

$$x = r \cos \theta, y = r \sin \theta$$

$$\frac{\partial}{\partial x} = \cos \theta \frac{\partial}{\partial r} - \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \quad \frac{\partial}{\partial y} = \sin \theta \frac{\partial}{\partial r} + \frac{\cos \theta}{r} \frac{\partial}{\partial \theta}$$

$$\frac{\partial^2}{\partial x^2} = \cos^2 \theta \frac{\partial^2}{\partial r^2} - \frac{2 \sin \theta \cos \theta}{r} \frac{\partial^2}{\partial r \partial \theta} + \frac{\sin^2 \theta}{r^2} \frac{\partial^2}{\partial \theta^2}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad r = \sqrt{x^2 + y^2} \quad \theta = \tan^{-1} \frac{y}{x}$$

$$f = f(x(r, \theta), y(r, \theta)) \quad \frac{\partial r}{\partial x} = \cos \theta \quad \frac{\partial r}{\partial y} = \sin \theta \quad \frac{\partial \theta}{\partial x} = -\frac{\sin \theta}{r} \quad \frac{\partial \theta}{\partial y} = \frac{\cos \theta}{r}$$

$$\frac{\partial f}{\partial x} = \cos \theta \frac{\partial f}{\partial r} - \frac{\sin \theta}{r} \frac{\partial f}{\partial \theta} \quad \frac{\partial f}{\partial y} = \sin \theta \frac{\partial f}{\partial r} + \frac{\cos \theta}{r} \frac{\partial f}{\partial \theta}$$

$$\frac{\partial^2 f}{\partial x^2} = \cos^2 \theta \frac{\partial^2 f}{\partial r^2} - \frac{2 \sin \theta \cos \theta}{r} \frac{\partial^2 f}{\partial r \partial \theta} + \frac{\sin^2 \theta}{r^2} \frac{\partial^2 f}{\partial \theta^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \sin^2 \theta \frac{\partial^2 f}{\partial r^2} + \frac{2 \sin \theta \cos \theta}{r} \frac{\partial^2 f}{\partial r \partial \theta} + \frac{\cos^2 \theta}{r^2} \frac{\partial^2 f}{\partial \theta^2}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{1}{r} \frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2}$$