

MAP estimate

$$\hat{h}^{\text{MAP}} = \underset{h}{\operatorname{argmax}} p(h|D) \quad \leftarrow \text{posterior}$$

$$= \underset{h}{\operatorname{argmax}} \underset{\substack{\uparrow \\ \text{likelihood}}}{p(D|h)} \cdot \underset{\substack{\uparrow \\ \text{prior}}}{p(h)}$$

$$= \underset{h}{\operatorname{argmax}} \log [p(D|h) \cdot p(h)]$$

$$= \underset{h}{\operatorname{argmax}} [\log(p(D|h)) + \log p(h)]$$

As we get more data, MAP estimate converges towards MLE

Ex:  $h_1 \triangleq$  powers of two =  $\{2, 4, 8, 16, 32, 64\}$

$h_2 \triangleq$  powers of two except 32 =  $\{2, 4, 8, 16, 64\}$

$$\underline{D_1 \rightarrow N_1 = 4}$$

$$p(h_1 | D_1) = \left(\frac{1}{6}\right)^4 \cdot p(h_1) \quad p(h_2 | D_1) = \left(\frac{1}{5}\right)^4 \cdot p(h_2)$$

$$\underline{D_2 \rightarrow N_2 = 1,000,000}$$

$$p(h_1 | D_2) = \left(\frac{1}{6}\right)^{1,000,000} \cdot p(h_1) \quad p(h_2 | D_2) = \left(\frac{1}{5}\right)^{1,000,000} \cdot p(h_2)$$

Ⓢ Data overwhelms the prior.