Design and Analysis of Algorithms
CS575, Fall 2024
Theory Assignment 4

Due on 5/1/24 (Wednesday) at 11:59pm (No late submission will be accepted!)

1. [20%] A set of inters A = {2, 5, 11, 14} is given. For the given set A, find **every subset** that sums to S = 16.
    a. [10%] Solve the problem using the depth first method. Draw a state space tree and clearly show every step. Also, number the nodes in the sequence of visiting them.
    b. [10%] Find the subsets via backtracking. Draw a (pruned) state space tree and clearly show every step. Number the nodes in the sequence of visiting them too.

2. [10%] Consider the following revised sum of subsets problem: **Given *n* positive integers w_1, …, w_n and two positive integers S_1 and S_2, find *all* subsets of w_1, …, w_n that add to a number between S_1 and S_2 (including S_1 and S_2)**. Suppose we follow the same general backtracking method for solving the original sum of the subsets problem (see the slides of the Backtracking chapter) and use the same notations for weightSoFar and totalPossibleLeft. Define the condition(s) for a node in the state space tree to be promising.

3. [30%] When the capacity of the knapsack is 13, solve the following **0-1 knapsack** problem using (a) [10%] the backtracking algorithm; (b) [10%] the Breadth-First search with branch and bound pruning algorithm and (c) [10%] the Best-first search with branch and bound pruning algorithm in Chapter 13. All algorithms use the optimal fractional knapsack algorithm to compute the upper bound of the profit. Show the actions step by step.

i	p_i	w_i	p_i / w_i
1	\$20	2	\$10
2	\$30	5	\$6
3	\$35	7	\$5
4	\$12	3	\$4
5	\$3	1	\$3

4. [20%] Consider a directed weighted graph G(V, E), where V is the set of vertices and E is the set of edges. Suppose we use adjacency list to implement the graph. Consider a sequence of |V| operations with each corresponding to a different vertex in V. For the operation for vertex v, the cost is assumed to be the number of outgoing edges from v.

    (a) [10%] What is the cost of the worst possible single operation in this sequence of operations?

    (b) [10%] What is the average amortized cost of the above sequence of operations based on aggregate analysis?

5. [20%] Consider **Problem A**: Given an undirected graph G = (V, E), determine whether G is a complete graph.

a. [10%] Does problem A belong to the class P? If no, explain why; if yes, describe and analyze a polynomial bound algorithm (in terms of |V|) that solves it. (Hint: check there are no missing edges).

b. [10%] Does problem A belong to the class NP? Explain.

6. **Extra Credit** [30%] Consider the following attempt to transform the Hamiltonian Cycle problem (HCP) to the Traveling Salesman problem (TSP). Let G = (V, E) be an instance of HCP. We construct an instance (G ′, d, B) of TSP as follows:

   a. G ′ = (V ′, E ′) is a graph with
      i. V ′ = V, i.e., no change to the set of vertices
      ii. E ′ = {(u, v) | u, v $\in$ V ′, u $\neq$ v}, i.e., form an edge between each pair of vertices in V ′, making G ′ a complete graph.

   b. d( ) is the distance function for each edge in E ′ defined below:

$$d(u,v) = \begin{cases} 1 & \text{if } (u,v) \in E \\ 2 & \text{if } (u,v) \notin E \end{cases}$$

   Now answer the following questions about this transformation:

   a. [15%] Find an appropriate value b for bound B for the TSP and show that for this bound G has a Hamiltonian cycle if and only if G ′ has a tour with a total distance at most b.

   b. [15%] Show that the above transformation takes polynomial time in terms of |V|.