Fourier Series

G V V Sharma*

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Contents

***Abstract*—This manual provides a simple introduction to Fourier Series**

1 Periodic Function

Let

$$x(t) = A_0 \left|\sin\left(2\pi f_0 t\right)\right| \tag{1.1}$$

1.1 Plot $x(t)$.
Solution:
Download the following python code for the plot of $x(t)$

```
wget 1_1.py
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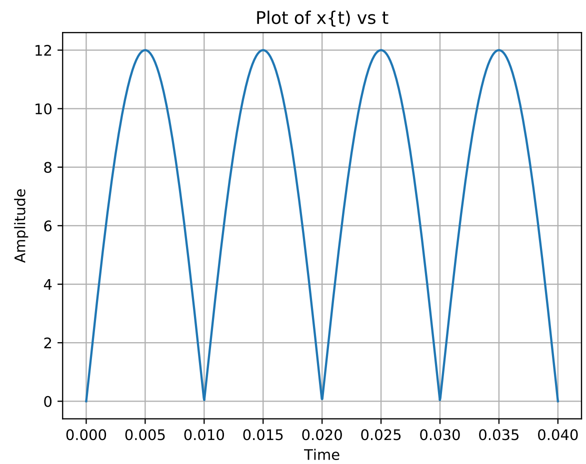

Fig. 1.1

1.2 Show that $x(t)$ is periodic and find its period.
Solution:

$$x(t+T) = A_0 \left|\sin\left(2\pi f_0 (t+T)\right)\right| \tag{1.2}$$
$$= A_0 \left|\sin\left(2\pi f_0 t + 2\pi f_0 T\right)\right| \tag{1.3}$$

If $x(t)$ is periodic, then $x(t) = x(t+T)$

$$x(t) = A_0 \left|sin\left(2\pi f_0 (t+T)\right)\right| \tag{1.4}$$
$$\left|\sin\left(2\pi f_0 t\right)\right| = \left|\sin\left(2\pi f_0 t + 2\pi f_0 T\right)\right| \tag{1.5}$$
$$2\pi f_0 T = n\pi \tag{1.6}$$
$$T = \frac{n}{f_0} \quad n = 1, 2, \dots \tag{1.7}$$

Fundamental period is

$$T = \frac{1}{2f_0} \tag{1.8}$$

2 Fourier Series

Consider $A_0 = 12$ and $f_0 = 50$ for all numerical calculations.

2.1 If

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_0 t} \tag{2.1}$$

show that

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-j2\pi k f_0 t}\, dt \tag{2.2}$$

Solution:

$$x(t)e^{-j2\pi n f_0 t} = \sum_{k=-\infty}^{\infty} c_k e^{-j2\pi (n-k) f_0 t} \tag{2.3}$$
$$\implies \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-j2\pi n f_0 t}\, dt \tag{2.4}$$
$$= \sum_{k=-\infty}^{\infty} c_k \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} e^{-j2\pi (n-k) f_0 t}\, dt \tag{2.5}$$

But

$$\int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} e^{-2\pi j(n-k)f_0 t}\, dt = \begin{cases} \frac{1}{f_0} & k = n \\ 0 & k \neq n \end{cases} \tag{2.6}$$

$$= \frac{1}{f_0}\delta(n-k) \tag{2.7}$$

$$\sum_{k=-\infty}^{\infty} c_k \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} e^{-2\pi j(n-k)f_0 t}\, dt \tag{2.8}$$

$$= \sum_{k=-\infty}^{\infty} \frac{c_k\,\delta(n-k)}{f_0} \tag{2.9}$$

$$= \frac{c_n}{f_0} \tag{2.10}$$

Therefore

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-2\pi k j f_0 t}\, dt \tag{2.11}$$

2.2 Find c_k for (1.1) **Solution:**

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} A_0 \left|\sin\left(2\pi f_0 t\right)\right| e^{-2\pi jkf_0 t}\, dt \tag{2.12}$$

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{0} A_0 \left(-\sin\left(2\pi f_0 t\right)\right) e^{-2\pi jkf_0 t}\, dt + f_0 \int_{0}^{\frac{1}{2f_0}} A_0 \left(\sin\left(2\pi f_0 t\right)\right) e^{-2\pi jkf_0 t}\, dt \tag{2.13}$$

$$c_k = f_0 \int_{0}^{\frac{1}{2f_0}} A_0 \sin\left(2\pi f_0 t\right) e^{2\pi jkf_0 t}\, dt + f_0 \int_{0}^{\frac{1}{2f_0}} A_0 \sin\left(2\pi f_0 t\right) e^{-2\pi jkf_0 t}\, dt \tag{2.14}$$

$$c_k = f_0 \int_{0}^{\frac{1}{2f_0}} A_0 \sin\left(2\pi f_0 t\right) \left(e^{2\pi jkf_0 t} + e^{-2\pi jkf_0 t}\right) \tag{2.15}$$

$$= f_0 A_0 \int_{0}^{\frac{1}{2f_0}} 2\sin\left(2\pi f_0 t\right) \cos\left(2\pi k f_0 t\right)\, dt \tag{2.16}$$

$$= f_0 A_0 \int_{0}^{\frac{1}{2f_0}} \left(\sin\left(2\pi(1+k)f_0 t\right)\right) + \int_{0}^{\frac{1}{2f_0}} \sin\left(2\pi(1-k)f_0 t\right) \tag{2.17}$$

$$= -f_0 A_0 \left[\frac{\cos\left(2\pi(1+k)f_0 t\right)}{2\pi(1+k)f_0} + \frac{\cos\left(2\pi(1-k)f_0 t\right)}{2\pi(1-k)f_0}\right] \tag{2.18}$$

$$= \frac{f_0 A_0}{2\pi f_0}\left[\frac{1-(-1)^{1+k}}{1+k} + \frac{1-(-1)^{1-k}}{1-k}\right] \tag{2.19}$$

$$= \frac{A_0}{2\pi}\left(1+(-1)^k\right)\left[\frac{1}{1+k} + \frac{1}{1-k}\right] \tag{2.20}$$

$$= \left(1+(-1)^k\right)\frac{A_0}{\pi(1-k^2)} \tag{2.21}$$

Therefore

$$c_k = \begin{cases} \frac{2A_0}{\pi(1-k^2)} & k \text{ is even} \\ 0 & k \text{ is odd} \end{cases} \tag{2.22}$$

2.3 Verify (1.1) using python
Solution: Download the following python code for verifying the plot of $x(t)$ using Fourier series.

