MBA PIONEER 2024

QUANTITATIVE APTITUDE

DPP: 11

Coordinate Geometry 2

Q1 Find the equation of a line passing through the point (5, 8) and parallel to the line 2x – 3y – 5 = 0 ?

(A) 2x – 3y + 14 = 0

(B) 2x + 3y – 15 = 0

(C) 3x – 2y + 14 = 0

(D) 3x + 2y – 15 = 0

Q2 Find the equation of a line passing through the point (–3, 7) and perpendicular to the line 5x + 8y + 6 = 0 ?

(A) 5x + 8y + 11 = 0

(B) 8x – 5y + 59 = 0

(C) 5x – 8y – 11 = 0

(D) 8x + 5y + 59 = 0

Q3 The ends of the diagonal AC of a rectangle ABCD are (6, 1) and (12, 9). If the other diagonal BD is parallel to the x-axis, what is the perpendicular distance (in units) of the diagonal BD from the x-axis ?

(A) 9 (B) 7

(C) 6 (D) 5

Q4 Find the angle between the diagonals of a parallelogram whose sides are 6x + 8y = 12, $5\sqrt{2x} + 5\sqrt{2y} = 12$, 6x + 8y = 40 and $5\sqrt{2x} + 5\sqrt{2y} = 40$

(A) 150 degree (B) 120 degree

(C) 90 degree (D) 45 degree

Q5 The vertices of a triangle OBC are O (0, 0), B (–3, –1) and C (–1, –3). What is the equation of the straight line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from point O is 0.5 units ?

(A) $3x + y - \sqrt{2} = 0$

(B) x + y – 3 = 0

(C) 3x + 3y +1 = 0

(D) $\sqrt{2x} + \sqrt{2y} + 1 = 0$

Q6 The distance between two parallel lines is 0.4 units. If one of the parallel lines is 9x + 12y = 10, then what would the equation of the other line be ?

(A) 9x + 12y = 4

(B) 3x + 4y = 5

(C) 12y + 9x = 16

(D) Either 9x + 12y = 4 or 12y + 9x = 16

Q7 A man walks from the point M(3, 3) in a straight line. After walking a certain distance, he takes exactly a left turn from point P and walks the same distance as he did before the turn to reach point N(-3, 1). What is the coordinate of point P ?

(A) (1,–1)

(B) (–1, 5)

(C) (3, -2)

(D) Either (1, –1) or (–1, 5)

Q8 4x - 3y + 12 = 0 is the equation of the diagonal of a rhombus. Point having coordinates (3, 10) is one of the vertices of the rhombus. What is the length of the diagonal of the rhombus having (3, 10) as one of the ends ?

(A) 2.2

(B) 2.4

(C) 6.2

(D) Cannot be determined

Q9 The points (3, 6) and (3, 18) are the ends of the diagonal of a square. One of the ends of the other diagonal of the square in the second quadrant will be ______.

(A) (9, 6) (B) (12, -3)

(C) (6, -9) (D) (-3, 12)

Q10 The points of intersection of the lines having equations 6x + 5y - 32 = 0 and kx - 2y + 4 = 0 have coordinates which are integral. The summation of the total possible number of integral values of k is equal to _____ ?

(A) 0 (B) -4

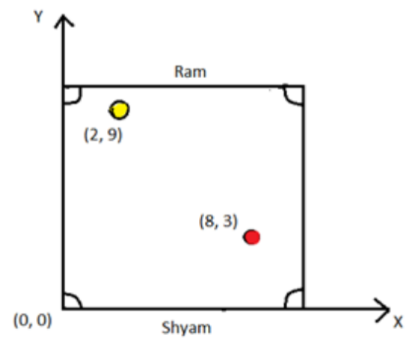
(C) 7 (D) -8

Q11 What will be the equation of the line cutting the lines 3x + 2y = 12 and 2x + 3y = 12 at the same point, and parallel to the line 4x + y = 0 ?

(A) x + 5y = -12 (B) y + 4x = 12

(C) 3x + 3y = -5 (D) 5y + 5x = 6

Q12 Ram and Shyam are playing a game of carrom, with the carrom board being in the form of a cartesian coordinate plane as shown in the diagram. When his turn to play came, Ram placed his striker at (2, 9) and wanted to hit the red queen carromman at (8, 3). But Ram wanted to hit the queen indirectly by first hitting the base on Shyam's side (the x-axis). What is the minimum distance the striker will have to travel to do that ?



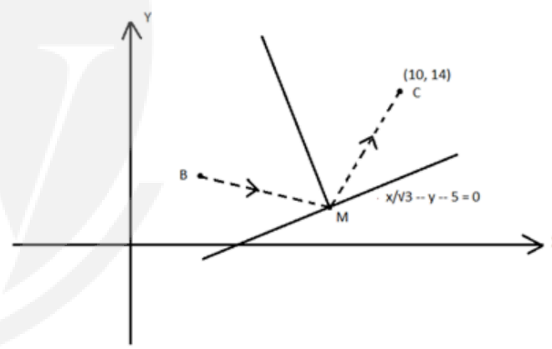
(A) 6 units

(B) $6\sqrt{2}\ units$

(C) $6\sqrt{5}\ unit$

(D) Cannot be determined

Q13 A ray of light originating from a bulb placed at B gets reflected by 120o from a point M on a mirror placed along a line having the equation $\frac{x}{\sqrt{3}} - y - 5 = 0$ and reaches the camera aperture placed at C (10, 14). What is the equation of the line MC ?



