B 24

C 18

D 54

Answer: C

Explanation:

$log_3 x \geq \frac{4*log_y 3 - 1}{log_y 3}$

$= log_3 x \geq 4 - \frac{1}{log_y 3}$

$= log_3 x \geq 4 - log_3 y$

$= log_3 x + log_3 y \geq 4$

$= log_3 xy \geq 4$

I.e. $xy \geq 3^4$

I.e. $xy \geq 81$

A.M $\geq$ G.M

Thus, $\frac{x+y}{2} \geq xy^{0.5}$

Thus, $\frac{x+y}{2} \geq 81$

Hence, $x + y \geq 18$

Hence, option C is the correct answer.

Question 63

For how many integral values does the inequality |x-||x-1|+x+3|| <4 satify

A 0

B 1

C 2

D 3

Answer: B

Explanation:

We have, |x-||x-1|+x+3|| <4

=> -4<x-||x-1|+x+3|<4

For x=1

-4<1-|4|<4, Hence x=1 satisfies

For x<1

-4<x-|1-x+x+3|<4

-4<x-4<4 => 0<x<8

=> x ϵ (0,1) Hence no integral value satisfies.

For x>1, -4<x-|x-1+x+3|<4

=> -4<x-(2x+2)<4 => -4<-x-2<4 => -4<x+2<4 => -6<x<2 For x>1 (0 integral values)

Total integral values = 1

Data Interpretation for CAT Questions (download pdf)

Question 64

If $a + 3b + 2c$ = 12 where a, b and c are positive numbers, then the maximum value of $a * b^3 * c^2$ is

Answer:64

Explanation:

AM $\geq$ GM

$\frac{a+b+b+b+c+c}{6} \geq \sqrt[6]{a*b^3*c^2}$

$\frac{12}{6} \geq \sqrt[6]{a*b^3*c^2}$

$2 \geq \sqrt[6]{a*b^3*c^2}$

$\sqrt[6]{a*b^3*c^2} \leq 2$

$a*b^3*c^2 \leq 2^6$

Maximum value is 64 which will occur when $a = b = c$

C is the correct answer.

Question 65

x + y = 8 and P = $5x^2$ + $11y^2$. What is the minimum possible value of P?

A 310

B 237.31

C 110

D 220

Answer: D

Explanation:

We have, x+y=8

x=8-y

Now, P = $5x^2$ + $11y^2$

= $5(8-y)^2$ + $11y^2$

= $320+5y^2-80y+11y^2$

= $(4y-10)^2$ +220

At y = 2.5, $(4y-10)^2$ +220

Hence minimum value = 220

Question 66

Find the smallest integer value of 'n' such that for all m >= n, the value$m^3 - 16m^2 + 81m - 126$ is positive.

Answer:8

Explanation:

The expression $m^3 - 16m^2 + 81m - 126$ can be written as $(m-3)(m-6)(m-7)$

(m - 3)(m - 6)(m - 7) >= 0 for m >= 7 U [3, 6].

So, the smallest integer value of 'n' such that for all m >= n, the value of (m - 3)(m - 6)(m - 7) is positive is n = 8.

Question 67

How many unit squares with integer coordinates are there inside $|x| + |y| = 4$

A 20

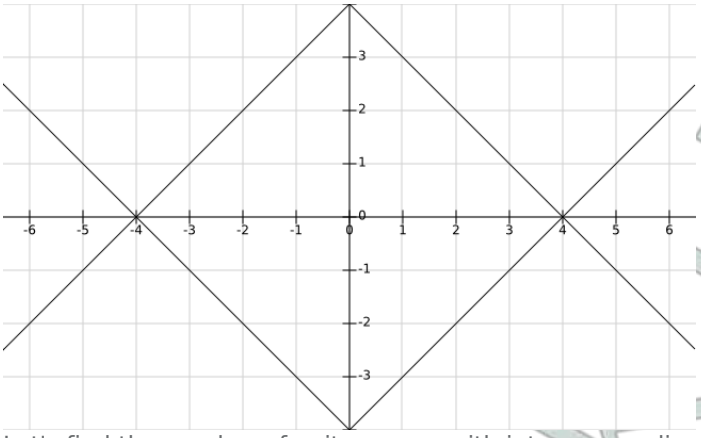
B 16

C 32

D 24

Answer: D

Explanation:
The graph of $|x| + |y| = 4$



Let's find the number of unit squares with integer coordinates in the first quadrant and multiply by 4.

The number of unit squares in the first quadrant = 3 + 2 + 1 = 6.

Hence, there are 6*4 = 24 unit squares.

Question 68

What is the sum of all the roots of the equation
$x^2 - 2x + |x - 1| - 5 = 0$

Answer:2

Explanation:
We have to take two cases:
Case I: When $x \geq 1$, then $|x - 1|$ will be positive
$x^2 - 2x + |x - 1| - 5 = 0$
$=> x^2 - 2x + x - 1 - 5 = 0$
$=> x^2 - x - 6 = 0$
$=> x = 3$ or $x = -2$
But, $x \geq 1$ So, $x = -2$ is neglected.
Case II: When $x < 1$, then $|x - 1| = 1 - x$
$x^2 - 2x + |x - 1| - 5 = 0$
$=> x^2 - 2x - x + 1 - 5 = 0$
$=> x^2 - 3x - 4 = 0$
$=> x = 4$ or $x = -1$
But, $x < 1$ So, $x = 4$ is ignored.
Therefore, the roots of the given equation are 3 and -1 and their sum is 2.
Hence, 2 is the correct answer.

Question 69

For what value of m, will the equation
$m^2(x^2 - x + 1) - m(3x + 1) - 2(2x^2 + x + 3)$ **= 0 have more than two solutions?**

A 1

B -2

C -3

D No such value of m exists

Answer: B

Explanation:
The given equation can be written as
$(m^2-4)x^2-(m^2+3m+2)x+m^2-m-6=0$
It will have more than two solutions when it is an identity.
=>$(m^2-4)=(m^2+3m+2)=m^2-m-6=0$
$(m^2-4)=0$ => $m=2$ or -2
$(m^2+3m+2)=0$ => $m=-2$ or -1
$(m^2-m-6)=0$ => $m=3$ or -2
Since, $m=-2$ satisfies all the equations, for $m=-2$, the given equation will have more than two solutions or infinite solutions.
Hence, option B is the correct answer.

Important Verbal Ability Questions for CAT (Download PDF)

Question 70

If $x^2+4ax+20-2a$ > 0 for all real x, then which of the following holds true for all the values of a?

A a > 0

B $\frac{5}{-2}$ < a< 2

C a>2

D -2 < a< $\frac{5}{2}$

Answer: B

Explanation:
The co-efficient of x^2 is greater than 0, so the discriminant should be less than zero, if the given expression is greater than 0 for all values of x.

$(4a)^2$ - 4*1*(20-2a) < 0

=> $16a^2-80+8a$ < 0

=> $2a^2+a-10$ < 0

=> $2a^2+5a-4a-10$ < 0

=> a(2a+5)-2(2a+5) < 0

=> (a-2)(2a+5)<0

$\frac{-5}{2}$ < a < 2

B is the correct answer.

