DBA3713 Analytics for Risk Management

Assignment 3 Report

Group 2

Name	**Matriculation Number**
Chern Tat Sheng	A0183739N
Chew Zhe Ming, Ryan	A0183631H
Lim Shu Min, Cambrie	A0204096J
Tan Jia Yi, Megan	A0188664L

Question 1 (Refer to notebook for detailed computation.)

(a)

	Probability	X	Y	X Return	Y Return	X^2 Return	Y^2 Return	Total Returns	ex	ey	ex^2	ey^2	Weighted Cov	Weighted Return	stdev
0	0.1	70	140	-0.30	0.166667	0.0900	0.027778	-0.024590	-0.030	0.016667	0.00900	0.002778	-0.005469	-0.002459	0.000457
1	0.5	105	125	0.05	0.041667	0.0025	0.001736	0.045082	0.025	0.020833	0.00125	0.000868	-0.000260	0.022541	0.000002
2	0.4	120	115	0.20	-0.041667	0.0400	0.001736	0.057377	0.080	-0.016667	0.01600	0.000694	-0.003125	0.022951	0.000082

Table 1. Tabulations across regulations

$$E[X] = 10\% * (-30\%) + 50\% * 5\% + 40\% * 20\% = \frac{3}{40} = 7.50\%$$
$$E[X^2] = 10\% * (-30\%)^2 + 50\% * (5\%)^2 + 40\% * (20\%)^2 = \frac{21}{800} = 2.62\%$$
$$E[Y] = 10\% * 16.67\% + 50\% * 4.17\% + 40\% * (-4.17\%) = \frac{1}{48} = 2.08\%$$
$$E[Y^2] = 10\% * (16.67\%)^2 + 50\% * (4.17\%)^2 + 40\% * (-4.17\%)^2 = 0.434\%$$

$$\sigma_X = \sqrt{\frac{21}{800} - (\frac{3}{40})^2} = 0.14361$$
$$\sigma_Y = \sqrt{0.434\% - (\frac{1}{48})^2} = 0.062500$$

$$cov_{X,Y} = -0.0088542$$

The correlation coefficient is:

$$\rho_{X,Y} = \frac{cov_{X,Y}}{\sigma_X \sigma_Y} = \frac{-0.0088542}{0.14361 * 0.062500} = -0.986\ (to\ 3\ s.f.)$$

(b)

The expected return of portfolio is:

$$E[Portfolio] = 10\% * 2.4\% + 50\% * 4.5\% + 40\% * 5.7\% = 0.0430 = 4.30\%\ (to\ 3\ s.f.)$$

The standard deviation of portfolio is:

$$\sigma_{portfolio} = 0.023274 = 0.0233 = 2.33\%\ (to\ 3\ s.f.)$$

The sharpe ratio of portfolio is:

$$Sharpe\ Ratio = \frac{E(portfolio) - 0.005}{\sigma_{portfolio}} = 1.63\ (to\ 3\ s.f.)$$

Question 2 (Refer to notebook for detailed computation.)

(a) (i) Let w_A and w_B be the proportion of investment in Portfolio A and B respectively, given that risk-free securities' standard deviation = 0 and the covariance of portfolio A and B = 0, the standard deviation of the portfolio is:

$$\sigma_{portfolio} = w_B \sigma_B$$

Since the standard deviation of Portfolio C is 0.15,

$$0.15 = 0.25 w_B$$
$$w_B = \frac{0.15}{0.25} = 0.6$$
$$w_A = 1 - 0.6 = 0.4$$

$$Amount\ to\ be\ invested\ in\ A = 0.4 * 20000 = \$8,000$$

$$Amount\ to\ be\ invested\ in\ B = 0.6 * 20000 = \$12,000$$

The amount to be invested in **A is $8,000** and the amount to be invested in **B is $12,000**.