```
wget 2_3.py
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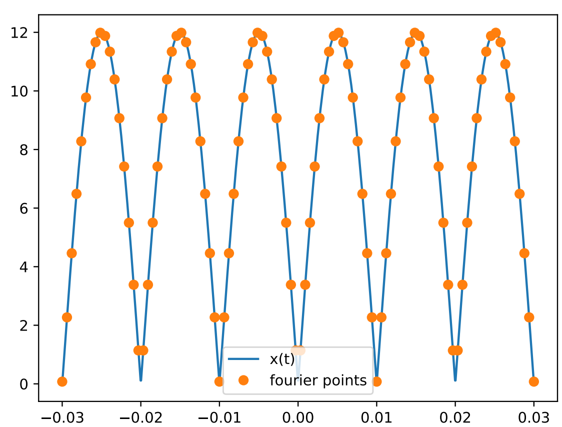


Fig. 2.3

2.4 Show that

$$x(t) = \sum_{k=0}^{\infty} \left(a_k \cos 2\pi jkf_0 t + b_k \sin 2\pi jkf_0 t\right) \tag{2.23}$$

and obtain the formulae for a_k and b_k.
Solution:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi jkf_0t} \tag{2.24}$$

$$x(t) = \sum_{k=-\infty}^{-1} c_k e^{2\pi jkf_0t} + c_0 + \sum_{k=1}^{\infty} c_k e^{2\pi jkf_0t} \tag{2.25}$$

Replacing k with $-k$ in the first summation, we have

$$x(t) = \sum_{k=1}^{\infty} c_{-k} e^{2\pi j-kf_0t} + c_0 + \sum_{k=1}^{\infty} c_k e^{2\pi jkf_0t} \tag{2.26}$$

$$x(t) = c_0 + \sum_{k=1}^{\infty} \left(c_k e^{2\pi jkf_0t} + c_{-k} e^{-2\pi jkf_0t} \right) \tag{2.27}$$

$$x(t) = c_0 + \sum_{k=1}^{\infty} (c_k + c_{-k}) \cos(2\pi jkf_0t) + \mathrm{j} \sum_{k=1}^{\infty} (c_k - c_{-k}) \sin(2\pi jkf_0t) \tag{2.28}$$

