MBA PIONEER PRO 2024

QUANTITATIVE APTITUDE

DPP: 6

Inequalities 1

Q1 For how many natural number values of x will $(x - 2)(x + 3)$ be positive?

(A) None or zero

(B) Only 1

(C) Only 2

(D) Infinitely Many

Q2 Let 'N' be a natural number. How many values of 'N' satisfy $2N + 5 > 3N$?

(A) 5 (B) 4

(C) 3 (D) 2

Q3 Find the range of x in $(x - 3)(x - 2) > 30$.

(A) $x < 2$ or $x > 3$

(B) (2, 3)

(C) (-3, 8)

(D) $x < -3$ or $x > 8$

Q4 If the number of integer values which satisfy the following inequality is n, then find the value of n^2 ?

$\frac{5}{3} < \frac{n}{n+4} < \frac{14}{3}$

(A) 1 (B) 16

(C) 3 (D) 4

Q5 Solve the quadratic inequality $x^2 - 15x + 56 \leq 0$

(A) (7, 8) (B) [7, 8)

(C) (7, 8] (D) [7, 8]

Q6 Solve the inequality $x^3 + 2x^2 - x - 2 < 0$

(A) $(-\infty, -2) \cup (-1, 1)$

(B) $(-\infty, -1) \cup (1, 2)$

(C) $(-2, -1) \cup (1, \infty)$

(D) $(-1, 1) \cup (2, \infty)$

Q7 Find the range of y if $y^6 - 37y^3 - 1728 \leq 0$

(A) [-27, 64] (B) [3, 8]

(C) [-8, 8] (D) [-3, 4]

Q8 Find the sum of all the integral solutions of the following inequality.

$\frac{6x^2-25x+11}{x^2+4x+5} < 0$

(A) 5 (B) 6

(C) 7 (D) 8

Q9 The least positive integer n for which $\sqrt{n+1} - \sqrt{n-1} < 0.2$ is:

(A) 24 (B) 26

(C) 28 (D) 30

Q10 If x and y are integers , then what is the maximum possible value of x that satisfied the inequality $3x+5y<7$?

(A) 0

(B) 1

(C) 2

(D) cann't be determined

Q11 Mr. S went to the market with Rs. 200. He needs to buy pens and pencils. Each pen costs Rs. 20 and each pencil costs Rs. 5 . He needs to buy at least 10 quantities of each kind and at least 25 quantities in total. What is the maximum number of pens he can buy?

(A) 11

(B) 12

(C) 13

(D) Impossible scenario

Q12 If (x-1) × (x-3) × (x-5) × (x-7) × (x-9) < 0, then find the integer values of x where x is a positive integer.

(A) 1 (B) 2

(C) 3 (D) 4

Q13 How many integral values of x satisfy the inequality x^2-7x+12$\leq$0

Q14 If x is real, the expression $\frac{-x^2+6x-1}{x}$ takes all values which do not lie between a and b (a < b), then a and b are:

(A) 4, 8 (B) 3, 7

(C) 4, 7 (D) None of these

Q15 If $6 \geq \frac{x+4}{2x+1} \geq 3$, then the number of integral values assumed by 55x will be _______.

(A) 20 (B) 21

(C) 22 (D) 23

Q16 If (x-2)(x-2^3)(x-2^4)(x-2^5) < 0 and x is a perfect square, then find the number of values that x can assume.

(A) 4 (B) 3

(C) 2 (D) 1

Q17 If (x-2^2)(x-2^4)(x-2^6) < 0 and (x^2 – 3x – 18) ≤ 0, then find the integral values that x can assume.

(A) 10 (B) 9

(C) 8 (D) 7

Q18 |6z - 3|< 3 is true for which values of z?

(A) z > 0

(B) 0 < z < 1

(C) -1 < z < 2

(D) 1 < z < 2

Q19 What are the real values of x that satisfy the inequation

$2x^2$ + 10x + 17 > 0?

(A) $(-\infty,\ \infty)$

(B) No real values

(C) $(-\infty,\ 0)$

(D) $(0,\ \infty)$

Q20 Find the maximum value of x such that $\sqrt{x}$ ≥ 5x.

(A) $\frac{1}{9}$

(B) $\frac{1}{36}$

(C) $\frac{1}{4}$

(D) $\frac{1}{25}$

Q21 How many integral values x can assume where $\frac{(x-10)(x-30)(x-50)}{(x-20)(x-40)(x-60)} \leq 0$

(A) 10 (B) 20

(C) 30 (D) 40

Q22 If (x^2 – 2x -8)(x^2 – x -12) ≤ 0, then how many integer values, (x^2 – 4x + 8) can be assumed?

