MBA PIONEER PRO 2024

QUANTITATIVE APTITUDE

DPP: 9

Functions 1

Q1 If |2x - 3| + 3|4x - 5| - 6 < 5x, then find the number of integer values that x can assume.

Q2 If|3x - 4| - 2|5x - 6| + 7 > 4x, then find the number of integer values x can assume.

Q3 If |6x + 5| - 4x < 11, then find the number of integral values 5x can assume.

Q4 Find the range of x where ||x - 5| - 6| > 5?

(A) $(-\infty,\ 6)$ or $(16,\ \infty)$

(B) (-∞, -1)∪ $(4,\ 6) \cup (7,\ 16)$

(C) (-5, 6)∪$(6,\ \infty)$

(D) (-∞, -6) or (4, 6) or (16, ∞)

Q5 Solve the inequation:

$\left|\frac{3}{x-5}\right| > 1,\ x \neq 5$

(A) (2, 3) ∪ (6, 7)

(B) (2, 5) ∪ (6, 8)

(C) (2, 3) ∪ (3, 5)

(D) (2, 5) ∪ (5, 8)

Q6 If $f\left(x\right) = \frac{1}{\sqrt{1-x^2}}, g\left(x\right) = \frac{1}{x}, 0 < x < 1,$ then find the value of $3\mathrm{f}\left(\mathrm{g}\left(\frac{5}{4}\right)\right)$.

Q7 If $f(x) = \frac{2^{-x}}{2^{-x} + 2^x}$ and $g(x) = \frac{5^x}{5^x + 5^{-x}}$, then find the value of $[f(x) + f(-x)]^{50} + [g(x) + g(-x)]^{50}$

Q8 Let $f(x) = x \cdot \frac{10^x - 1}{10^x + 1}$. Then, find the value of $f^3(x) - f^3(-x)$.

Q9 Let $f(x) = x^2 - x$ and $g(x) = x^{-1}$. Then find the value of $f(g(x)) + f(x)g(x^3)$.

Q10 Let $f(x) = 2x + 60$ and $g(x) = x^2 - 17x + 30$, then find the sum of zeroes of f(g(x)).

Q11 If f(1) = 1, f(4) = 73, f(9) = 753, then what is the remainder that f(15) leaves when it's divided by 15?

Q12 If $f(x) = \sqrt{x-2} + \sqrt{9-x}, g(x) = x+1,$ then find the domain of $f(g(x))$.

(A) (2, 9)

(B) [2, 9]

(C) (1, 8)

(D) [1, 8]

Q13 Let $f(x) = x + \frac{1}{x}$, $g(x) = x^2$, and $h(x) = x^3$. Then, $g(f(x)) + h(f(x)) = f(g(x)) + f(h(x))$ is possible when

1. $f\left(\frac{1}{x}\right) = -\frac{1}{3}f(1)$
2. $f(x) = -\frac{2}{3}$
3. $g\left(\frac{1}{x}\right) = -3f(2)$
4. $g(x) = -\frac{3}{2}$

(A) Only 1

(B) Only 1 and 2

(C) Only 1, 3 and 4

(D) Only 2 and 4

Q14 If $f(x) = \left(64 - x^4\right)^{\frac{1}{4}}$, for $0 < x < 2\sqrt{2}$, then find the value of $f\left(f\left(\frac{1}{3}\right)\right)$.

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

Q15

Let $f\left(x\right)=\frac{1}{1-\frac{1}{1-\frac{1}{1-x}}}, x\neq 1$ and $g\left(x\right)=\frac{x}{\sqrt{x^2-1}}, x\in(-\infty,-1)\cup(1,\infty)$.

Then find the domain of $g\left(f\left(f\left(\frac{1}{x}\right)\right)\right)$.

(A) x < 1

(B) –1 < x < 1

(C) x > 1

(D) x < – 1, x > 1

Q16 Let $f\left(x\right)=\frac{25^x}{25^x+5}, g\left(x\right)=f\left(x\right)+f\left(1-x\right)$, $h\left(x\right)=\sqrt{5+12x-9x^2}$ and $k(x)=g(x)+h(x)$. Then find the domain of $k(x)$.

(A) $-\frac{1}{3}\leq x\leq\frac{3}{5}$ (B) $-\frac{1}{5}\leq x\leq\frac{3}{5}$

(C) $-\frac{1}{5}\leq x\leq\frac{5}{3}$ (D) $-\frac{1}{3}\leq x\leq\frac{5}{3}$

Q17 Find the range of $f\left(x\right)=\frac{3+2x+2x^2}{1+x+x^2}$.

(A) $\left(\frac{2}{3},\ 10\right)$

(B) $\left[\frac{2}{3},\frac{10}{3}\right]$

(C) $[2,\ 10]$

(D) $\left[2,\ \frac{10}{3}\right]$

Q18 Let g(x) =|x–1| + |x–3| + |x–5|. Then, find the value of x for which the maximum value of g(x) is 12.