The coefficients a_k, b_k can be expressed as follows,

$$a_k = \begin{cases} c_0 & k = 0 \\ c_k + c_{-k} & k > 0 \end{cases} \tag{2.29}$$

$$b_k = \begin{cases} 0 & k = 0 \\ c_k - c_{-k} & k > 0 \end{cases} \tag{2.30}$$

2.5 Find a_k and b_k for (1.1)
Solution: Using (2.22) in the above coefficient expressions, we have

$$c_0 = \frac{2A_0}{\pi} \tag{2.31}$$

$$c_k + c_{-k} = \frac{2A_0}{\pi(1-k^2)} + \frac{2A_0}{\pi(1-(-k)^2)} \tag{2.32}$$

$$a_k = \begin{cases} \dfrac{2A_0}{\pi} & k = 0 \\ \dfrac{4A_0}{\pi(1-k^2)} & k > 0 \end{cases} \tag{2.33}$$

$$b_k = 0 \quad k \geq 0 \tag{2.34}$$

2.6 Verify (2.23) using python.
Solution: Download the following Python code for verifying the plot of Fourier series with coefficients obtained in (2.34).

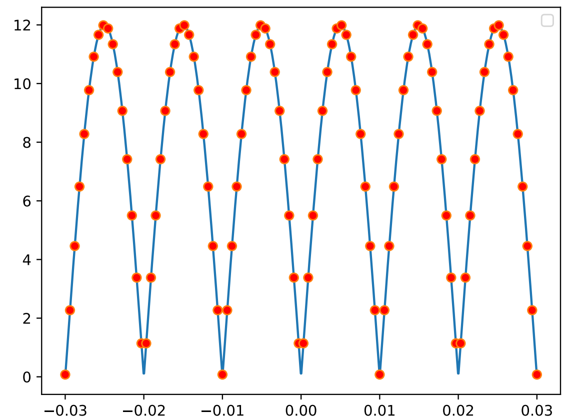


Fig. 2.6: Fourier Series with coefficients

3 Fourier Transform

3.1

$$\delta(t) = 0, \quad t \neq 0 \tag{3.1}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \tag{3.2}$$

3.2 The Fourier Transform of $g(t)$ is

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi jft}\, dt \tag{3.3}$$

3.3 Show that

$$g(t-t_0) \overset{\mathcal{F}}{\longleftrightarrow} G(f) e^{-2\pi jft_0} \tag{3.4}$$

Solution: Applying Fourier transform to $g(t-t_0)$, we obtain

$$\mathcal{F}(g(t-t_0)) = \int_{-\infty}^{\infty} g(t-t_0) e^{-2\pi jft}\, dt \tag{3.5}$$

$$\mathcal{F}(g(t-t_0)) = e^{-2\pi jft_0} \int_{-\infty}^{\infty} g(t-t_0) e^{-2\pi jf(t-t_0)}\, dt \tag{3.6}$$

Using definition of Fourier transform from (3.3), we have

$$\mathcal{F}(g(t-t_0)) = G(f) e^{-2\pi jft_0} \tag{3.7}$$

3.4 Show that

$$G(t) \overset{\mathcal{F}}{\longleftrightarrow} g(-f) \tag{3.8}$$

Solution: From the definition of Inverse Fourier transform, we have

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{2\pi j f t} \mathrm{df} \tag{3.9}$$

In (3.9), substituting $f \to t$ and $t \to -f$,

$$g(-f) = \int_{-\infty}^{\infty} G(t) e^{-2\pi j f t} \mathrm{dt} \tag{3.10}$$

From (3.3), we can conclude that

$$\mathcal{F}(G(t)) = g(-f) \tag{3.11}$$

3.5 $\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} ?$

Solution: Using the following property of δ function,

$$f(x)\delta(x-a) = f(a)\delta(x-a) \tag{3.12}$$

From (3.3), we have

$$\int_{-\infty}^{\infty} \delta(t-0) e^{-2\pi j f t} \mathrm{dt} \tag{3.13}$$

$$= \int_{-\infty}^{\infty} \delta(t) e^{0} \mathrm{dt} = \int_{-\infty}^{\infty} \delta(t) \mathrm{dt} \tag{3.14}$$

