

2、请计算如下两个向量与矩阵的卷积计算结果。

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(1) $[1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1] * [2\ 0\ -2]$

(2)

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 3 & 2 & 0 & 4 \\ 1 & 0 & 3 & 2 & 3 \\ 0 & 4 & 1 & 0 & 5 \\ 2 & 3 & 2 & 1 & 4 \\ 3 & 1 & 0 & 4 & 2 \end{bmatrix} =$$

(1) $1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

$$\begin{array}{cccccccccc} & & & & & & 2 & 0 & -2 \\ & & & & & & \hline & -2 & -4 & -6 & -8 & -10 & -8 & -6 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 6 & 8 & 10 & 8 & 6 & 4 & 2 & \\ \hline 2 & 4 & 4 & 4 & 4 & 0 & -4 & -4 & -4 & -4 & -2 \end{array}$$

故卷积结果为: $[2\ 4\ 4\ 4\ 4\ 0\ -4\ -4\ -4\ -4\ 2]$

(2) 由于卷积满足交换性, 故将 $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ 看作卷积核.

= 二维卷积计算方法为: $(A * B)(x, y) = \sum_i \sum_j A(i, j) B(x-i, y-j)$.

$$\begin{array}{ccc} \text{original} & x\text{-reflection} & xy\text{-reflection} \\ \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} & \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} & \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \end{array}$$

$$C_{00} = -1 \times 1 = -1, C_{01} = 0 \times 1 + 3 \times -1 = -3, C_{02} = 1 \times 1 + 0 \times 3 + -1 \times 2 = -1, C_{03} = 3, C_{04} = -2$$

$$C_{05} = 0, C_{06} = 4$$

$$C_{10} = -3, C_{11} = -6, C_{12} = -4, C_{13} = 4, C_{14} = -4, C_{15} = 2, C_{16} = 11$$

其余依此类推计算.

$$\begin{bmatrix} 1 & 3 & 2 & 0 & 4 \\ 1 & 0 & 3 & 2 & 3 \\ 0 & 4 & 1 & 0 & 5 \\ 2 & 3 & 2 & 1 & 4 \\ 3 & 2 & 0 & 4 & 2 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -1 & 3 & 2 & 0 & 4 \\ -3 & -6 & -4 & 4 & -4 & 2 & 11 \\ -3 & -7 & -6 & 3 & -6 & 4 & 15 \\ -3 & -11 & -4 & 8 & -10 & 3 & 17 \\ -7 & -11 & 2 & 5 & -10 & 6 & 15 \\ -8 & -5 & -6 & -4 & -6 & 9 & 8 \\ -3 & -1 & 3 & -3 & -2 & 4 & 2 \end{bmatrix}$$