CAT

MBA PIONEER 2024

QUANTITATIVE APTITUDE

DPP: 12

Functions 4

Q1 Find the range of the values of 'x' that satisfy the inequality $\mathrm{x}^2 - 15\mathrm{x} + 56 < 0$.

(A) $[7, 8]$ (B) $(7, 8)$

(C) $(7, 8]$ (D) $[7, 8)$

Q2 If $\log_2[\log_7(x^2 - x + 37)] = 1$, then what could be the value of 'x'?

(A) 3 (B) 5

(C) 4 (D) None of these

Q3 The sum of an integer and its square is 30. What can be the cube of the integer?

(A) 125

(B) – 126

(C) Both of a and b

(D) None of the above

Q4 Find the value of $\log_{10} xy$ if $x = 0.151515151515........$

$$\& y = \frac{1}{0.1212121212121212121212\ldots}$$

(Take $\log_{10} 2 = 0.310$ and $\log_{10} 5 = 0.699$)

(A) 0.081 (B) 0.079

(C) 0.054 (D) 0.078

Q5 For what value of k will the set of equations 3x + 2y = 5 and (k + 1)x + (k – 1)y = 10 have an infinite number of solutions?

(A) 9 (B) 5

(C) 10 (D) 7

Q6 Which of the following is the largest $81^{11}, (3^4)^{15}, 3^{14^{11}}, 27^{22}$?

(A) 81^{11}

(B) $(3^4)^{15}$

(C) $3^{14^{11}}$

(D) 27^{22}

Q7 The cost price of 4 apples and 5 pears is Rs. 55, and the cost price of 3 apples and 4 pears is Rs. 43. Find the cost price (in Rs.) of 5 apples.

(A) 20 (B) 15

(C) 25 (D) 30

Q8 Determine the common root of the quadratic equations:

$$x^2 - 3x - 18 = 0 \text{ and } x^2 - 8x + 12 = 0$$

(A) 3 (B) 5

(C) 6 (D) 9

Q9 If the roots of the quadratic equation $6\mathrm{x}^2 - \mathrm{bx} + \mathrm{k} = 0$ are reciprocal to each other, then find the value of k.

(A) 4 (B) 12

(C) 8 (D) 6

Q10 If $x^2 + 5x = 6$ and $x^2 + k = 5x$ has only one root in common, then what is the sum of the absolute value/s of all possible value/s of $\mathbf{k}$?

(A) 80 (B) 60

(C) 50 (D) 70

Q11 If $\log_7 \log_5(\sqrt{x+5} + \sqrt{x}) = 0$, then find the value of x.

(A) 1 (B) 0
(C) 2 (D) None of these

Q12 If $\log_{11}\left(x^2 - 11x + 19\right) = 0$, which of the following can be the value of $\log_x(8x)$?
(A) 2
(B) $\log_8 9 + 1$
(C) $2\log_9 8$
(D) 4

Q13 If $|a-2| \geq 4$ and $b^2 \leq 4$, then which of the following will be true?
(A) $ab \geq 2$
(B) $(a+1)(b-1) \geq 4$
(C) $|(a+1)b| + 1 \geq 1$
(D) $(\mathrm{ab} - 3) \leq 2$

Q14 If x and y are integers and $-5 \leq x \leq 6$ and $4 \leq y \leq 9$, then the minimum value of $\frac{2x-y}{x}$ is
(A) -7
(B) -2
(C) 1
(D) $\frac{19}{5}$

Q15 $216 \times \log_2 64 = 10x + 2 \times \log_3 y + 6^2$ and $\log_3(24 + \log_5 x) = 3$. Then find the value of $y - x$.
(A) 243 (B) 118
(C) 125 (D) 378

Q16 If a + b + ab = 1000, where a and b are positive integers, then how many ordered pairs (a, b) satisfy the given condition?
(A) 6 (B) 10
(C) 8 (D) 12

