Question 28

Ramesh, in order to achieve his sales target, started selling pens in packs of 15. To attract customers, he sells the pack at the marked price of 12 pens. He also started giving 1 pack free for every 4 packs purchased. Suresh bought 11 packets on which he got some packets free. He unpacked the pens and started selling them individually. He raised the marked price of an individual pen by 30% and offered a discount of 20%. What is the profit % earned by Suresh on selling all the pens?

A 58.33%

B 62.57%

C 53.64%

D 46.88%

Answer: C

Explanation:
Let the CP of 1 pen be Rs.10.
Thus, CP of 1 packet of pens = Rs.120.
He raises the CP by 30% and thus, MP = Rs. 13.
He is offering a discount of 20% and thus, the SP of 1 pen = Rs. 10.4.
Suresh buys 11 packets costing him 120*11 = Rs. 1320.
He got 2 packet free on these 11 packets and thus, CP of 1 packet for Suresh = Rs. 1320/13
Each packet has 15 pens and thus, CP of 1 pen = 1320/(13*15) = Rs. 6.77.

Thus, Profit percentage = $\frac{10.4-6.77}{6.77} = 53.64\%$

Hence, option C is the correct answer.

Question 29

In a class of 20 students, the average age decreases by 2 years when a new student joins. 3 more students whose ages are 12, 13 and 17 years join. The ratio of new average to the original average is $\frac{p}{q}$, where p,q are co-prime integers. Find p+q.

A 15

B 45

C 30

D 20

Answer: A

Explanation:
Assuming the original average is a.
After 1 student joins, the sum of the ages = 21(a-2) = 21a-42
After three more students join, the sum of age = 21a-42+12+13+17 = 21a
The new average age = $\frac{21a}{20+1+3} = \frac{21a}{24} = \frac{7a}{8}$
The ratio of new average to the original average = $\frac{7}{8} = \frac{p}{q}$
p+q = 7+8 = 15

Question 30

A right circular cylindrical tank is connected with pipe 1 and pipe 2. Pipe 1 is connected at the bottom and it is used to fill the tank. Pipe 2 is connected at half the height of the tank and used to drain the tank. It takes t_1 hours to fill the tank completely. If the pipe 2 was connected at three-fourth the height of the tank, it would have taken t_2 hours. The ratio of t_1 and t_2 is 18:11. Find the ratio of the flow rate of pipe 1 and pipe 2?

A 4:3

B 5:4

C 6:5

D 8:7

Answer: D

Explanation:
Assuming the flow rate of pipe 1 is a, the flow rate of pipe 2 is b and the capacity of the tank as 1 unit.
Also t_1 = 18t, t_2=11t
Time taken to fill half the tank = $\frac{Capacity}{Flow\,rate} = \frac{1}{2a}$
While filling the rest half, pipe 2 will start draining, hence the time taken to fill the rest of the half = $\frac{1}{2(a-b)}$
Total time taken = $\frac{1}{2a} + \frac{1}{2(a-b)} = t_1 = 18t$ ......(1)
Similarly, if pipe 2 was connected to the three-fourth of the tank,
Total time taken = $\frac{3}{4a} + \frac{1}{4(a-b)} = t_2 = 11t$ ......(2)
=> Multiplying equation (2) by 18 and equation (1) by 11 and equating, we get,
$\frac{54}{4a} + \frac{18}{4(a-b)} = \frac{11}{2a} + \frac{11}{2(a-b)}$
=> $\frac{32}{4a} = \frac{11}{2(a-b)} - \frac{9}{2(a-b)}$
=> $\frac{8}{a} = \frac{1}{(a-b)}$
=> $8(a-b) = a$
=> 7a = 8b
=> $\frac{a}{b} = \frac{8}{7}$

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Question 31

Ram invest 20% of his savings in a fixed deposit which gives 12.5% annually as simple interest. Of the remaining, he spends 25% to buy a car and rest he gives loan at an annual compound rate of x%. The rate by which the value of the car gets reduced each year is 3x%. After 2 years the total value of his investments increases by 20%. What is the value of x?

Answer:25

Explanation:
Assuming the original amount = 100
So, after 2 years, the fixed deposit will become 20+ $\frac{20 \times 12.5 \times 2}{100}$ = 25
Amount invested in car = $\frac{80 \times 25}{100}$ = 20
Amount invested in loan = 80-20 = 60
Assume, $\frac{x}{100} = r$
The value of car after 2 years = 20$(1-3r)^2$
The value of loan amount after 2 years = 60$(1+r)^2$
Total net investment after 2 years = 100$(1+\frac{20}{100})$
120 = 25+20*$(1-3r)^2$+60*$(1+r)^2$
=> 95 = 20-120r+180r^2+60+120r+60r^2
=> 15 = 240r^2
=> r^2 = $\frac{1}{16}$

$\Rightarrow r = \frac{1}{4}$

Hence, x = 100r = 25%

Question 32

In an election, the citizens vote for either candidate A or candidate B. Candidate A got 25% more female votes than candidate B. Candidate B got 50 less male votes than the candidate A. The ratio of the number of females who voted candidate A to the number of male voters of B is 3:2. Find the minimum possible population of the city if at least 1 male and at least 1 female voted to each candidate.

