

## Assignment

1. For any natural numbers  $a$ ,  $b$ , and  $c$ ,  $(a + b) \times (b + c) \times (c + a)$  is \_\_\_\_\_.  
1. Always Even      2. Always Odd      3. Sometimes Even, sometimes Odd
2. If,  $2^a = 4^b = 8^c$ , find the ratio of  $a : b : c$   
1.  $3 : 2 : 1$       2.  $4 : 2 : 1$       3.  $6 : 3 : 2$       4.  $1 : 2 : 4$       5. None of these
3. Which is greater of the two  $2^{300}$  or  $3^{200}$ ?  
1. Both are equal      2.  $3^{200}$       3.  $2^{300}$
4. The units digit of  $3^{1002} \times 7^{1002} \times 13^{1003}$  is  
1. 1      2. 3      3. 5      4. 7      5. 9
5. The smallest natural number  $n$  such that  $n!$  is divisible by 1000 is  
1. 20      2. 15      3. 10      4. 12      5. 25
6. Find the least natural number by which 72 should be multiplied so as to make it a multiple of 112.  
1. 14      2. 4      3. 7      4. 28      5. 2
7. Find the smallest number by which 3750 should be divided so that the quotient is a perfect square.  
1. 2      2. 3      3. 5      4. 6      5. 15
8. If  $n$  is the smallest perfect square which is divisible by 6, 8 and 15, find the sum of digits of  $n$ ?  
1. 11      2. 7      3. 15      4. 18      5. 9
9. Find the total number of prime factors of  $(6^{41} \times 3^{56} \times 11^7 \times 65^9)$ .  
1. 5      2. 9      3. 4      4. 3      5. 6
10. If both  $11^2$  and  $3^3$  are factors of the number  $a \times 4^3 \times 6^2 \times 13^{11}$ , then what is the smallest possible value of  $a$ ?  
1. 3      2. 121      3. 363      4. 3267      5. 33
11. All natural numbers from 101 to 200 are multiplied together. Find the number of zeroes at the end of the product.  
1. 24      2. 25      3. 33      4. 97      5. 20
12. Find the greatest number that will divide 365, 512 and 323 so as to leave a remainder 8 in each case.  
1. 2      2. 3      3. 21      4. 32      5. 42
13. Find the greatest number that on dividing 964, 1238, 1400 leaves a remainder of 41, 31 and 51 respectively?  
1. 13      2. 17      3. 19      4. 71      5. 142

14. Find the least number which when divided by 5, 6 and 7 and leaves a remainder 3, but when divided by 9 leaves no remainder.

1. 288      2. 213      3. 207      4. 423      5. 603

15. If eggs are removed from a basket two, three, four, five and six at a time, there remain, respectively, one, two, three, four and five eggs. But if the eggs are removed seven at a time no eggs remain. What is the least number of eggs that could have been in the basket?

1. 59      2. 119      3. 126      4. 147      5. 154

16.  $n$  is a number that when divided by 12 and 15 leave a remainder of 2 and 5 respectively. How many distinct three digit numbers can  $n$  assume

1. 14      2. 15      3. 16      4. 18      5. 17

17.  $X$  when divided by 899 leaves a remainder 63. What is the remainder when  $X$  is divided by 29?

1. 5      2. 7      3. 9      4. 24      5. Cannot be determined

18. A certain number when divided by 222 leaves a remainder of 35; another number when divided by 407 leaves remainder 47. What is the remainder when sum of these 2 numbers is divided by 37?

1. 35      2. 24      3. 18      4. 29      5. 8

19. If  $a = b^2 - b$  where  $b$  is any positive integer than 4, then the greatest number that definitely divided  $a^2 - 2a$ , for every possible value of  $b$  is:

1. 4      2. 8      3. 24      4. 72      5. 48

20. Find the remainder when  $7^{187}$  is divided by 800.

1. 7      2. 49      3. 343      4. 0      5. 1

21. Find the remainder when  $2^{1000}$  is divided by 25.

1. 1      2. 2      3. 4      4. 8      5. 24

22. Find the remainder when  $5^{100000}$  is divided by 18.

1. 13      2. 17      3. 12      4. 1      5. 5

23. What is the remainder when  $1044 \times 1047 \times 1050 \times 1053$  is divided by 33?

1. 3      2. 12      3. 21      4. 30      5. None of these

24. The natural numbers 1, 2, 3, ....., upto to 100 are written in that order to form the single number 1234...979899100. The number of times the digit 1 occurs in this number is:

