DBA3713 Analytics for Risk Management

Assignment 2 Report

Group 2

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Question 1

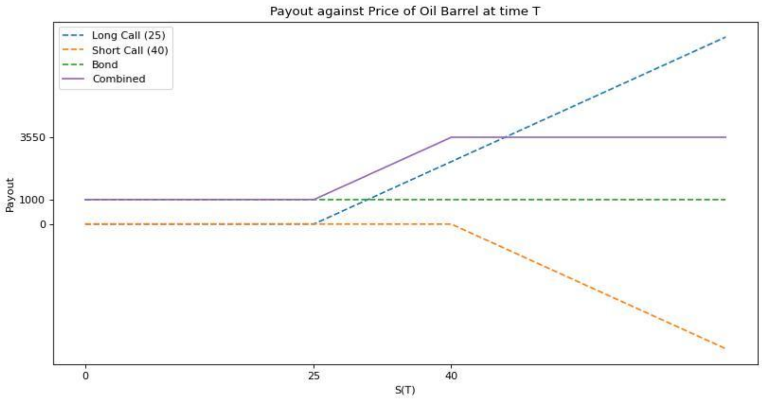

Figure 1. Payout against Price of Barrel of Oil / Strike Price at time T

Let S(T) be the price of the oil barrel at maturity.

For individual components of the "Portfolio" at S(T)

- S(T) < 25 (Blue) - your long call options will not be exercised.
- S(T) < 40 (Orange) - your short call options with strike 40 will not be exercised.
- The bond pays 1000 at maturity.

Bond offered by Standard Oil

- S(T) < \$25: The Payout is \$0 + 1000
- \$40 > S(T) > \$25: The Payout is \$170(S(T) - 25) + 1000
- S(T) > \$40: The Payout is \$2550 + 1000

As the combinations of the Bond offered by Standard Oil is exactly replicated by the combination of the individual components, the bond issued by Standard Oil is equal to the combination of a regular bond, a long position in 170 call options on oil with strike price of \$25 and a short position in 170 call options on oil with a strike price of \$40.

Such a bond payoff closely resembles a bull spread hedging strategy, where we long a call option with lower strike price and short a call with higher strike price. This causes the gains from loss from short call to be offset by gains from long call beyond the breakeven point, ensuring that losses are capped.

Question 2

	Cash Flow at t = 0	**Cash Flow at t = T**
Short Forward	0	1060 - S(T)
Long Stock	-1000	S(T)
Subtotal	-1000	1060
Borrow	1019.2307	-1060
Net Amount	**19.23 (2dp)**	**0**

Table 1. Cash flows at t = 0 vs t = T for given portfolio

The future is initially overvalued at 1000 - 1060/1.04 = -19.23. So you short the future - agree to find a party that will pay you 1060 at maturity for the stock. Then you borrow 1060/1.04 = 1019 from the bank, and buy the stock at 1000, locking in $19.23 profit now. At maturity, sell the stock to your long counterparty, receiving the futures forward price at 1060, and take that money to repay the bank your combined principal and interest of $1060 and get 0 cash flow at maturity.