A 101

B 97

C 93

D 87

Answer: B

Explanation:

Assume, the number of the female voter of B = 4x, then the number of female voters of A = $4x\left(1+\frac{25}{100}\right) = 5x$

Assume the number of the male voter of A = y, then the number of male voters of B = y-50

Now, $\frac{5x}{y-50} = \frac{3}{2}$

=> 3y-10x = 150

=> 3y = 10x+150 => y = $\frac{(10x+150)}{3}$

Population of the city = 4x+5x+y+y-50 = 9x+2y-50 = 9x+ $\frac{2(10x+150)}{3}$ -50

=100 + $9x + \frac{20x}{3}$ - 50

The minimum value of x can be 3 (x should be multiple of 3)

Hence, the minimum population of the city = 100+9*3+20 -50 = 97

Question 33

A train is approaching a tunnel XY. A man is seen at a point inside the tunnel XY, at a distance, which is $\frac{2}{5}$ of the length of the tunnel from X. If the man moves away from the train, the train meets the man at the exit of the tunnel i.e. Y. If the man moves towards the train, the train will meet the man at the entrance i.e. X. Find the ratio of the speeds of the train and the man.

A 4:1

B 5:1

C 4:3

D 5:3

Answer: B

Explanation:

Let the length of the tunnel = 5x

Let the speed of the train = t

Let the speed of the man = m

At the instant the distance of the train from the entrance of the tunnel, X = d

The initial distance of the man from X = $\frac{2}{5} \times 5x = 2x$

=> The distance of the man from Y = 5x-2x = 3x

If the man moves away from the train, the train meets the man at the exit of the tunnel i.e. Y.

$\frac{t}{m} = \frac{(d+5x)}{3x}$ ......(1)

If the man moves towards the train, the train will meet the man at the entrance i.e. X

$\frac{t}{m} = \frac{d}{2x}$ ....(2)

From (1) and (2), $\frac{d}{2x} = \frac{d+5x}{3x}$

=> d=10x

Now, $\frac{t}{m} = \frac{d}{2x} = \frac{10x}{2x} = \frac{5}{1}$

Option B is the correct answer.

Question 34

On an island, the male population is 25% more than the female population. The food is limited and enough for only 20 days for the given population. After 10 days, 20% of men leave and 25% more women join and the food lasts for 12 more days. Had all the male population left at the beginning and no one joined in the middle, how many days would the food have lasted?

Assume that the daily food consumption of all the men is the same. Same is the case for all the women.

A 240

B 300

C 250

D 270

Answer: D

Explanation:
Assume a male consumes m units of food per day and a female consumes w units of food per day.
Assume, the female population is 4x, then the male population will be $4x\left(1+\frac{25}{100}\right)$ = 5x
Now, the total amount of food = (5xm+4xw)*20
After 10 more days, food left = (20-10) = 10 days of food = 10(5xm+4xw)
20% men leave and 25% women join. So the new population of men = 5x($1-\frac{20}{100}$) = 4x
New population of women = $4x\left(1+\frac{25}{100}\right)$ = 5x
Hence, 10(5xm+4xw)=12(4xm+5xw)
=> 2xm = 20xw => m=10w
So the total amount of food in the beginning = (5xm+4xw)*20 = 20(50xw+4xw) = 1080xw
Had all the male population left in the beginning, food would have lasted for $\frac{1080xw}{4xw} = 270$ days

Question 35

The following analysis was made by an expert regarding the scores of 5 batsmen in a match. The score of Farukh was equal to the average score of Eman and Ganguli. The average score of Hasim and Farukh is equal to 4 times the score of Ganguli. The score of Hasim is equal to the average score of Imran and Ganguli. If the overall average of all 5 batsmen is 115, then find the score of Ganguli?

A 20

B 30

C 25

D 35

Answer: C

Explanation:
Representing the players by the initial letters of their names, we get E, F, G, H and I.
Now, F = $\frac{E+G}{2}$ => E+G=2F => E+F+G = 3F
H = $\frac{I+G}{2}$ => I+G=2H => G+H+I = 3H
Now, the sum E+F+G+H+I = 3F+3H-G
It is given that $\frac{(F+H)}{2} = 4G$ => F+H = 8G
Hence, the sum will be equal to 3(F+H)-G = 3*8G-G = 23G
E+F+G+H+I=115*5
=> G= 115*5/23
=> G = 25

Question 36

Mira and Amal walk along a circular track, starting from the same point at the same time. If they walk in the same direction, then in 45 minutes, Amal completes exactly 3 more rounds than Mira. If they walk in opposite directions, then they meet for the first time exactly after 3 minutes. The number of rounds Mira walks in one hour is

Answer:8

Video Solution

Explanation:
Considering the distance travelled by Mira in one minute = M,

The distance traveled by Amal in one minute = A.

Given if they walk in the opposite direction it takes 3 minutes for both of them to meet. Hence 3*(A+M) = C. (1)

C is the circumference of the circle.

Similarly, it is mentioned that if both of them walk in the same direction Amal completes 3 more rounds than Mira :

Hence 45*(A-M) = 3C. (2)

Multiplying (1)*15 we have :

45A + 45M = 15C.

45A - 45M = 3C.

Adding the two we have A = $\frac{18C}{90}$

Subtracting the two M = $\frac{12C}{90}$

Since Mira travels $\frac{12C}{90}$ in one minute, in one hour she travels : $\frac{12C}{90} \cdot 60 = 8C$

Hence a total of 8 rounds.