1. 11      2. 12      3. 20      4. 21      5. 19

25. 
$$\sqrt{8 + 3\sqrt{7} + \sqrt{12 + 5\sqrt{7} + \sqrt{23 + 8\sqrt{7}}}} =$$

1.  $2 + \sqrt{7}$       2. 5      3. 9      4.  $1 + \sqrt{7}$       5.  $3 + \sqrt{7}$

26. The number of prime numbers  $P$  such that  $1999! + 1 < P < 1999! + 1999$  is:  
 1. 1                      2. 1998                      3. 1999                      4. 2000                      5. 0
27. All the digits of a number is just 1, i.e. the number is made up of just 1s. Find the number of digits in the smallest such number that is divided by 7?  
 1. 6                      2. 7                      3. 9                      4. 10                      5. 11
28. The sum of the fourth powers of the first 100 natural numbers will have a unit's digit of \_\_\_\_.  
 1. 0                      2. 3                      3. 1                      4. 6                      5. 5
29. What is the unit digit of  $17^{27^{37}}$ ?  
 1. 1                      2. 3                      3. 7                      4. 9                      5. 5
30. Find the unit digit of  $1^{1!} + 2^{2!} + 3^{3!} + 4^{4!} + 5^{5!} + \dots + 10^{10!}$   
 1. 9                      2. 7                      3. 5                      4. 3                      5. 0
31. Find the number of factors of  $16!$ .  
 1. 16                      2. 34                      3. 376                      4. 5376                      5.  $2^{16} - 1$
32. What is the largest power of  $10!$  that can divide  $100!$   
 1. 10                      2. 11                      3. 12                      4. 20                      5. 16
33. In how many ways can 1 billion (1,000,000,000) be written as a product of two numbers such that neither of the numbers is a multiple of 10?  
 1. 2                      2. 1                      3. Not possible                      4. 3                      5. More than 3 ways
34. If all the factors of 720 be written in increasing order, the factor which occupies the 26<sup>th</sup> position is  
 1. 120                      2. 144                      3. 216                      4. 360                      5. 125
35. How many of the following statements are true?  
 i. Only squares of prime numbers have three factors  
 ii. If a number has 4 factors, the number must be a perfect cube  
 iii. If a number has 5 factors, the number must be a fourth power of a natural number  
 1. None                      2. One                      3. Two                      4. All three
36. The HCF and LCM of two numbers are 75 and 864 respectively. If one of the numbers is 240, find the other number.  
 1. 240                      2. 270                      3. 300                      4. 225                      5. Data inconsistent
37. When a number is divided by a particular divisor, the remainder obtained is 11. When another number is divided by the same divisor, the remainder obtained is 15. When yet another number is divided by the same divisor, the remainder obtained is 18. When the sum of the three numbers is divided by the same divisor, the remainder obtained is 2. How many such divisors are possible?  
 1. 0                      2. 1                      3. 2                      4. 3                      5. More than 3

38. When two numbers are individually divided by a particular divisor (greater than 1), they leave the same remainder. The difference between the numbers is 450. How many such divisors are possible?

1. 17      2. 12      3. 8      4. 1      5. 18

39. When 222, 333 and 444 are divided by a number  $n$ , the remainders obtained are 51, 48 and 45 respectively. What is the remainder obtained when  $(n + 4) \times (n + 7)$  is divided by 19?

1. 0      2. 1      3. 9      4. 18      5. 10

40. What is the minimum number of identical cubes with which one can construct a solid cuboid of dimensions  $20 \text{ cm} \times 16 \text{ cm} \times 12 \text{ cm}$ ?

1. 12      2. 40      3. 60      4. 80      5. 20

41. There are a certain number of identical cuboids of dimensions  $6 \text{ cm} \times 4 \text{ cm} \times 3 \text{ cm}$ . What is the volume of the smallest solid cube that can be made by stacking any number of these cuboids but each cuboid is stacked in the same orientation?

1. 216      2. 1000      3. 1296      4. 1728      5. 144

42. What is the remainder when  $9^1 + 9^2 + 9^3 + \dots + 9^{99}$  is divided by 6?

1. 0      2. 3      3. 4      4. 5      5. 1

43. Find the remainder when  $2^{50}$  is divided by 97.

1. 2      2. 4      3. 8      4. 16      5. 32

44. Find the remainder when  $3^{57}$  is divided by 41.

1. 3      2. 9      3. 27      4. 38      5. 14

45. Find the remainder when  $5^{25^{125}}$  is divided by 11.

1. 1      2. 2      3. 6      4. 10      5. 5

46. Find the remainder when  $63^{73^{83}}$  is divided by 13.

1. 0      2. 1      3. 11      4. 12      5. 2

47. A number when divided successively by 7, 5, and 3, leaves a remainder of 5, 3 and 1 respectively. Find the remainder when the number is divided by 105.

