MBA PIONEER-2024

QUANTITATIVE APTITUDE

HCF and LCM - 2

DPP: 08

Q1 There are twelve consecutive natural numbers arranged in descending order. The summation of the first six of them is 1677. What will be the summation of the last six of them?

(A) 1641
(B) 1713
(C) 1766
(D) Cannot be determined

Q2 The first and the last digits of a three digit number differ by 5. How many such pairs of numbers will have a difference of 495 among them

(A) 5 (B) 4
(C) 40 (D) 50

Q3 If x is a whole number and y an integer, and if $(x-y)^2 + x^2 = 25$, find the number of possible ordered pair solutions of (x, y)?

(A) 12 (B) 8
(C) 7 (D) 6

Q4 If the product of all the factors of a number is equal to the square of the number, and the sum of all the factors except the number itself is 21, find the number of possible values of the number in question?

(A) 6 (B) 4
(C) 3 (D) 2

Q5 When the digits of a two-digit number are reversed, it increases by 18. How many such two-digit numbers will increase by 18 when reversed?

(A) 6 (B) 7
(C) 8 (D) 9

Q6 There are three prime numbers A, B and C, such that their summation is 100 and the difference between any two of them is 36. Find the value of the product of the smallest two of them?

(A) 62 (B) 169
(C) 301 (D) 437

Q7 A natural number is obtained by adding 16 to the product of four consecutive even natural numbers. Is the natural number divisible by 2^n, where n is greater than four?

(A) Yes
(B) No
(C) May be, depends on the consecutive even natural numbers
(D) None of the above

Q8 A natural number is obtained by adding 16 to the product of four consecutive even natural numbers. Is the natural number a perfect square?

(A) Yes
(B) No
(C) May be, depends on the consecutive even natural numbers
(D) None of the above

Q9 If the number 5324661A is divisible by 18, what is the value of A?

(A) 8 (B) 6
(C) 4 (D) 0

Q10 If x is a natural number > 1, find the remainder when $\left(x^5 - x\right)$ is divided by 6?
(A) 0
(B) 2
(C) 3
(D) Cannot be determined

Q11 A number when divided by 5 leaves a remainder of 4. What is the remainder if 4 times the number is divided by 5?
(A) 3 (B) 2
(C) 1 (D) 0

Q12 A number when divided by a divisor which is 8 times the quotient gives 4 as the remainder. Find the number if the quotient is 6 times the remainder?
(A) 9604 (B) 8504
(C) 6405 (D) 4612

Q13 If x and y are natural numbers, find the number of ordered pair solutions for $\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$?
(A) 7 (B) 8
(C) 15 (D) 16

Q14 The sum of 2 numbers is 96 and their HCF is 12. How many such pairs of such numbers are possible?
(A) 2 (B) 4
(C) 5 (D) 8

Q15 What will be the last two digits of $\left(23^3 + 24^3 + 25^3 + 26^3 + 27^3\right)$?

Q16 What is the smallest natural number with precisely 27 divisors?
(A) 713 (B) 141
(C) 629 (D) 900

Q17 Find the sum of all factors of N = $2^3 \times 3^2 \times 5^4$ that are multiples of 14.
(A) 1200 (B) 18000
(C) 12600 (D) 1500

Q18 If x = $87^{459} + 23^{123} + 95^{267}$, then the unit digit of x is:
(A) 5 (B) 6
(C) 4 (D) 3

Q19 Find the last digit of $\frac{28^{458}}{53^{852}}$.
(A) 8 (B) 6
(C) 4 (D) 2

Q20 n! has x number of zeros at the end and (n + 1)! has (x + 3) zeroes at the end. 1 ≤ n ≤ 1000. How many solutions are possible for 'n'?
(A) 8 (B) 7
(C) 1 (D) 4

Q21 How many natural numbers 'n' are there, such that 'n!' ends with exactly 30 zeroes?
(A) 4 (B) 5
(C) 0 (D) 2

