CAT

MBA PIONEER PRO 2024

QUANTITATIVE APTITUDE

DPP: 5

Exponents 1

Q1 Simplify the expression $((x^{-27})^{\frac{1}{2}})^{\frac{1}{9}}$, x≥0.

(A) $\frac{1}{x^3}$

(B) $\frac{1}{\sqrt[3]{x}}$

(C) $\frac{1}{x\sqrt{x}}$

(D) $\sqrt{(x^3)}$

Q2 Simplify $\frac{9x^{-3}yz^{-4}}{3xy^{-4}z^3} \div \frac{6x^{-4}y^3z^{-2}}{2xy^{-3}z^4}$?

(A) $\frac{x}{\text{yz}}$

(B) $\frac{1}{3xyz}$

(C) $\frac{3y}{\text{xz}}$

(D) xyz

Q3 Simplify the expression $\sqrt[4]{216x^{-2}y^9 \times \frac{6}{x^6y^{-3}}}$

(A) $6y^3x^2$

(B) $\frac{x^3}{6y^2}$

(C) $\frac{6x^3}{y^2}$

(D) $\frac{6y^3}{x^2}$

Q4 What is the value of $x^{3/2}$ in the equation $3^{(5x+4)} = 729^4$?

(A) 10 (B) 2

(C) 8 (D) 4

Q5 Find the value of x which satisfies the equation:-

$5 \times 2^{x+3} - 21 \times 2^{x-1} = 236$

Q6 If $0.232^m = 116^n = 500$, find the value of $\frac{1}{m} - \frac{1}{n}$

Q7 If x = $(6561)^{5+2\sqrt{3}}$, then which of the following equals 81?

(A) $\frac{x^5}{x^{4\sqrt{3}}}$ (B) $\frac{x^{\frac{5}{26}}}{x^{2\sqrt{3}}}$

(C) $\frac{x^{\frac{5}{26}}}{x^{\frac{4}{\sqrt{3}}}}$ (D) $\frac{x^{\frac{5}{26}}}{x^{\frac{\sqrt{3}}{13}}}$

Q8 Let $2^a \cdot 3^b \cdot 5^c = \left(\frac{1}{2}\right)^2 (9)^3 (25)^4$. If $a^b b^c c^a = x^4$, then $\sqrt{x} = ?$

(A) 5 (B) 6

(C) 3 (D) 9

Q9 If $\frac{4^a}{5^b} = \left(\frac{32}{625}\right)^y$, then find $a + b$, where $y = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots \infty}}}$.

(A) 10 (B) 11

(C) 12 (D) 13

Q10 If $4^x \; x \; 3^y = 20736$, then what is the value of $\sqrt{12(x-y) + \sqrt{xy}}$?

(A) 2 (B) 4

(C) 6 (D) 8

Q11 What is the number of integer solutions of $3^x - 2^x + 12^x - 18^x = 0$?

(A) 1 (B) 3

(C) 5 (D) 7

Q12 Solve the equation for $x, (x+k)^2 = \sqrt{4096 + m}$, if $k + 2022^m = 8$ and $k - 3(2022)^m = 4$, where k and m are all integers.

(A) x = -1 (B) x = 1

(C) x = -15 (D) x = 1, -15

Q13

What is the value of 7^{2x} in $7^{4x}-7\left(\frac{10A}{11}\right)^{2x+1}-960=0$ if $2\left(\frac{10A}{11}\right)^{4}=4802$?

(A) 16 (B) 32

(C) 48 (D) 64

Q14 Let $\frac{\sqrt[4]{a}}{144}=\frac{54}{a}$, then if $a=4^{x}\times 9^{y}$, what is the value of xy ?

(A) 5 (B) 4

(C) 3 (D) 2

Q15 If $3^{3\times\frac{x^{2}(x+1)^{2}}{4}}=27^{729^{\frac{1}{3}}}$, what is the sum of the possible real values of x ?

