Polynomial Kernel

$$K(x,x') = (\delta x^T x' + \epsilon)^M$$

where r>0

## Example:

$$M=2$$
  $\delta=r=1$   $x,x' \in \mathbb{R}^2$ 

$$X = \begin{bmatrix} X_1 \\ x_2 \end{bmatrix} \qquad X' = \begin{bmatrix} X_1' \\ X_2' \end{bmatrix} \qquad X^T = \begin{bmatrix} X_1 & X_2 \end{bmatrix}$$

$$K(x_1x') = (x^Tx'+1)^2 = (x_1x_1'+y_2x_2'+1)^2$$

$$= (x_1 x_1')^2 + (x_2 x_2')^2 + 1 + 2x_1 x_1' + 2x_2 x_2' + 2x_1 x_1' x_2 x_2'$$

$$\phi(x)^T$$
  $\phi(x')$  where

$$\emptyset(x) = [1, [2x_1, [2x_2, x_1^2, x_2^2, [2x_1x_2]]$$

$$\phi(x) = \begin{bmatrix} 1 & \phi(x') = 1 \\ 2x_1 & (2x_2) \\ x_1^2 & (x_1')^2 \\ x_2^2 & (x_2')^2 \\ (2x_1x_2) & (2x_1'x_2') \end{bmatrix}$$

$$\phi(x') = \begin{bmatrix} 2x_1 \\ 2x_2 \\ (x_1) \end{bmatrix}$$

$$(2x_2 \\ (x_1) \end{bmatrix}$$

$$(x_2)^2$$

$$(2x_1 \\ x_2)$$

$$\emptyset(x)^{T} = \begin{bmatrix} 1 & 2x_1 & 2x_2 & x_1^2 & x_2^2 & 52x_1 & x_2 \end{bmatrix}$$

$$\emptyset(x)^{T}, \emptyset(x') = 1 + 2x_{1}x_{1}' + 2x_{2}x_{2}' + (x_{1}x_{1}')^{2} + (x_{2}x_{2}')^{2} + 2x_{1}x_{2}'$$