(A) $\sqrt{3x} - y + 2(7 - 5\sqrt{3}) = 0$

(B) $x/\sqrt{3} - y + 2(7\sqrt{3} + 5) = 0$

(C) $x/\sqrt{3} - y + 2(7 - 5\sqrt{3}) = 0$

(D) Cannot be determined

Q14 Two boys Ram and Shyam were standing at the coordinates (5, 8) and (11, 3) respectively of a football field resembling a coordinate plane. They both simultaneously noticed a box of chocolates lying at a point on the x-axis of the field, and immediately started running towards it at the same constant speed. They both reached the box simultaneously. How much distance away from the origin was the box of chocolates lying ?

(A) 2.38 units (B) 3.42 units

(C) 4.68 units (D) 5.52 units

Q15 P (1, -3), Q (0, 4) and R (-6, 7) form a triangle. What will be the equation of the altitude PM ?

(A) $2x - 3y + 5 = 0$

(B) $x - 2y - 3 = 0$

(C) $2x - y - 5 = 0$

(D) $3x + 2y + 3 = 0$

Q16 What will be the x-intercept made by the line having a slope of –(3/4) and having a y-intercept of –9 ?

(A) –12 (B) –13

(C) 14 (D) 12

Q17 There is a point P in the first quadrant of a coordinate plane at (5, 12). Find the slope of the line between the mirror images of P on the axes in the second and the fourth quadrant ?

(A) –2.4 (B) 3.6

(C) –0.48 (D) 0.36

Q18 Find the equation of the line perpendicular to $3x - 4y = 12$ and forming an area of 24 square units with the coordinate axes ?

(A) $4x + 3y = 24$

(B) $3x - 4y = 12$

(C) $4x + 3y = -24$

(D) Either $4x + 3y = 24$ or $4x + 3y = -24$

Q19 Find the area (in sq units) that the line parallel to the line $8x - 5y = 6$, and passing through (4, –7) would make with the coordinate axes ?

(A) 64.24 (B) 61.35

(C) 56.11 (D) 42.05

Q20 Find the area of the triangle (in square units) that the two lines $2x + 3y = 23$ and $3x - 4y = 9$ make with the x-axis ?

(A) 12.75 (B) 15

(C) 17.25 (D) 20

Q21 If the lines $x - 2y = 3$, $x + 3y = 3$ and $2x + y = 1$ form a right angled triangle, find the coordinates of the vertex of the triangle containing the right angle ?

(A) (3, 0)

(B) (1, –1)

(C) (0, 1)

(D) None of the above

Q22 What is the length of the diagonal of the square whose one opposite pair of sides have the equations of $8x + 6y + 3 = 0$ and $4x + 3y + 9 = 0$ respectively ?

(A) $\frac{9}{4}$ units (B) $\frac{3}{2}$units

(C) $\frac{3}{\sqrt{2}}$ units (D) $\frac{9}{2}$ units

Q23 If (4, 8), (–2, 4) and (10, –2) are three consecutive vertices of a parallelogram, what can be the fourth vertex out of the options given below ?

(A) (–6, 8) (B) (–8, 6)

(C) (12, 4) (D) (16, 2)

Q24 A line drawn through a point M(6, 7) makes an angle of 135° with the y-axis. If the line intersects the x-axis at point N, find the length of MN ?

(A) $7\sqrt{2}$ units (B) 14 units

(C) $6\sqrt{2}$ units (D) 12 units

Q25 What will be the equation of the line $8x + 6y = 25$ if the origin of the cartesian coordinate plane is shifted to the point (3, 4) ?

(A) $6x + 8y = 1$

(B) $8x + 6y = 73$

(C) $8x + 6y = 1$

(D) $4x + 3y = -19$

Q26 Find the centre of the circle having the equation $x^2 + y^2 + 4x - 6y - 36 = 0$

(A) (1, 2) (B) (2, −3)

(C) (−2, 3) (D) (3, −2)

Q27 There is a circle with centre at (2, 1) and radius 5 units. Which one among the options would best describe the coordinate of the point of intersection of the circle with the x-axis ?

(A) $\left(2+2\sqrt{6},\ 0\right)$

(B) $\left(2\sqrt{6},\ 0\right)$

(C) $\left(2–2\sqrt{6},\ 0\right)$

(D) Both $\left(2+2\sqrt{6},\ 0\right)$ and $\left(2–2\sqrt{6},\ 0\right)$

Q28 Find the radius of the circle passing through the points (1, 3), (-1, 1) and (2, 1) ?

(A) $\sqrt{\frac{5}{2}}$ units (B) 6 units

(C) $\sqrt{\frac{14}{15}}$ (D) 8 units

Q29 A circle lies on the cartesian coordinate plane such that the positive x and y axes are tangential to it. A point having coordinates (4, 2), lies on the circumference of the circle, on the segment created by both the points of tangency which is nearer to the origin. Find the area of the circle ? [Assume value of π to be 3.14)

(A) 628 units (B) 314 units

(C) 157 units (D) 94 units

Q30 Find the number of integral coordinates not lying outside the circle $x^2 + y^2 = 25$?

