MBA PRO BATCH

QUANTITATIVE APTITUDE

DPP: 8

Inequalities 3

Q1 If $8 \geq \frac{x+5}{3x+1} \geq 2$, then the number of integral values assumed by 115x will be _______.

Q2 If, a, b, c, and d are distinct natural numbers. Then, find the minimum positive integer value of x in $\frac{5x+3}{x+1} \geq k$, where $5k$ is the minimum possible value of (a + 2b + 3c + 4d).

Q3 Let f = x^4 - $6x^3$ + $12x^2$ - 8x. Determine the set of values of x for which the inequality f > 0 holds true:

(A) (-∞, 0) U (2, +∞)
(B) (-∞, 0) U (1, 3) U (3, +∞)
(C) (-∞, 1) U (2, 3) U (3, +∞)
(D) (-∞, 1) U (1, 2) U (3, +∞)

Q4 Let $g = x^5 - 10x^4 + 42x^3 - 92x^2 + 80x$. Determine the set of values of x for which the inequality g > 0 holds true:
(A) (-∞, 0) U (1, 4)
(B) (-∞, 0) U (2, 4)
(C) (-∞, 1) U (2, 4)
(D) (0, 2) U (4, +∞)

Q5 Consider the system of inequalities:
2x - 3y ≤ 6
x + y ≥ 5
Which of the following points is not in the solution set?
(A) (3, 2) (B) (2, 3)
(C) (1, 3) (D) (4, 1)

Q6 Given the following system of linear inequalities:
x + 3y ≤ 12
y - 2x ≥ -3
2x + y ≥ 2
Which of the following points is in the solution set?

1. (1, 2)
2. (2, 3)
3. (3, 1)
4. (4, 2)

(A) Only 2
(B) Both 1 and 2
(C) Both1 and 3
(D) All of the above

Q7 Consider the following system of linear inequalities:
I) x - y ≥ 1
II) 2x + y < 8
III) y > x^2 - 6x + 8
Which of the following points is in the solution set?

1. (1, 0)
2. (3, 1)
3. (2, 3)
4. (4, 3)

(A) Only 2
(B) Both 1 and 2
(C) Both 1 and 3
(D) None of the above

Q8

Consider the following system of linear inequalities:

5x - 3y ≥ 15

2x + 4y ≤ 12

x - y ≥ 0

x ≥ 0

y ≥ 0

Which of the following points (x, y) lies inside the feasible region of the system of inequalities?

1. (2, 1)
2. (5, 1)
3. (1, 1)
4. (3, 2)

(A) Only 2

(B) Both 1 and 2

(C) Both 1 and 3

(D) None of the above

Q9 If $2x + 5 < 5$(y+1) and $\frac{7y+6}{y+9}$ $\leq$1, then

(A) $x < \frac{5}{4}$

(B) $x > -\frac{5}{4}$

(C) $x < -\frac{5}{4}$

(D) $y > \frac{5}{4}$

Q10 Consider a polynomial f = (x - 1)(x + 2)(x - 3)(x + 4). Determine the solution for the following inequality:

f < 0

(A) (-∞, -4) ∪ (-2, 1) ∪ (3, +∞)

(B) (-4, -2) ∪ (1, 3)

(C) (-∞, -4) ∪ (-2, 1) ∪ (3, 5)

(D) (-∞, -4) ∪ (-2, 1) ∪ (2, 3)

Q11 If $9x + 2 = (6y + 2)$ and $\frac{5y+6}{y+1} \leq 2$, then which of the below options can be correct?

(A) $x \leq -\frac{2}{3}$

(B) $x > -\frac{2}{3}$

(C) $-\frac{8}{9} \leq x < -\frac{2}{3}$

(D) $y > \frac{2}{3}$

Q12 If (3x + 2y + 5z) ≤ 50, where x, y, and z are distinct odd prime numbers, then how many distinct values z can assume?

(A) 1 (B) 2

(C) 3 (D) 4

Q13 If $\frac{2x+3}{4x+1} \geq 2$, then the number of integral values assumed by 24x will be _______.

(A) 10 (B) 12

(C) 13 (D) 14

Q14 Prasoon went to a shop to purchase a few pens, pencils and erasers. The number of pens purchased is more than the number of pencils purchased which in turn is more than the number of erasers purchased. It is also given that the rates of the pens, pencils and erasers are distinct prime numbers when expressed in $ and the sum of rates of unit pen and pencil is equal to that of one eraser. If the total number of items purchased is 6 then find the minimum possible cost of purchase of all the items in $.

(A) 15 (B) 16

(C) 17 (D) 18

Q15 If $\frac{2x+9}{x+1} \geq 8$, then the number of integral values assumed by 12x will be _______.

(A) 12 (B) 13

(C) 14 (D) 15

Q16 If, a, b, c, d, e and f are distinct natural numbers. Then, find the minimum positive integer value of x in $\frac{8x+2}{x+2} \geq m$, where $8m$ is the minimum possible value of

(a + 2b + 3c + 4d + 5e + 6f).

(A) 10 (B) 11

(C) 12 (D) 13

Q17 If $x^4 - 6x^3 + 14x^2 - 15x + 6 > 0$. Choose the correct option below:

(A) (-∞, 1) ∪ (2, 3) ∪ (4, ∞)

(B) (-∞, 1) ∪ (2, ∞)
(C) (1, 2) ∪ (3, 4) ∪ (4, ∞)
(D) (1, 2) ∪ (4, ∞)

Q18 Determine the intervals for which $x^3 - 5x^2 + 8x < 4$. Choose the correct option below:
(A) (-∞, 1)
(B) (1, 2) ∪ (4, ∞)
(C) (1, 2)
(D) (-∞, 1) ∪ (4, ∞)

Q19 If $\left(2x^3 - x^2 - 13x\right) > 6$ then x must be from
(A) $(-\infty, -2) \cup \left(-\frac{1}{2}, 3\right)$
(B) $(-\infty, -2) \cup \left(-\frac{1}{2}, 3\right) \cup (3, \infty)$
(C) $\left(-2, -\frac{1}{2}\right) \cup (3, \infty)$
(D) (-∞, -2) ∪ (3, ∞)

Q20 Let h = $x^3 - 4x - x^2 + 4$. Determine the intervals where h is positive or negative.
(A) h > 0 for x ∈ (-∞, -3) ∪ (-1, 1) ∪ (2, 4) and h(x) < 0 for x ∈ (-3, -1) ∪ (1, 2) ∪ (4, +∞)
(B) h > 0 for x ∈ (-∞, -3) ∪ (-1, 1) ∪ (3, 4) and h(x) < 0 for x ∈ (-3, -1) ∪ (1, 3) ∪ (4, +∞)
(C) h > 0 for x ∈ (-2, 1) ∪ (2, ∞) and h(x) < 0 for x ∈ x ∈ (-∞, -2) ∪ (1, 2)
(D) h > 0 for x ∈ (-∞, -1) ∪ (1, 3) and h(x) < 0 for x ∈ (-1, 1) ∪ (3, +∞)

Q21 If $x^6 + \left(\frac{1}{x^6}\right) + 6 \leq 4\left(x^3 + \frac{1}{x^3}\right)$, determine the values of x for which the given inequality holds true.

