第六章作业:

1. $P(Y=1)=\frac{9}{15}$, $P(Y=-1)=\frac{6}{15}$

$P(X^{(1)}=1|Y=1)=\frac{2}{9}$, $P(X^{(1)}=2|Y=1)=\frac{3}{9}$, $P(X^{(1)}=3|Y=1)=\frac{4}{9}$

$P(X^{(2)}=S|Y=1)=\frac{1}{9}$, $P(X^{(2)}=M|Y=1)=\frac{4}{9}$, $P(X^{(2)}=L|Y=1)=\frac{4}{9}$

$P(X^{(1)}=1|Y=-1)=\frac{3}{6}$, $P(X^{(1)}=2|Y=-1)=\frac{2}{6}$, $P(X^{(1)}=3|Y=-1)=\frac{1}{6}$

$P(X^{(2)}=S|Y=-1)=\frac{3}{6}$. $P(X^{(2)}=M|Y=-1)=\frac{2}{6}$. $P(X^{(2)}=L|Y=-1)=\frac{1}{6}$.

对于测试样本 $x=(2,S)^T$, 有:

$$P(Y=1)P(X^{(1)}=2|Y=1)\,P(X^{(2)}=S|Y=1)=\frac{9}{15}\times\frac{3}{9}\times\frac{1}{9}=\frac{1}{45}$$

$$P(Y=-1)P(X^{(1)}=2|Y=-1)P(X^{(2)}=S|Y=-1)=\frac{6}{15}\times\frac{2}{6}\times\frac{3}{6}=\frac{1}{15}>\frac{1}{45}$$

故样本 $x=(2,S)^T$ 标签为 -1.

对于测试样本 $x=(1,N)^T$, 由于训练数据中无 N, 故利用 laplace 平滑:

则 $S_1=3$, $S_2=4$, $\lambda=1$, $k=2$ ∵ $P_\lambda(X^{(j)}=a_{jl}|Y=c_k)=\dfrac{\sum_{i=1}^{N}I(x_i^{(j)}=a_{jl},y_i=c_k)+\lambda}{\sum_{i=1}^{N}I(y_i=c_k)+S_j\lambda}$, 则有:

$P(Y=1)=\frac{10}{17}$, $P(Y=-1)=\frac{7}{17}$, $A_1=\{1,2,3\}$. $A_2=\{S,M,L,N\}$, $C=\{-1,1\}$

$P(X^{(1)}=1|Y=1)=\frac{3}{12}$. $P(X^{(1)}=2|Y=1)=\frac{4}{12}$. $P(X^{(1)}=3|Y=1)=\frac{5}{12}$.

$P(X^{(2)}=S|Y=1)=\frac{2}{13}$. $P(X^{(2)}=M|Y=1)=\frac{5}{13}$. $P(X^{(2)}=L|Y=1)=\frac{5}{13}$, $P(X^{(2)}=N|Y=1)=\frac{1}{13}$

$P(X^{(1)}=1|Y=-1)=\frac{4}{9}$, $P(X^{(1)}=2|Y=-1)=\frac{3}{9}$, $P(X^{(1)}=3|Y=-1)=\frac{2}{9}$

$P(X^{(2)}=S|Y=-1)=\frac{4}{10}$. $P(X^{(2)}=M|Y=-1)=\frac{3}{10}$. $P(X^{(2)}=L|Y=-1)=\frac{2}{10}$. $P(X^{(2)}=N|Y=-1)=\frac{1}{10}$.

故对 $x=(1,N)^T$ 判别:

$$P(Y=1)P(X^{(1)}=1|Y=1)P(X^{(2)}=N|Y=1)=\frac{10}{17}\times\frac{3}{12}\times\frac{1}{13}=\frac{5}{442}$$

$$P(Y=-1)P(X^{(1)}=1|Y=-1)P(X^{(2)}=N|Y=-1)=\frac{7}{17}\times\frac{4}{9}\times\frac{1}{10}=\frac{14}{765}>\frac{5}{442}.$$

故其标签为 $y=-1$.

