Assignment - 1

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Abstract—**This document contains the solution to Exercise 3.42 (c) of Oppenheimer.**

Problem 1. A causal and stable LTI system has the difference equation

$$y[n] + \sum_{k=1}^{10} \alpha_k y[n-k] = x[n] + \beta x[n-1] \tag{1}$$

Given $h[n] = (0.9)^n \cos\left(\frac{\pi n}{4}\right)$, sketch the pole-zero plot for the system function of S, and indicate the region of convergence.

Solution: The given equation is a linear constant-coefficient difference equation that can describe systems of upto order 10.
Given

$$h[n] = (0.9)^n \cos\left(\frac{\pi n}{4}\right) \tag{2}$$

can be expressed as

$$h[n] = (0.9)^n \cos\left(\frac{\pi n}{4}\right) u(n) \quad 0 \le n \le 10 \tag{3}$$

We perform a second-order approximation (similar to the Butterworth filter), so the given equation becomes a second-order difference equation.
So the coefficients of the difference equation are given by

$$\alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 0 \tag{4}$$

Applying Z-transform to the difference equation, we get

$$Y(z) + \alpha_2 Y(z-2) + \alpha_1 Y(z-1) = X(z) + \beta X(z-1) \tag{5}$$

$$Y(z) + \alpha_1 z^{-1} Y(z) + \alpha_2 z^{-2} Y(z) = X(z) + \beta z^{-1} X(z) \tag{6}$$

$$Y(z)\left(1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}\right) = X(z)\left(1 + \beta z^{-1}\right) \tag{7}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \beta z^{-1}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}} \tag{8}$$

From the given value of $h[n]$, we get

$$H(z) = \sum_{-\infty}^{\infty} h(n) z^{-n} \tag{9}$$

$$H(z) = \sum_{-\infty}^{\infty} (0.9)^n \cos\left(\frac{\pi n}{4}\right) u(n) z^{-n} \tag{10}$$

$$H(z) = \frac{1 - \left(\frac{0.9}{\sqrt{2}}\right) z^{-1}}{(1 - 0.9e^{\frac{j\pi}{4}} z^{-1})(1 - 0.9e^{\frac{-j\pi}{4}} z^{-1})} \tag{11}$$

The pole-zero plot is represented below and the shaded area represents the region of convergence.