Q17 Nitish booked movie tickets online which collects the same internet handling charges for each ticket of normal and 3D movie, whereas the basic price of each normal movie ticket is 80% of the 3D movie ticket. Nitish paid Rs. 1030 for two 3D movie tickets and 3 normal tickets and Rs. 610 for one 3D movie ticket and 2 normal tickets. The amount paid by Nitish is the sum of the basic price of a ticket and the internet handling charges. Find the internet handling charges (in Rs.) per ticket.
(A) 10 (B) 30
(C) 40 (D) 50

Q18 Ten three–digit numbers all having 9 in the hundred's place are added. But by mistake, the reverse of one of the numbers is added. As a result, the difference of the correct sum and the wrong sum is 6 more than 42 times the sum of the digits of the number which gets reversed. Find the sum of the number which got reversed and its reverse.
(A) 1222
(B) 1252
(C) 1555
(D) Can't be determined

Q19 Nitin raised a fund for the national environmental day. The amount (in thousands) on any given day after the start of fundraising event is given by a quadratic function $f(x) = ax^2 + bx + c$, where $f(x)$ is the money in thousand and x is the given day. If the amount collected on the first 3 days is 9000, 10000 and 13000 respectively, then find the amount raised on the 10th day.
(A) 70000 (B) 80000
(C) 90000 (D) 100000

Q20 If 5x + 2{x} = 24, where {x} is the fractional part function, then how many integer values x can assume?
(A) 3 (B) 2
(C) 1 (D) 0

Q21

Find the number of integral solutions of the given inequality.

$$\frac{(2x-17)}{(x-7)}-11\leq 15$$

(A) All positive integers
(B) All non–negative integers
(C) All integers except 7
(D) All negative integers except 7

Q22 Solve the given inequality for real

$$x, \frac{\left(x^2-13x+47\right)(x-2)}{(x-3)}\leq 0$$

(A) $x>3 \cup x\leq 2$
(B) $2\leq x\leq 3$
(C) $2<x\leq 3$
(D) $2\leq x<3$

Q23 If $\mathrm{f(x)=x-5}$, and $\mathrm{f^2(x)=f(f(x)), f^3(x)=f(f(f(x)))}$ and so on, then find the value of $f^8(9)$.

(A) -39 (B) -26
(C) -36 (D) -31

Q24 $3f\left(\frac{5a+3}{a-5}\right)+6f(a)=(a-3)^2$. Find the value of f(33).
(Note : 'a' is a real number not equal to 5)

(A) 199 (B) 198
(C) 98 (D) 99

Q25 Find the $\min f(x)$, if $f(x)=\max\left\{3x-2, x^2-6\right\}$

(A) -5 (B) 4
(C) -6 (D) 10

Q26 A function $\mathrm{f(x)}$ is defined for all real numbers x and y such that $f(x+y)=f(x)+f(y)-xy-5$.
If $\mathrm{f(1)}=4$, then find the value of $\mathrm{f}\left(-\frac{3}{2}\right)$

(A) 6.275 (B) 4.625
(C) 8.375 (D) 9.725

Q27 A function f(x) is defined for all real numbers x and y such that f(x + y) = f(x) + f(y) – xy – 5. If f(4) = 15, then find f(–2).

(A) -10 (B) 6
(C) -6 (D) 10

Q28 For natural numbers a, b, and c, if ab + bc = 17 and bc + ac = 65, then the minimum possible value of abc is:

Q29 If for any integer m, the equation $3x^2+mx+7=0$ has no real roots and the equation $x^2+(m-4)x+4=0$ has two distinct real roots for x, then how many values of m are possible?

Q30 In one of the management entrance exams, students will get +3 marks for every correct answer and lose 1 mark for every wrong answer. There was no deduction of marks for unattempted questions. The total numbers of questions were in the range of 25 to 30 in the exam. Ravi attempted all the questions in that exam and scored 66.67% of the maximum marks. What could be his score if he attempted $\frac{5}{7}$ of the total number of questions with 90% accuracy?