Question 3

Week	Stock Price	Delta	Shares Purchased	Cost of Shares Purchased ($000)	Cumulative Cash Outflow ($000)	Interest Cost ($000)
0	49.00	0.522	52,200	2,557.8	2,557.8	2.5
1	49.75	0.568	4,600	228.8	2,789.1	2.7
2	52.00	0.705	13,700	712.4	3,504.2	3.4
3	50.00	0.579	(12,600)	(630.0)	2,877.6	2.8
4	48.38	0.459	(12,000)	(580.6)	2,299.8	2.2
5	48.25	0.443	(1,600)	(77.2)	2,224.8	2.1
6	48.75	0.475	3,200	156.0	2,382.9	2.3
7	49.63	0.540	6,500	322.6	2,707.8	2.6
8	48.25	0.420	(12,000)	(579.0)	2,131.4	2.0
9	48.25	0.410	(1,000)	(48.2)	2,085.2	2.0
10	51.12	0.658	24,800	1,267.8	3,355.0	3.2
11	51.50	0.692	3,400	175.1	3,533.3	3.4
12	49.88	0.542	(15,000)	(748.2)	2,788.5	2.7
13	49.88	0.538	(400)	(20.0)	2,771.2	2.7
14	48.75	0.400	(13,800)	(672.8)	2,101.1	2.0
15	47.50	0.236	(16,400)	(779.0)	1,324.1	1.3
16	48.00	0.261	2,500	120.0	1,445.4	1.4
17	46.25	0.062	(19,900)	(920.4)	526.4	0.5
18	48.13	0.183	12,100	582.4	1,109.3	1.1
19	46.63	0.007	(17,600)	(820.7)	289.7	0.3
20	48.12	0.000	(700)	(33.7)	256.3	

Table 2. Dynamic delta hedging table under Sample Path 2 (Complete)

To create our dynamic delta hedging table (Table 2.) we created an algorithm following the step processes below:

1) Find the running Delta for week, using:
    a) Stock Prices from Week 0 - 21 (Sample Path 2)
    b) Strike Price K = $ 50;
    c) Duration to maturity, T = 20/52, which decreases by 1/52 for each week;
    d) Risk-free interest rate, r = 0.05
    e) Stock price volatility, σ = 0.2
2) Find the Cumulative shares and Shares purchased using:
    a) Cumulative shares: Initial number of shares (100,000) * Delta for each week
    b) Shares purchased: Cumulative shares for the current week - previous week

3) Find the Cost of shares:
    a) Shares purchased * Stock Price of shares for each week
4) Find Cash outflow and Interest cost:
    a) Week 0: Determine base values of Cash outflow and interest cost by using Cost of shares in Week 0 and Cost of shares in Week 0 * r/52
    b) Cash outflow (Week 1-20): Subsequently, the Cash outflow is the sum of Cost of shares for the current week, and the sum of Cash outflow and interest costs of the previous week
    c) Interest costs (Week 1-20): Computed using current week cash outflow * r/52, with the final interest cost for Week 20 being 0.

Question 4

(a) Terminologies used:

- Premium of the call option is \$c, premium of the put option is \$p
- Spot price at time = 0 is S(0), spot price at time = T is S(T)
- Loan at \$50
- Strike price of option is K

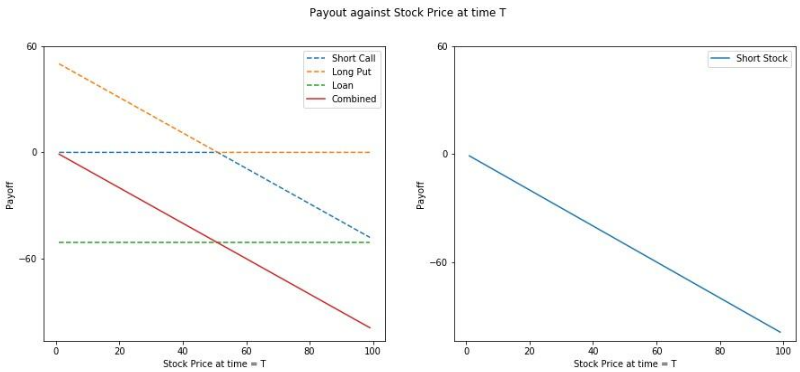

Figure 3. Payoff Curves of Short Call, Long Put (left) and Shorting Stock (right)

Our first observation is that the net payoff from the combination of Shorting a call, Longing a Put and taking a loan of 50 resembles that of a **Short Stock** Payoff Curve by being strictly downward sloping following the stock price at time T (Figure 3). Hence, the portfolio we have created is a synthetic short.