(A) 5 (B) 7

(C) 11 (D) 12

Q19 Let $5f\left(x^4\right)+2f\left(\frac{1}{x^4}\right)=x^4-1,\ \ x\neq 0,$ then $f(x^{16})$ equals:

(A) $\frac{(x^{16}+1)(5x^{16}-2)}{21x^{32}}$

(B) $\frac{(5x^{16}+2)(x^{16}-1)}{21x^{16}}$

(C) $\frac{(4x^{16}+2)(x^{16}-5)}{25x^{16}}$

(D) $\frac{(5x^{16}+16)(3x^{16}-5)}{21x^{16}}$

Q20 Let f(0) = 1, f(1) = 1, f(11) – f(10) = 1. What is the value of f(20)? (Assume f(x) as a quadratic function.)

(A) 30 (B) 20

(C) 501 (D) 0

Q21 Find the domain of f(x) = $\sqrt{\frac{x^2-7x+10}{x^2-5x+4}}$

(A) $(-\infty,\ 2)\cup[5,\ \infty)$

(B) $(1,\ 2)\cup(2,\ 4]\cup[5,\ \infty)$

(C) $(-\infty\ ,1)\cup[2,4)\cup[5,\ \infty)$

(D) $(-\infty,1]\cup(2,\ 4]\cup(5,\ \infty)$

Q22 Find the minimum value of $x^2 + 5x^8 + 3x$ when |x-1| + |x-2| + |x-3| = 9.

(A) 5 (B) 4

(C) 3 (D) 2

Q23 If f(x) = f(x-1) + f(x+1) and f(25) + f(26) = 9. Also, f(47) = -5, then what is the value of f(1) + f(2) + f(3)...........+ f(99)?

(A) 10 (B) 12

(C) 9 (D) 16

Q24 f(x) = px^2 +2x +1 and g(x) = x^2 +6x +2 are two quadratic functions such that p < 0. Then the value of p for which there is only one point of intersection between f(x) and g(x) is:

(A) -4 (B) -3

(C) -2 (D) -1

Q25 Find the range of the function f(x) = (x+4) (5-x) (x+1).

(A) [-2, 3]

(B) (-∞, 20]

(C) (-∞, +∞)

(D) [-20, ∞)

Q26 Let `g(x) = x^2–αx+β, α is an even positive integer. If one of the roots of g(x) = 0 is a prime number and the other is an even number and α +2β = 32, then, g(g(x)) equals:

(A) $x^4+8x^3-76x^2+110x+50$

(B) $x^4+18x^3-36x^2-12x+65$

(C) $x^4-8x^3+38x^2-156x+56$
(D) $x^4-16x^3+80x^2-128x+60$

Q27 Let $f(x) = |x - 2| + |x - 5| + |x - 7|$. Find the sum of the maximum and minimum integer values of x when $f(x) \leq 15$.

(A) 12
(B) 11
(C) 10
(D) 9

Q28 Let f is a function such that $f(0) = 3,\ f(1) = 4,\ f(2) = 6$ and $f(x+3) = 3f(x) - f(x+1)$. Then, the value of $f(6)$ is:

(A) 13
(B) -3
(C) 6
(D) 9

Q29 If $f(x^2 - 9) = 3x^2 + 2a + 3b$, $f(x-3) = x^3 - 3ax + 2b$ and $g(x) = x^2$, then what is the value of $g\left(\frac{b}{a}\right)$?

(A) -125
(B) 119
(C) 121
(D) 129

Q30 Consider the following two functions:
$f(x) = 3x^2 - 2x + 5$
$g(x) = 7x^3 + 4x^2 - x + 8$.
Then, the nature of the function $h(x) = f(g(2x - 1))$ is:

(A) Even
(B) Odd
(C) Neither
(D) Cannot be determined

Answer Key

Q1	2
Q2	1
Q3	22
Q4	(D)
Q5	(D)
Q6	5
Q7	2
Q8	0
Q9	0
Q10	17
Q11	12
Q12	(D)
Q13	(B)
Q14	(B)
Q15	(B)
Q16	(D)
Q17	(D)
Q18	(B)
Q19	(B)
Q20	(B)
Q21	(C)
Q22	(C)
Q23	(A)
Q24	(B)
Q25	(C)
Q26	(D)
Q27	(D)
Q28	(D)
Q29	(C)
Q30	(C)

Hints & Solutions

Q1 Text Solution:

To solve this inequality, we need to consider two cases, one when (2x - 3) is positive and the other when it's negative.

Case 1: $2x - 3 \geq 0$

In this case, we have $|2x - 3| = 2x - 3$. We can rewrite the inequality as follows:

$(2x - 3) + 3|4x - 5| - 6 < 5x$

Rearranging and simplifying, we get:

$|4x - 5| < x + 3$

Now, we need to consider two sub-cases: when 4x - 5 is positive and when it's negative.