Q22 Find the remainder when 8 divides 27^{17}
(A) 4 (B) 2
(C) 3 (D) 6

Q23 What is the remainder when $2859^{341} + 315^{19} + 716^{34} - 5^7$ is divided by 3?
(A) 2 (B) 3
(C) 7 (D) 5

Q24 What is the remainder when $n^7 - n$ is divided by 42?
(A) 6 (B) 5
(C) 2 (D) 0

Q25 What is the remainder when 1! + 2! + 3! … 100! is divided by 18?
(A) 9 (B) 6
(C) 7 (D) 3

Q26

What is the remainder when the infinite sum $(1!)^2 + (2!)^2 + (3!)^2 + \cdots$ is divided by 1152?

(A) 51 (B) 26
(C) 41 (D) 38

Q27 The number A39K2 is completely divisible by both 8 and 11. Here both A and K are single–digit natural numbers. Which of the following is a possible value of A + K?

(A) 8 (B) 10
(C) 12 (D) 14

Q28 Find the number of positive integers between 50 and 90 that are divisible by both the numbers 2 and 3.

(A) 7 (B) 9
(C) 12 (D) 8

Q29 Find the HCF and LCM of two numbers A and B. The sum of A and B is 800, and their difference is 400.

(A) HCF = 200, LCM = 400
(B) HCF = 300, LCM = 600
(C) HCF = 200, LCM = 600
(D) HCF = 400, LCM = 800

Q30 Two numbers are in the ratio 3 : 4. Their L.C.M. is 84. The greater number is

(A) 28 (B) 84
(C) 21 (D) 24

Answer Key

Q1	(A)	Q16	(D)
Q2	(C)	Q17	(C)
Q3	(C)	Q18	(A)
Q4	(D)	Q19	(C)
Q5	(B)	Q20	(B)
Q6	(A)	Q21	(C)
Q7	(B)	Q22	(C)
Q8	(A)	Q23	(A)
Q9	(D)	Q24	(B)
Q10	(A)	Q25	(A)
Q11	(C)	Q26	(C)
Q12	(D)	Q27	(B)
Q13	(C)	Q28	(A)
Q14	(A)	Q29	(C)
Q15	75	Q30	(A)

Hints & Solutions

Q1 Text Solution:

The difference between the first and the seventh consecutive natural number arranged in descending order = 6

Similarly the differences between the second and eighth, the third and ninth, the fourth and tenth, the fifth and eleventh, and the sixth and twelfth is also 6 each.

Thus the total difference of the summation of the first six numbers and the last six numbers

= 6 * 6 = 36

Since arranged in descending order, the summation of the last six will be less than the first six.

Hence, the summation of the last six of them

= 1677 - 36 = 1641

Q2 Text Solution:

495 = 99*5

We know that when we reverse a three digit number and take the difference of them (larger -- smaller), the difference is always 99* (difference of the first and last digits of that number).

Hence in this constraint, we are actually looking at the difference of the original three digit number and its reverse (larger one -- smaller one) whose first and the last digits of a three digit number differ by 5

The four pairs are :

(1) 6 __ 1 and 1 __ 6,

(2) 7 __ 2 and 2 __ 7,

(3) 8 __ 3 and 3 __ 8, and

(4) 9 __ 4 and 4 __ 9.

5 __ 0 and 0 __ 5 will not be possible as 0 __ 5 will become a two digit number

But each pair can have the middle digit anything from 0 to 9, that is 10 options each.

Hence, 4 * 10 = 40 pairs of three digit numbers whose first and the last digits differ by 5 will have a difference of 495 among them.

Q3 Text Solution:

$(x - y)^2 + x^2 = 25$

or, $(x - y)^2 = 25 - x^2$

or, $(x - y)^2 = (5 - x)(5 + x)$

Since x is a whole number, so, for y to be an integer the right hand side has to be a perfect square.