Alternatively,

Let the length of track be L
and velocity of Mira be a and Amal be b
Now when they meet after 45 minutes Amal completes 3 more rounds than Mira
so we can say they met for the 3rd time moving in the same direction
so we can say they met for the first time after 15 minutes
So we know Time to meet = Relative distance /Relative velocity
so we get $\frac{15}{60} = \frac{L}{a-b}$ (1)
Now When they move in opposite direction
They meet after 3 minutes
so we get $\frac{3}{60} = \frac{L}{a+b}$ (2)
Dividing (1) and (2)
we get $\frac{(a+b)}{(a-b)} = 5$
or 4a =6b
or a = 3b/2
Now substituting in (1)

we get :
$\frac{L}{b} \times 2 = \frac{15}{60}$
so $\frac{L}{b} = \frac{1}{8}$
So we can say 1 round is covered in $\frac{1}{8}$ hours
so in 1-hour total rounds covered = 8.

Important Verbal Ability Questions for CAT (Download PDF)

Question 37

In a textile factory, a man starts working on a project on day 1. On day 2, two more men joined him on the project. On day 3, three more men join the project and so on till the entire project is completed in exactly 10 days. In how many days can the project be completed by 20 women, if each woman is twice as fast as a man?

A 5 days

B 5.5 days

C 6 days

D 6.5 days

Answer: B

Explanation:
Let each man do one unit of work on a day.
So, total work required to complete the project = 1 + (1+2) + (1+2+3) + (1+2+3+4) + ... + (1+2+3+....+10) = 1*10 + 2*9 + 3*8 + 4*7 + 5*6 + 6*5 + 7*4 + 8*3 + 9*2 + 10*1 = 20 + 36 + 48 + 56 + 60 = 220 units
Work done by 1 woman in 1 day = 2 units
So, work done by 20 women in 1 day = 40 units
So, number of days required to complete the project = 220/40 = 5.5 days

Question 38

A given amount of wheat is to be sold in the market. Two-fifth of the total quantity is sold at a loss of 5%. Half of the remaining quantity is sold at a profit of 15%. For what % profit must the remaining amount be sold such that the overall gain is 30%?

A 121.67

B 106.67

C 91.67

D 66.67

Answer: C

Explanation:
Assume the total cost price of wheat = x
Then the cost price of $\frac{2}{5}$ quantity of wheat = $\frac{2x}{5}$
Selling price of this quantity = $\frac{2x}{5} \times \left(1 - \frac{5}{100}\right) = \frac{2x}{5} \times \frac{95}{100} = \frac{38x}{100} = \frac{19x}{50}$
The cost price of half the remaining quantity = $\frac{1}{2} \times \frac{3x}{5} = \frac{3x}{10}$
The selling price of this quantity = $\frac{3x}{10} \times \left(1 + \frac{15}{100}\right) = \frac{3x}{10} \times \frac{115}{100} = \frac{69x}{200}$
Now we have $\frac{3}{10}$ of the total quantity left to sell.
So, the cost price of this quantity = $\frac{3x}{10}$
To get an overall 30% selling price, the total selling price will be $x\left(1 + \frac{30}{100}\right) = 1.3x$
So the selling price of the remaining quantity left will be $\frac{13x}{10} - \frac{19x}{50} - \frac{69x}{200}$ = $\frac{260x}{200} - \frac{76x}{200} - \frac{69x}{200} = \frac{115x}{200} = \frac{23x}{40}$
Hence, profit % = $\frac{\frac{23x}{40} - \frac{3x}{10}}{\frac{3x}{10}} \times 100 = \frac{11x}{40} \times \frac{10}{3x} \times 100$ = 91.67%

Question 39

A metal trader sells zinc, copper and iron. On a particular day, the cost price of iron is 25% more than copper which in turn has cost price 33.33% more than zinc. The profit booked on zinc, copper and iron is 50%, 40% and 30% respectively. If the overall profit is 40%, what should be the ratio of quantities of zinc and iron sold on that day assuming that every metal was traded?

A 5/3

B 3/5

C 5/4

D Cannot be determined

Answer: A

Explanation:
Assuming the cost price of zinc/kg = a
Hence the price of copper/kg = 4a/3
Hence the price of iron/kg =5a/3

Assuming a/3 = b, The price per kg for zinc, iron and copper be 3b, 4b and 5b respectively.
The profit made on zinc/kg = 3b*0.5 = 1.5b
Profit on copper/kg = 4b*0.4 = 1.6b
Profit on iron/kg = 5b*0.3 =1.5b

Now assuming the quantities for zinc, iron and copper be x, y and z respectively.

Overall profit = (1.5bx+1.6by+1.5bz)/(3bx+4by+5bz) = 40/100 =0.4

=> (1.5bx+1.6by+1.5bz) = 1.2bx + 1.6by + 2bz

=> 0.3x = 0.5z

=> x/z =5/3

Data Interpretation for CAT Questions (download pdf)

Question 40

A, a fitness enthusiast, goes for a run everyday for 4 hours at a circular park. He runs at a speed of 5 km/hr and is able to finish a lap of the park in 36 mins. A starts running at 5 am. Another runner B arrives at 6:15 am and is 20% slower than A. At how many distinct points will A and B meet each other ? Assume that they both leave at the same time.

