MBA PIONEER PRO 2024

QUANTITATIVE APTITUDE

DPP: 3

Quadratic Equation - 1

Q1 What is the difference between the two roots of equation
$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{6}$?
(A) Zero (B) -1
(C) 1 (D) 5

Q2 In a quadratic equation of the form $ax^2 + bx + c = 0$, which of the following can be zero?
(A) Product of a and c
(B) Sum of a and c
(C) Difference between a and c
(D) All of the above

Q3 $2x^2 + bx + 8 = 0$ has non-real roots. How many integer values can 'b' take?
(A) 16 (B) 15
(C) 18 (D) 19

Q4 For how many natural number values of k will the equation $x^2 + x + k = 0$ have real roots?
(A) 2
(B) 0
(C) Many
(D) Cannot be determined

Q5 Let the quadratic equation $x^2 + \alpha x + \beta = 0$ have the distinct roots γ and δ. Then, sum of the roots of the quadratic equation $x^2 + (2\gamma + \alpha)x + \gamma^2 + \alpha\gamma + \beta = 0$ is :
(A) γ
(B) $\delta - \gamma$
(C) 2γ
(D) 2δ

Q6 The roots of the quadratic equation $ax^2 + bx + c = 0$ are real and equal. If the sum of the roots is equal to the sum of the coefficients a, b and c, what is the value of $(a + \frac{b}{2})^2$?
(A) -b (B) b
(C) -c (D) c

Q7 Construct a quadratic equation whose roots are 5 less than the roots of the equation $x^2 - 13x + 42 = 0$.
(A) $x^2 - 5x + 2 = 0$
(B) $x^2 - 2x + 5 = 0$
(C) $x^2 - 5x + 3 = 0$
(D) $x^2 - 3x + 2 = 0$

Q8 Compare the roots of the following quadratic equations:
I. $8x^2 + 34x - 117 = 0$
II. $12y^2 + 35y - 250 = 0$
(A) $x \le y$
(B) $x \ge y$
(C) $x < y$
(D) Relationship between x and y can't be established.

Q9 Compare the roots of the following quadratic equations:
I. $x^2 - 28x + 195 = 0$
II. $y^2 - 25y + 114 = 0$
(A) $x \le y$
(B) $x \ge y$
(C) $x < y$
(D) the relationship between x and y can't be established.

Q10 Compare the roots of the following quadratic equations:
I. $6x^2 - 37x + 56 = 0$
II. $27y^2 + 66y + 35 = 0$
(A) $x \le y$
(B) $x \ge y$
(C) $x > y$
(D)

Relationship between x and y can't be established.

Q11 Compare the roots of the following quadratic equations:

I. $121x^2 - 22x - 15 = 0$

II. $15y^2 + 22y - 121 = 0$

(A) $x \le y$

(B) $x \ge y$

(C) $x < y$

(D) Relationship between x and y can't be established.

Q12 Compare the roots of the following quadratic equations:

I. $84x^2 + 193x + 84 = 0$

II. $63y^2 + 130y + 63 = 0$

(A) $x \le y$

(B) $x \ge y$

(C) $x < y$

(D) Relationship between x and y can't be established.

Q13 Construct a quadratic equation whose roots are square roots of the roots of the equation $x^2 - 41x + 400 = 0$ if it is given that all the roots of the new equation are positive.

(A) $x^2 - 9x + 20 = 0$

(B) $x^2 - x \pm 20 = 0$

(C) $x^2 + 9x - 400 = 0$

(D) $x^2 \pm x \pm 20 = 0$

Q14 The common root of $6x^2 - 19x - 130 = 0$ and $12y^2 - 82x + 26 = 0$ is:

(A) $\frac{10}{3}$

(B) $-\frac{1}{3}$

(C) $\frac{19}{26}$

(D) $\frac{13}{2}$

Q15 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$ then find the equation whose roots are exactly 1 more than α, β.

(A) $(x^2 + 1) = 0$

(B) $(x^2 + 4x + 1) = 0$

(C) $(x^2 + x + 1) = 0$

(D) $(x^2 + 4) = 0$

Q16 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$ then find the equation whose roots are reciprocals of α, β having coefficient of x^2 as 2.

