CAT

MBA PIONEER 2024

Quantitative Aptitude

Sequence & Series Part I

DPP: 01

Q1 $If\ x+2y=12\ find\ the\ maximum\ value\ of\ x^2y^4$
given that x and y are positive real numbers
(A) 2^{12} (B) 2^{22}
(C) 2^4 (D) 2^{16}

Q2 The internal angles of a polygon are in AP with common difference as 10 degrees if the smallest angle is 100 degree find the minimum number of sides of the polygon
(A) 8 (B) 9
(C) 10 (D) 12

Q3 Sam is saving money in an account to buy a new laptop. He starts by depositing Rs. 2000 in the first month. Every month after that, he increases his deposit by Rs. 500. If the cost of the laptop is Rs. 30000, find the minimum number of months after which he could buy that laptop
(A) 8 (B) 9
(C) 15 (D) 7

Q4 The first term of an infinite GP is 3 times the sum of the following terms. Find the ratio of the 3rd term to the 5th term of that GP
(A) $\frac{16}{9}$ (B) 16
(C) $\frac{4}{3}$ (D) $\frac{3}{4}$

Q5 $if\ x+y=12\ find\ the\ maximum\ possible\ value\ of\ xy^2$
(A) 4 (B) 64
(C) 256 (D) 16

Q6
If 3 numbers a,b,c are in both AP and GP in the same order then,
find $\frac{a}{b}-\frac{b}{c}$
(A) 1 (B) 0
(C) b^2 (D) $\frac{a+c}{2}$

Q7 The wages of A,B,C are respectively in GP and Sum of the wages of A and B is 30 , also Sum of the wages of B and C is 60, then find the sum of the wages of A and C
(A) 50 (B) 45
(C) 40 (D) 55

Q8 If an employee gets 20 percent increment every year on his gross salary then the minimum number of years from the beginning to double the salary is approximately
(A) 3 (B) 4
(C) 5 (D) 6

Q9 If 11 times the 11th term of an AP is equal to 7 times the 7th term then find the value of 18th term of the AP
(A) 5 (B) 25
(C) 0 (D) 18

Q10 The product of first three numbers in a GP is 27 and the sum of 2nd and 4th terms is 15. Then which of the following can be the sum of 2nd,3rd and 4th terms of the GP
(A) 9 (B) 20
(C) 22 (D) 10

Q11

if a,c,b are in AP respectively and the equation $x^2 - (a+b)x + ab = 0$ *has only one solution then find* $a - c + b - c$

(A) 3 (B) 0
(C) 5 (D) 10

Q12 Sum of 11th to 20th (both included) terms in an AP is 1000 the common difference is 1 then find the sum of first 10 terms of the AP

(A) 900 (B) 800
(C) 600 (D) 300

Q13 If 3x+3y+z=12, the find the maximum value of xyz given that x, y and z are positive real numbers.

(A) $\frac{9}{16}$ (B) 27
(C) $\frac{64}{9}$ (D) 9

Q14 Sum upto infinity of a GP is 24, also the first term of the GP is 12, find the 4th term of the GP

(A) 1 (B) 1.5
(C) 3 (D) 6

Q15 $If\ \log(x),\ \ \log(2^x),\ \log(4^x)$ *are respectively in AP, then the value of x will be*

(A) 9 (B) 6
(C) 5 (D) 1

Q16 $If\ A = 2^{100} + 2^{99} + 2^{98} + \ldots\ldots + 2^0$ *then* $A - 1$ *is equal to*

(A) $2^{101} - 1$ (B) $2(2^{100} - 1)$
(C) $2^{100} - 2$ (D) $2^{100} - 1$

Q17 *In a GP sum of first 10 terms and sum of first 12 terms are equal, then what will be the sum of first 99 terms,*

if first term is 3

(A) 3 (B) 0
(C) 99 (D) 1

Q18 *In an AP, the sum of first 15 terms is 500 and the sum of next 15 terms is 1500, then what will be the common difference*