A 1 point

B 2 points

C 3 points

D 4 points

Answer: A

Explanation:

Circumference of the park = $\frac{36}{60} \times 5 = 3$ kms

When B arrives, distance run by A = $\frac{75}{60} \times 5 = 6.25$ kms

Thus, position of A when B arrives = $R(\frac{6.25}{3}) = 0.25$ kms from the starting point

Speed of B = $\frac{4}{5} \times 5 = 4$km/hr

Distance between A and B when B arrives = 0.25 kms

Thus, the 2 runners will meet only if the distance is either 0 or a multiple of 3 kms (circumference of the park)

Since A is faster than B, only the second case is a possibility.

Relative speed of A and B = 5-4 = 1 km/hr

Distance required between the runners to make sure they meet = 3 - 0.25 = 2.75 kms

Time needed to cover the required distance = 2.75 hrs = 2 hr 45 mins

Time when the runners meet for the first time = 6:15+2:45 = 9:00 AM

Since A runs for 4 hours starting from 5 AM, we know that he will stop at 9 AM.

Therefore, the runners meet only once at one distinct point.

Question 41

The value of $1^3 - 2^3 + 3^3 - 4^3 .... - 100^3$ **is**

A -507500

B -681750

C -676700

D -504000

Answer: A

Explanation:

$1^3 - 2^3 + 3^3 - 4^3 .... - 100^3$

$=1^3 + 2^3 + 3^3 ....100^3 - 2\left(2^3 + 4^3 + 6^3 ....100^3\right)$

$=1^3 + 2^3 + 3^3 ....100^3 - 2 \times 2^3\left(1^3 + 2^3 + 3^3 ....50^3\right)$

$=\left(\frac{100 \times 101}{2}\right)^2 - 16\left(\frac{50 \times 51}{2}\right)^2$

$=50^2 \times 101^2 - 4 \times 50^2 \times 51^2$

$=50^2\left(101^2 - 2^2 \times 51^2\right)$

$= 50^2(101 - 102)(101 + 102)$

=-507500

A is the correct answer.

Question 42

The number of integral solutions of the equation $2^x(4-x) = 2(x+2)$ **is**

Answer:3

Explanation:

$2^x > 0$

So $\frac{2(x+2)}{4-x} > 0$

$(x+2)(4-x) > 0$

-2 < x < 4

From this range of values only x = 0, 1, 2 satisfies the equation.

Hence 3 is the correct answer.

Question 43

If $\log_9(x-1) = \log_3(x-3)$**, then the sum of all the possible solutions of x is**

Answer:5

Explanation:
$\log_9(x-1) = \log_3(x-3)$

$\frac{\log(x-1)}{\log 9} = \frac{\log(x-3)}{\log 3}$

$\frac{\log(x-1)}{2\log 3} = \frac{\log(x-3)}{\log 3}$

$(x-1) = (x-3)^2$

$x^2 - 6x + 9 - x + 1 = 0$

x = 5, 2

Let's see if the values satifies the equation

If x = 2, the expression inside the log will become negative, which is invalid.

If x = 5, $\log_9 4 = \log_3 2$

The sum of the values of x which satisfies the equation is 5

Question 44

Which of the following can't be the sum of the squares of the roots of the equation: $x^2 + (p+6)x + (p + \frac{21}{4}) = 0$**, if both the roots are imaginary.**

A $\frac{21}{4}$

B $\frac{17}{4}$

C $\frac{15}{4}$

D $\frac{13}{4}$

Answer: A

Explanation:
We are given that root are imaginary, therefore,

$\Rightarrow (p+6)^2 - 4(p + \frac{21}{4}) < 0$

$\Rightarrow p^2 + 12p + 36 - 4p - 21 < 0$

$\Rightarrow p^2 + 8p + 15 < 0$

$\Rightarrow (p+5)(p+3) < 0$

Therefore, p = (-5, -3) ... (1)

Let 'a' and 'b' be the roots of the given quadratic equation,

$\Rightarrow a^2 + b^2 = (a+b)^2 - 2ab$

$\Rightarrow a^2 + b^2 = (p+6)^2 - 2(p + \frac{21}{4})$

$\Rightarrow a^2 + b^2 = p^2 + 10p + \frac{51}{2}$

$\Rightarrow a^2 + b^2 = p^2 + 10p + 25 - 25 + \frac{51}{2}$

$\Rightarrow a^2 + b^2 = (p+5)^2 + \frac{1}{2}$ ... (2)

We know that $-5 < p < -3$

$\Rightarrow -5+5 < p+5 < -3+5$
$\Rightarrow 0 < p+5 < 2$
$\Rightarrow 0 < (p+5)^2 < 4$
$\Rightarrow 0 + \frac{1}{2} < (p+5)^2 + \frac{1}{2} < 4 + \frac{1}{2}$
$\Rightarrow \frac{1}{2} < a^2 + b^2 < \frac{9}{2}$

Hence we can say that the sum of the squares of the roots of the given equation can't be $\frac{21}{4}$. Hence, option A is the correct answer.