(A) $2x^2 + 2x + 1$

(B) $2x^2 + x + 1$

(C) $x^2 + 2x + 1$

(D) $x^2 + x + 1$

Q17 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$, then find the equation whose coefficient of x^2 is 1 and whose roots are α^4 & β^4

(A) $(x^2 - 8x + 16) = 0$

(B) $(x^2 + 8x + 16) = 0$

(C) $(x^2 + 2x + 4) = 0$

(D) $(x^2 + 4x + 16) = 0$

Q18 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$, then find the equation whose coefficient of x^2 is 1 and whose roots are α^2 & β^2

(A) $(x^2 + 4) = 0$

(B) $(x^2 + 2) = 0$

(C) $(x^2 + 2x + 4) = 0$

(D) $(x^2 + 4x + 2) = 0$

Q19 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$ and γ, δ be the roots of the equation $x^2 - 2x + 2 = 0$, then find the value of $(\alpha^4 + \beta^4)(\gamma^6 + \delta^6)$.

(A) 2^3

(B) 0

(C) 2

(D) 2^5

Q20 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$, then find the equation whose co-efficient of x^2 is 1 and whose roots are $(\alpha + \beta)^2$ and $\alpha^2 + \beta^2$.

(A) $x^2 - 4x = 0$

(B) $(x^2 + 2^{2023}) = 0$

(C) $(x^2 + 8x + 2^{2023}) = 0$

(D) $(x^2 + 4x + 2^{2023}) = 0$

Q21 If α, β are the roots of the equation $x^2 + 2x + 2 = 0$, then find the equation whose coefficient of x^2 is 1 and whose roots are α^6 & β^6 ?

(A) $(x^2 + 4) = 0$

(B) $(x^2 + 64) = 0$

(C) $(x^2 + 8x + 16) = 0$

(D) $(x^2 + 4x + 8) = 0$

Q22 If $2^{(2x+2)} + 2^{(x+2)} = 24$, then find the value of $(x^{2023} + x^{2022} + x^{2021})$.
(A) $x^2 + 1$
(B) $2^x + x^2$
(C) $x + 2$
(D) Both option b & c

Q23 The roots of the equation $x^2 - 3x + 1 = 0$ are p & q then find the quadratic equation whose roots are p^2 & q^2
I. $(x^2 + 1) = 14x$
II. $(2x^2 + 2) = 14x$
III. $x = \frac{x^2+1}{7}$
(A) Only I
(B) Only I & III
(C) Only II & III
(D) Only III

Q24 Let p & q are the real roots of a quadratic equation where p > q. The quadratic equation having the roots $\frac{1}{p}$ and $\frac{1}{q}$ is $6x^2 - 5x + 1 = 0$. Then find the quadratic equation whose roots are $\frac{p+q}{p-q}$ and pq.
I. $3x^2 - 33x + 90 = 0$
II. $11x - x^2 - 30 = 0$
III. $11x + x^2 - 30 = 0$
(A) Only I
(B) Only I & II
(C) Only I & III
(D) Only III

Q25 The roots of the quadratic equation $ax^2 + bx - c = 0$ are real and unequal. If the square of the difference between the roots is equal to the product of the coefficients a, b and c divided by the square of a, what is the value $(b^2 + 4ac)$?
(A) The sum of the roots
(B) The product of the roots
(C) Sum of the coefficients
(D) Product of the coefficients

Q26 If equations $x^2 + ax + 20 = 0$ and $x^2 + bx - 20 = 0$ have one common root, then what is the value of $a^2 - b^2$?
(A) 75 (B) 80
(C) 85 (D) 90

Q27 Let the quadratic equation $x^2 - \alpha x + \beta = 0$ has the distinct roots p, q. If the roots of another quadratic equation are $p^2 - q^2$ and $p^3 - q^3$, then the product of the roots is equal to :
(A) $\alpha(\alpha^2 - \beta)^2$
(B) $\alpha(\alpha^2 - \beta)(\alpha^2 - 4\beta)$
(C) $\alpha(\alpha^2 - 4\beta)(\alpha^2 + \beta)$
(D) None of above

Q28 In the equation $x^2 + kx + 1 = 0$, if the roots are real, then what can be the possible range of values of k?
(A) $k \leq -2$
(B) $k \geq 2$
(C) $-2 \leq k \leq 2$
(D) Both A and B

