Package 'GeoModels'

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Title A Package for Geostatistical Gaussian and non Gaussian Data Analysis
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Depends R (>= $2.12.0$)
Description This package provides a set of procedures for a) simulation and estimation of some spatial and spatio-temporal random fields using standard likelihood and a likelihood approximation method called composite likelihood and b) prediction using best linear unbiased prediction. Spatio (temporal) bivariate data estimation involves estimation of both regression and covariance parameters. Gaussian and some non Gaussian Random fields can be analyzed using the GeoModels package. Among them, Weibull, logGaussian, skewGaussian, T, binomial, negative binomial and circular random fields can be analyzed.
Imports methods,
Suggests actuar, spam, pracma, scatterplot3d, dfoptim, dotCall64, fields, mapproj, gpuR, limma, gsl, optimParallel, parallel, pbivnorm, GoFKernel, zipfR, plot3D, sn, shape, numDeriv, hypergeo, sphereplot, ucminf, VGAM License GPL (>= 2)
<pre>URL https://vmoprojs.github.io/GeoModels-page/</pre>
Repository GitHub
Encoding UTF-8
<pre>BugReports https://github.com/vmoprojs/GeoModels/issues</pre>
R topics documented:
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2 anomalies

anoma	alies Annual precipitation anomalies in U.S.
ndex	100
	WlsStart
	winds.coords
	winds
	StartParam
	Prscores
	NuisParam
	MatSqrt, MatInv, MatLogDet
	MatDecomp
	Lik
	GeoWLS
	GeoVariogram
	GeoVarestbootstrap
	GeoTests
	GeoSim
	GeoScatterplot
	GeoResiduals
	GeoQQ
	GeoNeighborhood
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	GeoKrig
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	DeviceInfo
	CorrParam
	CorrelationPar
	CompLik2
	CompLik
	CkVarType
	CkType
	CkModel
	CkLikelihood
	CkInput
	CkCorrModel

Description

A (7252x3)-matrix containing lon/lat and yearly total precipitation anomalies registered at 7.352 location sites in USA. For more details see http://www.image.ucar.edu/Data/precip_tapering/.

Usage

data(anomalies)

austemp 3

Format

A numerical matrix of dimension 7252x3.

Source

Kaufman, C.G., Schervish, M.J., Nychka, D.W. (2008) Covariance tapering for likelihood-based estimation in large spatial data sets. *Journal of the American Statistical Association, Theory & Methods*, **103**, 1545–1555.

austemp

Maximum australian temperature

Description

A matrix containing maximum temperature in Australia in July 2011.

Usage

data(austemp)

Format

A (446×4) -matrix containing longitude, latitude, maximum temperature, and the 'so called' geometric temperature covariate.

Source

Bevilacqua M., Caamaño C., Morales-Oñate V., Arellano-Valle R. B. (2020) Non-Gaussian Geostatistical Modeling using (skew) t Processes, *Scandinavian Journal of Statistics*.

CheckBiv

Checking Bivariate covariance models

Description

The procedure control if the correlation model is bivariate.

Usage

CheckBiv(numbermodel)

Arguments

number model

numeric; the number associated to a given correlation model.

Value

Returns TRUE or FALSE depending if the correlation model is bivariate or not.

4 CheckDistance

Author(s)

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Examples

```
library(GeoModels)
CheckBiv(CkCorrModel("Bi_matern_sep"))
```

CheckDistance

Checking Distance

Description

The procedure controls the type of distance.

Usage

CheckDistance(distance)

Arguments

distance

String; the type of distance, for the description see GeoCovmatrix. Default is Eucl. Other possible values are Geod and Chor that is euclidean, geodesic and chordal distance.

Value

Returns 0,1,2 for euclidean, geodesic, chordal distances respectively. Otherwise returns NULL.

Author(s)

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See Also

GeoFit

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CheckSph

Checking if a covariance is valid only on sphere

Description

Subroutine called by InitParam. The procedure controls if a covariance model is valid only on the sphere.

Usage

CheckSph(numbermodel)

Arguments

numbermodel

Numeric; the code number for the covariance model.

Value

Returns TRUE or FALSE

Author(s)

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CheckST

Checking SpaceTime covariance models

Description

The procedure control if the correlation model is spacetime.

Usage

CheckST(numbermodel)

Arguments

numbermodel

numeric; the number associated to a given correlation model.

Value

Returns TRUE or FALSE depending if the correlation model is spatiotemporal or not.

Author(s)

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Examples

```
library(GeoModels)
CheckST(CkCorrModel("gneiting"))
```

CkCorrModel

Checking Correlation Model

Description

The procedure controls if the correlation model inserted is correct.

Usage

```
CkCorrModel(corrmodel)
```

Arguments

corrmodel

String; the name of a correlation model, for the description see GeoCovmatrix.

Value

Return a number associated to a given correlation model if the model is considered in the package. Otherwise return NULL.

Author(s)

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CkInput

Checking Input

Description

Subroutine called by the fitting procedures. The procedure controls the the validity of the input inserted by the users.

Usage

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Arguments

Χ

A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions coordx of coordinates or a numeric vector assigning 1-dimension of coordinates. A numeric vector assigning 1-dimension of coordinates; coordy is interpreted coordy only if coordx is a numeric vector otherwise it will be ignored. coordt A numeric vector assigning 1-dimension of temporal coordinates. String; the name of a correlation model, for the description see GeoFit. corrmodel A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordx_dyn coordinates. Optional argument, the default is NULL data A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations. distance String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details. fcall String; Fitting to call the fitting procedure and simulation to call the simula-A named list giving the values of the parameters that will be considered as fixed known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored. grid Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered. likelihood String; the configuration of the composite likelihood. Marginal is the default. Numeric; an optional positive value indicating the maximum spatial distance maxdist considered in the composite-likelihood computation. maxtime Numeric; an optional positive value indicating the maximum temporal lag separation in the composite-likelihood. radius Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth. String; the density associated to the likelihood objects. Gaussian is the default. model Numeric; the number of trials in a binomial random fields. Default is 1. String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the optimizer default. A numeric vector of parameters, needed only in simulation. See GeoSim. param start A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default. String; the name of the tapered correlation function. taper Numeric; an optional value indicating the separabe parameter in the space time tapsep quasi taper (see Details). String; the type of the likelihood objects. If Pairwise (the default) then the type marginal composite likelihood is formed by pairwise marginal likelihoods. varest Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default. String; the type of estimation method for computing the estimate variances, see vartype GeoFit. weighted Logical; if TRUE the likelihood objects are weighted. If FALSE (the default) the composite likelihood is not weighted.

Numeric; Matrix of space-time covariates in the linear mean specification.

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See Also

GeoFit

CkLikelihood

Checking Composite-likelihood Type

Description

Subroutine called by InitParam. The procedure controls the type of the composite-likelihood inserted by the users.

Usage

```
CkLikelihood(likelihood)
```

Arguments

likelihood

String; the configuration of the composite likelihood. Marginal is the default.

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See Also

GeoFit

CkModel

Checking Random Field type

Description

Subroutine called by InitParam. The procedure controls the type of random field inserted by the users.

Usage

```
CkModel(model)
```

Arguments

model

String; the density associated to the likelihood objects. Gaussian is the default.

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Author(s)

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See Also

GeoFit

CkType

Checking Likelihood Objects

Description

Subroutine called by InitParam. The procedure controls the type of likelihood objects inserted by

Usage

CkType(type)

Arguments

type

String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.

Author(s)

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See Also

GeoFit

CkVarType

Checking Variance Estimates Type

Description

Subroutine called by InitParam. The procedure controls the method used to compute the estimates' variances.

Usage

CkVarType(type)

Arguments

type

String; the method used to compute the estimates' variances. If SubSamp the estimates' variances are computed by the sub-sampling method, see GeoFit.

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See Also

GeoFit

CompLik	Optimizes the Composite log-likelihood	

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

Usage

Logical; if TRUE then the data come from a bivariate random field. Otherwise

Arguments

bivariate

	from a univariate random field.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details .
flagcorr	A numeric vector of binary values denoting which parameters of the correlation function will be estimated.
flagnuis	A numeric vector of binary values denoting which nuisance parameters will be estimated.
fixed	A numeric vector of parameters that will be considered as known values.
GPU	Numeric; if NULL (the default) no GPU computation is performed.

grid Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -

matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.

likelihood String; the configuration of the compositelikelihood, see GeoFit.

local Numeric; number of local work-items of the GPU

lower An optional named list giving the values for the lower bound of the space pa-

rameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of

the list must be the same of the names in the start list.

model Numeric; the id value of the density associated to the likelihood objects.

Numeric; number of trials in a binomial random fields.

String; the names of the correlation parameters. namescorr namesnuis String; the names of the nuisance parameters.

namesparam String; the names of the parameters to be maximised. numparam Numeric; the number of parameters to be maximised. numparamcorr Numeric; the number of correlation parameters.

optimizer String; the optimization algorithm (see optim for details). Nelder-Mead is the

> default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.

onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is

computed without optimizing. FALSE is the default.

Logical; if TRUE optmization is performed using optimParallel using the maxiparallel

mum number of cores, when optimizer is L-BFGS-B.FALSE is the default.

param A numeric vector of parameters' values.

spacetime Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field. type

String; the type of the likelihood objects. If Pairwise (the default) then the

marginal composite likelihood is formed by pairwise marginal likelihoods.

An optional named list giving the values for the upper bound of the space paupper

rameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names

of the list must be the same of the names in the start list.

Logical; if TRUE the estimate' variances and standard errors are returned. FALSE varest

is the default.

String; the type of estimation method for computing the estimate variances, see vartype

GeoFit.

weigthed Logical; if TRUE then decreasing weights coming from a compactly supported

correlation function with compact support maxdist (maxtime) are used.

winconst Numeric; a positive value for computing the spatial sub-window in the sub-

sampling procedure.

Numeric; a value in (0,1] for defining the proportion of overlapping in the winstp

spatial sub-sampling procedure.

Numeric; a positive value for computing the temporal sub-window in the subwinconst_t

sampling procedure.

Numeric; a value in (0,1] for defining the proportion of overlapping in the winstp_t

temporal sub-sampling procedure.

Numeric; Number of (dynamical) temporal instants. ns

Numeric; Matrix of space-time covariates in the linear mean specification. Χ

Logical; if TRUE then the sensitivy matrix is computed sensitivity

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See Also

GeoFit

CompLik2	Optimizes the Composite log-likelihood
CompLik2	Optimizes the Composite log-likelihood

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

Usage

```
CompLik2(bivariate, coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance, flagcorr, flagnu fixed, GPU,grid,likelihood, local,lower, model, n, namescorr, namesnuis, namesparam, numparam, numparamcorr, optimizer, onlyvar, parallel, param, spacetime, type, upper, varest, vartype, weigthed, winconst, winstp,winconst_t, winstp_t, ns, X,sensitivity colidx,rowidx)
```

Arguments

Logical; if TRUE then the data come from a bivariate random field. Otherwise from a univariate random field.
A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
Numeric; the id of the correlation model.
A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details .
A numeric vector of binary values denoting which parameters of the correlation function will be estimated.
A numeric vector of binary values denoting which nuisance parameters will be estimated.
A numeric vector of parameters that will be considered as known values.
Numeric; if NULL (the default) no GPU computation is performed.

grid Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -

matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.

likelihood String; the configuration of the compositelikelihood, see GeoFit.

local Numeric; number of local work-items of the GPU

lower An optional named list giving the values for the lower bound of the space pa-

rameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of

the list must be the same of the names in the start list.

model Numeric; the id value of the density associated to the likelihood objects.

n Numeric; number of trials in a binomial random fields.

namescorr String; the names of the correlation parameters.

String; the names of the nuisance parameters.

namesparam String; the names of the parameters to be maximised.

Numeric; the number of parameters to be maximised.

Numeric; the number of correlation parameters.

optimizer String; the optimization algorithm (see optim for details). Nelder-Mead is the

default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the

case of one-dimensional optimization, the function optimize is used.

onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is

computed without optimizing. FALSE is the default.

parallel Logical; if TRUE optmization is performed using optimParallel using the maxi-

mum number of cores, when optimizer is L-BFGS-B.FALSE is the default.

param A numeric vector of parameters' values.

spacetime Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field.

type String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.

An optional named list giving the values for the upper bound of the space pa-

rameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names

of the list must be the same of the names in the start list.

variest Logical; if TRUE the estimate' variances and standard errors are returned. FALSE

is the default.

vartype String; the type of estimation method for computing the estimate variances, see

GeoFit.

upper

weigthed Logical; if TRUE then decreasing weigths coming from a compactly supported

correlation function with compact support maxdist (maxtime) are used.

winconst Numeric; a positive value for computing the spatial sub-window in the sub-

sampling procedure.

winstp Numeric; a value in (0,1] for defining the proportion of overlapping in the

spatial sub-sampling procedure.

winconst_t Numeric; a positive value for computing the temporal sub-window in the sub-

sampling procedure.

winstp_t Numeric; a value in (0,1] for defining the proportion of overlapping in the

temporal sub-sampling procedure.

ns Numeric; Number of (dynamical) temporal instants.

X Numeric; Matrix of space-time covariates in the linear mean specification.

sensitivity Logical; if TRUE then the sensitivy matrix is computed colidx Numeric; Vector of indexes for spatial distances.

Numeric; Vector of indexes for spatial distances.

14 CorrParam

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See Also

GeoFit

CorrelationPar

Lists the Parameters of a Correlation Model

Description

Subroutine called by InitParam and other procedures. The procedure returns a list with the parameters of a given correlation model.

Usage

CorrelationPar(corrmodel)

Arguments

corrmodel

Integer; an integer associated to a given correlation model.

Author(s)

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See Also

GeoFit

CorrParam

Lists the Parameters of a Correlation Model

Description

The procedure returns a list with the parameters of a given correlation model.

Usage

CorrParam(corrmodel)

Arguments

corrmodel

String; the name of a correlation model.

CorrParam 15

Author(s)

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See Also

GeoCovmatrix

Examples

```
require(GeoModels)
### Example 1. Parameters of the Matern model
###
CorrParam("Matern")
### Example 2. Parameters of the Generalized Wendland model
CorrParam("GenWend")
###
### Example 3. Parameters of the Generalized Wendland model
CorrParam("GenCauchy")
###
### Example 4. Parameters of the space time Gneiting model
CorrParam("Gneiting")
### Example 5. Parameters of the bi-Matern separable model
CorrParam("Bi_Matern_sep")
```

DeviceInfo

Prints Device Information

Description

Prints the device details available in your computer. Device name, Max compute units, whether it supports double precision, among others.

