PRML 第四章作业

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1、设有如下三类模式样本集 ω_1，ω_2 和 ω_3，其先验概率相等，求 S_w 和 S_b

ω_1：$\{(1\ 0)^T, (2\ 0)^T, (1\ 1)^T\}$

ω_2：$\{(-1\ 0)^T, (0\ 1)^T, (-1\ 1)^T\}$

ω_3：$\{(-1\ -1)^T, (0\ -1)^T, (0\ -2)^T\}$

一. 有 $P(w_1) = P(w_2) = P(w_3) = P = \frac{1}{3}$

则：求得各均值向量为：

$$\vec{m_1} = \left(\frac{4}{3}, \frac{1}{3}\right)^T = \frac{1}{3}(4, 1)^T$$

$$\vec{m_2} = \left(\frac{-2}{3}, \frac{2}{3}\right)^T = \frac{1}{3}(-2, 2)^T$$

$$\vec{m_3} = \left(-\frac{1}{3}, \frac{-4}{3}\right)^T = -\frac{1}{3}(1, 4)^T$$

总体均值向量：

$$\vec{m_0} = E\{\vec{x}\} = \sum_{i=1}^{c} P(w_i)\vec{m_i} = \frac{1}{3}(\vec{m_1} + \vec{m_2} + \vec{m_3}) = \frac{1}{9}(1, -1)^T$$

① 计算 S_w：

以 w_1 为例：$P(w_1)E\{(\vec{x}-\vec{m_1})(\vec{x}-\vec{m_1})^T | w_1\}$

$$= \frac{1}{3}\left\{\frac{1}{3}\times\begin{bmatrix}\frac{1}{9} & \frac{1}{9}\\ \frac{1}{9} & \frac{1}{9}\end{bmatrix} + \frac{1}{3}\begin{bmatrix}\frac{4}{9} & \frac{-2}{9}\\ \frac{-2}{9} & \frac{1}{9}\end{bmatrix} + \frac{1}{3}\begin{bmatrix}\frac{1}{9} & \frac{-2}{9}\\ -\frac{2}{9} & \frac{4}{9}\end{bmatrix}\right\}$$

$$= \frac{1}{3}\begin{bmatrix}\frac{2}{9} & \frac{-1}{9}\\ \frac{-1}{9} & \frac{2}{9}\end{bmatrix}$$

同理，得：

$$S_w = \frac{1}{3}\begin{bmatrix}\frac{2}{9} & \frac{-1}{9}\\ \frac{-1}{9} & \frac{2}{9}\end{bmatrix} + \frac{1}{3}\begin{bmatrix}\frac{2}{9} & \frac{1}{9}\\ \frac{1}{9} & \frac{2}{9}\end{bmatrix} + \frac{1}{3}\begin{bmatrix}\frac{2}{9} & \frac{-1}{9}\\ \frac{-1}{9} & \frac{2}{9}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{2}{9} & \frac{-1}{27}\\ \frac{-1}{27} & \frac{2}{9}\end{bmatrix}$$

② 计算 $S_b = \sum_{i=1}^{c} P(w_i)(\vec{m_i} - \vec{m_0})(\vec{m_i} - \vec{m_0})^T$

以 w_1 为例：$P(w_1)(\vec{m_1} - \vec{m_0})(\vec{m_1} - \vec{m_0})^T$

$$= \frac{1}{3}\left\{\frac{1}{81}\begin{bmatrix}121 & 44\\ 44 & 16\end{bmatrix}\right\}$$

同理，得：

$$S_b = \frac{1}{3}\begin{bmatrix}\frac{121}{81} & \frac{44}{81}\\ \frac{44}{81} & \frac{16}{81}\end{bmatrix} + \frac{1}{3}\begin{bmatrix}\frac{49}{81} & \frac{-49}{81}\\ -\frac{49}{81} & \frac{49}{81}\end{bmatrix} + \frac{1}{3}\begin{bmatrix}\frac{16}{81} & \frac{44}{81}\\ \frac{44}{81} & \frac{121}{81}\end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix}62 & 13\\ 13 & 62\end{bmatrix}.$$

2、设有如下两类样本集，其出现的概率相等：

ω_1：$\{(0\ 0\ 0)^T, (1\ 0\ 0)^T,$

$(1\ 0\ 1)^T, (1\ 1\ 0)^T\}$

ω_2：$\{(0\ 0\ 1)^T, (0\ 1\ 0)^T,$

$(0\ 1\ 1)^T, (1\ 1\ 1)^T\}$

用 K-L 变换，分别把特征空间维数降到二维和一维，并画出样本在该空间中的位置

二. 先计算样本均值:

$$\vec{m} = \frac{1}{2}\left\{\frac{1}{4}[3\ 1\ 1]^T + \frac{1}{4}[1\ 3\ 3]^T\right\}$$

$$= \frac{1}{2}[1,\ 1\ 1]^T = [\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}]^T$$

平移样本:

$$\omega_1: \left\{[-\tfrac{1}{2}, -\tfrac{1}{2}\ -\tfrac{1}{2}]^T, [\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}]^T, [\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}]^T, [\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}]^T\right\}$$

$$\omega_2: \left\{[-\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}]^T, [-\tfrac{1}{2}, \tfrac{1}{2}\ -\tfrac{1}{2}]^T, [-\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}]^T, [\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}]^T\right\}.$$

求自相关矩阵: $\because P(\omega_1) = P(\omega_2) = \frac{1}{2}$, 故.

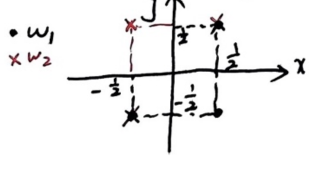
$$R = \sum_{i=1}^{2} P(\omega_i) E\{xx^T\} = \frac{1}{2}\left\{\frac{1}{4}\sum_{j=1}^{4} x_{1j}x_{1j}^T\right\} + \left\{\frac{1}{4}\sum_{j=1}^{4} x_{2j}x_{2j}^T\right\}\frac{1}{2}$$

$$= \frac{1}{8}\left\{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} + \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right\} = \frac{1}{4} I_{3\times3}.$$ 特征值为 $\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{4}$

特征向量为: $\phi_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $\phi_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\phi_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$.

(1) 降至二维, 选取 $\lambda_1 = \lambda_2 = \frac{1}{4}$, ~~对应特征~~ 特征向量取 ϕ_1 和 ϕ_2.

则 $\Phi = \begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}$ 故: $y = \Phi^T x$ (其中 x 为变换后的向量)



$$\omega_1 = \left\{[-\tfrac{1}{2}\ \tfrac{1}{2}]^T, [\tfrac{1}{2}\ -\tfrac{1}{2}]^T, [\tfrac{1}{2}\ -\tfrac{1}{2}]^T, [\tfrac{1}{2}\ \tfrac{1}{2}]^T\right\}$$

$$\omega_2 = \left\{[-\tfrac{1}{2}\ -\tfrac{1}{2}]^T\ [-\tfrac{1}{2}\ \tfrac{1}{2}]^T\ [-\tfrac{1}{2}\ \tfrac{1}{2}]^T, [\tfrac{1}{2}\ \tfrac{1}{2}]^T\right\}.$$

(2) 降至一维情况: 选 $\lambda_1 = \frac{1}{4}$, ϕ_1 为特征向量.

则 $\omega_1 = \{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$

$\omega_2 = \{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\}$