MBA PRO 2024

QUANTITATIVE APTITUDE

DPP: 11

Functions 3

Q1 If [x] denotes the greatest integer function and [1.2 + [2.7 + [-3.8]]] = [x + 7], then the value of x is, where $x \in Z$.

(A) 7 (B) 8

(C) -7 (D) -8

Q2 If [x] denotes the greatest integer function and f(x) is defined as f(x)= [x - 5]. Find the value of f(f(f(1.9)))

(A) 15 (B) 14

(C) -15 (D) -14

Q3 If $x \in (1, \frac{3}{2})$, then the value of f(x) = [x] + [x + 0.5] + [2x] , where [.] denotes a greatest integer function

(A) 2 (B) 4

(C) 0 (D) 1

Q4 If x and y satisfy the equations y = 2[x] + 3 and y = 3[x − 2] simultaneously, where [.] denotes the greatest integer not exceeding the real number x, then the value of [x + y] is

(A) 30 (B) 9

(C) 21 (D) 31

Q5 If [.] denotes the greatest integer function and 4[x - 1] – 10 = 3[x] + 1, then x can have the value

(A) 12 < x < 13

(B) 14 < x < 15

(C) 15 ≤ x < 16

(D) 16 < x < 17

Q6 How many real values of x will satisfy the equation $[x]^2 - 11[x] + 18 = 0$, where [.] denotes the greatest integer function.

A. Two

B. One

C. No value

D. Infinitely many values

(A) Two

(B) One

(C) No Value

(D) Infinitely many values

Q7 If [.] denotes the greatest integer functions then the value of

$\left[\frac{1}{2}+\frac{1}{2024}\right]+\left[\frac{1}{2}+\frac{2}{2024}\right]+\left[\frac{1}{2}+\frac{3}{2024}\right]+ \ldots\ldots\ldots +\left[\frac{1}{2}+\frac{2023}{2024}\right]$ is

(A) 2023 (B) 1012

(C) 1011 (D) 1013

Q8 If [.] denotes the greatest integer functions then the value of

$\left[\frac{1}{2}-\frac{1}{1000}\right]+\left[\frac{1}{2}-\frac{2}{1000}\right]+\left[\frac{1}{2}-\frac{3}{1000}\right]+ \ldots\ldots\ldots +\left[\frac{1}{2}-\frac{999}{1000}\right]$ is

(A) 499 (B) -499

(C) 1000 (D) -1000

Q9 Let $[x]^2 - 9[x] + 20 = 0$, then x will be

(A) x ∈ [5, 6]

(B) x ∈ (4, 5]

(C) x ∈ [4, 6)

(D) x ∈ (3, 4]

Q10 Let log ([x] - 3) + log ([x] – 4) – log (4[x] – 18) = 0 then x will be,

(A) x ∈ [5, 7)

(B) x ∈ [3, 5)

(C) $x \in (4, 5]$
(D) $x \in [2, 7)$

Q11 Let $([x]^2 - [x] - 6)^2 + ([y]^2 + [y] - 6)^2 = 0$. If P is equal to the maximum possible value of $(x-y)^2$, then find [P] where [.] represents the greater integer function.

(A) 42 (B) 44
(C) 46 (D) 48

Q12 Let a + b + c = $\log_6(5x + 4)$, where a = $\log_6(2x + 1)$, b = $\frac{1}{2}\log_{\sqrt{6}}(3x+2)$ and $c = \frac{1}{\log_{4x+3} 6}$. Then which of the following relations is correct?

(A) $11x^3 + 24x - 2 = 0$
(B) $24x^3 - 3x^2 + 12x + 10 = 0$
(C) $24x^3 + 46x^2 + 24x + 2 = 0$
(D) $12x^3 + 23x^2 + 11x + 1 = 0$

Q13 $If\ \log_5 \log_2 (x^3 + 5) = 1,\ then\ find\ value\ of\ x^3 + x^2 + 4x + 2.$

(A) 27 (B) 3
(C) 35 (D) 50

Q14 Solve for x, $\log_5 [\log_{256}(x) - \log_{81}(9^{\frac{1}{4}}) + 125^x] = 3x$

Q15 How many solutions the equation $|3x - 2| - |2x + 1| = x - 1$ have?