Answer Key

Q1	(A)
Q2	(B)
Q3	(D)
Q4	(C)
Q5	(D)
Q6	(D)
Q7	(D)
Q8	(B)
Q9	(D)
Q10	(A)
Q11	(B)
Q12	(C)
Q13	(A)
Q14	(B)
Q15	(C)
Q16	(A)
Q17	(A)
Q18	(D)
Q19	(C)
Q20	(A)
Q21	(B)
Q22	(C)
Q23	(D)
Q24	(A)
Q25	(B)
Q26	(C)
Q27	(D)
Q28	(A)
Q29	(B)
Q30	81

Hints & Solutions

Q1 Text Solution:

The general equation of a line parallel to the line 2x – 3y – 5 = 0 or, 2x – 3y = 5 would be of the form 2x – 3y = k

But this parallel line passes through (5, 8), which implies that the point (5, 8) satisfies the equation

Thus, 2*5 – 3*8 = k or, k = –14

Hence, the equation of a line passing through the point (5, 8) and parallel to the line 2x – 3y – 5 = 0 is 2x – 3y = –14 or, 2x – 3y + 14 = 0

Q2 Text Solution:

The general equation of a line perpendicular to the line 5x + 8y + 6 = 0 or, 5x + 8y = –6 = 0 would be of the form 8x – 5y = k

But this perpendicular line passes through (–3, 7), which implies that the point (–3, 7) satisfies the equation

Thus, 8*(–3) – 5*7 = k or, k = –59

Hence, the equation of a line passing through the point (–3, 7) and perpendicular to the line 5x + 8y + 6 = 0 is 8x – 5y = –59 or, 8x – 5y + 59 = 0

Q3 Text Solution:

Since BD is parallel to the x-axis, let the coordinates of its ends be (p, h) and (q, h)

Because ABCD is a rectangle, and the diagonals of a rectangle bisect each other, hence AC and BD will bisect each other.

Hence the y-coordinate of the point of intersection would be (h + h)/2 considering BD, and (1 + 9)/2 = 5 considering AC.

Both are the same point.

Hence, (h + h)/2 = 5

or, h = 5

Hence, the distance of the diagonal BD from the x-axis = 5 units

Q4 Text Solution:

The opposite sides of a parallelogram are parallel.

Hence, if the parallelogram is represented by ABCD, and if AB has the equation of 6x + 8y = 12, then opposite side CD must have the equation of 6x + 8y = 40.

Also if BC has the equation of $5\sqrt{2x} + 5\sqrt{2y} = 12$, then opposite side DA must have the equation of $5\sqrt{2x} + 5\sqrt{2y} = 40$

If the coordinates of B be (m, n), and since B satisfies the equation of both AB and BC,

then : 6m + 8n = 12 and $5\sqrt{2m} + 5\sqrt{2n} = 12.$

Also the diagonal BD from point D, which satisfies the equation of both CD and DA, must have the equation of

$(6x + 8y - 40) + k * (5\sqrt{2x} + 5\sqrt{2y} -, 40) = 0$

where k ≠ 0

But diagonal BD passes through both B and D, and so (m, n) satisfies this equation too.

Hence,

$(6m + 8n - 40) + k * (5\sqrt{2m} + 5\sqrt{2n} - 40) = 0$

or, (12 – 40) + k*(12 – 40) = 0

or, k = –1

Hence equation of BD is $(6x + 8y - 40) + (-1) * (5\sqrt{2x} + 5\sqrt{2y} - 40) = 0$

or, $(5\sqrt{2} - 6)x + (5\sqrt{2} - 8)y = 0$

Slope of $BD = (6 - 5\sqrt{2}) / (5\sqrt{2} - 8)$

Similarly,

If the coordinates of A be (p, q), and since A satisfies the equation of both AB and DA,

then : 6p + 8q = 12 and $5\sqrt{2p} + 5\sqrt{2q} = 40$.

Also the diagonal AC from point C, which satisfies the equation of both BC and CD, must have the equation of

$\left(5\sqrt{2x} + 5\sqrt{2y} - 12\right) + j$, where j ≠ 0

$* \left(6x + 8y - 40\right) = 0$

But diagonal AC passes through both A and C, and so (p, q) satisfies this equation too.

Hence,

$\left(5\sqrt{2p} + 5\sqrt{2q} - 12\right) + j$

$* \left(6p + 8q - 40\right) = 0$

or, (40 – 12) + J*(12 – 40) = 0

or, j = 1

Hence equation of AC is

$\left(5\sqrt{2x} + 5\sqrt{2y} - 12\right) + (1)$

$* \left(6x + 8y - 40\right) = 0$

or, $\left(5\sqrt{2} + 6\right)x + \left(5\sqrt{2} + 8\right)y = 58$

Slope of $AC = -\left(5\sqrt{2} + 6\right) / \left(5\sqrt{2} + 8\right)$

Multiplying the slopes of BD and AC, we get

$\left(6-5\sqrt{2}\right)*-\left(5\sqrt{2}+6\right)/\left(5\sqrt{2}-8\right)$

$*\left(5\sqrt{2}+8\right)$

= (50 – 36) / (50 – 64)

= -1

Hence the diagonals are perpendicular to each other

Q5 Text Solution:

Slope of line BC = {–3 – (–1)} / {–1 – (–3)} = (–2)/2 = –1

Thus equation of any line parallel to BC will be :

y = (–1)x + c or, x + y – c = 0

If we consider line BC, x + y – c = 0 will satisfy it.

So, (–3) + (–1) – c = 0 or, c = (–4)

So, the equation of line BC is : x + y + 4 = 0

The perpendicular distance of O (0, 0) from the line $BC = \left(0 + 0 + 4\right) / \sqrt{\left(12 + 12\right)} =$ $2\sqrt{2}\ units$

Since the straight line asked for is parallel to BC and is intersecting the sides OB and OC, so the perpendicular distance of O (0, 0) from the line must be less than 2√2 units.

Which 0.5 units is.

