MBA PIONEER 2024

QUANTITATIVE APTITUDE

DPP: 02

Sequence & Series Part II

Q1 *Let* x_1 , x_2 , x_3 ... *be in harmonic progression with* $x_1 = 5$ *and* $x_{20} = 25$. *Find the largest positive integer* n *for which* $x_n \geq 0$?

Q2 Three positive numbers are in a decreasing Geometric Progression such that if the middle term of the GP is doubled then all the terms will be in an Arithmetic Progression in that order. Find the common ratio of the GP

(A) $2+\sqrt{3}$ (B) $2-\sqrt{3}$
(C) $2+\sqrt{5}$ (D) $2-\sqrt{5}$

Q3 If the sum of the first 15 terms in an AP is 325 and the sum of the next 15 terms is 625, find the common difference of the AP.

(A) $\frac{4}{3}$ (B) $\frac{5}{3}$
(C) $\frac{7}{3}$ (D) $\frac{3}{7}$

Q4 *It is given that the nth term of AP is defined as* t_n, *and sum up to 'n' terms of the AP is defined as Sn. If* $|t_5| = |t_{15}|$ *and* t_2 *is not equal to* t_7, *what is* S_{19}?

(A) 3 (B) 1
(C) 2 (D) 0

Q5 There is a sequence A is defined by $a_n = a_{n-1} + 3$, $a_1 = 11$ and there is sequence B is defined as $b_n = b_{n-1} - 4$, $b_3 = 103$. If $a_k > b_{k+3}$, find the smallest value k can take?

(A) 12 (B) 15
(C) 14 (D) 16

Q6 There is a sequence A is defined by $a_n = a_{n-1} + 3$, $a_1 = 9$ and there is sequence B is defined as $b_n = b_{n-1} - 4$,such that $b_4 = 100$. If $a_k < b_{k+2}$, find the largest value k can take?

(A) 14 (B) 15
(C) 12 (D) 13

Q7 If the sum of first 30 terms in AP is 625 and the sum of the next 30 terms is 1425, then find the common difference

(A) $\frac{5}{9}$ (B) $\frac{8}{9}$
(C) $\frac{2}{9}$ (D) $\frac{7}{9}$

Q8 Two colleagues were figuring out their salary in different years of their career which were in Arithmetic Progression.The ratio of sum of their salaries after 'r' number of years is (4r+5) : (2r+15).Find the ratio of their salary after the seventh year.

(A) $\frac{57}{41}$ (B) $\frac{53}{41}$
(C) $\frac{53}{45}$ (D) $\frac{51}{43}$

Q9 Anaesthesia decides to invest in the stock of her husbands company for a period of 4 years such that her investments always increase with the same amount year by year and the sum of the investments for four years is 2000. If the sum of the squares of her investment is 1200000.Find the third investment made by her

(A) 500 (B) 400
(C) 800 (D) 600

Q10 Consider a set of positive integers $a_1, a_2, \ldots, a_{52}$ such that $a_1 < a_2 < \ldots < a_{52}$. It is told that their arithmetic mean is one less than the arithmetic mean of a_2, a_3, ..., a_{52}. If a_{52} = 120, then find the largest possible value that a_1 can take

Q11 The AM(arithmetic mean) of three numbers a,b,c is 60 and the AM of 5 numbers a,b,c,d,e is 70 it is given that d=$\frac{a+b}{2}$and e=$\frac{b+c}{2}$.It is also given that a is always greater than or equal to c,then find the minimum possible value of a

Q12 Consider three positive real numbers, a, b, and c, forming a geometric progression with a<b<c. Now, if the terms 5a, 16b, and 12c form an arithmetic progression, what is the common ratio of the initial geometric progression?

(A) 2.5 (B) 0.16
(C) 0.33 (D) 3.5

Q13 Let x1, x2, ..., x3n represent an arithmetic progression with x1=3 and x2=7. If the sum of these 3n terms amounts to 1830, what is the smallest positive integer m so that m^2(x1+x2+...+xn) goes beyond 1830?

Q14 In an infinite geometric progression x1, x2, x3, ... , each term xn satisfies the condition $x_n = 3(x_{n+1} + x_{n+2} + \ldots)$ for all n≥1. If the sum x1+x2+x3+...=32, what value does $\frac{x8}{x6}$ hold?