1. 1      2. 3      3. 28      4. 61      5. 52

48. How many three digit numbers exist which have at least one of the digits as 5?

1. 280      2. 252      3. 200      4. 171      5. 243

49. How many three digit numbers exist which have exactly one of the digits as 5?

1. 280      2. 271      3. 262      4. 243      5. None of these

50. In the following each of the letters represents a unique whole number from 0 to 9. No two letters represent the same digit and no digit is represented by two different letters. What is the sum  $V + I + E + R$ ?

1. 14                  2. 15                  3. 16                  4. 17                  5. 18

1. 8                      2. 9                      3. 10                      4. 11                      5. 12

1.1                      2.2                      3.3                      4.4                      5.0

1. 0                      2. 1                      3. 15                      4. 30                      5. 23

is the smallest positive integer such that  $n/2$  is a perfect square,  $n/3$  is a perfect cube and  $n/5$  is a perfect fifth power. If  $n$  is represented in its factorized form as  $p_1^a \times p_2^b \times p_3^c \times \dots$ , where  $p_1, p_2, p_3, \dots$  are all distinct prime numbers, then find the sum  $a + b + c + \dots$

1. 31                      2. 37                      3. 54                      4. 56                      5. 59

1. 24                      2. 40                      3. 64                      4. 120                      5. 48

1. 2024      2. 945      3. 1080      4. 1079      5. 935

1. 6                      2. 24                      3. 30                      4. 34                      5. 39

1. 2                      2. 6                      3. 12                      4. 18                      5. 28

1. 20 ways      2. 12 ways      3. 7 ways      4. 2 ways      5. 1 way

61. A trader has some pickle jars. When he divides these jars in groups of  $n$ , 7 jars are left at the end i.e. after forming all possible groups of  $n$  jars. If he had just one-fifth of the number of jars and had he divided these into groups of  $n$  jars, he would have been left with 32 jars after forming all possible groups of  $n$  jars. Assuming that  $n$  has the smallest possible value, how many jars will be left if he tries to pack 460 jars in the same way?

1. 0                      2. 1                      3. 3                      4. 6                      5. 9

62. A trader has some pickle jars that he needs to pack. When he divides these jars into groups of  $n$  jars, he is left with 7 jars at the end i.e. after forming all possible groups of  $n$  jars. If he had five times the number of jars he had and had he divided these into groups of  $n$  jars, he would have been left with 4 jars at the end i.e. after forming all possible groups of  $n$  jars. Find the value of  $n$ ?

1. 11                      2. 14                      3. 27                      4. 31                      5. 37

63. How many numbers less than 100 when divided by 7, 8 and 9 leave a remainder of 5, 4 and 3?

1. None                      2. 1                      3. 2                      4. 3                      5. More than 3

64. If  $n$  is the smallest number which when divided by 3, 5, 7 and 9 leave a remainder of 1, 2, 3 and 4 respectively, find the product of the digits of  $n$ .

1. 35                      2. 28                      3. 21                      4. 14                      5. 8

65. There are two traffic signals. The first one stays red for 5 minutes and green for 2 minutes whereas the second one stays red for 6 minutes and green for 2 minutes. If both the signals just turned red simultaneously, after how much time will they turn green simultaneously?

1. 32 minutes                      2. 56 minutes                      3. 54 minutes  
4. 28 minutes                      5. Never

66. When 23 divides  $461^n$  and  $485^n$ , the difference in the remainder is 1. Which of the following could be a value of  $n$ ?

1. 265                      2. 392                      3. 184                      4. 323                      5. 110

67. Find the remainder when 1444246849 by 792.

1. 1                      2. 7                      3. 9                      4. 2                      5. 3

68. Find the remainder when  $25^{35^{75}}$  is divided by 15

1. 0                      2. 5                      3. 10                      4. 14                      5. 1

Directions for Qs 69 & 70: Roopa was carrying  $n$  flowers to be offered to four temples that come on her way. Each of the temple has a magical pond before it which doubles the number of flowers dipped in it. Roopa dips all the flowers she is carrying in each of the pond before entering any temple and then offers the same number of flowers at each temple, turn by turn. In the end she is left with one fourth of the number of flowers she started with.

