MBA PIONEER PRO 2024
Quantitative Aptitude
Functions 2

DPP: 10

Q 1 Find the total number of digits in 2^{51}, if log 2 = 0.3010

(A) 14 (B) 15
(C) 16 (D) 17

Q 2 If log 3 = 0.477, log 5 = 0.699, and log 6 = 0.779, then find the number of digits in 90^{90}.

(A) 154 (B) 165
(C) 176 (D) 187

Q 3 Let $\log_5(x^3) - 2\log_5(x) = \log_5(3)$, where x is a positive real number. Then, find the number of digits in $(3x)^{22}$ if $\log_{10} 3 = 0.477$.

(A) 20 (B) 21
(C) 22 (D) 23

Q 4 Solve the following logarithmic equation for x:
$\log_3(x^2 - 4x + 4) - \log_3(x - 2) = 1$

(A) 1 (B) 2
(C) 3 (D) 5

Q 5 If $\log_7(a) + \log_7(b) = \log_7(5)$ and $\log_3(a) - \log_3(b) = \log_3(2)$, find the value of $(a+b)^2 + (a-b)^2$.

(A) 25 (B) 50
(C) 75 (D) 100

Q 6 Let a, b, and c be positive integers such that $\log_{10}(a) = 2$, $\log_{10}(bc) = 4$, and $\log_{10}(ab) = 3$. What is the value of c?

(A) 10 (B) 100
(C) 1000 (D) 10000

Q 7 Find the number of digits in $3^8 \times 7^5 \times 5^6$ if log 3 = 0.477, log 5 = 0.699 and log 7 = 0.845.

(A) 13 (B) 14
(C) 15 (D) 16

Q 8 Find the real root of the equation $3^{4x} + 243(3^{2x-5}) - 20 = 0$.

(A) $\frac{\log_3 7}{2}$
(B) $\log_3 5$
(C) $\frac{\log_3 4}{2}$
(D) $\log_3 25$

Q 9 Let $a^5 = b^8 = c^9 = d^{10}$, then find the value of $\log_{\sqrt{d}}(a^2b^3c^5)$.
[Approximate to the nearest integer].

Q 10 If $\log_2\left(\log_{64}x + \frac{1}{3} + 8^x\right) = 3x$, then find the value of x^{-1}.

(A) 2 (B) 3
(C) 4 (D) 5

Q 11 If $\log_{256}(16\log_2(1 + \log_6(3 + 3\log_3 x))) = \frac{1}{2}$, then find the value of x.

(A) 1 (B) 3
(C) 5 (D) 7

Q 12 Let x, y, and z be three positive real numbers such that $\log_3(x^2) + \log_{27}(y^2) + \log_{27}(z^4) = 1$. What is the value of $x^6\, y^2\, z^4$?

(A) 25 (B) 26
(C) 27 (D) 28

Q 13 Let a, b, and c be three positive real numbers such that $\log_5(a^2) + \log_{25}(b^3) + \log_{125}(c^4) = 12$. Find the value of $a^{12}\, b^9\, c^8$?

(A) 1 (B) 5^{12}
(C) 5^{72} (D) 72

Q 14 Given $\log_5(2x^3 - 4x^2) = 3\log_5(x)$, find the positive integer value of x.

(A) 1 (B) 2
(C) 3 (D) 4

Q 15 Given $\log_3(7x^2 - 12x + 8) - \log_3(7x - 4) = \log_3(3x - 2)$, find the positive integer value of x.

(A) 1 (B) 2
(C) 3 (D) 4

Q 16 If $(\log_2 x + 6)^2 + (\log_2 x - 1)^2 = (2\log_2 x + 5)^2$, then the integer value of x is:

(A) 1 (B) 2
(C) 3 (D) 4

Q 17 If $\log(5x - 15) - \log(x - 3) > \log(x - 2)$, then the number of positive integer values of x is:

(A) 1 (B) 2
(C) 3 (D) 4

Q 18 If $\log_3(28 - 3^x) \leq 3 - x$, then

(A) x ≤ 0, x ≥ 3
(B) 0 ≤ x ≤ 3
(C) $x \geq 0$
(D) $-3 < x < 0$

Q 19 Find the number of integral values of w in $(\log_2 w)^4 + 3(\log_2 w)^3 + (\log_2 w)^2 - (3\log_2 w + 2)^2 + 20 < 0$.

(A) 1 (B) 2
(C) 3 (D) 4

Q 20 Let $\log_2(x^2 - 5x + 7) - \log_2(x^2 + 2x + 13) > 2 - \log_2 12$. Then find the minimum positive integer value of x that can be assumed?

(A) 7 (B) 8
(C) 9 (D) 10

Q 21 If $\log_{\frac{1}{\sqrt{3}}}\left(2x^2+5x-3\right) = 2$, then how many positive integer values of (y, z) can be found in $y-11=k|z|$, where k is the sum of the possible values of x?

(A) 5 (B) 4
(C) 3 (D) 2

Q 22 If $\log_{81}\left(log_5 \text{ x }\right) + log_{\frac{1}{81}}\left(\log_5 \frac{1}{y}\right) = 0$, then find the value of xy.

(A) 1 (B) 2
(C) 3 (D) $\frac{1}{2}$

Q 23 Find the minimum integer value of x in $\log_e\left(x^2+28a^2\right) - \frac{1}{\log_{(a-3)} \text{ e}} = log_e\left(\frac{11ax}{a-3}\right)$ if a is an integer.