(A) 52 (B) 55
(C) 56 (D) 58

Answer Key

Q1	(B)
Q2	(C)
Q3	(C)
Q4	(B)
Q5	(B)
Q6	(C)
Q7	(C)
Q8	(C)
Q9	(D)
Q10	(D)
Q11	(D)
Q12	(D)
Q13	(C)
Q14	(A)
Q15	(B)
Q16	(A)
Q17	(B)
Q18	(B)
Q19	(C)
Q20	(D)
Q21	(C)
Q22	(A)
Q23	(D)
Q24	(A)
Q25	(A)
Q26	(B)
Q27	(C)
Q28	52
Q29	10
Q30	(A)

Hints & Solutions

Q1 Text Solution:

Topic - Functions

$$x^2 - 15x + 56 < 0$$
$$x^2 - 7x - 8x + 56 < 0$$
$$(x-7)(x-8) < 0$$

The critical points of the above inequality are 7 and 8 .

So, the inequality will hold true when $7 < x < 8$.

Q2 Text Solution:

Topic - Functions

Using the rule $\log_b a = x \Rightarrow b^x = a$, we get the following

$$\log_7\left(x^2 - x + 37\right) = 2^1 = 2$$
$$\left(x^2 - x + 37\right) = 7^2 = 49$$
$$x^2 - x + 37 - 49 = 0$$
$$x^2 - x - 12 = 0$$
$$x = 4, -3$$

Q3 Text Solution:

Topic - Functions

Let the integer be x.

So, we will get the equation as $x + x^2 = 30$.

$$x^2 + x - 30 = 0$$
$$(x+6)(x-5) = 0$$
$$x = 5 \text{ or } x = (-6)$$
$$\text{So, } x^3 = 125 \text{ or } (-216)$$

Q4 Text Solution:

Topic - Functions

$$x = 0.151515151515\ldots\ldots$$
$$x = \frac{15}{99}$$
$$y = \frac{1}{0.12121212121212121212\ldots}$$
$$y = \frac{99}{12}$$

Now, $xy = \frac{15}{99} \times \frac{99}{12}$

$$\text{xy} = \frac{5}{4}$$
$$\log_{10}\frac{5}{4} = \log_{10}5 - \log_{10}4 = \log_{10}5 - 2\log_{10}2$$
$$= .699 - .620$$
$$= 0.079$$

Q5 Text Solution:

Topic - Functions

To have an infinite number of solutions,

$$\frac{3}{k+1} = \frac{2}{k-1} = \frac{5}{10}$$
$$\text{or } 3(k-1) = 2(k+1)$$
$$\text{or } k = 5$$

After putting the value of k, we get:

$\frac{3}{6} = \frac{2}{4} = \frac{5}{10} = \frac{1}{2}$

Q6 Text Solution:

Topic - Functions

$$81^{11} = \left(3^4\right)^{11} = 3^{44}$$
$$\left(3^4\right)^{15} = 3^{60}$$
$$27^{22} = \left(3^3\right)^{22} = 3^{66}$$

Now if we observe, $3^{14^{11}}$, 14^{11} is clearly greater than 66, therefore the largest value will be $3^{14^{11}}$

Q7 Text Solution:

Topic - Functions

Let 'A' and 'P' be the cost price of one apple and
one pear, respectively

Then, 4A + 5P = 55 ... (1)

And 3A + 4P = 43... (2)

Multiplying equation (1) by 4 and equation (2) by 5, we get:

16A + 20P = 220 ... (3)

15A + 20 P = 215 ... (4)

Solving equations (3) and (4), we get:

A = 5 and P = 7

Required cost price of 5 apples = 25.

Q8 Text Solution:

Topic - Functions

One of the roots is common between the two equations.

Hence, $x^2 - 3x - 18 = x^2 - 8x + 12$

$$\mathrm{x} = 6$$

Therefore, the value of the common root is 6 .

Q9 Text Solution:

Topic - Functions

The roots are reciprocal of each other.

Let the roots of this equation be α and β

The relation between α and β, $\alpha = \frac{1}{\beta}$

$\alpha \times \beta = 1$

Product of the roots $= \frac{k}{6} = \alpha \times \beta = 1$

$\mathrm{k} = 6$

Q10 Text Solution:

Topic - Functions

From the first equation, we get $x^2 + 5x - 6 = 0$.

