第10次作业

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1. C . 2. A 3. C

4. (1). $Q=\{Box1, BOX2, Box3\}$.

$V=\{$苹果, 橘子$\}=\{a, o\}$. $\pi=[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$

$$A=\begin{bmatrix} \frac{1}{3} & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}, \quad B=\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}, \quad \lambda=\{A, B, \pi\}$$

(2) 使用viterbi算法, 序列 $O=\{a, a, o, o, o\}$.

初值: $\delta_1(i)=\pi_i b_i(O_1)$

$\therefore \delta_1(1)=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$. $\delta_1(2)=\frac{1}{3}\times\frac{3}{4}=\frac{1}{4}$, $\delta_1(3)=\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

$\psi_1(1)=0$ $\psi_1(2)=0$. $\psi_1(3)=0$.

递推: $\delta_2(i)=\max\limits_{1\le j\le 3}[\delta_1(j)a_{ji}]b_i(O_2)$

$\therefore \delta_2(1)=\max\{\frac{1}{6}\times\frac{1}{3}, \frac{1}{4}\times\frac{1}{3}, \frac{1}{12}\times\frac{1}{3}\}\times\frac{1}{2}=\frac{1}{24}$. $\psi_2(1)=2$.

$\delta_2(2)=\frac{1}{12}\times\frac{3}{4}=\frac{1}{16}$, $\psi_2(2)=2$

$\delta_2(3)=\frac{1}{12}\times\frac{1}{4}=\frac{1}{48}$, $\psi_2(3)=2$

$\delta_3(i)=\max\limits_{1\le j\le 3}[\delta_2(j)a_{ji}]b_i(O_3)$.

$\therefore \delta_3(1)=\frac{1}{3}\times\frac{1}{16}\times\frac{1}{2}=\frac{1}{96}$, $\psi_3(1)=2$.

$\delta_3(2)=\frac{1}{3}\times\frac{1}{16}\times\frac{1}{4}=\frac{1}{12\times16}$. $\psi_3(2)=2$

$\delta_3(3)=\frac{1}{3}\times\frac{1}{16}\times\frac{3}{4}=\frac{1}{64}$. $\psi_3(3)=2$.

$\delta_4(1)=\frac{1}{64}\times\frac{1}{3}\times\frac{1}{2}=\frac{1}{384}$, $\psi_4(1)=3$

$\delta_4(2)=\frac{1}{64}\times\frac{1}{3}\times\frac{1}{4}=\frac{1}{384\times4}$. $\psi_4(2)=3$

$\delta_4(3)=\frac{1}{64}\times\frac{1}{3}\times\frac{3}{4}=\frac{1}{256}$. $\psi_4(3)=3$

$\delta_5(1)=\frac{1}{3}\times\frac{1}{256}\times\frac{1}{2}$，$\psi_5(3)=3$

$\delta_5(2)=\frac{1}{3}\times\frac{1}{256}\times\frac{1}{4}$，$\psi_5(3)=3$

$\delta_5(3)=\frac{1}{3}\times\frac{1}{256}\times\frac{3}{4}$，$\psi_5(3)=3$

∴最佳序列为{BOX2, BOX2, BOX3, BOX3, BOX3}.

另外，由于各盒子独立，故该问题实际上为独立抽取问题，因此使用贪心找出各次各盒子到该状态最大概率选择盒子执行，最佳序列为{BOX2, BOX2, BOX3, BOX3, BOX3}.

5. 初始状态：$\pi=(1,0,0,0,0,0)^T$

则 $A=\begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$，$B=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0.4 & 0.1 & 0.2 & 0.3 & 0 & 0 \\ 0.4 & 0.1 & 0.1 & 0.4 & 0 & 0 \\ 0.2 & 0.3 & 0.3 & 0.2 & 0 & 0 \\ 0.1 & 0.4 & 0.4 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

观测序列为{begin, A, G, T, T, end}

则 $t=1$ 时，$x_1=begin$，且有：

$\alpha_1(1)=\pi_1 b_1(x_1)=1$，$\alpha_1(2)=\alpha_1(3)=\alpha_1(4)=\alpha_1(5)=\alpha_1(6)=0$.

当 $t=2$，有 $x_2=A$，$\alpha_2(j)=b_j(A)\sum_{i=1}^{6}a_{ij}\alpha_1(i)$

则 $\alpha_2(1)=0$，$\alpha_2(2)=0.2$，$\alpha_2(3)=0.2$，$\alpha_2(4)=\alpha_2(5)=\alpha_2(6)=0$.

$t=3$ 时：$x_3=G$，$\alpha_3(j)=b_j(G)\left[\sum_{i=1}^{6}a_{ij}\alpha_2(i)\right]$

且 $\alpha_3(1)=0$，$\alpha_3(2)=0.008$，$\alpha_3(3)=0.016$，$\alpha_3(4)=0.048$

$\alpha_3(5)=0.016$　$\alpha_3(6)=0$.

同理：当 $t=4$ 时：

$\alpha_{1\sim6}=[0,\ 0.00048,\ 0.00512,\ 0.00512,\ 0.00048,\ 0]$

当$t=5$时：

$$\alpha_{1\sim6}=[0,\ 0.0000288,\ 0.00164,\ 0.000486,\ 0.000107,\ 0]$$

当$t=6$时：

$$\alpha_{1\sim6}=[0,0,0,0,0,\ 0.000388]$$

故 $P(O|\lambda)=0.000388$.

(2) 序列$O=\{begin, T, A, T, A, end\}$.

当$t=1$. $x_1=begin$. 则 $V_1=[1,\ 0,\ 0,\ 0,\ 0,\ 0]$

$\psi_1=[0,\ 0,\ 0,\ 0,\ 0,\ 0]$

当$t=2$. $x_2=T$. 则有：$V_2=[0,\ 0.15,\ 0.2,\ 0,\ 0,\ 0]$

$\psi_2=[1,\ 1,\ 1,\ 1,\ 1,\ 1]$

当$t=3$. $x_3=A$：

$V_3=[0,\ 0.012,\ 0.064,\ 0.024,\ 0.004,\ 0]$

$\psi_3=[1,\ 2,\ 3,\ 2,\ 3,\ 1]$

当$t=4$. $x_4=T$：

$V_4=[0,\ 0.0072,\ 0.02048,\ 0.00192,\ 0.00128,\ 0]$

$\psi_4=[1,\ 2,\ 3,\ 2,\ 3,\ 4]$

当$t=5$. $x_5=T$：

$V_5=[0,\ 5.76\times10^{-5},\ 0.0065536,\ 0.001536,\ 0.0004096,\ 0]$

$\psi_5=[1,\ 2,\ 3,\ 4,\ 3,\ 5]$

当$t=6$. $x_6=end$：

$V_6=[0,\ 0,\ 0,\ 0,\ 0,\ 0.00036864]$

$\psi_6=[1,\ 2,\ 3,\ 4,\ 3,\ 5]$

故最佳序列为：$\{0,\ 2,\ 2,\ 2,\ 4,\ 5\}$.

概率值为：0.00036864