We can see that for

x = 0, y = 5 or −5,

x = 1, y = (1 −) or (− 1),

x = 2, y = (2 −) or (− 2),

x = 3, y = 7 or −1,

x = 4, y = 1 or 7 and

x = 5, y = 5.

For any value of x above 5, y will be imaginary.

Hence the number of possible ordered pair solutions are (0, 5), (0, −5), (3, 7), (3, −1), (4, 1), (4, 7) and (5, 5), that is 7 solutions

Q4 Text Solution:

Product of all factors of a number N = $N^{d/2}$ where d is number of factors of N

Here, $N^{d/2} = N^2$ or, $\frac{d}{2} = 2$ or, d = 4

So number of factors of the number N = 4

Now, a number having 4 factors can be only of the form P^3 or $P^1 * Q^1$

In the 1st case factors are 1, P, P^2 and P^3

So $1 + P + P^2 = 21$ or, $P(P + 1) = 20 = 4 * 5$ or P = 4

But P is prime, and so P = 4 is not acceptable.

In the 2nd case factors are 1, P, Q and PQ

So 1 + P + Q = 21, or P + Q = 20 = 3 + 17 or 7 + 13, or, either P = 3 and Q = 17 or else P = 7 and Q =

13

Both P and Q are prime numbers in this case

So no of possible values of the number (N) in question = 3 * 17 = 51 or, 7 * 13 = 91, that is 2.

Q5 Text Solution:

The difference between a two digit number and the number formed by reversing its digits = 9 * (the difference of the units and tens digit)

Thus 9*(the difference of the units and tens digit)

= 18

or, the difference of the units and tens digit = 2

Here since the number increases on reversing, in the original number the units digit must be greater than the tens digit by 2

The possible numbers are 13, 24, 35, 46, 57, 68 and 79, that is 7 numbers.

Q6 Text Solution:

Except for 2, all prime numbers are odd.

So if A, B and C are all odd prime numbers, then their summation should have been odd too.

But the summation is even (100)

But the summation of one even and two odd primes will be even

So it has to be that 2 is one of the prime numbers in question (say C) and the smallest one.

So A and B are odd prime numbers

Then A + B = 98

Only the difference between two odd numbers can be even.

So it must be that A – B = 36, assuming A to be the greater one.

Solving we can say that $A = \frac{(98+36)}{2} = 67$

and $B = \frac{(98-36)}{2} = 31$

Hence, the product of the smallest two of them = 31 * 2 = 62

Q7 Text Solution:

Let the four consecutive even natural numbers be (2k – 2), 2k, (2k + 2) and (2k + 4), where k is any natural number > 1.

The product of them = (2k – 2)2k(2k + 2)(2k + 4) = 16(k – 1)k(k + 1)(k + 2)

The natural number obtained by adding 16 to them

= 16(k – 1)k(k + 1)(k + 2) + 16

= 16*[(k – 1)k(k + 1)(k + 2) + 1]

Now if k is odd,

(k – 1)k(k + 1)(k + 2) is of the form of even*odd*even*odd = even.

Also if k is even,

(k – 1)k(k + 1)(k + 2) is of the form of odd*even*odd*even = even

Thus, [(k – 1)k(k + 1)(k + 2) + 1] is of the form of [even + 1], which is always odd, irrespective of the nature of k.

So the natural number 16*[(k – 1)k(k + 1)(k + 2) + 1] is = 16*odd number, which is = 2^4 * odd number

Hence, the natural number obtained by adding 16 to the product of four consecutive even natural numbers can never be divisible by 2^n, where n is greater than four

Q8 Text Solution:

Let the four consecutive even natural numbers be (2k – 2), 2k, (2k + 2) and (2k + 4), where k is any natural number > 1.