Q22 If $\left(x^2 - 4\right)\left(x^2 - x - 6\right) < 0$. Then find the number of integral values that 3x can assume.

Q23 If $x^4 + \left(\frac{1}{x^4}\right) - 2\left(x^2 + \frac{1}{x^2}\right) + 2 \leq 0$
Determine the values of x for which the given inequality holds true.
(A) 1
(B) 0
(C) -1
(D) Both option A & C

Q24 A metal wire company sells metal wires in bulk to various customers. The company has a policy that the total sales of metal wires should not exceed $12000 per day. The company charges $200 for every kilogram of metal wire it sells. The company also has a policy that the weight of metal wire sold to a single customer should not exceed 20 kilograms. What is the minimum number of customers the company per day can sell metal wires to, as per the above constraints?

Q25 If $x^4 - 10x^3 + 35x^2 - 50x + 24 < 0$, then find the integral number of values that 4x can take.

Q26 A farmer wants to fence a rectangular field using 200 meters of fencing material. The farmer wants to maximize the area of the field. What is the maximum area of the field that can be fenced in square meters?

Q27 If $x^3 - 6x^2 + 11x < 6$ and $x^3 - 11x^2 + 34x < 24$, then x the value of
(A) 1 < x < 6
(B) 1 < x < 3
(C) x > 1
(D) x < 1

Q28 If $x^3 - 8x^2 + 11x + 20 > 0$. Then x can be from which of the below intervals
(A) (3, 5) and (-∞, -1)
(B) (-1, 4) and (5, ∞)
(C) (-1, 3) and (5, ∞)
(D) (-∞, -2) and (3, 5)

Q29 If $(x-3)^2(x-5)^5(x-7)^3(x-9)^7 < 0$, then how many positive integral values of x can be found?

(A) 5 (B) 4
(C) 3 (D) 2

Q30 Let

$$P = \left(x^2 - 9\right)\left(x^2 - 4\right)\left(x^2 - 1\right)\left(x^2 - 16\right).$$

Find the intervals of x for which P > 0.

(A) (-∞, -4) ∪ (-3, -2) ∪ (-1, 1) ∪ (2, 3) ∪ (4, ∞)

(B) (-∞, -3) ∪ (-2, -1) ∪ (1, 2) ∪ (3, ∞)

(C) (-∞, -4) ∪ (-3, -2) ∪ (1, 2) ∪ (4, ∞)

(D) (-∞, -3) ∪ (-2, 1) ∪ (2, 3) ∪ (4, ∞)

Answer Key

Q1	85
Q2	1
Q3	(A)
Q4	(D)
Q5	(C)
Q6	(B)
Q7	(A, C)
Q8	(D)
Q9	(A)
Q10	(B)
Q11	(C)
Q12	(B)
Q13	(A)
Q14	(C)
Q15	(C)
Q16	(C)
Q17	(B)
Q18	(A)
Q19	(C)
Q20	(C)
Q21	1
Q22	2
Q23	(D)
Q24	3
Q25	6
Q26	2500
Q27	(D)
Q28	(B)
Q29	(B)
Q30	(A)

Hints & Solutions

Q1 Text Solution:

$\frac{x+5}{3x+1} \geq 2$

$\frac{x+5}{3x+1} - 2 \geq 0$

=> $\frac{x+5-2(3x+1)}{3x+1} \geq 0$

=> $\frac{-5x+3}{3x+1} \geq 0$

=> - 5x + 3 ≥ 0 and 3x + 1 > 0 [Since, at x = 0, the expression will not exist.]

=> 5x ≤ 3 and $3x > -1$

=> $x \leq \frac{3}{5}$ and $x > -\frac{1}{3}$

=> $-\frac{1}{3} < x \leq \frac{3}{5}$ .... (i)

Also,

$\frac{x+5}{3x+1} \leq 8$

$\frac{x+5}{3x+1} - 8 \leq 0$

=> $\frac{x+5-8(3x+1)}{3x+1} \leq 0$

=> $\frac{-23x-3}{3x+1} \leq 0$

=> 3x + 1 < 0 or -23x - 3 ≤ 0 [Since, at x = 0, the expression will not exist.]

=> $x < -\frac{1}{3}$ or $x \geq -\frac{3}{23}$ .... (ii)

So, combining (i) and (ii), we have

$-\frac{3}{23} \leq x \leq \frac{3}{5}$

-15 ≤ 115x ≤ 69

So, 115x can assume all the values from – 15 to 69.

Thus, the integral numbers that can be assumed by 115x will be 85.

Q2 Text Solution:

The minimum value of (a + 2b + 3c + 4d) can be obtained by making sure that the lower numbers are multiplied with the higher coefficients.

Also, a, b, c, and d can assume minimum values of 1, 2, 3, and 4 where d = 1, c = 2, b = 3, a = 4.

So, min (a + 2b + 3c + 4d) = 4 + (2 × 3) + (3 × 2) + (4 × 1)

- 5k = 20
- k = 4

So, $\frac{5x+3}{x+1} - 4 \geq 0$

$\frac{5x+3-4(x+1)}{x+1} \geq 0$

=> $\frac{x-1}{x+1} \geq 0$

Now, x = 1 and x = -1 are the critical points.

Note that x cannot be -1, since at x = -1, the expression will not exist.

Also, at x = 1, the inequality is satisfied.

Now, we need to determine whether x < -1 or -1 < x < 1 or x > 1.

<u>Case 1:</u> x < -1

Let x = -2, then we have

$\frac{-2-1}{-2+1} = 3$ ≥ 0

<u>Case 2:</u> -1 < x < 1

Let x = 0, then we have

$\frac{0-1}{0+1} = -1$, cannot be greater than equals 0.

<u>Case 3:</u> x > 1

Let x = 2, then we have

$\frac{2-1}{2+1} = \frac{1}{3} \geq 0$

So, we can conclude that,

x < -1 or x ≥ 1.

So, the minimum positive integer value of x is 1.

Q3 Text Solution:

To solve the inequality f > 0, we first need to find the critical points, which are the values of x where f = 0. Then, we will use the wavy curve method to determine the intervals where f > 0.

Step 1: Find the critical points

$f = x^4 - 6x^3 + 12x^2 - 8x$

$f = x(x^3 - 6x^2 + 12x - 8)$

Now, we need to find the roots of the cubic equation $x^3 - 6x^2 + 12x - 8 = 0$. Observe that x = 2 is a root:

$2^3 - 6(2)^2 + 12(2) - 8 = 0$

Using synthetic division or polynomial long division, we can factor the cubic equation:

$x^3 - 6x^2 + 12x - 8 = (x - 2)(x^2 - 4x + 4)$

Now, we can factor the quadratic equation as well:

$x^2 - 4x + 4 = (x - 2)(x - 2)$

So, the complete factorization of f(x) is:

$f = x(x - 2)^3$

The critical points are x = 0 and x = 2.