69. What is the least number of flowers she started with and what is the number of flowers she offered at each temple?

1. 30, 16                      2. 20, 21                      3. 40, 28                      4. 15, 18                      5. 40, 42

70. If she offered more than 80 but less than 100 flowers at each temple, how much did she start with?

1. 72                      2. 80                      3. 86                      4. 102                      5. 84

71. A number when successively divided by 8 and 9 leaves a remainder of 5 and 4 respectively. What will the remainders be if the order of division is reversed i.e. if the number is successively divided by 9 and 8?

1. 4 and 5              2. 2 and 3              3. 1 and 4              4. 2 and 4              5. 1 and 5

72. How many pairs of the natural numbers exist whose LCM is 540 and GCD is 18?

1. 2                      2. 4                      3. 6                      4. 8                      5. None of these

73. Find the number of natural numbers which are prime to  $2^3 \times 3^2 \times 5$  and less than it. (This question will also require the funda of set theory for more efficient solution)

1. 300                      2. 263                      3. 152                      4. 118                      5. 96

74. How many numbers from 1 to 100 are divisible only by 2 but neither by 3 nor by 7?

1. 34                      2. 31                      3. 29                      4. 27                      5. 25

75. In the following each of the letters represents a unique whole number from 0 to 9. No two letters represent the same digit and no digit is represented by two different letters. What is the sum  $M + O + N + E + Y$ ?

$$\begin{array}{r} S \ E \ N \ D \\ + \ M \ O \ R \ E \\ \hline M \ O \ N \ E \ Y \end{array}$$

1. 14                      2. 17                      3. 19                      4. 23                      5. 25

76. Find the digit in the ten's place in the expansion of  $129^{131}$ .

1. 0                      2. 1                      3. 2                      4. 3                      5. 4

77. The last two digits of  $6^{2005}$  are

1. 96                      2. 16                      3. 36                      4. 56                      5. 76

78. Which is the smallest number that can have 24 factors?

1. 516                      2. 420                      3. 360                      4. 480                      5. None of these

79. A natural number  $n$  is such that it has  $x$  factors, the number  $2n$  has  $2x$  factors and the number  $3n$  has  $\frac{3}{2}x$  factors. Find the number of factors of  $12n$ .

1.  $\frac{9}{4}x$                       2.  $\frac{9}{2}x$                       3.  $4x$                       4.  $12x$                       5.  $6x$

80. Find the number of possible pairs of co-prime factors of  $2^3 \times 3^2$

1. 2                      2. 9                      3. 13                      4. 17                      5. 22

81. Find the number of possible pairs of co-prime factors of  $2^3 \times 3^2 \times 5^3$

1. 11                      2. 110                      3. 122                      4. 144                      5. 160

82. Find the number of ways in which  $2^3 \times 3^4 \times 5^6 \times 7^2$  can be written as a product of two co-prime numbers.

1. 4                      2. 6                      3. 8                      4. 10                      5. 16

83. Every third natural number starting from 1 (i.e. 1, 4, 7, 10, ...) upto 1000 is multiplied. Find the number of zeroes at the end of the product.

1. 200                      2. 175                      3. 125                      4. 85                      5. 75

84. Find the HCF of  $37^{12} - 1$  and  $37^9 - 1$

1.  $37^3 - 1$                       2.  $37^3 + 1$                       3. 36                      4.  $37^2 + 1$                       5.  $37^2 - 1$

85. Find the HCF of  $11111\ldots1111_{100 \text{ ones}}$  and  $111\ldots111_{60 \text{ ones}}$ .

1. 11                      2. 111111                      3.  $111\ldots11_{12 \text{ ones}}$   
4.  $111\ldots11_{20 \text{ ones}}$                       5.  $111\ldots11_{60 \text{ ones}}$

86. How many pairs of numbers have their LCM as  $2^3 \times 3^2$ .

1. 18                      2. 21                      3. 24                      4. 27                      5. 30

87.  $\frac{\text{LCM of } 1, 2, 3, \dots, 200}{\text{LCM of } 105, 106, 107, \dots, 200} =$

1. 1                      2. 101                      3.  $101 \times 103$   
4.  $101 \times 102 \times 103$                       5.  $101 \times 102 \times 103 \times 104$

88. If the LCM of 1, 2, 3, ..., 120 is n, find the LCM of 1, 2, 3, ..., 125.

1. n                      2. 5n                      3. 11n                      4. 55n                      5. 110n

89. What is the remainder when  $128^{500}$  is divided by 153?

1. 4                      2. 16                      3. 64                      4. 67                      5. 89

90. What is the remainder when  $111\ldots111_{81 \text{ ones}}$  is divided by 81?

1. 0                      2. 1                      3. 9                      4. 80                      5. 72

91. What is the remainder when  $111\ldots111_{729 \text{ ones}}$  is divided by 728?

1. 1                      2. 11                      3. 121                      4. 111                      5. 727

92. The largest number amongst the following that will perfectly divide  $101^{100} - 1$  is

1. 100                      2. 10000                      3.  $100^{100}$                       4. 100000                      5.  $10^{10}$

93. From all the natural numbers from 2 to 10000, first all the perfect squares are erased, then all the perfect cubes are erased, then all the perfect  $4^{\text{th}}$  powers are erased and so on, in successive rounds, higher powers are eliminated. How many rounds are there such that atleast one number is erased in them?