Q16 Let f(x) be a function defined as $f(x) = [ax^2 + bx + 3]$, where [.] denotes the greatest integer function, and a, b, and c are integers. Given that f(1) = 2 and f(2) = 7, then which of the following can be one of the values of a and b respectively?

(A) 2, -3 (B) 3, -4
(C) -4, 5 (D) -5, 6

Q17 Let 2x + 3[x] + 5[x] + 4[x] = 42. Find the value of $([x]+x-2)^2$.

(A) 14 (B) 15
(C) 16 (D) 17

Q18 Let x + [x] + 2[x] + 3[x] = 28. Find the value of $([x]+x)^2$ where [x] denotes the greatest integer value of x not greater than x.

(A) 31 (B) 32
(C) 62 (D) 64

Q19 Evaluate:

$\left[1+ sin\frac{\pi}{1}\right]$ + $\left[1+ sin\frac{\pi}{2}\right]$ + $\left[1+ sin\frac{\pi}{3}\right]$ + $\left[1+ sin\frac{\pi}{4}\right]$ + $\left[1+ sin\frac{\pi}{5}\right]$+$\left[1+ sin\frac{\pi}{6}\right]$

where [.] denotes the greatest integer function.

(A) 5 (B) 6
(C) 7 (D) None of these

Q20 Let f(x) = $\left[\frac{50+x}{100}\right]$, where [.] denotes a greatest integer function. Find the value of $\sum_{x=1}^{100}$ f(x)

Q21 Consider the quadratic equation $2x^2 - 3x - 2 = 0$ and the greatest integer function f(x) = [x]. If the roots of the quadratic equation are p and q, find the sum of the possible integer values of f(p) and f(q). (where [x] denotes the greatest integer value of x not greater than x.].

(A) 3 (B) 1
(C) 2 (D) -1

Q22 $\log\left(\frac{x}{y}\right) + \log\left(\frac{y}{x}\right) = \log(x+3y)\ and$

$\log(xy) + \log\left(\frac{1}{xy}\right) = \log(3x+y)$

$then\ find\ the\ value\ of\ x+y$

(A) 1 (B) 2
(C) $\frac{1}{2}$ (D) 0

Q23 Consider the quadratic equation $x^2 - 11x + 28 = 0$ and the greatest integer function f(x) = $[\frac{x}{2}]$. Determine the value of x for which f(x) = 2., where [.] denotes the greatest integer function.

(A) 2 (B) 3
(C) 4 (D) 5

Q24 Let $([x]^2 - 2[x] - 35)^2 + ([y]^2 + [y] - 42)^2 = 0$. If M is equal to the maximum possible value of $(x + y)^2$, then find [M].

(A) 200 (B) 212
(C) 224 (D) 236

Q25 $(\log_5[x] - 3)^2 + (\log_3[y] - 5)^2 + (\log_2[z] - 3)^2 = 0$, then find the maximum possible value of $[(y - x + z)^2]$, where [.] denotes the greatest integer function.

(A) 11628 (B) 18126
(C) 12618 (D) 16383

Q26 What maximum positive integer value of x will satisfy the equation $\left[\frac{x}{5}\right]$ - $\left[\frac{x}{7}\right]$ = 1, where [.] denotes a greatest integer function.

Q27 For how many distinct positive integer values of x, the inequality $\left[\frac{2}{3} + \frac{x}{9}\right] < 3$ satisfied? [x] denotes the greatest integer value of x not greater than x

(A) 10 (B) 15
(C) 20 (D) 25

Q28 For how many positive integer values of x, $\left[\frac{x}{5}\right] = \left[\frac{x}{6}\right]$?
[[x] denotes the greatest integer value of x not greater than x.]

(A) 13 (B) 26
(C) 14 (D) 28

Q29 For how many positive integer values of x, $\left[\frac{x}{3}\right] - \left[\frac{x}{6}\right] < 5$ be satisfied?
[[x] denotes the greatest integer value of x not greater than x.]

(A) 25 (B) 26
(C) 27 (D) 28

Q30 Find the number of solutions of the quadratic equation $x^2 + [x] - 5x + 4 = 0$, where $[x]$ is representing the greatest integer function of x.