So, the Fourier transform of the δ-function is

$$\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} 1 \tag{3.15}$$

3.6 $e^{-j2\pi f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} ?$

Solution: From (3.3), we have

$$\mathcal{F}\left(e^{-j2\pi f_0 t}\right) = \int_{-\infty}^{\infty} e^{-2\pi j f_0 t} e^{-2\pi j f t} \mathrm{dt} \tag{3.16}$$

$$= \int_{-\infty}^{\infty} e^{-2\pi j (f+f_0) t} \mathrm{dt} \tag{3.17}$$

$$1 \overset{\mathcal{F}}{\longleftrightarrow} \delta(-f) \tag{3.18}$$

$$\implies \int_{-\infty}^{\infty} e^{-2\pi j f t} \mathrm{dt} = \delta(-f) \tag{3.19}$$

$$\implies \int_{-\infty}^{\infty} e^{-2\pi j (f+f_0) t} \mathrm{dt} = \delta(-(f+f_0)) \tag{3.20}$$

$$e^{-2\pi j f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} \delta(f+f_0) \tag{3.21}$$

3.7 $\cos(2\pi f_0 t) \overset{\mathcal{F}}{\longleftrightarrow} ?$

Solution: Using the linearity of the Fourier Transform and (3.3),

$$\cos(2\pi f_0 t) = \frac{1}{2}\left(e^{2\pi j f_0 t} + e^{-2\pi j f_0 t}\right) \tag{3.22}$$

$$e^{2\pi j f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} \delta(f+f_0) \tag{3.23}$$

$$e^{-2\pi j f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} \delta(f-f_0) \tag{3.24}$$

$$\cos(2\pi f_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}(\delta(f+f_0) + \delta(f-f_0)) \tag{3.25}$$

3.8 Find the Fourier Transform of $x(t)$ and plot it. Verify using python

Solution:

$$x(t) = A_0 \left|\sin 2\pi f_0 t\right| \tag{3.26}$$

$$\mathcal{F}(x(t)) = \int_{-\infty}^{\infty} A_0 \left|\sin 2\pi f_0 t\right| e^{-2\pi j f t} \mathrm{dt} \tag{3.27}$$

$$= \int_{0}^{\infty} A_0 \sin(2\pi f_0 t) e^{-2\pi j f t} \mathrm{dt} + A \tag{3.28}$$

$$= \frac{A_0}{2j} \int_{0}^{\infty} A_0 \left(e^{2\pi j f_0 t} - e^{-2\pi j f_0 t}\right) e^{-2\pi j f t} \mathrm{dt} + A \tag{3.29}$$

$$= \frac{A_0}{2j} \int_{0}^{\infty} e^{2\pi j (f_0 - f) t} \mathrm{dt} - \int_{0}^{\infty} e^{-2\pi j (f_0 + f) t} \mathrm{dt} + A \tag{3.30}$$

$$= -\frac{A_0}{4j^2\pi(f_0 - f)} - \frac{A_0}{4j^2\pi(f_0 + f)} + A \tag{3.31}$$

$$= \frac{A_0}{2\pi\left({f_0}^2 - f^2\right)} + A \tag{3.32}$$

Calculating the value of A,

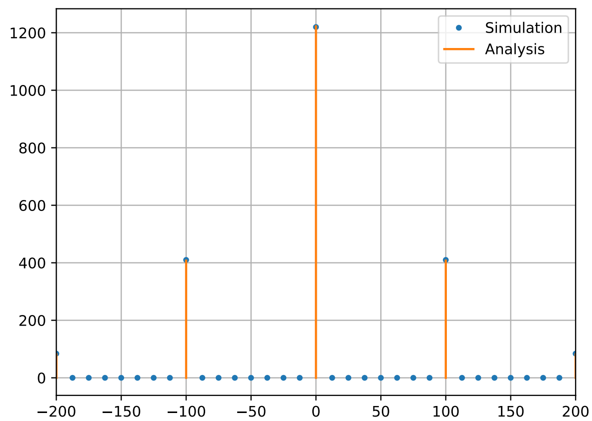
$$A = \int_{-\infty}^{0} -A_0 \sin(2\pi f_0 t) e^{-2\pi j f t} \mathrm{dt} \tag{3.33}$$