Or $\mathrm{x} = -6$ or 1

If -6 satisfies the second equation, then we get $(-6)^2 + \mathrm{k} = 5(-6)$.

Or $36 + \mathrm{k} = -30$

Or k $= -66$

If $(+1)$ satisfies the second equation, then we get $1 + \mathrm{k} = 5$ or $\mathrm{k} = 4$.

So, the two possible values of k are (-66) and $(+4)$. So, the sum is $|-66| + |4| = 70$.

Q11 Text Solution:

Topic - Functions

It is given that $\log_7 \log_5(\sqrt{x+5} + \sqrt{x}) = 0$

We know that $\log_{\mathrm{a}} \mathrm{b} = \mathrm{x} \Rightarrow \mathrm{a}^{\mathrm{x}} = \mathrm{b}$

$$\therefore \log_5(\sqrt{x+5} + \sqrt{x}) = 7^0$$
$$\log_5(\sqrt{x+5} + \sqrt{x}) = 1$$
$$\sqrt{x+5} + \sqrt{x} = 5^1$$

$$\sqrt{x+5} = 5 - \sqrt{x}$$

On squaring both sides, we get the following :

$$x + 5 = 25 + x - 2 \times 5\sqrt{x}$$
$$x + 5 = 25 + x - 10\sqrt{x}$$
$$10\sqrt{x} = 20$$
$$\sqrt{x} = 2$$
$$(\sqrt{x})^2 = (2)^2$$
$$x = 4$$

Q12 Text Solution:

Topic - Functions

$$\log_{11}(x^2 - 11x + 19) = 0$$
$$x^2 - 11x + 19 = 11^0$$
$$x^2 - 11x + 18 = 0$$
$$(x-2)(x-9) = 0$$

$$\mathrm{x} = 2 \text{ or } \mathrm{x} = 9$$

Now, $\log_2(16) = 4$

And, $\log_9(72) = \log_9 8 + 1$

Q13 Text Solution:

Topic - Functions

Given, $|a-2| \geq 4$

$a \geq 6$ and $a \leq -2$

Also, $b^2 \leq 4$

$$-2 \leq b \leq 2$$

Evaluating the options:

For $b=0$, option A is false.

For $a=-2$ and $b=0$, option B is false.

The minimum value of $|(a+1)b|$ is 0 , so the value of $|(a+1)b|+1$ will always be greater than or equal to 1 . Therefore, option c is correct.

For $a=7$ and $b=2$, option d is false.

Q14 Text Solution:

Topic - Functions

We need to find the minimum value of $\frac{2x-y}{x}$.

The value of $(2x-y)$ will be minimum when y is maximum. Also, we cannot assume ' x ' to be negative because in that case the whole expression will become positive. ' x ' cannot be 0 .

So, we can see that the minimum value of $\frac{2x-y}{x}$ will be at $x=1$ and $y=9$.

Required answer $=\frac{2x-y}{x}=\frac{2-9}{1}=-7$

Q15 Text Solution:

Topic - Functions

$\log_3(24+\log_5 x)=3$

$24+\log_5 x=3^3=27$

$\log_5 x=27-24=3$

$x=5^3=125$

Also, $216 \times \log_2 64=10x+2 \times \log_3 y+6^2$

Substituting $\log_2 64=6$ and $x=125$

$216 \times 6=10 \times 125+2 \times \log_3 y+36$

$1296=1250+2 \times \log_3 y+36$

$10=2 \times \log_3 y$

$5=\log_3 y$

$y=3^5=243$

$y-x=243-125=118$

Q16 Text Solution:

Topic - Functions

Given, a + b + ab = 1000

a + b + ab + 1 = 1000 + 1

a(1 + b) + 1(b + 1) = 1001

(a + 1)(b + 1) = 1001

Thus, we need to write 1001 as a product of two factors.

$$\text{i.e., } 1001=1 \times 1001$$
$$1001=7 \times 143$$
$$1001=11 \times 91$$
$$1001=13 \times 77$$

Now, ' a ' and ' b ' are positive integers, so the case of 1×1001 is not possible. Thus, the ordered pairs (a, b) can be $(6,142),(142,6),(10,90),(90,10),(12,76)$ and $(76,12)$.

