Example : Rolling a single die. (six-sided)

$$\mathcal{X} = \{1, 2, 3, 4, 5, 6\}$$

If a fair die (six-sided)

$$p(X=1) = 1/6$$

$$p(X=2) = 1/6$$

$$p(X=3) = 1/6$$

$$p(X=4) = 1/6$$

$$p(X=5) = 1/6$$

$$p(X=6) = 1/6$$

Probability of Union of Two Events

$$p(A \vee B) = p(A) + p(B) - p(A \wedge B)$$

Roll a die (fair, six-sided)

A = Die turns up 4

B = Die turns up 6

$$p(A \vee B) = \frac{1}{6} + \frac{1}{6} - 0$$

↳ $p(A \vee B) = p(A) + p(B)$

A & B are mutually exclusive.

↳ A & B cannot happen at the same time.

Roll two dice (fair, six-sided)

A = First die turns up 4

B = Second die turns up 6

$$p(A \vee B) = \frac{1}{6} + \frac{1}{6} - \frac{1}{36}$$

$$= \frac{11}{36}$$

Joint Probabilities

$$\left.\begin{aligned} p(A,B) &= \underbrace{p(A|B)}_{\text{conditional probability}} \cdot p(B) \\ p(A,B) &= p(B|A) \cdot p(A) \end{aligned}\right\} =$$

Example: A family has two children.

A : Both children are girls.

B : The first child is a girl.

$$p(A,B) = p(A|B) \cdot p(B)$$

$$= \frac{1}{2} \cdot \frac{1}{2} = 1/4$$

$$p(A,B) = p(B|A) \cdot p(A)$$

$$= 1 \cdot \frac{1}{4} = 1/4$$

Example : Medical Diagnosis

$p(x=1|y=1) = 0.8$ $\quad$ $x=1 \rightarrow$ mammogram is positive

$p(y=1|x=1) = ?$ $\quad$ $y=1 \rightarrow$ patient has breast cancer

$p(y=1) = 0.004$ $\leftarrow$ prior probability of having breast cancer

$p(x=1|y=0) = 0.1$ $\leftarrow$ false positive

$$p(y=1|x=1) = \frac{p(x=1|y=1) \cdot p(y=1)}{\sum_{y'} [p(x=1|y=y') \cdot p(y=y')]}$$

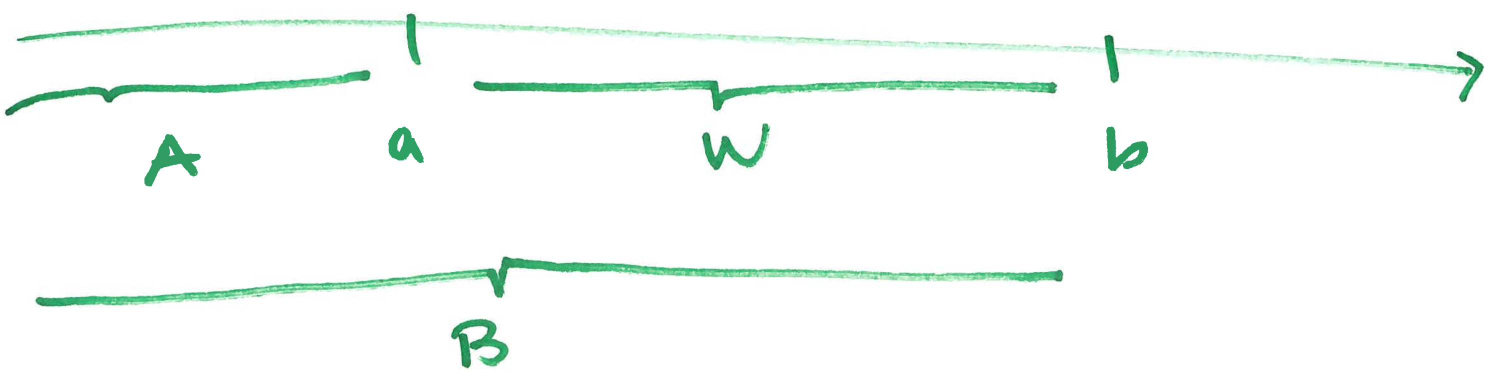
$$= \frac{0.8 \times 0.004}{p(x=1|y=0) \cdot p(y=0) + p(x=1|y=1) \cdot p(y=1)}$$

$$= \frac{0.8 \times 0.004}{0.1 \times 0.996 + 0.8 \times 0.004} = 0.031$$

$A = (x \leq a)$

$B = (x \leq b)$

$W = (a < x \leq b)$



$B = A \vee W$