Step 2: Apply the wavy curve method

We can now apply the wavy curve method to determine the intervals where f > 0. On a number line, we mark the critical points 0 and 2. Between these critical points, we'll test the intervals (-∞, 0), (0, 2), and (2, +∞).

Test the interval (-∞, 0): Choose x = -1

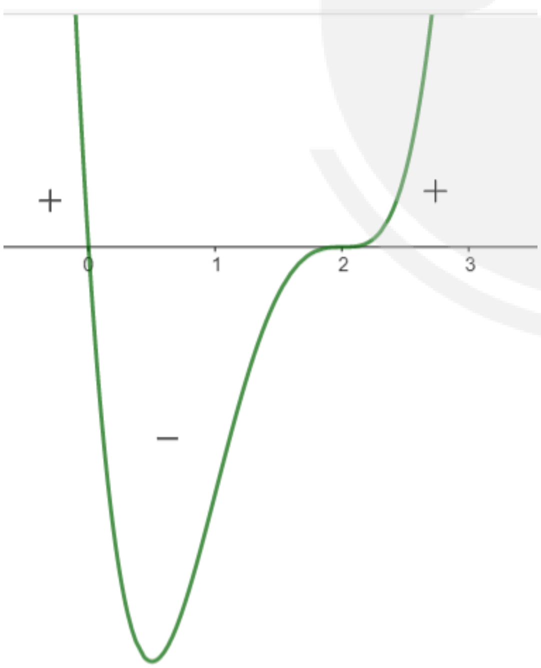
$f = -1(-1 - 2)^3 = -1(-3)^3 = 27 > 0$

Test the interval (0, 2): Choose x = 1

$f = 1(1 - 2)^3 = 1(-1)^3 = -1 < 0$

Test the interval (2, +∞): Choose x = 3

$f = 3(3 - 2)^3 = 3(1)^3 = 3 > 0$



Based on the wavy curve method, f > 0 in the intervals (-∞, 0) and (2, +∞). Thus, the correct answer is:

(A) (-∞, 0) U (2, +∞)

Q4 Text Solution:

To solve the inequality g > 0, we first need to find the critical points, which are the values of x where g = 0. Then, we will use the wavy curve method to determine the intervals where g > 0.

Step 1: Find the critical points

$g = x^5 - 10x^4 + 42x^3 - 92x^2 + 80x$

$g = x\left(x^4 - 10x^3 + 42x^2 - 92x + 80\right)$

Now, we need to find the roots of the quartic equation

$x^4 - 10x^3 + 42x^2 - 92x + 80 = 0.$

Notice that x = 2 and x = 4 are roots:

$2^4 - 10(2)^3 + 42(2)^2 - 92(2) + 80 = 0$

$4^4 - 10(4)^3 + 42(4)^2 - 92(4) + 80 = 0$

Using synthetic division or polynomial long division, we can factor the quadratic equation :

$x^4 - 10x^3 + 42x^2 - 92x + 80$

$= (x - 2)(x - 4)\left(x^2 - 4x + 10\right)$

Now, we have a quadratic equation $x^2 - 4x + 10$. Since its discriminant (Δ = $b^2 - 4ac$ = 16 - 40 = -24) is negative, the quadratic equation has no real roots.

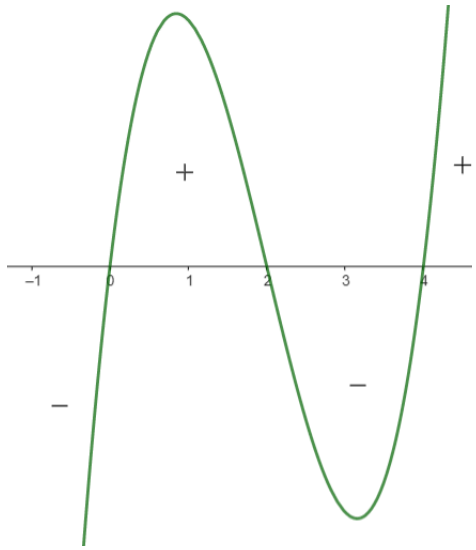
So, the complete factorization of g(x) is:

$g = x(x - 2)(x - 4)\left(x^2 - 4x + 10\right)$

The critical points are x = 0, x = 2, and x = 4.

Step 2: Apply the wavy curve method

We can now apply the wavy curve method to determine the intervals where g > 0. On a number line, we mark the critical points 0, 2, and 4. Between these critical points, we'll test the intervals (-∞, 0), (0, 2), (2, 4), and (4, +∞).



For (0, 2) & (4, +∞) we have a positive value of the function. So, the answer is (0, 2) U (4, +∞).

Q5 Text Solution:

To find which point is in the solution set, we need to test each option to see if they satisfy both inequalities.

A) (3, 2)

2(3) – 3(2) ≤ 6 → 6 – 6 ≤ 6 → 0 ≤ 6 (True)

3 + 2 ≥ 5 → 5 ≥ 5 (True)

B) (2, 3)

2(2) – 3(3) ≤ 6 → 4 – 9 ≤ 6 → -5 ≤ 6 (True)

2 + 3 ≥ 5 → 5 ≥ 5 (True)

C) (1, 3)

2(1) – 3(3) ≤ 6 → 2 – 9 ≤ 6 → -7 ≤ 6 (True)

1 + 3 ≥ 5 → 4 ≥ 5 (False)

D) (4, 1)

2(4) – 3(1) ≤ 6 → 8 – 3 ≤ 6 → 5 ≤ 6 (True)

4 + 1 ≥ 5 → 5 ≥ 5 (True)

Hence, option C is correct.

Q6 Text Solution:

To find which point is in the solution set, we need to test each option to see if they satisfy all inequalities.

1) (1, 2)

1 + 3(2) ≤ 12 → 1 + 6 ≤ 12 → 7 ≤ 12 (True)

2 – 2(1) ≥ -3 → 2 – 2 ≥ -3 → 0 ≥ -3 (True)

2(1) + 2 ≥ 2 → 2 + 2 ≥ 2 → 4 ≥ 2 (True)

2) (2, 3)

2 + 3(3) ≤ 12 → 2 + 9 ≤ 12 → 11 ≤ 12 (True)

3 – 2(2) ≥ -3 → 3 – 4 ≥ -3 → -1 ≥ -3 (True)

2(2) + 3 ≥ 2 → 4 + 3 ≥ 2 → 7 ≥ 2 (True)

3) (3, 1)

3 + 3(1) ≤ 12 → 3 + 3 ≤ 12 → 6 ≤ 12 (True)

1 – 2(3) ≥ -3 → 1– 6 ≥ -3 → -5 ≥ -3 (False)

2(3) + 1 ≥ 2 → 6 + 1 ≥ 2 → 7 ≥ 2 (True)

4) (4, 2)

4 + 3(2) ≤ 12 → 4 + 6 ≤ 12 → 10 ≤ 12 (True)

2 – 2(4) ≥ -3 → 2 – 8 ≥ -3 → -6 ≥ -3 (False)

2(4) + 2 ≥ 2 → 8 + 2 ≥ 2 → 10 ≥ 2 (True)

Hence, only 1 and 2 are correct.