Usage

DeviceInfo()

Details

The user can take this information into account so that the local parameter is set up in GeoFit when GPU computation is chosen.

Author(s)

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Examples

library(GeoModels)
DeviceInfo()

GeoCovariogram

Computes fitted covariance and/or variogram

Description

The procedure computes and plots covariance or variogram estimated fitting a Gaussian, and non Gaussian spatio (temporal) bivariate random fields. Allows to add the empirical estimates in order to compare them with the fitted model.

Usage

Arguments

fitted	A fitted object obtained from the GeoFit or GeoWLS procedures.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
answer.cov	Logical; if TRUE a vector with the estimated covariance function is returned; if FALSE (the default) the covariance is not returned.
answer.vario	Logical; if TRUE a vector with the estimated variogram is returned; if FALSE (the default) the variogram is not returned.
answer.range	Logical; if TRUE the estimated pratical range is returned; if FALSE (the default) the pratical range is not returned.
fix.lags	Integer; a positive value denoting the spatial lag to consider for the plot of the temporal profile.
fix.lagt	Integer; a positive value denoting the temporal lag to consider for the plot of the spatial profile.
show.cov	Logical; if TRUE the estimated covariance function is plotted; if FALSE (the default) the covariance function is not plotted.
show.vario	Logical; if TRUE the estimated variogram is plotted; if FALSE (the default) the variogram is not plotted.
show.range	Logical; if TRUE the estimated pratical range is added on the plot; if FALSE (the default) the pratical range is not added.
add.cov	Logical; if TRUE the vector of the estimated covariance function is added on the current plot; if FALSE (the default) the covariance is not added.
add.vario	Logical; if TRUE the vector with the estimated variogram is added on the current plot; if FALSE (the default) the correlation is not added.
pract.range	Numeric; the percent of the sill to be reached.
vario	A Variogram object obtained from the GeoVariogram procedure.
	other optional parameters which are passed to plot functions.

Value

The returned object is eventually a list with:

covariance The vector of the estimated covariance function; variogram The vector of the estimated variogram function;

Author(s)

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References

Cressie, N. A. C. (1993) *Statistics for Spatial Data*. New York: Wiley. Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. *Spring Verlang, New York*.

See Also

GeoFit.

Examples

```
library(GeoModels)
library(scatterplot3d)
### Example 1. Plot of fitted covariance and fitted
### and empirical variograms from a Gaussian RF
### with Matern correlation.
set.seed(21)
# Set the coordinates of the points:
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)
# Set the model's parameters:
corrmodel = "Matern"
model = "Gaussian"
mean = 0
sill = 1
nugget = 0
scale = 0.2/3
smooth=0.5
param=list(mean=mean,sill=sill, nugget=nugget, scale=scale, smooth=smooth)
# Simulation of the Gaussian random field:
\verb| data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model, param=param)$ \\ | data = GeoSim(coordx=coords, corrmodel=coords, corrmodel, model=model, param=param)$ \\ | data = GeoSim(coordx=coords, corrmodel=coords, corrmodel, cor
start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget,smooth=smooth)
# Maximum composite-likelihood fitting of the Gaussian random field:
fit = GeoFit(data=data,coordx=coords, corrmodel=corrmodel,model=model,
                         likelihood="Marginal",type='Pairwise',start=start,
                        optimizer="BFGS", fixed=fixed,maxdist=0.05)
# Empirical estimation of the variogram:
vario = GeoVariogram(data=data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE, show.vario=TRUE, vario=vario, pch=20)
### Example 2. Plot of fitted covariance and fitted
### and empirical variograms from a Binomial
### RF with exponential correlation.
set.seed(2111)
model="Binomial"; n=20
# Set the coordinates of the points:
x = runif(500, 0, 1)
y = runif(500, 0, 1)
```

```
coords=cbind(x,y)
# Set the model's parameters:
corrmodel = "exponential"
mean = 0
sill = 1
nugget = 0
scale = 0.2/3
param=list(mean=mean, sill=sill, nugget=nugget, scale=scale)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param,n=n)$data
start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget)
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
            likelihood="Marginal",type='Pairwise',start=start,n=n,
            optimizer="BFGS", fixed=fixed,maxdist=0.03)
# Empirical estimation of the variogram:
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE, show.vario=TRUE, vario=vario, pch=20)
### Example 3. Plot of fitted covariance and fitted
### and empirical variograms from a RF
### RF with Wend0 correlation.
###
set.seed(211)
model="Gamma"; shape=4
# Set the coordinates of the points:
x = runif(700, 0, 1)
y = runif(700, 0, 1)
coords=cbind(x,y)
# Set the model's parameters:
corrmodel = "Wend0"
mean = 0
sill = 1
nugget = 0
scale = 0.3
power2=4
param=list(mean=mean,sill=sill, nugget=nugget, scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data
start=list(mean=0,scale=scale,shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
```

```
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
           likelihood="Marginal",type='Pairwise',start=start,
           optimizer="BFGS", fixed=fixed,maxdist=0.03)
# Empirical estimation of the variogram:
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE, show.vario=TRUE, vario=vario,pch=20)
###
### Example 4. Plot of fitted and empirical variograms
### from a space time Gaussian random fields
### with double exponential correlation.
set.seed(92)
# Define the spatial-coordinates of the points:
x = runif(50, 0, 1)
y = runif(50, 0, 1)
coords=cbind(x,y)
# Define the temporal sequence:
time = seq(0, 15, 1)
# Simulation of the spatio-temporal Gaussian random field:
data = GeoSim(coordx=coords, coordt=time, corrmodel="Exp_Exp",param=list(mean=mean,
            nugget=nugget,scale_s=0.5/3,scale_t=2/2,sill=sill))$data
fixed=list(nugget=0, mean=0)
start=list(scale_s=0.2, scale_t=0.5, sill=1)
# Maximum composite-likelihood fitting of the space-time Gaussian random field:
fit = GeoFit(data, coordx=coords, coordt=time, corrmodel="Exp_Exp", maxtime=2,
           maxdist=0.1, likelihood="Marginal", type="Pairwise",
           optimizer="BFGS",fixed=fixed, start=start)
# Empirical estimation of spatio-temporal covariance:
vario = GeoVariogram(data,coordx=coords, coordt=time, maxtime=5,maxdist=0.5)
# Plot of the fitted space-time variogram
GeoCovariogram(fit,vario=vario,show.vario=TRUE)
# Plot of covariance, variogram and spatio and temporal profiles:
GeoCovariogram(fit,vario=vario,fix.lagt=1,fix.lags=1,show.vario=TRUE,pch=20)
### Example 5. Plot of parametric and empirical variograms
### estimated from a Bivariate Gaussian random fields with
### Matern correlation.
```

```
# Simulation of a bivariate spatial Gaussian random field:
set.seed(892)
# Define the spatial-coordinates of the points:
x = runif(200, -1, 1)
y = runif(200, -1, 1)
coords=cbind(x,y)
# Simulation of a bivariate Gaussian Random field
# with matern (cross) covariance function
scale_1 = 0.25/3
scale_2 = 0.2/3
scale_{12} = 0.15/3
sill_1=1
sill_2=1
smooth=0.5
pco1=0.3
param=list(mean_1=0,mean_2=0,scale_1=scale_1,scale_2=scale_2,scale_12=scale_12,
           sill_1=sill_1,sill_2=sill_2,nugget_1=0,nugget_2=0,
           smooth_1=smooth, smooth_12=smooth, smooth_2=smooth, pcol=pcol)
data = GeoSim(coordx=coords, corrmodel="Bi_Matern", param=param)$data
# Empirical bivariate variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=c(1,1,1))
\# selecting fixed and estimating parameters
fixed=list(mean_1=0, mean_2=0, nugget_1=0, nugget_2=0,
          smooth_1=smooth, smooth_12=smooth, smooth_2=smooth)
start=list(sill_1=var(data[1,]),sill_2=var(data[2,]),
          scale_1=scale_1, scale_2=scale_2, scale_12=scale_12,
          pcol=cor(data[1,],data[2,]))
# Maximum likelihood fitting of the bivariate random field:
fit= GeoFit(data, coordx=coords, corrmodel="Bi_Matern",likelihood="Marginal",
            optimizer="BFGS", type="Pairwise",
            start=start,fixed=fixed,maxdist=c(0.1,0.1,0.1))
{\tt GeoCovariogram(fit, vario=biv\_vario, show. vario=TRUE, pch=20)}
```

GeoCovmatrix

Spatial and Spatio-temporal Covariance Matrix of (non) Gaussian random fields

Description

The function computes the covariance matrix associated to a spatial or spatio-temporal or a bivariate spatial Gaussian or non Gaussian randomm field with given covariance model and a set of spatial location sites and temporal instants.

Usage

```
GeoCovmatrix(coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel,
```

distance="Eucl", grid=FALSE, maxdist=NULL, maxtime=NULL,
model="Gaussian", n=1, param, radius=6371, sparse=FALSE,
taper=NULL, tapsep=NULL, type="Standard", X=NULL)

Arguments

Χ

rguments	
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. At the moment implemented only for the Gaussian case. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of T numeric $(d_t \times 2)$ -matrices containing dynamical (in time) coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section Details .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid). See GeoFit.
maxdist	Numeric; an optional positive value indicating the marginal spatial compact support in the case of tapered covariance matrix. See GeoFit.
maxtime	Numeric; an optional positive value indicating the marginal temporal compact support in the case of spacetime tapered covariance matrix. See GeoFit.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
model	String; the type of RF. See GeoFit.
param	A list of parameter values required for the covariance model.
radius	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE the function return an object of class spam. This option should be used when a parametric compactly supporte covariance is used. Default is FALSE.
taper	String; the name of the taper correlation function if type is Tapering, see the Section Details .
tapsep	Numeric; an optional value indicating the separabe parameter in the space-time non separable taper or the colocated correlation parameter in a bivariate spatial taper (see Details).
type	String; the type of covariance matrix Standard (the default) or Tapering for tapered covariance matrix.

Numeric; Matrix of space-time covariates.

Details

In the spatial case, the covariance matrix of the random vector

$$[Z(s_1),\ldots,Z(s_n,)]^T$$

with a specific spatial covariance model is computed. Here n is the number of the spatial location sites.

In the space-time case, the covariance matrix of the random vector

$$[Z(s_1,t_1),Z(s_2,t_1),\ldots,Z(s_n,t_1),\ldots,Z(s_n,t_m)]^T$$

with a specific space time covariance model is computed. Here m is the number of temporal instants.

In the bivariate case, the covariance matrix of the random vector

$$[Z_1(s_1), Z_2(s_1), \dots, Z_1(s_n), Z_2(s_n)]^T$$

with a specific spatial bivariate covariance model is computed.

The location site s_i can be a point in the d-dimensional euclidean space with d=2 or a point (given in lon/lat degree format) on a sphere of arbitrary radius.

Here there is the list of all the implemented space and space-time and bivariate correlation models. The argument param is a list including all the parameters of a given correlation model specified by the argument corrmodel. For each correlation model one can check the associated correlation parameters using CorrParam. In what follows $\kappa>0,\,\beta>0,\,\alpha,\alpha_s,\alpha_t\in(0,2],$ and $\gamma\in[0,1].$ The associated parameters in the argument param are smooth, power2, power, power_s, power_t and sep respectively. Moreover let 1(A)=1 when A is true and 0 otherwise.

- Spatial correlation models:
 - 1. Cauchy defined as:

$$R(h) = (1+h^2)^{-\beta/2}$$

It is a special case of the Gencauchy model.

2. Exp defined as:

$$R(h) = e^{-h}$$

This model is a special case of the Matern and the Stable model.

3. GenCauchy (generalised Cauchy) defined as:

$$R(h) = (1 + h^{\alpha})^{-\beta/\alpha}$$

If h is the geodesic distance then $\alpha \in (0, 1]$.

4. Matern defined as:

$$R(h) = 2^{1-\kappa} \Gamma(\kappa)^{-1} h^{\kappa} K_{\kappa}(h)$$

If h is the geodesic distance then $\kappa \in (0, 0.5]$

5. Stable defined as:

$$R(h) = e^{-h^{\alpha}}$$

If h is the geodesic distance then $\alpha \in (0, 1]$.

6. Wave defined as:

$$R(h) = \sin(h)/h$$

This model is valid only for dimensions less than or equal to 3.

7. Wend0 defined as:

$$R(h) = (1-h)^{\mu} 1 (h \in [0,1])$$

where $\mu \geq 0.5(d+1)$. If h is the geodesic distance then $\mu \geq 2$.

8. Wend1 defined as:

$$R(h) = (1 - h)^{\mu + 1} (1 + (\mu + 1)h) 1(h \in [0, 1])$$

where $\mu \geq 0.5(d+1)+1$. If h is the geodesic distance then $\mu \geq 4$.

9. Wend2 defined as:

$$R(h) = (1-h)^{\mu+2}(1+(\mu+2)h+(1/3)((\mu+1)^2-1)h^2)1(h \in [0,1])$$

where $\mu \geq 0.5(d+1) + 2$. If h is the geodesic distance then $\mu \geq 6$.

10. GenWend (Generalized Wendland) defined as:

$$R(h) = \int_{h}^{1} [(1-x)^{\mu-1}(x^2 - h^2)^{\kappa-1} 1(h \in [0,1])] dx / B(2\kappa + 1, \mu)$$

where $\mu \geq 0.5(d+1) + \kappa$. The cases $\kappa = 0, 1, 2$ correspond to the Wend0, Wend1 and Wend2 respectively.

11. Multiquadric defined as:

$$R(h) = (1 - \alpha 0.5)^{2\beta} / (1 + (\alpha 0.5)^2 - \alpha \cos(h))^{\beta}, \quad h \in [0, \pi]$$

This model is valid on the unit sphere and h is the geodesic distance.

12. Sinpower defined as:

$$R(h) = 1 - (\sin(h/2))^{\alpha}, \quad h \in [0, \pi]$$

This model is valid on the unit sphere and h is the geodesic distance.

13. Smoke defined as:

$$R(h) = K * 1F2(1/\alpha, 1/\alpha + 0.5, 2/\alpha + 0.5 + \kappa), \quad h \in [0, \pi]$$

where $K = (\Gamma(a)\Gamma(i))/\Gamma(i)\Gamma(o)$). This model is valid on the unit sphere and h is the geodesic distance.