(a) (ii) The expected return of the portfolio is **13%.**

$$E[Portfolio] = 0.4 * 4\% + 0.6 * 19\% = 13\%$$

(b)

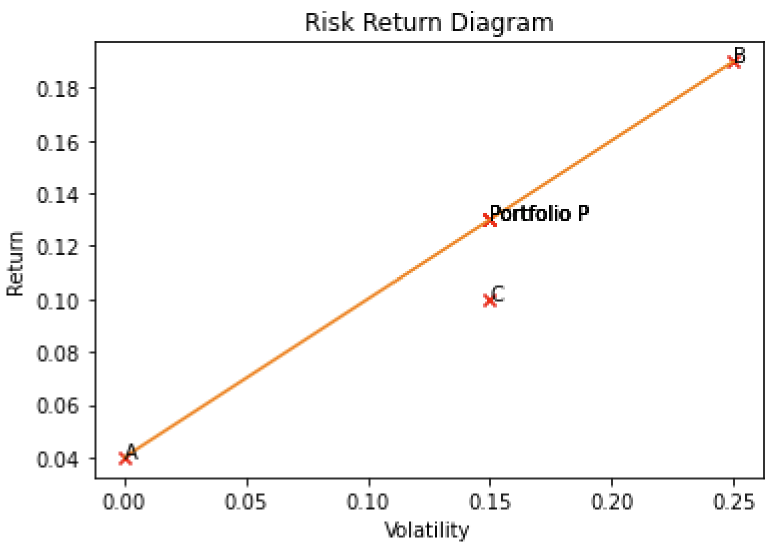

Figure 1. Risk Return Diagram, Q2

The new portfolio, consisting of 40% of A and 60% of B, gives a higher expected return of 13% or otherwise $2,600 of $20000, for the same volatility (Figure 1). Since the efficient frontier created by Portfolio A and B lies above Portfolio C, it has higher excess returns and a higher Sharpe Ratio.

Question 3
(a) (i) For the period 1/1/1985 through 12/31/1992, calculate:

The (in-sample) average excess return (i.e. the returns above the money market return) for each of the first four assets.

Average Excess Return = (Daily Return of Stock - Daily Money Market) / (Number of Days)

The average excess return of SB Non-US Bonds is 0.838 %. (3 d.p.)
The average excess return of MSCI EAFE is 1.029%. (3 d.p.)
The average excess return of CRSP VW US common stocks is 0.846%. (3 d.p.)
The average excess return of US Corp Bonds is 0.487%. (3 d.p.)

(a) (ii) The (in-sample) standard deviation for each of these excess returns.

Standard Deviation of Excess Returns = $\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}$

The standard deviation of excess return of SB Non-US Bonds is 3.669%. (3 d.p.)
The standard deviation of excess return of MSCI EAFE is 5.869%. (3 d.p.)
The standard deviation of excess return of CRSP VW US common stocks is 4.737%. (3 d.p.)
The standard deviation of excess return of US Corp Bonds is 1.564%. (3 d.p.)

(a) (iii) The (in-sample) covariance matrix (of the excess returns).

Cov(x,y)= $\frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{N-1}$

	SB Non-US Bonds	**MSCI EAFE**	**CRSP VW US common stocks**	**US Corp Bonds**
SB Non-US Bonds	0.133197%	0.121691%	-0.017705%	0.016848%
MSCI EAFE	0.121691%	0.340910%	0.115958%	0.017504%
CRSP VW US common stocks	-0.017705%	0.115958%	0.222041%	0.026104%
US Corp Bonds	0.016848%	0.017504%	0.026104%	0.024204%

(b) (i) Calculation of weights:
Global Minimum Variance Portfolio Weightage:

SB Non-US Bonds	MSCI EAFE	CRSP VW US common stocks	US Corp Bonds
0.06087	-0.00088	0.00208	0.93793

Tangency Portfolio Weightage:

SB Non-US Bonds	MSCI EAFE	CRSP VW US common stocks	US Corp Bonds
0.26592	-0.03509	0.14398	0.62519

Negative weights means that we take a **short position** on the stock for the portfolio instead.