Question 71

Sumitra has to solve 384 questions. She decides to solve a particular number of questions per day to complete the task in a certain number of days. Shalini, her friend, suggests that if she solves eight more questions per day than what Sumitra has decided, she will take eight fewer days to finish the task than what Sumitra has calculated. In how many days can the task be completed if Sumitra follows what Shalini suggested?

Answer:16

Explanation:
Let the number of questions to be solved per day be x, and the number of days required be y as per Sumitra's plan.

So, $xy=384$

As per Shalini,

$(x+8)(y-8)=384$

or, $xy - 8x + 8y - 64 = 384$

or, $8y - 8x = 64$

or, $y - x = 8$

Putting $y = \frac{384}{x}$, we get

$\frac{384}{x} - x = 8$

or, $384 - x^2 = 8x$

or, $x^2 + 8x - 384 = 0$

On solving, we get $x = -24$ or $x = 16$

But, x cannot be negative.

So, $x = 16$

=> $y = \frac{384}{x} = 24$

According to Shalini, number of days required = $(y - 8) = 16$ days

Hence, 16 is the required answer.

Question 72

'y' years ago, Priti's age was twice her sister's age and '4y' years ago, Priti's age was thrice her sister's age. If it is known that 'y' is a natural number, the difference between their present ages can be

A 20

B 43

C 36

D 25

Answer: C

Explanation:
Let the present age of Priti and her sister be P, Q respectively.

y years ago P-y=2*(Q-y)

P=2Q-y ----------Eq (1)

4y years ago, P-4y=3(Q-4y)

P=3Q-8y ----------Eq (2)

Equating Eq 1 & 2,we get

Q=7y

P=13y

Difference of their ages =13y-7y=6y

Among the given options, only 36 is a multiple of 6.
C is the correct answer.

Question 73

Two sums are formed by alternating the '+' and '*' signs between consecutive odd natural numbers as shown below.

P = 1*3+5*7+9*11+...+2017
Q = 1+3*5+7*9+11*....+2015*2017

What is the remainder when (Q-P) is divided by 1000?

A 512

B 256

C 128

D 64

Answer: C

Explanation:
Q - P = 1 + 3*4 + 7*4 + 11*4 + ... + 2015*4 - 2017
This equals 4*(3+7+11+....+2015) - 2016
This equals 4*504*1009 - 2016
This equals 2034144 - 2016 = 2032128

Hence, the correct answer is 128 which is option (c)

Question 74

The ratio of the sum of n terms of two arithmetic progressions is 1+3/n for all natural numbers 'n'. Then the ratio of the 8th term of both the series is

A 7/5

B 7/6

C 8/7

D 6/5

Answer: D

Explanation:
Assume a1, d1 and a2,d2

Ratio of the sum = $\frac{2a1+(n-1)d1}{2a2+(n-1)d2}$ = 1+3/n

=> $\frac{a1+((n-1)/2)d1}{a2+((n-1)/2)d2}$ =1+3/n .. ..(1)

Now, the ratio of 8th terms = (a1+7d1)/(a2+7d2)

On putting (n-1)/2 = 7 in (1), we can get the required ratio.

=> n=15

(a1+7d1)/(a2+7d2) = 1+3/15 = 6/5

Question 75

The sum of an infinite geometric progression is 5. If all the terms are of the GP are raised to the power 3, the sum of resulting series is 375. Then the common ratio of the GP is

A -1/2

B 1/5

C 2/3

D -2/5

Answer: A

Explanation:
Assuming the first term = a and the common ratio = r.

The sum = a/(1-r) = 5 ....(1)

Now after raising to the power 3, sum = $a^3/(1-r^3)$ = 375 ....(2)

After raising both sides of (1) to the power 3 and dividing by (2), we get

$(1+r+r^2)/(1-2r+r^2) = 1/3$

$=> 3+3r+3r^2 = 1-2r+r^2$

$=> 2r^2+5r+2=0$

=> r=-1/2 or r=-2,

Since GP is infinite, hence r=-2 will be rejected.

Question 76

The sum of 151 terms of a GP is 500 and the sum of 302 terms is 700. Then the sum of 604 terms of the GP is

A 862

B 900

C 812

D 875

Answer: C

Explanation:
Let the common ratio be r.
Hence sum of 302 terms = Sum of 151 terms + r^{151}(Sum of 151 terms)
$=>700=500+ r^{151}(500)$
$=>r^{151}=200/500 = 2/5$
Sum of 604 terms =
Sum of 302 terms + (r^{302})*(Sum of 302 terms)
Sum of 302 terms + $(r^{151}) * (r^{151})$*(Sum of 302 terms)= 700+700(4/25)=700+112=812

Question 77

How many points in the region enclosed by $x \geq 0$, $y \leq 0$ and $7x - 9y \leq 63$ have integral coordinates?

A 41

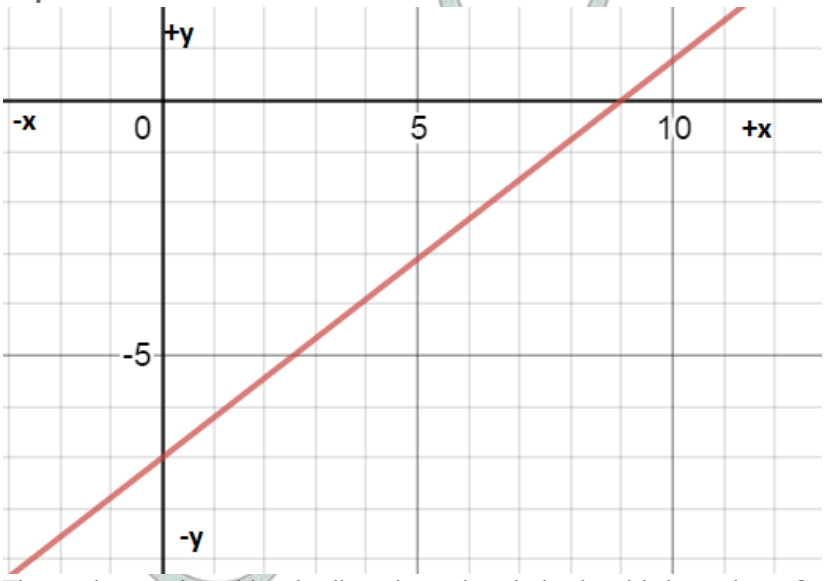
B 36

C 24

D 39

Answer: A

Explanation:



The region enclosed by the lines is a triangle in the third quadrant formed by the points (0,-7), (0,0) and (9,0). The number of coordinates in the region with x coordinate are as follows
x=0 => 8 points,
x=1, 7points,
x=2, 6points,
x=3, 5points,
x=4, 4points,
x=5, 4points and so on.
Total no. of points =41

Question 78

If f(x*y)= f(x)f(y), where x,y >0, and f(36)=16, the value of f(1).f(2).f(3).f(6).f(12).f(18) can be?