- Spatio-temporal correlation models.
 - Non-separable models:
 - 1. Gneiting defined as:

$$R(h, u) = e^{-h^{\alpha_s}/((1+u^{\alpha_t})^{0.5\gamma_{\alpha_s}})}/(1+u^{\alpha_t})$$

 $2. \ Gneiting_GC$

$$R(h, u) = e^{-u^{\alpha_t}/((1+h^{\alpha_s})^{0.5\gamma\alpha_t})}/(1+h^{\alpha_s})$$

where h can be both the euclidean and the geodesic distance

 $3.\ Iacocesare$

$$R(h, u) = (1 + h^{\alpha_s} + u_t^{\alpha})^{-\beta}$$

4. Porcu

$$R(h, u) = (0.5(1 + h^{\alpha_s})^{\gamma} + 0.5(1 + u^{\alpha_t})^{\gamma})^{-\gamma^{-1}}$$

5. Porcu1

$$R(h, u) = (e^{-h^{\alpha_s}(1 + u^{\alpha_t})^{0.5\gamma \alpha_s}}) / ((1 + u^{\alpha_t})^{1.5})$$

6. Stein

$$R(h,u) = (h^{\psi(u)}K_{\psi(u)}(h))/(2^{\psi(u)}\Gamma(\psi(u)+1))$$

where $\psi(u) = \nu + u^{0.5\alpha_t}$

7. $Wenx_space, x = 0, 1, 2$ defined as:

$$R(h, u) = \phi(u)^{3.5+2x} Wenx(h/\phi(u), \mu_s), \quad x = 0, 1, 2$$

where
$$\phi(u) = (1 + u^{0.5\alpha_t})^{-\gamma}$$
, $0 < \gamma \le \alpha_t/2$, $\mu_s \ge 0.5(d+5) + x$.

8. $Wenx_time$, x = 0, 1, 2 defined as:

$$R(h, u) = \phi(h)^{3.5+2x} Wenx(u/\phi(h); \mu_t), \quad x = 0, 1, 2$$

where
$$\phi(h) = (1 + h^{0.5\alpha_s})^{-\gamma}$$
, $0 < \gamma \le \alpha_s/2$, $\mu_t \ge 0.5(d+5) + x$.

9. *Multiquadric_st* defined as:

$$R(h,u) = ((1 - 0.5\alpha_s)^2 / (1 + (0.5\alpha_s)^2 - \alpha_s \psi(u) cos(h)))^{a_s}, \quad h \in [0,\pi]$$

where $\psi(u) = (1 + (u/a_t)^{\alpha_t})^{-1}$. This model is valid on the unit sphere and h is the geodesic distance.

10. Sinpower_st defined as:

$$R(h, u) = \left(e^{\alpha_s \cos(h)\psi(u)/a_s} \left(1 + \alpha_s \cos(h)\psi(u)/a_s\right)\right)/k$$

where $\psi(u)=(1+(u/a_t)^{\alpha_t})^{-1}$ and $k=(1+\alpha_s/a_s)exp(\alpha_s/a_s), \quad h\in[0,\pi]$ This model is valid on the unit sphere and h is the geodesic distance.

- Separable models.

Space-time separable correlation models are easly obtained as the product of a spatial and a temporal correlation model, that is

$$R(h, u) = R(h)R(u)$$

Several combinations are possible:

1. Exp_Exp defined as:

$$R(h, u) = Exp(h)Exp(u)$$

2. Matern_Matern defined as:

$$R(h, u) = Matern(h; \kappa_s) Matern(u; \kappa_t)$$

3. Stable_Stable defined as:

$$R(h, u) = Stable(h; \alpha_s)Stable(u; \alpha_t)$$

4. Wendx_Wendy defined as

$$R(h, u) = Wendx(h; \mu_s)Wendy(u; \mu_t), x, y = 0, 1, 2$$

Note that some models are nested. (The Exp_Exp with $Matern_Matern$ for instance.)

- Spatial bivariate correlation models (see below):
 - 1. Bi_Matern (Bivariate full Matern model)
 - 2. Bi_Matern_contr (Bivariate Matern model with contrainsts)
 - 3. Bi_Matern_sep (Bivariate separable Matern model)

- 4. Bi_LMC (Bivariate linear model of coregionalization)
- 5. Bi_LMC_contr (Bivariate linear model of coregionalization with constraints)
- 6. Bi_Wendx (Bivariate full Wendland model)
- 7. Bi_Wendx_contr (Bivariate Wendland model with contrainsts)
- 8. Bi_Wendx_sep (Bivariate separable Wendland model)
- 9. Bi_Smoke (Bivariate full Smoke model on the unit sphere)
- · Spatial taper.

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For spatial covariance tapering the taper functions are:

1. Bohman defined as:

$$T(h) = (1 - h)(\sin(2\pi h)/(2\pi h)) + (1 - \cos(2\pi h))/(2\pi^2 h)1_{[0,1]}(h)$$

GeoCovmatrix

2. Wendlandx, x = 0, 1, 2 defined as:

$$T(h) = Wendx(h; x + 2), x = 0, 1, 2$$

•

• Spatio-temporal tapers.

For spacetime covariance tapering the taper functions are:

1. $Wendlandx_Wendlandy$ (Separable tapers) x,y=0,1,2 defined as:

$$T(h, u) = Wendx(h; x + 2)Wendy(h; y + 2), x, y = 0, 1, 2.$$

- 2. $Wendlandx_time$ (Non separable temporal taper) x=0,1,2 defined as: $Wenx_time$, x=0,1,2 assuming $\alpha_t=2$, $\mu_s=3.5+x$ and $\gamma\in[0,1]$ to be fixed using tapsep.
- 3. $Wendlandx_space$ (Non separable spatial taper) x=0,1,2 defined as: $Wenx_space$, x=0,1,2 assuming $\alpha_s=2$, $\mu_t=3.5+x$ and $\gamma\in[0,1]$ to be fixed using tapsep.
- Spatial bivariate taper (see below).
 - 1. $Bi_Wendlandx$, x = 0, 1, 2

Remarks:

The associated parameters in param are sill, sill_1,sill_2, nugget, nugget_1,nugget_2, scale_s,scale_t, scale_1,scale_2,scale_12, smooth_1,smooth_2,smooth_12, a_1,a_12,a_21,a_2 respectively.

Let R(h) be a spatial correlation model given in standard notation. Then the covariance model applied with arbitrary variance, nugget and scale equals to:

$$C(h) = (\sigma^2 + \tau^2 1(h = 0))R(h/a, ...), \quad h \ge 0$$

Similarly if R(h, u) is a spatio-temporal correlation model given in standard notation, then the covariance model is:

$$C(h, u) = (\sigma^2 + \tau^2 1(h = 0, u = 0))R(h/a_s, u/a_t, ...)$$
 $h > 0, u > 0$

Here '...' stands for additional parameters.

Let R(h) be a spatial taper given in standard notation. Then the taper function applied with an arbitrary compact support (d_s) equals to:

$$T(h) = R(h/d_s)$$

Then the tapered covariance function is given by:

$$C^{tap}(h) = T(h)C(h)$$

Similarly if R(h, u) is a spatio-temporal taper given in standard notation, then the taper function applied with arbitrary compact supports $(d_s, d_t)^T$ equals to:

$$T(h, u) = R(h/d_s, u/d_t)$$

Then the tapered covariance function is given by:

$$C^{tap}(h, u) = T(h, u)C(h, u)$$

Compact supports d_s and d_t can be set by the user with maxdist and maxtime.

The bivariate models implemented are the following:

1. Bi_Matern defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i\sigma_j + \tau_i^2 1(i=j,h=0)) Matern(h/a_{ij},\kappa_{ij})$$
 $i,j=1,2.$ $h \ge 0$

where $\rho=\rho_{12}=\rho_{21}$ is the correlation colocated parameter and $\rho_{ii}=1$. The model Bi_Matern_sep (separable matern) is a special case when $a=a_{11}=a_{12}=a_{22}$ and $\kappa=\kappa_{11}=\kappa_{12}=\kappa_{22}$. The model Bi_Matern_contr (constrained matern) is a special case when $a_{12}=0.5(a_{11}+a_{22})$ and $\kappa_{12}=0.5(\kappa_{11}+\kappa_{22})$

2. Bi_Wendx (x = 0, 1, 2) defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i \sigma_j + \tau_i^2 1(i = j, h = 0)) Wendx(h/a_{ij}, \nu_{ij} + 1) \quad i, j = 1, 2. \quad h \ge 0$$

where $\rho=\rho_{12}=\rho_{21}$ is the correlation colocated parameter and $\rho_{ii}=1$. The model Bi_Wendx_sep (separable wendland) is a special case when $a=a_{11}=a_{12}=a_{22}$ and $\mu=\mu_{11}=\mu_{12}=\mu_{22}$. The model Bi_Wendx_contr (constrained matern) is a special case when $a_{12}=0.5(a_{11}+a_{22})$ and $\mu_{12}=0.5(\mu_{11}+\mu_{22})$

3. Bi_LMC defined as:

$$C_{ij}(h) = \sum_{k=1}^{2} (f_{ik}f_{jk} + \tau_i^2 1(i = j, h = 0))R(h/a_k)$$

where R(h) is a correlation model. The model Bi_LMC_contr is a special case when $f = f_{12} = f_{21}$. Bivariate LMC models, in the current version of the package, is obtained with R(h) equal to the exponential correlation model.

The bivariate spatial tapers implemented are the following:

1. $Bi_Wendlandx$, x = 0, 1, 2 defined as:

$$T_{ij}(h) = r_{ij}Wendx(h/d_{ij}, x), \quad i, j = 1, 2 \quad x = 0, 1, 2 \quad h \ge 0$$

with $r_{ii} = 1$ and $r_{12} = r_{21}$ to be fixed using tapsep.

If $T_{ii}(h)$ is a bivariate taper, Then the tapered bivariate covariance function is given by:

$$C_{ij}^{tap}(h) = T_{ij}(h)C_{ij}(h)$$

Compact supports d_{11}, d_{12}, d_{22} can be set by the user with maxdist.

Value

Returns an object of class CovMat. An object of class CovMat is a list containing at most the following components:

bivariate Logical:TRUE if the Gaussian random field is bivariaete otherwise FALSE;

coordx A d-dimensional vector of spatial coordinates; coordy A d-dimensional vector of spatial coordinates; coordt A t-dimensional vector of temporal coordinates;

coordx_dyn A list of t matrices of spatial coordinates;

covmatrix The covariance matrix if type isStandard. An object of class spam if type is

Tapering or Standard and sparse is TRUE.

corrmodel String: the correlation model; distance String: the type of spatial distance;

grid Logical:TRUE if the spatial data are in a regular grid, otherwise FALSE;

nozero In the case of tapered matrix the percentage of non zero values in the covariance

matrix. Otherwise is NULL.

maxdist Numeric: the marginal spatial compact support if type is Tapering;
maxtime Numeric: the marginal temporal compact support if type is Tapering;

n The number of trial for Binomial RFs

namescorr String: The names of the correlation parameters; numcoord Numeric: the number of spatial coordinates; numtime Numeric: the number the temporal coordinates;

model The type of RF, see GeoFit.

param Numeric: The covariance parameters;

tapmod String: the taper model if type is Tapering. Otherwise is NULL. spacetime TRUE if spatio-temporal and FALSE if spatial covariance model;

sparse Logical: is the returned object of class spam?;

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See Also

```
GeoKrig, GeoSim, GeoFit
```

Examples

```
library(GeoModels)
library(spam)
### Example 1. Spatial covariance matrix associated to
### a Matern correlation model
###
# Define the spatial-coordinates of the points:
x = runif(500, 0, 1)
y = runif(500, 0, 1)
coords = cbind(x,y)
# Correlation Parameters for Matern model
CorrParam("Matern")
# Matern Parameters
param=list(smooth=0.5, sill=1, scale=0.2, nugget=0)
matrix1 = GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param)
dim(matrix1$covmatrix)
### Example 2. Spatial tapered Covariance matrix associated to
### a Matern correlation model
###
matrix2 = GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param,
               maxdist=0.2,taper="Wendland1",type="Tapering")
# Tapered covariance matrix
as.matrix(matrix2$covmatrix)[1:15,1:15]
# Percentage of no zero values in the tapered matrix
matrix2$nozero
### Example 3. Spatial covariance matrix associated to
### a Generalized Wendland correlation model
# Gen Wendland Parameters
param=list(sill=1,scale=0.2,nugget=0,smooth=0,power2=4)
```

```
matrix3 = GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=param,sparse=TRUE)
# Percentage of no zero values in the tapered matrix
matrix3$nozero
### Example 4. Spatial covariance matrix associated to
### a Generalized Cauchy correlation model
# Gen Cauchy Parameters
param=list(sill=1,scale=0.2,nugget=0,power1=1,power2=1)
# Correlation Parameters for Gen Cauchy model
CorrParam("GenCauchy")
matrix4 = GeoCovmatrix(coordx=coords, corrmodel="GenCauchy", param=param)
matrix4$covmatrix[1:4,1:4]
### Example 5. Covariance matrix associated to
### a space-time double exponential correlation model
# Define the temporal-coordinates:
times = seq(1, 4, 1)
# Define covariance parameters
param=list(scale_s=0.3,scale_t=0.5,sill=1)
# Correlation Parameters for double exp model
CorrParam("Exp_Exp")
# Simulation of a spatial Gaussian random field:
matrix5 = GeoCovmatrix(coordx=coords, coordt=times, corrmodel="Exp_Exp",
               param=param)
dim(matrix5$covmatrix)
### Example 6. Covariance matrix associated to
### a skew gaussian RF with Exp correlation model
param=list(sill=1,scale=0.3/3,nugget=0,skew=4)
# Simulation of a spatial Gaussian random field:
```

```
matrix6 = GeoCovmatrix(coordx=coords, corrmodel="Exp", param=param,
                model="SkewGaussian")
# covariance matrix
matrix6$covmatrix[1:10,1:10]
### Example 7. Covariance matrix associated to
### a Weibull RF with Genwend correlation model
param=list(sill=1,scale=0.3,nugget=0,shape=4,mean=0,smooth=1,power2=5)
# Simulation of a spatial Gaussian random field:
matrix7 = GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=param,
               sparse=TRUE, model="Weibull")
# covariance matrix
matrix7$nozero
### Example 8. Covariance matrix associated to
### a binomial gaussian RF with Wendland correlation model
param=list(sill=1,scale=0.2,nugget=0,power2=4)
# Simulation of a spatial Gaussian random field:
matrix8 = GeoCovmatrix(coordx=coords, corrmodel="Wend0", param=param,n=5,
               model="Binomial")
# covariance matrix
matrix8$covmatrix[1:10,1:10]
### Example 9. Covariance matrix associated to
### a bivariate Matern exponential correlation model
set.seed(8)
# Define the spatial-coordinates of the points:
x = runif(10, -1, 1)
y = runif(10, -1, 1)
coords = cbind(x,y)
# Parameters
param=list(scale=0.3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
         nugget_1=0, nugget_2=0, smooth=0.5, pcol=-0.25)
# Covariance matrix
```