(b) (ii) Sharpe ratio maximizing portfolio with no-shorting constraint.
Sharpe Ratio Maximizing Portfolio Weightage:

SB Non-US Bonds	MSCI EAFE	CRSP VW US common stocks	US Corp Bonds
0.23056	-	0.12444	0.64500

(b) (iii) Plot of ex-post efficient frontier with all 3 portfolios and all 4 assets:
The in-sample tangency portfolio is: [0.26591555 -0.03508855 0.14397928 0.62519372]
The in-sample global minimum variance portfolio is: [6.08702525e-02 -8.80976812e-04 2.08125460e-03 9.37929470e-01]
The in-sample maximum sharpe ratio portfolio is: [2.30561690e-01 4.93859360e-17 1.24438181e-01 6.45000129e-01]

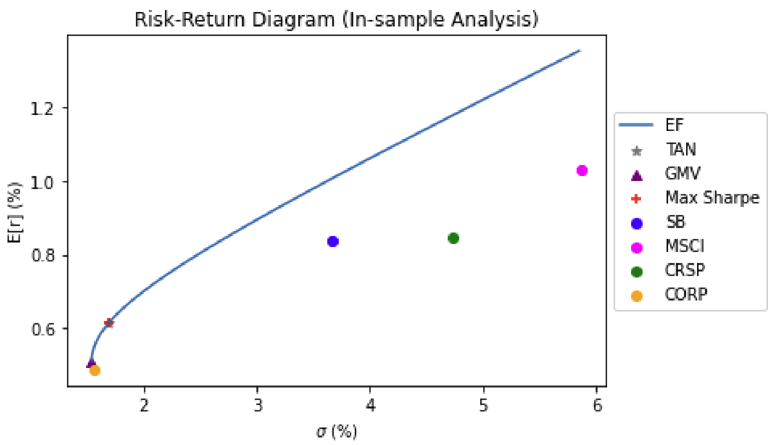

Figure 2. Risk Return Diagram, Q3

Question 4 (Refer to notebook for more detailed computation.)

(a) Variance of Stock 1 & 2:

- Var(Stock 1) = Cov(Stock 1, Stock 1)
- Var(Stock 2) = Cov(Stock 2, Stock 2)

Variance of portfolio, P:

$$w_{Stock\ 1}^{2} var(Stock\ 1) + w_{Stock\ 2}^{2} var(Stock\ 2) + w_{rf}^{2} \times 0 + 2 \times w_{Stock\ 1} w_{Stock\ 2} \times cov(Stock\ 1,\ Stock\ 2) = 0.046$$

(b) Betas of Stock 1 & 2:

- Beta 1: 1.6
- Beta 2: 0.8

Portfolio Beta: $\beta_{Stock\ 1} \times w_{Stock\ 1} + \beta_{Stock\ 2} \times w_{Stock\ 2} = 0.96$

(b) (i) Expected Return of portfolio, P (CAPM): 9.76%

(b) (ii) Solving

$$var(P) = w_{m}^{2} \times var(m) + (1 - w_{m})^{2} \times 0 + w_{m} \times (1 - w_{m}) \times 0$$

$$w_{m} = \sqrt{var(P) / var(m)}$$

Expected Return of new portfolio: $w_{m} \times r_{m} + (1 - w_{m}) \times r_{f} = 10.462\%$

Question 5

(a) (i) Calculating the average monthly excess returns and variance for each portfolio:

	Average Excess Returns	Variance
Portfolio 1	0.493	40.823
Portfolio 2	0.727	27.451
Portfolio 3	0.857	20.997
Portfolio 4	1.22	25.108

(a) (ii) Comparing linear regression slope against covariance definition of Beta:

	Regression Slope	Covariance Beta
Portfolio 1	0.761	0.761
Portfolio 2	0.720	0.720
Portfolio 3	0.778	0.778
Portfolio 4	0.931	0.931

We performed linear regression using the excess returns of each portfolio against the excess returns of the market to obtain the slope of the regression lines for each Portfolio. We then created a covariance matrix (Details in .ipynb) to obtain the covariance beta for each portfolio.