Transaction	**Cash flow today (\$)**	**Cash flow at Time = 0 (\$)** S(T) > K	**Cash flow at Time = T (\$)** S(T) < K
Short Call	c	51 - S(T)	0
Long Put	-p	0	51 - S(T)
Borrowing	50	-51	-51
Net Cash Flow	**c - p + 50**	- S(T)	**- S(T)**

Table 3. Payoff Table of Portfolio

The net cash flow of the portfolio at time T is:

$$Min\{51 - S(T), 0\} + Max\{51 - S(T), 0\} - 50 * (1 + 0.02) = -S(T)$$

Transaction today	Cash flow today ($)	Cash flow at Time = T ($)
Net Cash Flow of Shorting a Share	52	**- S(T)**

Table 4. Payoff Table of Short Call

Additionally, the net cash flow at time T is the **same** when shorting one share and for the portfolio (Table 3). The initial cash flow is **$(c - p + 50**).

(b) The new portfolio, Portfolio P, includes shorting a call, longing a put, borrowing $50 and **longing a share.**

Figure 4. Payoff Curve of Portfolio P

Transaction today	Cash flow today ($)	Cash flow at Time = T ($)
Short Call	c	Min{51 − S(T) , 0}
Long Put	-p	Max{51 - S(T), 0}
Borrowing	50	-51
Longing a share	**-52**	**S(T)**
Net Cash Flow	**3** (c-p-2 = 5-2 = 3)	**0**

Table 4. Arbitrage Table of Portfolio P

The initial net cash flow is positive, since:

$$c - p + 50 - S(0) = 5 + 50 - 52 = 3$$

The net cash flow at time t is always **zero** since the long position of the stock is combined with a synthetic short of the same stock (Figure 4). The **arbitrage opportunity is identified** since the initial net cash flow is always positive and there is zero net cash future inflow. A zero net cash future inflow has zero risk as it is not dependent on spot prices at time = T.

(c) The arbitrage opportunity disappears when net cash inflow = 0, where initial net cash flow is **c - p = 2.**

$$c - p + 50 - 52 = 0$$
$$c - p = 2$$

Question 5

EWMA, using the below metrics:

- λ = 0.94
- Volatility, σ = 0.013
- Yesterday's return = 300/298 - 1

Volatility (EWMA) = $\sqrt{\lambda \times \sigma^2 + (1 - \lambda) \times (300/298 - 1)^2} = 1.271\%$

GARCH(1,1), using the below metrics:

- Ω = 0.000002
- α = 0.04
- β = 0.94
- Volatility, σ = 0.013
- Yesterday's return = 300/298 - 1

Volatility (GARCH(1,1)) $= \sqrt{\omega + \alpha \times (300/298 - 1)^2 + \beta \times \sigma^2} = 1.275\%$
Long Term Volatility (GARCH(1,1)) = $\sqrt{\omega \div (1 - \alpha - \beta)} = 1.00\%$

Question 6

Optimal Parameter for EWMA Model:

- λ = **0.958**. (To 3 s.f.)

Optimal objective function value for EWMA = **-9.245**

Optimal Parameter for GARCH(1,1) Model (Basic):

- α = **0.0445**
- β = **0.953**
- ω = 1.33×10^{-7}

Optimal objective function value for Basic GARCH(1,1) = **-9.249**

Optimal Parameter for GARCH(1,1) Model (Variance Targeting Method):

- α = **0.0433**
- β = **0.953** (To 3 s.f.)

Optimal value for GARCH(1,1) + VT = **-9.249** (To 3 s.f.)