The perpendicular distance of O (0, 0) from the above line asked for

$= \left(0 + 0 - c\right) / \sqrt{\left(12 + 12\right)} = \left(-c\right)$ $/\sqrt{2}$

Hence,

$\left(-c\right) / \sqrt{2} = 0.5$

or, $c = -\left(1/\sqrt{2}\right)$

Hence, the equation of the straight line parallel to BC intersecting the sides OB and OC, whose perpendicular distance from point O is 0.5 units is :

$x + y = -\left(1/\sqrt{2}\right)$

or, $\sqrt{2x} + \sqrt{2y} + 1 = 0.$

Q6 Text Solution:

Any line parallel to 9x + 12y – 10 = 0 will have the equation as : 9x + 12y + k = 0

The distance between the two parallel straight lines $= \frac{|k-(-10)|}{\sqrt{(9^2+12^2)}} = \frac{|k+10|}{15}$

So,

$\frac{|k+10|}{15}$ = 0.4

or, $|k + 10|$= 6

Hence,

Either (k + 10) = 6 or, k = -4

Or, - (k + 10) = 6 or, k = -16

Hence, the equation of the other line would be :

Either 9x + 12y - 4 = 0

Or, 9x + 12y - 16 = 0

Q7 Text Solution:

The distance between the points M(3, 3) and

$N\left(-3,1\right) = \sqrt{\left[\{3-(-3)\}^2 + (3-1)^2\right]}$

$= \sqrt{\left(6^2+2^2\right)} = \sqrt{40} = 2\sqrt{10}\,units$

The trajectory of the man forms an isosceles right angled triangle MPN, where vertex P contains the right angle,

MP = NP, and the hypotenuse MN is of length $2\sqrt{10}\ units$

So half of the hypotenuse = $\sqrt{10}\ units$

Now, as per the property of a right angled isosceles triangle, the perpendicular dropped from the point containing the right angle onto the hypotenuse will perpendicularly bisect the hypotenuse and will be equal to the length of half the hypotenuse

So the perpendicular from P on MN will be of length $\sqrt{10}\ units$

Also mid-point of MN will have the coordinate of [(3 - 3)/2, (3 + 1)/2] = (0, 2)

The equation of the line MN would be :

$\frac{(y-1)}{\{x-(-3)\}} = \frac{(3-1)}{\{3-(-3)\}}$

or, $\frac{(y-1)}{(x+3)} = \frac{2}{6} = \frac{1}{3}$

or, 3y - 3 = x + 3

or, x - 3y + 6 = 0

Let the coordinate of P be (a, b)

So, the distance of P from

$MN = \frac{|(a-3b+6)|}{\sqrt{1^2+(-3)^2}} = \frac{|(a-3b+6)|}{\sqrt{10}}$

So, $\frac{|(a-3b+6)|}{\sqrt{10}} = \sqrt{10}$

$|a - 3b + 6| = 10$

Hence,

Either, a - 3b = 4 .... (A)

or, a - 3b = -16 .... (B)

Also the length of the distance of P (a, b) from mid-point of

$MN\left(0,\ 2\right)$

$= \sqrt{\left[(a-0)^2 + (b-2)^2\right]}$

$= \sqrt{\left[a^2 + (b-2)^2\right]}$

Thus,

$\sqrt{\left[a^2 + (b-2)^2\right]}$ = √10

or, $a^2 + (b - 2)^2 = 10$ .... (C)

From (C), the only eight possible values of (a, b) can be (1, -1), (-1, -1), (1, 5), (-1, 5), (3, 1), (-3, 1), (3, 3) and (-3, 3)

Out of them only (1, -1) satisfy (A)

and only (-1, 5) satisfy (B)

Q8 Text Solution:

The equation of one of the diagonals of the rhombus is : 4x - 3y + 12 = 0

It may be possible that point (3, 10) is one of the ends of this diagonal.

But then (3, 10) will have to satisfy the equation.

But 4*3 - 3*10 + 12 = 12 - 30 + 12 = -6, which is ≠ 0

So (3, 10) is not one of the ends of the diagonal 4x - 3y + 12 = 0

So (3, 10) must be one of the ends of the diagonal perpendicular to 4x - 3y + 12 = 0

The diagonals of a rhombus also bisect each other

The perpendicular distance of the point (3, 10) from the diagonal 4x - 3y + 12 = 0

$= \frac{|4*3-3*10+12|}{\sqrt{4^2+(-3)^2}}$

= $\frac{|(-6)|}{5}$

= 6/5 units

Hence, the length of the diagonal of the rhombus having (3, 10) as one of the ends = (6/5)*2 = 12/5 = 2.4 units

Q9 Text Solution:

Since the x coordinate of the ends of the diagonal is the same, it is parallel to the y-axis, and thus the length of the diagonal is the difference of the y coordinates = 18 - 6 = 12 units

Also the midpoint of the diagonal will be $\left(3, \frac{(6+18)}{2}\right)$ = (3, 12)

Since the other diagonal of the square is equal in length to the first, and is bisected perpendicularly by the first diagonal, hence the two possible coordinates of the ends of the second diagonal would be (3 + 6, 12) or (3 - 6, 12), that is (9, 12) or (-3, 12)

Out of them, (-3, 12) lies in the second quadrant.

Q10 Text Solution:

Let the points of intersection of the lines be M (a, b) say

Since M satisfies both the equations,

So, 6a + 5b – 32 = 0 or, 5b = 32 – 6a or, b = 32-6a/5 So, 6a + 5b – 32 = 0 or, (32 – 6a)/5

Also, ka – 2b + 4 = 0 or, 2b = ka + 4 or, b = (ka + 4)/2

Hence, (32 – 6a)/5 = (ka + 4)/2

or, 64 – 12a = 5ka + 20

or, a(5k + 12) = 44

or, a = 44/(5k + 12)

Similarly, since M satisfies both the equations,

So, 6a + 5b – 32 = 0 or, 6a = 32 – 5b or, a = (32 – 5b)/6

Also, ka – 2b + 4 = 0 or, ka = 2b + 4 or, a = (2b + 4)/k

Hence, (32 – 5b)/6 = (2b + 4)/k

or, 32k –5kb = 12b + 24

or, b(5k + 12) = 32k – 24

or, b = (32k – 24)/(5k + 12)

But a is integral.