(A) 0 (B) 1
(C) 2 (D) None of these

Answer Key

Q1	(D)
Q2	(D)
Q3	(B)
Q4	(A)
Q5	(C)
Q6	(D)
Q7	(B)
Q8	(B)
Q9	(C)
Q10	(A)
Q11	(D)
Q12	(C)
Q13	(D)
Q14	2
Q15	1
Q16	(B)
Q17	(C)
Q18	(D)
Q19	(C)
Q20	51
Q21	(B)
Q22	(C)
Q23	(C)
Q24	(C)
Q25	(D)
Q26	29
Q27	(C)
Q28	(C)
Q29	(B)
Q30	(C)

Hints & Solutions

Q1 Text Solution:

Given that

[1.2 + [2.7 + [-3.8]]] = [x + 7],

Or [1.2] + [2.7] + [-3.8] = [x] + 7

Or 1 + 2 - 4 = [x] + 7

Or -1 = [x] + 7

Or [x] = -8.

As x is an integer, so x = -8.

Option D

Q2 Text Solution:

f(1.9) = [1.9 – 5] = [-3.1] = - 4

f(f(1.9)) = f(-4) = [- 4 - 5] = [-9] = -9

f(f(f(1.9))) = f(-9) = [-9 – 5] =[-14] = -14.

Hence Option D.

Q3 Text Solution:

As 1< x < $\frac{3}{2}$,

[x] = 1

[x + 0.5] = 1 and [2x] = 2

Hence [x] + [x + 0.5] + [2x] = 1 + 1 + 2 = 4.

Option B.

Q4 Text Solution:

We have 2[x] + 3 = 3[x – 2]

Or 2[x] + 3 = 3[x] – 6

Or 6 + 3 = 3[x] – 2[x]

Or [x] = 9.

y = 2[x] + 3

= 2 × 9 + 3 = 21.

So, [x + y] = [9 + 21] = [30] = 30

Option A

Q5 Text Solution:

Given that 4[x - 1] – 10 = 3[x] + 1

Or 4[x] – 4 – 10 = 3[x] + 1

Or 4[x] – 14 = 3[x] + 1

Or 4[x] – 3[x] = 1 + 14

Or [x] = 15

Thus, either x = 15 or it lies between 15 and 16.

Hence Option C.

Q6 Text Solution:

Given that: $[x]^2$ - 11[x] +18 = 0

or ([x] - 2) $([x]-9)$ = 0

or [x] = 2 or 9

Hence, equation is satisfied when $2 \leq x < 3 \text{ and } 9 \leq x < 10$

There exist infinite real values of x.

Hence option D.

Q7 Text Solution:

Given that:

$\left[\frac{1}{2}+\frac{1}{2024}\right]$ + $\left[\frac{1}{2}+\frac{2}{2024}\right]$ + $\left[\frac{1}{2}+\frac{3}{2024}\right]$ + ...+ $\left[\frac{1}{2}+\frac{2023}{2024}\right]$

$\left[\frac{1}{2}+\frac{1}{2024}\right]$ + $\left[\frac{1}{2}+\frac{2}{2024}\right]$ + $\left[\frac{1}{2}+\frac{3}{2024}\right]$ + ...+ $\left[\frac{1}{2}+\frac{1012}{2024}\right]$ +

$\left[\frac{1}{2}+\frac{1013}{2024}\right]$ + $\left[\frac{1}{2}+\frac{1014}{2024}\right]$ + ...+ $\left[\frac{1}{2}+\frac{2023}{2024}\right]$

As can be seen, the first 1011 terms lie between 0 and 1 and the last 1012 terms lie between 1 and 2.

Hence, the value is (0 + 0 + 0 +......1011 times) + (1 + 1 + 1 +.....1012 times)

= 1012.

Hence Option B.