Sub-case 1: $4x - 5 \geq 0$

In this case, we have $|4x - 5| = 4x - 5$. Substituting this into the inequality, we get:

$4x - 5 < x + 3$

Solving for x, we get:

$3x < 8$

$x < \frac{8}{3}$ .... (i)

Sub-case 2: $4x - 5 < 0$

In this case, we have $|4x - 5| = -(4x - 5)$. Substituting this into the inequality, we get:

$-(4x - 5) < x + 3$

Solving for x, we get:

$5x - 2 > 0$

$5x > 2$

$x > \frac{2}{5}$ .... (ii)

Therefore, the solution for Case 1 is:

$\frac{2}{5} < x < \frac{8}{3}$ .... (iii)

Case 2: $2x - 3 < 0$

In this case, we have $|2x - 3| = -(2x - 3)$. We can rewrite the inequality as follows:

$-(2x - 3) + 3|4x - 5| - 6 < 5x$

Rearranging and simplifying, we get:

$3|4x - 5| < 7x + 3$

$|4x - 5| < \frac{7x}{3} + 1$

Now, we need to consider two sub-cases again: when 4x - 5 is positive and when it's negative.

Sub-case 1: $4x - 5 \geq 0$

In this case, we have $|4x - 5| = 4x - 5$. Substituting this into the inequality, we get:

$4x - 5 < \frac{7x}{3} + 1$

Solving for x, we get:

$x < \frac{18}{5}$ .... (iv)

Sub-case 2: $4x - 5 < 0$

In this case, we have $|4x - 5| = -(4x - 5)$. Substituting this into the inequality, we get:

$-(4x - 5) < \frac{7x}{3} + 1$

Solving for x, we get:

$x > \frac{12}{19}$ .... (v)

Therefore, the solution for Case 2 is:

$\frac{12}{19} < x < \frac{18}{5}$ .... (vi)

Hence, combining the conditions (iii) and (vi), we can conclude that,

$\frac{12}{19} < x < \frac{8}{3}$

Hence, the number of integral values x can assume is 2.

Q2 Text Solution:

To solve this inequality, we need to consider two cases, one when 3x - 4 is positive and the other when it's negative.

Case 1: $3x - 4 \geq 0$

In this case, we have $|3x - 4| = 3x - 4$. We can rewrite the inequality as follows:

$(3x - 4) - 2|5x - 6| + 7 > 4x$

Rearranging and simplifying, we get:

$-2|5x - 6| > x - 3$

$|5x - 6| < \frac{3}{2} - \frac{x}{2}$

Now, we need to consider two sub-cases: when 5x - 6 is positive and when it's negative.

Sub-case 1: $5x - 6 \geq 0$

In this case, we have |5x - 6| = 5x - 6. Substituting this into the inequality, we get:

5x - 6 < $\frac{3}{2} - \frac{x}{2}$

Solving for x, we get:

x < $\frac{15}{11}$ .... (i)

Sub-case 2: 5x - 6 < 0

In this case, we have |5x - 6| = —(5x - 6). Substituting this into the inequality, we get:

—(5x - 6) < $\frac{3}{2} - \frac{x}{2}$

Solving for x, we get:

x > 1 .... (ii)

Therefore, the solution for Case 1 is:

1 < x < $\frac{15}{11}$ .... (iii)

Case 2: 3x - 4 < 0

In this case, we have |3x - 4| = —(3x - 4). We can rewrite the inequality as follows:

—(3x - 4) - 2|5x - 6| + 7 > 4x

Rearranging and simplifying, we get:

- 2|5x - 6| > 7x - 11

|5x - 6| < $\frac{11}{2} - \frac{7x}{2}$

Now, we need to consider two sub-cases again: when 5x - 6 is positive and when it's negative.

Sub-case 1: 5x - 6 ≥ 0

In this case, we have |5x - 6| = 5x - 6. Substituting this into the inequality, we get:

5x - 6 < $\frac{11}{2}$ - $\frac{7x}{2}$

Solving for x, we get:

x < $\frac{23}{17}$ .... (iv)

Sub-case 2: 5x - 6 < 0

In this case, we have |5x - 6| = —(5x - 6). Substituting this into the inequality, we get:

—(5x - 6) < $\frac{11}{2} - \frac{7x}{2}$

Solving for x, we get:

x > $\frac{1}{3}$ .... (v)

Therefore, the solution for Case 2 is:

$\frac{1}{3}$ < x < $\frac{23}{17}$ .... (vi)

Hence, combining the conditions (iii) and (vi), we can conclude that,

$\frac{1}{3}$ < x < $\frac{15}{11}$

Q3 Text Solution:

To solve the inequality |6x + 5| - 4x < 11, we need to analyze the two possible scenarios based on the absolute value.

Case 1: (6x + 5) - 4x < 11 (when 6x + 5 ≥ 0)

2x + 5 < 11

2x < 6

x < 3

Case 2: —(6x + 5) - 4x < 11 (when 6x + 5 < 0)

—6x - 5 - 4x < 11

—10x < 16

x > - $\frac{8}{5}$

The solution to the inequality is - $\frac{8}{5}$ < x < 3.

- -8 < 5x < 15

Hence, 5x can assume 22 integral values. (From -7 to 14)

Q4 Text Solution:

Given that,

||x - 5| - 6| > 5

Case 1: When |x – 5| - 6 > 5

- |x – 5| > 5 + 6
- |x – 5| > 11
- x – 5 < -11 or, x – 5 > 11
- x < -11 + 5 or, x > 11 + 5
- x < -6 or, x > 16

Case 2: When |x – 5| - 6 < -5

- |x – 5| < -5 + 6
- |x – 5| < 1
- -1 < x – 5 < 1
- -1 + 5 < x < 1 + 5
- 4 < x < 6

So, the required range is $(-\infty, \ -6)$ or (4, 6) or (16, ∞).