(A) 10 (B) 11

(C) 12 (D) 13

Q23 (x-2)(x-4)(x-8) ≤ 0, where x is a positive integer. If a ≤ (x+2) ≤ b, then (a + b) =?

(A) 10 (B) 11

(C) 12 (D) 13

Q24 If x is a perfect square and also satisfies the below equation, then find the number of values that x can assume.

(x-1)(x-10)(x-10^2)(x-10^3) < 0

(A) 20 (B) 21

(C) 22 (D) 23

Q25 If (3x + 2y + 5z) < 34, where x, y, and z are distinct prime numbers, then how many distinct values z can assume?

(A) 1 (B) 2

(C) 3 (D) 4

Q26 The price of 3 pens, 2 pencils, 4 papers, 1 eraser is less than $21. If the price of each of the items are distinct integers when expressed in $, then how much cost will Bulbul incur who purchased 2 pens, 3 pencils, 10 papers, and 2 erasers?

(A) 30 (B) 31

(C) 32 (D) 33

Q27 What is the maximum value of $x^2 - 2x + 3$ if $\frac{1}{x-2} > \frac{1}{3}$ and x is a natural number?

(A) 11 (B) 4

(C) 3 (D) 5

Q28 Find the range of y such that $|y^2 - 11y + 24| > y^2 - 11y + 24$

(A) y ∈ (3, 8) (B) y ∈ (1, 3)

(C) y ∈ (3, 6) (D) None of these

Q29 If, a, b, c, d, e are distinct natural numbers. Then, find the range of x where $\frac{6x+2}{x+1} \geq f$, where $7f$ is the minimum possible value of (a + 2b + 3c + 4d + 5e).

(A) $(-\infty, -3) \cup (3, \infty)$

(B) $(-\infty, -1) \cup [3, \infty)$

(C) (-1,3]

(D) None of these

Q30 If (x-2)(x-5) ≤ 0, then for how many integer values $\frac{x+3}{5(x+1)} \leq \frac{2}{7}$

(A) 1 (B) 2

(C) 3 (D) 4

Answer Key

Q1	(D)
Q2	(B)
Q3	(D)
Q4	(B)
Q5	(D)
Q6	(A)
Q7	(D)
Q8	(B)
Q9	(B)
Q10	(D)
Q11	(D)
Q12	(B)
Q13	2
Q14	(A)
Q15	(C)
Q16	(C)
Q17	(D)
Q18	(B)
Q19	(A)
Q20	(D)
Q21	(C)
Q22	(A)
Q23	(D)
Q24	(D)
Q25	(B)
Q26	(B)
Q27	(A)
Q28	(A)
Q29	(B)
Q30	(B)

Hints & Solutions

Q1 Text Solution:

$(x-2)(x+3) > 0$

$-3 < x$ or $x > 2$

It will be true for all natural numbers greater than 2.

Thus, there are infinitely many solutions.

Hence, option d is the correct answer.

Q2 Text Solution:

$2N + 5 > 3N$

$N < 5$

So, the possible values of N are 1, 2, 3, and 4, i.e., four possible values.

Hence, option b is the correct answer.

Q3 Text Solution:

$(x-3)(x-2) > 30$

$x^2 - 5x + 6 > 30$

$x^2 - 5x - 24 > 0$

$(x-8)(x+3) > 0$

Now, for any quadratic inequality $ax^2 + bx + c < 0$

after factorizing it as $a(x - p)(x - q) < 0$, whenever a is greater than 0, the above inequality will hold good if x lies between p and q.

$a(x - p)(x - q)$ will be greater than 0, whenever x does not lie between p and q. In other words x should lie in the range $(-\infty, p)$ or (q, ∞).

Hence, $(x - 8)(x + 3) > 0$

=> $x < -3$ or $x > 8$.

Q4 Text Solution:

Let assume that n + 4 is positive i.e., $n > -4$.

$\Rightarrow 5n + 20 < 3n$ and $3n < 14n + 56$

$n < -10$ and $n > -\frac{56}{11}$, so no solution is possible.

Now let us assume that n + 4 is negative i.e., $n < -4$.

$\Rightarrow 5n + 20 > 3n$ and $3n > 14n + 56$

$n > -10$ and $n < -\frac{56}{11}$

So, n can take 4 integral values – 6, – 7, – 8, – 9.