(A) -1 (B) 0

(C) 1 (D) 2

Q16 Find the sum of the possible values of x in the following equation

$4x\ -\ 7\sqrt{x}+3\ =\ 0$

(A) 10

(B) $\frac{65}{4}$

(C) $\frac{25}{16}$

(D) $\frac{37}{16}$

Q17 Let $4^{\alpha}=16^{\beta}=64^{\gamma}$, then find the value of $\frac{1}{\alpha}+\frac{1}{\gamma}$.

(A) 2

(B) 1

(C) $\frac{2}{\beta}$

(D) $\frac{1}{\beta}$

Q18 If $\left(2^{x}-7\right)^{2}=6\left(2^{x}-\frac{5}{2}\right)$, find the sum of the possible values of x.

(A) 5 (B) 6

(C) 7 (D) 8

Q19 If $(3+2\sqrt{2})=a$, also if, $\left(\frac{\sqrt{a}}{2}+\frac{1}{2\sqrt{a}}\right)^{2}\left(a+\frac{1}{a}\right)^{3}=b^{2b}c^{c}$, then $(b+c)^{b}=?$

(A) 36 (B) 4

(C) 25 (D) 16

Q20 How many integer pairs (x, y) satisfy the following equation?

$x^{\frac{1}{2}}+y^{\frac{1}{2}}=\sqrt{3\times\sqrt{3\times\sqrt{3\times\sqrt{3\times 3}}}}$

(A) 3 (B) 4

(C) 5 (D) 2

Q21 Find the number of real solutions in the equation $(x^2 + 2)^{10} = 4x - x^2 - 6$.

(A) Only 1 solution

(B) 10 solutions

(C) 20 solutions

(D) Zero Solution

Q22 If $x^{\frac{a}{a+b+c}}\times x^{\frac{b}{a+b+c}}\times x^{\frac{c}{a+b+c}}=\frac{1011}{y^{\frac{c}{a+b+c}}\times y^{\frac{b}{a+b+c}}\times y^{\frac{a}{a+b+c}}}$ and $(x+y)=45$ what is the value of $x^{2}+y^{2}$?

(A) 1 (B) 2

(C) 3 (D) 4

Q23 If $7^m - 5^n = 117524$ and $7^{m-1} + 5^{n+1} = 17432$, then m+n equals:

(A) 10 (B) 9

(C) 8 (D) 7

Q24 Given that $x^{2024} y^{2023} = \frac{1}{3}$ and $x^{2022} y^{2025} = 27$, the value of $x^2 + y^3$ is

(A) $\frac{247}{9}$

(B) $\frac{241}{9}$

(C) $\frac{245}{9}$

(D) $\frac{244}{9}$

Q25 If $x^4 + x^3 + x^2 + x + 1 = 0$, then $x^{2050} + x^{2021} + x^{2022}$ equals which of the following:

(A) $x^2 + 1$ (B) x^2

(C) $-x^2$ (D) $-x^3(1+x)$

Q26 Let $a^p = b^q = c^r$, where $a, b, c > 1$. Then, if $(abc)^{\frac{1}{p^{-1}+q^{-1}+r^{-1}}} = a^x$, find the value of x.

(A) p (B) q

(C) r (D) 1

Q27 If $Y^{(4-X)} \times Z^{5X} = Y^{X+6} \times Z^{3X}$, what is the value of Z in terms of X and Y?

(A) 2XY

(B) XY^{x+1}

(C) $Y(Y)^{\frac{1}{X}}$

(D) $X^{\frac{1}{Y}+1}$

Q28 Let three numbers are $3^{1+a} + 3^{1-a}$, $\frac{x}{2}$ and $9^a + 9^{-a}$. If twice the middle number equals the sum of the rest of the numbers, then find the least positive value of x.

(A) 10 (B) 9

(C) 8 (D) 7

Q29 If $x = \left[1 + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{1024}+\sqrt{1023}}\right]$.

Then, $\sqrt{x\sqrt{x\sqrt{x\ldots\infty}}} = a^b$, where a, b are integers, then find the minimum value of $(a+b)$.