Q5 Text Solution:

It is given that,

$\left|\frac{3}{x-5}\right| > 1,\ x \neq 5$

- $\frac{3}{x-5} < -1$ or $\frac{3}{x-5} > 1$

- $\frac{x-2}{x-5} < 0$ or $\frac{8-x}{x-5} > 0$
- $2 < x < 5$ or 5 < x < 8, since x ≠ 5.

The solution set of given inequation (2,5) ∪ (5,8).

Q6 Text Solution:

Given that, $f\left(x\right) = \frac{1}{\sqrt{1-x^2}}$ and $g\left(x\right) = \frac{1}{x}$

Therefore, $f\left(g\left(x\right)\right) = f\left(\frac{1}{x}\right)$

$$= \frac{1}{\sqrt{1-\frac{1}{x^2}}}$$

$$= \frac{x}{\sqrt{x^2-1}}$$

Now,

$$f\left(g\left(\frac{5}{4}\right)\right) = \frac{\frac{5}{4}}{\sqrt{\left(\frac{5}{4}\right)^2-1}}$$

$$= \frac{\frac{5}{4}}{\sqrt{\frac{25}{16}-1}}$$

$$= \frac{\frac{5}{4}}{\sqrt{\frac{25-16}{16}}}$$

$$= \frac{\frac{5}{4}}{\frac{3}{4}}$$

$$= \frac{5}{3}$$

Therefore, $3f\left(g\left(\frac{5}{4}\right)\right) = 5$

Q7 Text Solution:

Given that,

$f\left(x\right) = \frac{2^{-x}}{2^{-x}+2^{x}}$ and $g\left(x\right) = \frac{5^{x}}{5^{x}+5^{-x}}$

Now, $f\left(x\right) + f\left(-x\right) = \frac{2^{-x}}{2^{-x}+2^{x}} + \frac{2^{-(-x)}}{2^{-(-x)}+2^{-x}}$

$= \frac{2^{-x}}{2^{-x}+2^{x}} + \frac{2^{x}}{2^{x}+2^{-x}}$

$= \frac{2^{-x}+2^{x}}{2^{-x}+2^{x}}$

= 1

Similarly,

$g\left(x\right) + g\left(-x\right) = \frac{5^{x}}{5^{x}+5^{-x}} + \frac{5^{-x}}{5^{-x}+5^{x}}$

$= \frac{5^{x}+5^{-x}}{5^{x}+5^{-x}}$

= 1

Now, $[f(x) + f(-x)]^{50} + [g(x) + g(-x)]^{50}$

$= (1)^{50} + (1)^{50}$

= 1 + 1

= 2

Q8 Text Solution:

Given that,

$f\left(x\right) = x \cdot \frac{10^{x}-1}{10^{x}+1}$

Then,

$f\left(x\right) - f\left(-x\right) = x \cdot \frac{10^{x}-1}{10^{x}+1} - \left(-x\right) \cdot \frac{10^{-x}-1}{10^{-x}+1}$

$= x \cdot \frac{10^{x}-1}{10^{x}+1} + x \cdot \frac{10^{x}(10^{-x}-1)}{10^{x}(10^{-x}+1)}$

$= x \cdot \frac{10^{x}-1}{10^{x}+1} + x \cdot \frac{1-10^{x}}{1+10^{x}}$

$= x \cdot \frac{10^{x}-1}{10^{x}+1} - x \cdot \frac{10^{x}-1}{10^{x}+1}$

= 0

Now,

$f^3(x) - f^3(-x)$

$= [f(x)]^3 - [f(-x)]^3$

$= [f(x) - f(-x)]^3 + 3f(x)f(-x) \times [f(x) - f(-x)]$

$= 0^3 + (3 \times f(x)f(-x) \times 0)$

= 0

Q9 Text Solution:

Given that,

$f(x) = x^2 - x$ and $g(x) = x^{-1}$

Now, $f(g(x)) = f(x^{-1})$

$= (x^{-1})^2 - x^{-1}$

$= x^{-2} - x^{-1}$

$= \frac{1}{x^2} - \frac{1}{x}$

$= \frac{1-x}{x^2}$

$= \frac{-(x-1)}{x^2}$

$= \frac{-(x^2-x)}{x^3}$

$= \frac{-f(x)}{x^3}$

$= -\left(\frac{1}{x}\right)^3 f\left(x\right)$

$= -\left(x^{-1}\right)^3 f\left(x\right)$

$= -\left(x^3\right)^{-1} f\left(x\right)$

$= -g\left(x^3\right) f\left(x\right)$

Therefore, $f\left(g\left(x\right)\right) = -g\left(x^3\right) f\left(x\right)$

- $f\left(g\left(x\right)\right) + g\left(x^3\right) f\left(x\right) = 0$

Q10 Text Solution:

Given that,

f(x) = 2x + 60 and g(x) = x^2 – 17x + 30

Then, f(g(x)) = f(x^2 – 17x + 30)

= 2(x^2 – 17x + 30) + 60

= 2x^2 – 34x + 60 + 60

= 2x^2 – 34x + 120

Now, for finding the zeroes of f(g(x)), we can have

f(g(x)) = 0

- 2x^2 – 34x + 120 = 0
- x^2 – 17x + 60 = 0
- x^2 – 12x – 5x + 60 = 0
- x(x-12) – 5(x-12) = 0
- (x-12)(x-5) = 0
- x = 5, 12

Hence, the sum of the required zeroes of f(g(x)) is (12+5) = 17

Q11 Text Solution:

f(1) = 1 = 1^3+ 3 × 1 – 3

f(4) = 73 = 4^3+ 3 × 4 – 3

f(9) = 753 = 9^3 + 3 × 9 – 3

∴ f(n) = n^3 + 3n – 3

∴ f(15) = 15^3 + 3 × 15 – 3

So, $\frac{(15^3 + 3 \times 15 - 3)}{15}$ = rem (12)

Q12 Text Solution:

Given that,

$f(x) = \sqrt{x-2} + \sqrt{9-x}$ and $g(x) = x+1$

Therefore,

$f(g(x)) = f(x+1)$

$= \sqrt{x+1-2} + \sqrt{9-(x+1)}$

$= \sqrt{x-1} + \sqrt{9-x-1}$

$= \sqrt{x-1} + \sqrt{8-x}$

Now, $f(g(x)$ will exist if

$x-1 \geq 0$ and $8-x \geq 0$

i.e., if x $\geq$ 1 and x $\leq$ 8

Hence, the required domain of $f(g(x))$ is $x \in [1, 8]$

Q13 Text Solution:

Given that,

$f(x) = x + \frac{1}{x}$, $g(x) = x^2$, and $h(x) = x^3$

Now,

$g(f(x)) = g\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)^2$

$= x^2 + \frac{1}{x^2} + 2$

$= f\left(x^2\right) + 2$

Also, $h(f(x)) = h\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)^3$

$= x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)$

$= f\left(x^3\right) + 3f\left(\frac{1}{x}\right)$ [Since, $f\left(\frac{1}{x}\right) = \frac{1}{x} + \frac{1}{\frac{1}{x}} = \frac{1}{x} + x = f(x)$]

Now,

$g(f(x)) + h(f(x)) = f\left(x^2\right) + 2 + f\left(x^3\right) + 3f\left(\frac{1}{x}\right)$

$= f(g(x)) + f(h(x)) + 3f\left(\frac{1}{x}\right) + 2$

$= f(g(x)) + f(h(x)) + 3f\left(\frac{1}{x}\right) + f(1)$

[Since, $f(1) = 1 + \frac{1}{1} = 2$]

Therefore,

$g(f(x)) + h(f(x)) = f(g(x)) + f(h(x))$ is possible when

$3f\left(\frac{1}{x}\right) + f(1) = 0$

i.e., if $f\left(\frac{1}{x}\right) = -\frac{1}{3}f(1)$

i.e., if $f(x) = -\frac{2}{3}$ [Since, $f(1) = 2$ and $f(x) = f\left(\frac{1}{x}\right)$]

Hence, only 1 and 2 are correct.

So, option B is correct.

Q14 Text Solution:

Given that,

$f(x) = \left(64 - x^4\right)^{\frac{1}{4}}$

Then,

$f\left(\frac{1}{3}\right) = \left(64 - \frac{1}{3^4}\right)^{\frac{1}{4}}$

$= \left(64 - \frac{1}{81}\right)^{\frac{1}{4}}$

$= \left(\frac{5183}{81}\right)^{\frac{1}{4}}$

Now,

$$f\left(f\left(\frac{1}{3}\right)\right) = f\left[\left(\frac{5183}{81}\right)^{\frac{1}{4}}\right]$$

$$= \left[64 - \left(\frac{5183}{81}\right)^{\frac{1}{4}\times 4}\right]^{\frac{1}{4}}$$

$$= \left(64 - \frac{5183}{81}\right)^{\frac{1}{4}}$$

$$= \left(\frac{1}{81}\right)^{\frac{1}{4}}$$

$$= \left(3^{-4}\right)^{\frac{1}{4}}$$

$$= 3^{-1}$$

$$= \frac{1}{3}$$

Q15 Text Solution:

Given that,

$$f\left(x\right) = \frac{1}{1-\frac{1}{1-\frac{1}{1-x}}}$$

$$= \frac{1}{1-\frac{1}{\frac{1-x-1}{1-x}}}$$

$$= \frac{1}{1-\frac{1-x}{-x}}$$

$$= \frac{1}{1+\frac{1-x}{x}}$$

$$= \frac{x}{x+1-x}$$

$$= x$$

Now,

$$f(f(x)) = f(x) = x$$

Therefore, $f\left(f\left(\frac{1}{x}\right)\right) = \frac{1}{x}$

So, $g\left(f\left(f\left(\frac{1}{x}\right)\right)\right) = g\left(\frac{1}{x}\right)$

$$= \frac{\frac{1}{x}}{\sqrt{\frac{1}{x^2}-1}}$$

$$= \frac{x\times\frac{1}{x}}{\sqrt{1-x^2}}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

Now,

$g\left(f\left(f\left(\frac{1}{x}\right)\right)\right)$ will exist if

$1 - x^2 > 0$

i.e., if $x^2 < 1$

i.e., if $-1 < x < 1$.

