Parcial 1: Señales y Sistemas

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Componentes teóricas de solución a mano

A. Distancia media de dos señales periódicas $d^2(x1-x2)^2$ con potencia

a. $d^2(x_1, x_2) = \overline{P_{x_1}} - P_{x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t) - x_2(t)|^2\, dt$: $x_1(t) = Ae^{j\omega_0 t}$, $x_2(t) = Be^{j5\omega_0 t}$

$A, B, T \in \mathbb{R}^+$ $\omega_0 = \frac{2\pi}{T}$

$$= \lim_{T\to\infty} \frac{1}{T}\int_T |Ae^{j\omega_0 t} - Be^{j5\omega_0 t}|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[\underbrace{\int_T (Ae^{j\omega_0 t})^2 dt}_{1} - 2AB\underbrace{\int_T e^{j\omega_0 t}\cdot e^{j5\omega_0 t}\, dt}_{2} + \underbrace{\int_T (Be^{j5\omega_0 t})^2 dt}_{3}\right]$$

$|x(t)|^2 = x(t)\cdot x(t)^*$ → conjugado.

∴ ① $\int_T |\underbrace{Ae^{j\omega_0 t}}_{\text{conjugado}}|^2 dt = A^2\int_T e^{j\omega_0 t}\cdot e^{-j\omega_0 t}\, dt$

$$= A^2\int_T e^0\, dt$$
$$= A^2\, e\Big|_0^T$$
$$= A^2T - \cancel{A^2 0}$$

$$\boxed{\int_T |Ae^{j\omega_0 t}|^2 dt = A^2 T}$$

② $\int_T (Be^{j5\omega_0 t})^2 dt = B^2\int e^{j5\omega_0 t}\cdot e^{-j5\omega_0 t}\, dt$

$$= B^2\int e^0\, dt$$
$$= B^2\, t\Big|_0^T$$
$$= B^2T - \cancel{B^2 0}^{\,0}$$

$$\boxed{\int_T (Be^{j5\omega_0 t})^2 dt = B^2 T}$$

③ $\int_T e^{j\omega_0 t}\cdot e^{j5\omega_0 t}\, dt = \int_T e^{j6\omega_0 t}\, dt$

$$= \int_T \cos(6\omega_0 t) + j\,sen(6\omega_0 t)\, dt$$

$$= \int_T \cos\left(\frac{12\pi}{T}t\right) + j\,sen\left(\frac{12\pi}{T}t\right)\cdot dt$$

$$= \frac{T}{12\pi}\left[\cancel{sen\left(\frac{12\pi}{\cancel{T}}\cdot \cancel{T}\right)}^{\,0} - \cancel{sen\left(\frac{12\pi}{T}\cdot 0\right)}^{\,0}\right] + \frac{T}{12}\left[-\left(\underbrace{\cos\left(\frac{12\pi}{\cancel{T}}\cdot\cancel{T}\right)}_{1} - \underbrace{\cos\left(\frac{12\pi}{T}\cdot 0\right)}_{1}\right)\right]$$

$$= \frac{T}{12\pi}[0 - 0] - \frac{T}{12\pi}\left[\cancel{1-1}\right]^{0}$$

$$\boxed{\int_T e^{j\omega_0 t}\cdot e^{j5\omega_0 t}\, dt = 0}$$

∴ $d^2(x_1, x_2) = \lim_{T\to\infty} \frac{1}{T}\left(A^2T - \cancel{2AB\cdot 0}^{\,0} + B^2T\right)$

$$= \lim_{T\to\infty} A^2 + B^2$$

$\boxed{d^2(x_1, x_2) = A^2 + B^2}$ → distancia de las dos señales.

media.

B. Señal obtenida en el tiempo discreto utilizando un conversor análogo digital a una frecuencia de muestreo $f_s = 5kHz$

b. $x(t) = 3\cos(\underbrace{1000\pi}_{\omega_1} t) + 5\,sen(\underbrace{2000\pi}_{\omega_2} t) + 10\cos(\underbrace{11000\pi}_{\omega_3} t)$: $F_s = 5KHz$

→ buscamos el periodo para cada señal

$\therefore T_1 = \frac{2\pi}{\omega_1} = \frac{\cancel{2\pi}}{1000\cancel{\pi}} = \frac{1}{500}\,[s]$

$T_2 = \frac{2\pi}{\omega_2} = \frac{\cancel{2\pi}}{2000\cancel{\pi}} = \frac{1}{1000}\,[s]$

$T_3 = \frac{2\pi}{\omega_3} = \frac{\cancel{2\pi}}{11000\cancel{\pi}} = \frac{1}{5500}\,[s]$

Nota: se puede observar que $x(t)$ es cuasiperiodica ∵ la division de sus frecuencias $\in \mathbb{Q}$

