CAT

MBA PIONEER 2024

QUANTITATIVE APTITUDE

DPP: 2

Factors

Q1 Find the number of factors of 540 .

(A) 24 (B) 36

(C) 42 (D) 48

Q2 Sum of all factors of 168 is . Choose the correct option

(A) 240 (B) 380

(C) 480 (D) 540

Q3 How many factors of 420 is odd?

(A) 8 (B) 16

(C) 24 (D) None of these

Q4 Find the number of even factors of 396 .

(A) 6 (B) 10

(C) 12 (D) 18

Q5 Find the number of composite factors of 4900

(A) 15 (B) 17

(C) 21 (D) 23

Q6 Find the sum of all factors of 1001 .

(A) 1216

(B) 1344

(C) 1440

(D) None of these

Q7 Find the sum of all even factors of 2592

(A) 6116 (B) 7502

(C) 8116 (D) 8372

Q8 Find the sum of all odd factors of 1344

(A) 16 (B) 26

(C) 32 (D) 38

Q9 The sum of perfect square factor of 936

(A) 7 (B) 13

(C) 49 (D) 50

Q10 The sum of perfect cubes factor of 10648 is

(A) 1331 (B) 11988

(C) 11987 (D) 1339

Q11 Find the number of factors of 2366 which is not divisible by 169 .

(A) 6 (B) 8

(C) 10 (D) 12

Q12 The 8^{th} position factor of 686070 from beginning is 10. If 113^{th} position factor from beginning is x. Find $\mathbf{x}$

(A) 2835 (B) 31185

(C) 68607 (D) 343035

Q13 Find the number of factors of $2^3 \times 3^2 \times 5^2 \times 7$ which is also multiples of 56 .

(A) 9 (B) 12

(C) 15 (D) 18

Q14 Number of factors of $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ is 49,40 and 70 respectively. Which of them can be a perfect cube?

(A) $\mathbf{x}$ and $\mathbf{y}$

(B) y and z

(C) $\mathbf{x}$ and $\mathbf{z}$

(D) All three

Q15 ab is a two digit number, which has 3 factors. How many factors can abab (4 digit number) have? (Consider all cases)

(A) 4
(B) 6
(C) 9
(D) Inadequate data

Q16 What is the smallest number which has total 14 factors?

Q17 Find the number of numbers less than 50 which are multiple of a perfect cube greater than 1 .
(A) 3 (B) 5
(C) 7 (D) 9

Q18 Number of factors of $2^a \times 3^b \times 7^7$ is 96 . Find the least possible value of $(a+b)$
(A) 3 (B) 5
(C) 8 (D) 6

Q19 How many factors of 414 , less than it is co-prime with 414 ?
(A) 132 (B) 98
(C) 84 (D) 72

Q20 What is the product of all factors of 18 ?
(A) 324 (B) 512
(C) 4096 (D) 5832

Q21 Find the number of prime factors of 17017.
(A) 2 (B) 4
(C) 6 (D) 8

Q22 How many numbers are multiple of 64^{112} but at the same time factor of 64^{120} ?
(A) 36 (B) 40
(C) 46 (D) 49

Q23 Find the number of factors of the largest 2 digit triangular number.
(A) 2 (B) 3
(C) 4 (D) 6

Q24 Number of factors of P is 18 . Find the maximum possible number of factors of P^2
(Given no. of prime factors of $\mathrm{E}=2$)

Q25 A number N has only three least possible prime factors. What can be the greatest factor of N which is the multiple of product of lowest and highest prime factors? (Given $\mathrm{N}<1000$)
(A) 960 (B) 990
(C) 570 (D) 940

Q26 Suppose $\mathrm{A}=2^{16} \times 5^{31}$. Find the number of factors of A^2 less than A, which completely divides A^2.
(A) 540 (B) 542
(C) 543 (D) 544