(A) 5 (B) 6

(C) 7 (D) 8

Q30 The number of integer solutions of the equation $(x^2 - 10)^{(x^3+x^2-14x-24)} = 1$ is:

(A) 5 (B) 4

(C) 3 (D) 2

Answer Key

Q1	(C)
Q2	(A)
Q3	(D)
Q4	(C)
Q5	3
Q6	-1
Q7	(D)
Q8	(B)
Q9	(D)
Q10	(A)
Q11	(A)
Q12	(D)
Q13	(D)
Q14	(B)
Q15	(A)
Q16	(C)
Q17	(C)
Q18	(B)
Q19	(C)
Q20	(B)
Q21	(D)
Q22	(C)
Q23	(B)
Q24	(D)
Q25	(D)
Q26	(A)
Q27	(C)
Q28	(C)
Q29	(C)
Q30	(B)

Hints & Solutions

Q1 Text Solution:

$((x^{-27})^{\frac{1}{2}})^{\frac{1}{9}}$ = $((x^{-27\times\frac{1}{2}})^{\frac{1}{9}}$

= $(x^{-\frac{27}{2}})^{\frac{1}{9}}$

= $x^{-\frac{27}{2}\times\frac{1}{9}}$

= $x^{-\frac{3}{2}}$

= $\frac{1}{x^{\frac{3}{2}}}$

= $\frac{1}{x\sqrt{x}}$

Hence, option (3) is the correct answer.

Q2 Text Solution:

$\frac{9x^{-3}yz^{-4}}{3xy^{-4}z^{3}} \div \frac{6x^{-4}y^{3}z^{-2}}{2xy^{-3}z^{4}}$

= $\frac{9x^{-3}yz^{-4}}{3xy^{-4}z^{3}} \times \frac{2xy^{-3}z^{4}}{6x^{-4}y^{3}z^{-2}}$

= $\frac{3y^{5}}{x^{4}z^{7}} \times \frac{1x^{5}z^{6}}{3y^{6}}$

= $\frac{y^5}{y^6}\times\frac{x^5}{x^4}\times\frac{z^6}{z^7}$

= $\frac{x}{\mathrm{yz}}$

Hence, option (1) is the correct answer.

Q3 Text Solution:

The given expression $\sqrt[4]{216x^{-2}y^{9}\times\frac{6}{x^{6}y^{-3}}}$ can be written as

= $\sqrt[4]{216\times 6\times\frac{x^{-2}y^{9}}{x^{6}y^{-3}}}$

= $\sqrt[4]{6\times 6\times 6\times 6\times\frac{x^{-2}y^{9}}{x^{6}y^{-3}}}$

= $6\sqrt[4]{\frac{y^{9+3}}{x^{6+2}}}$

= $6\sqrt[4]{\frac{y^{12}}{x^{8}}}$

= $6\times\frac{y^{12\times\frac{1}{4}}}{x^{8\times\frac{1}{4}}}$

= $\frac{6y^3}{x^2}$

Thus, option (4) is the correct answer.

Q4 Text Solution:

The given equation is $3^{(5x+4)} = 729^4$

Therefore, $3^{(5x+4)} = (3^6)^4$

$3^{5x+4} = 3^{24}$

5x+4 = 24

5x = 20

x = 4

Then,

$x^{\frac{3}{2}} = 4^{\frac{3}{2}} = 8$

Option (3) is correct.

Q5 Text Solution:

Here we are given the equation $5\times 2^{x+3}-21\times 2^{x-1}=236$ First we will rewrite the equation to get

$$5\times 2^4\times 2^{x-1}-21\times 2^{x-1}=236$$

Now taking 2^{x-1} common we get

$$2^{x-1}(80-21)=236$$

or, $2^{x-1}\times 59=236$

Taking 59 to RHS we get

$$2^{x-1}=\frac{236}{59}$$

or, $2^{x-1}=4$

or, $2^{x-1}=2^2$

Since the bases are same, we can equate the powers to get

$$x-1=2$$

or, $x=3$

Hence the value of x is 3 .

