

TA

Prove that there is no invertible 3×3 matrix that has all the same cofactors



To prove that there is no invertible 3×3 matrix with all the same cofactors, we can use the fact that the determinant of a matrix is the product of its eigenvalues.



Let A be a 3×3 matrix with all the same cofactors, and let's denote the cofactor of A in row i and column j as C_{ij} .

If all the cofactors are the same, then $C_{ij} = k$ for all i and j , where k is some constant.

Now, consider the determinant of A , which is given by:

$$\det(A) = \sum_{i=1}^3 (a_{ik} * C_{ik}) \text{ for } i = 1 \text{ to } 3$$

where a_{ik} is the (i, k) -th entry of A .

Since all the cofactors are the same ($C_{ij} = k$), we can factor k out of the summation:

$$\det(A) = k * \sum_{i=1}^3 (a_{ik}) \text{ for } i = 1 \text{ to } 3$$



Now, consider the eigenvalues of A . The eigenvalues of a matrix are the roots of its characteristic polynomial, which is given by:



$$\det(A - \lambda I) = 0$$

where λ is the eigenvalue we're trying to find, and I is the identity matrix.

In our case, we have:

$$\det(A - \lambda I) = k * \sum_{i=1}^3 (a_{ik} - \lambda) \text{ for } i = 1 \text{ to } 3$$

Since the eigenvalues of A are the roots of this characteristic polynomial, if all the eigenvalues are the same, then this polynomial should have a repeated root. However, we can see that this is not the case.

If all the eigenvalues were the same, the characteristic polynomial would be of the form:

$$P(\lambda) = k * \prod_{i=1}^3 (a_{ik} - \lambda) \text{ for } i = 1 \text{ to } 3$$

where \prod denotes the product.

Now, consider the product of the roots of this polynomial, which should be equal to the determinant of A by Vieta's formulas:

$$\text{product of roots} = k * \prod_{i=1}^3 (a_{ik} - \lambda) \text{ for } i = 1 \text{ to } 3$$

However, this product of roots should be equal to $\det(A)$, which we previously computed as $k * \sum_{i=1}^3 (a_{ik})$ for $i = 1$ to 3 .

Since $\prod_{i=1}^3 (a_{ik} - \lambda)$ and $\sum_{i=1}^3 (a_{ik})$ are not equal (the first is a product of three factors, and the second is a sum of three terms), we conclude that the eigenvalues of A cannot all be the same.

Therefore, there is no invertible 3×3 matrix that has all the same cofactors because such a matrix would require all of its eigenvalues to be the same, which we have shown is not possible.

Regenerate

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