The product of them = (2k – 2) 2k(2k + 2) (2k + 4)

= 16(k – 1) k(k + 1) (k + 2)

The natural number obtained by adding 16 to them = 16(k – 1) k(k + 1) (k + 2) + 16

Now, 16(k – 1)k (k + 1) (k + 2) + 16

= 16*[(k – 1)k (k + 1) (k + 2) + 1]

= 16*[(k^2 – k) (k^2 + 3k + 2) + 1]

= 16*[k^4 + $2k^3$ – k^2 – 2k + 1]

= 16*[(k^4 + $2k^3$ + k^2) – 2(k^2 + k) + 1]

= 16*[$(k^2 + k)^2$ – 2(k^2 + k) + 1^2]

= 4^2*$(k^2 + k - 1)^2$

Hence, a natural number obtained by adding 16 to the product of four consecutive even natural numbers is always a perfect square.

Q9 Text Solution:

When a number is divisible by 18, it is also divisible by the co-prime factors of 18, that is 2 and 9

For the number 5324661A to be divisible by 2, the digit A must be divisible by 2

Also for the number 5324661A to be divisible by 9, the summation of the digits = (27 + A) must be divisible by 9

From both, we can see that the only solution is A = 0

Q10 Text Solution:

$(x^5 - x) = (x - 1) \times x \times (x + 1)(x^2 + 1)$

But $(x - 1) \times x \times (x + 1)$ are three consecutive natural numbers for any value of $x > 1$

So at least one of them is even, and hence $(x - 1) \times x \times (x + 1)$ is divisible by 2

Also being three consecutive numbers, $(x - 1) \times x \times (x + 1)$ is divisible by 3

So $(x - 1) \times x \times (x + 1)$ must be perfectly divisible by 2*3 = 6

Thus $(x - 1) \times x \times (x + 1)(x^2 + 1)$ must also be perfectly divisible by 6

Hence, the remainder when $(x^5 - x)$ is divided by 6 = 0

Q11 Text Solution:

Since the number when divided by 5 leaves a remainder of 4, it can be expressed as (5q + 4), where q is the quotient

Four times the number = 4(5q + 4) = (20q + 16)

Remainder on dividing (20q + 16) by 5

= Remainder on dividing 20q by 5 + Remainder on dividing 16 by 5

= 0 + 1

= 1

Q12 Text Solution:

Let the quotient be q

So divisor = 8q

Thus the number can be expressed as {(8q)*q + 4}

The quotient is 6 times the remainder.

Thus the quotient q = 6*4 = 24

Hence the number = {(8*24)*24+4} = 8*576 + 4 = 4612

Q13 Text Solution:

$\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$

or, $12x - xy + 12y = 0$

or, $x(12 - y) + 12y = 0$

or, $x(12 - y) + 12y - 144 = -144$

or, $x(12 - y) - 12(12 - y) = -144$

or, $(x - 12)(12 - y) = -144$

or, $(x - 12)(y - 12) = 144$

Now $144 = 2^4 * 3^2$

Thus the number of factors of 144 = (4 + 1)(2 + 1) = 15

Thus the number of distinct pairs of factors $\frac{(15-1)}{2} = 7$

and the number of identical pairs of factors = 1

For each distinct pair of factors there will be two ordered pair solutions of (x, y)

Example with 1 * 144 :

(x – 12)(y – 12) = 1 * 144

Thus x – 12 = 1 or, x = 13

And y – 12 = 144 or, y = 156

Solution = (13, 156)

Similarly,

(x – 12)(y – 12) = 144 * 1

Thus x – 12 = 144 or, x = 156

And y – 12 = 1 or, y = 13

Solution = (156, 13)

Hence for 7 distinct pair of factors there will be 2*7 = 14 ordered pair solutions of (x, y)

For the only identical pair of factors there will be only one ordered pair solution of (x, y)

Example with 12 * 12 :

(x – 12)(y – 12) = 12 * 12

Thus x – 12 = 12 or, x = 24

And y – 12 = 12 or, y = 24

Solution = (24, 24)

Hence, the number of ordered pair solutions for $\frac{1}{x}+\frac{1}{y}=\frac{1}{12}$

when x and y are natural numbers = 14 + 1 = 15

Q14 Text Solution:

Because the HCF of the two numbers is 12, the numbers can be expressed as 12x and 12y where x and y are natural numbers co-prime to each other

Thus, 12x + 12y = 96

or, x + y = 8

So (x, y) = (7, 1) and (5, 3) can be the only possible pairs of natural numbers co-prime to each other

Hence 2 pairs of such numbers are possible.