Question 45

If $x = 8 - \sqrt{32}$ and $y = 2 + \sqrt{2}$, then $\left(x + \frac{1}{y}\right)^2$ is given by:

A $\frac{16}{25}x^2$

B $\frac{64}{81}y^2$

C $\frac{25}{16}y^2$

D $\frac{81}{64}x^2$

Answer: D

Explanation:
$x = 8 - \sqrt{32}$ and $y = 2 + \sqrt{2}$,

We have to find the value of $\left(x + \frac{1}{y}\right)^2$

$\left(8 - \sqrt{32} + \frac{1}{2+\sqrt{2}}\right)^2$

$\left(\frac{8(2+\sqrt{2}) - \sqrt{32}(2+\sqrt{2}) + 1}{2+\sqrt{2}}\right)^2$

$\left(\frac{9}{2+\sqrt{2}}\right)^2$

$\frac{81}{6+2\sqrt{2}}$

$\left(\frac{1}{y}\right)^2 = \left(\frac{1}{2+\sqrt{2}}\right)^2 = 6 + 2\sqrt{2}$

$= \left(\frac{2-\sqrt{2}}{2}\right)^2$

$= \frac{6-4\sqrt{2}}{4}$

$= \frac{3-2\sqrt{2}}{2}$

$x^2 = 64 + 32 - 64\sqrt{2}$

$= 96 - 64\sqrt{2}$

$= 32(3 - 2\sqrt{2}) = 32*2y^2$

we get, $x^2 = 64y^2$

$\frac{81}{y^2} = \frac{81}{64}x^2$

D is the correct answer.

Alternative solution,

$$xy = \left(8 - \sqrt{32}\right)\left(2 + \sqrt{2}\right) = 4\sqrt{2}\left(\sqrt{2} - 1\right) \times \sqrt{2}\left(\sqrt{2} + 1\right) = 8\left(2 - 1\right) = 8$$

x

(As $xy = 8 \longrightarrow y = 8$)

$$\left(x + \frac{1}{y}\right)^2 = \left(\frac{(xy+1)}{y}\right)^2 = \left(\frac{(xy+1) \times x}{8}\right)^2 = \left(\frac{(8+1) \times x}{8}\right)^2 = \frac{81}{64} \times x^2$$

Quantitative Aptitude for CAT Questions (download pdf)

Question 46

The number of distinct pairs of integers (m,n), satisfying $\mid 1 + mn \mid < \mid m + n \mid < 5$ is:

Answer:12

Video Solution

Explanation:
Let us break this up into 2 inequations [Let us assume x as m and y as n]

| 1 + mn | < | m + n |

| m + n | < 5

Looking at these expressions, we can clearly tell that the graphs will be symmetrical about the origin.

Let us try out with the first quadrant and extend the results to the other quadrants.

We will also consider the +X and +Y axes along with the quadrant.

So, the first inequality becomes,

1 + mn < m + n

1 + mn - m - n < 0

1 - m + mn - n < 0

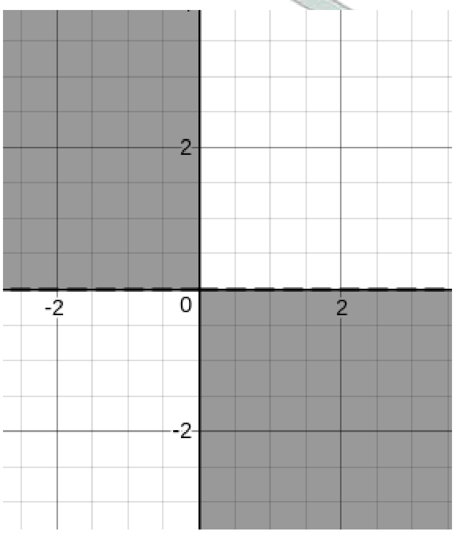
(1-m) + n(m-1) < 0

(1-m)(1-n) < 0

(m - 1)(n - 1) < 0

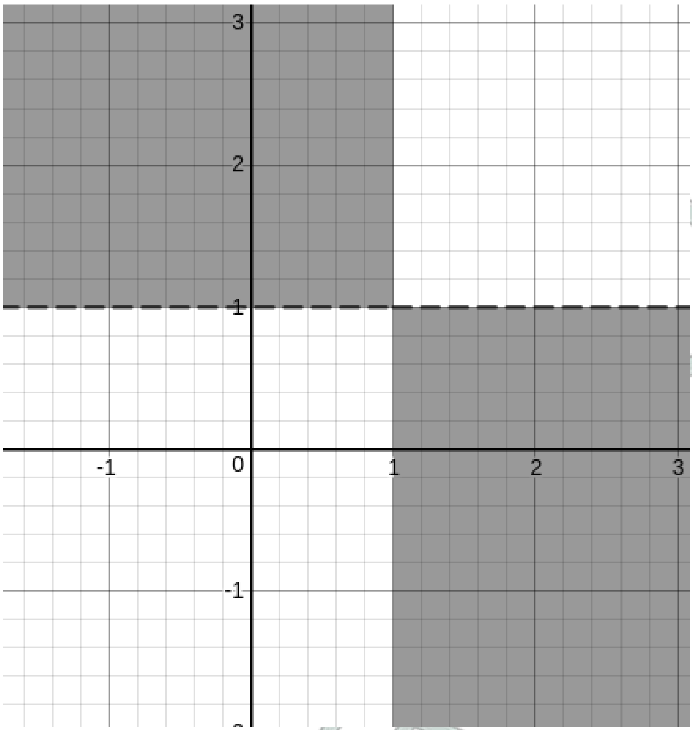
Let us try to plot the graph.