Q7 Text Solution:

To find which point is in the solution set, we need to test each option to see if they satisfy all inequalities.

1) (1, 0)

1 – 0 ≥ 1 → 1 ≥ 1 (True)

2(1) + 0 < 8 → 2 < 8 (True)

0 > 1^2 – 6(1) + 8 → 0 > 3 (False)

2) (3, 1)

3 – 1 ≥ 1 → 2 ≥ 1 (True)

2(3) + 1 < 8 → 7 < 8 (True)

1 > 3^2 – 6(3) + 8 → 1 > -1 (True)

3) (2, 3)

2 – 3 ≥ 1 → -1 ≥ 1 (False)

4) (4, 3)

4 – 3 ≥ 1 → 1 ≥ 1 (True)

2(4) + 3 < 8 → 11 < 8 (False)

Option A is the correct answer.

Q8 Text Solution:

We will examine the coordinates given in the answer choices:

1) (2, 1)

5(2) – 3(1) = 7 ≥ 15 (False)

2) (5, 1)

2(5) + 4(1) = 14 ≤ 12 (False)

3) (1, 1)

5(1) - 3(1) = 2 ≥ 15 (False)

4) (3, 2)

5(3) - 3(2) = 9 ≥ 15 (False)

The only point that lies inside the feasible region of the system of inequalities is the one that satisfies all the inequalities. From the analysis, none of the given points lies inside the feasible region.

Hence, option D is correct.

Q9 Text Solution:

Given, $2x + 5 < 5$(y+1)

=> 2x + 5 < 5y + 5

=> 2x - 5y < 0

=> 2x < 5y ..... (i)

Now, $\frac{7y+6}{y+9} \leq 1$

- $\frac{7y+6}{y+9} - 1 \ \leq \ 0$
- $\frac{7y+6-y-9}{y+9} \leq \ 0$
- $\frac{6y-3}{y+9} \ \leq \ 0$

Now, the critical points are $\frac{1}{2}$ and -9.

Note that, since, at y = -9, the expression will not exist. Also, at $y = \frac{1}{2}$, the inequality will be satisfied.

Now, we need to determine, whether y < -9, or $-9 < y < \frac{1}{2}$

<u>Case 1:</u> y < -9.

Let $y = -10$, then we have

$\frac{6(-10)-3}{-10+9} = 63$, cannot be less or equals 0.

<u>Case 1:</u> $-9 < y < \frac{1}{2}$

Let y = 0, then we have

$\frac{6(0)-3}{0+9} = -\frac{1}{3} \leq \ 0$

So, we can conclude that,

$-9 < y \ \leq \ \frac{1}{2}$

=> $-45 < 5y \ \leq \ \frac{5}{2}$ ...... (ii)

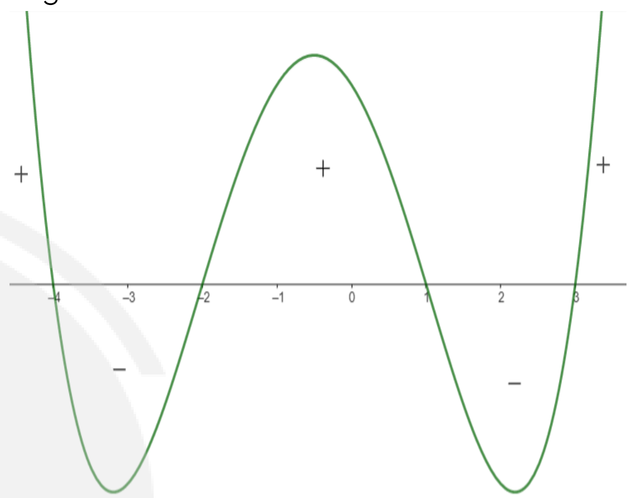
Using (i) and (ii), we have

2x < 5y $\leq \ \frac{5}{2}$

- $2x < \frac{5}{2}$
- $x < \frac{5}{4}$

Q10 Text Solution:

We have the polynomial function f = (x - 1)(x + 2)(x - 3)(x + 4). The critical points for this function are x = 1, x = -2, x = 3, and x = -4. The wavy curve method can be applied to determine the intervals where the function is positive or negative.



The critical points divide the real line into 5 intervals:

1. (-∞, -4)
2. (-4, -2)
3. (-2, 1)
4. (1, 3)
5. (3, +∞)

We will now determine the sign of f in each interval:

- (-∞, -4): Since all factors are negative in this interval, f is positive, i.e., f > 0.
- (-4, -2): Only the factor (x + 4) becomes positive, and the other factors are still negative. Therefore, f(x) is negative, i.e., f < 0.
- (-2, 1): Now, the factors (x + 4) and (x + 2) become positive, while (x - 1) and (x - 3)

remain negative. Thus, f(x) is positive, i.e., f > 0.

- (1, 3): In this interval, the factors (x + 4), (x + 2), and (x - 1) are positive, and only (x - 3) is negative. Hence, f(x) is negative, i.e.,
- f < 0.
- (3, +∞): All factors are positive in this interval, and so f is positive, i.e., f > 0.

We are asked to find the intervals where f < 0.
From our analysis, these intervals are:
(-4, -2) ∪ (1, 3)
Thus, the correct answer is option (B).

Q11 Text Solution:

Given, $9x + 2 = (6y + 2)$

- 3x = 2y .... (i)

Now, $\frac{5y+6}{y+1} \leq 2$

=> $\frac{5y+6}{y+1} - 2 \leq 0$

=> $\frac{5y+6-2(y+1)}{y+1} \leq 0$

=> $\frac{3y+4}{y+1} \leq 0$

Now, the critical points are $-\frac{4}{3}$ and -1.

Note that, since, at y = -1, the expression will not exist. Also, at $y = -\frac{4}{3}$, the inequality will be satisfied.

Now, we need to determine, whether y < $-\frac{4}{3}$, or $-\frac{4}{3} < y < -1$ or, $y > -1$.

<u>**Case 1:**</u> y < $-\frac{4}{3}$

Let y = -2, then we have

$\frac{3(-2)+4}{-2+1} = 2$, cannot be ≤ 0.

<u>**Case 2:**</u> $-\frac{4}{3} < y < -1$

Let $y = -\frac{6}{5}$, then we have

$\frac{3(-\frac{6}{5})+4}{-\frac{6}{5}+1} = -2 \leq 0$

<u>**Case 3:**</u> $y > -1$

Let y = 0, then we have

$\frac{3(0)+4}{0+1} = 4$, cannot be ≤ 0.