Thus, six ordered pairs satisfy the given condition.

Q17 Text Solution:

Topic - Functions

Let the internet handling charges for each ticket be Rs. I and the price of each 3D movie ticket be Rs. X. So, the price of each normal movie ticket

$$= \text{Rs. } \frac{4}{5}X$$

$$2 \times X + 3 \times \frac{4}{5} \times X + 5 \times I = 1030$$

$$22 \times X + 25 \times I = 5150$$

$$\text{Also, } X + 2 \times \frac{4}{5} \times X + 3 \times I = 610$$

$$13 \times X + 15 \times I = 3050$$

Equation $(1) \times 3 - (2) \times 5$ gives:

$$66\mathrm{X} + 75\mathrm{I} - 65\mathrm{X} - 75\mathrm{I} = 15450 - 15250$$
$$\mathrm{X} = 200$$

So, $\mathrm{I} = 30$

i.e., the internet handling charges for each ticket is Rs. 30 .

Q18 Text Solution:

Topic - Functions

Let N = 9ab be the number which gets reversed while adding and S be the sum of the remaining 9

numbers.

Sum of all ten numbers = S + (900 + 10a + b)

Sum of all ten numbers after N gets reversed

= S + (100b + 10a + 9)

As per the information,

$$\mathrm{S} + (900 + 10\mathrm{a} + \mathrm{b})$$
$$- \{\mathrm{S} + (100\,\mathrm{b} + 10\mathrm{a} + 9)\} = 6+$$
$$42(9 + \mathrm{a} + \mathrm{b})$$
$$891 - 99\,\mathrm{b} = 384 + 42\mathrm{a} + 42\,\mathrm{b}$$
$$507 - 141\,\mathrm{b} = 42\mathrm{a}$$
$$169 - 47\,\mathrm{b} = 14\mathrm{a}$$
$$\mathrm{b} = \frac{169 - 14\mathrm{a}}{47}$$
$$\mathrm{a} = 2 \text{ and } \mathrm{b} = 3$$
$$\text{Required sum } = 923 + 329 = 1252.$$

Q19 Text Solution:

Topic - Functions

From the given values, we get the following :

f(1) = $a(1)^2$ + b(1) + c = 9000 .....(1)

f(2) = $a(2)^2$ + b(2) + c = 10000 .....(2)

f(3) = $a(3)^2$ + b(3) + c = 13000 .....(3)

Then, (2) – (1) gives 3a + b = 1000 .....(4)

(3 – 2) gives 5a + b = 3000 .....(5)

(5) – (4) gives 2a = 2000; thus, a = 1000, b = -2000

Substituting the values of a and b, we have c = 10000

Thus, f(10) = 100000 – 20000 + 10000 = 90000.

Q20 Text Solution:

Topic - Functions

5x + 2{x} = 24

=> 2{x} = 24 – 5x

=> {x} = $\frac{24-5x}{2}$

Now, we know that,

0 ≤ {x} < 1

=> 0 ≤ $\frac{24-5x}{2}$ < 1

=> 0 ≤ 24 – 5x < 2

=> -24 ≤ -5x < 2 – 24

=> - 24 ≤ -5x < - 22

=> 24 ≥ 5x > 22

=> 24/5 ≥ x > 22/5

Then, there is no integer value of x between this range.

Q21 Text Solution:

Topic - Functions

$$\frac{(2x-17)}{(x-7)}-11 \leq 15$$
$$\frac{(2x-17)}{(x-7)}-26 \leq 0$$
$$\frac{(2x-17)-26(x-7)}{(x-7)} \leq 0$$
$$\frac{2x-17-26x+182}{(x-7)} \leq 0$$
$$\frac{-24x+165}{(x-7)} \leq 0$$
$$\frac{24x-165}{(x-7)} \geq 0$$
$$\frac{(24x-165)(x-7)}{(x-7)^2} \geq 0$$

As $(x-7)^2 \geq 0$

So,

$$(24x-165)(x-7) \geq 0$$

The critical points are $\frac{165}{24}$ and 7

For $x \geq 7$, say 8 , the value of $(24x-165)(x-7)$ is positive which is greater than zero.