→ Ahora buscamos T con T_1, T_2 y T_3

$T = K T_1 = \ell T_2 = \gamma T_3$

$T = K\frac{1}{500} = \ell\frac{1}{1000} = \gamma\frac{1}{5500}$

$11000T = \underbrace{22K = 11\ell = 2\gamma}$

500	1000	5500	10
50	100	550	10
5	10	55	5
1	2	11	2
	1	11	11
		1	

$10^2 \times 11 \times 5 \times 2 = \underline{\underline{11000}}$

(M.C.M) ⇒

22	11	2	2
11	11	1	11
1	1		

$2 \times 11 = 22$

Igualamos a T

$11000T = 22$

$T = \frac{22}{11000} \rightarrow \boxed{T = \frac{1}{500}}$

$\therefore$ $T = K T_1$; $T = \ell T_1$; $T = \gamma T_3$

$k = \frac{1/500}{1/500}$; $\ell = \frac{1/500}{1/1000}$; $\gamma = \frac{1/500}{1/5500}$

$\boxed{K = 1}$ $\boxed{\ell = 2}$ $\boxed{\gamma = 11}$

→ Miremos si fs de 5KHz cumple con Nyquist

$fs \geqslant 2F_{Max}$: $\omega = \frac{2\pi}{T} = 2\pi f \therefore f = \frac{\omega}{2\pi}$

$fs \geqslant 2F(f_1, f_2, f_3)$

$fs \geqslant 2(5500)$

$fs \geqslant 11000\ Hz$

$f_1 = \frac{1000\pi}{2\pi} = 500\,[Hz]$

$f_2 = \frac{2000\pi}{2\pi} = 1000\,[Hz]$

$f_3 = \frac{11000\pi}{2\pi} = 5500\,[Hz]$

$\therefore$ 5KHz no cumple con Nyquist ⟶ Posible frecuencia a utilisar $fs = 20f_3$

$\boxed{fs = 110KHz}$

cumple Nyquist

→ Ahora discretizamos la señal para 5KHz

$$x(t) = 3\cos(1000\pi t) + 5\,sen(2000\pi t) + 10\cos(11000\pi t)$$

$$x\left(t \to \frac{n}{Fs}\right) = 3\cos\left(1000\pi \frac{n}{Fs}\right) + 5\,sen\left(2000\pi \frac{n}{Ts}\right) + 10\cos\left(11000\pi \frac{n}{Ts}\right)$$

$$x[n] = 3\cos\left(\frac{1000\pi n}{5000}\right) + 5\,sen\left(\frac{2000\pi n}{5000}\right) + 10\cos\left(\frac{11000\pi n}{5000}\right)$$

$$x[n] = 3\cos\left(\underbrace{\frac{\pi}{5}}_{\Omega_1} n\right) + 5\,sen\left(\underbrace{\frac{2\pi}{5}}_{\Omega_2} n\right) + 10\cos\left(\underbrace{\frac{11\pi}{5}}_{\Omega_3} n\right)$$

buscamos copias

$$\underbrace{-\pi < \Omega_1 < \pi}_{cumple} \;;\; \underbrace{-\pi < \Omega_2 < \pi}_{cumple} \;;\; \underbrace{-\pi < \frac{11\pi}{5} < \pi}_{}$$

No cumple ∴ $\hat{\Omega}_3 = \frac{11\pi}{5} \overbrace{-2\pi}^{\text{quitamos una vuelta}}$

$$\hat{\Omega}_3 = \frac{\pi}{5} \;//$$

$$\therefore\; x[n] = 3\cos\left(\frac{\pi}{5} n\right) + 5\,sen\left(\frac{2\pi}{5}\right) + 10\cos\left(\frac{\pi}{5} n\right)$$

$$\boxed{x[n] = 13\cos\left(\frac{\pi}{5} n\right) + 5\,sen\left(\frac{2\pi}{5}\right)}$$

C. Encontrar T para discretisar

C. $x(t) = 20\cos\left(\underbrace{\tfrac{t}{3}}_{w_1}\right) + \cos\left(\underbrace{\tfrac{t}{4}}_{w_2}\right)$

→ comprobamos es cuasiperiodica

$\frac{w_1}{w_2} = \frac{1/3}{1/4} = \frac{4}{3} \in \mathbb{Q}$ → ∴ si es cuasiperiodica

→ buscamos T

$T_1 = \frac{2\pi}{w_1} = \frac{2\pi}{1/3} = 6\pi$

$T_2 = \frac{2\pi}{w_2} = \frac{2\pi}{1/4} = 8\pi$

∴ $T = \ell T_1 = k T_2$

$T = \ell 6\pi = k 8\pi$

$\frac{T}{\pi} = \ell 6 = k 8$

M.C.M $(6,8) = 24$

$\frac{T}{\pi} = 24$ → $\boxed{T = 24\pi}$

∴ $\ell = \frac{T}{T_1}$; $k = \frac{T}{T_2}$

$\ell = \frac{24\pi}{1/3\,\pi}$ $\qquad k = \frac{24\pi}{1/4}$

$\boxed{\ell = 72}$ $\qquad \boxed{k = 96}$