Hence, $n^2 = 4^2 = 16$.

Q5 Text Solution:

$x^2 - 15x + 56 \le 0$

$x^2 - 8x - 7x + 56 \le 0$

$x(x-8) - 7(x-8) \le 0$

$(x-8)(x-7) \le 0$

$7 \le x \le 8$ or $[7, 8]$

Hence, option d is the correct answer

Q6 Text Solution:

$$x^3 + 2x^2 - x - 2 < 0$$
$$(x-1)\left(x^2 + 3x + 2\right) < 0$$
$$(x-1)(x+2)(x+1) < 0$$

The range at which the given expression will be negative is

$$(-\infty, -2) \cup (-1, 1)$$

Hence, option a is the correct answer.

Q7 Text Solution:

Let $y^3 = x$, then $y^6 - 37y^3 - 1728$ becomes $x^2 - 37x - 1728 \le 0$

$x^2 - 64x + 27x - 1728 \le 0$

$x(x-64) + 27(x-64) \le 0$

The range of x will be $[-27, 64]$

Then, the range of y is $[-3, 4]$

Hence, option d is the correct answer

Q8 Text Solution:

$\frac{6x^2 - 25x + 11}{x^2 + 4x + 5} < 0$

Now the denominator $x^2 + 4x + 5$ is always greater than zero for any value of x.

It means the numerator should be less than zero.

$6x^2 - 25x + 11 < 0$

$6x^2 - 22x - 3x + 11 < 0$

$\Rightarrow (2x-1)(3x-11) < 0$

$\Rightarrow \frac{1}{2} < x < \frac{11}{3}$

The only integers satisfying the above inequality are 1, 2, and 3.

So, required Sum = (1 + 2 + 3) = 6.

Q9 Text Solution:

$$\sqrt{n+1} - \sqrt{n-1} < 0.2$$
$$\sqrt{n+1} < 0.2 + \sqrt{n-1}$$

Now, squaring both the sides

$$n+1 < 0.04 + n - 1 + 0.4\sqrt{n-1}$$
$$1.96 < 0.4\sqrt{n-1}$$
$$\text{Again, on squaring both the sides}$$
$$3.8416 < 0.16(n-1)$$
$$\Rightarrow 24.01 + 1 < n$$
$$\Rightarrow 25.01 < n$$

Hence, the minimum value of n is 26 .

Q10 Text Solution:

3x+5y<7

or, x<$\frac{7-5y}{3}$

Since y can take only integral values, we can put y= any negative integer

by doing so , we can see that the value of x in the above inequality becomes larger and larger

.

Thus , we cannot determine the maximum possible value of 'x'

Hence option d is correct answer

Q11 Text Solution:

Let the number of pens and pencils bought by Mr. S be ‘x’ and ‘y’, respectively.

So, 20x + 5y ≤ 200 ....(1)

Or 4x + y ≤ 40 ....(2)

Also, x ≥ 10, y ≥ 10, x + y ≥ 25...............(3)

To maximize the number of pens, we need to buy the minimum number of pencils.

Let's take y = 10.

So, from (2), we get:

4x ≤ 30

Or, x ≤ 7.5

So, x + y comes out to be less than 25 and it violates (3).

So, we have to check with higher values of y, i.e., 11, 12, 13, etc.

If we put y = 11, we get:

x < 7.25

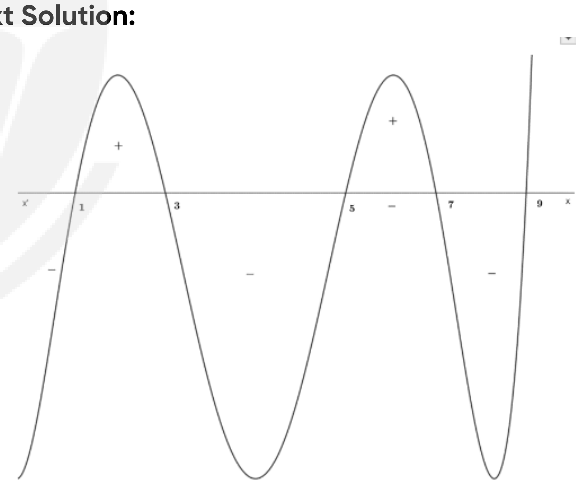
Here, we can see that we are getting even lesser values of x.

So, the condition of (3) will definitely get violated here.

Thus, there are no possible values of ‘x’ and ‘y’.

Hence, option d is the correct answer.