(A) 0.0625 (B) 0.125
(C) 0.25 (D) 0.5

Q15 $Consider\ a\ series\ where\ a_i\ is\ greater\ than\ 0\ and\ i = 1, 2, 3, \ldots, 50$.
$It\ is\ Given\ that\ a_1 + a_2 + a_3 + \cdots + a_{50} = 50,\ then\ find\ the\ minimum\ value\ of$
$1/a_1 + 1/a_2 + 1/a_3 + \cdots 1/a_{50}$

Q16 Find the approximate value of 's' if $s = 1 + \frac{2}{7} + \frac{6}{7^2} + \frac{10}{7^3} \ldots\ldots\ldots\ldots \infty$

(A) 3.44 (B) 2.44
(C) 1.44 (D) 0.44

Q17 In a geometric progression the sum of the first two terms is 12 and the sum of the fifth and sixth term is 192.The terms of the GP are alternatively positive and negative find the first term of the GP.

(A) 12 (B) -12
(C) 24 (D) -24

Q18 In a geometric progression (GP), the sum of an infinite number of terms is 48, and the sum of their squares is 768. Find the common ratio of the GP.

(A) 0.25 (B) 0.5
(C) 0.75 (D) 1.25

Q19 $x + y + z = 9\ find\ the\ minimum\ possible\ value\ of\ xy + yz + zx$
given that x, y and z are positive

(A) xyz (B) $(xyz)^2$
(C) $(xyz)^3$ (D) $(xyz)^4$

Q20 $If\ 3\ positive\ numbers\ a, b, c\ are\ in\ AP\ and\ abc = 8$
$then\ find\ the\ ratio\ \frac{HM}{GM}\ where\ HM$
$s\tan ds\ for\ \ Harmonic\ mean\ and\ GM$
$s\tan ds\ for\ GM$

(A) 2/b (B) 3/c
(C) 4/b (D) 5/a

Q21 If GM of x,y,z is 3 then what will be the minimum possible value for xy+yz+zx given that all x, y and z are positive.

(A) 3 (B) 9
(C) 27 (D) 1

Q22 3 positive numbers a,b,c are in GP if their arithmetic mean is $\frac{1}{ac}$ then find the harmonic mean of these numbers.

(A) a^2c^2 (B) b^2
(C) abc (D) $\frac{ac}{b^3}$

Q23 If the sides of a right angle triangle are in AP and the sum of the sides is 60 units then find the area of the triangle (in sq. units)

(A) 300 (B) 150
(C) 600 (D) 450

Q24 In a different planet if we drop a ball to the ground, after colliding with ground the ball will lose half of its energy and only rise back to half of the previous height . If it is dropped from 100m height initially then find the total distance covered by the ball (in m) for the entire bouncing process till it reaches to a state of rest.

(A) 300 (B) 400
(C) 900 (D) 600

Q25 A bouncing ball will lose 50 percent of its energy while colliding with ground. If the total distance travelled by the ball when it was dropped from a height 'h' before coming to complete rest is 1000m find the height in km at which the ball was first dropped

(A) 0.666 (B) 666.66
(C) 333.33 (D) 0.333

Q26 In a polygon the internal angles are in AP with common difference 5 and least angle is 120 degrees find the maximum possible number of diagonals for this polygon

(A) 27 (B) 54
(C) 104 (D) 35

Q27 $If\ a, b, c\ are\ in\ HP\ ,\ 2b = a\ and\ a + b = 9\ then\ find\ the\ sum\ of\ a, b, c$

(A) 9 (B) 10
(C) 11 (D) 12

Q28 $find\ the\ sum\ to\ infinity\ of\ the\ series$
$1 + \frac{2}{5} + \frac{6}{5^2} + \frac{10}{5^3} \ldots\ldots\ up\ to\ infinity$

(A) 2 (B) $\frac{7}{5}$
(C) $\frac{7}{4}$ (D) 3.5

Q29 Two colleagues were figuring out their salary in different years of their career which were in Arithmetic Progression.The ratio of sum of their salaries in 'r' number of years from start of their career is (2r+3) : (7r+19). find the ratio of their salary after 5 year