Q29 The roots of the equation $x^2 + kx = k$ are imaginary. If k is real, then find its possible range.
(A) $k < (-4)$, and $0 < k$
(B) $-4 < k < 0$
(C) $k = (-4)$
(D) Both B and C

Q30 $(x - a)(x - b) = 0$ is a quadratic polynomial whose roots are a and b.
If $x(4) = -\frac{1}{4}x(1)$ and b = 5, then find the value of 4a.
(A) 4 (B) 6
(C) 8 (D) 10

Answer Key

Q1	(D)
Q2	(D)
Q3	(B)
Q4	(B)
Q5	(B)
Q6	(A)
Q7	(D)
Q8	(D)
Q9	(D)
Q10	(C)
Q11	(D)
Q12	(D)
Q13	(A)
Q14	(D)
Q15	(A)
Q16	(A)
Q17	(B)
Q18	(A)
Q19	(B)
Q20	(A)
Q21	(B)
Q22	(D)
Q23	(C)
Q24	(B)
Q25	(D)
Q26	(B)
Q27	(B)
Q28	(D)
Q29	(B)
Q30	(D)

Hints & Solutions

Q1 Text Solution:

$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{6}$$
$$\frac{x+1-x}{x^2+x} = \frac{1}{6}$$
$$6 = x^2 + x$$
$$x^2 + x - 6 = 0$$
$$(x-2)(x+3) = 0$$
$$x = 2 \text{ or } x = (-3)$$

So, the difference is $2-(-3)=5$ or $-3-2=-5$.

Required difference will be 5 and -5 .

-5 is not in the options so +5 will be the correct choice here.

Q2 Text Solution:

From the theory of quadratic equations, we know that 'a' cannot be zero. But 'c' can be zero. So, obviously, the product of 'a' and 'c' can be zero.

Now, both 'a' and 'c' can be equal. So, their difference can be zero.

In , a – c = 0

Again, a + c can also be zero. In , a + c = 0

Hence, all the options can be zero.

Hence, option (D) is the correct answer.

Q3 Text Solution:

For non-real roots, discriminant D < 0.

$b^2 - 4ac < 0$

$b^2 - 4 \times 2 \times 8 < 0$

$b^2 < 64$

$-8 < b < 8$

Then, the possible values of b are −7, −6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6, and 7.

Hence, option B is the correct answer.

Q4 Text Solution:

The discriminant D = $1^2 - 4(1)(k) = 1 - 4k$

The roots to be real, D must be non-negative.

So, $1 - 4k \geq 0$

$\Rightarrow -4k \geq -1$

$\Rightarrow 4k \leq 1$

$\Rightarrow k \leq$

So, k must be less than or equal to

So, there are no natural number values of k.

Hence, option (B) is the correct answer.

Q5 Text Solution:

Given that, roots of the equation $x^2 + \alpha x + \beta = 0$ are γ and δ.

So, $\gamma + \delta = -\alpha$ .... (i)

$\gamma\delta = \beta$ .... (ii)

The sum of the roots of the equation $x^2 + (2\gamma + \alpha)x + \gamma^2 + \alpha\gamma + \beta = 0$ is

$-(2\gamma + \alpha) = -(2\gamma - \gamma - \delta) = \delta - \gamma$

Option (b) is correct.

Q6 Text Solution:

The given quadratic equation is $ax^2 + bx + c = 0$

Now, since the roots are real and equal, so D = 0.

$b^2 = 4ac$ ..... (i)

Also, it is given that, the sum of the roots = a + b + c

Then, $-\frac{b}{a} = a + b + c$

=> $-b = a(a+b+c)$

=> $-b = a^2 + ab + ac$

On multiplying by 4 on both sides we get,

=> $-4b = 4a^2 + 4ab + 4ac$

=> $-4b = 4a^2 + 4ab + b^2$ [By using (i)]

=> $-4b = (2a+b)^2$

=> $(a + \frac{b}{2})^2 = -b$

Q7 Text Solution:

The given equation is $x^2 - 13x + 42 = 0$.