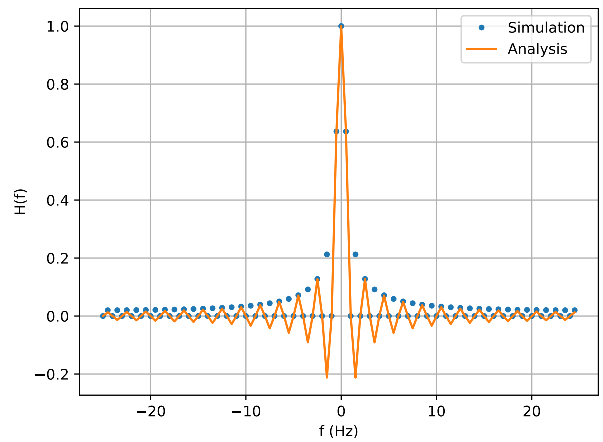
$$A = -\frac{A_0}{2j}\left[\int_{-\infty}^{0} e^{2\pi j (f_0 - f) t} \mathrm{dt} - \int_{-\infty}^{0} e^{-2\pi j (f_0 + f) t} \mathrm{dt}\right] \tag{3.34}$$

$$A = -\frac{A_0}{2j}\left(\frac{1}{2\pi j (f_0 - f)} + \frac{1}{2\pi j (f_0 + f)}\right) \tag{3.35}$$

$$A = \frac{A_0}{2\pi\left(f_0^2 - f^2\right)} \tag{3.36}$$

$$A_0 \left|\sin 2\pi f_0 t\right| \overset{\mathcal{F}}{\longleftrightarrow} \frac{A_0}{\pi\left(f_0^2 - f^2\right)} \tag{3.37}$$





3.9 Show that

$$\mathrm{rect}\,(t) \xleftrightarrow{\mathcal{F}} \mathrm{sinc}\,(t) \tag{3.38}$$

Verify using python.
Solution: solution We write

$$\mathcal{F}\,(\mathrm{rect}\,(t)) = \int_{-\infty}^{\infty} \mathrm{rect}\,(t)\, e^{-2\pi jft}\, dt \tag{3.39}$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-2\pi jft}\, dt \tag{3.40}$$

$$= \frac{e^{\pi jf} - e^{-\pi jf}}{2\pi jf} \tag{3.41}$$

$$= \frac{\sin \pi f}{\pi f} = \mathrm{sinc}\,(f) \tag{3.42}$$

Verifying using plot in Python,

3.10 $\mathrm{sinc}\,(t) \xleftrightarrow{\mathcal{F}} ?$. Verify using python.
Solution: From (3.9), we have

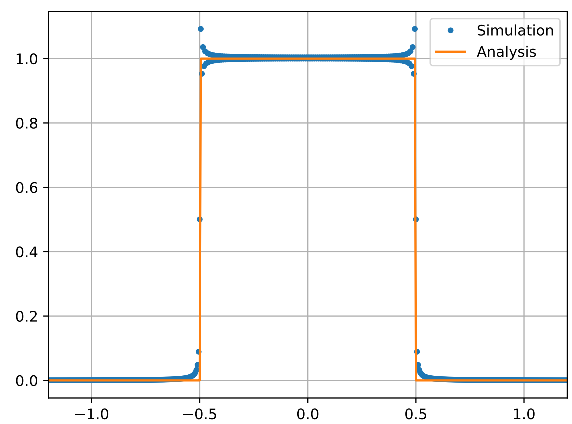
$$\mathrm{sinc}\,(t) \xleftrightarrow{\mathcal{F}} \mathrm{rect}\,(-f) \tag{3.43}$$

Since $\mathrm{rect}\,(f)$ is an even function, we have

$$\mathrm{sinc}\,(t) \xleftrightarrow{\mathcal{F}} \mathrm{rect}\,(f) \tag{3.44}$$

4 Filter

4.1 Find $H(f)$ which transforms $x(t)$ to DC 5V.
Solution: The function $H(f)$ is a low pass filter which filters out even harmonics and leaves the zero frequency component behind.
From the previous Fourier transform, we can see that the $\mathrm{rect}\,(t)$ function represents an ideal



low pass filter. Suppose the cutoff frequency is $f_c = 50$ Hz, then

$$H(f) = \mathrm{rect}\left(\frac{f}{2f_c}\right) = \begin{cases} 1 & |f| < f_c \\ 0 & \text{otherwise} \end{cases} \tag{4.1}$$

Multiplying by a scaling factor to get DC 5V,

$$H(f) = \frac{\pi V_0}{2A_0}\mathrm{rect}\left(\frac{f}{2f_c}\right) \tag{4.2}$$

where $V_0 = 5$ V.