Q8 Text Solution:

Given that:

$\left[\frac{1}{2}-\frac{1}{1000}\right]$ + $\left[\frac{1}{2}-\frac{2}{1000}\right]$ + $\left[\frac{1}{2}-\frac{3}{1000}\right]$ + ...+ $\left[\frac{1}{2}-\frac{999}{1000}\right]$

$\left[\frac{1}{2}-\frac{1}{1000}\right]$ + $\left[\frac{1}{2}-\frac{2}{1000}\right]$ + $\left[\frac{1}{2}-\frac{3}{1000}\right]$ + ...+ $\left[\frac{1}{2}-\frac{500}{1000}\right]$ +

$\left[\frac{1}{2}-\frac{501}{1000}\right]$ +.........+ $\left[\frac{1}{2}-\frac{999}{1000}\right]$

As can be seen, the first 500 terms lie between 0.5 and 0 and the next 499 terms lie between 0 and -1.

Hence, we can get

(0 + 0 + 0 + ...500 times) + (-1 -1 -1 - ....499 times)

= - 499 Answer.

Q9 Text Solution:

The given equation is

$[x]^2 - 9[x] + 20 = 0$

=> $[x]^2 - 4[x] - 5[x] + 20 = 0$

=> $[x]([x] - 4) - 5([x] - 4) = 0$

=> $([x] - 4)([x] - 5) = 0$

=> $[x] = 4, 5$

So, $x \in [4, 6)$

Q10 Text Solution:

The given equation is

$\log([x]-3) + \log([x] - 4) - \log(4[x] - 18) = 0$

$\log([x]-3)([x]-4) = \log(4[x] - 18)$

$([x]-3)([x]-4) = (4[x] - 18)$

$[x]^2 - 7[x] + 12 - 4[x] + 18 = 0$

$[x]^2 - 11[x] + 30 = 0$

$[x]^2 - 5[x] - 6[x] + 30 = 0$

$([x] - 5)([x] - 6) = 0$

$[x] = 5, 6$

$x \in [5, 7)$

Q11 Text Solution:

$([x]^2 - [x] - 6)^2 + ([y]^2 + [y] - 6)^2 = 0$

=> $([x]^2 - [x] - 6)^2 = 0$; $([y]^2 + [y] - 6)^2 = 0$

=> $[x]^2 - [x] - 6 = 0$; $[y]^2 + [y] - 6 = 0$

=> $([x] - 3)([x] + 2) = 0$; $([y] + 3)([y] - 2) = 0$

=> $[x] = -2, 3$; $[y] = -3, 2$

$x \in [-2,-1) \cup [3, 4)$ and $y \in [-3,-2) \cup [2, 3)$

=> $P < 49$

So, $[P] = 48$

Q12 Text Solution:

We are given $a + b + c = \log_6(5x + 4)$....... (i)

Also, $\log_6(2x + 1) = a$ .... (ii)

$\frac{1}{2}\log_{\sqrt{6}}(3x + 2) = b$

- $\log_{(\sqrt{6})^2}(3x + 2) = b$
- $\log_6(3x + 2) = b$ .... (iii)

And, $c = \frac{1}{\log_{4x+3}6}$

- $\log_6(4x + 3) = c$ .... (iv)

Adding equations (ii), (iii) and (iv), we get

$a + b + c = \log_6(2x + 1) + \log_6(3x + 2) + \log_6(4x + 3)$

- $a + b + c = \log_6[(2x+1)(3x+2)(4x+3)]$ ...... (v)

Now, we can equate the two expressions (i) and (v) for a + b + c:

$\log_6((2x + 1)(3x + 2)(4x + 3)) = \log_6(5x + 4)$

- $(2x + 1)(3x + 2)(4x + 3) = (5x + 4)$

Expanding both sides of the equation, we get:

$24x^3 + 46x^2 + 29x + 6 = 5x + 4$

Rearranging the equation into a cubic equation, we get:

$24x^3 + 46x^2 + 24x + 2 = 0$

Q13 Text Solution:

$\log_5 \log_2\left(x^3 + 5\right) = 1$

$\Rightarrow \log_2\left(x^3 + 5\right) = 5$

$\Rightarrow x^3 + 5 = 2^5 = 32$

$\Rightarrow x^3 = 27$

$\Rightarrow x = 3$

$so,\ x^3 + x^2 + 4x + 2 = 3^3 + 3^2 + 4 \times 3 + 2$

$= 27 + 9 + 12 + 2 = 50$

Q14 Text Solution:

The given equation is

$\log_5[\log_{256}(x) - \log_{81}(9^{\frac{1}{4}}) + 125^x] = 3x$

=> $\log_5[\log_{256}(x) - \frac{1}{4}\log_{81}(9) + 125^x] = 3x$

- $\log_{256} x - \frac{1}{8} + 125^x = 5^{3x}$
- $\log_{256} x - \frac{1}{8} + 125^x = 125^x$
- $\log_{256} x = \frac{1}{8}$
- $x = 256^{\frac{1}{8}}$
- $x = 2$

Q15 Text Solution:

$|3x - 2| - |2x + 1| = x - 1$

Case 1: $3x - 2 \geq 0$ and $2x + 1 \geq 0$

In this case, we can rewrite the equation as:

$(3x - 2) - (2x + 1) = x - 1$

Solve for x:

$x - 3 = x - 1$

In this case, there is no solution, as the equation is not true.

Case 2: 3x - 2 ≥ 0 and 2x + 1 < 0

In this case, we can rewrite the equation as:

$(3x - 2) - (-(2x + 1)) = x - 1$

Solve for x:

$3x - 2 + 2x + 1 = x - 1$

$5x - 1 = x - 1$

$4x = 0$

$x = 0$

Now, check if the solution fits the conditions of this case:

$3(0) - 2 = -2$, which is not greater than or equal to 0. Thus, this solution is invalid for this case.

Case 3: 3x - 2 < 0 and 2x + 1 ≥ 0

In this case, we can rewrite the equation as:

$(-(3x - 2)) - (2x + 1) = x - 1$

Solve for x :

$-3x + 2 - 2x - 1 = x - 1$

$-5x + 1 = x - 1$

$-6x = -2$

$x = \frac{1}{3}$

Now, check if the solution fits the conditions of this case:

$2(\frac{1}{3}) + 1 = \frac{5}{3}$, which is greater than or equal to 0, and

$3(\frac{1}{3}) - 2 = -1$, which is less than 0. Thus, this solution is valid for this case.

Case 4: 3x - 2 < 0 and 2x + 1 < 0

We can write this as,

$-(3x - 2) - \{-(2x + 1)\} = x - 1$

$-3x + 2 + 2x + 1 = x - 1$

$-x + 3 = x - 1$

$-2x = -4$

$x = 2$

$3x - 2 = 3 \times 2 - 2 = 4 > 0$

$2x + 1 = 2 \times 2 + 1 = 5 > 0.$

After evaluating all cases, the only valid solution is $x = \frac{1}{3}$.

Q16 Text Solution:

Given that, $f(x) = [ax^2 + bx + 3]$

So, $f(1) = [a + b + 3] = 2$

$\Rightarrow 2 \le a + b + 3 < 3$

$\Rightarrow -1 \le a + b < 0$ .... (i)

Also,

$f(2) = [4a + 2b + 3] = 7$

$\Rightarrow 7 \le 4a + 2b + 3 < 8$

$\Rightarrow 4 \le 4a + 2b < 5$ ...... (ii)

If we take $a + b = -1$ .... (iii)

and $4a + 2b = 4$

i.e., $2a + b = 2$ .... (iv)

Solving (iii) and (iv), we have

$a = 3, b = -4$

Q17 Text Solution:

$2x + 3[x] + 5[x] + 4[x] = 42$

$\Rightarrow 2x = 42 - 12[x]$

$\Rightarrow x = 21 - 6[x]$

As, [x] is an integer, so 21 - 6[x] will also be an integer.

So, x is an integer.

Hence, x = [x]

Thus,

$x = 21 - 6x$

$\Rightarrow x = 3$

Therefore, $([x] + x - 2)^2 = (2x - 2)^2$

$= (2 \times 3 - 2)^2$

$= 4^2$

$= 16.$

Q18 Text Solution:

$x + [x] + 2[x] + 3[x] = 28$

$\Rightarrow x = 28 - 6[x]$

As, [x] is an integer, so 28 - 6[x] will also be an integer.

So, x is an integer.