Then, $x^2 - (6 + 7)x + 42 = 0$

=> $x^2 - 6x - 7x + 42 = 0$

=> $x(x - 6) - 7(x - 6) = 0$

=> $(x - 7)(x - 6) = 0$

=> $x = 7, 6$

Then, the roots of the quadratic equation whose roots are 5 less than the roots of the equation $x^2 - 13x + 42 = 0$, are 2, 1.

So, the required quadratic equation is

=> $x^2 – (2 + 1)x + 2 = 0$

=> $x^2 – 3x +2 = 0$.

Q8 Text Solution:

The given equation is $8x^2 + 34x - 117 = 0$

=> $8x^2 + (52 - 18)x - 117 = 0$

=> $8x^2 + 52x – 18x -117 = 0$

=> $4x(2x + 13) – 9(2x + 13) = 0$

=> $(4x – 9)(2x+13) = 0$

=> $x = \frac{9}{4}, -\frac{13}{2}$

Also, the other equation is $12y^2 + 35y - 250 = 0$

=> $12y^2 + (75 - 40)y -250 = 0$

=> $12y^2 +75y – 40y - 250 = 0$

=> $3y(4y + 25) – 10(4y +25) = 0$

=> $(3y – 10)(4y + 25) = 0$

=> $y = \frac{10}{3}, y = -\frac{25}{4}$

Since, $\frac{10}{3} > \frac{9}{4}$, and $\frac{10}{3} > -\frac{13}{2}$, but $-\frac{25}{4} < \frac{9}{4}$.

As one of the values of x is more than one value of y and the other one is less than the other value of y, we cannot establish any relation between x & y.

So, the relation cannot be established.

Q9 Text Solution:

The given equation is $x^2 – 28x + 195 = 0$

=> $x^2 – (13 + 15)x + 195 = 0$

=> $x^2 – 13x – 15x + 195 = 0$

=> $x(x-13) – 15(x-13) = 0$

=> $(x-15)(x-13) = 0$

=> $x = 15, 13$

The other equation is $y^2 – 25y + 114 = 0$

=> $y^2 – (6+19)y + 114 = 0$

=> $y^2 – 6y -19y + 114 = 0$

=> $y(y-6) - 19(y-6) = 0$

=> $(y-19)(y-6) = 0$

=> $y = 6, 19$

As one of the values of x is more than one value of y and the other one is less than the other value of y, we cannot establish any relation between x & y.

Therefore, the roots of the two equations cannot be compared.

Q10 Text Solution:

The given equation is $6x^2 – 37x + 56 = 0$

=> $6x^2 – (16 + 21)x + 56 = 0$

=> $6x^2 – 16x - 21x + 56 = 0$

=> $2x(3x – 8) – 7(3x – 8) = 0$

=> $(2x – 7) (3x – 8) = 0$

=> $x = \frac{7}{2}, \frac{8}{3}$

Also, the other equation is $27y^2 + 66y + 35 = 0$

=> $27y^2 + 45y +21y + 35 = 0$

=> $9y(3y+5) + 7(3y+5) = 0$

=> $(9y+7)(3y+5) = 0$

=> $y = -\frac{7}{9}, -\frac{5}{3}$

Clearly, it can be seen that, $x > y$.

Hence, option C is correct.

Q11 Text Solution:

The given equation is $121x^2 - 22x - 15 = 0$

=> $121x^2 – (55 – 33)x - 15 = 0$

=> $121x^2 – 55x + 33x -15 = 0$

=> $11x(11x – 5) + 3(11x – 5) = 0$

=> $(11x+3)(11x-5) = 0$

=> $x = -\frac{3}{11}, \frac{5}{11}$

Also, the other equation is $15y^2 + 22y - 121 = 0$

=> $15y^2 + (55 – 33)y -121 = 0$

=> $15y^2 + 55y – 33y – 121 = 0$

=> $5y(3y + 11) – 11(3y + 11) = 0$

=> $(5y -11)(3y+11) = 0$

=> $y = \frac{11}{5}, -\frac{11}{3}$

It can be clearly seen that, $\frac{11}{5} > -\frac{3}{11}$ and $\frac{11}{5} > \frac{5}{11}$, but $-\frac{11}{3} < -\frac{3}{11}$.

As one of the values of x is more than one value of y and the other one is less than the other value of y, we cannot establish any relation between x & y.