So (5k + 12) must be a factor of 44

Factors of 44 = +1, –1, +2, –2, +4, –4, +11, –11, +22 and –22 and + - 44.

or, (5k + 12) must be = +1, –1, +2, –2, +4, –4, +11, –11, +22 –22 and + - 44.

or, 5k must be = –11, –13, –10, –14, –8, –16, –1, –23, +10 , –34, 32, -56

But for k to be integral, as per given data, the value of 5k must be a multiple of 5

This is possible only for 5k = –10 and +10

or k = –2 and 2

Now, b also must be integral.

When k = –2,

b = {32*(–2) – 24}/{5*(–2) + 12} = (–88)/2 = –44, which is an integral value

When k = 2,

b = (32*2 – 24)/(5*2 + 12) = 40/2 = 20, which is an integral value

So k is indeed = –2 and 2

Q11 Text Solution:

If our line has to cut both the lines 3x + 2y = 12 and 2x + 3y = 12 at the same point, then it has to pass through the point of intersection of both the lines.

Adding both the equations we get 5x + 5y = 24

or, x + y = 24/5

and subtracting, x - y = 0

Solving them we get the point of intersection of the lines as (2.4, 2.4)

Our line is parallel to 4x + y = 0.

Hence, the equation of our line : 4x + y + k = 0

But it passes through (2.4, 2.4).

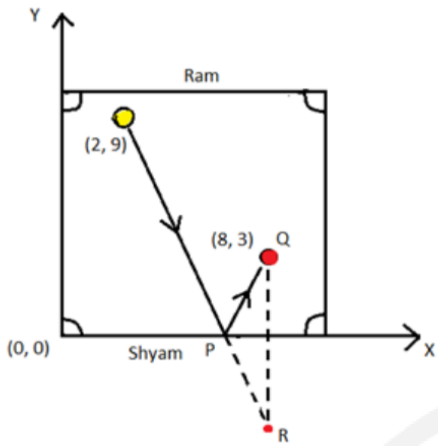
Hence, 4*2.4 + 2.4 + k = 0

or, k = -12

Thus, the equation of the line cutting the lines 3x + 2y = 12 and 2x + 3y = 12 at the same point, and parallel to the line 4x + y = 0 is 4x + y - 12 = 0

Q12 Text Solution:

To hit the red queen carromman at (8, 3) from (2, 9) via the x-axis by travelling the minimum distance, the striker has to reach the red queen after reflection on the x-axis



If the point of reflection of the striker on the x-axis is P, then let the line of travel be extended to meet the perpendicular dropped from the red queen (Q) at R.

Hence, as per basic geometry rules, the x-axis bisects the perpendicular QR.

Thus the coordinate of R will be (8, –3)

Also triangle PQR is an isosceles triangle with PQ = PR

Hence, the minimum distance the striker will have to travel to reach the red queen after reflection on the x-axis

= Length of the path from (2, 9) to P + PQ

= Length of the path from (2, 9) to P + PR

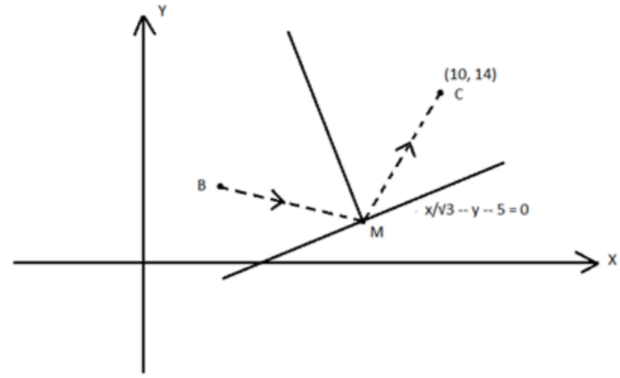
= Length of the path from (2, 9) to R (8, –3)

$= \sqrt{[(2–8)2 + 9–(–3)2]}$

$= \sqrt{180}$

$= 6\sqrt{5}\ unit$

Q13 Text Solution:



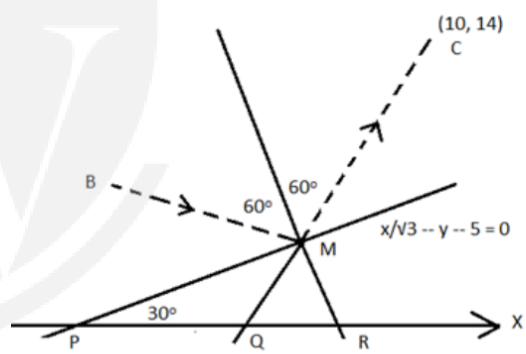
The equation of the line on which the mirror is placed is : $x/\sqrt{3} - y - 5 = 0\ or,\ y = x/\sqrt{3} - 5$

Thus the slope of the line on which the mirror is placed is $1/\sqrt{3} = \tan 30^\circ$

Hence the angle of the line on which the mirror is placed with the x-axis = 30°

The perpendicular to the line on which the mirror is placed from M will bisect the angle of reflection of 120° to 60°

Geometrically, the angles will be as below :



CM and the perpendicular at M extended meet the x-axis at Q and R respectively

Thus in triangle PMR, since angle PMR = 90° and angle MPR =30°, angle MRP = 60°.

