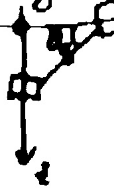


$m_1 = 0.1\,Kg \quad m_2 = 0.3\,Kg \quad d = 0.5\,m \quad \theta_0 = \frac{\pi}{6}$

$m g\, d(1-\sin\theta) + m_2 g \frac{d}{2}(1-\sin\theta) = g\, d(1-\sin\theta_0)\left(m + \frac{m_2}{2}\right) = 0.61\,J = E_0$

$E_0 = \frac{1}{2} m v_1^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v_1^2 + \frac{1 \cdot 1}{2 \cdot 3} m_2 d^2 \omega^2 = v_1^2\left(\frac{m}{2} + \frac{m_2}{6}\right) \quad v_1 = \sqrt{\frac{6 E_0}{3m + m_2}} = 2.47\,\frac{m}{s}$

$m_1 = 2\,Kg \quad r = 0.15\,m \quad d = 0.12\,m \quad m_2 = 0.3\,Kg \quad \mu \quad M = 0.23\,Nm \quad t_0 = 1.2\,s$

$I_1 \alpha_1 = M - f d \quad I_2 \alpha_2 = f d \qquad I_1 \alpha_1 + I_2 \alpha_2 = M \qquad \theta_1 = \frac{1}{2}\alpha_1 t_0^2 \quad \theta_2 = \frac{1}{2}\alpha_2 t_0^2$

$\Delta L = M t_0 = I_1 \omega_1 + I_2 \omega_2 \qquad -\frac{\theta_1 - \theta_2}{2\pi} \cdot 2\pi d\, m_2 g \mu + M \theta_1 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2$

$m = 2\,Kg \quad r = 0.5\,m \quad M = 14.4\,Nm \quad K = 8\,Nm\frac{s}{rad} \qquad m_2 = \frac{m}{4} \quad \mu = 0.2$

a) $\frac{dL}{dt} = M_{tot} \qquad I\frac{d\omega}{dt} = M - K\omega \qquad \int_0^\omega \frac{d\omega}{M - K\omega} = \int_0^t \frac{dt}{I}$

$\frac{t}{I} = \left[-\frac{1}{K}\ln(M - K\omega)\right]_0^\omega = -\frac{1}{K}\ln(M-K\omega) + \frac{1}{K}\ln M = \frac{1}{K}\ln\frac{M}{M-K\omega} \qquad \frac{M}{M-K\omega} = e^{\frac{Kt}{I}}$

$M e^{-\frac{Kt}{I}} = M - K\omega \qquad \omega = \frac{M}{K}\left(1 - e^{-\frac{K}{I}t}\right) \qquad \lim_{t\to\infty} \omega = \frac{M}{K} = 1.8\,\frac{rad}{s}$

b) m_2 $\quad I\omega = I'\omega' \quad \frac{m r^2}{2}\omega = \left(\frac{m r^2}{2} + \frac{m}{4} r^2\right)\omega' \quad \frac{m r^2}{2}\omega = \frac{3}{4}\frac{m r^2}{2}\omega' \quad \omega' = \frac{2}{3}\omega = 1.2\,\frac{rad}{s}$

$m = 10\,Kg \quad r = 0.2 \quad \mu = 0.2 \quad M_A = 3.88\,Nm \quad n_A = 20\ giri$

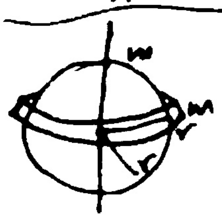
$M_{att} = \mu m g \frac{r}{2} = 1.96\,Nm \qquad I = \frac{m r^2}{2} = I_A = I_B = 0.2\,Kg\,m^2$

$I\alpha_A = M_A - M_{att} \quad \alpha_A = 19.6\,\frac{rad}{s^2} \quad I\alpha_B = M_{att} \quad \alpha_B = 9.8\,\frac{rad}{s^2}$

$20 \cdot 2\pi = \frac{1}{2}\alpha_A t^2 \qquad t = \sqrt{\frac{80\pi}{\alpha_A}} = 3.6\,s \qquad L = M_A t = I\omega_A + I\omega_B$

$40\pi M_A = \frac{1}{2} I(\omega_A^2 + \omega_B^2) + 2\pi (n_A - n_B) M_{ATT}$

$80\pi M_A = I t^2(\alpha_A^2 + \alpha_B^2) + 4\pi n_B M_{att} \quad n_A - n_B = \frac{80\pi M_A - I t^2(\alpha_A^2 + \alpha_B^2)}{4\pi M_{att}} = 10 \quad n_B = 10$