A 4096

B 1024

C 256

D 64

Answer: B

Explanation:
f(1*1)=f(1).f(1)

$f(1)=f(1)^2$

Thus f(1)= 1 ...(f(1) cannot be 0 as then other values will become 0 because f(n)=f(1).f(n)

f(36)=f(12)f(3)=16

f(36)= f(18).f(2)=16

f(36)=f(6).f(6)=16

f(6)=+4,-4

Hence, one of the possible values is f(1).f(2).f(3).f(6).f(12).f(18) = 1x16x16x4=1024

Question 79

A quadratic function f(x) has a value 5 at x=-2. If the maximum value of the function is 8 at x=-1, then f(-3) is

A 3

B -6

C -4

D 5

Answer: C

Explanation:
Assuming the quadratic function to be f(x) = ax^2 +bx+c

We have, f(-2) = 4a-2b+c = 5

f(-1) = a-b+c = 8

=> 3a-b=-3 .....(1)

Now the maximum value will occur at -b/2a = -1

=> b=2a .....(2)

From (1) and (2), we get

3a-2a=-3 =>a=-3

b=3a+3 = -9+3=-6

c = 8+b-a = 8-6+3 = 5

Therefore f(-3) = 9a-3b+c = 9*-3-3*-6+5 = -4

Alternate Solution:

As the quadratic equation reaches its maximum when x=-1, it is of the form a(x+1)^2+c.
As this maximum value equals 8, the value of c = 8. Hence, the quadratic equation is of the form a(x+1)^2+8
The value of this function when x=-2 is 5. Hence, a+8=5 or a=-3
So, the quadratic equation is -3(x+1)^2+8.
When, x=-3, it equals -3*4+8 = -12+8 = -4

Question 80

If p(x) = min(8-x, x-7)+1, q(x)=max(6-x,x-9)-1. Find the area of region bounded by p(x) and q(x).

A 16

B 8

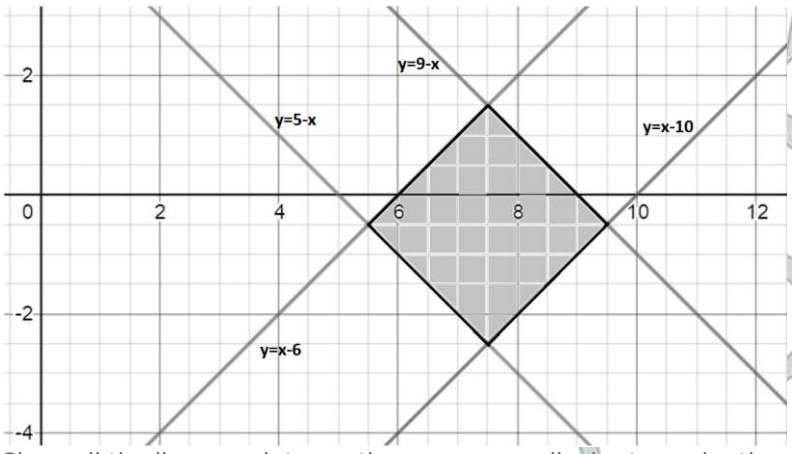
C $8\sqrt{2}$

D $6\sqrt{2}$

Answer: B

Explanation:
p(x) = min(8-x, x-7)+1 and q(x)=max(6-x,x-9)-1

=> p(x) = min(9-x, x-6) and q(x)=max(5-x,x-10)



Since all the lines are intersecting are perpendicular to each other, the area bound will be rectangular.

The distance between parallel lines x+y=9 and x+y=5 is equal to $|\frac{9-5}{\sqrt{1^2+1^2}}|=2\sqrt{2}$

The distance between parallel lines x-y= 6 and x-y=10 is equal to $|\frac{10-6}{\sqrt{1^2+1^2}}|=2\sqrt{2}$

Hence the area will be $2\sqrt{2}*2\sqrt{2}$ = 8

Question 81

If f(n)= 6+4g(n), where g(n) = minimum of {4n+3, 24-n}. Find number of possible integral value of n if f(n) > 0?

A 7

B 27

C 15

D 13

Answer: B

Explanation:

g(n)=min(4n+3, 24-n)

Both are the equation of a line,

assume at x both are equal g(x_1)= 4x+3

g(x_2)= 24-x

4x+3=24-x

x=4.2

Case 1

For n>4.2

g(n)= 24-n

f(n)=6 + 4(24-n)

f(n)= 102-4n

f(n)>0

102-4n>0

$\therefore$ n$\leq$25

Case 2

n<4.2

g(x)= 4n+3

f(n)= 6+4(4n+3)

f(n)>0

n > -18/16

Hence every integer from {-1, 25} both inclusive will give f(n)>0

Hence 27 integers is the correct answer.

Question 82

A quadratic function f(x) is defined such that x^2-6x+4 $\leq$ f(x) $\leq$ $2x^2$-12x+13. If f(7)=19, then the value of f(9) is

A 39

B 49

C 41

D 29

Answer: B

Explanation:

x^2-6x+4 = $(x-3)^2$-5

$2x^2$-12x+13 = $2(x-3)^2$-5

Both the functions reach their minimum value i.e. -5 at x=3

Hence, f(x) should also be -5 at x=3 =>f(3)=-5

The equation f(x) should be of form $k(x-3)^2$-5.

Now f(7)=19, $k(4)^2$-5=19 =>k=3/2=1.5

f(9)=$1.5(9-3)^2$-5=1.5*36-5=54-5 = 49

Question 83

If the function f(x) is defined for all the positive values of x and y such that f(x*y)=f(x)+f(y) and f(4)=8. Then the value of f(8)+f(16)+f(32)+.....+f(128)+f(256) is

A 136

B 120

C 132

D 108

Answer: C

Explanation:

f(4)=(2*2)=f(2)+f(2)

f(4)=2*f(2)=8

∴ f(2)=4

f(2*1)=f(2)+f(1)

∴ f(1)=0

f(8)=f(4*2)

=f(4)+f(2)

=2*f(2)+f(2)=3*f(2)

f(16)=f(4*4) = f(4)+f(4)

=4*f(2)

Similarly, f(32) = 5f(2), f(64)=6f(2), f(128)=7f(2), f(256)=8f(2)

f(8)+f(16)+f(32)+f(64)+f(128)+f(256)

= 3*f(2)+4*f(2)+....8*f(2)

=33*f(2)

=33*4=132

C is the correct answer.