(A) 15 (B) 16
(C) 17 (D) 18

Q 24 If $x^{1+\ log_{2012} \text{x}} = 2012x$, then how many distinct rational values of x can be obtained ?

(A) 1 (B) 2
(C) 3 (D) 4

Q 25 If the solution of the inequation $(\log_3 x)^4 - 35(\log_3 x)^2 + (\log_3 x + 15)^2 < 30(\log_3 x)$ is (a, b), where a, b are positive integers, then find the value of $\frac{1}{2}(b-a)$.

(A) 105 (B) 106
(C) 107 (D) 108

Q 26 If $(\log_{10}x\ +4)^3 + (\log_{10}x\ -1)^3 = (2\log_{10}x\ +3)^3$, then x can be a:

1. natural number
2. rational number
3. irrational number
4. can't be determined.

(A) Only 1 (B) Only 2 and 3
(C) Only 1, 2 and 3 (D) Only 4

Q 27 Let $\log_5(3x + 5) - \log_5(x - 1) = 2$. Then, how many digits will the number $(121x)^{76}$ have if $\log_{10} 3 = 0.477$, $\log_{10} 5 = 0.699$ and $\log_{10} 11 = 1.041$?

(A) 151 (B) 156
(C) 163 (D) 169

Q 28 If $5^{\log_a\{\log_b(\log_c\ x)\}} = 1$and $\log_b\{\log_c\ (\log_a x)\} = 0$, then find the value of $\left(\frac{1}{b}+\frac{1}{c}\right)$.

(A) $\log_a\ ac$ (B) $\log_{ac}\ a$
(C) $\log_{a^c}\ ac$ (D) $\log_{c^a}\ ac$

Q 29 If $\log_{81}(log_5 x\ ) + log_{\frac{1}{81}}\left(\log_5\frac{1}{y}\right) = 0$ and $4x^2 = 41 - 4y^2$, then find the value of 2|x + y|.

Q 30 If $(x-2)^{\log_2 \text{x}^2 - 3log_x 8} = \frac{1}{(x-2)^{17}}$ then find the number of rational roots of the equation.

Answer Key

Q1	C
Q2	C
Q3	B
Q4	D
Q5	A
Q6	C
Q7	A
Q8	C
Q9	27
Q10	C
Q11	B
Q12	C
Q13	C
Q14	D
Q15	A
Q16	B
Q17	C
Q18	A
Q19	A
Q20	C
Q21	D
Q22	A
Q23	B
Q24	B
Q25	D
Q26	C
Q27	D
Q28	C
Q29	7
Q30	2

Hints & Solutions

Q 1 Text Solution:

To find the number of digits we can use the logs

$\log 2^{51} = 51 \log 2 = 51 \times 0.3010 = 15.351$

The characteristic value here is 15, therefore total number of digits in 2^{51} is (characteristic value + 1)

Hence, the total number of digits in $2^{51} = 15 + 1 = 16$

Thus, option (C) is correct.

Q 2 Text Solution:

Let $k = 90^{90}$

Then, $\log k = \log 90^{90}$

$\log k = 90 \log 90$

$= 90 \times \log (3 \times 5 \times 6)$

$= 90 \times [\log(3) + \log(5) + \log(6)]$

$= 90 \times [0.477 + 0.699 + 0.779]$

$= 90 \times 1.955$

$= 175.95$

Since, the characteristic of log k is 175, so the number of digits in 90^{90} is (characteristic + 1), i.e., 176.

Q 3 Text Solution:

Given: $\log_5(x^3) - 2\log_5(x) = \log_5 (3)$

We can use the properties of logarithms to simplify the left side of the equation:

$\log_5(x^3) - \log_5(x^2) = \log_5 (3)$

Now, we can use the quotient rule of logarithms:

$\log_5\left(\frac{x^3}{x^2}\right) = log_5(3)$

$\frac{x^3}{x^2} = 3$

Dividing the powers of x, we get:

x = 3

Now, we need to find out the number of digits in $(3x)^{22}$, i.e., in 9^{22}.

Let $M = 9^{22}$

$\Rightarrow \log_{10} M = \log_{10} 9^{22}$

$\Rightarrow \log_{10} M = \log_{10} (3)^{2 \times 22}$

$\Rightarrow \log_{10} M = \log_{10} 3^{44}$

$\Rightarrow \log_{10} M = 44 \times \log_{10} 3 = 44 \times 0.477 = 20.988$

So, the number of digits in $(3x)^{22}$ is (characteristic of $\log_{10}M$) + 1

The number of digits in $(3x)^{22}$ is 20 + 1 = 21.

Q 4 Text Solution:

To solve the logarithmic equation, we can first apply the properties of logarithms to simplify the equation.