2.1、$P(Y=1)=\pi$，$P(Y=0)=1-\pi$，

$$P(x_i|y=k)=\frac{1}{\sqrt{2\pi}\sigma_i}exp\left\{\frac{(x-\mu_{ik})^2}{2}\right\}.$$

则有 $P(y|x)=\frac{P(x,y)}{P(x)}=\frac{P(x|y)P(y)}{P(x)}.$

则有 $$P(y=k|x)=\frac{P(x,y=k)}{P(x)}=\frac{P(x|y=k)P(y=k)}{\sum_{i=1}^{c}P(x|y=i)P(y=i)}.$$

$$=\frac{P(x|y=k)P(y=k)}{\pi\cdot\frac{1}{\sqrt{2\pi}\sigma_i}exp\left\{\frac{(x-\mu_{i1})^2}{2}\right\}+(1-\pi)\frac{1}{\sqrt{2\pi}\sigma_i}exp\left\{\frac{(x-\mu_{i0})^2}{2}\right\}}$$

$$\therefore P(y=1|x)=\frac{\pi\, exp\left\{\frac{(x-\mu_{i1})^2}{2}\right\}}{\pi\, exp\left\{\frac{(x-\mu_{i1})^2}{2}\right\}+(1-\pi)exp\left\{\frac{(x-\mu_{i0})^2}{2}\right\}}=\frac{1}{1+exp\left\{ln\frac{1-\pi}{\pi}+\sum_i ln\frac{P(x_i|y=0)}{P(x_i|y=1)}\right\}}$$

$$P(y=0|x)=\frac{(1-\pi)\,exp\left\{\frac{(x-\mu_{i0})^2}{2}\right\}}{\pi\, exp\left\{\frac{(x-\mu_{i1})^2}{2}\right\}+(1-\pi)\,exp\left\{\frac{(x-\mu_{i0})^2}{2}\right\}}$$ 符合LR模型.

2.2.同理得到：

$$P(y=1|x)=\frac{\frac{\pi}{\sigma_{i1}}exp\left\{\frac{(x-\mu_{i1})^2}{2\sigma_{i1}}\right\}}{\frac{\pi}{\sigma_{i1}}exp\left\{\frac{(x-\mu_{i1})^2}{2\sigma_{i1}}\right\}+\frac{1-\pi}{\sigma_{i0}}exp\left\{\frac{(x-\mu_{i0})^2}{2\sigma_{i0}}\right\}}=\frac{1}{1+exp\left\{\frac{1-\pi}{\pi}\frac{\sigma_{i1}}{\sigma_{i0}}+\sum_i ln\frac{P(x_i|y=0)}{P(x_i|y=1)}\right\}}$$

$$P(y=0|x)=\frac{\frac{1-\pi}{\sigma_{i0}}exp\left\{\frac{(x-\mu_{i0})^2}{2\sigma_{i0}}\right\}}{\frac{\pi}{\sigma_{i1}}exp\left\{\frac{(x-\mu_{i1})^2}{2\sigma_{i1}}\right\}+\frac{1-\pi}{\sigma_{i0}}exp\left\{\frac{(x-\mu_{i0})^2}{2\sigma_{i0}}\right\}}$$ 不符合LR模型.

$$P(y=1|x_1,x_0)=\frac{P(x_1,x_0|y=1)P(y=1)}{P(x_1,x_0|y=0)P(y=0)+P(x_1,x_0|y=1)P(y=1)}=\frac{1}{1+\frac{P(y=0)}{P(y=1)}\cdot\frac{P(x_0,x_1|y=0)}{P(x_0,x_1|y=1)}}$$

而：$$\frac{P(x_1,x_2|y=0)}{P(x_1,x_2|y=1)}=\frac{N(\mu_{10},\mu_{20},\sigma_1,\sigma_2,\rho)}{N(\mu_{11},\mu_{21},\sigma_1,\sigma_2,\rho)}.$$

$$=exp\left\{-\frac{\sigma_2^2(2x_1-\mu_{10}-\mu_{11})(\mu_{11}-\mu_{10})+\sigma_2^2(2x_2-\mu_{20}-\mu_{21})-2\rho\sigma_1\sigma_2(\mu_{10}\mu_{20}-\mu_{11}\mu_{21}-(\mu_{10}-\mu_{20})x_2-(\mu_{11}-\mu_{21})x_1)}{2(1-\rho^2)\sigma_1^2\sigma_2^2}.\right\}$$

可以看出，关于x_1、x_2二都为一次项

故符合LR模型.