Thus in triangle MRQ, angle MRQ = 60° and angle QMR = vertically opposite angle 60°. Hence angle MQR = 60° too

Thus the slope of the line QMC is tan 60° = $\sqrt{3}$

Thus the equation of the line MC is : √3x + c

But MC satisfies the point C (10, 14)

So $14 = 10\sqrt{3} + c$

or, $c = 2\left(7 - 5\sqrt{3}\right)$

Hence, the equation of the line MC is : $\sqrt{3}x - y + 2\left(7 - 5\sqrt{3}\right) = 0$

Q14 Text Solution:

Since the box of chocolates was lying on the x-axis, let the coordinate of its position be (x, 0)

Since both Ram and Shyam started to run towards the box simultaneously at the same constant speed, and managed to reach it simultaneously, the distance of the box from Ram and Shyam must have been the same.

Hence, $\sqrt{(5-x)^2 + (8-0)^2}$ must be $= \sqrt{(11-x)^2 + (3-0)^2}$

or, $(5 - x)^2 + (8 - 0)^2 = (11 - x)^2 + (3 - 0)^2$

or, $25 + x^2 - 10x + 64 = 121 + x^2 - 22x + 9$

or, 12x = 41

or, x = 3.42 units approx.

Hence, the box of chocolates lying at a distance of 3.42 units from the origin

Q15 Text Solution:

PM is perpendicular to the side QR of the triangle PQR.

Slope of QR = (7 – 4)/(–6 –0) = 3/–6 = –(1/2)

Hence slope of PM = 2

Thus the equation of PM is : y = 2x + c

But this line satisfies P (1, –3)

So, –3 = 2*1 + c or c = –5

Hence the equation of PM is : y = 2x – 5 or, 2x – y – 5 = 0

Q16 Text Solution:

The equation of the line having a slope of –(3/4) and a y-intercept of –9 is :

y = –(3/4)x + (–9) or, (3/4)x + y + 9 = 0

At the x-intercept, the value of y = 0

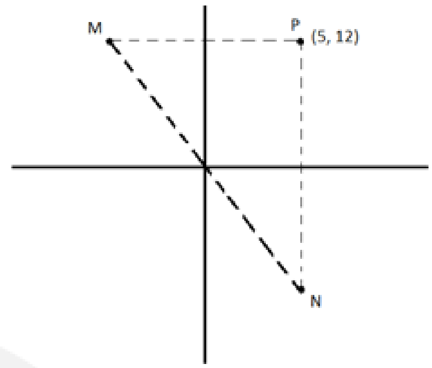
Hence, (3/4)x + 0 + 9 = 0

or, x = (–9)*(4/3)

or, x = –12

Hence the x-intercept made by the line having a slope of –(3/4) and a y-intercept of –9 is -12 units.

Q17 Text Solution:



The image of P in the second quadrant is M, whose coordinate will be (–5, 12)

The image of P in the fourth quadrant is N, whose coordinate will be (5, –12)

Hence the slope of the line MN = {12 – (–12)}/ {(–5) – 5} = 24/(–10) = –(12/5) = –2.4

Q18 Text Solution:

The equation of the line perpendicular to 3x – 4y = 12 is :

4x + 3y = k or, (4/k)x + (3/k)y = 1 or, x/(k/4) + y/(k/3) = 1

From the above equation we can say that the x-intercept is k/4, and the y-intercept is k/3

Thus the area the line forms with the coordinate axes is 1/2 * k/4 * k/3 = k^2/24 square units

But k^2/24 = 24

or k = $\pm$ 24

Hence the equation of the line perpendicular to 3x – 4y = 12 and forming an area of 24 square units with the coordinate axes is either 4x + 3y = 24 or 4x + 3y = –24

Q19 Text Solution:

Equation of the line parallel to 8x – 5y = 6 is : 8x – 5y = k

But this line passes through (4, –7)

Hence, 8*4 – 5*(–7) = k or, k = 67

Thus Equation of the parallel line is : 8x – 5y = 67 or, (8/67)x + (–5/67)y = 1 or, x/(67/8) + y/(–67/5) = 1

Hence the x and y intercepts are 67/8 and 67/5 respectively.

Thus the area that the line parallel to the line 8x – 5y = 6, and passing through (4, –7) would make with the coordinate axes

= 1/2 * 67/8 * 67/5

= 67^2/80

= 56.11 square units

Q20 Text Solution:

The line 2x + 3y = 23 intersects with the x-axis (equation of y = 0) at $\left(\frac{23}{2}, 0\right)$

The line 3x – 4y = 9 intersects with the x-axis (equation of y = 0) at (3, 0)

Hence the length between the intersection points = $\frac{23}{2} - 3 = \frac{17}{2}$ units

On solving, it can be seen that the two lines 2x + 3y = 23 and 3x – 4y = 9 intersect each other at the point (7, 3)

The perpendicular of this point (7, 3) to the x-axis will have the value of 3 units

Hence the area of the triangle that the two lines 2x + 3y = 23 and 3x – 4y = 9 make with the x-axis

$= \frac{1}{2} * \frac{17}{2} * 3$

= 51/4

= 12.75 square units

Q21 Text Solution:

By observation we can see that the lines x – 2y = 3 and 2x + y = 1 are the sides that are perpendicular to each other, and hence will contain the right angle in the triangle

On solving the two equations we get x = 1 and y = –1

Thus, the coordinates of the vertex of the triangle containing the right angle is (1, –1)

Q22 Text Solution:

The equations of one opposite pair of sides of the square are 8x + 6y + 3 = 0 and 4x + 3y + 9 = 0 respectively, or, 8x + 6y + 3 = 0 and 8x + 6y + 18 = 0 respectively

The opposite pair of sides can now clearly be observed to be parallel to each other.