Alternate Explanation:

Using Cauchy's Functional Equation: The solution of f(x*y)=f(x+y) is f(x) = $log_k x$

Hence, f(4) = $log_k 4$ = 8 =>2$log_k 2$=8 => $log_k 2$ = 4

We have, f(8)+f(16)+f(32)+.....+f(128)+f(256) = $log_k 8$ + $log_k 16$ + $log_k 32$ + ...+ $log_k 256$

=> 3$log_k 2$ + 4$log_k 2$ + 5$log_k 2$ + ...+ 8$log_k 2$ = (3+4+5+....+7+8)*$log_k 2$ = 33*4=132

Question 84

If f(x)=$\frac{9^x}{9^x+3}$, then the value of f(1/99)+f(2/99)+f(3/99)+.....f(98/99) is

Answer:49

Explanation:

f(1-x)= $\frac{9^{1-x}}{9^{1-x}+3}$ = $\frac{9}{3*9^x+9}$ = $\frac{3}{9^x+3}$

f(1-x)+f(x)=1

Hence f(1/99)+f(98/99)=1

Since 49 such pairs are there, sum = 49

How to prepare for Data Interpretation for CAT

Question 85

If a function F(x) is defined such that F(x)+F($x-1$)=x^2 and F(10)=2019. Then the value of F(52).

Answer:3342

Explanation:

F(x)+F($x-1$)=x^2

F(11)+F(10)=11^2 ----- Eq (1)

F(12)+F(11)=12^2 ----- Eq (2)

Eq(2) - Eq(1)

F(12)-F(10)=12^2 - 11^2 ----- Eq (3)

F(13)+F(12)=13^2 ----- Eq (4)

F(14)+F(13)=14^2 ----- Eq (5)

Eq(5) - Eq(4)

F(14)-F(12)=14^2 - 13^2 ----- Eq (6)

Adding Eq (6) and Eq(3), we get

F(14) - F(10)=14^2 - 13^2 + 12^2 - 11^2

Similarly, F(52) - F(10) = 52^2 - 51^2 + 50^2 - 49^2.....+ 12^2 - 11^2

F(52) = $(52+51)(52-51)+(50+49)(50-49)+$.....$(12+11)(12-11)$ + F(10)

$=52+51+50+49+$ .....$12+11$+2019

=1323+2019

=3342

3342 is the correct answer.

Question 86

Find the area of the region bounded by the curve $f(x)=|x+3|+|x+7|$ and the line $y=6$.

A 10 Sq. units

B 24 Sq. units

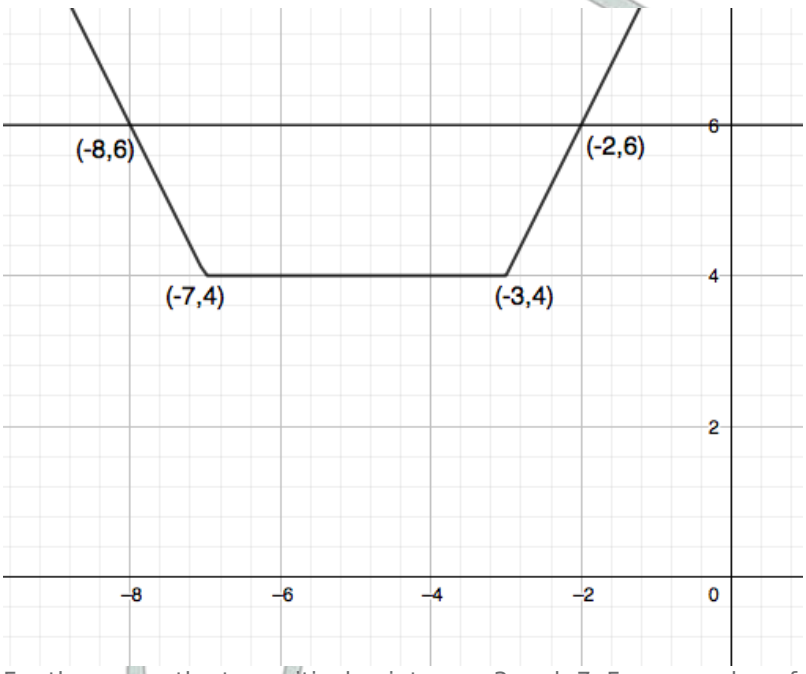
C 12 Sq. units

D 16 Sq. units

Answer: A

Explanation:

Below is the graph for the given curve and line:



For the curve, the two critical points are -3 and -7. For any value of 'x' between -3 and -7, the value of y will always be 4. The enclosed area is that of a trapezium with two parallel bases of length 4 and 6 units and height 2 units.

$\therefore$ area of bounded region = $\frac{1}{2}\times(4+6)\times 2$ Sq. units = 10 Sq. units.

Hence, option A is the correct answer.

Question 87

A = {1, 2, 4, 5, 7, 8, 9}, B = {2, 3, 5, 7, 8, 9}
P#Q is defined as a set of all those elements which are either in P or in Q, but not in the both.
P@Q is defined as a set of all those elements which are present in both P and Q.
How many elements will (A@B)#(A#B) have?

A 9

B 8

C 7

D 4

Answer: B

Explanation:
A = {1, 2, 4, 5, 7, 8, 9}
B = {2, 3, 5, 7, 8, 9}
A@B = {2, 5, 7, 8, 9}
A#B = {1, 3, 4}
(A@B)#(A#B) = {1, 2, 3, 4, 5, 7, 8, 9}
There, there are 8 elements in (A@B)#(A#B).
Hence, option B is the correct answer.

How to prepare for Quantitative aptitude for CAT

Question 88

A function f(n) can be written as f(n)-f(n-1) = n-1 for all n which are integers and greater than 1. If f(1) = 1, find f(50).

A 1251

B 1275

C 1201

D 1226

Answer: D

Explanation:
f(2)-f(1) = 1
f(2) = 2
f(3)-f(2) = 2
f(3) = 4
f(4) = 7, f(5) = 11 and so on.
The series is 1,2,4,7,11,16 and so on in which the dofference of the series is in AP.
S = 1+ 2 + 4 + 7 +11+.....+f(50)
S = 1 + 2 + 4 + 7+.....+f(49)+f(50)
Substract both the equations
0 = 1+1+2+3+4+....+49-f(50)
f(50) = 1+(49*50/2) = 1225+1 = 1226

Alternatively,

We know that the series is 1,2,4,7,11,16...
We know that the series 1,(1+2),(1+2+3)... is 1,3,6,10,15
If we compare the 2 series, If we add 1 to second series, it is equivalent to first series from second term.
Hence f(50) = 1+(49*50/2) = 1225+1 = 1226

f(50) = 1+(49*50/2) = 1225+1 = 1226

Question 89

If $2 < a < 4$ and [a] = The greatest integer less than a, What is the probability (p) of $[a^2] = [a]^2$?

A 0.75<p<1

B 0.5<p<0.75

C 0.25<p<0.5

D 0<p<0.25

Answer: D

Explanation:

Given function is $[a^2] = [a]^2$
Let us consider RHS
If $[a]^2$ = 4
then $2 \leq a < 3$
If $[a]^2$ = 9
Then $3 \leq a < 4$

Let us consider LHS
If $[a^2] = 4$
Then $2 < a \leq \sqrt{5}$

If $[a^2] = 9$
Then $3 \leq a < \sqrt{10}$
Hence, the range satisfying $[a^2] = [a]^2$ is $(2, \sqrt{5})$ and $[3, \sqrt{10})$
Required probability = $\frac{\sqrt{5}-2+\sqrt{10}-3}{4-2}$ = $\frac{2.23-2+3.16-3}{2}$ = $\frac{0.39}{2}$ ~0.2
Hence, option D is the correct answer.