By using quotient rule of logarithms, which states that $\log_a(m) - \log_a(n) = \log_a(\frac{m}{n})$, we can rewrite the given equation:

$\log_3(\frac{x^2 - 4x + 4}{x-2}) = 1$

Now, we can use the property of logarithms, which states that if $\log_a(m) = n$, then $a^n = m$:

$3^1 = \frac{x^2 - 4x + 4}{x-2}$

Simplifying the equation, we get:

$3 = \frac{x^2 - 4x + 4}{x-2}$

Now, let's get rid of the fraction by multiplying both sides of the equation by (x - 2):

$3(x - 2) = x^2 - 4x + 4$

Expanding and simplifying the equation, we get:

$3x - 6 = x^2 - 4x + 4$

Rearranging the terms to form a quadratic equation:

$x^2 - 7x + 10 = 0$

Now, let's try to factor the quadratic equation:

$(x - 2)(x - 5) = 0$

This gives us two potential solutions:

$x - 2 = 0 \Rightarrow x = 2$

$x - 5 = 0 \Rightarrow x = 5$

However, we need to check if these solutions are valid in the original logarithmic equation. It's important to remember that the argument of a logarithm must always be greater than zero.

For x = 2, the first term of the original equation becomes $\log_3(0)$, which is undefined. Therefore, x = 2 is not a valid solution.

For x = 5, both terms in the original equation are defined:

$\log_3(\frac{5^2 - 4\times5 + 4}{5-2}) = 1$

So, x = 5 satisfies the original equation.

Thus, option D is correct.

Q 5 Text Solution:

We have two equations:

$\log_7(a) + log_7(b) = log_7(5)$

$\log_3(a) - log_3(b) = log_3(2)$

Using the properties of logarithms, we can rewrite the equations as follows:

$\log_7(ab) = log_7(5)$

$\log_3\left(\frac{a}{b}\right) = log_3(2)$

Since the logarithms are equal, we can equate the arguments:

ab = 5 ….(1)

$\frac{a}{b} = 2$ ….. (2)

From equation 2, we can find a:

a = 2b …. (3)

Substitute the value of a in equation 1:

(2b)b = 5

$2b^2 = 5$

$b^2 = \frac{5}{2}$

Now, find the value of a^2:

$a^2 = (2b)^2 = 4b^2 = 4\left(\frac{5}{2}\right) = 10$

$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2) = 2\left(10 + \frac{5}{2}\right) = 2 \times \frac{25}{2} = 25$

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Q 6 Text Solution:

Given:

$\log_{10}(a) = 2$……….(1)

$\log_{10}(bc) = 4$………..(2)

$\log_{10}(ab) = 3$…………(3)

From the first equation, we can write:

$a = 10^2 = 100$

From the third equation, we can write:

$ab = 10^3$

Since we know the value of a, we can find the value of b:

$100 \times b = 10^3$

$b = 10$

Now we can use the second equation:

$\log_{10}(bc) = 4$

Substitute the values of b and c:

$\log_{10}(10 \times c) = 4$

Using the properties of logarithms:

$\log_{10}(10) + \log_{10}(c) = 4$

Since $\log_{10}(10) = 1$:

$1 + \log_{10}(c) = 4$

$\log_{10}(c) = 3$

- $c = 10^3 = 1000$

Q 7 Text Solution:

Given that,

$\log 3 = 0.477$,

$\log 5 = 0.699$ and

$\log 7 = 0.845$

Now, let $P = 3^8 \times 5^6 \times 7^5$

$\Rightarrow \log P = \log(3^8 \times 5^6 \times 7^5)$

$= \log(3^8) + \log(5^6) + \log(7^5)$

$= 8\log(3) + 6\log(5) + 5\log(7)$

$= 8 \times 0.477 + 6 \times 0.699 + 5 \times 0.845$

$= 3.816 + 4.194 + 4.225$

$= 12.235$

So, the number of digits in $3^8 \times 7^5 \times 5^6$

$= 12 + 1$

$= 13$.

Q 8 Text Solution:

Given that,

$3^{4x} + 243(3^{2x-5}) - 20 = 0$

$\Rightarrow (3^{2x})^2 + 3^5 \times 3^{2x-5} - 20 = 0$

$\Rightarrow (3^{2x})^2 + 3^{2x} - 20 = 0$

$\Rightarrow (3^{2x})^2 + 5(3^{2x}) - 4(3^{2x}) - 20 = 0$

$\Rightarrow 3^{2x}(3^{2x} + 5) - 4(3^{2x} + 5) = 0$

$\Rightarrow (3^{2x} - 4)(3^{2x} + 5) = 0$

$\Rightarrow 3^{2x} = 4$, but $3^{2x} \neq -5$

$\Rightarrow \log_3(3^{2x}) = \log_3 4$

$\Rightarrow 2x \log_x 3 = \log_x 4$

$\Rightarrow 2x = \log_3 4$

$\Rightarrow x = \frac{\log_3 4}{2}$

Q 9 Text Solution:

Let $a^5 = b^8 = c^9 = d^{10} = k$ (say) $(k \neq 0)$

Therefore,

$a = k^{\frac{1}{5}}$

$b = k^{\frac{1}{8}}$

$c = k^{\frac{1}{9}}$

$d = k^{\frac{1}{10}}$

Therefore,

$\log_{\sqrt{d}}(a^2 b^3 c^5)$

$= \log_{d^{\frac{1}{2}}}\left(k^{\frac{2}{5}}\, k^{\frac{3}{8}}\, k^{\frac{5}{9}}\right)$

$= \log_{k^{\frac{1}{20}}}\left(k^{\frac{2}{5}+\frac{3}{8}+\frac{5}{9}}\right)$

$= \log_{k^{\frac{1}{20}}} k^{\frac{479}{360}}$

$= \frac{479}{18}\log_{k^{\frac{1}{20}}}\left(k^{\frac{1}{20}}\right)$

$= \frac{479}{18}$

$= 27$ (approx.)