Q16 Text Solution:

Given that,

$$f\left(x\right) = \frac{25^x}{25^x+5}$$

then $f\left(1-x\right) = \frac{25^{1-x}}{25^{1-x}+5}$

$$= \frac{25.25^{-x}}{25.25^{-x}+5}$$

$$= \frac{25}{25+5.25^x}$$

$$= \frac{5}{5+25^x}$$

Now,

$$g\left(x\right) = f\left(x\right) + f\left(1-x\right)$$

$$= \frac{25^x}{25^x+5} + \frac{5}{5+25^x}$$

$$= \frac{25^x+5}{25^x+5}$$

$$= 1$$

So, $k(x) = g(x) + h(x)$

$$= 1 + \sqrt{5+12x-9x^2}$$

$$= 1 + \sqrt{-\left(9x^2-12x-5\right)}$$

$$= 1 + \sqrt{-\left(9x^2-15x+3x-5\right)}$$

$$= 1 + \sqrt{-(3x+1)(3x-5)}$$

Now, $k(x)$ will exist if

$$-3\left(3x+1\right)\left(3x-5\right) \geq 0$$

$$\Rightarrow \left(3x+1\right)\left(3x-5\right) \leq 0$$

$$\Rightarrow -\frac{1}{3} \leq x \leq \frac{5}{3}$$

Q17 Text Solution:

Let $y = \frac{3+2x+2x^2}{1+x+x^2}$

- $y\left(1+x+x^2\right) = 3+2x+2x^2$
- $y + xy + x^2y = 3+2x+2x^2$
- $x^2y - 2x^2 + xy - 2x + y - 3 = 0$
- $x^2(y-2) + x(y-2) + y - 3 = 0$ .... (i)

Since, the discriminant of (i) ≥ 0, so

$(y-2)^2 - 4(y-2)(y-3) \geq 0$

- $(y-2)(y-2-4y+12) \geq 0$
- $(y-2)(10-3y) \geq 0$
- $\left(y-2\right)\left(3y-10\right) \leq 0$
- $\left(y-2\right)\left(y-\frac{10}{3}\right) \leq 0$
- $2 \leq y \leq \frac{10}{3}$
- $2 \leq f\left(x\right) \leq \frac{10}{3}$

Q18 Text Solution:

To find the maximum value of x in the inequality $|x-1| + |x-3| + |x-5| \le 12$, we will consider the different possible cases for the absolute value expressions and find the intervals where the inequality is satisfied. Then, we'll find the maximum value of x from these intervals.

First, let's analyze the different cases for the absolute value expressions:

$|x-1|$: This expression will change sign when x = 1

$|x-3|$: This expression will change sign when x = 3

$|x-5|$: This expression will change sign when x = 5

Based on these critical points (1, 3, and 5), we will have 4 intervals to analyze:

Interval 1: x < 1

In this interval, all absolute value expressions are negative, so we have:

$-(x-1) - (x-3) - (x-5) \le 12$

Solving this inequality, we get:

$-3x + 9 \le 12$

$-3x \le 3$

$x \ge -1$ .... (i)

Interval 2: 1 ≤ x < 3

In this interval, the expression $|x-1|$ is positive, while the other two are negative:

$(x-1) - (x-3) - (x-5) \le 12$

Solving this inequality, we get:

$-x + 7 \le 12$

$-x \le 5$

$x \ge -5$ .... (ii)

Interval 3: 3 ≤ x < 5

In this interval, the expressions $|x-1|$ and $|x-3|$ are positive, while $|x-5|$ is negative:

$(x-1) + (x-3) - (x-5) \le 12$

Solving this inequality, we get:

$x + 1 \le 12$

$x \le 11$ .... (iii)

Interval 4: x ≥ 5

In this interval, all absolute value expressions are positive, so we have:

$(x-1) + (x-3) + (x-5) \le 12$

Solving this inequality, we get:

$3x - 9 \le 12$

$3x \le 21$

$x \le 7$

Interval 1: $x \ge -1$ (for $x < 1$)

Interval 2: $x \ge -5$ (for $1 \le x < 3$)

Interval 3: $x \le 11$ (for $3 \le x < 5$)

Interval 4: $x \le 7$ (for $x \ge 5$)

We're looking for the maximum value of x that satisfies the inequality. Since the inequality is non-strict (i.e., it contains the "less than or equal to" sign), we can also consider the boundary points of the intervals.

In Interval 1, the maximum value for x is < 1 (the boundary point of this interval). In Interval 2, the maximum value for x is < 3 (the boundary point of this interval). In Interval 3, the maximum value for x is 11 (from the constraint $x \le 11$). Finally, in Interval 4, the maximum value for x is 7 (from the constraint $x \le 7$).