Question 90

Two functions A(x) and B(x) are such that $4A^2(x) - 2B(x)B(-x) = B^2(x) + B^2(-x)$. If $A(4) = 24$ what is the value of $A(-4)$?

A 24

B -24

C 32

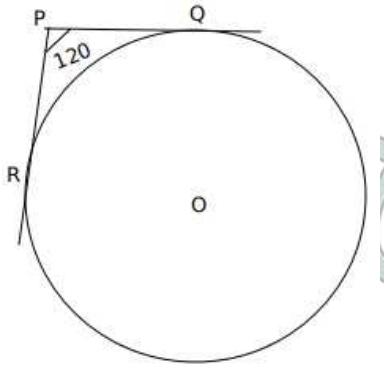
D Cannot be determined

Answer: D

Explanation:
$4A^2(x) - 2B(x)B(-x) = B^2(x) + B^2(-x)$
$=> 4A^2(x) = [B(x) + B(-x)]^2$
$=> 4A^2(x) = 4A^2(-x)$
$=> A(x) = \pm A(-x)$
$=> A(-4) = \pm A(4) = \pm 24$

Question 91

In the figure given below, O is the centre of the circle of radius 5cm. Two tangents are drawn from an external point P. What is the length of QR (in cm) if QPR is 120^0



Answer:5

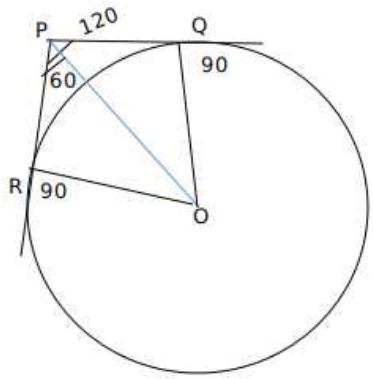
Explanation:

The length of the tangent drawn from an external point to a circle is equal.

length of PQ=length of PR

The angular bisector of angle QPR passes through the centre of the circle.

The tangent to the circle makes right angle with the radii of the circle.



Therefore the $\angle POQ$ =180-(60+90)

$=30^0$

Angle QOR = 2*$\angle POQ$

$=60^0$

In the triangle QOR,the lengths of QO and OR are equal. So the angles opposite to them should be equal.

$\angle RQO + \angle QRO = 180^0 - 60^0 = 120^0$

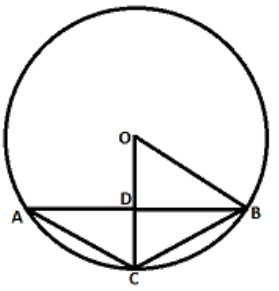
$\angle RQO = \angle QRO = 60^0$

$\therefore$ Triangle QOR is an equilateral triangle, hence the length of QR = 5 cm

5 is the correct answer.

Question 92

In given circle, O is the center, OB and OC are the radii. It is given that OD:DC = 2:3, angle BDC=105^0 and AC=11.52 cm. If the radius of the circle is equal to 10 cm, find the area of the triangle BOC (Take sin75=0.96).



A $25\sqrt{3}$

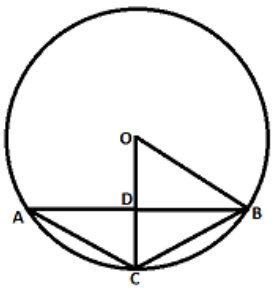
B 50

C 48

D $25\sqrt{2}$

Answer: A

Explanation:



From question, OD:DC = 2:3, OD=4 cm, DC = 6cm

$\angle BDC = 105 => \angle ADC = 180 - 105 = 75$

In the circle, using sin rule in triangle ADC, $\frac{AC}{sin\angle ADC} = \frac{DC}{sin\angle DAC}$

=> $\frac{11.52}{0.96} = \frac{6}{sin\angle DAC}$

=> $sin\angle DAC = 0.5$

$\angle DAC = 30^0$,

Now $\angle DAC$ and $\angle BOC$ are subtended from same chord, $2\angle DAC = \angle BOC$

$\angle BOC = 2 \times 30^0 = 60^0$

Area of $\triangle BOC = \frac{1}{2} \times 10 \times 10 \times sin60^0 = 25\sqrt{3}$

A is the answer.

Question 93

If the values of inradius and circumradius of a right-angled triangle are 3 cm, 8.5 cm. Then the area of the triangle(in cm^2) is

A 60

B 30

C 45

D Cannot be determined

Answer: A

Explanation:

Let a,b,c be the base, height and hypotenuse of the triangle

We know that inradius r= $\frac{a+b-c}{2}$

r= $\frac{a+b+c-2c}{2}$

r= $\frac{2s-2c}{2}$

r=s-c

r=s-2R

r+2R=s

we know that r= $\frac{Area\ of\ the\ triangle}{s}$

Area of the triange =r(r+2R)

On substituting the values,we get

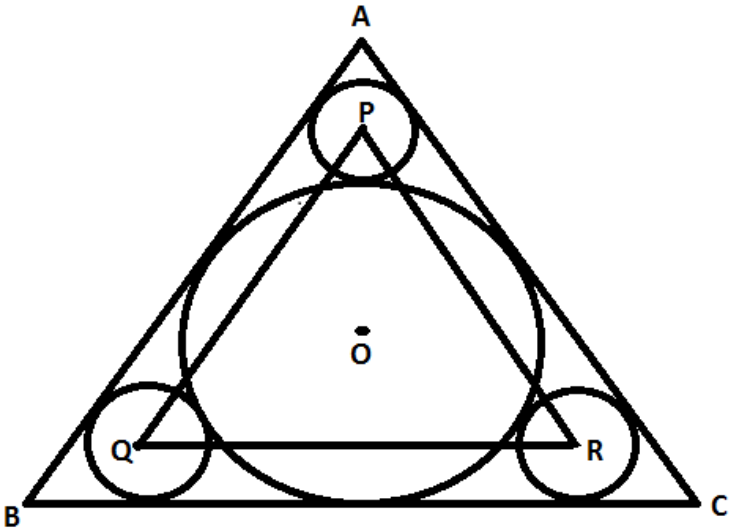
=3(3+17)

=60

A is the correct answer.

Question 94

A circle with radius 6 cm is inscribed inside an equilateral triangle ABC. Three smaller circle are drawn touching the incircle and the sides of ABC as shown in the figure. Another triangle is formed by joining centres P,Q and R of these smaller circles. What is the perimeter of triangle PQR?