Q6 Text Solution:

Here we are given $0.232^m=116^n=500$

- From the above relation we can say

$$0.232^m = 500$$

dividing the exponents by $\frac{1}{m}$ gives us

$$0.232 = 500^{\frac{1}{m}} (1)$$

- Similarly,

$$116^n = 500$$

on dividing the exponents by $\frac{1}{n}$ gives us

$$116 = 500^{\frac{1}{n}} (2)$$

- On dividing equation [2] by [1] we get

$$500 = 500^{\frac{1}{n} - \frac{1}{m}}$$

since the bases are same we can equate the powers to get $\frac{1}{n} - \frac{1}{m} = 1$
or, $\frac{1}{m} - \frac{1}{n} = -1$
Hence, $\frac{1}{m} - \frac{1}{n} = -1$ has to be typed.

Q7 Text Solution:

x = $(6561)^{5+2\sqrt{3}}$

$x^{\frac{1}{(5+2\sqrt{3})}}$ = 6561

On rationalizing $\frac{1}{(5+2\sqrt{3})}$ we get

$\frac{1}{(5+2\sqrt{3})}$ =

$$\frac{(5-2\sqrt{3})}{(5+2\sqrt{3})(5-2\sqrt{3})} = \frac{(5-2\sqrt{3})}{25-12} = \frac{(5-2\sqrt{3})}{13}$$

So, $x^{\frac{(5-2\sqrt{3})}{13}}$ = 6561 = 81^2

=> $81 = x^{\frac{(5-2\sqrt{3})}{13\times2}}$

=> 81 = $\frac{x^{\frac{5}{26}}}{x^{\frac{\sqrt{3}}{13}}}$

Q8 Text Solution:

Given that, $2^a \cdot 3^b \cdot 5^c = \left(\frac{1}{2}\right)^2 (9)^3 (25)^4$

$$2^a 3^b 5^c = 2^{-2} \times 3^6 \times 5^8$$

On comparing both sides we get,

$$\begin{aligned} \text{So, } a &= -2, \\ b &= 6, \\ c &= 8 \end{aligned}$$

Therefore,

$$\begin{aligned} a^b b^c c^a &= (-2)^6 (6)^8 (8)^{-2} \\ &= 2^6 \times 2^8 \times 3^8 \times 2^{-6} \\ &= 2^8 \times 3^8 = x^4 \text{ (Given)} \end{aligned}$$

So, $x = (2 \times 3)^{\frac{8}{4}} = 36$
So, $\sqrt{x} = 6$

Q9 Text Solution:

$$y = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots \infty}}}$$

$$\begin{aligned} &\Rightarrow y^2 = 2 + y \\ &\Rightarrow y^2 - y - 2 = 0 \\ &\Rightarrow y^2 - 2y + y - 2 = 0 \\ &\Rightarrow y = 2, -1 \\ &\Rightarrow y = 2[\text{ Since, } y > 0, \text{ so } y \neq -1] \end{aligned}$$

Therefore,

$$\frac{4^a}{5^b} = \left(\frac{32}{625}\right)^y$$

$$\Rightarrow \frac{2^{2a}}{5^b} = \frac{2^{5y}}{5^{4y}} = \frac{2^{10}}{5^8}$$

So, $2^{2a} = 2^{10}$

$$\begin{aligned} &\Rightarrow 2a = 10 \\ &\Rightarrow a = 5 \end{aligned}$$

Also, $5^b = 5^8$

$$\Rightarrow b = 8$$

So, $(a+b)=5+8=13$

Q10 Text Solution:

Given that,

$$4^x \times 3^y = 20736$$

Now, $20736 = 2^8 \times 3^4$

$$= \left((2^2)^2\right)^2 \times 3^4$$
$$= (4^2)^2 \times 3^4$$
$$= 4^4 \times 3^4$$

Therefore,

$$4^x \times 3^y = 4^4 \times 3^4$$

Which Implies that, $x=4=y$

Now,

$$\sqrt{12(x-y)+\sqrt{xy}}$$
$$= \sqrt{0+\sqrt{16}}$$
$$= \sqrt{4}$$
$$= 2$$

Q11 Text Solution:

Let 3^x be a and let 2^x be b.