(a) (iii)

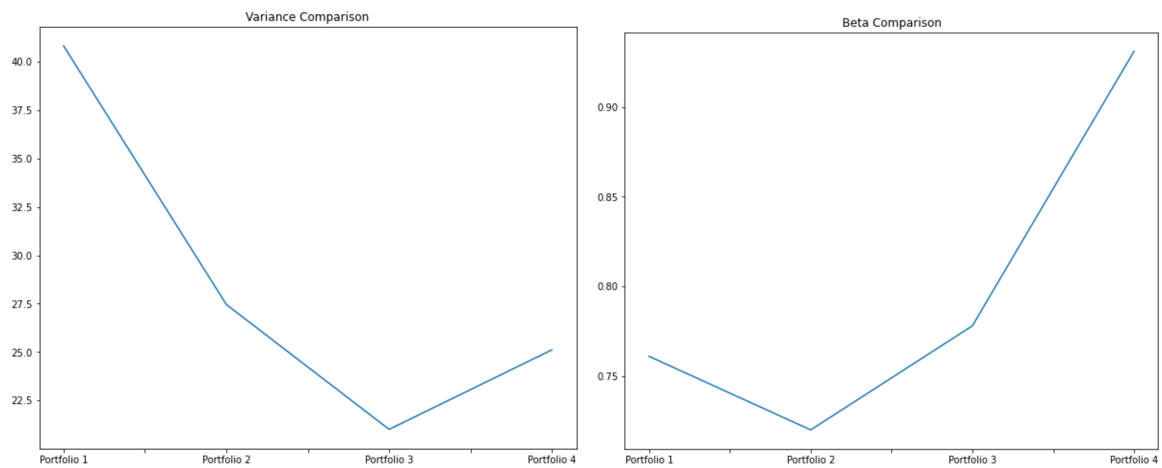

Figure 3. Variance Comparison & Beta Comparison (Based on 4 portfolios)

Portfolios 1 - 4 are arranged in an increasing number of stocks in a manner similar to Diversification. From Figure 3, we see that as the number of stocks in the portfolio increases, the variance decreases. However, in Portfolio 4, as we include all 13 stocks, the variance tends to follow the variance of the market (systematic risk), a risk inherent to the entire market or market segment. Conversely, as the beta of the portfolio is the weighted sum of the individual stock betas, it changes based on each stock added to the portfolio.

(b) (i) Comparing stock beta to linear regression slope and covariance definition of Beta:

	Regression Slope	Covariance Beta	Stock Beta
Portfolio 1	0.761	0.761	0.761
Portfolio 2	0.720	0.720	0.720
Portfolio 3	0.778	0.778	0.778
Portfolio 4	0.931	0.931	0.931

Using the covariance plot, we found the variance of the "World: Market", and using the covariance of each stock divided by the variance of the "World: Market", we obtained the Beta of each stock. Using a weighted average of each stock's Beta, we find that the results are similar for all methods.

(b) (ii) Comparing average excess returns against CAPM returns:

Stock Name	Beta	Returns	CAPM Returns
TOYOTA	0.954	1.248	1.282
GM	0.848	0.556	1.191
BMW	1.032	1.439	1.349
FORD	0.929	1.426	1.260
CHRYSLER	1.113	1.401	1.419
APPLE	0.864	0.415	1.204
IBM	0.674	0.430	1.041
COMPAQ	1.070	3.580	1.382
HP	1.049	1.350	1.364
BUSCH	0.638	1.195	1.010
HEINEKEN	0.768	1.479	1.122
KIRIN	1.460	0.874	1.717
MOLSON	0.708	0.516	1.070

Following the chart above, we see that for a plethora of stocks, the CAPM calculated returns are largely different from that of the excess returns. CAPM returns are more uniform and this does not capture the fat-tails nature of returns.