(A) $\frac{40}{9}$ (B) $\frac{9}{20}$
(C) $\frac{3}{4}$ (D) $\frac{4}{3}$

Q19 *In a AP if* $|T_{13}| = |T_{17}|$ *then find the sum of first 30 terms it is given that* $T_{30} = 100$
and no other term in this AP is 100

(A) 0 (B) 70
(C) 30 (D) 100

Q20 *if 5 times 5th term of an AP is equal to 8 times 8th term of the same AP, also the first term in the AP is 24, then sum of first 25 terms of this AP is*

(A) 0 (B) 25
(C) 600 (D) 13

Q21 *If sum up to n terms of an AP is equal to* p^{th} *term of the AP, also 3 times the 3rd term is equal to 7th term then which of the following is true*

(A) $n^2 - n - 2p + 2 = 0$ (B) $n^2 - 2n - p + 2 = 0$
(C) $2n^2 - n - 2p + 2 = 0$ (D) $n^2 - n - 2p + 1 = 0$

Q22 *If p, q, r are in AP then qr, pr, qp are in*

(A) AP
(B) GP

(C) HP
(D) none of the above

Q23 2 friends Raj and Manoj work for the same company for the last couple of years, if the total amount they earned during this period is in the ratio 3n+2 : 2n+3 respectively where 'n' is the number of years they worked for the company, also it is known that every year they get d_1 *and* d_2 *amount respectively as*, find the *increment* ratio of the money they earned respectively in the 9th year

(A) $\frac{29}{20}$ (B) $\frac{53}{37}$
(C) $\frac{29}{18}$ (D) none

Q24 Find the sum of the series 0.5+0.55+0.555+0.5555..........up to n terms

(A) $\frac{5}{81}\left[9n-1+\frac{1}{10^n}\right]$ (B) $\frac{5}{81}\left[9n+1+\frac{1}{10^n}\right]$
(C) $\frac{5}{9}\left[9n-1+\frac{1}{10^n}\right]$ (D) $\frac{5}{9}\left[9n-1+\frac{9}{10^n}\right]$

Q25 **$ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have one common solution , also a,b,c are in GP then** $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ *are in*

(A) AP (B) HP
(C) GP (D) can't say

Q26 *In this series are* $5, 9, 13, 17, \ldots\ldots\ldots$ *let sum of first n terms is denoted by* S_n *then find* $\sum_{n=1}^{15} S_n$

(A) 2840 (B) 2480
(C) 3200 (D) 3440

Q27 In a AP sum to n terms is $4n^2 - 2n$ then find the least possible value for n such that n th term is divisible by 13

(A) 4 (B) 6
(C) 3 (D) 5

Q28 The m^{th} term and n^{th} term of an increasing GP are 4 and 36 respectively. If the common ratio 'r' is an integral value then find the least possible value of 2r+n-m

(A) 6 (B) 8
(C) 3 (D) 2

Q29 In an infinite GP the total sum is 33 also it has another property

$T_n = 2\left(T_{n+1} + T_{n+2} + T_{n+3} + \ldots\ldots \textit{up to infinit}\right.$

where T_n *denotes n th term of the GP*
find the 4th term of the GP

(A) $\frac{22}{81}$ (B) $\frac{9}{22}$
(C) $\frac{3}{22}$ (D) $\frac{22}{27}$

Q30 The sum of 13th and 15th term of an AP is 2024 find the sum of first 27 terms of the AP

(A) 27324 (B) 27624
(C) 27424 (D) 28424

Answer Key

Q1	(A)
Q2	(A)
Q3	(A)
Q4	(B)
Q5	(C)
Q6	(B)
Q7	(A)
Q8	(C)
Q9	(C)
Q10	(A)
Q11	(B)
Q12	(A)
Q13	(C)
Q14	(B)
Q15	(D)
Q16	(B)
Q17	(A)
Q18	(A)
Q19	(D)
Q20	(A)
Q21	(A)
Q22	(C)
Q23	(B)
Q24	(A)
Q25	(A)
Q26	(A)
Q27	(A)
Q28	(B)
Q29	(D)
Q30	(A)