Q 10 Text Solution:

Given that,

$\log_2\left(\log_{64} x + \frac{1}{3} + 8^x\right) = 3x$

- $\log_{64} x + \frac{1}{3} + 8^x = 2^{3x}$
- $\log_{64} x + \frac{1}{3} + 8^x = \left(2^3\right)^x$
- $\log_{64} x + \frac{1}{3} + 8^x = 8^x$
- $\log_{64} x + \frac{1}{3} = 0$
- $\log_{64} x = -\frac{1}{3}$
- $x = 64^{-\frac{1}{3}}$
- $x = \left(4^3\right)^{-\frac{1}{3}} = 4^{-1}$
- $x^{-1} = 4$

Q 11 Text Solution:

$\log_{256}(16\log_2(1 + \log_6(3 + 3\log_3 x))) = \frac{1}{2}$

$\Rightarrow 256^{\frac{1}{2}} = 16\log_2(1 + \log_6(3 + 3\log_3 x))$

$\Rightarrow 16\log_2(1 + \log_6(3 + 3\log_3 x)) = 16$

$\Rightarrow \log_2(1 + \log_6(3 + 3\log_3 x)) = 1$

$\Rightarrow (1 + \log_6(3 + 3\log_3 x)) = 2$

$\Rightarrow \log_6(3 + 3\log_3 x) = 1$

$\Rightarrow 3 + 3\log_3 x = 6$,

$\Rightarrow 3\log_3 x = 3$

Hence $x = 3$.

Q 12 Text Solution:

Given: $\log_3(x^2) + \log_{27}(y^2) + \log_{27}(z^4) = 1$

We can rewrite the logarithms to the base 3:

$2 \times \log_3(x) + (\frac{2}{3}) \times \log_3(y) + (\frac{4}{3}) \times \log_3(z) = 1$

Using properties of logarithms, we can combine them:

$\log_3(x^2 \times y^{\frac{2}{3}} \times z^{\frac{4}{3}}) = 1$

Now, we can use the property of logarithms, $\log_b(a) = c$, where $b^c = a$:

$3^1 = x^2 \times y^{\frac{2}{3}} \times z^{\frac{4}{3}}$

To find the value of $x^6 y^2 z^4$, we can raise both sides of the equation to the power of 3:

$(x^2 \times y^{\frac{2}{3}} \times z^{\frac{4}{3}})^3 = 3^3$

$\Rightarrow x^6 y^2 z^4 = 27$

Q 13 Text Solution:

Given: $\log_5(a^2) + \log_{25}(b^3) + \log_{125}(c^4) = 12$

We can rewrite the logarithms to the base 5:

$2 \times \log_5(a) + (\frac{3}{2}) \times \log_5(b) + (\frac{4}{3}) \times \log_5(c) = 12$

Now, let's multiply the entire equation by 6 to remove the fractions:

$12 \times \log_5(a) + 9 \times \log_5(b) + 8 \times \log5(c) = 72$

We can now rewrite the equation using the properties of logarithms:

$\log_5(a^{12}) + \log_5(b^9) + \log_5(c^8) = 72$

$\log_5(a^{12}\, b^9\, c^8) = 72$

$a^{12}\, b^9\, c^8 = 5^{72}$

Q 14 Text Solution:

Using the properties of logarithms, we can simplify the equation:

$\log_5(2x^3 - 4x^2) = 3\log_5(x)$

$\log_5(2x^2(x - 2)) = \log_5(x^3)$

Since the logarithms have the same base, we can equate the arguments:

$\Rightarrow 2x^2(x - 2) = x^3$

Now, solve the equation:

$\Rightarrow 2x^2(x - 2) - x^3 = 0$

$\Rightarrow x^3 - 2x^2(x - 2) = 0$

$\Rightarrow x^3 - 2x^3 + 4x^2 = 0$

$\Rightarrow 4x^2 - x^3 = 0$

Factor out the common term x^2:

$x^2(4 - x) = 0$

So, x = 0 or x = 4.

Q 15 Text Solution:

Using the properties of logarithms, we can combine the logs:

$\log_3 \frac{7x^2 - 12x + 8}{7x - 4} = log_3\,(3x - 2)$

Since the logarithms have the same base, we can equate the arguments:

$\frac{7x^2 - 12x + 8}{7x - 4} = 3x - 2$

Now, cross-multiply:

$7x^2 - 12x + 8 = (3x - 2)(7x - 4)$

Expanding the right side:

$7x^2 - 12x + 8 = 21x^2 - 14x - 12x + 8$

Now, move all terms to one side:

$14x^2 - 14x = 0$

Factor the equation:

$14x(x - 1) = 0$

So, x = 0 or x = 1.

Q 16 Text Solution:

Given that,

$(\log_2 x + 6)^2 + (\log_2 x - 1)^2 = (2\log_2 x + 5)^2$ …. (i)

Let $\log_2 x + 6 = a$, $\log_2 x - 1 = b$.