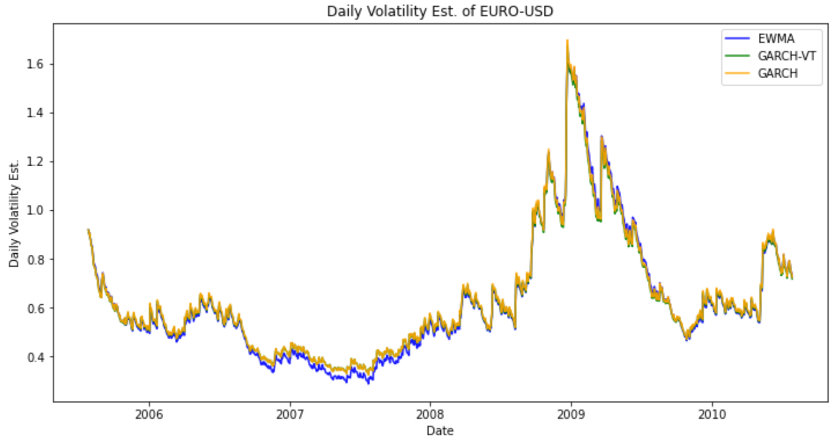

Figure 5. Daily Volatility Estimates of the EURO-USD on three(3) models

Hence, **all 3 models are similar** in estimating the daily volatility of EURO-USD (Figure 5).

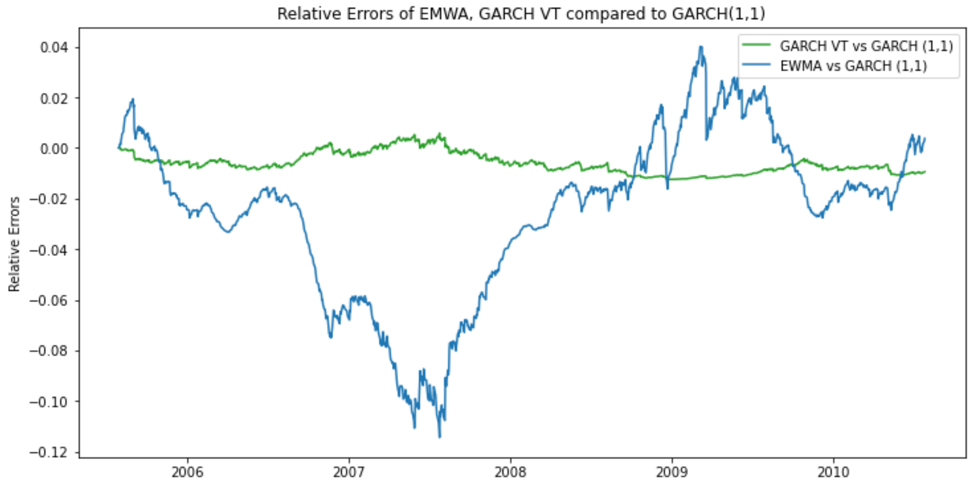

Figure 6. Relative Errors of EWMA and GARCH VT compared to GARCH(1,1)

For a more detailed analysis, the relative error of GARCH and GARCH-VT are close to zero, which means that they are almost identical (Figure 6). The relative error of EWMA compared to GARCH(1,1) is approximately **-0.08**, which is a reasonably small value.

$$(\sigma_i^{EWMA} - \sigma_i^{GARCH})/\sigma_i^{GARCH} \approx -0.08$$

Explorations

To test the effects of changing the decision tree's maximum depth and minimum sample split, we take a range of values for depth and split respectively to visualise how they affect the key performance evaluation metrics of the decision tree.

The test metrics (5 Levels each) we used are as below:

- Depth ranging from 1 to 10, in steps of 1.
- Splits ranging from 1 to 1000, in steps of 50.

We obtained the below results after performing our testing on the following performance evaluation metrics: Loss function and AUC.