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```
matrix9 = GeoCovmatrix(coordx=coords, corrmodel="Bi_matern_sep", param=param)$covmatrix
head(matrix9)
```

GeoCV

n-fold kriging Cross-validation

Description

The procedure use the $\mathsf{GeoKrig}$ function to compute n-fold kriging cross-validation using informations from a GeoFit object.

Usage

Arguments

fit	An object of class GeoFit.
K	The number of iterations in cross-validation.
n.fold	Numeric; the percentage of data to be deleted (and predicted) in the cross-validation procedure.
local	Logical; If local is TRUE, then local kriging is performed. The default is FALSE.
maxdist	Numeric; an optional positive value indicating the distance in the spatial neighborhood if local kriging is performed.
maxtime	Numeric; an optional positive value indicating the distance in the temporal neighborhood if local kriging is performed.
sparse	Logical; if TRUE kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
which	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be $1\ \text{or}\ 2$
seed	Numeric; The seed used in the n-fold kriging cross-validation. Default is 1. Comparison between different models in terms of n-fold kriging cross-validation must be performed using the same seed

Value

Returns an object containing the following informations:

predicted A list of the predicted values in the CV procedure;
data_to_pred A list of the data to predict in the CV procedure;
mae The vector of mae in the CV procedure;
rmse The vector of rmse in the CV procedure;

Author(s)

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See Also

GeoKrig.

GeoFit

Max-Likelihood-Based Fitting of Gaussian and non Gaussian RFs.

Description

Maximum weighted composite-likelihood fitting for Gaussian and some Non-Gaussian univariate spatial, spatio-temporal and bivariate spatial RFs The function returns the model parameters' estimates and the estimates' variances. Moreover the function allows to fix any of the parameters and setting upper/lower bound in the optimization.

Usage

```
GeoFit(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL,corrmodel, distance="Eucl",
    fixed=NULL,GPU=NULL, grid=FALSE, likelihood='Marginal', local=c(1,1),
    lower=NULL,maxdist=NULL,
    maxtime=NULL, memdist=TRUE,method="cholesky", model='Gaussian',n=1, onlyvar=FALSE,
    optimizer='Nelder-Mead', parallel=FALSE,
    radius=6371, sensitivity=FALSE,sparse=FALSE, start=NULL, taper=NULL, tapsep=NULL,
    type='Pairwise', upper=NULL, varest=FALSE, vartype='SubSamp', weighted=FALSE, winconst=NULL,
    winconst_t=NULL, winstp_t=NULL,X=NULL)
```

Arguments

data	A d -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid). For the description see the Section Details .
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section Details .

distance String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details. fixed An optional named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated. **GPU** Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed Logical; if FALSE (the default) the data are interpreted as spatial or spatialgrid temporal realisations on a set of non-equispaced spatial sites (irregular grid). likelihood String; the configuration of the composite likelihood. Marginal is the default, see the Section Details. local Numeric; number of local work-items of the OpenCL setup lower An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list. maxdist Numeric; an optional positive value indicating the maximum spatial distance considered in the composite or tapered likelihood computation. See the Section **Details** for more information. maxtime Numeric; an optional positive value indicating the maximum temporal separation considered in the composite or tapered likelihood computation (see Details). Logical; if TRUE then the distances in the composite likelihood are computed memdist before the optimization. String; the type of matrix decomposition used in the simulation. Default is method cholesky. The other possible choices is svd. mode1 String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section Details. Numeric; number of trials in a binomial RF; number of successes in a negative binomial RF onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default. String; the optimization algorithm (see optim for details). Nelder-Mead is the optimizer default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used. parallel Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default. Numeric; the radius of the sphere in the case of lon-lat coordinates. The default radius is 6371, the radius of the earth. Logical; if TRUE then the sensitivy matrix is computed sensitivity Logical; if TRUE then maximum likelihood is computed using sparse matrices sparse algorithms (spam packake). It should be used with compactly supported covariance models.FALSE is the default. An optional named list with the initial values of the parameters that are used start by the numerical routines in maximization procedure. NULL is the default (see

Details).

taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see Details).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods (see Details).
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; (SubSamp the default) the type of method used for computing the estimates' variances, see the Section $\textbf{Details}$.
weighted	Logical; if TRUE the likelihood objects are weighted, see the Section Details . If FALSE (the default) the composite likelihood is not weighted.
winconst	Numeric; a bivariate positive vector for computing the spatial sub-window in the sub-sampling procedure. See Details for more information.
winstp	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure. The case 1 correspond to no overlapping. See Details for more information.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure. See Details for more information.
winstp_t	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure. The case 1 correspond to no overlapping. See Details for more information.
Χ	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

Details

Note, that the standard likelihood may be seen as particular case of the composite likelihood. In this respect GeoFit provides maximum likelihood fitting. Only composite likelihood estimation based on pairs are considered. Specifically marginal pairwise likelihood is considered for each type of random field (Gaussian and not Gaussian). In the Gaussian case other types of estimation are available: conditional and difference pairwise likelihood, covariance tapering and cross-validation method.

The optimization method is specified using optimizer. The default method is Nelder-mead and other available methods are ucminf, nlm, BFGS. L-BFGS-B and nlminb. In the last two cases upper and lower bounds constraints in the optimization can be specified using lower and upper.

Depending on the dimension of data and on the name of the correlation model, the observations are assumed as a realization of a spatial, spatio-temporal or bivariate RF. Specifically, with data, coordx, coordy, coordt parameters:

- If data is a numeric d-dimensional vector, coordx and coordy are two numeric d-dimensional vectors (or coordx is $(d \times 2)$ -matrix and coordy=NULL), then the data are interpreted as a single spatial realisation observed on d spatial sites;
- If data is a numeric $(t \times d)$ -matrix, coordx and coordy are two numeric d-dimensional vectors (or coordx is $(d \times 2)$ -matrix and coordy=NULL), coordt is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on d spatial sites and for t times.

 If data is a numeric (2×d)-matrix, coordx and coordy are two numeric d-dimensional vectors (or coordx is (d×2)-matrix and coordy=NULL), then the data are interpreted as a single spatial realisation of a bivariate RF observed on d spatial sites.

• If data is a list, coordxdyn is a list and coordt is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on dynamical spatial sites (different locations sites for each temporal instants) and for t times.

It is possible consider data on a regular grid using option grid=TRUE. Specifically:

- If data is a numeric $(d \times d)$ -matrix, coordx and coordy are two numeric d-dimensional vectors, grid=TRUE, then the data are interpreted as a single spatial RF realisation observed on d equispaced spatial sites (named regular grid).
- If data is a numeric (d × d × t)-array, coordx and coordy are two numeric d-dimensional vectors, coordt is a numeric t-dimensional vector, grid=TRUE, then the data are interpreted as a single spatial-temporal realisation of a RF observed on d equispaced spatial sites and for t times.
- If data is a numeric $(d \times d \times 2)$ -array, coordx and coordy are two numeric d-dimensional vectors, grid=TRUE, then the data are interpreted as a single realisation of a bivariate RF observed on d equispaced spatial sites.

When grid=FALSE is is possible to specify a matrix of covariates using X. Specifically:

- In the spatial case X must be a $(d \times k)$ covariates matrix associated to data a numeric d-dimensional vector;
- In the spatiotemporal case X must be a $(N \times k)$ covariates matrix associated to data a numeric $(t \times d)$ -matrix, where $N = t \times d$;
- In the spatiotemporal case X must be a $(N \times k)$ covariates matrix associated to data a numeric $(t \times d)$ -matrix, where $N = 2 \times d$;

The corrmodel parameter allows to select a specific correlation function for the RF. (See GeoCovmatrix).

The distance parameter allows to consider differents kinds of spatial distances. The settings alternatives are:

- 1. Eucl, the euclidean distance (default value);
- 2. Chor, the chordal distance;
- 3. Geod, the geodesic distance;

The likelihood parameter represents the composite-likelihood configurations. The settings alternatives are:

- $1. \ \ Conditional, the \ composite-likelihood \ is \ formed \ by \ conditionals \ likelihoods;$
- 2. Marginal, the composite-likelihood is formed by marginals likelihoods;
- 3. Full, the composite-likelihood turns out to be the standard likelihood;

The model parameter indicates the type of RF considered. The available options are:

RF with marginal symmetric distribution:

- Gaussian, for a Gaussian RF
- Logistic, for a Logistic RF (see Bevilacqua, M. Caamano C., Gaetan, C. 2018)
- StudentT, for a StudentT RF (see Bevilacqua, M. Caamano C., 2018)

RF with positive right skewed marginal distribution:

- Gamma for a Gamma RF (see Bevilacqua, M. Caamano C., 2018)
- Gamma for a Weibull RF (see Bevilacqua, M. Caamano C., 2018)
- LogGaussian for a LogGaussian RF
- LogLogistics for a LogLogistic RF (see Bevilacqua, M. Caamano C., Gaetan, C. 2018)

RF with with possibly asymmetric marginal distribution:

- SkewGaussian for a skew Gaussian RF (see Hao Zhang, H. and El-Shaarawi A. (2010))
- SinhAsinh for Sinh-arcsinh RF (see Bevilacqua, M. Caamano C., 2018)

RF with for directional data

• Wrapped for a wrapped Gaussian RF (see Alegria A., Bevilacqua, M., Porcu, E. (2016))

Rf with marginal counts data

- Binomial for a Binomial RF (see Bevilacqua, M. Caamano C., Gaetan C. (2018))
- BinomialNeg for a negative Binomial RF(see Bevilacqua, M. Caamano C., Gaetan C. (2018))

For a given model the associated parameters are given by nuisance and covariance parameters. In order to obtain the nuisance parameter use NuisParam. In order to obtain the covariance parameter associated to a given covariance model use CorrParam.

All the nuisance and covariance parameters must be specified by the user using the start and the fixed parameter. Specifically:

The start parameter allows to specify (as starting values for the optimization) the parameters to be estimated. The fixed parameter allows to fix some of the parameters.

Regression parameters in the linear specification must be specified as mean, mean1,..meank (see NuisParam). In this case a matrix of covariates with suitable dimension can be specified using the parameter X. In the case of a single mean then X should not be specified and it is interpreted as a vector of ones.

The taper parameter, optional in case that type=Tapering, indicates the type of taper correlation model. (See GeoCovmatrix)

The type parameter represents the type of likelihood used in the composite-likelihood definition. The possible alternatives are listed in the following scheme.

If a Gaussian or (any) non Gaussian RF is considered then the possible combination is marginal pairwise likelihoods (likelihood=Marginal) and type="Pairwise").

If a Gaussian RF is considered (model=Gaussian) then:

- If the composite is formed by marginal likelihoods (likelihood=Marginal):
 - Pairwise, the composite-likelihood is defined by the pairwise likelihoods;
 - Difference, the composite-likelihood is defined by likelihoods which are obtained as difference of the pairwise likelihoods.
- If the composite is formed by conditional likelihoods (likelihood=Conditional)
 - Pairwise, the composite-likelihood is defined by the pairwise conditional likelihoods.
- If the composite is formed by a full likelihood (likelihood=Full):
 - Standard, the objective function is the classical multivariate likelihood;
 - Restricted, the objective function is the restricted version of the full likelihood (e.g. Harville 1977, see **References**);

- Tapering, the objective function is the tapered 2 (unbiased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);

- Tapering1, the objective function is the tapered 1 (biased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);
- CV, the objective function is the cross validation estimation method;

The maxdist parameter set the maximum spatial distance below which pairs of sites with inferior distances are considered in the composite-likelihood. This can be lower of the maximum spatial distance. **Note** that this corresponds to use a weighted composite-likelihood with binary weights. Pairs with distance less than maxdist have weight 1 and are included in the likelihood computation, instead those with greater distance have weight 0 and then excluded. The default is NULL, in this case the effective maximum spatial distance between sites is considered.

The same arguments of maxdist are valid for maxtime but here the weighted composite-likelihood regards the case of spatial-temporal field. At the moment is implemented only for Gaussian RFs. The default is NULL, in this case the effective maximum temporal lag between pairs of observations is considered.

In the case of tapering likelihood maxdist and maxtime describes the spatial and temporal compact support of the taper (see GeoCovmatrix). If they are not specified then the maximum spatial and temporal distances are considered. In the case of space time adaptive taper the tapsep parameter allows to specify the spatio temporal compact support (see GeoCovmatrix).

The varest parameter specifies if the standard error estimation of the estimated parameters must be computed. For Gaussian RF and standard (restricted) likelihood estimation, standard errors are computed as square root of the diagonal elements of the Fisher Information matrix (asymptotic covariance matrix of the estimates under increasing domain). For Gaussian RF and tapered and composite likelihood estimation, standard errors estimate are computed as square root of the diagonal elements of the Godambe Information matrix. (asymptotic covariance matrix of the estimates under increasing domain (see Shaby, B. and D. Ruppert (2012) for tapering and Bevilacqua et. al. (2012), Bevilacqua and Gaetan (2013) for weighted composite likelihood)). The vartype parameter specifies the method used to compute the estimates' variances in the composite likelihood case. In particular for estimating the variability matrix J in the Godambe expression matrix. This parameter is considered if varest=TRUE. The options are:

• SubSamp (the default), indicates the Sub-Sampling method;

The weighted parameter specifies if the likelihoods forming the composite-likelihood must be weighted. If TRUE the weights are selected by opportune procedures that improve the efficient of the maximum composite-likelihood estimator (not implemented yet). If FALSE the efficient improvement procedure is not used.

For computing the standard errors by the sub-sampling procedure, winconst and winstp parameters represent respectively a positive constant used to determine the sub-window size and the the step with which the sub-window moves.

In the spatial case (subset of R^2), the domain is seen as a rectangle $B \times H$, therefore the size of the sub-window side b is given by $b = winconst \times \sqrt(B)$ (similar is of b). For a complete description see Lee and Lahiri (2002). By default winconst is set $B/(4 \times \sqrt(B))$. The winstp parameter is used to determine the sub-window step. The latter is given by the proportion of the sub-window size, so that when winstp=1 there is not overlapping between contiguous sub-windows. In the spatial case by default winstp=0.5. The sub-window is moved by successive steps in order to cover the entire spatial domain. Observations, that fall in disjoint or overlapping windows are considered indipendent samples.

In the spatio-temporal case winconst_t represents the length of the temporal sub-window. By default the size of the sub-window is computed following the rule established in Li et al. (2007). By default winstp is the time step.

Value

Returns an object of class GeoFit. An object of class GeoFit is a list containing at most the following components:

bivariate Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;

clic The composite information criterion, if the full likelihood is considered then it

coincides with the Akaike information criterion;

coordx A *d*-dimensional vector of spatial coordinates; coordy A *d*-dimensional vector of spatial coordinates; coordt A *t*-dimensional vector of temporal coordinates; coordx_dyn A list of dynamical (in time) spatial coordinates; convergence A string that denotes if convergence is reached;

corrmodel The correlation model;

data The vector or matrix or array of data;

distance The type of spatial distance; fixed The vector of fixed parameters;

iterations The number of iteration used by the numerical routine;

likelihood The configuration of the composite likelihood;

logCompLik The value of the log composite-likelihood at the maximum;

maxdist The maximum spatial distance used in the weighted composite likelihood. If no

spatial distance is specified then it is NULL;

maxtime The maximum temporal distance used in the weighhed composite likelihood. If

no spatial distance is specified then it is NULL;

message Extra message passed from the numerical routines; model The density associated to the likelihood objects;

missp True if a misspecified Gaussian model is ued in the composite likelihhod;

n The number of trials in a binominal RF; the number of successes in a negative

Binomial RFs;

ns The number of (different) location sites in the bivariate case;

nozero In the case of tapered likelihood the percentage of non zero values in the covari-

ance matrix. Otherwise is NULL.

numcoord The number of spatial coordinates;

numtime The number of the temporal realisations of the RF;

param The vector of parameters' estimates;

param The radius of the sphere in the case of great circle distance;

stderr The vector of standard errors;

sensmat The sensitivity matrix;

varcov The matrix of the variance-covariance of the estimates;

varimat The variability matrix;

vartype The method used to compute the variance of the estimates;

type The type of the likelihood objects.

winconst The constant used to compute the window size in the spatial sub-sampling;

winstp The step used for moving the window in the spatial sub-sampling;

winconst_t The constant used to compute the window size in the spatio-temporal sub-sampling;

winstp_ The step used for moving the window in the spatio-temporal sub-sampling;

X The matrix of covariates;

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Examples