Q27 Prime factors of a number N are only $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and c. It's known that $(a+b+c)=10$ and number of factors of N is 8 . Find the maximum possible value of N.
(A) 30
(B) 24
(C) 12
(D) Inadequate date

Q28 Find the sum of all natural numbers which has highest possible number of factors
(A) 308
(B) 402
(C) 482
(D) None of these

Q29 $\mathrm{N}=2^{42} \times 3^{34}$. How many factors of N, excluding 1, are perfect cubes?
(A) 120 (B) 143
(C) 170 (D) 179

Q30 Find the least number which has only 2,3 and 7 as prime factor and the total number of factor it has is 20.
(A) 168
(B) 252

(C) 336

(D) None of these

Answer Key

Q1	(A)
Q2	(C)
Q3	(A)
Q4	(C)
Q5	(D)
Q6	(B)
Q7	(B)
Q8	(C)
Q9	(D)
Q10	(B)
Q11	(B)
Q12	(C)
Q13	(A)
Q14	(D)
Q15	(B)
Q16	192
Q17	(C)
Q18	(B)
Q19	(A)
Q20	(D)
Q21	(B)
Q22	(D)
Q23	(C)
Q24	55
Q25	(A)
Q26	(C)
Q27	(A)
Q28	(B)
Q29	(D)
Q30	(C)

Hints & Solutions

Q1 Text Solution:

$$540 = 2^2 \times 3^3 \times 5$$

So, number of factors

$$\begin{aligned} &= (2+1)(3+1)(1+1) \\ &= (3 \times 4 \times 2) = 24 \end{aligned}$$

Q2 Text Solution:

(c)

$$168 = 2^3 \times 3 \times 7$$

Sum of all factor of 168

$$\begin{aligned} &= \left(2^0 + 2^1 + 2^2 + 2^3\right) \times \left(3^0 + 3^1\right) \\ &\qquad \times \left(7^0 + 7\right) \\ &= (1+2+4+8) \times (1+3) \times (1+7) \\ &= (15 \times 4 \times 8) \end{aligned}$$

= 480

Q3 Text Solution:

(a)

$$420 = 2^2 \times 3 \times 5 \times 7$$

Number of odd factors of 420

$$\begin{aligned} &= (1 \times 2 \times 2 \times 2) \\ &= 8 \end{aligned}$$

Q4 Text Solution:

(c)

$$396 = 2^2 \times 3^2 \times 11$$

Number of even factors of 396

$$= (2 \times 3 \times 2) = 12$$

Q5 Text Solution:

(d)

$$4900 = 2^2 \times 5^2 \times 7^2$$

Number of prime factors $= 3$

Total number of factors

$$\begin{aligned} &= (3 \times 3 \times 3) \\ &= 27 \end{aligned}$$

Since '1' is neither prime nor composite so we have to subtract '1' from it.

So, composite number of factors

$$\begin{aligned} &= (27 - 3 - 1) \\ &= 23 \end{aligned}$$

Q6 Text Solution:

(b)

$$1001 = 7 \times 11 \times 13$$

Sum of all factors of 1001

$$\begin{aligned} &= \left(\tfrac{7^{1+1}-1}{7-1}\right) \times \left(\tfrac{11^{1+1}-1}{11-1}\right) \times \left(\tfrac{13^{1+1}-1}{13-1}\right) \\ &= \left(\tfrac{48}{6} \times \tfrac{120}{10} \times \tfrac{168}{12}\right) \\ &= (8 \times 12 \times 14) \\ &= 1344 \end{aligned}$$

Q7 Text Solution:

(b)

$$2592 = 2^5 \times 3^4$$

Sum of all even factors

$$= \left(2^1+2^2+2^3+2^4+2^5\right) \times \left(3^0+3^1+3^2+3^3+3^4\right)$$
$$= (2+4+8+16+32) \times (1+3+9+27+81)$$
$$= (62 \times 121)$$
$$= 7502$$

Q8 Text Solution:

(c)

$$1344 = 2^6 \times 3 \times 7$$

Sum of all odd factors

$$= 2^0 \times \left(3^0+3^1\right) \times \left(7^0+7^1\right)$$
$$= (4 \times 8)$$
$$= 32$$

Q9 Text Solution:

(d)

$$936 = 2^3 \times 3^2 \times 13$$

Perfect squares factors

$$= 2^2 \times 3^2$$

So, sum of perfect square factor

$$= \left(2^0+2^2\right) \times \left(3^0+3^2\right)$$
$$= (5 \times 10)$$
$$= 50$$

Q10 Text Solution:

(b)

$$10648 = 2^3 \times 11^3$$

So, sum of perfect cubes factor

$$= \left(2^0+2^3\right)\left(11^0+11^3\right)$$
$$= (9 \times 1332)$$
$$= 11988$$

Q11 Text Solution:

$$2366 = 2 \times 7 \times 13^2$$

So, number of factors not divisible by

$$169 = (2 \times 2 \times 2)$$
$$= 8$$

Q12 Text Solution:

$$686070 = 2 \times 3^4 \times 5 \times 7 \times 11^2$$

The factor from beginning are $1, 2, 3, 5, 6, 7, 9, 10, 11$ and so on We know that, 1^{st} factor from beginning $\times$ last factor from beginning $= \mathrm{n}^{\text{th}}$ factor from beginning $\times \mathrm{n}^{\text{th}}$ factor from end

Q13 Text Solution:

$$2^3 \times 3^2 \times 5^2 \times 7$$
$$= (8 \times 7) \times \left(3^2 \times 5^2\right)$$
$$= 56 \times \left(3^2 \times 5^2\right)$$
$$\text{Number of Factors of } \left(3^2 \times 5^2\right)$$
$$= (2+1)(2+1)$$
$$= 9$$

Q14 Text Solution:

Number of factors of $a^p b^q c^r =$

$$(p+1)(q+1)(r+1)$$

Hence, p, q and r should be of $3\ \mathrm{m}, 3\mathrm{n}$ and $3\ \mathrm{s}$ type

or Number of factors should be of $(3m+1)(3n+1)(3s+1)$ type Now, Let's check,

$$
\begin{aligned}
49 &= 7 \times 7 \\
&= (6+1)(6+1) \quad \rightarrow \text{satisfy} \\
40 &= 4 \times 10 \\
&= (3+1) \times (9+1) \quad \rightarrow \text{satisfy} \\
70 &= 7 \times 10 \\
&= (6+1) \times (9 \times 1) \quad \rightarrow \text{satisfy}
\end{aligned}
$$

Q15 Text Solution:

Given, ab has 3 factors

So, ab is a square of prime number

or ab can be 25,49

Number of factors of $2525 = (3 \times 2) = 6$

And same goes with 4949

So, number of factors $= 6$

Q16 Text Solution:

$$14 = 2 \times 7 \text{ or } 14 \times 1$$

To get the smallest number, we will take smallest prime factor with highest power or $\mathrm{N}($ say $) = 2^6 \times 3$

It has number of factors

$$
\begin{aligned}
&= (6+1)(1+1) \\
&= 14
\end{aligned}
$$

Q17 Text Solution:

Perfect cube greater than 1 but less than 50 $= 2^3, 3^3$

Multiples of 23 less than 50

$$= (8 \times 1, 8 \times 2, 8 \times 3, 8 \times 4, 8 \times 5, 8 \times 6)$$

and multiples of 2^3 less than 50

$$= (1 \times 3^3)$$

So, number of such numbers

$$
\begin{aligned}
&= (6+1) \\
&= 7
\end{aligned}
$$

Q18 Text Solution:

Topic - Number System (Factors)

Number of factors of

$$
\begin{aligned}
&2^a \times 3^b \times 7^7 \\
&(a+1)(b+1)(7+1) \\
&= 8(a+1)(b+1)
\end{aligned}
$$