Hence, the relation cannot be established.

Q12 Text Solution:

The given equation is $84x^2 +193x + 84 = 0$

=> $84x^2 + (144 + 49)x + 84 = 0$

=> $84x^2 + 144x + 49x + 84 = 0$

=> $12x(7x + 12) + 7(7x + 12) = 0$

=> $(12x +7)(7x + 12) = 0$

=> $x = -\frac{7}{12}, -\frac{12}{7}$

The other equation is $63y^2 + 130y + 63 = 0$

=> $63y^2 + (81 + 49)y + 63 = 0$

=> $63y^2 + 81y + 49y + 63 = 0$

=> $9y(7y + 9) + 7(7y + 9) = 0$

=> $(9y + 7)(7y + 9) = 0$

=> $y = -\frac{7}{9}, -\frac{9}{7}$

It is clear that, $-\frac{7}{9} < -\frac{7}{12}$, but $-\frac{7}{9} > -\frac{12}{7}$

As one of the values of x is more than one value of y and the other one is less than the other value of y, we cannot establish any relation between x & y.

Hence, the relation cannot be established.

Q13 Text Solution:

The given equation is $x^2 - 41x + 400 = 0$.

Then, $x^2 - (25 + 16)x + 400 = 0$

=> $x^2 - 25x - 16x + 400 = 0$

=> $x(x - 25) - 16(x - 25) = 0$

=> $(x - 16)(x - 25) = 0$

=> $x = 16, 25$

Then, the roots of the quadratic equation whose roots are square roots of the roots of the equation $x^2 - 41x + 400 = 0$, are 4, 5.

(One can directly eliminate the negative roots as per the last condition given in the question)

Therefore, the required quadratic equation is

$x^2 - (4 + 5)x + 20 = 0$

=> $x^2 - 9x + 20 = 0$.

Q14 Text Solution:

The given equations are

$6x^2 - 19x - 130 = 0$ .... (i)

$12y^2 - 82x + 26 = 0$ .... (ii)

From (i), we have

$6x^2 - 19x - 130 = 0$

=> $6x^2 - (39 - 20)x - 130 = 0$

=> $6x^2 - 39x + 20x - 130 = 0$

=> $3x(2x - 13) + 10(2x - 13) = 0$

=> $(3x + 10)(2x - 13) = 0$

=> $x = -\frac{10}{3}, \frac{13}{2}$

From equation (ii), we have

$12y^2 - 82y + 26 = 0$

=> $2(6y^2 - 41y + 13) = 0$

=> $6y^2 - (39 + 2)y + 13 = 0$

=> $6y^2 - 39y - 2y + 13 = 0$

=> $3y(2y - 13) - 1(2y - 13) = 0$

=> $(3y - 1)(2y - 13) = 0$

=> $y = \frac{1}{3}, y = \frac{13}{2}$

Hence, the common root is $\frac{13}{2}$.

Q15 Text Solution:

Given that from equation $x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

The roots of the new equation are $(\alpha + 1)$ and $(\beta + 1)$

So,

$(\alpha + 1) + (\beta + 1)$

$= 0$

$(\alpha + 1)(\beta + 1)$

$= \alpha\beta + (\alpha + \beta) + 1$

$= 1$

So, the equation whose roots are $(\alpha + 1)$ and $(\beta + 1)$ is

$(x^2 + 1) = 0$

Q16 Text Solution:

Given that from equation $x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

The roots of the new equation are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

So,

$(\frac{1}{\alpha} + \frac{1}{\beta})$

$= \frac{\alpha + \beta}{\alpha\beta}$

$= -1$

The product of the roots are $\frac{1}{\alpha\beta} = \frac{1}{2}$

So, the equation is

$2(x^2 + x + \frac{1}{2})$

$= 2x^2 + 2x + 1$.