If we consider only mn < 0, then we get

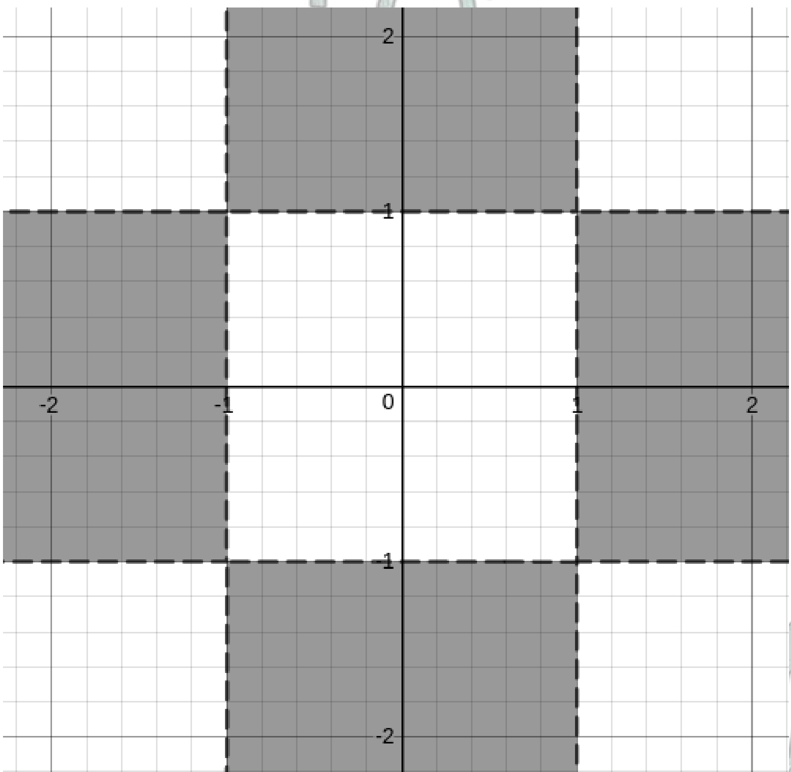


But, we have (m - 1)(n - 1) < 0, so we need to shift the graphs by one unit towards positive x and positive y.

So, we have,



But, we are only considering the first quadrant and the +X and +Y axes. Hence, if we extend, we get the following region.



So, if we look for only integer values, we get

(0,2), (0,3),.......

(0,-2), (0, -3),......

(2,0), (3,0), ...

(-2,0), (-3,0), ...

Now, let us consider the other inequation as well, in which |x + y| < 5

Since one of the values is always zero, the modulus of the other value is less than or equal to 4.

Hence, we get

(0,2), (0,3), (0,4)

(0,-2), (0, -3), (0, -4)

(2,0), (3,0), (4,0)

(-2,0), (-3,0), (-4,0)

Hence, a total of 12 values.

Question 47

If $\log_2 2 - 2\log_{\sqrt{2}} 4 + 3\log_{\sqrt[3]{2}} 8 - \ldots\ldots\ldots\ldots\ldots\ldots\ldots - 10\log_{\sqrt[10]{2}} 1024 = k$**, then what is the value of** $|k|$**?**

Answer:575

Explanation:
$\log_2 2 - 2\log_{\sqrt{2}} 4 + 3\log_{\sqrt[3]{2}} 8 - \ldots\ldots\ldots\ldots\ldots\ldots\ldots - 10\log_{\sqrt[10]{2}} 1024$ can be written as
$\log_2 2 - 2\log_{\sqrt{2}} 2^2 + 3\log_{\sqrt[3]{2}} 2^3 - \ldots\ldots\ldots\ldots\ldots\ldots\ldots - 10\log_{\sqrt[10]{2}} 2^{10}$
=> $\log_2 2 - 2*2\log_{\sqrt{2}} 2 + 3*3\log_{\sqrt[3]{2}} 2 - \ldots\ldots\ldots\ldots\ldots\ldots\ldots - 10*10\log_{\sqrt[10]{2}} 2$
=> we know that $\log_{\sqrt[n]{y}} x = n\log_y x$
Thus, $1 - 2*2*2\log_2 2 + 3*3*3\log_2 2 - \ldots\ldots\ldots\ldots\ldots\ldots\ldots - 10*10*10\log_2 2$
=> $1 - 2^3 + 3^3 - 4^3 + 5^3 - \ldots\ldots\ldots\ldots - 10^3$
=> $1 - 8 + 27 - 64 + 125 - 216 + 343 - 512 + 729 - 1000$
= -575 = k
Thus, $|k| = 575$

Question 48

Ravi has coins of denominations Rs 5 and Rs 16 only. Find the sum of digits of the maximum whole number amount that he cannot pay using these denominations and also the number of such amounts that he cannot pay using the coins he has.