(A) $\frac{21}{82}$ (B) $\frac{17}{82}$
(C) $\frac{19}{82}$ (D) $\frac{23}{82}$

Q30 If the sum of first 15 terms in AP is 625 and the sum of the next 15 terms is 825, find the common difference

(A) $\frac{8}{9}$ (B) $\frac{1}{9}$
(C) $\frac{2}{9}$ (D) $\frac{4}{9}$

Answer Key

Q1	24
Q2	(B)
Q3	(A)
Q4	(D)
Q5	(C)
Q6	(A)
Q7	(B)
Q8	(A)
Q9	(D)
Q10	43
Q11	10
Q12	(A)
Q13	3
Q14	(A)
Q15	50
Q16	(C)
Q17	(B)
Q18	(B)
Q19	(A)
Q20	(A)
Q21	(C)
Q22	(A)
Q23	(B)
Q24	(A)
Q25	(D)
Q26	(C)
Q27	(C)
Q28	(C)
Q29	(A)
Q30	(A)

Hints & Solutions

Q1 Text Solution:

Topic - Sequence and Series

Sub-topic - Harmonic Progression

$If\ \ x_1, x_2, x_3 \ldots .\ are\ in\ harmonic$
$progression,\ then\ \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3} \ldots \ldots$
$are\ in\ AP.$

$First\ term\ of\ \ AP$

$\frac{1}{x_1} = \frac{1}{5}$

$20th\ term\ of\ \ AP,\ \frac{1}{x_{20}} = \frac{1}{25} \Rightarrow \frac{1}{5} + 19$

$d = \frac{1}{25}$

$\Rightarrow d = \frac{-4}{19 \times 25}$

$We\ have\ to\ find\ the\ largest\ positive$
$integer\ n\ for\ which\ x_n \geq 0$

$\Rightarrow \frac{1}{5} + \left(n - 1\right) d \geq 0$

$\Rightarrow \frac{1}{5} + \left(n - 1\right) d \geq 0$

$\Rightarrow \frac{1}{5} + \left(n - 1\right) \frac{-4}{19 \times 25} \geq 0$

$\Rightarrow n \leq 24.75$

$answer = 24$

Q2 Text Solution:

$a, ar, ar^2\ \ are\ in\ GP.$

$\sin ce\ \ the\ middle\ term\ of\ the\ GP\ is$
$doubled,$

$a, 2ar, ar^2\ \ are\ in\ AP.$

$\Rightarrow 4ar = a + ar^2 \Rightarrow r^2 - 4r + 1$
$= 0$

$\Rightarrow r = 2 \pm \sqrt{3}$

$\sin ce\ the\ series\ is\ an\ decrea\sin g\ \ GP$

$so,\ \ \Rightarrow r = 2 - \sqrt{3}$

Q3 Text Solution:

$the\ sum\ of\ the\ first\ 15\ terms,\ S_{15} =$
325

$The\ sum\ of\ the\ next\ 15\ terms,\ K_{15} =$
625

$a_{16} = a_1 + 15d$

$a_{17} = a_2 + 15d$

$K_{15} = a_{16} + a_{17} + \ldots\ldots + a_{30}$

$K_{15} = a_1 + 15d + a_2 + 15d + \ldots\ldots$
$+ a_{15} + 15d$

$K_{15} = a_1 + a_2 + \ldots\ldots + a_{15} +$
$15\left(15d\right)$

$K_{15} = S_{15} + 15\left(15d\right)$

$i.e.,\ 625 = 325 + 15 \times 15d$

$300 = 15 \times 15d$

$d = \frac{4}{3}$

Q4 Text Solution:

$Given\ \left|t_5\right| = \left|t_{15}\right|.$

$t_5 = t_{15}\ or\ t_5 = -t_{15}.$

$If\ t_5 = t_{15},\ that\ is\ d = 0$

$therefore\ t_2\ would\ be\ equal\ to\ t_7\ \ which$
$is\ not\ possible$

$t_5 = -t_{15}\ Or,\ t_5 + t_{15} = 0.$

$then\ t_{10} = 0.$

$\sin ce\ the\ terms\ are\ in\ AP\ t_{10} = \frac{t_5 + t_{15}}{2}$
$= 0$

$S_{19} = 19 * t_{10} = 0$

$the\ average\ of\ n\ terms\ in\ an\ A.P.\ is\ the$
$middle\ term.$

$sum\ of\ n\ terms\ in\ an\ A.P,\ is\ n\ times$
$the\ middle\ term.$

Q5 Text Solution:

$Sequence\ A\ is\ an\ A.P.\ with\ a = 11,$
$and\ common\ difference\ 3.$
$So,\ a_k = 11 + (k-1)3.$
$Sequence\ B\ is\ an\ A.P\ \ with\ third\ term$
$103\ and\ common\ difference - 4.$
$t_3 = a + 2d$
$103 = a + 2(-4)\ or\ a = 111$
$b_{k+3} = 111 + (k+2)(-4) = 111 - 4k$
$-8 = 103 - 4k$
$a_k > b_{k+3}$
$11 + (k-1)3 > 103 - 4k$
$8 + 3k > 103 - 4k$
$7k > 95$
$k > \frac{95}{7}$
$k\ has\ to\ be\ an\ integer,\ so\ smallest\ value$
$k\ can\ take\ is\ 14.$

Q6 Text Solution:

$Sequence\ A\ is\ an\ A.P.\ with\ a = 9,\ and$
$common\ difference\ 3.$
$So,\ a_k = 9 + (k-1)3.$
$Sequence\ B\ is\ an\ A.P\ \ with\ fourth$
$term\ 100\ and\ common\ difference - 4.$
$t_4 = a + 3d$
$100 = a + 3(-4)\ or\ a = 112$
$b_{k+2} = 112 + (k+1)(-4) = 112 - 4k$
$-4 = 108 - 4k$
$a_k < b_{k+2}$
$9 + (k-1)3 < 108 - 4k$
$6 + 3k < 108 - 4k$
$7k < 102$
$k < \frac{102}{7}$
$k\ has\ to\ be\ an\ integer,\ so\ largest\ value\ k$
$can\ take\ is\ 14$

Q7 Text Solution:

$the\ sum\ of\ the\ first\ 30\ terms,\ S_{30} =$
625
$The\ sum\ of\ the\ next\ 30\ terms,\ K_{30} =$
1425
$a_{31} = a_1 + 30d$
$a_{32} = a_2 + 30d$
$K_{30} = a_{31} + a_{32} + \ldots\ldots + a_{60}$
$K_{30} = a_1 + 30d + a_2 + 30d + \ldots\ldots$
$+ a_{30} + 30d$
$K_{30} = a_1 + a_2 + \ldots\ldots + a_{30} +$
$30(30d)$
$K_{30} = S_{30} + 30(30d)$
$i.e.,\ 1425 = 625 + 30 \times 30d$
$800 = 30 \times 30d$
$d = \frac{8}{9}$

Q8 Text Solution:

$Let\ a1,\ a2\ be\ the\ salaries\ of\ their\ first$
$year\ and\ d1\ and\ d2\ be\ the\ increment\ of$
$salaries\ of\ the\ employee.$
$Now,\ the\ sum\ of\ r\ terms\ of\ their$
$salaries\ are$

$$S_a = \frac{r}{2} * \left[2a_1 + \left(r-1\right)d_1\right]$$

$$S'_a = \frac{r}{2} * \left[2a_2 + \left(r-1\right)d2\right]$$

$$\frac{\frac{r}{2} * \left[2a_1 + \left(r-1\right)d_1\right]}{\frac{r}{2} * \left[2a_2 + \left(r-1\right)d2\right]} = \frac{4r+5}{2r+15}$$

$$\frac{[2a_1 + (r-1)d_1]}{[2a_2 + (r-1)d2]} = \frac{4r+5}{2r+15}$$

$Ratio\ of\ Salaries\ after\ the\ 7th\ year$

$$= \frac{a_1+6d_1}{a_2+6d_2}$$

$$= \frac{2a_1+(13-1)d_1}{2a_2+(13-1)d_2}$$

$$= \frac{4*13+5}{2*13+15}$$

$= 57 : 41$

Q9 Text Solution:

$let\ the\ investment\ be\ \ of\ the\ form$

$\left(a-3d\right),\ \left(a-d\right),\ \left(a+d\right),\left(a+3d\right)$

$\left(a-3d\right)\ +\ \left(a-d\right)\ +\ \left(a+d\right)$

$+\ \left(a+3d\right)\ =\ 2000$

$=>\ 4a\ =\ 2000\ =>$

$a\ =\ 500$

$(a-3d)^2\ +\ (a-d)^2\ +\ (a+d)^2$

$+\ (a+3d)^2\ =\ 1200000$

$=>\ 4\left(a^2\ +\ 5\ d^2\right)\ =\ 1200000$

$=>\ \left(a^2\ +\ 5\ d^2\right)\ =\ 300000$

$=>\ 250000\ +\ 5d^2\ =\ 300000$

$=>\ 5d^2\ =\ 50000$

$=>\ d2\ =\ 10000$

$=>\ d\ =\ \pm 100$

$\therefore\ d\ =\ 100\ \Big(\ As\ the\ investment$

$increases\ every\ year\ so\ d\ =\ \ -100\ not$

$possible\Big)$

$Hence\ the\ investments\ made\ are\ 200,$

$400, 600, 800$

Q10 Text Solution:

$AM\ of\ a_2, a_3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, a_{52}\ be$

M

$therefore\ the\ AM\ of\ a_1, a_2, a_3, \ldots\ldots\ldots$

$\ldots\ldots, a_{52}\ will\ be\ M-1$

$\frac{a_1+51M}{52} = M-1$

$a_1 = M-52$

$a_1\ will\ be\ maximum\ when\ the\ value\ of$

$M\ is\ maximum$

$M\ is\ maximum\ when\ the\ numbers\ are$

$70, 71, \ldots\ldots\ldots 120$

$a_1 = 95-52$

$= 43$

Q11 Text Solution:

$\frac{a+b+c}{3} = 60$

$a+b+c = 180$

$\frac{a+b+c+d+e}{5} = 70$

$a+b+c+d+e = 350$

$therefore\ \ d+e = 350-180$

$= 170$

$d+e = \frac{a+b}{2} + \frac{b+c}{2} = \frac{a+b+c}{2} + \frac{b}{2} = 170$

$\frac{180}{2} + \frac{b}{2} = 170$

$b = 160$

$which\ implies\ a+c = 20$

$a\ has\ the\ min\ value\ when\ a = c$

$therefore\ min\ value\ of\ a\ \ = 10$

Q12 Text Solution:

Let the terms in GP be

$\frac{\mathrm{a}}{\mathrm{r}}, \mathrm{a}, \mathrm{ar}$

$Also$, then,

$32ar = 5a + 12ar^2$

$Dividing\ the\ both\ sides\ by\ 'a'$

on symplifying we get the equation

as $12\mathrm{r}^2 - 32\mathrm{r} + 5 = 0.$

therefore $\mathrm{r} = \frac{1}{6}, \frac{5}{2}.$

since $\mathrm{a} < \mathrm{b} < \mathrm{c}$ the common ration should be greater than 1

$\mathrm{r} = \frac{5}{2}$

Q13 Text Solution:

$Let\ us\ assume\ 3n = k$

$\Longrightarrow \frac{k}{2}\left(2a + (k-1)d\right) = 1830$

$we\ know\ that\ a = 3,\ d = 4$

$\Longrightarrow \frac{k}{2}\left(2(3) + (k-1)4\right) = 1830$

$\Longrightarrow \frac{k}{2}\left(6 + 4k - 4\right) = 1830$

$\Longrightarrow k\left(2k + 1\right) = 1830$

$\Longrightarrow 2k^2 + k = 1830$

$\Longrightarrow 2k^2 + k - 1830 = 0$

$By\ factorizing\ we\ can\ find\ that\ k = 30,$
$n = 10$

$\Longrightarrow 102\ \left(2(3) + 4(9)\right) = 5(6 + 36)$

$\Longrightarrow 5(42) = 210$

$\Longrightarrow m^2\left(a1 + a2 + \ldots.. + an\right) >$
1830

$\Longrightarrow 210 \times m^2 > 1830 \Longrightarrow m^2 = 9,$
$\sin ce\ 210 \times 9 = 1890$