Comparing the maximum values from all intervals, we find that the overall maximum value of x is 11, which is from Interval 3, but at x = 11, the given inequation is not satisfied.

So, the required maximum value of x = 7.

Q19 Text Solution:

Given that,

$$5f\left(x^4\right)+2f\left(\frac{1}{x^4}\right)=x^4-1$$

Replace x by x^4, then we have

$$5f\left(x^{16}\right)+2f\left(\frac{1}{x^{16}}\right)=x^{16}-1 \text{ .... (i)}$$

Replace x by $\frac{1}{x}$, then we have

$$5f\left(\frac{1}{x^{16}}\right)+2f\left(x^{16}\right)=\frac{1}{x^{16}}-1 \text{ .... (ii)}$$

Now, multiplying (i) by 5 and (ii) by 2 and then subtracting them, we get

$$25f\left(x^{16}\right)+10f\left(\frac{1}{x^{16}}\right)-10f\left(\frac{1}{x^{16}}\right)-4f\left(x^{16}\right)=5x^{16}-5-\frac{2}{x^{16}}+2$$

- $21f\left(x^{16}\right)=5x^{16}-\frac{2}{x^{16}}-3$
- $f\left(x^{16}\right)=\frac{5x^{32}-2-3x^{16}}{21x^{16}}$

$=\frac{5x^{32}-5x^{16}+2x^{16}-2}{21x^{16}}$

$=\frac{5x^{16}\left(x^{16}-1\right)+2\left(x^{16}-1\right)}{21x^{16}}$

$=\frac{\left(5x^{16}+2\right)\left(x^{16}-1\right)}{21x^{16}}$

Q20 Text Solution:

Let f(x) = $ax^2 + bx + c$

f(0) = 1 ⇒ c = 1

∴ f(x) = $ax^2 + bx + 1$

f(1) = 1 ⇒ a + b = 0 ... (i)

f(11) – f(10) = 1

⇒ (121a + 11b + 1) – (100a + 10b + 1) = 1

⇒ 21a + b = 1 ... (ii)

From (i) and (ii),

a = $\frac{1}{20}$, b = $-\frac{1}{20}$

⇒ f(20) = 400a + 20b + 1 = 20 – 1 + 1 = 20

Option (B) is correct.

Q21 Text Solution:

Given that,

f(x) = $\sqrt{\frac{x^2-7x+10}{x^2-5x+4}}$

= $\sqrt{\frac{(x-2)(x-5)}{(x-1)(x-4)}}$

Now, f(x) will exist if

(x -2)(x – 5) ≥ 0 and (x-1)(x-4) > 0

i.e., if x ≤ 2, x ≥ 5 and x < 1, x > 4

i.e., if x < 1,2 ≤ x < 4,x ≥ 5

i.e., if $x \in (-\infty, 1) \cup [2,4) \cup [5, \infty)$

Q22 Text Solution:

|x-1| + |x-2| + |x-3| = 9

The value of the expression at x = 3 is 3, and the value at x = 1 is also 3, while at x = 2 is 2.

So, the value of the expression equals 9 when x is greater than 3 or when x is less than 1.

Case 1: x > 3

x-1 + x-2 + x-3 = 9

or x = $\frac{15}{3}$ = 5

Case 2: x < 1

1 –x + 2–x + 3– x = 9

- 6 – 3x = 9
- 3x = - 3
- x = -1

Now, the minimum value of $x^2 + 5x^8 + 3x$ can be obtained at x = -1.

The required minimum value of $x^2 + 5x^8 + 3x$ = $(-1)^2 + 5(-1)^8 + 3(-1) = 3$

Q23 Text Solution:

Here any term is equal to the sum of its neighbours except for the first and the last term.

So, if f(1) = a, f(2) = b, then f(3) = b–a, f(4) = -a, f(5) = -b and f(6) = a–b, f(7) =a, f(8) =b and so on...

Thus, terms repeat after a gap of 6 i.e., there is a cyclicity of 6

So, f(25) = a and f(26) = b or a +b = 9

Also, f(47) = f(5) = -b = -5. Or b = 5 or a = 4

The sum of first six terms is zero, i.e., groups of 6 terms starting from the first will be zero. So, sum up-to 96 terms will be zero. Thus, we need to calculate f(97) +f(98)+f(99) = a + b + b–a = 2b = 2 × 5 = 10

So, f(1) + f(2) + f(3)...........+ f(99) = 10

Hence option (A) is correct.

Q24 Text Solution:

Equating the two functions we get: $px^2 + 2x + 1 = x^2 + 6x + 2$

Or, $(p-1)x^2 - 4x - 1 = 0$

As there is only one point of intersection, so the above equation must have equal roots, or the discriminant must be equal to zero.

$4^2 + 4(p-1) = 0$

4 + p -1 =0

or, p = -3

Hence option (B) is correct.

Q25 Text Solution:

At x= 0, the value of the function is 20 and this value rejects the first option. Taking some higher values of x, we realize that on the positive side, the value of the function will become negative when we take x greater than 5 since the value of (5 -x) would be negative. Also, the value of f(x) would start tending to $-\infty$ as we take the bigger value of x.