```
library(GeoModels)
library(fields)
### Example 1, 2 : Estimation of a spatial Gaussian RF with
### Matern correlation using pairwise likelihood and
### maximum likelihood with BGGS and nlminb optimization
# Define the spatial-coordinates of the points:
set.seed(3)
N=400 # number of location sites
x <- runif(N, 0, 1)
set.seed(6)
y <- runif(N, 0, 1)
coords <- cbind(x,y)</pre>
# Define spatial matrix covariates
X=cbind(rep(1,N),runif(N))
# Set the covariance model's parameters:
corrmodel <- "Matern"
mean <- 0.2
mean1 <- -0.5
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5
param<-list(mean=mean,mean1=mean1,sill=sill,nugget=nugget,scale=scale,smooth=smooth)</pre>
# Simulation of the spatial Gaussian RF:
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param,X=X)$data</pre>
fixed<-list(nugget=nugget,smooth=smooth)</pre>
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)</pre>
### Example 1. Maximum pairwise likelihood fitting of
### Gaussian RFs with exponential correlation.
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,</pre>
               optimizer="BFGS", maxdist=0.05, likelihood="Marginal",
               type="Pairwise", start=start,fixed=fixed,X=X)
print(fit1)
```

```
### Example 2. Standard Maximum likelihood fitting of
### Gaussian RFs with exponential correlation.
I=Inf
lower<-list(mean=-I,mean1=-I,scale=0,sill=0)</pre>
upper<-list(mean=I,mean1=I,scale=I,sill=I)</pre>
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,</pre>
                optimizer="nlminb", upper=upper, lower=lower,
                likelihood="Full", type="Standard",
                start=start,fixed=fixed,X=X)
print(fit2)
######### Examples of spatial non-Gaussian RFs ###########
### Example 3. Maximum pairwise likelihood fitting of spatial
### Gamma and Weibull RFs with Generalized Wendland correlation
### using Nelder-Mead and BFGS
set.seed(524)
# Define the spatial-coordinates of the points:
N=500
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)</pre>
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
shape=2
scale=0.2
smooth=0
model="Weibull"
corrmodel="GenWend"
param=list(mean=mean, mean1=mean1, sill=1-nugget, scale=scale,
                 shape=shape,nugget=nugget,power2=4,smooth=smooth)
# Simulation of a non stationary weibull RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model=model, X=X,</pre>
         param=param)$data
fixed<-list(nugget=nugget,power2=4,sill=1-nugget,smooth=smooth)</pre>
start<-list(mean=mean, mean1=mean1, scale=scale, shape=shape)</pre>
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
                optimizer="Nelder-Mead",
```

```
start=start,fixed=fixed)
print(fit$param)
model="Gamma"
start<-list(mean=mean, mean1=mean1, scale=scale)</pre>
fixed<-list(nugget=nugget,power2=4,sill=1-nugget,shape=6)</pre>
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel="Wend0", model=model,</pre>
                   maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
                     optimizer="BFGS",
                   start=start,fixed=fixed)
print(fit$param)
### Example 4. Maximum pairwise likelihood fitting of
### StudendT spatial RFs with Wendland correlation
set.seed(15274)
# Define the spatial-coordinates of the points:
N=300
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)</pre>
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
sill=0.5
scale=0.2
df=4 ## degrees of freedom
model="StudentT"
corrmodel="Wend0"
# Simulation of a studentT RF:
param=list(mean=mean,mean1=mean1,sill=sill,scale=scale,df=1/df,nugget=nugget,power2=4)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,</pre>
          param=param)$data
## estimation assuming df unknown
fixed<-list(nugget=nugget,power2=4)</pre>
start<-list(mean=mean, mean1=mean1, scale=scale, sill=sill, df=1/df)
lower<-list(mean=-I, mean1=-I, scale=0, sill=0, df=0)</pre>
upper<-list(mean=I,mean1=I,scale=I,sill=I,df=0.5)</pre>
# Maximum pairwise composite-likelihood fitting of the RF:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
              maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
              lower=lower,upper=upper,optimizer="nlminb",start=start,fixed=fixed)
print(fit1$param)
```

```
## df must be rounded and fixed
df=round(1/(as.numeric(fit1$param['df'])))
 fixed<-list(nugget=nugget,power2=4,df=1/df)</pre>
 start<-list(mean=mean, mean1=mean1, scale=scale, sill=sill)</pre>
lower<-list(mean=-I,mean1=-I,scale=0,sill=0)</pre>
upper<-list(mean=I,mean1=I,scale=I,sill=I)</pre>
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                   maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
                 lower=lower,upper=upper, optimizer="nlminb" , start=start,fixed=fixed)
print(fit$param)
### Example 5. Maximum pairwise likelihood fitting of
### SinhAsinh-Gaussian spatial RFs with Wendland correlation
set.seed(261)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
x <- runif(500, 0, 1)
y < - runif(500, 0, 1)
coords <- cbind(x,y)</pre>
corrmodel="Wend0"
mean=0;nugget=0
sill=1
skew=-0.5
tail=1.5
power2=4
c_supp=0.2
# model parameters
param=list(power2=power2, skew=skew, tail=tail,
            mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param)$data</pre>
plot(density(data))
fixed=list(power2=power2,nugget=nugget)
start=list(scale=c_supp, skew=skew, tail=tail, mean=mean, sill=sill)
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                   maxdist=0.05,likelihood="Marginal",type="Pairwise",
                  optimizer="BFGS" , start=start,fixed=fixed)
print(fit1$param)
# Maximum likelihood:
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                   likelihood="Full", type="Standard",
                  optimizer="BFGS" , start=start,fixed=fixed)
```

```
print(fit2$param)
### Example 6. Maximum pairwise likelihood fitting of
### Binomial and negative Binomial
### with exponential correlation
set.seed(422)
N=350
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)</pre>
coords <- cbind(x,y)</pre>
mean=0.1; mean1=0.8; mean2=-0.5 # regression parameters
X=cbind(rep(1,N),runif(N),runif(N)) # marix covariates
corrmodel <- "Wend0"</pre>
param=list(mean=mean, mean1=mean1, mean2=mean2, sill=1, nugget=0, scale=0.2, power2=4)
# Simulation of the spatial Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="Binomial", n=10,X=X,</pre>
           param=param)$data
fixed <- list(nugget=nugget,power2=4,sill=1)</pre>
start <- list(scale=0.2,mean=mean,mean1=mean1,mean2=mean2)</pre>
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel,n=10, X=X, start=start,</pre>
                 maxdist=0.05,model="Binomial", fixed=fixed, optimizer="BFGS")
print(fit1)
set.seed(220)
# Simulation of the spatial Negative Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="BinomialNeg", n=5,X=X,</pre>
            param=param)$data
# Maximum pairwise likelihood:
fit2 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel, n=5,X=X, start=start,</pre>
            maxdist=0.05, model="BinomialNeg", fixed=fixed, optimizer="BFGS")
print(fit2)
####### Examples of spatio-temporal RFs #########
set.seed(52)
# Define the temporal sequence:
time <- seq(1, 10, 1)
# Define the spatial-coordinates of the points:
x < -runif(20, 0, 1)
set.seed(42)
y <- runif(20, 0, 1)
coords=cbind(x,y)
```

```
# Set the covariance model's parameters:
corrmodel="Exp_Exp"
scale_s=0.2/3
scale_t=1
sill=1
nugget=0
mean=0
param<-list(mean=0,scale_s=scale_s,scale_t=scale_t,</pre>
          sill=sill,nugget=nugget)
# Simulation of the spatial-temporal Gaussian RF:
data <- GeoSim(coordx=coords,coordt=time,corrmodel=corrmodel,</pre>
            param=param)$data
### Example 7. Maximum pairwise likelihood fitting of a
### space time Gaussian RF with double-exponential correlation
### using BFGS
###
# Fixed parameters
fixed<-list(nugget=nugget)</pre>
# Starting value for the estimated parameters
start<-list(mean=mean,scale_s=scale_s,scale_t=scale_t,sill=sill)</pre>
# Maximum composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,coordt=time,</pre>
                 corrmodel="Exp_Exp", maxtime=1, maxdist=0.3,
                 likelihood="Marginal", type="Pairwise",
                 optimizer="BFGS" , start=start,fixed=fixed)
print(fit)
###
### Example 8. Maximum standard likelihood fitting of a
### space time Gaussian RF observed on dynamical spatial coordinates
### with double-exponential correlation using BFGS
coordx_dyn=list()
set.seed(31)
for(k in 1:length(time))
NN=sample(1:maxN,size=1)
x <- runif(NN, 0, 1)
y <- runif(NN, 0, 1)</pre>
coordx_dyn[[k]]=cbind(x,y)
}
data <- GeoSim(coordx_dyn=coordx_dyn, coordt=time, corrmodel="Exp_Exp",</pre>
             param=param)$data
I=Inf
```

```
lower<-list(mean=-I,scale_s=0,scale_t=0,sill=0)</pre>
upper<-list(mean=-I,scale_s=I,scale_t=I,sill=I)</pre>
fit <- GeoFit(data=data,coordx_dyn=coordx_dyn,coordt=time,</pre>
            corrmodel="Exp_Exp",likelihood="Full",type="Standard",
            optimizer="BFGS", lower=lower,upper=upper,
            start=start,fixed=fixed)
print(fit)
####### Examples of spatial bivariate RFs #########
### Example 9. Maximum, and pairwise likelihood fitting of a
### bivariate Gaussian RF with separable Bivariate matern
### (cross) correlation model using BFGS and a parallel
### implementation of L-BFGS-B
# Define the spatial-coordinates of the points:
set.seed(5)
x <- runif(250, 0, 1)
y <- runif(250, 0, 1)
coords=cbind(x,y)
# parameters
param=list(mean_1=0, mean_2=0, scale=0.1, smooth=0.5, sill_1=1, sill_2=1,
         nugget_1=0,nugget_2=0,pcol=0.2)
# Simulation of a spatial Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel="Bi_Matern_sep",</pre>
            param=param)$data
# selecting fixed and estimated parameters
fixed=list(nugget_1=0, nugget_2=0, smooth=0.5)
start=list(mean_1=0,mean_2=0,sill_1=var(data[1,]),sill_2=var(data[2,]),
         scale=0.1,pcol=cor(data[1,],data[2,]))
# Maximum pairwise likelihood
fitcl<- GeoFit(data=data, coordx=coords, corrmodel="Bi_Matern_sep",</pre>
                  likelihood="Marginal",type="Pairwise",
                  optimizer="BFGS" , start=start,fixed=fixed,
                  \max dist = c(0.05, 0.05, 0.05))
print(fitcl)
T=Tnf
lower=list(mean_1=-I,mean_2=-I,sill_1=0,sill_2=0,
         scale=0,pcol=-1)
upper=list(mean_1=I,mean_2=I,sill_1=I,sill_2=I,
         scale=I,pcol=1)
# Maximum likelihood :
require(optimParallel)
```

GeoKrig

Spatial (bivariate) and spatio temporal optimal linear prediction for Gaussian and non Gaussian RFs.

Description

For a given set of spatial location sites and temporal instants, the function computes optimal linear prediction and associated mean square error for the Gaussian and non Gaussian case.

Usage