Hints & Solutions

Q1 Text Solution:

$x+2y=12$

split this in to 6 parts

$\frac{x}{2}+\frac{x}{2}+\frac{y}{2}+\frac{y}{2}+\frac{y}{2}+\frac{y}{2}=12$

now, AM of these 6 will be

$AM=\frac{\frac{x}{2}+\frac{x}{2}+\frac{y}{2}+\frac{y}{2}+\frac{y}{2}+\frac{y}{2}}{6}$

$=\frac{12}{6}$

$=2$

$GM=\left(\left(\frac{x}{2}\right)^2\left(\frac{y}{2}\right)^4\right)^{\frac{1}{6}}$

$AM\geq GM$

$\left(\left(\frac{x}{2}\right)^2\left(\frac{y}{2}\right)^4\right)^{\frac{1}{6}}\leq 2$

$\left(\left(\frac{x}{2}\right)^2\left(\frac{y}{2}\right)^4\right)\leq 2^6$

$x^2y^4\leq 2^6\times 2^4\times 2^2=2^{12}$

Maximum value of x^2y^4 *is* 2^{12}

Q2 Text Solution:

The sum of internal angles of a polygon having n sides is

$(n-2)\times 180$

Sum of AP whose first term is 100 and c.d is 10 up to n terms is

$\frac{n}{2}(2\times 100+(n-1)\times 10)$

these 2 are equal so

$(n-2)\times 180=\frac{n}{2}(2\times 100+(n-1)\times 10)$

on solving we get

$10n^2-17n+720=0$

or $n^2-17n+72=0$

$n=8$ *or* 9

Therefore,

we can say that the minimum number of sides in the polygon will be 8.

Q3 Text Solution:

$a=2000$

$d=500$

$\frac{n}{2}(2a+(n-1)d)\geq 30000$

$n(4000+(n-1)500)\geq 60000$

$3500n+500n^2\geq 60000$

$7n+n^2\geq 120$

$n^2+7n-120\geq 0$

$(n-8)(n+15)\geq 0$

$n\geq 8;$

as in the other case 'n' will be negative.

Thus, he can buy the laptop after a minimum period of 8 months

Q4 Text Solution:

let the GP be $a, ar, ar^2\ldots.$

here

$a=3\left(ar+ar^2+ar^3\ldots..up\ to\ infinity\right)$

$a=\frac{3ar}{1-r}$

$\frac{r}{1-r}=\frac{1}{3}$

$4r=1$

$r=\frac{1}{4}$

$3rd\ term\ =ar^2$

$5th\ term\ =ar^4$

$\frac{3rd\ term}{5th\ term}=\frac{1}{r^2}$

$=\frac{1}{\left(\frac{1}{4}\right)^2}$

$=\frac{1}{\frac{1}{16}}$

$=16$

Q5 Text Solution:

Here

$x+y=12$ *splitting it to* 3 *parts*

$x+\frac{y}{2}+\frac{y}{2}=12$

$\frac{x+\frac{y}{2}+\frac{y}{2}}{3}=4$

$AM=4$

as $AM\geq GM$

$GM\ of\ these\ =\left(x\left(\frac{y}{2}\right)^2\right)^{\frac{1}{3}}\leq 4$

$x\left(\frac{y}{2}\right)^2\leq 4^3$

$xy^2\ \leq\ 256$

Therefore, the maximum possible value of xy^2 *has to be* 256.