Q15 Text Solution:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Let us find the last two digits of $(23^3 + 27^3)$

So, $(23^3 + 27^3)$

$= (23 + 27)(23^2 - 23*27 + 27^2)$

= 50 * (odd – odd + odd)

= 50 * (odd)

Hence last two digits of $(23^3 + 27^3) = 50$

Let us find the last two digits of $(24^3 + 26^3)$

So, $(24^3 + 26^3)$

$= (24 + 26)(26^2 - 26*28 + 28^2)$

= 50 * (even – even + even)

= 50 * (even)

Hence last two digits of $(24^3 + 28^3) = 00$

Last two digits of $25^3 = 25$

Hence, the last two digits of $(23^3 + 24^3 + 25^3 + 26^3 + 27^3) = 50 + 00 + 25 = 75$

Q16 Text Solution:

To discover the smallest natural number with precisely 27 divisors, we ought to express 27 as a product of its prime factors:

$27 = 3^3$

To have 27 divisors, the number ought to be within the form $N = p^{(a-1)} \times q^{(b-1)} \times r^{(c-1)}$,

where p, q, and r are distinct prime numbers, and a, b, and c are positive integers.

To minimize the value of N, we take the smallest three distinct prime numbers: p = 2, q = 3, and r = 5.

Therefore, the smallest natural number with precisely 27 divisors is:

$N = 2^{(3-1)} \times 3^{(3-1)} \times 5^{(3-1)}$

$N = 2^2 \times 3^2 \times 5^2$

N = 4 × 9 × 25

N = 900

Therefore, the smallest natural number with precisely 27 divisors is 900.

Hence, (D) 900 is the correct option.

Q17 Text Solution:

For an integer N of the form $p^a \times q^b \times r^c$, the sum of the factors is $(p^0 + p^1 + p^2 + p^3 + ... + p^a)(q^0 + q^1 + q^2 + q^3 + ... + q^b)(r^0 + r^1 + r^2 + r^3 + ... + r^c)$.

To find the sum of all factors of a number $N = 2^3 \times 3^2 \times 5^4$ that are multiples of 14, we need to consider all possible combinations of powers of 2, 3, and 5. Each combination represents a factor of N. Next, determine the factor that is divisible by 14. The factors of 14 are 1, 2, 7, and 14. Multiply each of these factors by various combinations of powers of 2, 3, and 5 to find the factors of N that are multiples of 14.

Then multiply these coefficients by their respective powers of 2, 3, and 5.

$14 \times 2^0 \times 3^0 \times 5^0 = 14$

$14 \times 2^1 \times 3^0 \times 5^0 = 28$

$14 \times 2^2 \times 3^0 \times 5^0 = 56$

$14 \times 2^3 \times 3^0 \times 5^0 = 112$
$14 \times 2^0 \times 3^1 \times 5^0 = 42$
$14 \times 2^1 \times 3^1 \times 5^0 = 84$
$14 \times 2^2 \times 3^1 \times 5^0 = 168$
$14 \times 2^3 \times 3^1 \times 5^0 = 336$
$14 \times 2^0 \times 3^2 \times 5^0 = 126$
$14 \times 2^1 \times 3^2 \times 5^0 = 252$
$14 \times 2^2 \times 3^2 \times 5^0 = 504$
$14 \times 2^3 \times 3^2 \times 5^0 = 1008$
$14 \times 2^0 \times 3^0 \times 5^1 = 70$
$14 \times 2^1 \times 3^0 \times 5^1 = 140$
$14 \times 2^2 \times 3^0 \times 5^1 = 280$
$14 \times 2^3 \times 3^0 \times 5^1 = 560$
$14 \times 2^0 \times 3^1 \times 5^1 = 210$
$14 \times 2^1 \times 3^1 \times 5^1 = 420$
$14 \times 2^2 \times 3^1 \times 5^1 = 840$
$14 \times 2^3 \times 3^1 \times 5^1 = 1680$
$14 \times 2^0 \times 3^2 \times 5^1 = 630$
$14 \times 2^1 \times 3^2 \times 5^1 = 1260$
$14 \times 2^2 \times 3^2 \times 5^1 = 2520$
$14 \times 2^3 \times 3^2 \times 5^1 = 5040$