Then, $a + b = 2\log_2 x + 5$

Now, the equation will become

$a^2 + b^2 = (a + b)^2$

The above equation when expanded implies that 2ab = 0

Therefore, a = 0 or b = 0

- $\log_2 x + 6 = 0$ or, $\log_2 x - 1 = 0$
- $\log_2 x = -6$ or, $\log_2 x = 1$
- $x = 2^{-6}$ or, $x = 2^1$
- $x = \frac{1}{64}$ or, x = 2

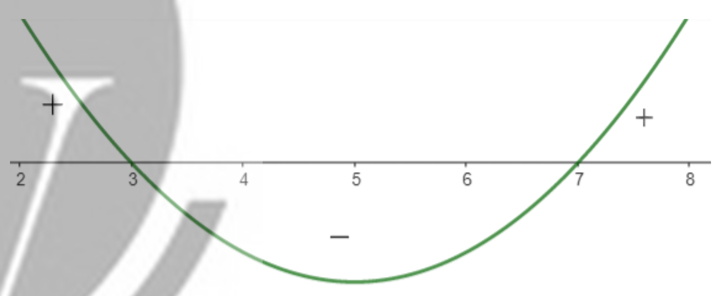
So, the only integer value of x = 2.

Q 17 Text Solution:

Given that,

$\log\ (5x - 15) - \log(x - 3) > \log(x - 2)$

- $\log(x - 3) + \log(x - 2) < \log(5x - 15)$
- $(x - 3)(x - 2) < (5x - 15)$
- $x^2 - 5x + 6 + 21 - 5x - 6 < 0$
- $x^2 - 10x + 21 < 0$
- $x^2 - 7x - 3x + 21 < 0$
- $(x - 7)(x - 3) < 0$



- $3 < x < 7$

So, the number of possible positive integer values of x are 4, 5, and 6.

Q 18 Text Solution:

$\log_3\ (28 - 3^x) \leq 3 - x$

- $28 - 3^x \leq 3^{3 - x}$
- $28 - 3^x \leq 3^3\,(3^{-x})$
- $28 - 3^x \leq 27(3^{-x})$
- $3^x - 28 + 27(3^{-x}) \geq 0$
- $3^{2x} - 28(3^x) + 27 \geq 0$
- $3^{2x} - 3^x - 27(3^x) + 27 \geq 0$
- $(3^x - 27)(3^x - 1) \geq 0$
- $3^x \leq 1,\ 3^x \geq 27$
- $x \leq 0,\ x \geq 3$

Q 19 Text Solution:

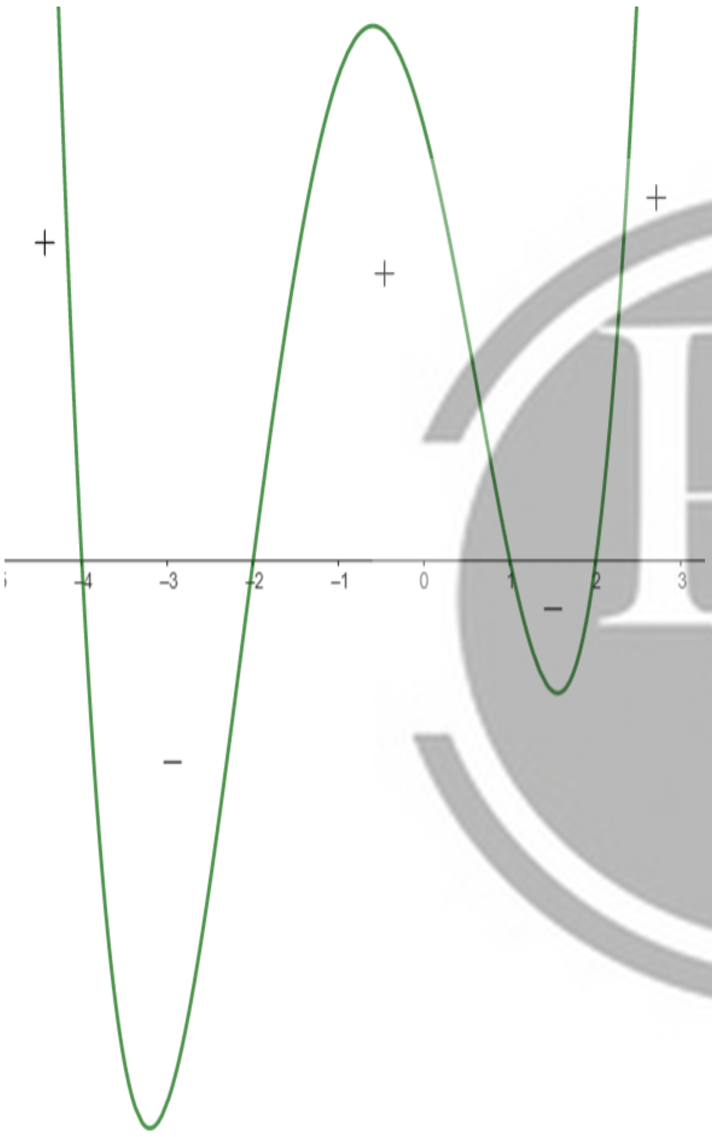
Given inequality: $(\log_2 w)^4 + 3(\log_2 w)^3 + (\log_2 w)^2 - (3\log_2 w + 2)^2 + 20 < 0$

Let's denote $\log_2 w$ as L, then the inequality becomes:

$L^4 + 3L^3 + L^2 - (3L + 2)^2 + 20 < 0$

- $L^4 + 3L^3 + L^2 - 9L^2 - 2 \times 3L \times 2 - 4 + 20 < 0$
- $L^4 + 3L^3 - 8L^2 - 12L + 16 < 0$
- $(L - 2)(L + 2)(L + 4)(L - 1) < 0$

Now, we will find the intervals of L where the inequality holds true:
$(L - 2)(L + 2)(L + 4)(L - 1) < 0$



- $-4 < L < -2$ and $1 < L < 2$
- $-4 < \log_2 w < -2$ and $1 < \log_2 w < 2$
- $2^{-4} < w < 2^{-2}$ and $2^1 < w < 2^2$
- $\frac{1}{16} < w < \frac{1}{4}$ and $2 < w < 4$

Hence, the only integral value of w satisfying the given inequation is 3.

