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Course code : MATH 337

Course title : Mathematical Analysis for Computer Science

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Assignment 01

1. Prove $a^0 + a^1 + a^2 + \dots + a^n = \frac{a^{n+1} - 1}{a-1}$ using Mathematical Induction Method.

Solution:

Basis:

$$a^0 = \frac{a^{0+1} - 1}{a-1}$$

$$\Rightarrow a^0 = \frac{a^1 - 1}{a-1}$$

$$\Rightarrow a^0 = \frac{a-1}{a-1}$$

$$\therefore a^0 = 1$$

Hypothesis: $a^0 + a^1 + a^2 + \dots + a^n = \frac{a^{n+1} - 1}{a-1}$

Induction: $a^0 + a^1 + a^2 + \dots + a^n + a^{n+1}$

$$= \frac{a^{n+1} - 1}{a-1} + a^{n+1}$$

$$= \frac{a^{n+1} - 1 + (a^{n+1})(a-1)}{a-1}$$

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$$= \frac{a^{n+1} - 1 + a^{n+2} - a^{n+1}}{a-1}$$

$$= \frac{a^{n+2} - 1}{a-1}$$

[Proved]

2. Derive the open and closed form of Non-Serializable Double tower of Hanoi. Prove the closed form using Mathematical Induction method.

Solution:

Non-Serializable Double Tower of Hanoi

$T_0 = 0$

open form is, $T_n = T_{n-1} + 2 + T_{n-1}$

$= 2T_{n-1} + 2$

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Now, $T_n = 2T_{n-1} + 2$

$= 2(T_{n-2} + 2) + 2$

$= 2^2 T_{n-2} + 2 \cdot 2 + 2$

$= 2^2 (2T_{n-3} + 2) + 2^1 \cdot 2 + 2$

$= 2^3 T_{n-3} + 2^2 \cdot 2 + 2^1 \cdot 2 + 2^0 \cdot 2$

$\vdots$

$= 2^n T_{n-n} + 2^{n-1} \cdot 2 + 2^{n-2} \cdot 2 +$
$2^{n-3} \cdot 2 + \dots + 2^2 \cdot 2 + 2^1 \cdot 2 + 2^0 \cdot 2$

$= 2^n T_0 + 2^{n-1} \cdot 2 + 2^{n-2} \cdot 2 + 2^{n-3} \cdot 2 +$
$\dots + 2^2 \cdot 2 + 2^1 \cdot 2 + 2^0 \cdot 2$

$= 2^0 \cdot 2 + 2^1 \cdot 2 + 2^2 \cdot 2 + \dots + 2 \cdot 2^{n-3} +$
$2 \cdot 2^{n-2} + 2 \cdot 2^{n-1}$

$= 2(2^0 + 2^1 + 2^2 + \dots + 2^{n-3} + 2^{n-2} + 2^{n-1})$

$T_n = 2(2^n - 1)$

$\therefore T_n = 2^{n+1} - 2$ is called the closed form solution of non-serializable double tower of hanoi.

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Now, proving the closed form using mathematical induction method.

Basis: $T_0 = 2^{0+1} - 2$

$= 2 - 2$

$= 0$

Hypothesis: Let, $T_n = 2^{n+1} - 2$

Induction: $T_{n+1} = 2\,T_{n+1-1} + 2$

$= 2\,T_n + 2$

$= 2(2^{n+1} - 2) + 2$

$= 2^{n+2} - 4 + 2$

$= 2^{n+2} - 2$

[Proved]

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3. Derive the open and closed form of Serializable Double Tower of Hanoi. Prove the closed form using Mathematical Induction Method.

Solution:

Serializable Double tower of Hanoi

$T_0 = 0$

$T_1 = 3$

$T_2 = 11$

$T_3 = 27$

$\therefore T_n = T_{n-1} + 1 + T_{n-1} + 1 + T_{n-1} + 1 + T_{n-1}$

$T_n = 4T_{n-1} + 3$ is Open form

Now, $T_n = 4T_{n-1} + 3$

$= 4(4T_{n-2} + 3) + 3$

$= 4^2 T_{n-2} + 4 \cdot 3 + 3$

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$= 4^2(4T_{n-3}+3)+4.3+3$

$= 4^3 \; T_{n-3}+4^2.3+4^1.3+4^0.3$

$= 4^n T_{n-n}+4^{n-1}.3+4^{n-2}.3+\cdots+4^2.3+$
$4^1.3+4^0.3$

$= 4^n T_0+4^{n-1}.3+4^{n-2}.3+\cdots+4^2.3+$
$4^1.3+4^0.3$

$T_n = 3(4^0+4^1+4^2+\cdots+4^{n-2}+4^{n-1})$

$= \dfrac{3(4^n-1)}{3}$

$\therefore T_n = 4^{n-1}$ is called closed form

Now, Proving the closed form using Mathematical Induction Method.

