MEAN

Discrete

$$\mathbb{E}[X] \triangleq \sum_{x \in \mathcal{X}} x \cdot p(x)$$

For a six-sided die (fair)

$$\mathcal{X} = \{1,2,3,4,5,6\}$$

$$\mathbb{E}[X] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$$

$$= \frac{21}{6} = 3.5$$

VARIANCE

$$\mathrm{var}[X] \triangleq \mathbb{E}[(X-\mu)^2]$$

Discrete Random Variables

$$\begin{aligned}
\mathrm{var}[X] &= E[(X-\mu)^2] \\
&= \mathbb{E}[X^2 - 2X\mu + \mu^2] \\
&= E[X^2] - E[2X\mu] + E[\mu^2] \\
&= E[X^2] - 2\mu E[X] + \mu^2 \\
&= E[X^2] - 2\mu^2 + \mu^2 \\
&= E[X^2] - \mu^2
\end{aligned}$$

Continuous Random Variables

$$\text{Var}[X] \triangleq E[(X-\mu)^2]$$

$$= \int_x (x-\mu)^2 \cdot p(x) \cdot dx$$

$$= \int_x x^2 \cdot p(x) \cdot dx - \int_x 2x\mu\, p(x) dx + \int_x \mu^2\, p(x) \cdot dx$$

$$= E[x^2] - 2\mu\, E[x] + \mu^2$$

$$= E[x^2] - \mu^2$$

$$= E[x^2] - (E[x])^2$$

Binomial Distribution

$n = 2$ $\quad p(\text{heads}) = \theta$

$$Bin(0 \mid 2, \theta) = (1-\theta)(1-\theta)$$

$$Bin(1 \mid 2, \theta) = \theta \cdot (1-\theta) + (1-\theta) \cdot \theta$$

$$Bin(2 \mid 2, \theta) = \theta \cdot \theta = \theta^2$$

$$Bin(k \mid n, \theta) = \binom{n}{k} \theta^k \cdot (1-\theta)^{n-k}$$

$$= \frac{n!}{k!\,(n-k)!} \theta^k (1-\theta)^{n-k}$$

Multinomial Distribution

Ex: 4-sided die

Die is rolled 10 times

3 of side-1

3 of side-2

4 of side-3

0 of side-4

$x = (3, 3, 4, 0)$

$$Mu(x \mid n, \theta) = \frac{10!}{3!\,3!\,4!\,0!} \cdot \theta_1^3 \cdot \theta_2^3 \cdot \theta_3^4 \cdot \theta_4^0$$

Poisson Distribution

$$Poi(x|\lambda) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$$

Ex: $\lambda = 1$

$$Poi(0|\lambda) = e^{-1} \cdot \frac{1^0}{0!} = e^{-1} \approx 0.37$$

$$Poi(1|\lambda) = e^{-1} \cdot \frac{1^1}{1!} = e^{-1} \approx 0.37$$

$$Poi(2|\lambda) = e^{-1} \cdot \frac{1^2}{2!} = \frac{e^{-1}}{2}$$

$$Poi(3|\lambda) = e^{-1} \cdot \frac{1^3}{3!} = \frac{e^{-1}}{6}$$

Covariance

$$\operatorname{cov}[X,Y] \triangleq E[(X-E(X))\cdot(Y-E[Y])]$$

$$= E[XY - X\cdot E[Y] - E[X]\cdot Y + E[X]\cdot E[Y]]$$

$$= E[XY] - E[X]\cdot E[Y] - E[X]\cdot E[Y] + E[X]\cdot E[Y]$$

$$= E[XY] - E[X]\cdot E[Y]$$