Hence, x = [x]

Thus,

x = 28 - 6x

=> x = 4

Therefore, $([x] + x)^2 = (2x)^2 = (2 \times 4)^2 = 64$

Q19 Text Solution:

Given that:

=> $\left[1+\ sin\frac{\pi}{1}\right]$ + $\left[1+\ sin\frac{\pi}{2}\right]$ + $\left[1+\ sin\frac{\pi}{3}\right]$ + $\left[1+\ sin\frac{\pi}{4}\right]$ + $\left[1+\ sin\frac{\pi}{5}\right]$ + $\left[1+\ sin\frac{\pi}{6}\right]$

=> 1+ $\left[\sin\frac{\pi}{1}\right]$ + 1+ $\left[\sin\frac{\pi}{2}\right]$ + 1 + $\left[\sin\frac{\pi}{3}\right]$ + 1+ $\left[\sin\frac{\pi}{4}\right]$ +

1 + $\left[\sin\frac{\pi}{5}\right]$ + 1+ $\left[\sin\frac{\pi}{6}\right]$

=> (1 + 1 + 1 + 1 + 1 + 1)+ $\left[\sin\frac{\pi}{1}\right]$ + $\left[\sin\frac{\pi}{2}\right]$ + $\left[\sin\frac{\pi}{3}\right]$ + $\left[\sin\frac{\pi}{4}\right]$ + $\left[\sin\frac{\pi}{5}\right]$ + $\left[\sin\frac{\pi}{6}\right]$

= 6 + 0 + 1+ 0+0+0+0

= 7

Hence option C.

Q20 Text Solution:

Given that:

f(x) = $\left[\frac{50+x}{100}\right]$ = $\left[\frac{50}{100}+\frac{x}{100}\right]$ = $\left[\frac{1}{2}+\frac{x}{100}\right]$

Now, $f(x)$ = $\left[\frac{1}{2}+\frac{x}{100}\right]$

= $\left[\frac{1}{2}+\frac{1}{100}\right]$+ $\left[\frac{1}{2}+\frac{2}{100}\right]$ + $\left[\frac{1}{2}+\frac{3}{100}\right]$+........+ $\left[\frac{1}{2}+\frac{50}{100}\right]$+.......+ $\left[\frac{1}{2}+\frac{100}{100}\right]$

For all x < 50, f(x) = 0 and for all $x \geq 50$ and , f(x) = 1

So, $f(x)$ = (0 + 0 + 0 +....49 times) + (1 + 1 + 1 +...... 51 times)

= 51. Answer

Q21 Text Solution:

To solve this question, we need to find the roots p and q of the quadratic equation $2x^2 - 3x - 2 = 0$, and then find the greatest integer function values f(p) and f(q).

Step 1: Use the quadratic formula to find the roots p and q.

The quadratic formula for finding the roots of a quadratic equation $ax^2 + bx + c = 0$ is given by:

x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

In our case, a = 2, b = -3, and c = -2. Substituting these values into the quadratic formula, we get:

$x\ =\frac{3\pm5}{2(2)}$

Step 2: Compute the values of p and q.

p = $\frac{3+5}{4}$ = $\frac{8}{4}$ = 2

q = $\frac{3-5}{4}$ = - $\frac{2}{4}$ = - $\frac{1}{2}$

So, the roots of the quadratic equation are p = 2 and q = - $\frac{1}{2}$.

Step 3: Determine the greatest integer function value for each root, i.e., f(p) and f(q).

Now, we need to find the greatest integer function values for p and q.

f(p) = f(2) = [2] = 2 (since 2 is an integer, the greatest integer function value is the same as the number itself)

f(q) = f(- $\frac{1}{2}$) = [- $\frac{1}{2}$] = –1 (the greatest integer less than or equal to - $\frac{1}{2}$ is –1)

So, the possible integer values of f(p) and f(q) are 2 and –1, respectively.