Q17 Text Solution:

Given that $- x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 4$

$= 0$

$(\alpha^4 + \beta^4) = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= -2^3$

$= -8$

Hence, the sum of the roots is -8

Also,

$\alpha^4\beta^4 = (\alpha\beta)^4 = 16$

Thus, the equation having roots α^4 & β^4 is

$K(x^2 + 8x + 16) = 0$

Given that K = 1, we get

$(x^2 + 8x + 16) = 0$

Q18 Text Solution:

Given that – $x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 4$

$= 0$

Also,

$\alpha^2\beta^2 = (\alpha\beta)^2 = 4$

Thus, the equation having roots α^2 & β^2 is

$K(x^2 + 4) = 0$

Given that K = 1, we get

$(x^2 + 4) = 0$

Q19 Text Solution:

Given that from equation $x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 4$

$= 0$

$(\alpha^4 + \beta^4) = (\alpha^2 + \beta^2)^2 - 2\,\alpha^2\beta^2$

$\alpha^4 + \beta^4 = -2^3$

Also given that from equation $x^2 - 2x + 2 = 0$

$(\gamma + \delta) = 2$

$\gamma\delta = 2$

$(\gamma^2 + \delta^2) = (\gamma + \delta)^2 - 2\gamma\delta$

$= 4 - 4$

$(\gamma^2 + \delta^2) = 0$

On cubing both sides we get,

$(\gamma^2 + \delta^2)^3 = 0^3$

$\gamma^6 + \delta^6 + 3\gamma^2\delta^2 (\gamma^2 + \delta^2) = 0$

$\gamma^6 + \delta^6 = 0$

Hence,

$(\gamma^6 + \delta^6)(\alpha^4 + \beta^4) = 0$

Q20 Text Solution:

Given that, $x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

So, $(\alpha + \beta)^2 = (-2)^2 = 4$

And $(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 4$

$= 0$

Required roots are 4 and 0.

Required Equation will be,

$k(x^2 - (4 + 0)x + 4 \times 0) = 0$

Here k = 1 then required equation will be,

$x^2 - 4x = 0$

Q21 Text Solution:

Given that –: $x^2 + 2x + 2 = 0$

$(\alpha + \beta) = -2$

$\alpha\beta = 2$

$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 4$

$(\alpha^2 + \beta^2) = 0$

On cubing both sides we get,

$(\alpha^2 + \beta^2)^3 = 0^3$

$\alpha^6 + \beta^6 + 3\alpha^2\beta^2(\alpha^2 + \beta^2) = 0$

$\alpha^6 + \beta^6 = 0$

$\alpha^6\beta^6 = (\alpha\beta)^6 = 64$

Thus, the equation having roots α^6 & β^6 is

$K(x^2 + 64) = 0$

Given that K = 1, we get

$(x^2 + 64) = 0$

Q22 Text Solution:

$2^{(2x + 2)} + 2^{(x + 2)} = 24$

$\Rightarrow 4 \times 2^{(2x)} + 4 \times 2^x = 24$

$\Rightarrow 2^{(2x)} + 2^x = 6$

Let us assume that $2^x = p$

So,

$2^{(2x)} + 2^x = 6$

$\Rightarrow p^2 + p = 6$

$\Rightarrow p^2 + 3p - 2p - 6 = 0$

$\Rightarrow p(p + 3) - 2(p + 3) = 0$

$\Rightarrow p = -3, 2$

2^x cannot assume a negative value.

So,

$2^x = 2$

$\Rightarrow x = 1$

So,

$(x^{2023} + x^{2022} + x^{2021}) = 1 + 1 + 1 = 3$

Option a is $x^2 + 1 = 1 + 1 = 2$

Option b is $2^x + x^2 = 2 + 1 = 3$ [Correct]

Option c is (x + 2) = 1 + 2 = 3 [Correct]
So, both options b & c are correct.
Hence, option d is the right choice.

Q23 Text Solution:

From the equation we can get that-
p + q = 3
p q = 1
=> $p^2q^2 = 1$
So,
$p^2 + q^2 = (p+q)^2 - 2pq = 7$
Thus the equation should be $K(x^2 - 7x + 1) = 0$ where K ≠ 0
If we put K = $\frac{1}{7}$ we get $\frac{x^2}{7} - x + \frac{1}{7} = 0$
=> x = $\frac{x^2+1}{7}$
If we put K = 2 we get –
$(2x^2 - 14x + 2) = 0$
=> $(2x^2 + 2) = 14x$
Thus, both option II & III are right.