A $24\sqrt{3}$

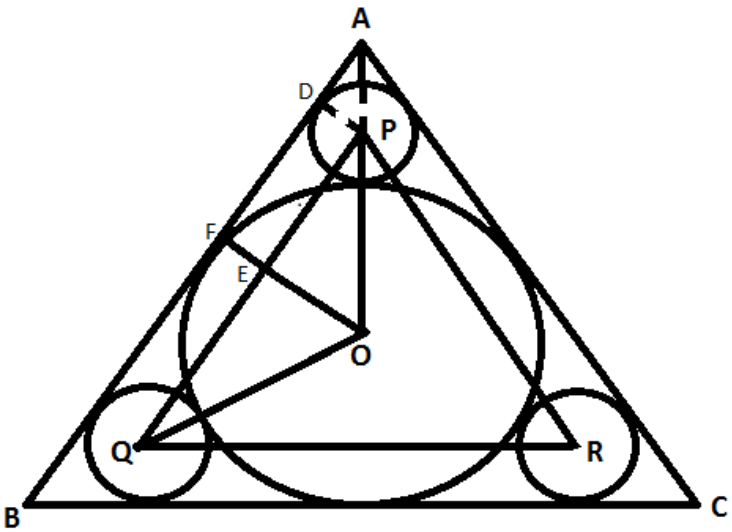
B $36\sqrt{3}$

C $8\sqrt{3}$

D $12\sqrt{3}$

Answer: A

Explanation:



Consider radius of each smaller circle be r and that of the larger circle be R = 6 cm.

Construct PD such that PD is perpendicular to AB.

In right triangle ADP, AP = DP/cos(APD) = DP/ cos(60) = 2DP = 2r

In right triangle OFA, OF/cos(AOF) = AO => R/(1/2)=AP+OP =>2R=2r+r+R => R=3r

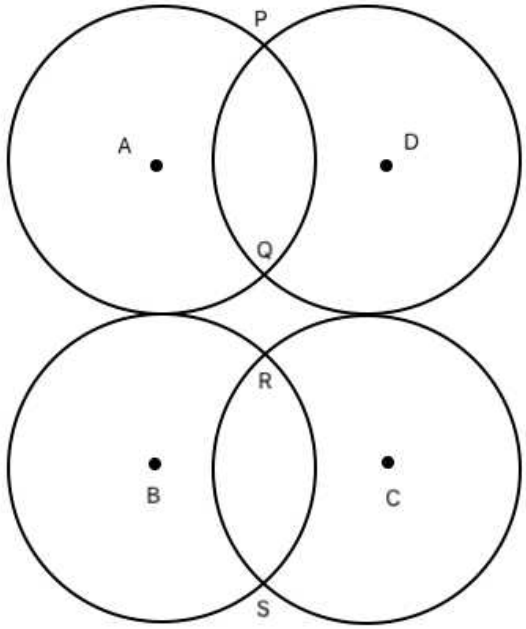
In right triangle OEP, EP=OPcos(EPO) = (R+r)(($\sqrt{3}$)/2) = (4R/3)(($\sqrt{3}$)/2) = 2R/($\sqrt{3}$)

PQ=2EP = 4R/($\sqrt{3}$)

Perimeter = 12R/($\sqrt{3}$) = $24\sqrt{3}$ cm (R= 6cm)

Question 95

Four circles of radius 2cm each are arranged as shown in the figure. A, B, C and D are the centres of the given circles. Also, $\angle PAQ = 60°$. The given figure is symmetrical. Find out the area bounded by the circles that does not lie inside any of the circles.



A $4\sqrt{3} + \frac{3\pi}{2}$

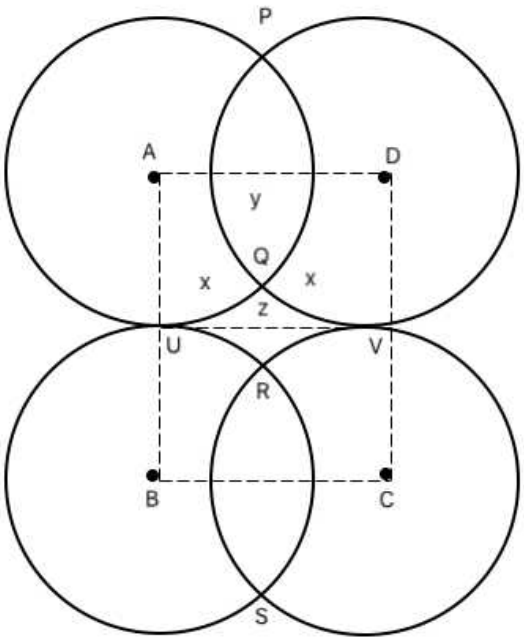
B $3\sqrt{3} - \frac{4\pi}{3}$

C $4\sqrt{3} - \frac{3\pi}{2}$

D $6\sqrt{3} - \frac{8\pi}{3}$

Answer: D

Explanation:



Since, the given figure is symmetrical so, areas denoted by 'x' are equal.
In $\triangle APD$, $\angle PAD = \angle PDA = 30^\circ$ & $\angle APD = 120^\circ$
$AP = PD = 2$cm & $AD = 2\sqrt{3}$cm (Property of $30^\circ - 30^\circ - 120^\circ$ triangle)

Now, $2x + y + z = 2 \times 2\sqrt{3} = 4\sqrt{3}$ _____(1)
$x + y = \frac{1}{4} \times \pi \times 2^2 = \pi$ _____(2)

So, $x + z = 4\sqrt{3} - \pi$

Area of segment $PQ = \frac{60^\circ}{360^\circ} \times \pi \times 2^2 - \frac{1}{2} \times 2^2 \times sin(60^\circ)$
$= \frac{2\pi}{3} - \sqrt{3}$

So, $y = \frac{2\pi}{3} - \sqrt{3}$

From (2)
$x = \pi - y = \pi - \frac{2\pi}{3} + \sqrt{3} = \frac{\pi}{3} + \sqrt{3}$

So, $z = 4\sqrt{3} - \pi - x$
=> $z = 3\sqrt{3} - \frac{4\pi}{3}$

Required area = $2z = 6\sqrt{3} - \frac{8\pi}{3}$

Hence, option D is the correct answer.

Question 96

An isosceles trapezium circumscribed over a circle has one of its parallel sides thrice the other. If the perimeter of the trapezium is 16 cms. The area of the trapezium in sq cm is

A $4\sqrt{3}$

B 8

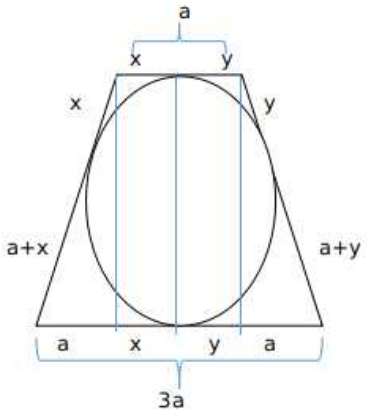
C $8\sqrt{3}$

D $16\sqrt{3}$

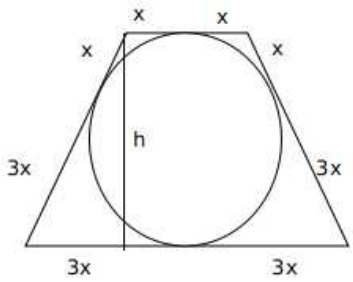
Answer: C

Explanation:
Let the length of one of the parallel sides = 2x

Length of other parallel side = 6x



Since it is isosceles trapezium, the non parallel sides are equal
a+2x=a+2y
∴ x=y
a=2x



Given Perimeter = 16 cms

16x=16

x=1

Let h be the length of the height dropped to the base

$(4x)^2=h^2+(2x)^2$

h=$2\sqrt{3}$

Area of the trapezium= $\frac{1}{2}$*height*(Sum of length of the parallel sides)

$=\frac{1}{2}*2\sqrt{3}*(8)$

$=8\sqrt{3}$

Hence C is the correct answer.