For $\frac{165}{24} \leq x \leq 7$, say 6.99 , the value of $(24x-165)$
$(x-7)$ is negative which is less than zero.

For $x \leq \frac{165}{24}$, say 1 , the value of $(24x-165)(x-7)$ is positive which is greater than zero.

But $\mathrm{x}=7$ will make the denominator of Eq (1) zero which is not allowed.

Hence, the required solution is $\left(x>7 \cup x \leq \frac{165}{24}\right)$.

Q22 Text Solution:

Topic - Functions

$$\frac{\left(x^2-13x+47\right)(x-2)}{(x-3)} \leq 0$$
$$\frac{\left(x^2-13x+47\right)(x-2)(x-3)}{(x-3)^2} \leq 0$$

As $(x-3)^2 \geq 0$ (Denominator is 0 at $x=3$ which is not allowed)

$$\left(\mathrm{x}^2-13\mathrm{x}+47\right)(\mathrm{x}-2)(\mathrm{x}-3) \leq 0$$

$x^2-13x+47$ is always positive at any value of x.

Now, we have $(x-2)(x-3) \leq 0$

The critical points are 2 and 3 .

For ≥ 3, the value of $(x-2)(x-3)$ is either zero or positive which is equal to zero or greater than zero.

For $2 \leq x \leq 3$, the value of $(x-2)(x-3)$ is either

zero or negative which is equal to zero or less than zero.

For ≤ 2, the value of $(x-2)(x-3)$ is either zero or positive which is equal to zero or greater than zero. But $\mathrm{x}=3$ will make the denominator of eq (1) zero which is not allowed.

Hence, the required solution is $x>3 \cup x \leq 2$

Q23 Text Solution:

Topic - Functions

$$f(x) = x - 5$$
$$\text{so, } f^2(x) = (x-5) - 5 = x - 10$$
$$\text{Similarly, } f^8(x) = x - 15$$
$$\text{So, } f^8(x) = x - 40$$
$$\text{Now, } f^8(9) = 9 - 40 = -31$$

Q24 Text Solution:

Topic - Functions

Substituting a = 6, $3f\left(\frac{5\times6+3}{6-5}\right) + 6f(6) = (6-3)^2$

$$\Rightarrow 3f(33) + 6f(6) = 9$$

Substituting $a = 33, 3f\left(\frac{5\times33+3}{33-5}\right) + 6f(33)$

$$= (33-3)^2 \Rightarrow 3f(6) + 6f(33) = 900$$

On solving (1) and (2),

$$f(6) = -98$$
$$f(33) = 199$$

Q25 Text Solution:

Topic - Functions

Min $\left(\max\left\{3x-2, x^2-6\right\}\right)$ is observed when we get the following cases:

$$3x - 2 = x^2 - 6$$
$$x^2 - 3x - 4 = 0$$
$$x = 4 \text{ or } x = -1$$
$$\text{Min}\left(\max\left\{3(-1)-2, (-1)^2 - 6\right\}\right) = \text{Min}(\max\{-5, -5\}) = \text{Min}(-5) = -5$$

$$\text{Min } \left(\max\left\{3(4)-2, (4)^2 - 6\right\}\right) = \text{Min}(\max\{10, 10\}) = \text{Min}(10) = 10$$
$$\text{Minimum is } -5 \text{ at } x = -1$$

Q26 Text Solution:

Topic - Functions

Given, $f(x+y) = f(x) + f(y) - xy - 5$

Put $x = 1$ and $y = 0$, we get

$$f(1) = f(1) + f(0) - 0 - 5$$
$$f(0) = 5$$

Put $x = \frac{1}{2}$ and $y = \frac{1}{2}$, we get

$$f\left(\frac{1}{2}+\frac{1}{2}\right) = f\left(\frac{1}{2}\right) + f\left(\frac{1}{2}\right) - \left(\frac{1}{2}\times\frac{1}{2}\right) - 5$$
$$f(1) = 2f\left(\frac{1}{2}\right) - \frac{1}{4} - 5$$
$$4 = 2f\left(\frac{1}{2}\right) - \frac{21}{4}$$
$$f\left(\frac{1}{2}\right) = \frac{37}{8}$$