Q12 Text Solution:



As seen from the wavy curve,

at x = 4, 8 the inequation satisfies.

So, the answer is 2.

Q13 Text Solution:

$x^2 - 7x + 12 \leq 0 \Rightarrow$(x-4)(x-3)$\leq$0

$3 \leq x \leq 4$; Hence x can take 2 integral values

Q14 Text Solution:

Let y = $\frac{-x^2+6x-1}{x}$

xy = -x^2 + 6x - 1

$x^2+(y-6)x+1 = \ 0$

If x is real then,

$(y-6)^2-4 \ \geq \ 0$

$y^2-12y+36-4 \ \geq \ 0$

$y^2-12y+32 \ \geq \ 0$

$(y-4)(y-8) \ \geq \ 0$

$y \ \leq \ 4$ or, y ≥ 8

∴ a = 4 and b = 8

Thus, the required option (A) is correct.

Q15 Text Solution:

$\frac{x+4}{2x+1} \geq 3$

- x + 4 ≥ 6x + 3
- 5x ≤ 1
- $x \leq \frac{1}{5}$ .... (i)

Also,

$\frac{x+4}{2x+1} \leq 6$

- x+4 ≤ 12x + 6
- 11x ≥ -2
- $x \geq -\frac{2}{11}$ .... (ii)

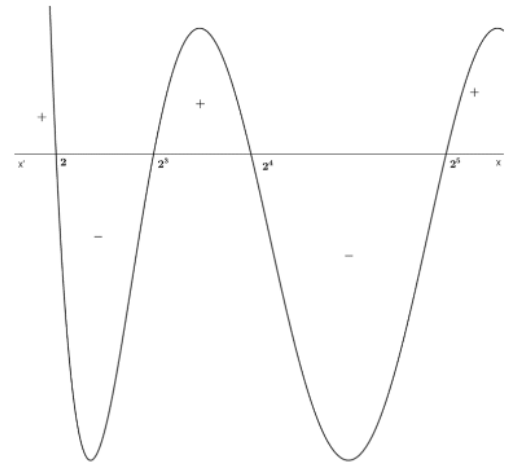
So, combining (i) and (ii), we get,

-10 ≤ 55x ≤ 11

So, 55x can assume all the values from – 10 to 11.

Thus, the integral numbers that can be assumed by 55x will be 22.

Q16 Text Solution:

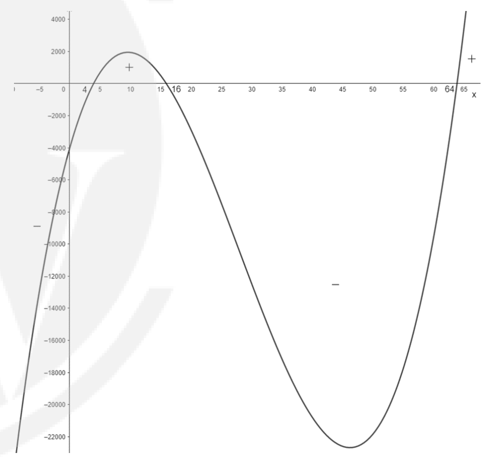


Using wavy curve method, we can say that, if 2 < x < 8, or, 16 < x < 32, then it will satisfy the inequation.

So, x can be 4, 25.

Thus, x can take 2 values.

Q17 Text Solution:



From the above wavy curve, we can say that if 16 < x < 64 or x < 4, then the inequation is satisfied.

x^2 – 3x – 18 ≤ 0

(x–6)(x+3) ≤ 0

-3 ≤ x ≤ 6

So, x = -3, -2, -1, 0, 1, 2, 3.

Thus, x can assume 7 values.

Q18 Text Solution:

We have

6z - 3 < 3 or 6z - 3 > -3

6z < 3 + 3 or 6z > -3 + 3

6z < 6 or 6z > 0

z < 1 or z > 0

Thus, 0 < z < 1.

Thus, option (B) is correct.

Q19 Text Solution:

It is given that $2x^2 + 10x + 17 > 0$

$\Rightarrow x^2 + 5x + \frac{25}{4} - \frac{25}{4} + \frac{17}{2} > 0$

$\Rightarrow (x + \frac{5}{2})^2 + \frac{9}{4} > 0$

Since, $[x + \frac{5}{2}]^2$ is always greater than 0 for any value of x.

The required value of x is in $(-\infty,\ \infty)$.

Q20 Text Solution:

It is given that,

$\geq 5x$

Let $x = y^2$.