<u>Basis:</u> $T_0 = 4^0-1 = 0$

<u>Hypothesis:</u> Let, $T_n = 4^n-1$

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Induction: $T_{n+1} = 4\,T_{n+1-1} + 3$

$$= 4\,T_n + 3$$

$$= 4(4^n - 1) + 3$$

$$= 4^{n+1} - 4 + 3$$

$$= 4^{n+1} - 1$$

[Proved]

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4. Derive the open and closed form of Triple of Tower of Hanoi. Prove the closed form using Mathematical Induction method.

Solution:

Triple Tower of Hanoi

$T_0 = 0,$

$T_1 = 3$

$T_2 = 9$

$T_3 = 21$

$\therefore T_n = T_{n-1} + 3 + T_{n-1}$

$T_n = 2T_{n-1} + 3$ is called open form

Now, $T_n = 2T_{n-1} + 3$

$= 2(2T_{n-2} + 3) + 3$

$= 2^2 . T_{n-2} + 2.3 + 3$

$= 2^2 (2T_{n-3} + 3) + 2.3 + 3$

$= 2^3 T_{n-3} + 2^2 . 3 + 2^1 . 3 + 2^0 . 3$

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$= 2^n T_{n-1} + 2^{n-1} \cdot 3 + 2^{n-2} \cdot 3 + \cdots + 2^2 \cdot 3 +$
$2^1 \cdot 3 + 2^0 \cdot 3$

$= 2^n T_0 + 2^{n-1} \cdot 3 + 2^{n-2} \cdot 3 + \cdots + 2^2 \cdot 3 + 2^1 \cdot 3 +$
$2^0 \cdot 3$

$= 3 (2^0 + 2^1 + 2^2 + \cdots + 2^{n-2} + 2^{n-1})$

$T_n = 3 \cdot \frac{2^n - 1}{2 - 1}$

$\therefore T_n = 3(2^n - 1)$ is called closed form

Now, Proving closed form using Mathematical Induction Method.

<u>Basis</u>: $T_0 = 3(2^0 - 1)$
$= 3(1-1)$
$= 3 \cdot 0$
$= 0$

<u>Hypothesis</u>: Let $T_n = 3(2^n - 1)$

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Induction: $T_{n+1} = 2T_{n+1-1} + 3$

$= 2T_n + 3$

$= 2\{3(2^n - 1)\} + 3$

$= 6(2^n - 1) + 3$

$= 3 \cdot 2 \cdot 2^n - 6 + 3$

$= 3(2^{n+1} - 1)$

[Proved]

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5. Derive the open and closed from for n intersecting zigzag shapes. Prove the closed form using Mathematical Induction Method.

Solution:

Zigzag Shapes

Zigzag	Lines	Line region	Zigzag region	Lost region
1	3	7	2	5
2	6	22	12	10
3	9	46	31	15

Open form, $z_n = l_{3n} - 5n$

$$= \frac{3n(3n+1)}{2} + 1 - 5n$$

$$= \frac{3^2n^2 + 3n}{2} + 1 - 5n$$

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$$= \frac{9n^2 + 3n + 2 - 10n}{2}$$

$$= \frac{9n^2 - 7n + 2}{2}$$

$\therefore$ closed form is, $z_n = \dfrac{9n^2 - 7n + 2}{2}$

Now, Proving the closed form using Mathematical induction Method.

Basis: $z_0 = \dfrac{9.0 - 7.0 + 2}{2}$

$$= \frac{2}{2}$$

$$= 1$$

Hypothesis: Let,

$$z_n = \frac{9n - 7n + 2}{2}$$

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Induction:

$$2n+1 = l_{3(n+1)} - 5(n+1)$$

$$= l_{3n+3} - 5n - 5$$

$$= \frac{(3n+3)(3n+4)}{2} + 1 - 5n - 5$$

$$= \frac{9n^2+12n+9n+12n+2-10n-10}{2}$$

$$= \frac{9n^2+11n+4}{2}$$

[Proved]