Similarly, on the negative side, when we take the value of x lower than -4, f(x) becomes positive and when we take it further away from 0 on the negative side, the value of f(x) would continue tending to $+\infty$.

Hence, option (C) is the answer.

Q26 Text Solution:

Let m, n be the roots of $x^2-\alpha x+\beta=0$, then m+ n = α.

Therefore, one root is 2, as one of the roots is prime and the other is an even number, but α is an even integer.

[since m is prime and n is even, so m must be 2]

Therefore, g(2) = 0

i.e., $2\alpha - \beta = 4$ and $\alpha + 2\beta = 32$

Solving both of these equations, we have

So, α=8, β=12

Hence, the function becomes

$g(x) = x^2-8x+12$

So, $g(g(x)) = g(x^2 - 8x + 12)$

$= (x^2 - 8x + 12)^2 - 8(x^2 - 8x + 12) + 12$

$= x^4-16x^3+80x^2-128x+60$

Q27 Text Solution:

To find the maximum value of x in the inequality $|x-2| + |x-5| + |x-7| \leq 15$, we will consider the different possible cases for the absolute value expressions and find the intervals where the inequality is satisfied. Then, we'll find the maximum value of x from these intervals.

First, let's analyze the different cases for the absolute value expressions:

|x-2|: This expression will change sign when x < 2

|x-5|: This expression will change sign when x < 5

|x-7|: This expression will change sign when x < 7

Based on these critical points (2, 5, and 7), we will have 4 intervals to analyze:

Interval 1: x < 2

Then, we have

$-(x-2) - (x-5) - (x-7) \leq 15$

$\Rightarrow -3x + 2 + 5 + 7 \leq 15$

$\Rightarrow -3x + 14 \leq 15$

$\Rightarrow -3x \leq 1$

$\Rightarrow x \geq -1/3$

Interval 2: 2 ≤ x < 5

Then, we have

$(x - 2) - (x-5) - (x - 7) \leq 15$

- $-x + 10 \leq 15$
- $-x \leq 5$
- $x \geq -5$

Interval 3: 5 ≤ x < 7

Then, we have

$(x - 2) + (x-5) - (x - 7) \leq 15$

- $x \leq 15$

Interval 3: x ≥ 7

$(x - 2) + (x-5) + (x - 7) \leq 15$

- $3x - 14 \leq 15$
- $3x \leq 29$
- $x \leq 29/3 = 9.67$ (approx.)

Hence, the maximum integer value of x in $|x - 2| + |x - 5| + |x - 7| \leq 15$ is 9 and the minimum integer value of x in $|x - 2| + |x - 5| + |x - 7| \leq 15$ is 0.

So, the required sum = 9 + 0 = 9

Q28 Text Solution:

It is given that,

$f(x+3) = 3f(x) - f(x+1)$ ...... (1)

Substituting $x = 0$, we have

$f(3) = 3f(0) - f(1)$

$= 3 \times 3 - 4$

$= 5$

Again, substituting $x = 1$, we have

$f(4) = 3f(1) - f(2)$

$= 3 \times 4 - 6$

$= 6$

Similarly, substituting $x = 3$, we have

$f(6) = 3f(3) - f(4)$

$= 3 \times 5 - 6$

$= 9$

Hence, option (D) is the correct answer.

Q29 Text Solution:

Given that,

$f\left(x^2 - 9\right) = 3x^2 + 2a + 3b$

At $x^2 = 9$, we have

f(0) = 3 × 9 + 2a + 3b

- f(0) = 27 + 2a + 3b ..... (i)

Also, $f(x - 3) = x^3 - 3ax + 2b$

At x = 3, we have

f(0) = 3^3 – 3a(3) + 2b

- f(0) = 27 – 9a + 2b ..... (ii)

Equating equations (i) and (ii), we have

27 + 2a + 3b = 27 – 9a + 2b

- 11a + b = 0
- $\frac{b}{a} = -11$

Now, we are also given that,

g(x) = x^2

Therefore,

$g\left(\frac{b}{a}\right) = (-11)^2 = 121$

Q30 Text Solution:

First, let's find the function g(2x - 1):

g(2x - 1) = 7$(2x - 1)^3$ + 4$(2x - 1)^2$ - (2x - 1) + 8

Now let's substitute g(2x - 1) into the function f(x):

h(x) = f(g(2x - 1)) = 3$[g(2x - 1)]^2$ - 2[g(2x - 1)] + 5

To find the nature of the function h(x), we'll evaluate h(-x) and compare it to h(x):

h(-x) = f(g(-2x - 1))

Now let's find g(-2x - 1):

g(-2x - 1) = 7$(-2x - 1)^3$ + 4$(-2x - 1)^2$ - (-2x - 1) + 8

= -7$(2x+1)^3$ + 4$(2x+1)^2$ + (2x+1) + 8

So, neither g(2x-1) = g(-2x-1), nor g(-2x – 1) = -g(2x-1)

Therefore, since, g(2x- 1) is neither even, nor an odd function, so, h(x) is neither even, nor an odd function.