Q 20 Text Solution:

$\log_2(x^2 - 5x + 7) - \log_2(x^2 + 2x + 13) > 2 - \log_2 12$

$\log_2 \frac{x^2-5x+7}{x^2+2x+13} > 2 - \log_2 12$

Using the property of logarithm, we can rewrite the inequality:

$\log_2 \frac{x^2-5x+7}{x^2+2x+13} + \log_2 12 - \log_2 4 > 0$

$\log_2 \frac{x^2-5x+7}{x^2+2x+13} + \log_2 \frac{12}{4} > 0$

$\log_2 \frac{x^2-5x+7}{x^2+2x+13} + \log_2 3 > 0$

$\log_2 \frac{3(x^2-5x+7)}{x^2+2x+13} > 0$

Now, we need to find the range of x values for which the expression inside the logarithm is greater than 1:

$\frac{3(x^2-5x+7)}{x^2+2x+13} > 1$

Let's rewrite the inequality:

$3\left(x^2 - 5x + 7\right) > x^2 + 2x + 13$

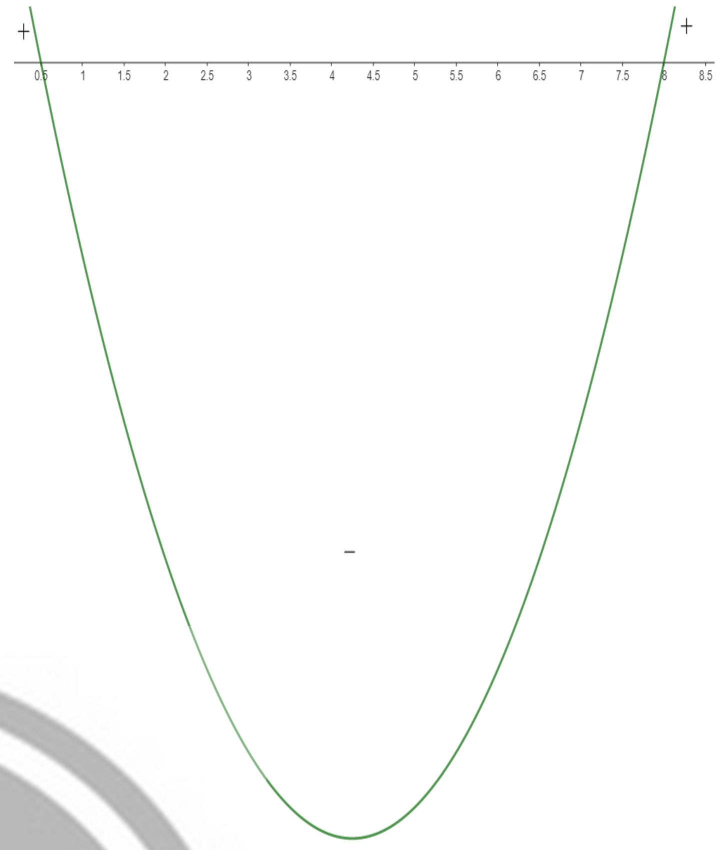
Now, simplify:

$2x^2 - 17x + 8 > 0$

Next, factor the quadratic:

$(2x - 1)(x - 8) > 0$

Using the Wavy Curve method, we have



The inequality is satisfied when $x < \frac{1}{2}$ or $x > 8$.
Therefore, the minimum positive integer value of x that can be assumed is 9.

Q 21 Text Solution:

$\log_{\frac{1}{\sqrt{3}}}\left(2x^2 + 5x - 3\right) = 2$

$\left(2x^2 + 5x - 3\right) = \left[\left(\frac{1}{\sqrt{3}}\right)^2\right]$

- $2x^2 + 5x - 3 = \frac{1}{3}$
- $3(2x^2 + 5x - 3) = 0$
- $2x^2 + 6x - x - 3 = 0$
- $(2x - 1)(x+3) = 0$
- $x = \frac{1}{2}, \ -3$

Now, the required sum of the possible values
$k = \frac{1}{2} - 3 = -\frac{5}{2}$
Therefore, the given equation $y - 11 = k|z|$ becomes
$y - 11 = -\frac{5}{2}|z|$

- $2y = 22 - 5|z|$
- $y = \frac{22-5|z|}{2}$

Therefore, since for any values of z, |z| gives positive value, so z will give a positive integral value if z is a multiple of 2 such that $22 - 5|z| > 0$.
So, z can be 2 and 4 because when $z > 4$, then y becomes negative.
So, (y, z) can be (6, 2), and (1, 4).
Thus, a total of 2 positive integer values of (y, z) can be obtained.