Question 97

A point on the circumference of a semicircle is joined with the endpoints of the diameter of the semicircle. It is found that the sides of the triangle so formed are in an arithmetic progression. If it is known that the length of the sides of the triangle are integers, which of the following can be the perimeter of the semicircle? (Take $\pi = \frac{22}{7}$)

A 120 units

B 140 units

C 160 units

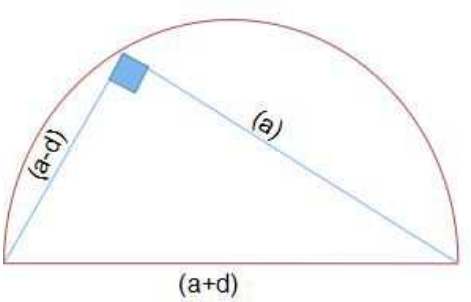
D 180 units

Answer: D

Explanation:

It has been given that a point on the circumference of the circle is joined with the end points of the semicircle. Therefore, the triangle so formed should be a right-angled triangle (Since the angle subtended by the diameter of the circle on the circumference is 90°.

It has been given that the sides of the triangle are in an arithmetic progression. Let us assume the sides to be $a-d, a,$ and $a+d$ units. $a+d$ must be the length of the hypotenuse of the triangle.



Applying Pythagoras theorem, we get,
$(a+d)^2 = a^2 + (a-d)^2$
$a^2 + d^2 + 2ad = a^2 + a^2 + d^2 - 2ad$
$4ad = a^2$
$4d = a$

Therefore, the 3 sides of the triangle will be of the form $3d, 4d$ and $5d$.
$5d$ is the diameter of the semicircle.
=> Radius of the semicircle = $2.5d$

Perimeter of the semicircle = $\pi * r + 2r$
$= \frac{22}{7} * r + 2r$
$= r * \frac{36}{7}$
$= 2.5d * \frac{36}{7}$
Perimeter of the semicircle = $\frac{90*d}{7}$ units.

We know that 'd' has to be an integer. Therefore, the perimeter has to be a multiple of 90/7. Only option D satisfies this condition and hence, option D is the right answer.

Question 98

A flight starts from Delhi at 9:00 am local time and reaches Dubai at 1 am local time. The same flight starts from Dubai at 3pm local time and reaches Delhi and 3 am local time. If the flight undertook both the journies at the same speed, find the time difference between Dubai and Delhi?

A 3 hours

B 1.5 hours

C 2 hours

D 2.5 hours

Answer: C

Explanation:
Let Delhi be x hours ahead of Dubai.

Time taken ignoring time difference = 24 hours - (9am - 1am) = 16 hours

Actual time taken = 16 + x

Time taken from Dubai to Delhi ignoring time difference = 3 am - 3 pm = 12 hours

Actual time taken = 12 - x

As the same distance is covered in each case at same speed, time taken would be the same.

Hence, 16 + x = 12 - x

x = -2 hours

Hence, Dubai is ahead of Delhi by 2 hours.

Question 99

When dropped from a height 'h', balls of Type 1 bounce to height 2h/3 while balls of Type 2 bounce to height h/2. Two balls, one of Type 1 and one of Type 2, are dropped from 12m and 36m respectively. Then what is the sum of the total distance (in metres) they will travel after bouncing indefinitely?

Answer:168

Explanation:
Initially, the distance traveled by Type 1 ball from the drop till the ground = 12m

The ball will bounce back two-third of the height= $\frac{2\times12}{3}$

The distance it will cover after the first bounce = $2\times\frac{2\times12}{3}$

The total distance covered by Type 1 ball= $12+2\times\frac{2\times12}{3}+2\times\frac{2}{3}\times\frac{2\times12}{3}+\ldots$

The total distance covered by Type 1 ball= $12+2\times12\{\frac{\frac{2}{3}}{1-\frac{2}{3}}\}$

The total distance covered by Type 1 ball= 60m

Similarly,

The total distance covered by Type 2 ball= $36+2\times\frac{1\times36}{2}+2\times\frac{1}{2}\times\frac{1\times36}{2}+\ldots$

The total distance covered by Type 2 ball=108m

Total distance they will travel= 108+60=168m

Question 100

Anand and Mahesh are travelling from place A to place B which are 100 km apart. Anand stops for 10 minutes after each km and Mahesh stops for 25 minutes after each 4 km. The ratio of speed of Anand and Mahesh is 11:2. From A, Anand started 1 hour later than Mahesh and both reach B at the same time. Find the time taken(in minutes) by Anand to travel from A to B.

A 890

B 1090

C 1190

D 1200

Answer: B

Explanation:
Consider the velocity of Anand 11v and the velocity of Mahesh be 2v.
Then time taken used in traveling(without break) by Anand = 100/(11v)
Total breaks taken by Anand = 99 Time spent in breaks = 99*10 =990
Total time taken by Anand = 100/(11v)+ 990
Time taken used in traveling (without break) by Mahesh = 100/(2v)
Total breaks taken by Mahesh = 24 Time spent in breaks = 25*24 = 600
Total time taken by Mahesh =100/(2v)+600

Since both reach at same time, 100/(11v)+ 990 + 60 = 100/(2v)+600 (Anand started 60 minutes late)
=>100/(2v)- 100/(11v)=450
=> 100/v = 50*22 = 1100

Time taken by Anand = 100/(11v)+ 990 = 100+990 = 1090 minutes

Question 101

A and B start from a point P on a circular track. Both move in the opposite direction such that ratio of their speeds is 7:11. If circumference of the 44 meters. What is the shortest distance between the points where they meet for 5th time and the point where they meet for the 8th time?

A 7

B $7\sqrt{3}$

C 3.5

D $3.5\sqrt{3}$

Answer: A

Explanation:
Circumference = 2π= 44

=> r= 7 m

Divide the circumference into 7+11 =18 equal parts.

Then they will meet for the first time when A moves 7 parts in his direction and B moves 11 parts in the opposite direction.

Now consider the movement of A, A moves 7 parts between two points, hence between 5th and 8th meeting point he will travel (8-5)*7 = 21 parts.

Now the distance along the circumference between two points = 21 mod 18 = 3 parts

Now 3 parts will subtend angle (3/18)*360 = 60 at the centre.

Therefore the radii joining two points and the line joining two points will form an equilateral triangle.

Hence the distance between two points = radius = 7m

Question 102

Muthu starts from point A on a circular track with speed 10m/s and meets his friend Vivek after 15 seconds. Now, Muthu and Vivek travel in opposite directions and Vivek reaches Point A in 10 s. Vivek then meets Muthu exactly 14 seconds after crossing point A. Find the circumference of the circular track.