Q6 Text Solution:

Numbers are a,b,c

$let\ a=a$

$b=\ ar$

$c=ar^2\ \left(a,b,c\ in\ GP\right)$

$\sin ce\ \ a,b,c\ in\ AP\ also$

so

$b=\frac{a+c}{2}$

$ar=\frac{a+ar^2}{2}$

$=r^2+1$

$r^2\ -\ 2r+1\ =\ 0$

By solving Quadratic Equation

$r=1$

$a=a$

$b=a$

$c=a$

$\frac{a}{b}-\frac{b}{c}=1-1=0$

Q7 Text Solution:

$Let\ A=a,\ B=ar,\ C=ar^2$

$a+ar=30\to equation\ 1$

$ar+ar^2=60\to equation\ 2$

$\frac{equation\ 2}{equation\ 1}=\frac{60}{30}$

$\frac{ar(1+r)}{a(1+r)}=2$

$r=2$

$a+ar=30$

$3a=30$

$a=10$

$ar=20$

$ar^2=40$

$A+C=10\ +\ 40$

$=50$

Q8 Text Solution:

$a=a$

$r=1.2$

GP

$a,ar,ar^2.....$

let after n years salary got doubled then

$2a=ar^{n-1}$

$2=(1.2)^{n-1}$

now we need to find the minimum number of years it will take to double the salary

Putting $n=4$ *we get the term as* 1.728

Now,

putting $n\ =\ 5$,

we get the term as 2.0736

Therefore,

a minimum of 5 *years are required such that the salary gets doubled.*

Q9 Text Solution:

if

$11(a+10d)=7(a+6d)$

$11a+110d=7a+42d$

$4a+68d=0$

Dividing the above equation by 4, we have,

$a+17d=0$

$18th\ term=0$

Q10 Text Solution:

Let GP first 4 terms are

$\frac{a}{r}, a, ar, ar^2$

Product of first 3 terms

$=a^3=27$

so $a=3$

Now sum of 2nd and 4th

$= a+ar^2=15$

$3\left(1+r^2\right)=15$

$r=\pm 2$

now the GP can be either

$\frac{-3}{2}, 3, -6, 12, -24.....$

or

$\frac{3}{2}, 3, 6, 12, 24......$

taking case 1

the sum of 2nd, 3rd and 4th terms is

$= 3-6+12$

$=9$

taking case 2

the sum of 2nd, 3rd and 4th terms is

$= 3+6+12$

$=21$

As only 9 is present among the answer options,

it will be correct answer choice

Q11 Text Solution:

The equation

$x^2-\left(a+b\right)x+ab=0$

has only one solution

$a+b=2c \quad (AP)$

Therefore,

$a-c\ +\ b-c$

$=\ a+b\ -\ 2c$

$=\ 2c\ -\ 2c$

$=\ 0$

Q12 Text Solution:

Sum of 11th to 20th (both included) terms in an AP is 1000,

the common difference is 1

$T_{11}=a+10d=a+(10d)$

$T_{12}=a+11d=a+d+(10d)$

$......$

$T_{20}=a+19d=a+9d+(10d)$

So sum from 11 to 20 th term is

$=S_{10}+10(10d)$

$1000=S_{10}+100\times 1$

$S_{10}=900$

Q13 Text Solution:

As per question

$3x+3y+z=12$

taking AM of $3x,\ 3y,\ z$

$\frac{3x+3y+z}{3}=4$

$AM=4$

Taking GM

$(3x\times 3y\times z)^{\frac{1}{3}}$

$AM\geq GM$

$(3x\times 3y\times z)^{\frac{1}{3}}\leq 4$

$3x\times 3y\times z\leq 64$

$xyz\leq \frac{64}{9}$

Therefore,

the maximum possible value of xyz will be $64/9$

Q14 Text Solution:

Topic - Sequence and Series

$\frac{a}{1-r} = 24$

$a = 12$

$r = 0.5$

$12 \times 0.5^3 = 1.5$

Q15 Text Solution:

$\log(x),\ \log(2^x),\ \log(4^x)$ *are in AP*

$2\ \log(2^x) = \log(4^x) + \log(x)$

$2\ \log(2^x) = \log(2^{2x}) + \log(x)$

$\log\ (2^{2x}) = \log\ (2^{2x}\ .\ x)$

$2^{2x} = 2^{2x}\ .\ x$

$Therefore,\ x = 1$

Q16 Text Solution:

$A = 2^{100} + 2^{99} + 2^{98} + \ldots\ldots + 2^0$

total 101 *terms*

The terms here form a GP

where $a = 1,\ r = 2$

$\left(\text{when 'A' is seen in reverse order}\right)$

$S_{101} = \frac{2^{101}-1}{2-1} = A$

$A = 2^{101} - 1$

$A - 1 = 2^{101} - 1 - 1$

$= 2^{101} - 2$

$= 2\left(2^{100} - 1\right)$

Q17 Text Solution:

Sum of first 10 *terms* $=$ *sum of first* 12 *terms*

$=$ *sum of first* 10 $+$ $11th$ *term* $+$ $12\ th$ *term*

so $11th$ *term* $+$ $12th$ *term* $= 0$

let $11\ th$ *term* $= a$

$12\ th$ *term* $= ar$

$a + ar = 0$

$a = 3$ *given so* $r = -1$

$GP = 3, -3, 3, -3, 3, -3\ldots\ldots\ldots\ldots$

sum up to 99 *terms* $= 3$

(*sum up to odd number of terms is* 3

and sum up to even number of terms is 0)

Q18 Text Solution:

$s_{15} = 500$

$s_{30} = 500 + 1500 = 2000$

$15\left(2a + 29d\right) = 2000\ (i)$

$\frac{15}{2}\left(2a + 14d\right) = 500\ (ii)$

Dividing equation (i) *by* (ii),

we have,

$\frac{(2a+29d)}{(2a+14d)} = 2$

$2a + 29d = 4a + 28d$

$d = 2a.$

Substituting the value of $2a = d$

in equation (i), *we have,*

$15\left(30d\right) = 2000$

$450d = 2000$

$d = \frac{40}{9}$

Q19 Text Solution:

$if\ |T_{13}| = |T_{17}|$
$then\ a + 12d = \pm(a + 16d)$
$either\ d = 0\ or\ 2a + 28d = 0$
$if\ d = 0$
$then\ all\ terms\ would\ be\ same$
$= T_{30} = 100$
$but\ no\ other\ term\ is\ 100\ so\ d\ is\ not\ 0$
$2a + 28d = 0$
$so\ T_{15} = 0$
$so\ sum\ to\ 29\ terms\ is\ 0$
$(\sin ce\ middle\ term\ is\ 0)$
$so\ S_{30} = 0 + T_{30} = 100$

Q20 Text Solution:

$5(a + 4d) = 8(a + 7d)$
$= 3a + 36d = 0$
$= a + 12d = 0$
$also\ 2a + 24d = 0$
so
$T_{13} = 0$
$if\ T_{13} = 0$
$then\ S_{25} = 0$

Q21 Text Solution:

$\frac{n}{2}(2a + (n-1)d) = a + (p-1)d$
$also\ 3(a + 2d) = a + 6d$
$a = 0$
$\frac{n}{2}((n-1)d) = (p-1)d$
$n(n-1) = 2(p-1)$
$n^2 - n - 2p + 2 = 0$

Q22 Text Solution:

$p, q, r\ in\ AP\ then$
$\frac{p}{pqr}, \frac{q}{pqr}, \frac{r}{pqr}\ are\ also\ in\ AP$
$\frac{1}{qr}, \frac{1}{pr}, \frac{1}{qp}\ are\ also\ in\ AP$
$then\ qr, pr, qp\ are\ in\ HP$

Q23 Text Solution:

$Raj : Manoj\ = 3n + 2 : 2n + 3$
$also\ \sin ce\ they\ are\ getting\ cons\tan t$
$increment$
$the\ salaries\ on\ yearly\ basis\ are\ in\ AP$
$S_R : S_M\ = 3n + 2 : 2n + 3$
$\frac{\frac{n}{2}(2a_1+(n-1)d_1)}{\frac{n}{2}(2a_2+(n-1)d_2)} = \frac{3n+2}{2n+3}$
$salaries\ earned\ in\ 9th\ year$
$\frac{T_R}{T_M} = \frac{a_1+8d_1}{a_2+8d_2}$
$= \frac{2a_1+16d_1}{2a_2+16d_2}$
$= \frac{2a_1+(17-1)d_1}{2a_2+(17-1)d_2} = \frac{3n+2}{2n+3} = \frac{3\times17+2}{2\times17+3}$
$= \frac{53}{37}$