1. 13                      2. 12                      3. 8                      4. 6                      5. 5

Directions for Qs. 94 & 95: A diamond thief after stealing n diamonds had to pass through 3 doors with security guard. At the first door, the thief had to give half the diamond he was carrying plus one additional diamond. At the second he had to part with one-third of the diamonds he was now carrying plus two more diamonds. At the third door he had to give one-fourth of the diamonds he was now carrying plus 3 diamonds. At last he was left with



just

$m$  ( $m > 0$ ) diamonds.

94. What is the least value that  $m$  can assume?

1. 1                      2. 2                      3. 3                      4. 4                      5. 5

95. If  $m = 9$ , what is the value of  $n$ ?

1. 32                      2. 38                      3. 44                      4. 56                      5. 60

96. How many solutions exist for the equation  $\frac{3}{a} - \frac{5}{b} = \frac{1}{15}$ , where  $a$  and  $b$  are integers.

1. 31                      2. 32                      3. 15                      4. 16                      5. 8

97. Each of three friends independently think of a number, which is the product of two different primes. Which of the following could be the product of three numbers thought by them, if no two persons think of the same pair of primes?

1. 120                      2. 144                      3. 12100                      4. 3000                      5. 420

98. A student comes and writes the first 100 even numbers on a black board, one besides the other. Find the number of times the digit '2' is written.

1. 40                      2. 38                      3. 31                      4. 30                      5. None of these

99. The product of the first 100 odd numbers is equal to:

1.  $\frac{100!}{2^{100} \times 50!}$       2.  $\frac{200!}{2^{100} \times 50!}$       3.  $\frac{100!}{2^{200} \times 50!}$       4.  $\frac{200!}{2^{100} \times 100!}$       5.  $\frac{200!}{2^{200} \times 100!}$

| Assignment |   |    |   |    |   |    |   |
|------------|---|----|---|----|---|----|---|
| 1          | 1 | 26 | 5 | 51 | 1 | 76 | 3 |
| 2          | 3 | 27 | 1 | 52 | 2 | 77 | 5 |
| 3          | 2 | 28 | 1 | 53 | 1 | 78 | 3 |
| 4          | 4 | 29 | 2 | 54 | 5 | 79 | 2 |
| 5          | 2 | 30 | 1 | 55 | 1 | 80 | 4 |
| 6          | 1 | 31 | 4 | 56 | 2 | 81 | 3 |
| 7          | 4 | 32 | 3 | 57 | 3 | 82 | 3 |
| 8          | 5 | 33 | 2 | 58 | 5 | 83 | 4 |
| 9          | 1 | 34 | 2 | 59 | 2 | 84 | 1 |
| 10         | 3 | 35 | 3 | 60 | 4 | 85 | 4 |
| 11         | 2 | 36 | 5 | 61 | 2 | 86 | 1 |
| 12         | 3 | 37 | 3 | 62 | 4 | 87 | 3 |
| 13         | 4 | 38 | 1 | 63 | 2 | 88 | 4 |
| 14         | 4 | 39 | 3 | 64 | 1 | 89 | 4 |
| 15         | 2 | 40 | 3 | 65 | 3 | 90 | 1 |
| 16         | 2 | 41 | 4 | 66 | 1 | 91 | 4 |
| 17         | 1 | 42 | 2 | 67 | 1 | 92 | 2 |
| 18         | 5 | 43 | 2 | 68 | 3 | 93 | 4 |
| 19         | 3 | 44 | 4 | 69 | 2 | 94 | 3 |
| 20         | 3 | 45 | 1 | 70 | 2 | 95 | 4 |
| 21         | 1 | 46 | 3 | 71 | 3 | 96 | 1 |
| 22         | 1 | 47 | 4 | 72 | 2 | 97 | 3 |
| 23         | 4 | 48 | 2 | 73 | 5 | 98 | 3 |
| 24         | 4 | 49 | 5 | 74 | 3 | 99 | 4 |
| 25         | 1 | 50 | 4 | 75 | 1 |    |   |