$3^x - 2^x + 12^x - 18^x = 0$

=> $3^x - 2^x + (3^x)(4^x) - (9^x)(2^x) = 0$

=> $a - b + ab^2 - ba^2 = 0$

=> $b(ab-1) - a(ab-1) = 0$

=> $(b-a)(ab-1)=0$

=> $ab=1$

=> $3^x 2^x = 6^x = 1$

=> $x = 0$

The number of integer solutions of $3^x - 2^x + 12^x - 18^x = 0$ is 1.

Q12 Text Solution:

Given that,

$$k + 2022^m = 8$$

and

$$k - 3(2022)^m = 4$$

Let $2022^m = y$. Then, we have

$$k + y = 8(i)$$

$k - 3y = 4$ (ii)

Solving (i) and (ii), we have $k=7$, and $y=1$

Therefore, $2022^m = 1 = 2022^\circ$

$$\Rightarrow m = 0$$

So, the given equation can be written as

$$(x+7)^2 = \sqrt{4096}$$

Now,

$$(x+7)^2 = \sqrt{64 \times 64}$$

$$(x+7)^2 = 64$$
$$x+7 = \sqrt{64}$$
$$x+7 = \pm 8$$
$$x = 8-7, -8-7$$
$$x = 1, -15$$

Thus, option (4) is the correct answer.

Q13 Text Solution:

Given that, $2\left(\frac{10A}{11}\right)^4 = 4802$

$$\Rightarrow \left(\frac{10A}{11}\right)^4 = 2401$$
$$\Rightarrow \left(\frac{10A}{11}\right)^4 = 7^4$$
$$\Rightarrow \frac{10A}{11} = 7$$

Now, we will try to convert the given equation into a quadratic equation.

$$
\begin{aligned}
& 7^{4x}-7\left(\frac{10A}{11}\right)^{2x+1}-960=0 \\
& 7^{4x}-7 \cdot 7^{2x+1}-960=0 \\
& \left(7^{2x}\right)^2-7 \cdot 7^{2x} \cdot 7^1-960=0 \\
& \left(7^{2x}\right)^2-49 \cdot 7^{2x}-960=0
\end{aligned}
$$

Let $7^{2x}=a \Rightarrow$ The equation becomes

$$
\begin{aligned}
& a^2-49a-960=0 \\
& (a-64) \times(a+15)=0 \\
& a=64 \text{ or } a=-15 \\
& 7^{2x}=a \text{ will always be } \geq 0 .
\end{aligned}
$$

$\therefore a=-15$ is not a valid solution.

$a=64$

$7^{2x}=64$

Q14 Text Solution:

Given that, $\frac{\sqrt[4]{a}}{144}=\frac{54}{a}$

$$
\begin{aligned}
& \Rightarrow \frac{a^{\frac{1}{4}}}{144}=\frac{54}{a} \\
& \Rightarrow a^{\frac{1}{4}} \cdot a=54 \times 144 \\
& \Rightarrow a^{\frac{5}{4}}=2^5 \times 3^5 \\
& \Rightarrow a^{\frac{1}{4}}=6 \\
& \Rightarrow a=6^4=1296
\end{aligned}
$$

Now, $a=4^x \times 9^y$

$$
\begin{aligned}
& \Rightarrow 1296=4^x \times 9^y \\
& \Rightarrow 6^4=4^x \times 9^y \\
& \Rightarrow(2 \times 3)^4=4^x \times 9^y \\
& \Rightarrow 2^4 \times 3^4=4^x \times 9^y \\
& \Rightarrow 4^2 \times 9^2=4^x \times 9^y \\
& \Rightarrow x=2=y
\end{aligned}
$$