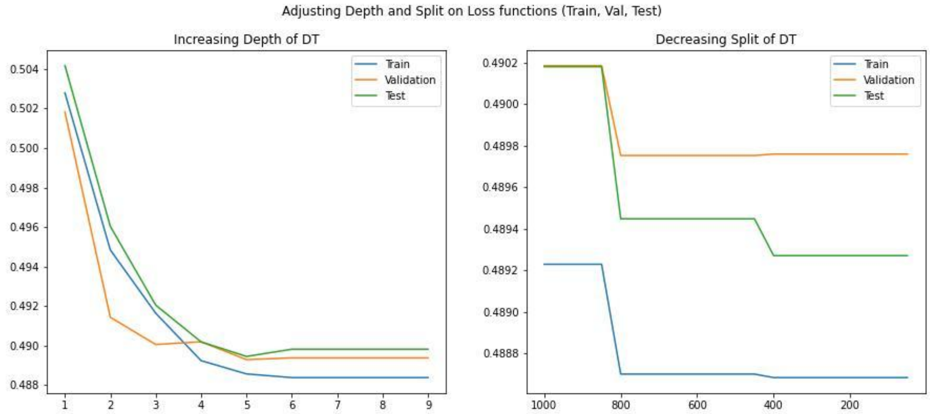

Figure 7. Adjusting Depth and Split on Loss function

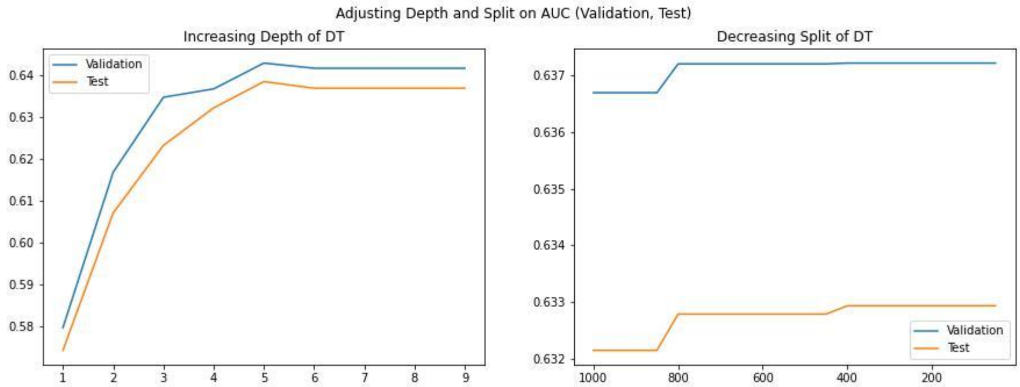

Figure 8. Adjusting Depth and Split on AUC

The hyperparameter max_depth controls how many levels the decision tree splits. Increasing max_depth too much will result in a complex, overfitted tree as the classifier will be allowed to split on every variable. While the model's training error may decrease, the model will be too specific to the set of data it is trained on and would not be very helpful in classifying other sets of data. On the other hand, decreasing max_depth too much will result in an overly simplistic model. The tree will not be flexible enough to capture any meaningful patterns in the data.

The min_samples_split hyperparameter looks at the number of samples in a node, and will not split further if the number of samples is less than min_samples_split. If min_samples_split is too large, then the decision tree may be under-fitted as the first few nodes would contain a majority of the samples and then not allowed to split further. On the other hand, if min_samples_split is too small, the nodes would be allowed to split all the way down the tree, resulting in an overfitted and complex tree that would not be helpful in classifying other sets of data.

From our explorations, we see that the optimal depth for this particular decision tree is 5, yielding low MLE loss functions (In train, validation and test sets) and the highest AUC (In validation and test sets), while the optimal number of sample splits is 400, yielding the lowest MLE loss functions and Highest AUC values (In validation and test sets).

Thus, the ideal hyperparameters for this decision tree are as follows:

- Optimal Depth : 5
- Optimal number of Splits: 400

Evaluating our “ideal” decision tree:

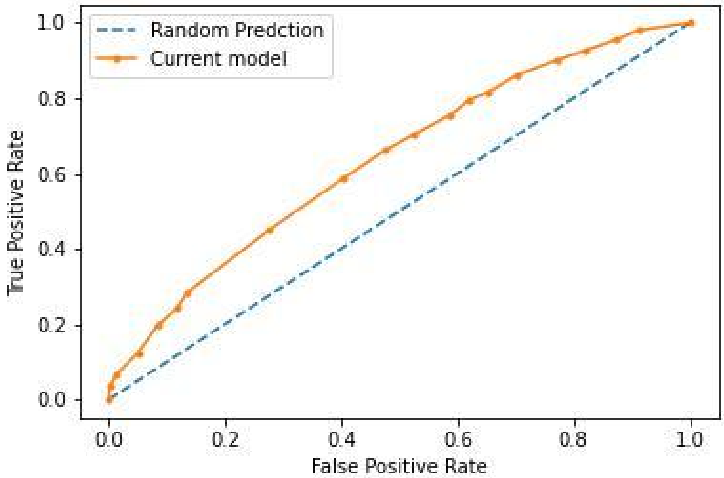

Figure 9. Plot of TPR against FPR (Demonstrating AUC predictions)

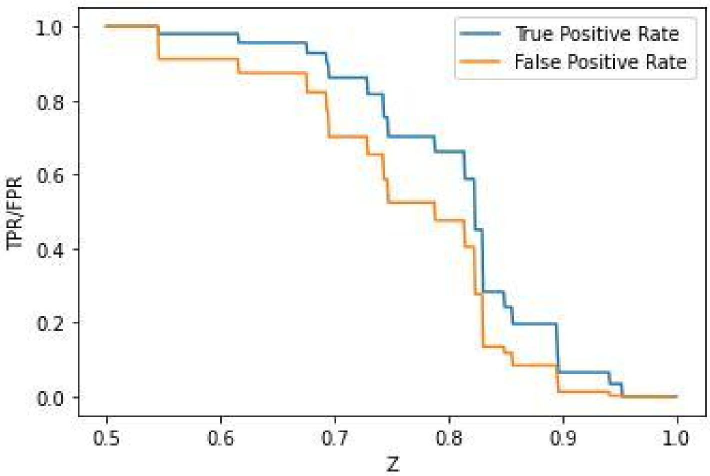

Figure 10. Plot of TPR/FPR against Z

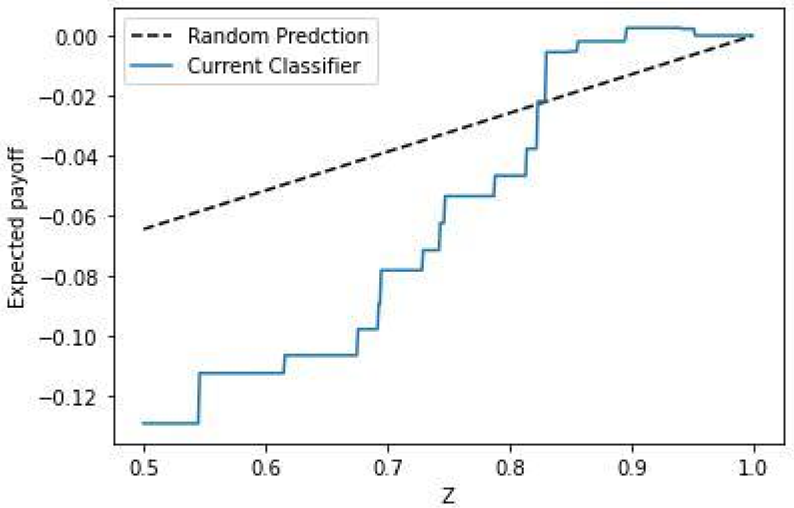

Figure 11. Plot of Expected Payoff against Z

From the ROC curve, the AUC predictions from the current classifier is 0.637 (3sf), which is higher than the original lecture code's AUC value of 0.632 (3sf). Additionally, from the TPR-FPR Curve, we can effectively illustrate the performance of the model on test data. Lastly, the optimal Z our group obtained is 0.896 (3sf), compared to the original optimal Z of 0.879 (3sf).

Thus, by changing the maximum depth and the minimum number of splits, we are able to find parameters that lead to better results from performance measures and higher values for Optimal Z, as well as improved AUC (Higher) and MLE loss (Lower) results.