Polynomial kernel

$$K(x, x') = (\gamma x^T x' + r)^M \quad \text{where } r > 0$$

Example:

$M = 2 \quad \gamma = r = 1 \quad x, x' \in \mathbb{R}^2$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad x' = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} \quad x^T = [x_1 \ x_2]$$

$$K(x, x') = (x^T x' + 1)^2 = (x_1 x_1' + x_2 x_2' + 1)^2$$

$$= (x_1 x_1')^2 + (x_2 x_2')^2 + 1 + 2x_1 x_1' + 2x_2 x_2' + 2x_1 x_1' x_2 x_2'$$

$\phi(x)^T \cdot \phi(x')$ where

$$\phi(x) = [1, \sqrt{2}x_1, \sqrt{2}x_2, x_1^2, x_2^2, \sqrt{2}x_1 x_2]$$

$$\phi(x) = \begin{bmatrix} 1 \\ \sqrt{2}x_1 \\ \sqrt{2}x_2 \\ x_1^2 \\ x_2^2 \\ \sqrt{2}x_1x_2 \end{bmatrix} \qquad \phi(x') = \begin{bmatrix} 1 \\ \sqrt{2}x_1' \\ \sqrt{2}x_2' \\ (x_1')^2 \\ (x_2')^2 \\ \sqrt{2}x_1'x_2' \end{bmatrix}$$

$$\phi(x)^T = \begin{bmatrix} 1 & \sqrt{2}x_1 & \sqrt{2}x_2 & x_1^2 & x_2^2 & \sqrt{2}x_1x_2 \end{bmatrix}$$

$$\phi(x)^T \cdot \phi(x') = 1 + 2x_1x_1' + 2x_2x_2' + (x_1x_1')^2 + (x_2x_2')^2 + 2x_1x_2x_1'x_2'$$