Put x = 1, y = -1/2

We get the value of f(-1/2) = 41/8

Further, put x = -1/2 and y = -1/2, we get

f(-1) = 5

Further, put x = -1 and y = -1/2, we get

f(-3/2) = 37/8 or 4.625

Q27 Text Solution:

Topic - Functions

Given, f(x + y) = f(x) + f(y) – xy – 5

Put x = 2 and y = 2, we get

f (2 + 2) = f(2) + f(2) – 4 – 5

f (4) = 2f(2) – 9

15 + 9 = 2f (2)

f(2) = 12

Put x = 4 and y = –2, we get

f(4 – 2) = f(4) + f(–2) +8 –5

f(2) = f(4) + f(–2) +3

12 = 15 + f(− 2) +3
12 = 18 + f(-2)
f(-2) = -6

Q28 Text Solution:

Topic - Functions

Given that, ab + bc = 17
⇒ b(a + c) = 17
Since, a, b, and c are all natural numbers, so
b = 1 and a + c = 17
Now, bc + ac = 65
⇒ c + ac = 65, since b = 1
⇒ c(1 + a) = 65
⇒ c(1 + 17 − c) = 65
⇒ c(18 − c) = 65
⇒ 18c − c^2 = 65
⇒ c^2 − 18c + 65 = 0
⇒ (c − 5)(c − 13) = 0
⇒ c = 5, 13
Now, when c = 5, then a = 17 − 5 = 12
When c = 13, then a = 17 − 13 = 4
Therefore, the minimum value of abc = 4 × 1 × 13 = 52.

Q29 Text Solution:

Topic - Functions

For an equation to have no real roots or two distinct
real roots, we need to analyze the discriminant (D)
of the quadratic equation.
The discriminant is given by $\mathrm{D}=\mathrm{b}^2-4\mathrm{ac}$, where a, b, and c are the coefficients of the quadratic equation $\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}=0$.
For the first equation, $3\mathrm{x}^2+\mathrm{mx}+7=0$, we have: $\mathrm{a}=3,\ \mathrm{b}=\mathrm{m}$, and $\mathrm{c}=7$
The discriminant is $\mathrm{D}_1=\mathrm{m}^2-4\times3\times7=\mathrm{m}^2-84$. Since this equation has no real roots, the discriminant must be negative:

$$\mathrm{D}_1<0$$

Further, the discriminant of the second quadratic equation must be positive as the roots are real and distinct

$$(\mathrm{m}-4)^2-16>0$$
$$\Rightarrow(\mathrm{m}-4)^2>16$$
$$\Rightarrow\mathrm{m}-4<-4\text{ or }\mathrm{m}-4>4$$
$$\Rightarrow\mathrm{m}<0\text{ or }\mathrm{m}>8$$

To find the number of possible values of m, we need to find the range of m where both $D_1<0$ and $D_2>0$ are true.
Upon analyzing these inequalities and considering
that m must be an integer, we find that there are 10
possible values of m that satisfy both conditions.
These values are m = {−9, −8, −7, −6, −5, −4, −3, −
2, −1 and 9}.
Hence, the number of possible values of m is 10.

Q30 Text Solution:

Topic - Functions

Let us assume that the total number of questions in the exam are N.
Also x is the number of correct answers and $(\mathrm{N}-\mathrm{x})$ is the number of wrong answers.
As per the information, $3\mathrm{x}-1(\mathrm{N}-\mathrm{x})=3\mathrm{N}\times\frac{2}{3}$

$$x=\frac{3N}{4}$$

N must be a multiple of 4 and in the range of 25 to
30.

$$N = 28$$

Number of questions attempted

$$= \frac{5}{7} \times 28 = 20$$

Score $= \frac{9}{10} \times 20 \times 3 - 2$
New score $= 52$.