黄磊　计 702　 2022E8013282156

1. 完成课本数字图像处理第二版 116 页，习题 3.25，即拉普拉斯算子具有理论上的旋转不变性。

★3.25 **证明如式(3.7.1)所示的拉普拉斯变换是各向同性的(旋转不变)。需要下列轴旋转 θ 角的坐标方程：**

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

其中(x,y)为非旋转坐标，而(x',y')为旋转坐标。

拉普拉斯变换为：

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

证明各向同性，即证明：

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}$$

有：

$$\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial x'} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial x'} = \frac{\partial f}{\partial x}\cos\theta \ + \frac{\partial f}{\partial y}\sin\theta$$

$$\frac{\partial f}{\partial y'} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial y'} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial y'} = -\frac{\partial f}{\partial x}\sin\theta \ + \frac{\partial f}{\partial y}\cos\theta$$

$$\frac{\partial^2 f}{\partial x'^2} = \frac{\partial^2 f}{\partial x^2}\cos^2\theta + \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\sin\theta\cos\theta + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\cos\theta\sin\theta \ + \frac{\partial^2 f}{\partial y^2}\sin^2\theta$$

$$\frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial x^2}\sin^2\theta - \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\cos\theta\sin\theta - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\sin\theta\cos\theta \ + \frac{\partial^2 f}{\partial y^2}\cos^2\theta$$

因此将后两个式子相加，有:

$$\frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

证毕。