The distance between the parallel sides $= mod\frac{(18-3)}{\sqrt{(82+62)}} = \frac{15}{10} = \frac{3}{2}$ units

This distance is the length of the side of the square, which is equal for all the four sides

Hence the length of the diagonal of the square $= \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}}$ units

Q23 Text Solution:

Let (4, 8), (–2, 4) and (10, –2) be denoted by A, B and C respectively

Let the coordinates of the fourth point D be (a, b)

Slope of $AB = \frac{(4-8)}{(-2-4)} = \frac{4}{6} = \frac{2}{3}$

Slope of $BC = \frac{(-2-4)}{10-(-2)} = \frac{-6}{12} = \frac{-1}{2}$

In a parallelogram, the opposite pair of sides are parallel

Thus, CD is parallel to AB

So their slopes are equal

Slope of $CD = \frac{b-(-2)}{(a-10)} = \frac{(b+2)}{(a-10)}$

So, $\frac{(b+2)}{(a-10)} = \frac{2}{2}$ or, 3b + 6 = 2a – 20 or, 2a – 3b = 26 ……… (Eqn 1)

Also, DA is parallel to BC

So their slopes are equal

Slope of DA = (8 – b)/(4 – a)

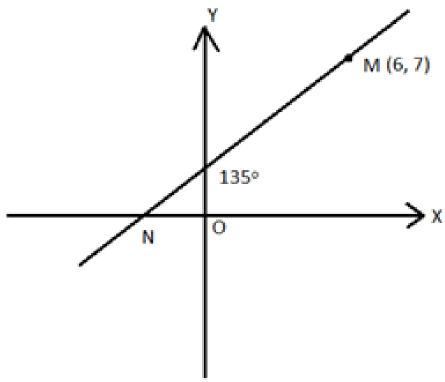
So, $\frac{(8-b)}{(4-a)} = \frac{-1}{2}$ or, 16 – 2b = –4 + a or, a + 2b = 20 ……… (Eqn 2)

Solving the above two equations we get :

b = 2 and a = 16

Hence, the fourth vertex out of the options given below is (16, 2)

Q24 Text Solution:



Since the line MN makes an angle of 135° with the y-axis, 135° is the exterior angle of the triangle formed by the point of intersection with the y-axis, N and the origin O.

Since the x and y axis are perpendicular to each other, angle MNO = 135° – 90° = 45°

Hence the slope of the line MN = tan 45° = 1

Thus the equation of MN is : y = x + c

But it passes through (6, 7)

So 7 = 6 + c or, c = 1

Hence equation is : y = x + 1

N lies on the x-axis, that is y = 0

Hence 0 = x + 1 or, x = –1

Thus the coordinate of N = (–1, 0)

Hence, the length of $MN = \sqrt{6-(-1)2+(7-0)2} = \sqrt{98} = 7\sqrt{2}$ units

Q25 Text Solution:

When (0, 0) becomes (3, 4), every value of x in (0, 0) will become (x – 3) and every value of y in (0, 0) will become (y – 4)

So, the equation of the line 8x + 6y = 25 if the origin of the cartesian coordinate plane is shifted to the point (3, 4) will be

8*(x – 3) + 6*(y – 4) = 25

or, 8x – 24 + 6y – 24 = 25

or, 8x + 6y = 73

Q26 Text Solution:

$x^2 + y^2 + 4x – 6y – 36 = 0$ can be written as

$(x^2 + 2*2*x + 2^2) + (y^2 – 2*3*y + 3^2) – 2^2 – 3^2 – 36 = 0$

or, $(x + 2)^2 + (y – 3)^2 = 49$

Hence the centre of the circle is (–2, 3)

Q27 Text Solution:

The equation of the circle with centre at (2, 1) and radius 5 units is : $(x -- 2)^2 + (y -- 1)^2 = 5^2$

At the intersection of the circle with the x-axis, the value of y = 0

So $(x -- 2)^2 + (0 -- 1)^2 = 5^2$

or, $x^2 – 2 * 2 * x + 2^2 + 1 = 25$

or, $x^2 -- 4x -- 20 = 0$

or,

$$x = \frac{\left[--(--4) \frac{+}{--} \sqrt{\left\{(--4)^2 --4*1*(--20)\right\}}\right]}{2*1}$$

$$= \frac{\left[4+/--\sqrt{96}\right]}{2} = \frac{\left[4+/--4\sqrt{6}\right]}{2} = 2+/--2\sqrt{6}$$

Hence the coordinate of the points of intersection of the circle with the x-axis are $\left(2+2\sqrt{6},0\right)$ and $\left(2--2\sqrt{6},0\right)$

Q28 Text Solution:

The equation of a circle with centre (a, b) and radius r is :

$(x-a)^2 + (y-b)^2 = r^2$

This equation is satisfied by the points (1, 3), (–1, 1) and (2, 1)

So,

$(1-a)^2 + (3-b)^2 = r^2$ .... (1)

$(-1-a)^2 + (1-b)^2 = r^2$ .... (2)

and $(2-a)^2 + (1-b)^2 = r^2$ .... (3)

From (1) and (2) :

$\left(1-a\right)2 + (3-b)^2 = (-1-a)^2 + (1-b)^2$

or, $1-2a + a^2 + 9-6b + b^2 = 1 + 2a + a^2 + 1 - 2b + b^2$

or, 4a + 4b = 8

or, a + b = 2 .... (4)

From (2) and (3) :

$(-1-\ a)^2 + (1-b)^2 = (2-a)^2 + (1-b)^2$

or, $(-1 - a)^2 = (2 - a)^2$

or, $1 + 2a + a^2\ =\ 4-4a + a^2$

or, 6a = 3

or, $a\ =\ \frac{1}{2}\sqrt{\frac{5}{2}}$

From eqn (4) :

b = 2 – 1/2

or, b = 3/2

Hence, $\left(x-\frac{1}{2}\right)^2 +\ \left(y-\ \frac{3}{2}\right)^2\ =\ r^2$

It passes through (1, 3)