Q 22 Text Solution:

Given that,

$\log_{81}(\log_5 x) + \log_{\frac{1}{81}}\left(\log_5 \frac{1}{y}\right) = 0$

- $\log_{3^4}(\log_5 x) + \log_{3^{-4}}\left(\log_5 \frac{1}{y}\right) = 0$
- $\frac{1}{4}\log_3(\log_5 x) - \frac{1}{4}\log_3(-\log_5 y) = 0$
  [Since, $\log_{A^C} B = \frac{1}{C}\log_A B$]
- $\frac{1}{4}\left[\log_3\left(-\frac{\log_5 x}{\log_5 y}\right)\right] = 0$
- $\log_3\left(-\frac{\log_5 x}{\log_5 y}\right) = 0$

- $-\frac{\log_5 x}{\log_5 y} = 3^0 = 1$
- $\log_5 x = -\log_5 y$
- $x = \frac{1}{y}$
- $x = \frac{1}{y}$
- xy = 1

Q 23 **Text Solution:**

Given that,
$\log_e (x^2 + 28a^2) - \frac{1}{\log_{(a-3)} e} = \log_e \left(\frac{11ax}{a-3}\right)$

- $\log_e (x^2 + 28a^2) - \log_e (a-3) = \log_e \left(\frac{11ax}{a-3}\right)$ …. (i)

The above equation is defined if a – 3 > 0, i.e., if a > 3. Also, if $\frac{11ax}{a-3} > 0$.
Now, since, a > 3, so, ax > 0, which implies x > 0.
Therefore, equation (i) will become
$\log_e \left(\frac{x^2+28a^2}{a-3}\right) = \log_e \left(\frac{11ax}{a-3}\right)$

- $x^2 + 28a^2 = 11ax$
- $x^2 - 4ax - 7ax + 28a^2 = 0$
- (x – 4a)(x – 7a) = 0
- x = 4a, 7a

Now, since, a > 3, so the minimum value of a = 4.
So, at a = 4, the minimum value of x = 4a = 4 × 4 = 16

Q 24 **Text Solution:**

Given that
$x^{1 + \log_{2012} x} = 2012x$

- $x^1 \cdot x^{\log_{2012} x} = 2012x$
- $x\left(x^{\log_{2012} x} - 2012\right) = 0$
- $x \neq 0, \left(x^{\log_{2012} x} - 2012\right) = 0$
- $x^{\log_{2012} x} = 2012$
- $\log_{2012} x = \log_x 2012$
- $\log_{2012} x = \frac{1}{\log_{2012} x}$
- $(\log_{2012} x)^2 = 1$
- $\log_{2012} x = \pm 1$
- $x = 2012^{\pm 1}$
- $x = 2012, \frac{1}{2012}$

Hence, only 2 distinct rational values of x can be obtained.

Q 25 **Text Solution:**

Given that,
$(\log_3 x)^4 - 35(\log_3 x)^2 + (\log_3 x + 15)^2 < 30(\log_3 x)$

- $(\log_3 x)^4 - 35(\log_3 x)^2 + (\log_3 x)^2 + 30(\log_3 x) + (15)^2 < 30 \log_3 x$
- $(\log_3 x)^4 - 34(\log_3 x)^2 + 225 < 0$
- $(\log_3 x)^4 - 9(\log_3 x)^2 - 25(\log_3 x)^2 + 225 < 0$
- $[(\log_3 x)^2 - 9][(\log_3 x)^2 - 25] < 0$
- $9 < (\log_3 x)^2 < 25$
- $3 < \log_3 x < 5$ or, $-5 < \log_3 x < -3$
- $3^3 < x < 3^5$ or, $3^{-5} < x < 3^{-3}$
- $27 < x < 243$ or, $\frac{1}{243} < x < \frac{1}{27}$

Now, since, $x \in$ (a, b), where a, b are positive integers, so $x \in (27, 243)$
Therefore, a = 27 and b = 243.
Then, $\frac{1}{2}(b-a) = \frac{1}{2}(243-27) = 108$.

Q 26 **Text Solution:**

Given that,
$(\log_{10} x + 4)^3 + (\log_{10} x - 1)^3 = (2\log_{10} x + 3)^3$ …. (i)
Let $\log_{10} x + 4 = a$, $\log_{10} x - 1 = b$
Then, a + b = 2 $\log_{10}$ x + 3
Therefore, (i), we have
$a^3 + b^3 = (a+b)^3$
The above when expanded implies the following,

- 3ab(a + b) = 0
- a = 0, or, b = 0 or, a + b = 0
- $\log_{10} x + 4 = 0$ or, $\log_{10} x - 1 = 0$ or, $2 \log_{10} x + 3 = 0$
- $\log_{10} x = -4$ or, $\log_{10} x = 1$ or, $\log_{10} x = -\frac{3}{2}$
- $x = 10^{-4}$ or, $x = 10^1$ or, $x = 10^{-\frac{3}{2}}$
- $x = \frac{1}{10000}$ or, x = 10 or, $x = \frac{1}{10\sqrt{10}}$

Hence, x can be a natural number, rational number or irrational number.