Hence, $xy=4$

Q15 Text Solution:

Given that,

$$
\begin{aligned}
& 3^{3 \times \frac{x^2(x+1)^2}{4}}=27^{729^{\frac{1}{3}}} \\
& 3^{3 \times \frac{x^2(x+1)^2}{4}}=27^9 \\
& 27^{\frac{x^2(x+1)^2}{4}}=27^9 \\
& \frac{x^2(x+1)^2}{4}=9 \\
& \frac{x(x+1)}{2}=3 \\
& x^2+x-6=0 \\
& (x+3)(x-2)=0 \\
& x=2,-3
\end{aligned}
$$

Hence, the sum of the possible values of $x=-3+2=-1$.

Q16 Text Solution:

The given equation can be written as

$$
\begin{aligned}
& 4(\sqrt{x})^2-7\sqrt{x}+3=0 \\
& 4(\sqrt{x})^2-4\sqrt{x}-3\sqrt{x}+3=0 \\
& 4\sqrt{x}(\sqrt{x}-1)-3(\sqrt{x}-1)=0 \\
& (4\sqrt{x}-3)(\sqrt{x}-1)=0 \\
& \sqrt{x}=\frac{3}{4}, \sqrt{x}=1 \\
& x=\frac{9}{16}, 1
\end{aligned}
$$

So, the sum of the possible values of $x=\frac{9}{16}+1=\frac{25}{16}$ Thus, option (C) is the correct answer.

Q17 Text Solution:

Given that, $4^\alpha=16^\beta=64^\gamma$

That is, $2^{2\alpha}=2^{4\beta}=2^{6\gamma}$

Therefore, by the laws of indices, we have

$$
\begin{aligned}
& 2\alpha=4\beta=6\gamma \\
& \alpha=2\beta=3\gamma=k(\text{ say}) \\
& \beta=\frac{k}{2}, \gamma=\frac{k}{3}
\end{aligned}
$$

Now, $\frac{1}{\alpha}+\frac{1}{\gamma}=\frac{1}{k}+\frac{3}{k}=\frac{4}{k}=\frac{2}{\beta}$

Q18 Text Solution:

$$
\begin{aligned}
&(2^x-7)^2=6\left(2^x-\frac{5}{2}\right)\\
&\Rightarrow 2^{2x}-14\cdot 2^x+49=6\cdot 2^x-15\\
&\Rightarrow 2^{2x}-20\cdot 2^x+64=0\\
&\Rightarrow y^2-20y+64=0\,[\text{ Let } 2^x=y]\\
&\Rightarrow (y-16)(y-4)=0\\
&\Rightarrow y=4,16
\end{aligned}
$$

Therefore, $2^x=16$, or $2^x=4$

$$\Rightarrow x=4, \text{ or } x=2.$$

Hence, the sum of the possible values of $x=4+2=6$

Q19 Text Solution:

Given that, $(3+2\sqrt{2})=a$

$$
\begin{aligned}
&\Rightarrow a=(\sqrt{2})^2+1^2+2\sqrt{2}=(\sqrt{2}+1)^2\\
&\Rightarrow \sqrt{a}=(\sqrt{2}+1)\\
&\Rightarrow \frac{1}{\sqrt{a}}=\frac{1}{\sqrt{2}+1}=\sqrt{2}-1
\end{aligned}
$$

Then,

$$
\begin{aligned}
&\left(\frac{\sqrt{a}}{2}+\frac{1}{2\sqrt{a}}\right)^2\left(a+\frac{1}{a}\right)^3\\
&=\left(\frac{\sqrt{2}+1}{2}+\frac{\sqrt{2}-1}{2}\right)^2(3+2\sqrt{2}+3\\
&\qquad -2\sqrt{2})^3\\
&=2\times 6^3\\
&=2^4\times 3^3=b^{2b}\times c^c\text{ (Given)}
\end{aligned}
$$