$therefore\ m = 3$

Q14 Text Solution:

$Given\ the\ infinite\ geometric$
$progression\ with\ the\ sum\ x_1 + x_2 + \ldots..$
$\ldots.\infty = 32$

$\frac{x}{1-r} = 32$

$x_n = 3\left(x_{n+1} + x_{n+2} + \ldots\right)\ for\ \ n \geq 1.$

$When\ \ n = 1,\ applying\ the\ formula$
$gives\ us$

$x_1 = 3\left(x_2 + x_3 + \ldots\ldots.\right)$

$x_1 = \frac{3x_1 r}{1-r}$

$1 - r = 3r$

$r = 0.25$

$therefore\ x = 32 * 0.75 = 24$

$\frac{x_8}{x_6} = \frac{24\times r^7}{24\times r^5} = r^2 = 0.0625$

Q15 Text Solution:

$A\ M \geq H\ M$

$AM\ of\ first\ 50\ terms =$
$\frac{a\,1 + a\,2 + a\,3 + \ldots + a50}{50}$

$HM\ of\ the\ first\ 50\ terms =$
$\frac{50}{\left(\frac{1}{a\,1} + \frac{1}{a\,2} + \frac{1}{a\,3} + \ldots + \frac{1}{a\,50}\right)}$

$\frac{a\,1 + a\,2 + a\,3 + \ldots + a50}{50}$

$\geq \frac{50}{\left(\frac{1}{a\,1} + \frac{1}{a\,2} + \frac{1}{a\,3} + \ldots + \frac{1}{a\,50}\right)}$

$\Rightarrow 1 \geq \frac{50}{\left(\frac{1}{a\,1} + \frac{1}{a\,2} + \frac{1}{a\,3} + \ldots + \frac{1}{a\,50}\right)}$

$\Rightarrow \left(\frac{1}{a\,1} + \frac{1}{a\,2} + \frac{1}{a\,3} + \ldots + \frac{1}{a\,50}\right)$
≥ 50

$Hence,\ minimum\ value\ of$
$\left(\frac{1}{a\,1} + \frac{1}{a\,2} + \frac{1}{a\,3} + \ldots + \frac{1}{a\,50}\right) =$
50

Q16 Text Solution:

Topic - Sequence and Series

Sub - topic - Special Series

$s = 1 + \frac{2}{7} + \frac{6}{7^2} + \ldots\ldots\ldots..\infty$

$\frac{s}{7} = \frac{1}{7} + \frac{2}{7^2} + \ldots\ldots\ldots..\infty$

$s - \frac{s}{7} = \frac{6s}{7} = 1 + \frac{1}{7} + \frac{4}{7^2} + \ldots\ldots\ldots..\infty$

$\frac{6s}{7} - \frac{8}{7} = \frac{4}{7^2} + \frac{4}{7^3} + \ldots\ldots.\infty$

$= \frac{\frac{4}{7^2}}{1-\frac{1}{7}} = \frac{4}{6\times 7} = \frac{6s-8}{7}$

$s = \frac{4/6 + 8}{6}$

$s = 1.44$

Q17 Text Solution:

$a + ar = 12 \cdots (i)$

$ar^4 + ar^5 = 192 \cdots (ii)$

$dividing\ (ii)\ by\ (i),$

$we\ have\ \frac{ar^4+ar^5}{a+ar}$

$\Rightarrow r^4 = 16 \ \Rightarrow \ \pm\ 2\ \ r = -2\Big(\sin ce\ terms$

$are\ alternatively\ positive\ and\ negative\Big)$

$substituting\ in\ equation\ 1\ we\ get\ a =$
-12

Q18 Text Solution:

$In\ a\ geometric\ progression\ \left(GP\right),\ the$

$sum\ of\ an\ infinite\ number\ of\ terms\ is$
$48,$
$and\ the\ sum\ of\ their\ squares\ is\ 768.$
$Find\ the\ common\ ratio\ of\ the\ GP.$

$let\ GP\ = \ a, ar, ar^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \infty$