Therefore, the sum of possible integer values of f(p) and f(q) = 2 – 1 = 1

Q22 Text Solution:

$$\log\left(\frac{x}{y}\right)+\log\left(\frac{y}{x}\right)=\log(x+3y)\Rightarrow eq1$$

$$\log(xy)+\log\left(\frac{1}{xy}\right)=\log(3x+y)\Rightarrow eq2$$

$$\log\left(\frac{x}{y}\right)+\log\left(\frac{y}{x}\right)=\log(x+3y)\Rightarrow eq1$$

$$=\log(x)-\log(y)+\log(y)-\log(x)=0$$

$$=\log(x+3y)$$

$$\Rightarrow x+3y=1\ because\ \log\Big(1\Big)=0$$

$$\log(xy)+\log\left(\frac{1}{xy}\right)=\log(3x+y)\Rightarrow eq2$$

$$\log\left(\frac{1}{xy}\right)=-\log(xy)$$

$$eq2\Rightarrow\log(xy)-\log(xy)=0$$

$$=\log(3x+y)$$

$$\Rightarrow 3x+y=1\ because\ \log\Big(1\Big)=0$$

$$now\ we\ have$$

$$x+3y=1\ and$$

$$3x+y=1$$

$$adding\ these\ we\ get$$

$$4x+4y=2$$

$$x+y=\frac{1}{2}$$

Q23 Text Solution:

To solve this question, we need to determine the interval of x for which the greatest integer function f(x) = $[\frac{x}{2}]$ equals 2.

Step 1: Identify the interval in which x lies.

Since f(x) = 2 and f(x) = $[\frac{x}{2}]$, we need to find the interval of x for which the greatest integer value of $\frac{x}{2}$ is equal to 2. We can represent this as:

$2 \le \frac{x}{2} < 3$

Now, to find the interval for x, we simply multiply each part of the inequality by 2:

$4 \le x < 6$

So, x must lie in the interval [4, 6).

Step 2: Check if the roots of the quadratic equation lie within the identified interval.

The given quadratic equation is $x^2 - 11x + 28 = 0$.

We can factor this equation as:

$(x - 4)(x - 7) = 0$

This gives us roots: x = 4, 7

Step 3: Determine if the root lies within the specified interval.

Now, we need to check if the root x = 4 lies within the interval [4, 6). Since x = 7 is not within this interval, the given quadratic equation has only one value x = 4 for which f(x) = 2.

Q24 Text Solution:

$([x]^2 - 2[x] - 35)^2 + ([y]^2 + [y] - 42)^2 = 0$

=> $([x]^2 - 2[x] - 35)^2 = 0$; $([y]^2 + [y] - 42)^2 = 0$

=> $[x]^2 - 2[x] - 35 = 0$; $[y]^2 + [y] - 42 = 0$

=> $([x] - 7)([x] + 5) = 0$; $([y] + 7)([y] - 6) = 0$

=> [x] = -5, 7 ; [y] = -7, 6

$x \in [-5, -4) \cup [7,8)$ and $y \in [-7, -6) \cup [6,7)$

=> M < 225

So, [M] = 224.

Q25 Text Solution:

$(\log_5[x] - 3)^2 + (\log_3[y] - 5)^2 + (\log_2[z] - 3)^2 = 0$

=> $\log_5[x] - 3 = 0$, $\log_3[y] - 5 = 0$, $\log_2[z] - 3 = 0$

=> $\log_5[x] = 3$, $\log_3[y] = 5$, $\log_2[z] = 3$

Therefore, we have

=> $[x] = 5^3 = 125$ => $x \in [125, 126)$,

$[y] = 3^5 = 243$ => $y \in [243, 244)$, and

$[z] = 2^3 = 8$ => $z \in [8, 9)$

To get the maximum value of (y - x + z) we have to take the maximum value of y and z and minimum value of x.

Let's for example -

y = 243.9999

x = 125

z = 8.9999

=> (y – x + z) = (243.9999 - 125 + 8.9999) = 127.9998

=> $[(y - x + z)^2] = [(127.9998)^2] = [(16383.948)]$

= 16383

=> Maximum possible value of [(y – x + z)²] = 16383

Q26 Text Solution:

Since $\left[\frac{x}{5}\right]$ and $\left[\frac{x}{7}\right]$ are both integers, and given that their difference is 1,