So the sum of the multiples are = 14 + 28 + 56 + 112 + 42 + 84 + 168 + 336 + 126 + 252 + 504 + 1008 + 70 + 140 + 280 + 560 + 210 + 420 + 840 + 1680 + 630 + 1260 + 2520 + 5040

Total = 12600

Doing the calculation, the sum of the factors of $N = 2^3 \times 3^2 \times 5^4$, which are multiples of 14, is about 12,600.

Hence, (C) 12600 is the correct option.

Q18 Text Solution:

To find the unit digit of any number, we determine the cyclicity of the unit digit of the given number.

Cyclicity is the number of times the given number takes to repeat itself if it is undergoing a power operation.

Here, the cyclicity of 7 is 4. The remainder after dividing 459 by 4 is 3.

The cyclicity of 3 is 4. The remainder after dividing 123 by 4 is 3.

Any power of 5 ends only with 5. The unit digit of 95 remains constant at 5 for any positive power.

Therefore, the unit digit of x can be determined by taking the remainder as the power of the unit digits of the given number respectively.

$x = 7^3 + 3^3 + 5$
$x = 343 + 27 + 5$
$x = 375$

So, the unit digit of x is 5.

Hence, (A) 5 is the correct option.

Q19 Text Solution:

To find the unit digit of any number, we determine the cyclicity of the unit digit of the given number.

Cyclicity is the number of times the given number takes to repeat itself if it is undergoing a power operation.

Here, the cyclicity of 8 is 4. The remainder after dividing 458 by 4 is 2.

The cyclicity of 3 is 4. The remainder after dividing 852 by 4 is 0.

Therefore, the unit digit of the number can be determined by taking the remainder as the power of the unit digits of the given number respectively.

$= \frac{8^2}{3^0}$
$= \frac{64}{1}$
$= 64$

So, the unit digit of the number is 4.

Hence, (C) 4 is the correct option.

Q20 Text Solution:

We can see that increasing the natural number by 1, we are gathering 3 more powers of 5.

Therefore, (n + 1) is a multiple of 125 but not a multiple of 625 as it would result in 4 powers of 5.

Therefore, (n + 1) will be equal to all the multiples of 125 minus 625.

Total number of multiples of 125 less than 1000 = 8

Total number of multiples of 625 less than 1000 = 1

The required answer is (8 – 1) =7

Hence, the correct answer is (B) 7.

Q21 Text Solution:

According to the question, n! should have 30 zeroes in the end. If n = 100 (taking randomly keeping in view 30 is the number of zeros]

$100!\ has\ \frac{100}{5} + \frac{100}{5^2} = 24$

So n should be greater than 100.

Next multiple of 5 is 105. But 105 = 5 × 21, has only one extra 5. Number of zeroes will increase by 1 only. Similarly 110, 115 and 120 also have one extra 5. Number of zeroes (from 120! to 124!) = 28.

Now, the next multiple of 5 is 125, and 125 contains three 5's. So, the number of zeroes will increase by 3. Number of zeros in 125! = 28 + 3 = 31. So, there is no factorial of a number that ends with 30 zeros.

The correct answer is (C) 0.

Q22 Text Solution:

First, simplify the base number, i.e. 27

$27 = 3^3$

$\therefore 27^{17} = (3^3)^{17}$

By applying the properties of exponents: $(2^3 \times 3^3)^{17} = (2^3)^{17} \times (3^3)^{17}$

$(3^3)^{17} = 3^{(3*17)}$

$(3^3)^{17} = 3^{51}$.