$S_\infty = \frac{a}{1-r} = 48\ so\ a = \ 48 - 48r$

$S^2{}_\infty = a^2 + a^2r^2 + a^2r^4 + \ldots\ldots\ldots\ldots \infty$

$= \ A\ GP\ with\ first\ term\ a^2\ and\ c.r = r^2$

$sum\ will\ be = \frac{a^2}{1-r^2} = 768$

$= \frac{(48-48r)^2}{1-r^2} = 768$

$= \frac{48^2 \times (1-r)^2}{(1-r)(1+r)} = 768$

$2304 \times (1-r) = 768\,(1+r)$

$2304 - 2304r = 768 + 768r$

$3072r = 1536$

$r = \frac{1}{2}$

Q19 Text Solution:

$x + y + z = 9$

$AM = \frac{x+y+z}{3} = \frac{9}{3} = 3$

$Since,\ AM\ \geq\ HM,\ we\ hace$

$3\ \geq\ \frac{3xyz}{xy+yz+zx}$

$1\ \geq\ \frac{xyz}{xy+yz+zx}$

$xy + yz + zx\ \geq\ xyz$

Q20 Text Solution:

$a, b, c\ are\ in\ AP\ so\ AM = b$

$abc = 8\ \ so$

$GM = \ (abc)^{\frac{1}{3}} = (8)^{\frac{1}{3}}$

$= 2$

now

$AM \times HM = GM^2$

$b \times HM = 4$

$HM = \frac{4}{b}$

$\frac{HM}{GM} = \frac{2}{b}$

Q21 Text Solution:

$GM = 27$

$= (xyz)^{\frac{1}{3}} = 3$

$xyz = 27$

$GM \geq HM$

$HM = \frac{3xyz}{xy+yz+zx} \leq 3$

$xy + yz + zx \geq xyz = 27$

$xy + yz + zx \geq 27$

Q22 Text Solution:

$a, b, c\ are\ in\ GP\ so\ GM = b\ also\ ac = b^2$

$AM = \frac{1}{ac} = \frac{1}{b^2}$

$as\ per\ relation$

$AM \times HM = GM^2$

$\frac{1}{b^2} \times HM = b^2$

$HM = b^4$

$= (ac)^2$

Q23 Text Solution:

$let\ sides\ are\ a-d, a, a+d$

$(a-d)^2+a^2=(a+d)^2$

$solving\ this$

$-2ad+a^2=2ad$

$a^2=4ad$

$a=4d$

$now\ sum=3a=60\ so\ a=20\ d=5$

$sides\ length\ are\ =15, 20, 25$

$area\ =\frac{1}{2}[20\times 15]=150\ sq\ units$

Q24 Text Solution:

$dropped\ from\ 100m\ h=100$

$first\ fall\ =h$

$first\ rise\ =\ {}^{h}/_{2}\ after\ losing\ half\ of\ its\ energy$

$second\ fall\ =\ h/2$

$second\ rise\ =\ {}^{h}/_{4}\ldots\ldots\ldots\ldots\ldots\ldots$

$up\ to\ infinity$

$total\ distance\ travelled=\ h+2\times\left[\frac{h}{2}+\frac{h}{4}+\ldots\ldots\ldots\ldots up\ to\ infinity\right]$

$$=\ h+2\times\left[\frac{\frac{h}{2}}{1-{}^{1}/_{2}}\right]=h+2h=3h$$

$=3\times 100\ =300m$

Q25 Text Solution:

$on\ first\ fall\ it\ will\ travel\ distance\ h$

$on\ the\ subsequent\ rise\ it\ will\ rise\ up\ to\ \frac{h}{2}$

$since\ it\ wil\ lose\ half\ of\ its\ energy$

$on\ the\ subsequent\ fall\ it\ will\ fall\ \frac{h}{2}$

$on\ the\ subsequent\ rise\ it\ will\ rise\ up\ to\ \frac{h}{4}$

$since\ it\ wil\ lose\ half\ of\ its\ energy$

$on\ the\ subsequent\ fall\ it\ will\ fall\ \frac{h}{4}\ etc$

so

$distance\ travelled=\ h+2\left[\frac{h}{2}+\frac{h}{4}+\ldots\ldots\ up\ to\ infinity\right]$

$$=h+2\left[\frac{\frac{h}{2}}{1-\frac{1}{2}}\right]=h+2h=3h=1000$$

$h=333.33m$

$=0.333\ km$

Q26 Text Solution:

$a=120,\ d=5$

$sum\ of\ all\ internal\ angles=\ (n-2)\times 180=\frac{n}{2}\ [2\times 120+(n-1)5]$

$360n-720=240n+5n^2-5n$

$5n^2-125n+720=0$

$n^2-25n+144=0$

$(n-16)(n-9)=0\Rightarrow n=16,\ 9$

$also$

$Therefore,\ for\ maximum\ number\ of\ diagonals\ we\ will\ take\ n\ =\ 16.$

$So\ the\ number\ of\ diagonals\ will\ be$

$16(13)/2$

$=\ 104$

Q27 Text Solution:

Topic - Sequence and Series

Sub-topic - Harmonic Progression

a, b, c *are in HP then* $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ *are in AP*

$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

also $2b = a$ *and* $a + b = 9$

so solving this we get $b = 3$ *and* $a = 6$

$\frac{2}{3} = \frac{1}{6} + \frac{1}{c}$

$\frac{1}{c} = \frac{3}{6} = \frac{1}{2}$

$c = 2$

$a + b + c = 6 + 3 + 2 = 11$

Q28 Text Solution:

$s = 1 + \frac{2}{5} + \frac{6}{5^2} + \frac{10}{5^3} \ldots\ldots\ldots\ldots\ldots\ldots$ *up to* ∞

$\frac{s}{5} = \frac{1}{5} + \frac{2}{5^2} + \frac{6}{5^3} \ldots\ldots\ldots\ldots$ *up to* ∞

$s - \frac{s}{5} = \frac{4s}{5} = 1 + \frac{1}{5} + \frac{4}{5^2} + \frac{4}{5^3} + \ldots\ldots\ldots.$ *upto*∞

$\frac{4s}{5} = 1 + \frac{1}{5}$

$+ 4\left[\frac{1}{5^2} + \frac{1}{5^3} + \ldots\ldots\ldots\ldots upto\infty\right]$

$= 1 + \frac{1}{5} + 4 \times GP$ *whose* $a = \frac{1}{5^2}$ *and* r

$= \frac{1}{5}$

$= 1 + \frac{1}{5} + 4\left[\frac{\frac{1}{5^2}}{1 - \frac{1}{5}}\right] = 1 + \frac{1}{5} + 4 \times \left[\frac{\frac{1}{5^2}}{\frac{4}{5}}\right]$

$= 1 + \frac{1}{5} + \frac{1}{5}$

$\frac{4s}{5} = \frac{7}{5}$

$s = \frac{7}{4}$

Q29 Text Solution:

Let a1, a2 be the salaries of their first year and d1 and d2 be the increment of salaries of the employee.

Now, the sum of r terms of their salaries are

$S_a = \frac{r}{2} * \left[2a_1 + \left(r - 1\right)d_1\right]$

$S'_a = \frac{r}{2} * \left[2a_2 + \left(r - 1\right)d2\right]$

$\frac{\frac{r}{2} * \left[2a_1 + \left(r-1\right)d_1\right]}{\frac{r}{2} * \left[2a_2 + \left(r-1\right)d2\right]} = \frac{2r+3}{7r+19}$

$\frac{[2a_1 + (r-1)d_1]}{[2a_2 + (r-1)d2]} = \frac{2r+3}{7r+19}$

Ratio of Salaries earned by them in the 5th year

$= \frac{a_1+4d_1}{a_2+4d_2} = \frac{2a_1+(9-1)d_1}{2a_2+(9-1)d_2} = \frac{21}{82}$

Q30 Text Solution:

The sum of the first 15 terms, $S_{15} = 625$

The sum of the next 15 terms, $K_{15} = 825$

$a_{16} = a_1 + 15d$

$a_{17} = a_2 + 15d$

etc. let K_{15} *denote the sum of next 15 terms then*

$K_{15} = a_1 + 15d + a_2 + 15d + \ldots\ldots + a_{15} + 15d$

$K_{15} = a_1 + a_2 + \ldots\ldots + a_{15} + 15(15d)$

$K_{15} = S_{15} + 15(15d)$

i.e., $825 = 625 + 15 \times 15d$

$200 = 15 \times 15d$

$d = \frac{8}{9}$