We can have the following values

For x = 5 and 6,

$\left[\frac{x}{5}\right]$ - $\left[\frac{x}{7}\right]$ = 1- 0 = 1

For x = 10, 11, 12 and 13,

$\left[\frac{x}{5}\right]$ - $\left[\frac{x}{7}\right]$ = 2 – 1 = 1

For x = 15, 16, 17, 18 and 19,

$\left[\frac{x}{5}\right]$ - $\left[\frac{x}{7}\right]$ = 3 – 2 = 1

For x = 21, 22, 23 and 24,

$\left[\frac{x}{5}\right]$ - $\left[\frac{x}{7}\right]$ = 4 – 3 = 1

For x = 28 and 29,

$\left[\frac{x}{5}\right]$ - $\left[\frac{x}{7}\right]$ = 5 – 4 = 1

Equation will not be satisfied for x > 29.

So the highest value of x is 29. Answer

Q27 Text Solution:

For x = 1 to 2

$\left[\frac{2}{3}+\frac{x}{9}\right]=0$ < 3 [Total 2 positive integers.]

For x = 3 to 11 [Total 9 positive integers.]

$\left[\frac{2}{3}+\frac{x}{9}\right]=1$ < 3

For x = 12 to 20 [Total 9 positive integers.]

$\left[\frac{2}{3}+\frac{x}{9}\right]=2$ < 3

Hence, for a total 20 positive integer values of x, the given inequality is satisfied.

Q28 Text Solution:

For x = 1 to 4, $\left[\frac{x}{5}\right]=\left[\frac{x}{6}\right]=0$ [4 possible positive integers]

For x = 6 to 9, $\left[\frac{x}{5}\right]=\left[\frac{x}{6}\right]=1$ [4 possible positive integers]

For x = 12 to 14, $\left[\frac{x}{5}\right]=\left[\frac{x}{6}\right]=2$ [3 possible positive integers]

For x = 18 to 19, $\left[\frac{x}{5}\right]=\left[\frac{x}{6}\right]=3$ [2 possible positive integers]

For x = 24, $\left[\frac{x}{5}\right]=\left[\frac{x}{6}\right]=4$ [1 possible positive integer]

Hence, the number of possible positive integers = 4 + 4 + 3 + 2 + 1 = 14

Q29 Text Solution:

For x = 1 to 2,

$\left[\frac{x}{3}\right]-\left[\frac{x}{6}\right]=0<5$

For x = 3 to 8,

$\left[\frac{x}{3}\right]-\left[\frac{x}{6}\right]=1<5$

For x = 9 to 14,

$\left[\frac{x}{3}\right]-\left[\frac{x}{6}\right]=2<5$

For x = 15 to 20,

$\left[\frac{x}{3}\right]-\left[\frac{x}{6}\right]=3<5$

For x = 21 to 26,

$\left[\frac{x}{3}\right]-\left[\frac{x}{6}\right]=4<5$

So, for 26 positive integer values of x, the given inequality will be satisfied.

Q30 Text Solution:

The given equation can be written as

$x^2+(x-f)-5x+4=0$, where f is representing the fraction portion.

$\Rightarrow\left(x^2-4x+4\right)-f=0$

$\Rightarrow f=x^2-4x+4$

Now, the fraction f is such that $0\ \leq f<1$.

Therefore, $0\leq x^2-4x+4<1$

First consider, $x^2-4x+4=0$

Then, $(x-2)^2=0$

$\Rightarrow x=2,\ 2$

Again, let $x^2-4x+4<1$

Then, $x^2-4x+3<0$

$x^2-3x-x+3<0$

$(x-3)(x-1)<0$

$\Rightarrow\ 1<x<3$

Hence, we have $[x]=1$ or 2.

If $[x]=1$, then the given equation becomes

$x^2+1-5x+4=0$

$x^2-5x+5=0$

$=\frac{5+\sqrt{5}}{2},\ \frac{5-\sqrt{5}}{2}$

$\approx 3.62,\ 1.38$

So, $x \neq \frac{5+\sqrt{5}}{2},\ \ x = \frac{5-\sqrt{5}}{2}$

Again, if $[x] = 2,$ then the given equation becomes

$x^2 + 2 - 5x + 4 = 0$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$\Rightarrow x \neq 3,\ \ x = 2$

Hence, the number of solutions to the equation is 2.