Q24 Text Solution:

$0.5 + 0.55 + 0.555 + \ldots..$
$= \frac{5}{9}(0.9 + 0.99 + \ldots.)$
$= \frac{5}{9}((1 - 0.1) + (1 - 0.01) + \ldots.)$
$\frac{5}{9}((1 + 1 + \ldots up\ to\ n) - (0.1 + 0.01 + \ldots..))$
$the\ second\ part\ is\ in\ GP\ with\ a = 0.1\ and$
$r = 0.1$
$\frac{5}{9}\left[(n) - \frac{0.1(0.1^n-1)}{0.1-1}\right]$
$= \frac{5}{9}\left[n - \frac{10^n-1}{9\times10^n}\right]$
$= \frac{5}{81}\left[9n - 1 + \frac{1}{10^n}\right]$

Q25 Text Solution:

$ax^2 + 2bx + c = 0$, $dx^2 + 2ex + f = 0$

have common solution

also a, b, c *are in* GP

so $b^2 = ac$

$4b^2 = 4ac$ *so the equation* 1 *is having only one root which is*

$\frac{-2b}{2a} = \frac{-b}{a}$

now this is one of the roots of equation 2

putting this in equation 2

$d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + f = 0$

$db^2 - 2eba + fa^2 = 0$

dividing by ab^2

$\frac{d}{a} - 2\frac{e}{b} + \frac{f}{c} = 0$

$\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ *are in* AP

Q26 Text Solution:

$5, 9, 13, 17 \ldots.$ *is a* AP

$d = 4,\ a = 5$

$S_n = \frac{n}{2}(2 \times 5 + (n-1)4)$

$= \frac{n}{2}(10 + 4n - 4)$

$= \frac{n}{2}(6 + 4n)$

$= n\left(3 + 2n\right)$

$\sum_{n=1}^{15} S_n = \sum_{n=1}^{15}\left(3n + 2n^2\right)$

$= 3\sum_{n=1}^{15} n + 2\sum_{n=1}^{15} n^2$

$= 3\frac{(n(n+1))}{2} + 2\frac{n(n+1)(2n+1)}{6}$

putting $n = 15$

answer will be

$= 360 + 2480$

$= 2840$

Q27 Text Solution:

$S_n = 4n^2 - 2n$

$T_n = S_n - S_{n-1} = \left(4n^2 - 2n\right)$

$-\left(4(n-1)^2 - 2\left(n-1\right)\right)$

$= 8n - 6$

series is $2, 10, 18, 26 \ldots$

26 *is the first term that is divisible by* 13 *in this series*

$n = 4$

Q28 Text Solution:

$ar^{m-1} = 4$

$ar^{n-1} = 36$

$r^{n-m} = \frac{36}{4} = 9 = 3^2$

(*because* r *is a positive integer, positive because it is an increasing* GP)

$r = 3$ *and* $n - m = 2$

$2r = 6$

$2r + n - m = 8$

Q29 Text Solution:

$S_\infty = 33$

$\frac{a}{1-r} = 33$

also

$ar^{n-1} = 2\left(ar^n + ar^{n+1} + ar^{n+2} + \ldots..\infty\right)$

$ar^{n-1} = \frac{2ar^n}{1-r}$

$1 - r = 2r$

$r = \frac{1}{3}$

$\frac{a}{1-r} = 33$

so $a = 33 \times \frac{2}{3} = 22$

$T_4 = 22 \times \left(\frac{1}{3}\right)^3 = \frac{22}{27}$

Q30 Text Solution:

$(a + 12d) + (a + 14d) = 2024$
$2a + 26d = 2024$
$sum\ to\ 27\ terms\ is$
$\frac{27}{2}(2a + 26d) = \frac{27}{2} \times 2024$
$= 27324$