So, we can conclude that,

$-\frac{4}{3} \leq y < -1$

=> $-\frac{8}{3} \leq 2y < -2$ ..... (ii)

Using (i) and (ii), we have

3x = 2y < -2

- 3x < -2
- $x < -\frac{2}{3}$

$-\frac{8}{3} \leq 3x$

=> $-\frac{8}{9} \leq x < -\frac{2}{3}$

Q12 Text Solution:

Min (3x + 2y + 5z) can be obtained by multiplying lower numbers with the highest coefficient.

So, min (3x + 2y + 5z) = (3 × 5) + (2 × 7) + (5 × 3) = 44 ≤ 50

If we make z = 5, then y = 7, x = 3

Then, (3x + 2y + 5z) = 3(3) + 2(7) + 5(5) = 48 ≤ 50

If we make z = 7 then y = 5 , x = 3

then (3x + 2y + 5z) = 3(3) + 2(5) + 5(7) = 54 $\geq$ 50

Hence, Z can assume two values.

Q13 Text Solution:

$\frac{2x+3}{4x+1} \geq 2$

$\frac{2x+3}{4x+1} - 2 \geq 0$

=> $\frac{2x+3-2(4x+1)}{4x+1} \geq 0$

=> $\frac{-6x+1}{4x+1} \geq 0$

Now, the critical points are $\frac{1}{6}$ and $-\frac{1}{4}$.

Note that, since, at x = $-\frac{1}{4}$, the expression will not exist. Also, at $x = \frac{1}{6}$, the inequality will be satisfied.

Now, we need to determine, whether x < $-\frac{1}{4}$, or $-\frac{1}{4} < x < \frac{1}{6}$ or, $x > \frac{1}{6}$.

<u>**Case 1:**</u> x < $-\frac{1}{4}$

Let x = -1, then we have

$\frac{-6(-1)+1}{4(-1)+1} = -\frac{7}{3}$, cannot be ≥ 0.

<u>**Case 2:**</u> $-\frac{1}{4} < x < \frac{1}{6}$

Let $x = 0$, then we have

$\frac{-6(0)+1}{4(0)+1} = 1 \geq 0$

<u>**Case 3:**</u> $x > \frac{1}{6}$

Let x = 1, then we have

$\frac{-6(1)+1}{4(1)+1} = -1$, cannot be $\geq\ 0$.

So, we can conclude that,

$-\frac{1}{4} < x\ \leq \frac{1}{6}$ ...... (i)

=> $-6 < 24x\ \leq\ 4$

So, 24x can assume all the values from – 5 to 4.

Thus, the integral numbers that can be assumed by 24x will be 10.

Q14 Text Solution:

- To minimize the total cost we need to minimize the cost of the items. As it is given that the rates of the pens, pencils and erasers are distinct prime numbers when expressed in $ and the sum of rates of unit pen and pencil is equal to that of one eraser, so we can conclude that one of the rates between pen or pencil has to be $2. If all three prime numbers are odd, then it will not hold the above condition true as the sum of two odds is even.
- As the number of pens purchased is the most out of all the items, we can say that to minimize the total cost we need to make sure the minimum cost/unit is assigned to the item having the highest quantity.
- So, the price of the pen = $2/unit
- It is also given that the total items purchased is 6. So, the number of pens purchased is 3, pencils purchased will be 2 and the eraser will be 1.
- To minimize the total cost, the other costs will be $3 and $5. Also, to minimize the overall cost, we need to make sure the higher number is getting multiplied with the lower coefficients.
- So, the price of a pencil is $3/unit and that of eraser is $5/unit.
- Hence, the total cost is ($2 $\times$3 + 3\times$2 + 5\times$ 1) = $17
- Thus the minimum total cost is $17.

Q15 Text Solution:

$\frac{2x+9}{x+1} - 8\ \geq\ 0$

=> $\frac{2x+9-8x-8}{x+1}\ \geq\ 0$

=> $\frac{-6x+1}{x+1}\ \geq\ 0$

Now, the critical points are $\frac{1}{6}$ and -1.

Note that, since, at x = -1, the expression will not exist. Also, at $x = \frac{1}{6}$, the inequality will be satisfied.

Now, we need to determine, whether x < -1, or $-1 < x < \frac{1}{6}$ or, $x > \frac{1}{6}$.

Case 1: x < -1

Let x = -2, then we have

$\frac{-6(-2)+1}{-2+1} = -13$, cannot be $\geq\ 0$.

Case 2: $-1 < x < \frac{1}{6}$

Let x = 0, then we have

$\frac{-6(0)+1}{0+1} = 1 \geq\ 0$

Case 3: $x > \frac{1}{6}$

Let x =1, then we have

$\frac{-6(1)+1}{1+1} = -\frac{5}{2}$, cannot be $\geq\ 0$.

So, we can conclude that,

$-1 < x\ \leq\ \frac{1}{6}$

=> $-12 < 12x\ \leq\ 2$

So, 12x can assume all the values from – 11 to 2.

Thus, the integral values that can be assumed by 12x will be 14.

Q16 Text Solution:

The minimum value of (a + 2b + 3c + 4d + 5e + 6f) can be obtained by making sure that the lower numbers are multiplied with the higher coefficients.

Also, a, b, c, d, e and f can assume minimum values of 1, 2, 3, 4, 5, and 6, where f=1, e = 2, d = 3, c = 4, b = 5, a = 6.

So, min (a + 2b + 3c + 4d + 5e + 6f) = 6 + (2 × 5) + (3 × 4) + (4 × 3) + (5 × 2) + (6 × 1)

- 8m = 56

- m = 7

So, $\frac{8x+2}{x+2} - 7 \geq 0$

=> $\frac{8x+2-7(x+2)}{x+2} \geq 0$

=> $\frac{x-12}{x+2} \geq 0$

Now, x = 12 and x = -2 are the critical points.

Note that x cannot be -2, since at x = -2, the expression will not exist.

Also, at x = 12, the inequality is satisfied.

Now, we need to determine whether x < -2 or -2 < x < 12 or x > 12.

<u>Case 1:</u> x < -2

Let x = -3, then we have

$\frac{-3-12}{-3+2} = 15 \geq 0$

<u>Case 2:</u> -2 < x < 12

Let x = 0, then we have

$\frac{0-12}{0+2} = -6$, cannot be greater than equals 0.

<u>Case 3:</u> x > 12

Let x = 13, then we have

$\frac{13-12}{13+2} = \frac{1}{15} \geq 0$

So, we can conclude that,

x < -2 or x ≥ 12.

Thus, x can assume a minimum positive integer value of 12.

Q17 Text Solution:

To solve the inequality f(x) > 0 using the wavy curve method, we first need to find the critical points of f(x), which are the zeros of the function.

$f(x) = x^4 - 6x^3 + 14x^2 - 15x + 6$

The critical points can be found by setting f(x) to zero:

$x^4 - 6x^3 + 14x^2 - 15x + 6 = 0$

x = 1 and x = 2 are two roots of the equation.