```
GeoKrig(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel, distance="Eucl",
    grid=FALSE, loc, maxdist=NULL, maxtime=NULL, method="cholesky",
    model="Gaussian", n=1,nloc=NULL,mse=FALSE, lin_opt=TRUE,
    param, radius=6371, sparse=FALSE,taper=NULL,tapsep=NULL,
    time=NULL, type="Standard",type_mse=NULL,
    type_krig="Simple",weigthed=TRUE,which=1, X=NULL,Xloc=NULL)
```

Arguments

data	A d -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d -dimensional vector giving 1-dimension of spatial coordinates used for prediction. qndd-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates used for prediction; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section Details .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details of GeoFit.

grid Logical; if FALSE (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid). lin_opt Logical; If TRUE (default) then optimal (pairwise) linear kriging is computed. Otherwise optimal (pairwise) kriging is computed in the mean square sense. A numeric $(n \times 2)$ -matrix (where n is the number of spatial sites) giving 2loc dimensions of spatial coordinates to be predicted. Numeric; an optional positive value indicating the maximum spatial compact maxdist support in the case of covariance tapering kriging. Numeric; an optional positive value indicating the maximum temporal compact maxtime support in the case of covasriance tapering kriging. String; the type of matrix decomposition used in the simulation. Default is method cholesky. The other possible choices is svd. n Numeric; the number of trials in a binomial random fields. Default is 1. nloc Numeric; the number of trials of the locations sites to be predicted in a binomial random fields type II. Default is 1. Logical; if TRUE (the default) MSE of the kriging predictor is computed mse model String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section Details. param A list of parameter values required for the correlation model. See the Section Details. Numeric: the radius of the sphere if coordinates are passed in lon/lat format; radius Logical; if TRUE kriging is computed with sparse matrices algorithms using spam sparse package. Default is FALSE. It should be used with compactly supported covari-String; the name of the taper correlation function, see the Section **Details**. taper tapsep Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see Details). A numeric $(m \times 1)$ vector (where m is the number of temporal instants) giving the time temporal instants to be predicted; the default is NULL then only spatial prediction is performed. String; if Standard then standard kriging is performed; if Tapering then krigtype ing with covariance tapering is performed; if Pairwise then pairwise kriging is performed String; if Theoretical then theoretical MSE pairwise kriging is computed. If type_mse SubSamp then an estimation based on subsampling is computed. String; the type of kriging. If Simple (the default) then simple kriging is pertype_krig formed. (See the Section Details). weigthed Logical; if TRUE then decreasing weights coming from a compactly supported correlation function with compact support maxdist (maxtime) are used in the pairwise kriging. which Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2 Χ Numeric; Matrix of spatio(temporal)covariates in the linear mean specification. Xloc Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.

Details

Best linear unbiased predictor and associated mean square error is computed for Gaussian and some non Gaussian cases. Specifically, for a spatial or spatio-temporal or spatial bivariate dataset, given a set of spatial locations and temporal istants and a correlation model corrmodel with some fixed parameters and given the type of RF (model) the function computes simple kriging, for the specified spatial locations loc and temporal instants time, providing also the respective mean square error. For the choice of the spatial or spatio temporal correlation model see details in GeoCovmatrix function. The list param specifies mean and covariance parameters, see CorrParam and GeoCovmatrix for details. The type_krig parameter indicates the type of kriging. In the case of simple kriging, the known mean can be specified by the parameter mean in the list param (See examples). In the Gaussian case, it is possible to perform kriging based on covariance tapering for simple kriging (Furrer et. al, 2008). In this case, space or space-time tapered function and spatial or spatio-temporal compact support must be specified. For the choice of a space or space-time tapered function see GeoCovmatrix. When performing kriging with covariance tapering, sparse matrix algorithms are exploited using the package spam.

Value

Returns an object of class Kg. An object of class Kg is a list containing at most the following components:

bivariate TRUE if spatial bivariate cokriging is performed, otherwise FALSE; coordx A d-dimensional vector of spatial coordinates used for prediction; coordy A d-dimensional vector of spatial coordinates used for prediction; coordt A t-dimensional vector of temporal coordinates used for prediction;

corrmodel String: the correlation model;

covmatrix The covariance matrix if type is Standard. An object of class spam if type is

Tapering

data The vector or matrix or array of data used for prediction

distance String: the type of spatial distance;

grid TRUE if the spatial data used for prediction are observed in a regular grid, other-

wise FALSE;

loc A $(n \times 2)$ -matrix of spatial locations to be predicted.

n The number of trial for Binomial RFs

nozero In the case of tapered simple kriging the percentage of non zero values in the

covariance matrix. Otherwise is NULL.

numcoord Numeric:he number d of spatial coordinates used for prediction; numloc Numeric: the number n of spatial coordinates to be predicted; numtime Numeric: the number d of the temporal instants used for prediction; numt Numeric: the number m of the temporal instants to be predicted;

model The type of RF, see GeoFit.

param Numeric: The covariance parameters;

pred A $(m \times n)$ -matrix of spatio or spatio temporal kriging prediction;

radius Numeric: the radius of the sphere if coordinates are pssed in lon/lat format;

spacetime TRUE if spatio-temporal kriging and FALSE if spatial kriging;

tapmod String: the taper model if type is Tapering. Otherwise is NULL.

time A m-dimensional vector of temporal coordinates to be predicted;

type String: the type of kriging (Standard or Tapering).

type_krig String: the type of kriging.

mse A $(m \times n)$ -matrix of spatio or spatio temporal mean square error kriging predic-

tion;

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References

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York. Furrer R., Genton, M.G. and Nychka D. (2006). *Covariance Tapering for Interpolation of Large Spatial Datasets*. Journal of Computational and Graphical Statistics, **15-3**, 502–523.

See Also

GeoCovmatrix

Examples

```
library(GeoModels)
library(fields)
library(hypergeo)
# Define the spatial-coordinates of the points:
set.seed(79)
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel_1 = "exponential"
mean=0
sill=1
nugget=0
scale=0.3/3
param=list(mean=mean, sill=sill, nugget=nugget, scale=scale)
# Set the wendland parameters (two compatible correlations):
corrmodel_2 = "Wend0"
mean=0
sill=1
nugget=0
power2=3
c_supp=0.3
param_wen=list(mean=mean, sill=sill, nugget=nugget, scale=c_supp, power2=power2)
# Simulation of the spatial Gaussian random field:
```

```
data = GeoSim(coordx=coords, corrmodel=corrmodel_1,
           param=param)$data
# locations to predict
xx = seq(0,1,0.025)
loc_to_pred=as.matrix(expand.grid(xx,xx))
### Example 1. Spatial simple kriging of n sites of a
### Gaussian random fields with exponential correlation.
pr=GeoKrig(loc=loc_to_pred, coordx=coords, corrmodel=corrmodel_1,
     param= param, data=data,mse=TRUE)
### Example 3. Spatial simple kriging of n sites of a
### Gaussian random fields using a compatible Wendland model
param=list(mean=mean, sill=sill, nugget=nugget, scale=scale, power2=power2)
pr_wen=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_2,data=data,
     param=param_wen, sparse=TRUE, mse=TRUE)
colour = rainbow(100)
par(mfrow=c(2,2))
zlim=c(-2.6, 2.6)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
        zlim=zlim,xlab="",ylab="",
        main="Simple Kriging with exponential model ")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr\$mse,ncol=length(xx)),col=colour,
        xlab="",ylab="",main="Std error")
# simple kriging map prediction
image.plot(xx, xx, matrix(pr_wen$pred,ncol=length(xx)),col=colour,
        zlim=zlim,xlab="",ylab="",main="Simple Kriging with Wendland model")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr_wen$mse,ncol=length(xx)),col=colour,
        xlab="",ylab="",main="Std error")
### Example 4. Spatial simple kriging of a binomial
###
           random field
set.seed(312)
```

```
model="Binomial";n=6
# Define the spatial-coordinates of the points:
x = runif(800)
y = runif(800)
coords=cbind(x,y)
#### mean and covariance parameters ###
mean=0
sill=1
nugget=0
scale=0.2
##################
param=list(mean=mean, sill=sill, nugget=nugget, scale=scale, power2=4)
# Simulation of the Binomial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel="Wend0", model=model, n=n,
             \verb|sparse=TRUE,param=param| \$ data
par(mfrow=c(1,2))
#### map of simulated data
quilt.plot(x, y, data,nlevel=n+1,col=rainbow(n+1),zlim=c(0,n), main="Data")
## estimation with pairwise likelihood
fixed=list(nugget=nugget,power2=4,sill=1)
start=list(mean=0,scale=scale)
# Maximum pairwise likelihood fitting :
fit = GeoFit(data, coordx=coords, corrmodel="Wend0",model=model,n=n,
                  likelihood='Marginal', type='Pairwise',maxdist=0.03,
                  optimizer="BFGS", start=start,fixed=fixed)
# locations to predict
xx = seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))
## simple kriging
pr=GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,n=n,
      sparse=TRUE,param= as.list(c(fit$param,fixed)))
#standard binomial kriging
map_binom=matrix(pr$pred,ncol=length(xx))
image.plot(xx, xx, map_binom,col=rainbow(n+1),zlim=c(0,n),
          xlab="",ylab="",main="Simple Kriging ")
###
### Example 5. Spatial simple kriging of the residuals of a
###
              Weibull random field
set.seed(312)
model="Weibull"
corrmodel = "GenWend"
# Define the spatial-coordinates of the points:
```

```
NN=800
coords=cbind(runif(NN),runif(NN))
## matrix covariates
a0=rep(1,NN)
a1=runif(NN,-1,1)
X=cbind(a0,a1)
# Set model parameters
shape=2
## regression parameters
mean = 1
mean1 = -0.2
# correlation parameters
sill = 1
nugget = 0
power2=4
scale = 0.3
smooth=0
## simulation
param=list(shape=shape,nugget=nugget,mean=mean,mean1=mean1,
  scale=scale,sill=sill,power2=power2,smooth=smooth)
data = GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
                model=model,sparse=TRUE,X=X)$data
####starting and fixed parameters
fixed=list(nugget=nugget,power2=power2,smooth=smooth,sill=sill)
start=list(mean=mean, mean1=mean1, scale=scale, shape=shape)
## estimation with pairwise likelihood
maxdist=0.1
fit2 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel,X=X,
              maxdist=maxdist,likelihood="Marginal",type="Pairwise",
              optimizer="BFGS",start=start,fixed=fixed, model = model)
## computing residuals
res=GeoResiduals(fit2)
# locations to predict
xx = seq(0,1,0.025)
loc_to_pred=as.matrix(expand.grid(xx,xx))
#optimal linear kriging for residuals
pr=GeoKrig(data=res$data, coordx=coords,loc=loc_to_pred,corrmodel=corrmodel=model=model,mse=TRUE,
       sparse=TRUE,param=as.list(c(res$param,res$fixed)))
## map of residuals
par(mfrow=c(1,3))
quilt.plot(coords,res$data,main="Residuals")
map=matrix(pr$pred,ncol=length(xx))
mapmse=matrix(pr$mse,ncol=length(xx))
image.plot(xx, xx, map,
           xlab="",ylab="",main="Residuals Kriging ")
image.plot(xx, xx, mapmse,
           xlab="",ylab="",main="Simple Kriging ")
```

```
### Example 5. Spatial simple kriging of a t
###
            random field
###
model="StudentT"
df=7
corrmodel = "Wend0"
nsel=700
coords=cbind(runif(nsel),runif(nsel))
mean = 0
sill = 1
nugget = 0
power2=4
scale = 0.2
# Starting value for the estimated parameters
set.seed(3132)
param=list(nugget=nugget,mean=mean, scale=scale,sill=sill,df=1/df,power2=power2)
data = GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
    model=model,sparse=TRUE)$data
fixed=list(nugget=nugget,power2=4,df=1/df)
start=list(mean=mean, scale=scale,sill=sill)
I=Inf
upper=list(mean=I, scale=I,sill=I)
lower=list(mean=-I, scale=0,sill=0)
# Maximum pairwise composite-likelihood fitting of the RF:
fit = GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
          maxdist=0.03, likelihood="Marginal", type="Pairwise",
          lower=lower,upper=upper,
          optimizer="nlminb", start=start,fixed=fixed, model = model)
# locations to predict
xx = seq(0,1,0.05)
loc_to_pred=as.matrix(expand.grid(xx,xx))
#optimal linear kriging
pr=GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,
     sparse=TRUE,param= as.list(c(fit$param,fit$fixed)))
par(mfrow=c(1,2))
#### map of simulated data
quilt.plot(coords[,1], coords[,2], data, main="Data")
map_t=matrix(pr$pred,ncol=length(xx))
image.plot(xx, xx, map_t,
         xlab="",ylab="",main="Simple Kriging ")
######## Examples of spatio temporal kriging ##########
```

```
model="Gaussian"
# Define the spatial-coordinates of the points:
x = runif(80, 0, 1)
y = runif(80, 0, 1)
coords=cbind(x,y)
times=1:8
# Define model correlation and associated parameters
corrmodel="exp_exp"
param=list(nugget=0, mean=0, scale_s=0.2/3, scale_t=1, sill=1)
# Simulation of the space time Gaussian random field:
set.seed(31)
data=GeoSim(coordx=coords,coordt=times,corrmodel=corrmodel,
          param=param)$data
# Maximum pairwise likelihood fitting of the space time random field:
start = list(scale_s=0.2/3,scale_t=1,sill=1,mean=0)
fixed = list(nugget=0)
I=Inf
lower=list(scale_s=0,scale_t=0,sill=0,mean=-I)
upper=list(scale_s=I,scale_t=I,sill=I,mean=I)
fit = GeoFit(data, coordx=coords, coordt=times, model=model, corrmodel=corrmodel,
            likelihood='Marginal', type='Pairwise',start=start,fixed=fixed,
            optimizer="nlminb", maxdist=0.1,maxtime=1)
### Example 6. Spatio temporal simple kriging of n locations
### sites and m temporal instants for a Gaussian random fields
### with estimated double exponential correlation.
###
set.seed(3)
param=as.list(c(fit$param,fit$fixed))
# locations to predict
xx = seq(0,1,0.05)
loc_to_pred=as.matrix(expand.grid(xx,xx))
# Define the times to predict
times_to_pred=1:2
pr=GeoKrig(loc=loc_to_pred,time=times_to_pred,coordx=coords,coordt=times,
      corrmodel=corrmodel, param=param,data=data,mse=TRUE)
par(mfrow=c(2,2))
zlim=c(-2.5, 2.5)
colour = rainbow(100)
for(i in 1:2) {
image.plot(xx, xx, matrix(pr$pred[i,],ncol=length(xx)),col=colour,
          zlim=zlim, main = paste(" Kriging Time=" , i),ylab="")
image.plot(xx, xx, matrix(pr$mse[i,],ncol=length(xx)),col=colour,
          main = paste("Std err Time=" , i),ylab="")
}
```

