit digit will be 2^1 = **2**

3 also has a cyclicity of 4 for its units digits, 444/4, leaves a remainder of 0, therefore the unit digit will be same as that of 3^4 which is **1**

As 4 is raised to odd power, it will always end with **4**

Further, the powers of 5 always end in **5**

Thus, the unit digit of the complete product will be same as that of the product

1 x 2 x 1 x 4 x 5

= 0

Q29 Text Solution:

The maximum possible units digit is 9. Let us see, by hit and trial, if there is any value of n which satisfies the given criteria otherwise we will move onto 8 as the units digit.

If n = 2, $2^{2!}$ ends with 4

If n = 3, $3^{3!}$ ends with 9

We will not try n = 4, as the powers of 4 can only have the units digit of 4 or 6.

Similarly, the powers of 5 always end with 5 and the powers of 6 always end with 6.

If n =7, $7^{7!}$ ends with 1 as 7 has a cyclity of 4 for its units digits and 7! is a multiple of 4.

No power of 8 can ever end with 9, so we need not try that.

If n = 9, $9^{9!}$ would end with 1, as 9! is even (divisible by 2)

Therefore, for n < 10, only 1 value satisfies.

Q30 Text Solution:

7 has a cyclicity of 4 for its units digits, remainder when 1002 is divided by 4 is 2.

So the units digit of 7^{1002} will be 9

As the power of 4 is even, it will always end with 6.

So, the required units digit will be 9 - 6

= 3