This equation factors into:

$(x - 1)(x - 2)\left(x^2 - 3x + 3\right) = 0$

From this, we can identify the critical points as x1 = 1 and x2 = 2. For the quadratic term, x^2 - 3x + 3, we can use the discriminant to determine if there are any real roots:

$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(3) = 9 - 12 = -3$

Since the discriminant is negative, the quadratic term has no real roots.

Now, we'll test intervals around the critical points to determine the sign of f(x):

x < 1: Choose x = 0, then f(x) = 6 > 0.

1 < x < 2: Choose x = 1.5, then f(x) = -0.125 < 0.

x > 2: Choose x = 3, then f(x) = 6 > 0.

Considering the inequality f(x) > 0, the intervals that satisfy this inequality are:

(-∞, 1) ∪ (2, ∞)

Thus, the correct answer is (B) (-∞, 1) ∪ (2, ∞).

Q18 Text Solution:

To factorize the cubic polynomial $x^3 - 5x^2 + 8x - 4$, we can try to find a common factor or use synthetic division to find one of its linear factors.

In this case, we can use the rational root theorem to test potential rational roots.

The rational root theorem states that if a rational number p/q is a root of a polynomial with integer coefficients, then p must be a factor of the constant term and q must be a factor of the leading coefficient.

For the given polynomial, the possible factors of the constant term, -4, are ±1, ±2, and ±4, and the possible factors of the leading coefficient, 1, are ±1. Therefore, the possible rational roots are ±1, ±2, and ±4.

Now we can test these potential roots in the polynomial:

1: $(1)^3 - 5(1)^2 + 8(1) - 4 = 1 - 5 + 8 - 4 = 0$

We find that x = 1 is a root of the polynomial. The result of the synthetic division is the quadratic factor $x^2 - 4x + 4$. Now, we can factor this quadratic:

$x^2 - 4x + 4 = (x - 2)^2$

So, the complete factorization of the cubic polynomial is:

$x^3 - 5x^2 + 8x - 4 = (x - 1)(x - 2)^2$

To solve the inequality f(x) < 0 using the wavy curve method, we first need to find the critical points of f(x), which are the zeros of the function.

$f(x) = (x - 1)(x - 2)^2$

The critical points can be found by setting f(x) to zero:

$(x - 1)(x - 2)^2 = 0$

From this, we can identify the critical points as 1 and 2. Since 2 has an even multiplicity, it does not change the sign of the function.

Now, we'll test intervals around the critical points to determine the sign of f(x):

x < 1: Choose x = 0, then f(x) = $(-1)(2)^2$ = -4, so f(x) < 0.

Considering the inequality f(x) < 0, the intervals that satisfy this inequality are (-∞, 1)

Q19 Text Solution:

To factorize the cubic polynomial $2x^3 - x^2 - 13x - 6$, we can try to find a common factor or use synthetic division to find one of its linear factors. In this case, we can use the rational root theorem to test potential rational roots.

The rational root theorem states that if a rational number p/q is a root of a polynomial with integer coefficients, then p must be a factor of the constant term and q must be a factor of the leading coefficient.

For the given polynomial, the possible factors of the constant term, -6, are ±1, ±2, ±3, and ±6, and the possible factors of the leading coefficient, 2, are ±1 and ±2. Therefore, the possible rational roots are ±1, ±2, ±3, ±6, ±1/2, and ±3/2.

Now we can test these potential roots in the polynomial:

1: $2(-1)^3 - (-1)^2$ - 13(-1) - 6 = 2 + 1 + 13 - 6 ≠ 0

2: $2(2)^3 - (2)^2$ - 13(2) - 6 = 16 - 4 - 26 - 6 ≠ 0

3: $2(3)^3 - (3)^2$ - 13(3) - 6 = 54 - 9 - 39 - 6 = 0

We find that x = 3 is a root of the polynomial. The result of the synthetic division is the quadratic factor $2x^2$ + 5x + 2. Now, we can factor this quadratic:

$2x^2 + 5x + 2 = (2x + 1)(x + 2)$

So, the complete factorization of the cubic polynomial is:

$2x^3 - x^2 - 13x - 6$

$= (x - 3)(2x + 1)(x + 2)$

To solve the inequality using the wavy curve method, we first identify the critical points, which are the points where the inequality is equal to 0. This occurs when any of the factors are equal to 0:

x - 3 = 0 => x = 3

x + 2 = 0 => x = -2

2x + 1 = 0 => x = $-\frac{1}{2}$

Next, we create a number line including these critical points (-2, $-\frac{1}{2}$, and 3) and divide the number line into intervals based on these points:

Interval 1: x < -2

Interval 2: -2 < x < $-\frac{1}{2}$

Interval 3: - $\frac{1}{2}$ < x < 3

Interval 4: x > 3

Now, we check the sign of the inequality expression in each interval by selecting a test point from each interval:

Interval 1 (x < -2): Test point x = -3

(-3 - 3)(-3 + 2)(2(-3) + 1) = (-6)(-1)(-5) < 0

Interval 2 (-2 < x < - $\frac{1}{2}$): Test point x = -1

(-1 - 3)(-1 + 2)(2(-1) + 1) = (-4)(1)(-1) > 0

Interval 3 (-1/2 < x < 3): Test point x = 0

(0 - 3)(0 + 2)(2(0) + 1) = (-3)(2)(1) < 0

Interval 4 (x > 3): Test point x = 4

(4 - 3)(4 + 2)(2(4) + 1) = (1)(6)(9) > 0

Based on the signs obtained in each interval, the inequality (x - 3)(x + 2)(2x + 1) > 0 is true for intervals 2 and 4. Therefore, the solution set for the inequality is:

$x \in (-2, -\frac{1}{2}) \cup (3, \infty)$

The correct answer is option C.

Q20 Text Solution:

To find the intervals where h is positive or negative, we first need to find the critical points of the two factors. The critical points are the roots of the equations

$x^3 - 4x - x^2 + 4 = 0$

$x^3 - 4x - x^2 + 4$

$= x^3 - x^2 - 4x + 4$

$= x^2(x - 1) - 4(x - 1)$

$= (x^2 - 4)(x - 1)$

$= (x + 2)(x - 1)(x - 2)$

Now, we will use the wavy curve method.

Thus, h is positive for $x \in (-2, 1) \cup (2, \infty)$

h is negative for $x \in (-\infty, -2) \cup (1, 2)$.