$m, r \quad M = 8.6\,Nm \quad t_0 = 6.3\,s \quad t' = 10.5\,s \qquad \omega_A(t') = \omega_B(t') \quad M(t > t_0) = 0$

$I_s \alpha_s = M - M_a \quad I_A \alpha_A = M_a \qquad \omega_A(t > t') = 0$

$\omega_A(t_0) = \alpha_A t_0 \quad \omega_s(t_0) = \alpha_s t_0$

$\frac{\alpha_A}{\alpha_s} = \frac{\omega_A}{\omega_s} \qquad \omega_{A0} + \frac{M_a}{I_A} t' = \omega_{s0} - \frac{M_a}{I_s} t'$

$M_a t'\left(\frac{I_A + I_s}{I_A I_s}\right) = \omega_{s0} - \omega_{A0} \qquad \frac{I_A + I_s}{I_A I_s} = \frac{m r^2 + \frac{2}{5} m r^2}{\frac{2}{5} m^2 r^4} = \frac{\frac{7}{5}}{\frac{2}{5} m r^2} = \frac{7}{2 m r^2} \qquad M_a = \frac{\omega_{s0} - \omega_{A0}}{7 t'}(2 m r^2)$

$M_A = \frac{2 m r^2}{7 t'} t_0 (\alpha_s - \alpha_A) = \frac{2 m r^2 t_0}{7 t'}\left(\frac{M - M_a}{\frac{2}{5} m r^2} - \frac{M_a}{m r^2}\right) = \frac{2 t_0}{7 t'}\left(\frac{5M - 5M_a - 2M_a}{2}\right) = \frac{t_0}{7 t'}(5M - 7M_a)$

$M_A + \frac{t_0}{t'} M_A = \frac{5 t_0}{7 t'} M \qquad M_A = \frac{5 t_0 M}{7 t'\left(1 + \frac{t_0}{t'}\right)} = \frac{5 t_0 M}{7(t' + t_0)} = 2.3\,Nm$ $(3.69\,Nm)$ $\quad \omega_s(t) = \alpha_s t_0 - \frac{M_a}{I_s}(t' - t_0)$

$\omega_s = 0 \quad \alpha_s t_0 = \frac{M_a t}{I_s} \qquad \frac{M - M_a}{I_s} t_0 = \frac{M_a}{I_s} t \qquad t = 8.34 = t' + 4.14\,s \qquad \omega = \frac{1200 \cdot 2\pi}{60}\frac{rad}{s} = 125.7\,\frac{rad}{s}$

$\rho = 7 \cdot 10^3\,\frac{Kg}{m^3} \quad r_1 = 0.3 \quad r_2 = 0.5 \qquad$ fase 1: M_2 cost $\omega_{20} = 1200\,rpm$, M_1 opposto, $\omega_1 = 0$, $t = 0, t_0 = 100\,s$ $\qquad$ fase 2 $\quad M_1, M_2 = 0$

$d_1 = 1\,m \quad d_2 = 0.2\,m \quad M_a$ se c'è rotaz. relativa $\qquad t_0 = 100\,s,\ t_1 = 150\,s \quad \omega_{11} = \omega_{21}$

$m_1 = \rho \pi r_1^2 d_1 = 1979\,Kg \quad m_2 = \rho(\pi r_2^2 d_2 - \pi r_1^2 d_2) = 704\,Kg \quad I_1 = \frac{1}{2} m_1 r_1^2 = 89.1\,Kg\,m^2 \quad I_2 = \frac{1}{2}\rho\pi d_2 (r_2^4 - r_1^4) = 119.6\,Kg\,m^2$

1): $I_1 \alpha_1 = 0 = M_a - M_1 \quad I_2 \alpha_2 = M_2 - M_a \qquad \omega_{20} = \alpha_2 t_0 \quad \alpha_2 = 1.26\,\frac{rad}{s}$

2): $I_2 \omega_{20} = (I_1 + I_2)\omega_1 \quad \omega_1 = 72\,\frac{rad}{s} \qquad \omega_{20} - \frac{M_a}{I_2}(t_1 - t_0) = \omega_1 \quad M_a = \frac{I_2(\omega_{20} - \omega_1)}{t_1 - t_0} = 128.4\,Nm = M_1$

$M_2 = M_a + I_2 \alpha_2 = 279.1\,Nm$