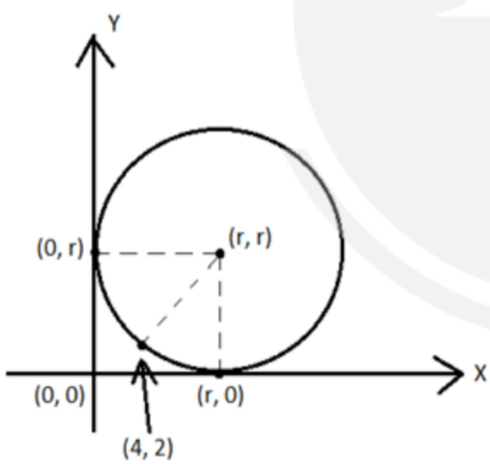
So $\left(1-\frac{1}{2}\right)^2 + \left(3-\frac{3}{2}\right)^2\ =\ r^2$

or, $r^2\ = \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \frac{5}{2}$

or, $r\ =\ \sqrt{\frac{5}{2}}$

Hence, the radius of the circle passing through the points (1, 3), (–1, 1) and (2, 1) = $\sqrt{\frac{5}{2}}$ units.

Q29 Text Solution:



Let the radius of the circle be r

Thus the coordinate of the centre of the circle will be (r, r), and the coordinates of the point of tangencies (0, r) and (r, 0)

The distance of (r, r) from (4, 2) will also be r, as it is the radius of the same circle.

So $\sqrt{(4-r)^2 + (2-r)^2} = r$

or, $(4-r)^2 + (2-r)^2 = r^2$

or, $16-8r + r^2 + 4-4r + r^2 = r^2$

or, $r^2-12r + 20 = 0$

or, (r – 10)(r – 2) = 0

Thus r = either 10 or 2

But point (4, 2) lies on the segment created by both the points of tangency which is nearer to the origin (as shown in diagram)

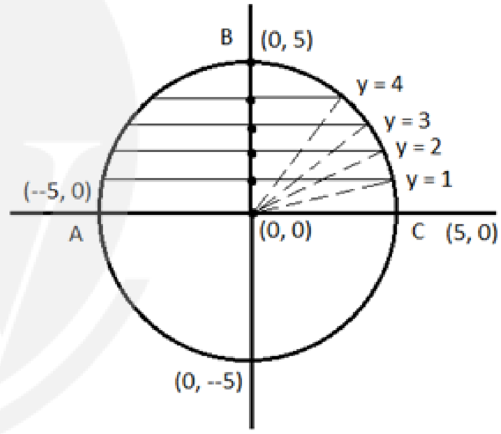
So the x–coordinate of (4, 2) lies between the x–coordinates of (0, 0) and (r, 0)

Hence r cannot be 2

Thus the radius of the circle is 10

Hence, the area of the circle = π*(10)^2 = 3.14*100 = 314 units

Q30 Text Solution:



The circle $x^2 + y^2 = 25$ has the centre at (0, 0) and the radius as 5 units

Refer to the above diagram which represents such a circle, with special attention to the semi–circle ABC

AC is the diameter, with A as (–5, 0) and C as (5, 0)

There are Five {(–5, 0), (–4, 0), (–3, 0), (–2, 0) & (–1, 0)} + One {(0, 0)} + Five {(1, 0), (2, 0), (3, 0), (4, 0) & (5, 0)}

= (2*5 + 1) or 11 integral points on the diameter AC

Let us now consider the chord parallel to the diameter through y = 1

The length of half the chord will be given by applying Pythagoras Theorem in the right angled triangle made by half the chord, one unit on y–axis and the radius of the circle connecting (0, 0) and the extremity of the chord, and will be $\sqrt{\left(5^2-1^2\right)}$

The number of integral points on the chord will be 2*[The largest integer ≤ $\sqrt{\left(5^2-1^2\right)}$] + 1

Similarly, let us now consider the chord parallel to the diameter through y = 2

The length of half the chord will be given by applying Pythagoras Theorem in the right angled triangle made by half the chord, two units on y–axis and the radius of the circle connecting (0, 0) and the extremity of the chord, and will be $\sqrt{\left(5^2-2^2\right)}$

The number of integral points on the chord will be 2*[The largest integer ≤ $\sqrt{\left(5^2-2^2\right)}$] + 1

This will continue till y = 5

The same will be repeated for the semi–circle below ABC, excepting the diameter AC, which has already been accounted for.

Thus the number of integral coordinates not lying outside the circle $x^2+y^2=25$ which basically means number of integral coordinates lying inside and on the circle $x^2+\ y^2=25$ will be given by :

(2*5 + 1) + 2*[(2*{The largest integer ≤ $\sqrt{\left(5^2-1^2\right)}$} + 1) + (2*{The largest integer ≤ $\sqrt{\left(5^2-2^2\right)}$} + 1) + (2*{The largest integer ≤ $\sqrt{\left(5^2-3^2\right)}$} + 1) + (2*{The largest integer ≤ $\sqrt{\left(5^2-4^2\right)}$} + 1) + (2*{The largest integer ≤ $\sqrt{\left(5^2-5^2\right)}$)} + 1)]

= 11 + 2* [2*(4) + 1 + 2*(4) + 1 + 2*(4) + 1 + 2*(3) + 1 + 2*(0) + 1]

= 11 + 2* [9 + 9 + 9 + 7 + 1]

= 11 + 2*35

= 81