Q21 Text Solution:

Let's rewrite the inequality by making a substitution: let $y = x^3$. Then $\frac{1}{x^3} = y^{-1}$. The inequality becomes:

$y^2 + y^{-2} - 4(y + y^{-1}) + 6 \leq 0$

Now, let's multiply throughout by y^2 to get rid of the fractions:

$y^4 + 1 - 4y^3 - 4y + 6y^2 \leq 0$

Rearrange the terms to form a polynomial:

$y^4 - 4y^3 + 6y^2 - 4y + 1 \leq 0$

The above inequality can be written as,

$y^4 + y^2 - 2y^3 + y^2 + 1 - 2y - 2y^3 - 2y + 4y^2 \leq 0$

$y^2(y^2 - 2y + 1) + 1(y^2 - 2y + 1) - 2y(y^2 - 2y + 1) \leq 0$

$(y^2 - 2y + 1)(y^2 - 2y + 1) \leq 0$

$(y - 1)^2 (y - 1)^2 \leq 0$

Now, we want to find the critical points of this polynomial. Notice that the polynomial can be factored as:

$(y - 1)^4 \leq 0$

The critical point is y = 1. Since $y = x^3$, we have $x^3 = 1$, which gives us x = 1 (since the real cube root of 1 is 1). The inequality holds true at this critical point, as $(y - 1)^4$ is always non-negative and equals zero only when y = 1.

Q22 Text Solution:

To solve the inequality $(x^2 - 4)(x^2 - x - 6) < 0$ using the wavy curve method, we first need to find the critical points.

$(x^2 - 4)(x^2 - x - 6) < 0$

The critical points can be found by setting the inequality to zero:

$(x^2 - 4)(x^2 - x - 6) = 0$

This equation factors further into:

(x - 2)(x + 2)(x - 3)(x + 2) = 0

From this, we can identify the critical points as $x_1 = -2$, $x_2 = 2$, and $x_3 = 3$.

Now, we'll test intervals around the critical points to determine the sign :

x < -2: Choose x = -3, then f(x) = -5 × (-1)(-1)(-6) = 30 > 0.

-2 < x < 2: Choose x = 0, then f(x) = (-4)(-6) = 24 > 0.

2 < x < 3: Choose x = 2.5, then f(x) = (.5)(4.5)(-.5)(4.5) = -5.0625 < 0.

x > 3: Choose x = 4, then f(x) = 2 × 6 × 1 × 6 = 72 > 0.

Considering the inequality f(x) < 0, the intervals that satisfy this inequality are: (2, 3)

Thus, the number of integral values that 3x can assume is 2.

Q23 Text Solution:

$x^4 + \left(\frac{1}{x^4}\right) - 2\left(x^2 + \frac{1}{x^2}\right) + 2 \leq 0$

- $\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right)\leq 0$
- $(x^2+\frac{1}{x^2})(x^2+\frac{1}{x^2}-2)\leq 0$
- $0\leq\left(x^2+\frac{1}{x^2}\right)\leq 2$
- $0+2\leq\left(x^2+\frac{1}{x^2}+2\right)\leq 2+2$
- $2\leq\left(x^2+\frac{1}{x^2}+2\right)\leq 4$
- $2\leq(x+\frac{1}{x})^2\leq 4$

The above inequality holds true for x = 1 and -1

Hence, option D is correct.

Q24 Text Solution:

Let x be the number of customers the company per day can sell metal wires to. If it needs to sell to the least number of customers, then it needs to make sure that it sells the maximum possible weight to all the customers.

Then, the total weight of metal wire sold to all the customers is 20x kilograms, and the total cost is 200 × 20x = $4000x dollars. The constraints can be written as:

4000x ≤ 12000

=> x ≤ 3.

Q25 Text Solution:

To solve the inequality by using the wavy curve method, we first need to find the critical points.

$x^4-10x^3+35x^2-50x+24<0$

The critical points can be found by setting the inequality to zero:

$x^4-10x^3+35x^2-50x+24=0$

If we put x = 1, then we get the above expression equal to 0.

To divide the polynomial $x^4-10x^3+35x^2-50x+24$ by (x - 1), we can use synthetic division or polynomial long division.

Therefore,

$\frac{x^4-10x^3+35x^2-50x+24}{x-1}=x^3-9x^2+26x-24$

Similarly at x = 2 also it gives 0 for $x^3-9x^2+26x-24$

To divide the polynomial $x^3-9x^2+26x-24$ by (x - 2), we can use synthetic division or polynomial long division.

Therefore, $\frac{x^3-9x^2+26x-24}{x-2}=x^2-7x+12$

$=(x-3)(x-4)$

This equation factors into:

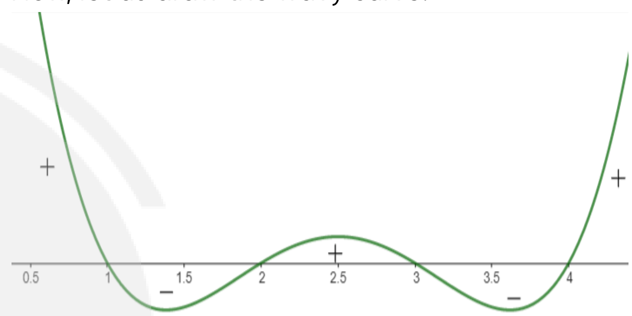
(x - 1)(x - 2)(x - 3)(x - 4) = 0

Therefore, we have

$x^4-10x^3+35x^2-50x+24<0$

=> (x - 1)(x - 2)(x - 3)(x - 4) < 0

Now, let us draw the wavy curve:



Considering the inequality is < 0, the intervals that satisfy this inequality are:

$x\in$ (1, 2) ∪ (3, 4)

So, 4x is in (4, 8) ∪ (12, 16)

Thus, the number of integral values of 4x can be assumed is 6.

Q26 Text Solution:

Let x m be the width of the field, then the length of the field is (100 – x)

The area of the field which is fenced A = x(100 – x) = 100x – x^2

So,

A = 2500 + 100x – x^2 – 2500

- A = 2500 – (50 – x)^2

A will be maximum when x = 50.

- Max(A) = 2500 - $(50 - 50)^2$

Thus, the maximum area which can be fenced is 2500 square meters.

Q27 Text Solution:

Given that,

$x^3 - 6x^2 + 11x < 6$

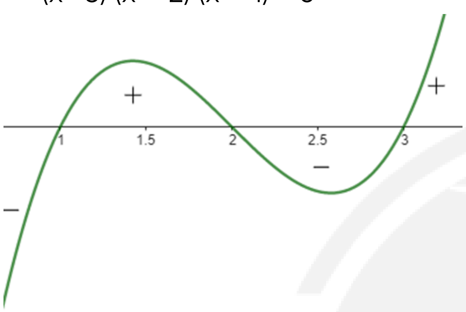
=> $x^3 - x^2 - 5x^2 + 6x + 5x - 6 < 0$

=> $x^3 - 5x^2 + 6x - x^2 + 5x - 6 < 0$

=> $(x^2 - 5x + 6)(x-1) < 0$

=>$(x^2 - 2x - 3x + 6)(x - 1) < 0$

=> $(x - 3)(x - 2)(x - 1) < 0$



i.e., $x < 1$ and $2 < x < 3$

Again, $x^3 - 11x^2 + 34x < 24$

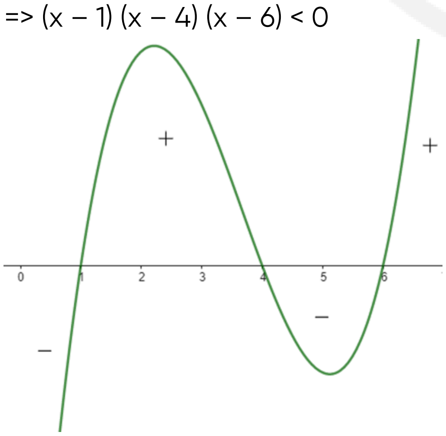
=> $x^3 - x^2 - 10x^2 + 24x + 10x - 24 < 0$