Now, we find the remainder when (3^{51}) is divided by 8.

We use 3^{51} for finding the remainder.

When powers of 3 are divided by 8, the remainders follow the pattern 3, 1, 3, 1,...

i.e. 3^1 gives a remainder of 3, 3^2 gives a remainder of 1, 3^3 gives a remainder of 3, and so on.

As 51 is an odd number, 3^{51} will have a remainder of 3 when divided by 8.

Therefore, the remainder when 27^{17} is divided by 8 is the same as the remainder when 351 is divided by 8, which is 3.

The correct answer is (C) 3.

Q23 Text Solution:

We calculate the remainder when the terms are individually divided by 3.

2859^{341}: The remainder when 2859 is divided by 3 = 0. Hence, 2859^{341} will also have a remainder of 0 when divided by 3.

315^{19}: The remainder when 315 is divided by 3 = 0. Hence, 315^{19} will also have a remainder of 0 when divided by 3.

716^{34}: The remainder when 76 is divided by 3 = 2. Hence, we calculate the remainder when 2^{34} is divided by 3.

The pattern of powers of 2 for the remainder when divided by 3, is 2, 1, 2, 1, ... (2^1 gives a remainder of 2, 2^2 gives a remainder of 1, 2^3 gives a remainder of 2, and so on).

By observing the pattern, it repeats every 2 powers. 34 is an even number,so the remainder will be 1.

Therefore, by dividing 716^{34} by 3 we get the remainder of 1.

5^7: The remainder when 5 is divided by 3 = 2. Hence, we calculate the remainder when 5^7 is divided by 3.

The pattern of powers of 5 for the remainder when divided by 3, is 2, 2, 2, 2, ... (5^1 gives a remainder of 2, 5^2 gives a remainder of 2, 5^3 gives a remainder of 2, and so on). Observing the pattern, it repeats every power. Hence, the remainder will be 1.

Therefore, by dividing 5^7 by 3 we get the remainder of 2. Now, we find the remainder when the expression is divided by 3:

Remainder = (Remainder when 2859^{341} is divided by 3) + (Remainder when 315^{19} is divided by 3) + (Remainder when 716^{34} is divided by 3) – (Remainder when 5^7 is divided by 3)

Remainder = 0 + 0 + 1 – 2 = –1

–1 is not divisible by 3. We find the remainder when –1 is divided by 3. It is 2.

Therefore the remainder when $2859^{341} + 315^{19} + 716^{34} - 5^7$ is divided by 3 is 2.

The correct answer is (A) 2.

Q24 Text Solution:

Since 7 is prime, $n^7 - n$ is divisible by 7.

$n^7 - n = n(n^6 - 1) = n(n + 1)(n - 1)(n^4 + n^2 + 1)$.

Now $(n - 1)(n)(n + 1)$ is divisible by $3! = 6$.

Hence $n^7 - n$ is divisible by $6 \times 7 = 42$.

Hence the remainder is 0.

The correct answer is (B) 0.

Q25 Text Solution:

We have to find out Remainder of when divided by 18.

$$= \text{Rem}\,\frac{(1!+2!+3!....100!)}{18}$$

6! is divisible by 18

7! is divisible by 18

100! is divisible by 18

= We have to find out $\left[\left(\frac{1!+2!+3!+4!+5!}{18}\right)\right]$

$$= \text{Rem}\left(\frac{1+2+6+24+120}{18}\right)$$

$$= \text{Rem}\,\frac{153}{18} = 9$$

Hence, (A) 9 is the correct option.