A 600m

B 450m

C 800m

D 350m

Answer: A

Explanation:
Muthu meets Vivek after 15 seconds with speed 10m/s. Distance covered is 150m.

Vivek travels till point A in 10s. Hence, Vivek travels at a speed of 15m/s.

When Vivek reaches A, Muthu would have covered a Distance of 100m.

They meet together after 14 seconds. Relative speed is 25m/s. The total distance covered is 25*14=350m.

Circumference= 150+100+350=600m

Question 103

If A beats B by 300 meters in a 1.5 km race and B beats C by 2 minutes 30 seconds in the same race. What is the sum of speeds of A and C if A beats C by 6 minutes 40 seconds in a 3 km race?

A 12.75 m/s

B 11.25 m/s

C 9.75 m/s

D 13.5 m/s

Answer: B

Explanation:

Consider speeds(m/s) of A, B and C be a,b and c respectively.

If A beats B by 300 meters in a 1.5 km race,

(1500/a)=(1200/b)

or, (a/b)=5/4.....(1)

If B beats C by 2 minutes 30 seconds (=150 seconds)

(1500/c)=(1500/b)+150

(1/c)=(1/b)+(1/10).....(2)

If A beats C by 6 minutes 40 seconds(=400 seconds) in a 3 km race,

(3000/a)+(400)=(3000/c)

(1/a)+(1/7.5)=(1/c).....(3)

From 1,2 and 3, we get a=7.5 m/s b=6 m/s and c=3.75 m/s

Sum of speeds of A and C= 7.5+3.75=11.25 m/s

B is the answer.

Question 104

2 trains pass through a tunnel at an equal speed of 10 m/s. The first train takes twice as much time as the second train to cross the tunnel completely. The trains can cross each other completely in 2 minutes if they are travelling in the opposite directions on parallel tracks. How much time (in seconds) will a train thrice the length of the shorter train take to cross the tunnel travelling at the same speed as these 2 trains?
(Enter 0 if the answer cannot be determined)

Answer:240

Explanation:
We know that the 2 trains travel with the same speed.
Let the length of the tunnel be T m.
Let the length of the shorter train be 'x' and the length of the longer train be 'y'.

The 2 trains cross each other completely in 2 minutes (120 seconds) if they are travelling on opposite tracks.
When 2 trains travel in the opposite directions, the total distance that should be traveled by the 2 trains to cross each other completely will be equal to the sum of the length of the trains.

We know that both the trains travel at 10 m/s. Since the trains are moving in the opposite directions, the relative velocity is 10+10 = 20 m/s.

Sum of the lengths of the trains, x+y = 120*20
=> x+y = 2400 m
y = 2400-x

It has been given that the longer train takes twice as long as the shorter train to cross the tunnel.
Distance traveled by a train to completely cross a tunnel = Length of the train + length of the tunnel.

2*(T+x)/10 = (T+2400-x)/10

2T+2x = T-x+2400
T+3x = 2400 m

We have to find out the time taken by a train thrice as longer as the shorter train to cross the tunnel at the same speed as these 2 trains. Therefore, we have to find the time taken by a train of length 3x to cross the tunnel at 10 m/s.

A train of length 3x will have to cover a distance of T+3x to cross the tunnel completely.
We know that T+3x = 2400 m
=> Time taken to cross the tunnel = 2400/10 = 240 seconds.

Therefore, 240 is the right answer.

Question 105

Akhilesh and Mamata left from Lucknow and Kolkata towards Kolkata and Lucknow respectively. They took the same route and started simultaneously. After meeting each other on the way, Mamata took another 8 hours to reach her destination, while Akhilesh took 18 hours to reach his destination. If the speed of Akhilesh is 40 km/hr then find out the speed(in km/hr) of Mamata.

Input -1 if the answer can not be determined

Answer:60

Explanation:
It is given that they started simultaneously and took the same route. Hence, we can say that they must have started $\sqrt{18 * 8} = 12$ hours earlier from the meeting point.

Akhilesh took another 18 hours to complete the remaining distance. Hence, total time taken by Akhilesh to complete entire journey = 18 + 12 = 30 hours.

Mamata took another 8 hours to complete the remaining distance. Hence, total time taken by Mamata to complete entire journey = 8 + 12 = 20 hours.

We know the speed of Akhilesh. Hence, the distance between Lucknow and Kolkata = 30*40 = 1200 km

The same distance is covered by Mamata in 20 hours hence the speed of Mamata = $\frac{1200}{20}$ = 60 km/hr

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Question 106

Anunay is climbing up the moving escalator that is going up. He takes 90 steps to reach the top while Vineet is coming down the same escalator. The ratio of the speed of Anunay and Vineet is 5:7. If both of them start together and take the same time to reach the other end of the escalator then find out the number of steps in the escalator.

Answer:108

Explanation:
Let the speed of Anunay and Vineet be '$5x$' and '$7x$' steps per second respectively.
Let escalator's speed be "y" steps per second.
Let "t" be the time taken to reach at the other end by Anunay and Vineet.
Total equivalent number of steps taken by Anunay considering the help of escalator = $5xt + yt$
Total equivalent number of steps taken by Vineet considering the resistance from escalator = $7xt - yt$

On equating the number of steps, we get
$\Rightarrow 5xt + yt = 7xt - yt$
$\Rightarrow 2xt = 2yt$
$\Rightarrow x = y$

When Anunay takes 90 steps, the escalator would have moved by $\frac{90y}{5x}$ steps i.e. $\frac{90}{5} = 18$ steps in the same time.
Therefore, the number of steps in the escalator = 90+18 = 108.

Question 107

Two swimmers Michael Phelps and Matt Biondi started swimming towards each other from opposite ends of a river across the width. They first met at a point 1500m away from one shore. They crossed each other, touched the opposite end and returned immediately. They met each other again at 900m from the other shore. Find the width(in m) of the river.

Assume that the speed of water in the river is negligible.

Answer:3600

Explanation:
Let 'd' be the width of the river. When they first meet one swimmer must have covered a distance of 1500m and other

swimmer must have covered a distance of $d - 1500$ m. Let us assume that Michael Phelps covered a distance of 1500m with a speed of A m/hr whereas Matt Biondi covered a distance of $d - 1500$ m with a speed of B m/hr.

$$\frac{1500}{A} = \frac{d-1500}{B}$$ ... (1)

In second case, when they meet 900m from the other shore then total distance covered by Michael Phelps = $d + 900$m whereas total distance covered by Matt Biondi = $2d - 900$m. Therefore,

$$\frac{d+900}{A} = \frac{2d-900}{B}$$ ... (2)

From the equation (1) and (2) we can see that,

$$\frac{1500}{d-1500} = \frac{d+900}{2d-900}$$

$$\Rightarrow 3000d - 1350000 = d^2 - 600d - 135000$$

$$\Rightarrow d^2 - 3600d = 0 \text{ or } d = 0, 3600$$

'd' can't be 0. Hence, d = 3600m.