So, $2b=4$

$$\Rightarrow b=2$$

Also, $c=3$

Therefore, $(b+c)^b=5^2=25.$

Q20 Text Solution:

$$
\begin{aligned}
&x^{\frac{1}{2}}+y^{\frac{1}{2}}=\sqrt{3\times\sqrt{3\times\sqrt{3\times\sqrt{3\times 3}}}}\\
&=> x^{\frac{1}{2}}+y^{\frac{1}{2}}=\sqrt{3\times\sqrt{3\times\sqrt{3\times 3}}}\\
&\Rightarrow x^{\frac{1}{2}}+y^{\frac{1}{2}}=\sqrt{3\times\sqrt{3\times 3}}\\
&\Rightarrow> x^{\frac{1}{2}}+y^{\frac{1}{2}}=\sqrt{3\times 3}=3
\end{aligned}
$$

Possible values

$$
\begin{aligned}
&x=9,y=0\\
&x=0,y=9\\
&x=1,y=4\\
&x=4,y=1
\end{aligned}
$$

Hence, 4 solutions.

Q21 Text Solution:

Completing the square on the RHS of the equation gives

$(x^2 + 2)^{10} = 4x - x^2 - 6$

$(x^2 + 2)^{10} = - 2 - (x^2 - 4x + 4)$

$(x^2 + 2)^{10} = -2 - (x-2)^2$

For real x, the LHS is always positive and the RHS is always negative and so the equation has no real solutions.

Q22 Text Solution:

$$x^{\frac{a}{a+b+c}} \times x^{\frac{b}{a+b+c}} \times x^{\frac{c}{a+b+c}} = \frac{1011}{y^{\frac{c}{a+b+c}} \times y^{\frac{b}{a+b+c}} \times y^{\frac{a}{a+b+c}}}$$

$$\Rightarrow x^{\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}} = \frac{1011}{y^{\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}}}$$

$$\Rightarrow x^{\frac{a+b+c}{a+b+c}} = \frac{1011}{y^{\frac{a+b+c}{a+b+c}}}$$

$$\Rightarrow xy = 1011$$

Now, $x^2 + y^2 = (x+y)^2 - 2xy$

$= 45^2 - 2 \times 1011$

$= 2025 - 2022$

$= 3.$

Q23 Text Solution:

It is given that $7^m - 5^n = 117524$ and $7^{m-1} + 5^{n+1} = 17432$

Let $7^{m-1} = p$ and $5^n = q$

So, 7p - q = 117524 ---- (1)

p + 5q = 17432 ---- (2)

Substitute p = $\frac{117524+q}{7}$ into the equation (2).

$\frac{117524+q}{7}$ + 5q = 17432

$\frac{117524+36q}{7}$ = 17432

q = 125 = 5^3

Now, we know that, q = $5^n = 5^3$

So, n = 3.

Also, p = $\frac{117524+125}{7}$ = 16807 = 7^5

we know that, p = $7^{m-1} = 7^5$

m−1 = 5

=> m = 6

Thus, m + n = 6 + 3 = 9.

Q24 Text Solution:

$x^{2024}\, y^{2023} = \frac{1}{3}$ ---(1)

$x^{2022}\, y^{2025} = 27$ ---(2)

Divide (1) by (2) we get $(\frac{x}{y})^2 = \frac{1}{81}$

y = 9x

Substituting the value of y in (2)

$x^{2022} \,.\, x^{2025} \,.\, 9^{2025} = 3^3$

$x^{4047} \,.\, 3^{4050} = 3^3$

$x^{4047} = 3^{-4047}$ => x = $\frac{1}{3}$, y = 3

$x^2 + y^3 = \frac{1}{9} + 27 = \frac{244}{9}$

Q25 Text Solution:

We know that,

$x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

Since, $x^4 + x^3 + x^2 + x + 1 = 0$

$\therefore x^5 - 1 = 0$

$\Rightarrow x^5 = 1$

Now, $x^{2050} + x^{2021} + x^{2022}$

$= x^{2020}\,(x^{30} + x + x^2)$

$= (x^5)^{404}\,[(x^5)^6 + x + x^2]$

$= 1 \times (1 + x + x^2)$

$= 1 + x + x^2$

$= -x^3(1+x)$ [Since, $x^4+x^3+x^2+x+1= 0$]

Hence, option D.