So, $\geq 5y^2$

$\Rightarrow y \geq 5y^2$

or, $5y^2 - y \leq 0$

$y(5y - 1) \leq 0$

So, $0 \leq y \leq \frac{1}{5}$ satisfies the above inequality.

Maximum value of y is $\frac{1}{5}$.

Maximum value of x is $(\frac{1}{5})^2 = \frac{1}{25}$.

Q21 Text Solution:

Here it is given that, $\frac{(x-10)(x-30)(x-50)}{(x-20)(x-40)(x-60)} \leq 0$

this can be written as,

$\frac{(x-10)(x-30)(x-50)(x-20)(x-40)(x-60)}{(x-20)(x-40)(x-60)(x-20)(x-40)(x-60)} \leq 0$

Critical points are : 10, 20, 30, 40, 50, 60.

Possible solutions lie between [10, 20], [30,40], [50,60] = 33 solutions

But at x = 20, 40, 60 denominators become zero and expressions become infinite.

The possible number of solutions are 33 - 3 = 30 solutions.

Q22 Text Solution:

$(x^2 - 2x - 8)(x^2 - x - 12) \leq 0$

- $(x^2 - 4x + 2x - 8)(x^2 - 4x + 3x - 12) \leq 0$
- $(x-4)(x+2)(x-4)(x+3) \leq 0$
- $(x-4)^2(x+2)(x+3) \leq 0$

as, x is a real number, $(x-4)^2 \geq 0$

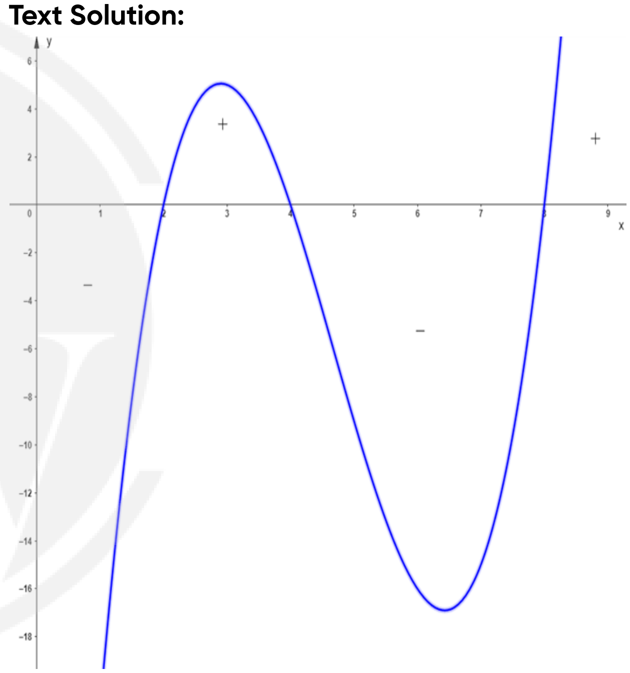
So, $(x+2)(x+3) \leq 0$

- $-3 \leq x \leq -2$
- $-5 \leq (x-2) \leq -4$
- $25 \geq (x-2)^2 \geq 16$
- $29 \geq x^2 - 4x + 8 \geq 20$

So, $(x^2 - 4x + 8)$ can assume all integers from 20 to 29.

Hence, the number of values $(x^2 - 4x + 8)$ can assume is 10.

Q23 Text Solution:



From the above wavy curve, we can say that,

$(x-2)(x-4)(x-8) \leq 0$

- $4 \leq x \leq 8$ or, $x \leq 2$

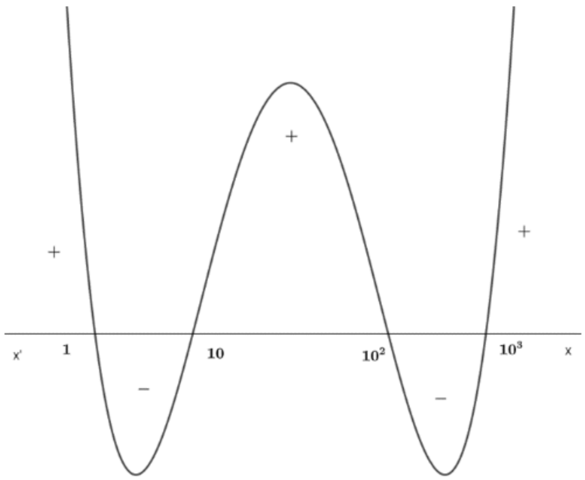
Also, x is a positive integer.