Q 27 **Text Solution:**

Using the properties of logarithms, we can rewrite the equation as follows:
$\log_5 \left(\frac{3x+5}{x-1}\right) = 2$
Now, we can use the property of logarithms, $\log_b(a)$ = c, where $b^c = a$:
$5^2 = \frac{3x+5}{x-1}$
$25 = \frac{3x+5}{x-1}$
Now, we can cross-multiply to remove the fraction:
25(x - 1) = 3x + 5
25x - 25 = 3x + 5
22x = 30
$x = \frac{30}{22}$
$x = \frac{15}{11}$

Now, we need to find out the number of digits in $(121x)^{76}$, i.e., in $(165)^{76}$.

So, let P = 165^{76}

Then, $\log_{10} P = \log_{10} 165^{76}$

=> $\log_{10} P = 76 \times \log_{10} (165)$

=> $\log_{10} P = 76 \times \log_{10} (5 \times 3 \times 11)$

=> $\log_{10} P = 76 \times [\log_{10}(5) + \log_{10}(3) + \log_{10}(11)]$

=> $\log_{10} P = 76 \times [0.699 + 0.477 + 1.041]$

=> $\log_{10} P = 76 \times 2.217$

=> $\log_{10} P = 168.492$

So, the number of digits in $(121x)^{76}$ is (characteristic of $\log_{10}$P) + 1,

i.e., the number of digits in $(121x)^{76}$ is (168 + 1) = 169

Q 28 Text Solution:

Given that, $5^{\log_a\{\log_b(\log_c x)\}} = 1$

$\Rightarrow 5^{\log_a\{\log_b(\log_c x)\}} = 5^0$

$\Rightarrow \log_a\{\log_b(\log_c x)\} = 0$

$\Rightarrow \log_b (\log_c x) = a^0 = 1$

$\Rightarrow \log_c x = b^1 = b$

$\Rightarrow \frac{1}{b} = \frac{1}{\log_c x} = \log_x c \quad \ldots\ldots (i)$

Also, $\log_b\{\log_c (\log_a x)\} = 0$

$\log_c (\log_a x) = b^0 = 1$

$\Rightarrow \log_a x = c^1 = c \quad \ldots (ii)$

$\Rightarrow \frac{1}{c} = \frac{1}{\log_a x} = log_x a$

Again, from (ii), we have

x = a^c …. (iv)

Now, $\left(\frac{1}{b} + \frac{1}{c}\right) = log_x c + log_x a$

$= \log_x ac$

$= \log_{a^c} ac$ [using (iv)]

Hence, option (C) is correct.

Q 29 Text Solution:

Given that,

$\log_{81}(log_5 x) + log_{\frac{1}{81}}\left(\log_5 \frac{1}{y}\right) = 0$

- $\log_{3^4}(\log_5 x) + log_{3^{-4}}\left(\log_5 \frac{1}{y}\right) = 0$
- $\frac{1}{4}\log_3(\log_5 x) - \frac{1}{4}\log_3(-log_5 y) = 0$
- [Since $\log_{A^C} B = \frac{1}{C}\log_{A} B$]
- $\frac{1}{4}\left[\log_3\left(-\frac{\log_5 x}{\log_5 y}\right)\right] = 0$
- $\log_3\left(-\frac{\log_5 x}{\log_5 y}\right) = 0$
- $-\frac{\log_5 x}{\log_5 y} = 3^0 = 1$
- $\log_5 x = -log_5 y$
- $x = \frac{1}{y}$
- $x = \frac{1}{y}$
- xy = 1 …. (i)

Now,

$4x^2 = 41 - 4y^2$

- $4x^2 + 4y^2 = 41$
- $x^2 + y^2 = \frac{41}{4}$
- $(x+y)^2 - 2xy = \frac{41}{4}$
- $(x+y)^2 - 2 = \frac{41}{4}$ [Using (i)]
- $(x+y)^2 = \frac{41}{4} + 2$
- $(x+y)^2 = \frac{49}{4}$
- $(x+y) = \pm\frac{7}{2}$

Now, $2|x+y| = 2 \times \frac{7}{2} = 7$

Q 30 Text Solution:

Given that,

$(x-2)^{\log_2 x^2 - 3log_x 8} = \frac{1}{(x-2)^{17}}$

- $(x-2)^{\log_2 x^2 - 3log_x 8} = (x-2)^{-17}$

Taking log on both sides, we have

- $(\log_2 x^2 - 3log_x 8) \log (x-2) = -17 \log (x-2)$
- $\log (x-2) [\log_2 x^2 - 3log_x 2^3 + 17] = 0$
- $\log (x-2) \left[2log_2 x - \frac{9}{\log_2 x} + 17\right] = 0$
- $\log (x-2) \left[2(\log_2 x)^2 + 17log_2 x - 9\right] = 0$
- $\log (x-2) \left[2(\log_2 x)^2 + 18log_2 x - log_2 x - 9\right] = 0$
- $\log (x-2) (2log_2 x - 1)(\log_2 x + 9) = 0$
- $\log (x-2) = 0$ or, $(2log_2 x - 1) = 0$ or, $(\log_2 x + 9) = 0$
- x – 2 = 1 or, $\log_2 x = \frac{1}{2}$ or, $\log_2 x = -9$
- x = 3 or, $x = 2^{\frac{1}{2}}$ = or, $x = 2^{-9} = \frac{1}{2^9} = \frac{1}{512}$

Hence, the number of rational roots of the given equation is 2.