A 12, 25

B 14, 30

C 12, 30

D 14, 25

Answer: B

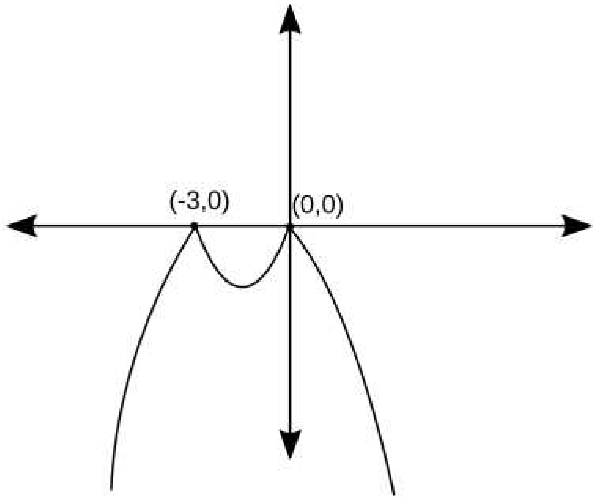
Explanation:
The problem can be re-written as "find the largest whole number that cannot be represented by the linear equation 5x + 16y such that both x and y are non-negative integers".
Here, let's use the Chicken McNugget's Theorem which states that if a and b are positive co-prime integers, then the largest positive integer that cannot be represented by ma + nb is (ab - a - b). The number of such positive integers that cannot be represented by ma + nb is equal to $\frac{(a-1)(b-1)}{2}$.
5x + 16y => Largest amount that cannot be represented by these denominations = (16)(5) - 16 - 5 = 59 => Sum of the digits = 5 + 9 = 14
=> Number of amounts that cannot be represented by these denominations = $\frac{(16-1)(5-1)}{2}$ = 30.
Hence B is the answer.

Alternate Method:
16 = 1(Mod 5) => Any number greater than 16 and of the form 5k+1 can be represented by 5x + 16y
32 = 2(Mod 5) => Any number greater than 32 and of the form 5k+2 can be represented by 5x + 16y
48 = 3(Mod 5) => Any number greater than 48 and of the form 5k+3 can be represented by 5x+16y
64 = 4(Mod 5) => Any number greater than 64 and of the form 5k+4 can be represented by 5x+16y
=> Last 5k+4 number less than 64 cannot be represented by 5x+16y => 59 is the answer.

The number of numbers of the form 5k+1 which cannot be represented is 1, 6, 11. => 3
The number of numbers of the form 5k+2 which cannot be represented is 2, 7, 12 ... 27 => 6
The number of numbers of the form 5k+3 which cannot be represented is 3, 8, 13, ... 43 => 9
The number of numbers of the form 5k+4 which cannot be represented is 4, 9, 14 . . 59 => 12
Therefore, the number of such numbers = 3 + 6 + 9 + 12 = 30

Question 49

If f(x) = x^2 +3x, then which of the following options represents the graph given below?



A **- f(x)**

B **-|f(x)|**

C **- f(-x)**

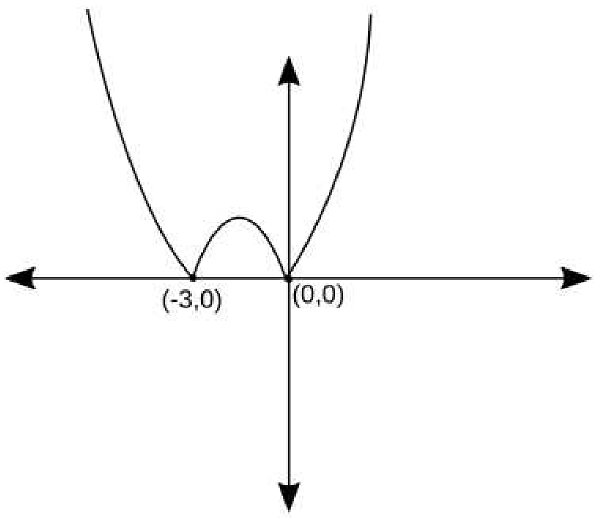
D **|f(x)|**

Answer: B

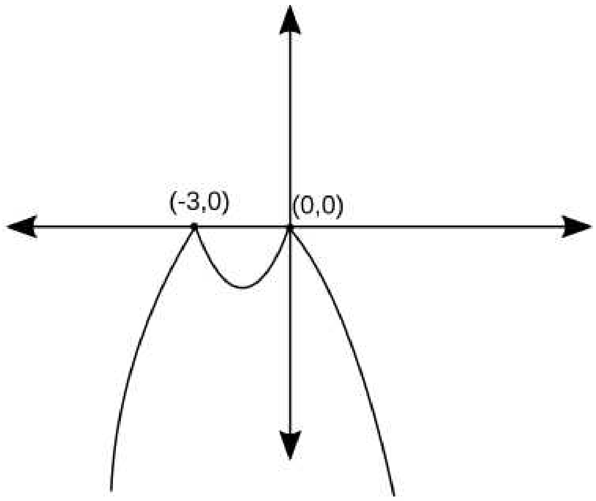
Explanation:
f(x)= x(x+3).

f(x) = 0 at x = 0, -3

The graph of the function |f(x)|:



We can invert the graph by - |f(x)|.



Option B is the correct answer.

Question 50

For real x, if $x^2-5x+\frac{1}{x^2}-\frac{5}{x}-4\le0$. Find the difference of the largest and smallest possible integral value of x.