```
######## Examples of spatial bivariate cokriging ##########
###
### Example 7. Bivariate simple cokriging of n locations
### for a Gaussian random fields with separable Matern correlation
# Define the spatial-coordinates of the points:
set.seed(12)
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)
# Simulation of a spatial bivariate Gaussian random field
# with Matern separable covariance model
param=list(scale=0.3/3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
        nugget_1=0,nugget_2=0,pcol=0.7,smooth=0.5)
data = GeoSim(coordx=coords, corrmodel="Bi_matern_sep", param=param)$data
fixed=list(nugget_1=0,nugget_2=0,smooth=0.5,mean_1=0,mean_2=0)
start=list(sill_1=var(data[,1]),sill_2=var(data[,2]),scale=0.3/3,
     pcol=cor(data[1,],data[2,]))
# Maximum Composite likelihood fitting of the random field:
fitcl= GeoFit(data, coordx=coords, corrmodel="Bi_matern_sep",
       likelihood="Marginal", type="Pairwise", maxdist=0.1,
       optimizer="BFGS", start=start,fixed=fixed)
# locations to predict
xx = seq(0,1,0.04)
loc_to_pred=as.matrix(expand.grid(xx,xx))
colour = rainbow(100)
pr1=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
      param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=1,mse=TRUE)
pr2=GeoKrig(loc=loc_to_pred, coordx=coords, corrmodel="Bi_matern_sep",
      param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=2,mse=TRUE)
par(mfrow=c(2,3))
quilt.plot(coords,data[1,])
# simple kriging map prediction of the first variable
image.plot(xx, xx, matrix(pr1$pred,ncol=length(xx)),col=colour,
         xlab="",ylab="",main="First Simple coKriging")
# simple kriging map prediction variance of the first variable
image.plot(xx, xx, matrix(pr1$mse,ncol=length(xx)),col=colour,
         xlab="",ylab="",main="Std error")
```

GeoKrigloc

Spatial (bivariate) and spatio temporal optimal linear local prediction for Gaussian and non Gaussian RFs.

Description

For a given set of spatial location sites and temporal instants, the function computes optimal linear prediction and associated mean square error for the Gaussian and non Gaussian case using a spatial (temporal) neighborhood

Usage

Arguments

`	9	
	data	A d -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.
	coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d -dimensional vector giving 1-dimension of spatial coordinates used for prediction. qndd-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
	coordy	A numeric vector giving 1-dimension of spatial coordinates used for prediction; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
	coordt	A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is NULL then a spatial random field is expected.
	coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
	corrmodel	String; the name of a correlation model, for the description see the Section Details .
	distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details of GeoFit.

grid	Logical; if FALSE (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
loc	A numeric $(n \times 2)$ -matrix (where n is the number of spatial sites) giving 2-dimensions of spatial coordinates to be predicted.
maxdist	Numeric; an optional positive value indicating the distance in the spatial neighborhood.
maxtime	Numeric; an optional positive value indicating the distance in the temporal neighborhood.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
nloc	Numeric; the number of trials of the locations sites to be predicted in a binomial random fields type II. Default is 1.
mse	Logical; if TRUE (the default) MSE of the kriging predictor is computed
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section Details .
param	A list of parameter values required for the correlation model. See the Section Details .
radius	Numeric: the radius of the sphere if coordinates are passed in lon/lat format;
sparse	Logical; if TRUE kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
time	A numeric $(m \times 1)$ vector (where m is the number of temporal instants) giving the temporal instants to be predicted; the default is NULL then only spatial prediction is performed.
type	String; if Standard then standard kriging is performed; if Tapering then kriging with covariance tapering is performed; if Pairwise then pairwise kriging is performed
type_mse	String; if Theoretical then theoretical MSE pairwise kriging is computed. If SubSamp then an estimation based on subsampling is computed.
type_krig	String; the type of kriging. If Simple (the default) then simple kriging is performed. (See the Section Details).
weigthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime) are used in the pairwise kriging.
which	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2
Χ	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
Xloc	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.

Details

This function use the GeoKrig with a spatial or spatio-temporal neighborhood computed using the function GeoNeighborhood. The neighborhood is specified with the option maxdist and maxtime.

Value

Returns an object of class Kg. An object of class Kg is a list containing at most the following components:

bivariate TRUE if spatial bivariate cokriging is performed, otherwise FALSE; coordx A d-dimensional vector of spatial coordinates used for prediction; coordy A d-dimensional vector of spatial coordinates used for prediction; coordt A t-dimensional vector of temporal coordinates used for prediction;

corrmodel String: the correlation model;

covmatrix The covariance matrix if type is Standard. An object of class spam if type is

Tapering

data The vector or matrix or array of data used for prediction

distance String: the type of spatial distance;

grid TRUE if the spatial data used for prediction are observed in a regular grid, other-

wise FALSE;

loc A $(n \times 2)$ -matrix of spatial locations to be predicted.

n The number of trial for Binomial RFs

nozero In the case of tapered simple kriging the percentage of non zero values in the

covariance matrix. Otherwise is NULL.

numcoord Numeric:he number d of spatial coordinates used for prediction; numloc Numeric: the number n of spatial coordinates to be predicted; numtime Numeric: the number d of the temporal instants used for prediction;

numt Numeric: the number m of the temporal instants to be predicted;

model The type of RF, see GeoFit.

param Numeric: The covariance parameters;

pred A $(m \times n)$ -matrix of spatio or spatio temporal kriging prediction;

radius Numeric: the radius of the sphere if coordinates are pssed in lon/lat format;

spacetime TRUE if spatio-temporal kriging and FALSE if spatial kriging; tapmod String: the taper model if type is Tapering. Otherwise is NULL. time A m-dimensional vector of temporal coordinates to be predicted;

type String: the type of kriging (Standard or Tapering).

type_krig String: the type of kriging.

mse $A(m \times n)$ -matrix of spatio or spatio temporal mean square error kriging predic-

tion;

Author(s)

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References

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York. Furrer R., Genton, M.G. and Nychka D. (2006). *Covariance Tapering for Interpolation of Large Spatial Datasets*. Journal of Computational and Graphical Statistics, **15-3**, 502–523.

See Also

GeoCovmatrix

Examples

```
######### Examples of Spatial local kriging ##########
require(GeoModels)
require(fields)
####
model="Gaussian"
# Define the spatial-coordinates of the points:
set.seed(79)
x = runif(800, 0, 1)
y = runif(800, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel = "exponential"
mean=0
sill=1
nugget=0
scale=0.3/3
param=list(mean=mean,sill=sill,nugget=nugget,scale=scale)
# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,
           param=param)$data
# locations to predict
xx = seq(0,1,0.025)
loc_to_pred=as.matrix(expand.grid(xx,xx))
###
### Example 1. Comparing spatial kriging with local kriging for
### a Gaussian random field with exponential correlation.
pr=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel,
     model=model,param= param, data=data,mse=TRUE)
pr_loc=GeoKrigloc(data=data,loc=loc_to_pred,coordx=coords,corrmodel=corrmodel,
    model=model,maxdist=0.1, param= param,mse=TRUE)
colour = rainbow(100)
par(mfrow=c(2,2))
zlim=c(-2.6, 2.6)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
         zlim=zlim,xlab="",ylab="",
         main="Kriging with exponential model ")
```

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```
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
          xlab="",ylab="",main="Std error")
# simple kriging map prediction
image.plot(xx, xx, matrix(pr_loc$pred,ncol=length(xx)),col=colour,
          zlim=zlim,xlab="",ylab="",main="Local Kriging with exponential model")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr_loc$mse,ncol=length(xx)),col=colour,
          xlab="",ylab="",main="Std error")
#### Example: spatio temporal Gaussian local kriging #####
require(GeoModels)
require(fields)
set.seed(78)
coords=cbind(runif(100),runif(100))
coordt = seq(0, 5, 0.25)
corrmodel="Exp_Exp"
param=list(nugget=0, mean=0, scale_s=0.2/3, scale_t=0.25/3, sill=2)
data = GeoSim(coordx=coords, coordt=coordt,corrmodel="Exp_Exp",
                       param=param)$data
## two location to predict
loc_to_pred=matrix(runif(8),4,2)
## three temporal instants to predict
time=c(0.5,1.2,3.7)
pr=GeoKrig(data=data,loc=loc_to_pred,time=time,coordx=coords,coordt=coordt,corrmodel=corrmodel,
  model="Gaussian", param= param, mse=TRUE)
pr_loc=GeoKrigloc(data=data,loc=loc_to_pred,time=time,coordx=coords,coordt=coordt,corrmodel=corrmodel,
 maxdist=0.15,maxtime=1.5,model="Gaussian", param= param, mse=TRUE)
### full and local prediction
#pr$pred
#pr_loc$pred
```

GeoNeighborhood

Spatio (temporal) neighborhood selection for local kriging.

Description

The procedure select a spatio (temporal) neighborhood for given spatial (temporal) locations.

Usage

```
GeoNeighborhood(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, bivariate=FALSE, distance="Eucl", grid=FALSE, loc, maxdist=NULL, maxtime=NULL, radius=6371, time=NULL,
```

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Arguments

data A d-dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal

realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on

regular grid).

coordx A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-

dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius

radius are passed in lon/lat format expressed in decimal degrees.

coordy A numeric vector giving 1-dimension of spatial coordinates; coordy is inter-

preted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be

numeric a $(d \times 2)$ -matrix.

coordt A numeric vector giving 1-dimension of temporal coordinates. Optional argu-

ment, the default is NULL then a spatial RF is expected.

coordx_dyn A list of m numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial

coordinates. Optional argument, the default is NULL

bivariate If TRUE then data is considered as spatial bivariate data.

distance String; the name of the spatial distance. The default is Eucl, the euclidean

distance. See the Section **Details** of GeoFit.

grid Logical; if FALSE (the default) the data are interpreted as spatial or spatial-

temporal realisations on a set of non-equispaced spatial sites (irregular grid).

loc $A(1 \times 2)$ -matrix giving the spatial coordinate of the location for which a neigh-

borhood is computed.

maxdist Numeric; a positive value indicating the maximum spatial distance considered

in the spatial neighborhood selection.

maxtime Numeric; a positive value indicating the maximum temporal distance considered

in the temporal neighborhood selection.

radius Numeric; a value indicating the radius of the sphere when using the great circle

distance. Default value is the radius of the earth in Km (i.e. 6371)

time Numeric; a value giving the temporal instant for which a neighborhood is com-

puted.

X Numeric; Matrix of space-time covariates.

Value

Returns a list containing the following informations:

coordx A list of the matrix coordinates of the computed spatial neighborhood;

coordt A vector of the computed temporal neighborhood;

data A list of the vector of data associated with the spatio (temporal) neighborhood;

distance The type of spatial distance;

numcoord The vector of numbers of location sites involved the spatial neighborhood;
numtime The vector of numbers of temporal insttants involved the temporal neighbor-

hood;

radius The radius of the sphere if coordinates are passed in lon/lat format;

spacetime TRUE if spatio-temporal and FALSE if spatial RF;

X The matrix of spatio (temporal) covariates associated with the computed spatio

(temporal) neighborhood;

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Author(s)

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Examples

```
library(GeoModels)
#### Example: spatial neighborhood ######
set.seed(7)
coords=cbind(runif(500),runif(500))
param=list(nugget=0, mean=0, scale=0.2, sill=1,
          power2=4,smooth=1)
data_all = GeoSim(coordx=coords, corrmodel="GenWend",
                     param=param)$data
##two locations
loc_{to\_pred=matrix}(c(0.3,0.5,0.7,0.2),2,2)
neigh=GeoNeighborhood(data_all, coordx=coords,
               loc=loc_to_pred,maxdist=0.075)
# two Neighborhoods
neigh$coordx
# associated data
neigh$data
#### Example: spatio temporal spatial neighborhood#
set.seed(78)
coords=matrix(runif(10),5,2)
coordt = seq(0, 4, 0.25)
param=list(nugget=0,mean=0,scale_s=0.2/3,scale_t=0.25/3,sill=2)
data_all = GeoSim(coordx=coords, coordt=coordt,corrmodel="Exp_Exp",
                     param=param)$data
## two location to predict
loc_to_pred=matrix(runif(4),2,2)
## three temporal instants to predict
time=c(1,2,3)
plot(coords,xlim=c(0,1),ylim=c(0,1))
points(loc_to_pred,pch=20)
neigh=GeoNeighborhood(data_all, coordx=coords, coordt=coordt,
               loc=loc_to_pred,time=time,maxdist=0.6,maxtime=0.5)
# first spatio-temporal neighborhoods
# with associated data
neigh$coordx[[1]]
```

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GeoQQ

Quantile-quantile plot of residuals

Description

The procedure plots a quantile-quantile plot for the residuals associated to a fitted model

Usage

GeoQQ(fit)

Arguments

fit

A GeoFit object obtained from GeoResiduals.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

Examples

```
library(GeoModels)
set.seed(211)

model="Weibull"; shape=4
N=700 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)
```

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```
# regression parameters
mean = 5
mean1=0.8
X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
nugget = 0
scale = 0.3
power2=4
param=list(mean=mean, mean1=mean1, sill=sill, nugget=nugget,
           scale=scale, shape=shape, power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data
start=list(mean=mean, mean1=mean1, scale=scale, shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
                    likelihood="Marginal", type='Pairwise', start=start,
                    fixed=fixed,maxdist=0.1)
res=GeoResiduals(fit)
GeoQQ(res)
```

GeoResiduals

Computes fitted covariance and/or variogram

Description

The procedure return a GeoFit object associated to the estimated residuals

Usage

```
{\tt GeoResiduals(fit)}
```

Arguments

fit

A fitted object obtained from the GeoFit.

Value

A GeoFit object with the estimated residuals

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>,https://sites.google.com/site/moralesonatevictor/

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See Also

GeoFit.

Examples