=> $x^3 - 10x^2 + 24x - x^2 + 10x - 24 < 0$

=> $(x^2 - 10x + 24)(x - 1) < 0$

=> $(x^2 - 4x - 6x + 24)(x - 1) < 0$

=> $(x - 1)(x - 4)(x - 6) < 0$



Therefore, $x < 1$, $4 < x < 6$

So, combining the two conditions, we get $x < 1$.

Q28 Text Solution:

To solve the inequality we will first find the critical points, which we can find by finding the roots of the equation,

$x^3 - 8x^2 + 11x + 20 > 0$

To find the roots, we can try to factor the polynomial:

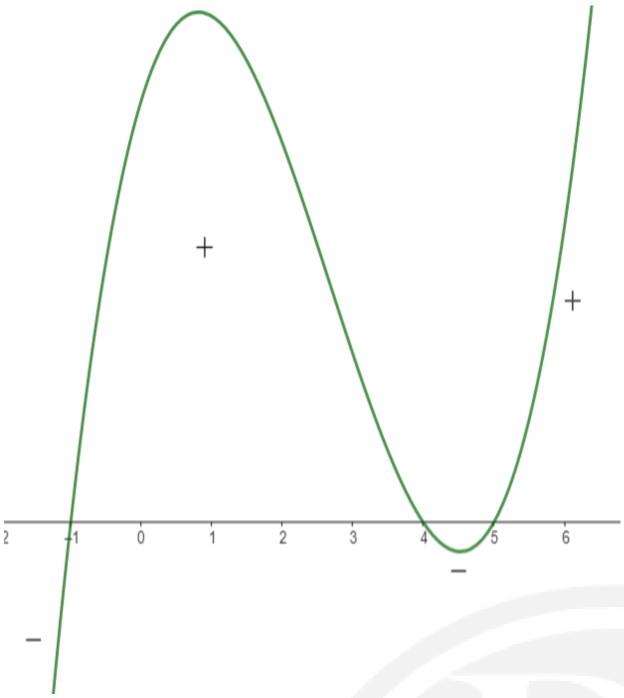
$(x - 5)(x + 1)(x - 4) > 0$

Now we have three roots: $x = -1$, $x = 4$, and $x = 5$. These are our critical points.

Next, we will use the wavy curve method to determine the sign of the polynomial in each interval created by the critical points:

1. $(-\infty, -1)$: Choose a test point, say $x = -2$, the value is $= (-2 - 5)(-2 + 1)(-2 - 4) = (-)(-)(-) < 0$ (Not possible)
2. $(-1, 4)$: Choose a test point, say $x = 0$. The value is $= (0 - 5)(0 + 1)(0 - 4) = (-)(+)(-) > 0$. (Possible)
3. $(4, 5)$: Choose a test point, say $x = 4.5$. The value is $= (4.5 - 5)(4.5 + 1)(4.5 - 4) = (-)(+)(+) < 0$. (Not possible)
4. $(5, \infty)$: Choose a test point, say $x = 6$. The value is $= (6 - 5)(6 + 1)(6 - 4) = (+)(+)(+) > 0$ (Possible)

Now we know that the expression is positive in intervals $(-\infty, -1)$ and $(4, \infty)$.



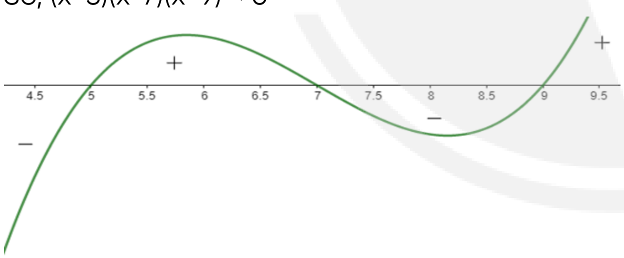
Therefore, the correct answer is: option B) (-1, 4) and (5, ∞)

Q29 Text Solution:

$(x-3)^2(x-5)^5(x-7)^3(x-9)^7 < 0$

As, $(x-3)^2 \geq 0$, $(x-5)^4 \geq 0$, $(x-7)^2 \geq 0$ and $(x-9)^6 \geq 0$ [Since, even powered x is always greater than equals zero]

So, $(x-5)(x-7)(x-9) < 0$



So, $x < 5$ and $7 < x < 9$ and $x \neq 3$

Thus, the positive integer values that x can assume are 1, 2, 4, and 8.

Q30 Text Solution:

To solve this inequality, we need to factorize the expression and determine the critical points. We will use the wavy curve method to determine the intervals of x where P > 0.

First, we factorize the given expression:

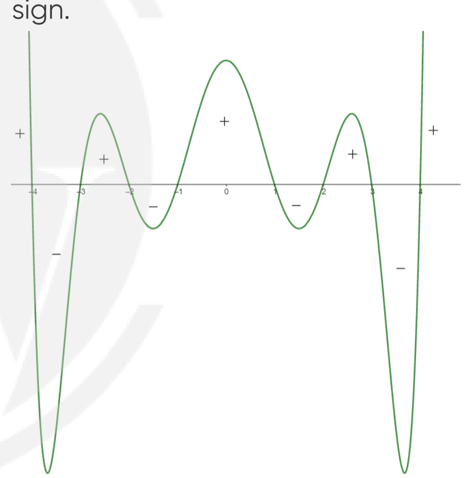
$$P = (x^2 - 9)(x^2 - 4)(x^2 - 1)(x^2 - 16)$$

The critical points occur where each factor is equal to zero:

1. $x^2 - 9 = 0 \Rightarrow x = \pm 3$
2. $x^2 - 4 = 0 \Rightarrow x = \pm 2$
3. $x^2 - 1 = 0 \Rightarrow x = \pm 1$
4. $x^2 - 16 = 0 \Rightarrow x = \pm 4$

Now, we will construct a wavy curve based on these critical points:

We start with a positive sign at the rightmost end and then alternate signs while moving towards the left side. The signs indicate the overall value of the expression in each interval. Since we are looking for the intervals where P > 0, we will select the intervals with a positive sign.



The intervals where P > 0 are: (-∞, -4) ∪ (-3, -2) ∪ (-1, 1) ∪ (2, 3) ∪ (4, ∞)

The correct answer is option (A).