So, x can assume a minimum value of 1 and a maximum value of 8.

So, $3 \leq (x+2) \leq 10$

So, $(a + b) = (3 + 10) = 13$

Q24 Text Solution:



$(x-1)(x-10)(x-10^2)(x-10^3) < 0$

As seen from the wavy curve, any one digit and 3-digit number will satisfy the inequation.

So, $x = 2^2, 3^2, 11^2, 12^2, \ldots, 31^2$

So, x can assume a total of = 2 + (31 – 11 +1) = 23 values.

Q25 Text Solution:

Min (3x + 2y + 5z) can be obtained by multiplying lower numbers with the highest coefficient.

So, put x=3,y=5,z=2. min (3x + 2y + 5z) = (3 × 3) + (2 × 5) + (5 × 2) = 29

If we make z = 3, then y = 7, x = 5

Then, (3x + 2y + 5z) = 15 + 14 + 15 = 44 which is more than 34.

If we take z = 3, y = 2 and x = 5 then we get (3x + 2y + 5z) = 15 + 4 + 15 = 34 which is not satisfying the condition.

But if we take x =2, y=5 and z=3 , then Min(3x2+2x5+5x3) = 6+10+15= 31 < 34 , So, z can assume only 2 value. satisfied . Answer is 2 .

Q26 Text Solution:

Let the price of 1 pen, 1 pencil, 1 paper and 1 eraser are \$a, \$b, \$c, and \$d respectively.

So, 3a + 2b + 4c + d < 21.

- 3a + 2b + 4c + d ≤ 20

As, a, b, c and d are distinct integers, let's try to find the minimum value that can be assumed by (3a + 2b + 4c + d).

So, to minimize the value, we need to make sure that the lower values are getting multiplied with higher coefficients.

Hence, Min(c) = 1

Min (a) = 2

Min (b) = 3

Min (d) = 4

So, Min (3a + 2b + 4c + d) = 6 + 6 + 4 + 4 = 20

Hence, a, b, c, and d assume their minimum values.

So, Bulbul's cost is (2a + 3b + 10c + 2d) = \$(4 + 9 + 10 + 8) = \$31.

Q27 Text Solution:

$\frac{1}{x-2} > \frac{1}{3}$

$= \frac{1}{x-2} - \frac{1}{3} > 0$

$= \frac{-x+5}{3(x-2)} > 0$

$= \frac{x-5}{3(x-2)} < 0$

Here we don't know about the polarity of (x-2) so we can write the above quadratic equation as,

$= \frac{(x-2)\,(x-5)}{3(x-2)(x-2)} < 0$

So, 2 < x < 5

So, max value of $x^2 - 2x + 3 = (4^2 - 8 + 3) = 11$.

Q28 Text Solution:

We know that,

If for any expression a, |a| > a, a has to be negative.

∴ $y^2 - 11y + 24 < 0$

⇒ (y – 3) (y – 8) < 0

⇒ y ∈ (3, 8)

Q29 Text Solution:

The minimum value of (a + 2b + 3c + 4d + 5e) can be obtained by making sure that the lower numbers are multiplied with the higher coefficients.

Also, a, b, c, d, e can assume minimum values of 1, 2, 3, 4, 5, where e = 1, d = 2, c = 3, b = 4, a = 5.

So, min (a + 2b + 3c + 4d + 5e) = 5 + (2 × 4) + (3 × 3) + (4 × 2) + (5 × 1)

- 7f = 35
- f = 5

So, $\frac{6x+2}{x+1} \geq 5$

- $\frac{(6x+2)}{(x+1)} - 5 \geqslant 0$

  $\frac{(6x+2)-5(x+1)}{(x+1)} \geq 0$

  $\frac{(6x+2-5x-5)}{(x+1)} \geq 0$

  $\frac{(x-3)}{(x+1)} \geq 0$

  $\frac{(x-3)(x+1)}{(x+1)^2} \geq 0$

  so, range is $(-\infty$,-1)∪[3.∞)

Q30 Text Solution:

Using wavy curve method, we can say that,

(x-2)(x-5) ≤ 0

- 2 ≤ x ≤ 5

Also,

$\frac{x+3}{5(x+1)} \leq \frac{2}{7}$

- 7x + 21 ≤ 10x + 10
- 3x ≥ 11
- x ≥ $\frac{11}{3}$

Also, $5 \geq x \geq \frac{11}{3}$

- x = 4, 5
- Thus, x can assume only 2 integer values.