4.2 Find $h(t)$.
Solution: Suppose $g(t) \xleftrightarrow{\mathcal{F}} G(f)$. Then, for some nonzero $a \in \mathbb{R}$, let $u = at$,

$$g(at) \xleftrightarrow{\mathcal{F}} \frac{1}{a}G\left(\frac{f}{a}\right) \tag{4.3}$$

From (4.2), we have

$$\implies h(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{\pi V_0}{2A_0}\text{rect}\left(\frac{f}{2f_c}\right) \tag{4.4}$$

$$h(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{\pi V_0 2f_c}{2A_0}\frac{1}{2f_c}\text{rect}\left(\frac{f}{2f_c}\right) \tag{4.5}$$

$$h(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{2\pi f_c V_0}{A_0}\text{rect}\left(\frac{f}{2f_c}\right) \tag{4.6}$$

$$h(t) = \frac{2\pi V_0}{A_0} f_c \text{sinc}\left((2f_c t)\right) \tag{4.7}$$

4.3 Verify your result using through convolution.
Solution: Fourier transform of $x(t)$ and $h(t)$ respectively is

$$X(f) = \sum_{k=-\infty}^{\infty} \frac{2A_0}{\pi}\frac{\delta\left(f - 2kf_0\right)}{1 - 4k^2} \tag{4.8}$$

$$H(f) = \frac{\pi V_0}{2A_0}\text{rect}\left(\frac{f}{2f_c}\right) \tag{4.9}$$

$$X(f) \times H(f) = \sum_{k=-\infty}^{\infty} V_0 \frac{\delta\left(f - 2kf_0\right)}{1 - 4k^2}\text{rect}\left(\frac{f}{2f_c}\right) \tag{4.10}$$

$$X(f) \times H(f) = \sum_{k=0}^{0} V_0 \frac{\delta\left(f - 2kf_0\right)}{1 - 4k^2} \tag{4.11}$$

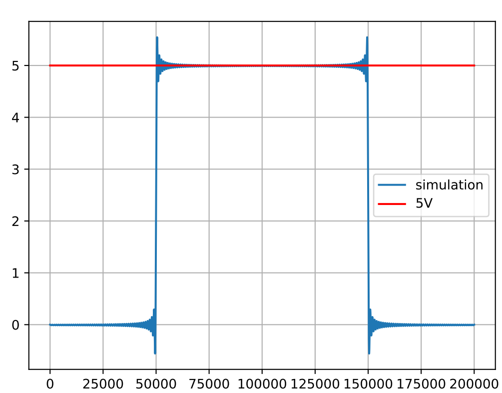
Hence,

$$X(f) \times H(f) = V_0 \frac{\delta\left(f\right)}{1 - 4 \times 0} \tag{4.12}$$

$$X(f) \times H(f) = V_0 \delta\left(f\right) \tag{4.13}$$

$$V_0 \delta\left(t\right) \overset{\mathcal{F}^{-1}}{\longleftrightarrow} V_0 \times 1 \tag{4.14}$$

$$\implies H(t) * x(t) = V_0 \tag{4.15}$$



5 Filter Design

5.1 Design a Butterworth filter for $H(f)$.
Solution: Butterworth filters are designed to have a frequency response as flat as possible in the passband. Here, we design a low-pass filter of order n. As the order of filter increases, excessive ripple is produced in the passband. Generalized form of frequency response for n^{th} order Butterworth low-pass filter is

$$H(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}} \tag{5.1}$$

- n is the order
- w = operating frequency
- w_c = cutoff frequency
- ϵ = maximum passband gain

To find the order of the filter, we consider the following values

- Passband frequency = 50Hz
- Stopband frequency = 100Hz
- Attenuation between -1 dB and -5 dB

$$A_p = 10\log_{10}\left|H_n(f_p)\right|^2 \tag{5.2}$$

$$= -10\log_{10}\left(1 + \left(\frac{f_p}{f_c}\right)^{2n}\right) \tag{5.3}$$

$$A_s = -10\log_{10}\left(1 + \left(\frac{f_s}{f_c}\right)^{2n}\right) \tag{5.4}$$

$$\implies n = \frac{\log\left(\frac{10^{-\frac{A_p}{10}} - 1}{10^{-\frac{A_s}{10}} - 1}\right)}{2\log\left(\frac{f_p}{f_s}\right)} \approx 1.53 \tag{5.5}$$