Q24 Text Solution:

$6x^2 - 5x + 1 = 0$
=> $6x^2 - 2x - 3x - 1 = 0$
=> $2x(3x - 1) - (3x - 1) = 0$
=> x = ½ , ⅓
So, $\frac{1}{q}$ = ½ , $\frac{1}{p}$ = ⅓
=> q = 2, p = 3
=> $\frac{p+q}{p-q} = \frac{2+3}{3-2} = 5$
=> pq = 6
So, the quadratic equation having roots pq, $\frac{p+q}{p-q}$ is
K(x – 5) (x – 6) = 0 where K ≠ 0
= $K(x^2 - 11x + 30) = 0$
If we put K = 3 we will get $3x^2 - 33x + 90 = 0$
If we put K = -1 we will get $-x^2 + 11x - 30 = 0$
Thus, option I & II are right.

Q25 Text Solution:

The given quadratic equation is $ax^2 + bx - c = 0$
Let the roots of the quadratic equation be p and q.
Then, (p + q) = $-\frac{b}{a}$
and pq = $-\frac{c}{a}$
Therefore, $(p - q)^2 = (p + q)^2 - 4pq$
$(p - q)^2 = (-\frac{b}{a})^2 - 4 \times (-\frac{c}{a})$
$(p - q)^2 = \frac{b^2}{a^2} + \frac{4c}{a}$
$(p - q)^2 = \frac{b^2 + 4ac}{a^2}$
Given that, the square of the difference between the roots is equal to the product of the coefficients a, b and c divided by the square of a.
So, $(p - q)^2 = \frac{abc}{a^2}$
=> $\frac{b^2 + 4ac}{a^2} = \frac{abc}{a^2}$
=> $b^2 + 4ac$ = abc
Hence, option D is correct.

Q26 Text Solution:

Let α is the common root.
$\alpha^2 + a\alpha + 20 = \alpha^2 + b\alpha - 20$
$(a - b)\alpha = -40$
$\Rightarrow \alpha = \frac{40}{b-a}$
Put the value of α in $\alpha^2 + a\alpha + 20 = 0$.
$\frac{1600}{(b-a)^2} + \frac{40a}{b-a} + 20 = 0$
$\frac{1600+40ab-40a^2}{(b-a)^2} = -20$
$1600 + 40ab - 40a^2 = -20a^2 - 20b^2 + 40ab$
$1600 = 20a^2 - 20b^2$
$a^2 - b^2 = 80$

Q27 Text Solution:

By the given condition,
$p + q = \alpha$
$pq = \beta$
$(p - q)^2 = (p + q)^2 - 4pq$
$(p - q)^2 = \alpha^2 - 4\beta$
Therefore, the product of the roots $p^2 - q^2$ and $p^3 - q^3$ is
$(p^2 - q^2)(p^3 - q^3) = (p + q)(p - q)(p - q)(p^2 + pq + q^2)$
$= \alpha(\alpha^2 - 4\beta)\{(p+q)^2 - pq\}$
$= \alpha(\alpha^2 - 4\beta)(\alpha^2 - \beta)$
Hence, option (b) is correct.

Q28 Text Solution:

For the equation $x^2 + kx + 1 = 0$, the discriminant is $k^2 - 4$
If the roots are real, then D ≥ 0
so $k^2 \geq 4$
So, k ≤ -2 or 2 ≤ k
Hence, option (D) is the correct answer.

Q29 Text Solution:

The given equation is $x^2 + kx = k$.

Or $x^2 + kx - k = 0$

Discriminant = D = $k^2 - 4(1)(-k) = k^2 + 4k$

The roots are imaginary. So, $D < 0$

Or $k^2 + 4k < 0$

Or $k(k + 4) < 0$

So, $(-4) < k < 0$ is the answer.

Hence, option (B) is the correct answer.

Q30 Text Solution:

Given, a and b are roots of the quadratic equation $(x - a)(x - b) = 0$.

So, $(x - a)(x - b) = 0$

Now, $x(4) = (4 - a)(4 - 5) = k(a - 4)$

Also, $x(1) = (1 - a)(1 - 5) = k(4a - 4)$

According to the question,

$x(4) = -$ of $x(1)$

$(a - 4) = -(4a - 4)$

$4a - 16 = -4a + 4$

$8a = 20$

$a =$

Required value of $4a = 4 \times = 10$

Hence, option D is the correct answer.