Design and Analysis of Algorithms
CS575, Spring 2024

# Theory Assignment 3.3

Due on 4/22/2024 (Monday) at 11:59pm

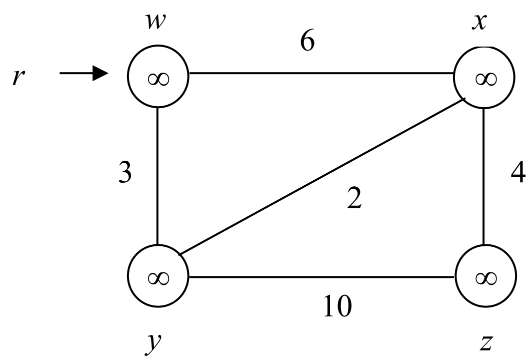
Remember to include the following statement at the start of your answers with a signature by the side. "I have done this assignment completely on my own. I have not copied it, nor have I given my solution to anyone else. I understand that if I am involved in plagiarism or cheating I will have to sign an official form that I have cheated and that this form will be stored in my official university record. I also understand that I will receive a grade of 0 for the involved assignment for my first offense and that I will receive a grade of "F" for the course for any additional offense.

1. *[25 points]* Jack plans to drive from city $A$ to city $B$ along a highway. Suppose $g_0, g_1, g_2, \ldots, g_k$ are the gas stations along the highway and are ordered in increasing distance (in miles) from city $A$, where $g_0$ is located at the starting place (city $A$). Let $d_i$ be the distance (in miles) between $g_i$ and $g_{i+1}$, $i = 0, \ldots, k - 1$, and we assume that these distances are known to Jack. Each time Jack fills gas to his car, he gets a full tank of gas. Jack also knows the number of miles, $m$, his car can drive with a full tank of gas. Jack's goal is to minimize the number of times he needs to stop at gas stations for his trip. You can assume that Jack starts from $g_0$ with a full tank.
    a. *[8 points]* Design a greedy algorithm to solve the above problem, i.e., to minimize the number of stops for gas.
    b. *[12 points]* Show that your greedy algorithm has the *greedy choice property*, i.e., each local decision will lead to the optimal solution. Basically, you need to argue for the correctness of your algorithm.
    c. *[5 points]* What is the running time of your algorithm?

2. *[20 points]* Given the Prim's algorithm shown below (a min-priority queue is used in the implementation):

```
PRIM(G, w, r)
  Q = ∅
  for each u ∈ G.V
      u.key = ∞
      u.π = NIL
      INSERT(Q, u)
  DECREASE-KEY(Q, r, 0)        // r.key = 0
  while Q ≠ ∅
      u = EXTRACT-MIN(Q)
      for each v ∈ G.Adj[u]
          if v ∈ Q and w(u, v) < v.key
              v.π = u
              DECREASE-KEY(Q, v, w(u, v))
```

Apply the algorithm to the weighted, connected graph below (the initialization part has been done). Show a new intermediate graph after each vertex is processed in the while loop. For each intermediate graph and the final graph, you need to show the vertex being processed, the new key value for each vertex and edges in the current (partial) MST (draw a directed edge from vertex $v$ to $u$ if $v.\pi = u$).




3. *[20 points]* Apply Kruskal's algorithm to the graph below. Show new intermediate graphs with the shaded edges belong to the forest being grown. The algorithm considers each edge in sorted order by weight. An arrow points to the edge under consideration at each step of the algorithm. If the edges joins two distinct trees in the forest, it is added to the forest, thereby merging the two trees.