This is equals to 96

So, $8(a+1)(b+1) = 96$

or $(a+1)(b+1) = 12$

or $(a+1)(b+1) = 3 \times 4$

So, a can be $= 2$ and b can be $= 3$

Least value of $(a+b) = (2+3) = 5$

Q19 Text Solution:

$$414 = 2 \times 3^2 \times 23$$

So, required number

$$
\begin{aligned}
&= 414\left(1-\tfrac{1}{2}\right)\left(1-\tfrac{1}{3}\right)\left(1-\tfrac{1}{23}\right) \\
&= \left(414 \times \tfrac{1}{2} \times \tfrac{2}{3} \times \tfrac{22}{23}\right) \\
&= 132
\end{aligned}
$$

Q20 Text Solution:

$$18 = 2 \times 3^2$$

Number of factors $= (2 \times 3) = 6$

Product of all factors of 18

$$
\begin{aligned}
&= (18)^{\frac{6}{2}} \\
&= 18^3 \\
&= 5832
\end{aligned}
$$

Q21 Text Solution:

$$17017 = 7 \times 11 \times 13 \times 17$$

So, number of prime factors $= 4$

Q22 Text Solution:

Topic - Number System

$$64^{112}=\left(2^{6}\right)^{112}=2^{672}$$

and $64^{120}=\left(2^{6}\right)^{120}=2^{720}$
multiple of 2^{672} and factor of 2^{720}

$$=2^{672}, 2^{673}, 2^{674} \ldots . . 2^{720}$$

Number of such numbers

$$\begin{aligned} & =(720-672)+1 \\ & =48+1 \\ & =49 \end{aligned}$$

Q23 Text Solution:

We know that triangular number is a number, which can be expressed as the sum of consecutive natural numbers starting with 1,

So, $1+2+\ldots\ldots+\underset{(\text{say})}{\mathrm{K}}<100$

At, $\mathrm{k}=13, \frac{13(13+1)}{2}<2$

or $91<100$,

So, the number is 91

$$91=7 \times 13$$

Number of factors

$$\begin{aligned} & =(2 \times 2) \\ & =4 \end{aligned}$$

Q24 Text Solution:

$$18=2 \times 3^{2}$$

Factors of $18=1,2,3,6,9,18$

If we take, 3 and 6 (close the each other) then $P=a^{2} \times b^{5}$ (say a and b are the two prime factors) or $\mathrm{P}^{2}=\left[\mathrm{a}^{2} \times \mathrm{b}^{5}\right]^{2} \quad=\mathrm{a}^{4} \times 5^{10}$

Number of factors of P^{2}

$$\begin{aligned} & =(4+1)(10+1) \\ & =(5 \times 11) \\ & =55 \end{aligned}$$

Q25 Text Solution:

Three least possible prime numbers are $=2,3$ and 5

So, $\mathrm{N}=2^{\mathrm{a}} \times 3^{\mathrm{b}} \times 5^{\mathrm{c}}$

(Let a, b and c are the power)

Also $\mathrm{N}<1000$

or $1000>2^{\mathrm{a}} \times 3^{\mathrm{b}} \times 5^{\mathrm{c}}$

product of least and highest prime factors $=2 \times 5$

$$=10$$

If $\mathrm{c}=3$

Then, for $5^{3} \times 2^{\mathrm{a}} \times 3^{5}<1000$

a and b can be 1 and 1

So, $\mathrm{N}=750$

If $\mathrm{c}=2$

then, $5^{2} \times 2^{\mathrm{a}} \times 3^{\mathrm{b}}<1000$

or $2^{a} \times 3^{b}<40$

a can $be=3$ and $b=1$

So, $\mathrm{N}=600$

If $\mathrm{c}=1$

Then, $5 \times 2^{\mathrm{a}} \times 3^{\mathrm{b}}<1000$

or $2^{\mathrm{a}} \times 3^{\mathrm{b}}<200$

a can be 6 and $b=1$

So, $\mathrm{N}=960$

Clearly N is maximum at 960

So, greatest factor of N which is multiple of 10 is 960

Q26 Text Solution:

Given, $A=2^{16} \times 5^{31}$

$$\begin{aligned} & \text{So, } A^{2}=2^{16 \times 2} \times 5^{31 \times 2} \\ & =2^{32} \times 5^{62} \end{aligned}$$

To divide A^{2}, number less than A should only contain 2,5 or both as prime factor.

or number of such factors

$$= [(16+1)\times(31+1)]-1$$
$$= (17\times 32)-1$$
$$= 543$$

Q27 Text Solution:

We have $\mathrm{N} = \mathrm{a^p} \times \mathrm{b^q} \times \mathrm{c^r}$

And, $(p+1)(q+1)(r+1) = 8$

This is only possible when $\mathrm{p} = \mathrm{q} = \mathrm{r} = 1$.

So, $\mathrm{N} = \mathrm{a} \times \mathrm{b} \times \mathrm{c}$

As, a, b and c are prime number

so, a can be 2 , b can be 3 and c can be 5 .

Only 2,3 and 5 satisfy the criteria) of their sum $= 10$

Therefore, $\mathrm{N} = (2\times 3\times 5) = 30$

Q28 Text Solution:

Maximum number of prime factor that can be used simultaneously is $(2,3),(5,2,3)$ and $(7,2,3)$

So, if the number is N

Then, $\mathrm{N} = 2^{\mathrm{a}} \times 3^{\mathrm{b}} \times 5^{\mathrm{c}}$

At $\mathrm{a} = 2,\ \mathrm{b} = 1, \mathrm{c} = 1$

We get $\mathrm{N} = 60$,

If $\mathrm{N} = 2^{\mathrm{a}} \times 3^{\mathrm{b}} \times 7^{\mathrm{c}}$

Then at $\mathrm{a} = 2,\ \mathrm{b} = 1, \mathrm{c} = 7$

We get $\mathrm{N} = 84$

If $\mathrm{N} = 2^{\mathrm{a}} \times 3^{\mathrm{b}}$

Then at $\mathrm{a} = 5,\ \mathrm{b} = 1$, we get $\mathrm{N} = 96$

Also, at $\mathrm{a} = 3,\ \mathrm{b} = 2$, we get

$$\mathrm{N} = 72$$

If $\mathrm{N} = 2^{\mathrm{a}} \times 3^{\mathrm{b}} \times 5^{\mathrm{c}}$

Then at $\mathrm{a} = 1,\ \mathrm{b} = 2, \mathrm{c} = 1$,

$$\mathrm{N} = 90$$

For all these numbers, number of factors $= 12$ (highest possible)

Therefore, $\quad \Sigma \mathrm{N} = (60+84+96+72+90)$
$= 402$

Q29 Text Solution:

Topic - Number System

Numbers of ways of choosing powers of 2 starting from 0 till all the multiples of 3 uptil 42 will be 15.

Similarly, the number of ways of choosing powers of 3 starting from 0 till all the multiples of 3 uptil 34 will be 12.

Thus, the number of perfect cube factors will be

15 x 12

= 180

Now, 1 needs to be excluded from the above.

Therefore, the required answer will be

180 - 1

= 179

Q30 Text Solution:

Let the number is N,

So, $\mathrm{N} = 2^{\mathrm{a}} \times 3^{\mathrm{b}} \times 7^{\mathrm{c}}$

Given, $(a+1)\times(b+1)\times(c+1) = 20$

Factor of 20 are 1, 2, 4, 5, 10, 20.

If $\mathrm{c} = 1$, then

$$(a+1)\times(b+1) = 10$$

If $b = 1$, then

$$(a+1) = 5$$
$$\text{So, a} = 4$$

Least value of N

$$= 2^4 \times 3 \times 7$$
$$= 336$$