Q26 Text Solution:

Let $a^p = b^q = c^r = k^{pqr}$

Then, $a = k^{qr}$

$$b = k^{pr}$$
$$c = k^{pq}$$

So, $abc = k^{pq+qr+pr}$

Therefore,

$$(abc)^{\frac{1}{p^{-1}+q^{-1}+r^{-1}}} = \left(k^{pq+qr+pr}\right)^{\frac{pqr}{pq+pr+qr}} = k^{pqr}$$
$$= a^p = a^x \text{ (Given)}$$

Hence, $x = p$

Q27 Text Solution:

$Y^{(4-X)} \times Z^{5X} = Y^{X+6} \times Z^{3X}$

$Y^{4-X-X-6} = Z^{3X-5X}$

$Y^{-2X-2} = Z^{-2X}$

$\frac{1}{Y^{2(X+1)}} = \frac{1}{Z^{2X}}$

$Y^{2(X+1)} = Z^{2X}$

$Y^{(X+1)} = Z^X$

$Z = Y^{1+\frac{1}{X}}$

$= Y\,(Y)^{\frac{1}{X}}$

Q28 Text Solution:

By the given condition,

$$2 \times \frac{x}{2} = 3^{1+a} + 3^{1-a} + 9^{a} + 9^{-a} \Rightarrow x$$
$$= 3^{1+a} + 3^{1-a} + 3^{2a} + 3^{-2a}$$

Now, x will give a least positive value if $a = 0$.

Then, we have

$$x = 3 + 3 + 1 + 1 = 8.$$

Q29 Text Solution:

$$x$$
$$= \left[1 + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{1024}+\sqrt{1023}}\right]$$
$$= 1 + \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \ldots + \sqrt{1024} - \sqrt{1023}$$
$$= 2^{5}$$

Also,

$$y = \sqrt{x\sqrt{x\sqrt{x \ldots \infty}}}$$
$$=> y^{2} = xy$$
$$=> y = x = 2^{5} [\text{ Since, } y \neq 0]$$

Therefore, $2^{5} = a^{b}$

$$\Rightarrow a = 2, b = 5 \text{ or, } a = 32, b = 1$$

So, $(a + b)$ is having a minimum value of $(2 + 5) = 7$.

Q30 Text Solution:

The given equation is $(x^{2} - 10)^{(x^{3}+x^{2}-14x-24)} = 1$

We know that, $a^b = 1$ implies 3 cases:

Case 1: When b = 0.

$x^3 + x^2 - 14x - 24 = 0$

$x^3 + 3x^2 - 2x^2 - 6x - 8x - 24 = 0$

=> $x^2(x+3) - 2x(x+3) - 8(x+3) = 0$

=> $(x+3)(x^2-2x-8)=0$

=> x = -3,

=> x = -3, -2, 4

Case 2: When a = 1.

$x^2 - 10 = 1$

$x^2 = 11$

=> x is not an integer.

Case 3: When a = -1 and b is even.

$x^2 - 10 = -1$

=> $x^2 = 9$

=> $x = \pm 3$

At x = 3,

$b = x^3 + x^2 - 14x - 24$

$= 3^3 + 3^2 - 14(3) - 24$

$=27 + 9 - 42 - 24$

$=36 - 66$

$= -30$

At x = -3,

$b=x^3+x^2-14x-24=(-3)^3+(-3)^2-14(-3)-24$

$= -27 + 9 + 42 - 24$

$= 51 - 51$

$= 0$

Hence, the possible solutions are x = 3, -3, -2, 4.

So, the number of possible integer solutions is 4.