Hence, we choose a 2^{nd} order Butterworth filter with

$$f_c = \frac{f_p}{\left(10^{-\frac{A_p}{10}} - 1\right)^{\frac{1}{2n}}} \approx 77.74Hz \tag{5.6}$$

5.2 Design a Chebyschev filter for $H(f)$.
Solution: Chebyshev filters are used for distinct frequencies of one band from another. They are carried out by recursion rather than convolution.
Frequency response of Type-1 Chebyshev filter is given by

$$|H_n(f)| = \frac{1}{\sqrt{1 + \epsilon^2 T_n^2\left(\frac{f}{f_c}\right)}} \tag{5.7}$$

- ϵ = ripple factor
- f_c = cutoff frequency
- T_n = Chebyshev polynomial of n^{th} order

Assuming the same parameters as before along with a ripple of $0.1Db$, we get

$$\epsilon = \sqrt{10^{\frac{\delta}{10}} - 1} \approx 0.15 \tag{5.8}$$

Also, assume that $f_c = f_p \implies \frac{f_s}{f_c} > 1$

$$A_s = -10\log_{10}\left(1 + \epsilon^2 T_n^2\left(\frac{f_s}{f_c}\right)\right) \tag{5.9}$$

$$\implies T_n\left(\frac{f_s}{f_c}\right) = \frac{\sqrt{10^{-\frac{A_s}{10}} - 1}}{\epsilon} \tag{5.10}$$

$$\implies \cosh\left(n\cosh^{-1}\left(\frac{f_s}{f_c}\right)\right) = \frac{\sqrt{10^{-\frac{A_s}{10}} - 1}}{\epsilon} \tag{5.11}$$

Thus

$$n = \frac{\cosh^{-1}\left(\frac{\sqrt{10^{-\frac{A_s}{10}}-1}}{\epsilon}\right)}{\cosh^{-1}\left(\frac{f_s}{f_c}\right)} \approx 2.26 \tag{5.12}$$

Hence, we choose a 3rd order Chebyshev filter.

5.3 Design a circuit for your Butterworth filter
Solution: Using the table of normalized Butterworth coefficients, we can see that for a 2nd order Butterworth filter

$$C_1 = 1.4142F \tag{5.13}$$

$$L_2 = 1.4142H \tag{5.14}$$

On denormalizing these values, we get

$$C_1' = \frac{C_1}{2\pi f_c} = 2.89mF \tag{5.15}$$

$$L_2' = \frac{L_2}{2\pi f_c} = 2.89mH \tag{5.16}$$

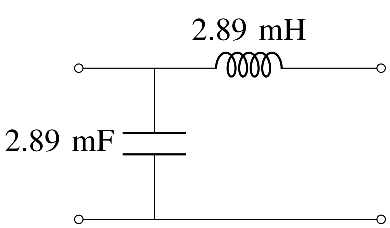


Fig. 5.3: 2nd order Butterworth filter circuit

5.4 Design a circuit for your Chebyschev filter
Solution: Using the table of normalized Chebyshev coefficients, we can see that for a 3rd order Chebyshev filter

$$C_1 = 1.432F \tag{5.17}$$

$$L_2 = 1.5937H \tag{5.18}$$

$$C_3 = 1.4328F \tag{5.19}$$

On denormalizing these values, we get

$$C_1' = \frac{C_1}{2\pi f_c} = 4.56mF \tag{5.20}$$

$$L_2' = \frac{L_2}{2\pi f_c} = 5.07mH \tag{5.21}$$

$$C_3' = \frac{C_3}{2\pi f_c} = 4.56mF \tag{5.22}$$

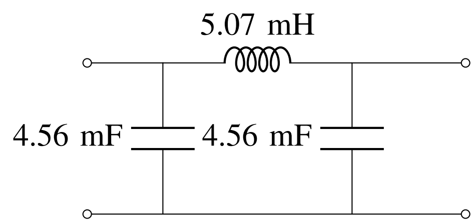


Fig. 5.4: 3rd order Chebyshev filter circuit