Answer:4

Explanation:
We have, $x^2-5x+\frac{1}{x^2}-\frac{5}{x}-4\le0$ which can be rearranged as:
$x^2+\frac{1}{x^2}+2-2-5x-\frac{5}{x}-4\le0$
=> $x^2+\frac{1}{x^2}+2-5x-\frac{5}{x}-6\le0$
=> $\left(x+\frac{1}{x}\right)^2-5\left(x+\frac{1}{x}\right)-6\le0$
Assuming $x+\frac{1}{x}=t$
=> $t^2-5t-6\le\ 0$
=> (t-6)(t+1)$\le\ 0$
=> $-1\le\ t\le\ 6$
=> $-1\le\ x+\frac{1}{x}\le\ 6$
Using AM-GM, the minimum value of $x+\frac{1}{x}$ for x>0 will be 2.
=> $x+\frac{1}{x}\ \ge2$
Hence, in $2\le\ x+\frac{1}{x}\le\ 6$
$x+\frac{1}{x}\ \ge2$, will be true for all positive real values of x. =>x>0
The smallest integral value satisfying will be x=1
Also, $x+\frac{1}{x}\ \le6$, the largest value satisfying the equation will be 5 because x=6, then 6+1/6(a positive value) will be greater than 6.
Hence the smallest value integral value satisfying is 1 and the largest integral value satisying is 5.
Difference = 5-1 = 4

Question 51

The average of 33 consecutive 3 digit even numbers increases by 6 if the digits of 30th number in series are reversed. If digits of the 30th term are in strictly increasing or strictly decreasing order and hundreds digits of all numbers are the same, what is the sum of digits of the 3rd term in series?

A 15

B 12

C 21

D 18

Answer: B

Explanation:

Consider average of given numbers = a

Total sum = 33a

Assuming x,y and z are hundreds digit, tens digit and units digit of 30th term of the series respectively.

After reversing the digits, new average = $\frac{33a-(100x+10y+z)+((100z+10y+z))}{33}$ = $a+6$

⇒ a + $\frac{99(z-x)}{33}$ = $a+6$

⇒ $z-x$ = 2

x cannot be 0 as all are 3 digit numbers

Case 1: z= 8 x= 6 y=7 The digits are in strictly increasing or strictly decreasing order.

30th number = 678 =a+2*29 Hence 1st term of series = 620, Last term = a+32*d = 684

Hundreds digits of all numbers are the same.

Case 2: z= 6 x= 4 y=5 The digits are in strictly increasing or strictly decreasing order.

30th number = 456 =a+2*29 Hence 1st term of series = 398, Last term = a+32*d = 462

Hundreds digits of all numbers are not the same.

Case 3: z= 4 x= 2 y=3 The digits are in strictly increasing or strictly decreasing order.

30th number = 234 =a+2*29 Hence 1st term of series = 176, Last term = a+32*d = 240

Hundreds digits of all numbers are not the same.

Only 1st case is possible.

3rd term = a+2*2 = 624

Sum of digits of 624 = 6+2+4 = 12

Question 52

Consider the quadratic equation $4x^2-a^2x+12=0$ where a is a positive integer and at least one root is an integer. Another quadratic equation $x^2-ax-18=0$ has both its roots as integers. The value of a is

Enter -1 if the answer can't be found.

Answer:7

Explanation:
We have, $4x^2-a^2x+12=0$
For at least 1 integral roots, the factors of 12*4 = 48 are taken.
48 = 1*48 = 2*24 = 3*16 = 4*12 = 6*8
Sum of these factors should be a^2
Only two possibilities = 1+48 = 49 and 4+12 = 16
Hence, a=7 or a = 4
$x^2-7x-18\ =\ 0\ or\ x^2-4x-18\ =0$
Consider the first equation: $x^2-7x-18\ =0$ => $x^2-9x+2x-18\ =0$ => (x-9)(x+2)=0 => x=9 or -2 (Both roots are integral)
In the second equation, D = $4^2+4\times\ 18$ = 88 which is not a perfect square. Hence integral roots are not possible.
=> a =7

Question 53

The given system of linear equations has no solution:
9ax+2y=3
4x+(2-a)y=2
Then which of the following is true?

A 3a-4=0

B 3a-2=0

C 3a+2=0

D 3a+4=0

Answer: A

Explanation:
We have, 9ax+2y=3
4x+(2-a)y=2
Since, given system of equation does not have any solution:
$\frac{9a}{4} = \frac{2}{2-a} \neq \frac{3}{2}$
=> 9a(2-a)=8
=> $9a^2 - 18a + 8 = 0$
=> $9a^2 - 12a - 6a + 8 = 0$
=> (2a-3)(4a-3)=0
=> a =2/3 or a=4/3
Now, if we put a=2/3, $\frac{2}{2-a}$ will be equal to 3/2. Hence it will be rejected.
=> a= 4/3

Question 54

Suppose that $a = \frac{2b}{3}$ and $a^b = b^a$. The sum of a and b can be expressed as a rational number $\frac{p}{q}$, where p and q are co-prime positive integers. Find the value of $p+q$?

A 35

B 91

C 47

D 53

Answer: D

Explanation:
Let us assume that $a = kb$ where k can take any positive value.

We are given that $a^b = b^a$

$\Rightarrow (kb)^b = b^{kb}$

$\Rightarrow (k^b) \times (b^b) = (b^b)^k$

$\Rightarrow (k)^b = (b^b)^{k-1}$

Taking b^{th} root each side

$\Rightarrow k = (b)^{k-1}$

$\Rightarrow b = (k)^{\frac{1}{k-1}}$

Bu substituting $k = \frac{2}{3}$

$\Rightarrow b = (\frac{2}{3})^{-3}$

$\Rightarrow b = (\frac{3}{2})^3$

$\Rightarrow b = \frac{27}{8}$

Hence $a = \frac{2}{3} \times \frac{27}{8}$

$\Rightarrow a = \frac{9}{4}$

We can calculate $a + b = \frac{9}{4} + \frac{27}{8}$