Q26 Text Solution:

We have to find out the remainder when $(1!)^2 + (2!)^2 + (3!)^2 + \cdots$ is divided by 1152

$1152 = 2^7 \times 3^7$

= $(6!)^2$ is divisible by 1152

= All $(n!)^2$ are divisible by 1152 as long as n > 5

So, our problem is now reduced to

$$\text{Rem}\left[\frac{\left((1!)^2+(2!)^2+(3!)^2+(4!)^2+(5!)^2\right)}{1152}\right]$$

$$= \text{Rem}\left[\frac{(1+4+36+576+14400)}{1152}\right]$$

$$= \text{Rem}\left[\frac{15017}{1152}\right]$$

= 41

Therefore the remainder when the infinite sum $(1!)^2 + (2!)^2 + (3!)^2 + \cdots$ is divided by 1152 is 41.

The correct answer is (C) 41.

Q27 Text Solution:

The number is divisible by 11, so the difference between the sum of the digits at the odd places and the digits at the even places is either 0 or a multiple of 11.

Let the difference be a 0, so 11 + A = 3 + K

=> K – A = 8, the only possible value is 9,1.

Now we have to check if it satisfies the divisibility by 8 tests.

K= 9 makes the last 3 digits 992. This is divisible by 8.

Let's check for other cases when the difference is 11

11 + A – 3 – K = 11 => A – K = 3.

The possible values in this case are (9, 6), (8, 5), (7, 4), (6, 3), (5, 2), (4, 1).

Among these cases only (8, 5) and (4, 1) will be divisible by 8. So the possible values of the sum are 13, 5, and 10.

Now the difference between the sum of odd and even places cannot be 22.

11 + A – 3 – K = 22 => A – K = 14

Since both A and K are single–digit natural numbers, this is not possible. Thus the only possible values of the sum are 5, 10, and 13. Of the given options, only 10 is present.

Hence, the correct answer is (B) 10.

Q28 Text Solution:

First, the least common multiple (LCM) of 2 and 3 is calculated:

LCM (2, 3) = 6.

Next, we will find the multiples of 6 ranging between 50 and 90.

After dividing 50 by 6, we get the quotient 8 with a remainder of 2.

After dividing 90 by 6, we get the quotient of 15 with a remainder of 0.

The multiples of 6 in the given range are 54, 60, 66, 72, 78, 84, and 90.

There are 7 positive integers between 50 and 90 that are divisible by both 2 and 3.

Hence, the correct option is (A) 7.

Q29 Text Solution:

To find the HCF and LCM of A and B, we need to use the formula:

HCF × LCM = A × B

Given, A + B = 800 and A – B = 400

Adding the two equations,

2A = 1200

A = 600

Now, since A + B = 800, we can find B:

600 + B = 800

B = 800 – 600

B = 200

Now, we have A = 600 and B = 200.

HCF = HCF of 600 and 200

Prime factorization of 600: $2^3 \times 3 \times 5^2$

Prime factorization of 200: $2^3 \times 5^2$

HCF = $2^3 \times 5^2$ = 200

LCM = $\frac{A \times B}{HCF}$

LCM = $\frac{600 \times 200}{200}$

LCM = $\frac{120{,}000}{200}$

LCM = 600

The correct answer is (C) HCF = 200, LCM = 600.

Q30 Text Solution:

Given, the two numbers are in the ratio 3:4, which means they can be written as 3x and 4x, where x is a common factor.

Let's calculate the values again:

LCM(3x, 4x) = 84

To find the HCF of 3x and 4x, we need to find their prime factorization.

Prime factorization of 3x: 3 × x

Prime factorization of 4x: $2^2 \times x$

HCF = x

Now, we can find the greater number by comparing 3x and 4x:

Since 4x is the greater number, we can equate it to the options:

4x = 28 (option a)

4x = 84 (option b)

4x = 21 (option c)

4x = 24 (option d)

Now, let us see which option satisfies the condition of LCM = 84:

For option a:

4x = 28

$x = \frac{28}{4}$

x = 7

LCM(3x, 4x) = LCM(37, 47) = LCM(21, 28) = 84

4x = 28 satisfies the condition that the LCM of the two numbers is 84.

Therefore, the correct answer is (A) 28.