```
library(GeoModels)
set.seed(211)
model="Weibull";shape=4
N=700 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)
# regression parameters
mean = 5
mean1=0.8
X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
nugget = 0
scale = 0.3
power2=4
param=list(mean=mean, mean1=mean1, sill=sill, nugget=nugget,
           scale=scale, shape=shape, power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data
start=list(mean=mean, mean1=mean1, scale=scale, shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
                    likelihood="Marginal", type='Pairwise', start=start,
                    fixed=fixed,maxdist=0.1)
res=GeoResiduals(fit)
mean(res$data) # should be 1
# Empirical estimation of the variogram for the residuals:
vario = GeoVariogram(res$data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(res, show.vario=TRUE, vario=vario,pch=20)
```

GeoScatterplot

Description

The function produces h-scatterplots for the spatial, spatio-temporal and bivariate setting.

Usage

Arguments

data	A d -dimensional vector (a single spatial realisation) or a $(n \times d)$ -matrix $(n \text{ iid spatial realisations})$ or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array $(n \text{ iid spatial realisations on regular grid})$ or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$ -array $(n \text{ iid spatial-temporal realisations})$ or or an $(d \times d \times t \times n)$ -array $(n \text{ iid spatial-temporal realisation on regular grid})$ or an $(d \times d \times t \times n)$ -array $(n \text{ iid spatial-temporal realisations})$ or regular grid). See GeoFit for details.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details of GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.
maxdist	A numeric value denoting the spatial maximum distance, see the Section Details .
times	A numeric vector denoting the temporal instants involved Details .
numbins	A numeric value denoting the numbers of bins, see the Section Details .
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
bivariate	Logical; if FALSE (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are interpreted as a realization from a bivariate field.

Details

some details

Value

The returned object is eventually a list with:

covariance The vector of the estimated covariance function; variogram The vector of the estimated variogram function;

Author(s)

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Examples

GeoSim

Simulation of Gaussian and non Gaussian Random Fields.

Description

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields. The function return a realization of a Random Field for a given covariance model and covariance parameters. Simulation is based on Cholesky decomposition.

Usage

Arguments

٠	•	
	coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
	coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
	coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
	coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
	corrmodel	String; the name of a correlation model, for the description see the Section Details .
	distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section $\bf Details$ of $\bf GeoFit$.
	GPU	Numeric; if NULL (the default) no GPU computation is performed.
	grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
	local	Numeric; number of local work-items of the GPU
	method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
	model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section Details .
	n	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1 .
	param	A list of parameter values required in the simulation procedure of RFs, see Examples .
	radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
	sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake). It should be used with compactly supported covariance models.FALSE is the default.
	X	Numeric; Matrix of space-time covariates.

Value

Returns an object of class GeoSim. An object of class GeoSim is a list containing at most the following components:

 $\label{eq:logical:TRUE} \textbf{Logical:TRUE} \ \ \textbf{if the Gaussian} \ \ \textbf{RF} \ \textbf{is bivariate}, \ \textbf{otherwise} \ \ \textbf{FALSE};$

 $\begin{array}{lll} {\sf coordx} & {\sf A} \ d\text{-dimensional vector of spatial coordinates;} \\ {\sf coordy} & {\sf A} \ d\text{-dimensional vector of spatial coordinates;} \\ {\sf coordt} & {\sf A} \ t\text{-dimensional vector of temporal coordinates;} \\ {\sf coordx_dyn} & {\sf A} \ \text{list of dynamical (in time) spatial coordinates;} \\ \end{array}$

corrmodel The correlation model; see GeoCovmatrix.

data The vector or matrix or array of data, see GeoFit;

distance The type of spatial distance; model The type of RF, see GeoFit.

n The number of trial for Binomial RFs; the number of successes in a negative

Binomial RFs;

numcoord The number of spatial coordinates;

numtime The number the temporal realisations of the RF;

param The vector of parameters' estimates;

with underlying exponential correlation

radius The radius of the sphere if coordinates are passed in lon/lat format;

randseed The seed used for the random simulation;

spacetime TRUE if spatio-temporal and FALSE if spatial RF;

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Examples

```
library(GeoModels)
library(mapproj)
library(fields)
###
### Example 1. Simulation of a spatial Gaussian RF on a regular grid
# Define the spatial-coordinates of the points:
x < - seq(0,1,0.045)
y < - seq(0,1,0.045)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Matern", param=list(smooth=0.5,</pre>
          mean=0,sill=1,scale=0.4/3,nugget=0))$data
# Simulation of a spatial Gaussian RF with Generalized Wendland correlation function
# using sparse alghorithm matrices
set.seed(261)
data2 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend", param=list(smooth=0,</pre>
            power2=4,mean=0,sill=1,scale=0.4,nugget=0))$data
par(mfrow=c(1,2))
image.plot(x,y,data1,col=terrain.colors(100),main="Matern",xlab="",ylab="")
image.plot(x,y,data2,col=terrain.colors(100),main="Wendland",xlab="",ylab="")\\
###
### Example 2. Simulation of a spatial binomial RF
```

```
### on a regular grid
###
# Define the spatial-coordinates of the points:
x < - seq(0, 1, 0.022)
y < - seq(0, 1, 0.022)
coords <- cbind(x,y)</pre>
set.seed(251)
# Simulation of a spatial Binomial RF:
sim <- GeoSim(x,y,grid=TRUE, corrmodel="Wend0",</pre>
          model="Binomial", n=n, sparse=TRUE,
          param=list(nugget=0, mean=0, scale=.2, sill=1, power2=4))
image.plot(x,y,sim$data,nlevel=n+1,col=terrain.colors(n+1),zlim=c(0,n))
### Example 3. Simulation of a spatial Weibull RF
### with underlying exponential correlation
# Define the spatial-coordinates of the points:
x \leftarrow seq(0,1,0.032)
y < - seq(0,1,0.032)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Exponential",model="Weibull",</pre>
       param=list(shape=1.2,mean=0,sill=1,scale=0.3/3,nugget=0))$data
image.plot(x,y,data1,col=terrain.colors(200),main="Weibull RF",xlab="",ylab=""")
###
### Example 4. Simulation of a spatial t RF
### with with underlying exponential correlation
# Define the spatial-coordinates of the points:
x < - seq(0,1,0.03)
y < - seq(0,1,0.03)
set.seed(268)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend",model="StudentT", sparse=TRUE,</pre>
       param=list(df=1/4,mean=0,sill=1,scale=0.3,nugget=0,smooth=1,power2=5))$data
image.plot(x,y,data1,col=terrain.colors(100),main="Student-t RF",xlab="",ylab="")
###
### Example 5. Simulation of a sinhasinh RF
### with underlying Wend0 correlation.
```

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```
###
# Define the spatial-coordinates of the points:
x <- runif(800, 0, 2)
y <- runif(800, 0, 2)
coords <- cbind(x,y)</pre>
set.seed(261)
corrmodel="Wend0"
# Simulation of a spatial Gaussian RF:
param=list(power2=4,skew=0,tail=1,
            mean=0,sill=1,scale=0.2,nugget=0) ## gaussian case
data0 <- GeoSim(coordx=coords, corrmodel=corrmodel,</pre>
             model="SinhAsinh", param=param, sparse=TRUE)$data
plot(density(data0),xlim=c(-7,7))
param=list(power2=4,skew=0,tail=0.7,
           mean=0,sill=1,scale=0.2,nugget=0) ## heavy tails
data1 <- GeoSim(coordx=coords, corrmodel=corrmodel,</pre>
             model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data1),lty=2)
param=list(power2=4, skew=0.5, tail=1,
           mean=0,sill=1,scale=0.2,nugget=0) ## asymmetry
data2 <- GeoSim(coordx=coords, corrmodel=corrmodel,</pre>
             model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data2),lty=3)
### Example 6. Simulation of a bivariate Gaussian RF
### with separable bivariate exponential correlation model
### on a regular grid.
# Define the spatial-coordinates of the points:
x \leftarrow seq(-1,1,0.08)
y < - seq(-1,1,0.08)
# Simulation of a bivariate spatial Gaussian RF:
# with a separable Bivariate Matern
set.seed(12)
param=list(mean_1=0, mean_2=0, scale=0.12, smooth=0.5,
          sill_1=1, sill_2=1, nugget_1=0, nugget_2=0, pcol=0.5)
data <- GeoSim(x,y,grid=TRUE,corrmodel="Bi_matern_sep",</pre>
             param=param)$data
par(mfrow=c(1,2))
image.plot(x,y,data[,,1],col=terrain.colors(100),main="1",xlab="",ylab="")
image.plot(x,y,data[,,2],col=terrain.colors(100),main="2",xlab="",ylab="")
```

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```
### Example 7. Simulation of a spatio temporal Gaussian RF.
### observed on dynamical location sites with double exponential correlation
# Define the dynamical spatial-coordinates of the points:
coordt=1:5
coordx_dyn=list()
maxN=40
set.seed(8)
for(k in 1:length(coordt))
NN=sample(1:maxN,size=1)
x <- runif(NN, 0, 1)
y <- runif(NN, 0, 1)</pre>
coordx_dyn[[k]]=cbind(x,y)
coordx_dyn
param<-list(nugget=0, mean=0, scale_s=0.2/3, scale_t=2/3, sill=1)</pre>
data <- GeoSim(coordx_dyn=coordx_dyn, coordt=coordt, corrmodel="Exp_Exp",</pre>
                  param=param)$data
## spatial realization at first temporal instants
data[[1]]
## spatial realization at third temporal instants
data[[3]]
###
### Example 8. Simulation of a Gaussian RF
### with a Wend0 correlation on the planet earth
###
require(plot3D)
require(sphereplot)
distance="Geod"; radius=6371
NN=4500 ## total point on the sphere on lon/lat format
set.seed(80)
coords=pointsphere(NN,c(-180,180),c(-90,90),c(radius,radius))
## Set the wendland parameters
corrmodel <- "Wend0"
param<-list(mean=0, sill=1, nugget=0, scale=radius*0.3, power2=3)</pre>
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(coordx=coords,corrmodel=corrmodel,sparse=TRUE,</pre>
       distance=distance, radius=radius, param=param) $ data
#converting in 3d cartesian coordinates
b=sph2car(coords[,1], coords[,2], radius = radius, deg = TRUE)
```

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```
x0 = b[,1]; y0 = b[,2]; z0 = b[,3]
# plotting
scatter3D(x0,y0,z0,colvar=data,clim=c(min(data),max(data)),pch=20,cex=0.8,
col=rainbow(200))
### Example 9. Simulation of a Gaussian RF
### with Wend0 model on USA
###
distance="Geod"; radius=6378.88
NN=40
x=seq(-125,-64,length.out=NN)
y=seq(27,50, length.out =NN)
nrow(expand.grid(x,y))
## Set the wendland parameters
corrmodel <- "Wend0"</pre>
param<-list(mean=0,sill=1,nugget=0,scale=radius*0.3,power2=3)</pre>
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(x,y,grid=TRUE,corrmodel=corrmodel,sparse=TRUE,</pre>
      distance=distance,radius=radius,param=param)$data
image.plot(x,y,data,col=terrain.colors(100),xlab="",ylab="")
map("usa", add = TRUE)
###
### Example 10. Simulation of a Wrapped RF
### with underlying exponential correlation
### on a regular grid
###
# Define the spatial-coordinates of the points:
x <- runif(200,0, 1)
y <- runif(200,0, 1)
coords <- cbind(x,y)</pre>
set.seed(251)
# Simulation of a spatial wrapped RF:
sim <- GeoSim(coordx=coords, corrmodel="Exp",</pre>
           model="Wrapped",
           param=list(nugget=0, mean=0, scale=.1, sill=1))$data
long <- 0.08;
x1 <- coords[,1] + long*cos(sim)</pre>
y1 <- coords[,2] + long*sin(sim)</pre>
eps <- 0.1
plot(0,xlim=c(0-eps,1+eps),ylim=c(0-eps,1+eps));
```

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```
require(shape)
Arrows(coords[,1], coords[,2], x1, y1, arr.length = 0.2, code = 2, arr.type = "triangle",col=1)
```

GeoTests

Statistical Hypothesis Tests for Nested Models

Description

The function performs statistical hypothesis tests for nested models based on composite or standard likelihood versions of Wald-type and Wilks-type (likelihood ratio) statistics.

Usage

```
GeoTests(object1, object2, ..., statistic)
```

Arguments

object1 An object of class GeoFit.

object2 An object of class GeoFit that is a nested model within object1.

Further successively nested objects.

String; the name of the statistic used within the hypothesis test (see **Details**).

Details

The implemented hypothesis tests for nested models are based on the following statistics:

- 1. Wald-type (Wald);
- 2. Likelihood ratio or Wilks-type (Wilks under standard likelihood); For composite likelihood available variants of the basic version are:
 - Rotnitzky and Jewell adjustment (WilksRJ);
 - Satterhwaite adjustment (WilksS);
 - Chandler and Bate adjustment (WilksCB);
 - Pace, Salvan and Sartori adjustment (WilksPSS);

More specifically, consider an p-dimensional random vector \mathbf{Y} with probability density function $f(\mathbf{y};\theta)$, where $\theta\in\Theta$ is a q-dimensional vector of parameters. Suppose that $\theta=(\psi,\tau)$ can be partitioned in a q'-dimensional subvector ψ and q''-dimensional subvector τ . Assume also to be interested in testing the specific values of the vector ψ . Then, one can use some statistical hypothesis tests for testing the null hypothesis $H_0:\psi=\psi_0$ against the alternative $H_1:\psi\neq\psi_0$. Composite likelihood versions of 'Wald' and 'score' statistics have the usual asymptotic chi-square distribution with q' degree of freedom. The Wald-type statistic is

$$W = (\hat{\psi} - \psi_0)^T (G^{\psi\psi})^{-1} (\hat{\theta}) (\hat{\psi} - \psi_0),$$

where $G_{\psi\psi}$ is the $q'\times q'$ submatrix of the Godambe or Fisher information pertaining to ψ and $\hat{\theta}$ is the maximum likelihood estimator from the full model. This statistic can be called from the routine GeoTests assigning at the argument statistic the value: Wald.

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Alternatively to the Wald-type statistic one can use the composite version of the Wilks-type or likelihood ratio statistic, given by

$$W = 2[C\ell(\hat{\theta}; \mathbf{y}) - C\ell\{\psi_0, \hat{\tau}(\psi_0); \mathbf{y}\}].$$

In the composite likelihood case, the asymptotic distribution of the composite likelihood ratio statistic is given by

$$W \dot{\sim} \sum_{i} \lambda_{i} \chi^{2},$$

for $i=1,\ldots,q'$, where χ_i^2 are q' iid copies of a chi-square one random variable and $\lambda_1,\ldots,\lambda_{q'}$ are the eigenvalues of the matrix $(H^{\psi\psi})^{-1}G^{\psi\psi}$. There exist several adjustments to the composite likelihood ratio statistic in order to get an approximated $\chi_{q'}^2$. For example, Rotnitzky and Jewell (1990) proposed the adjustment $W'=W/\bar{\lambda}$ where $\bar{\lambda}$ is the average of the eigenvalues λ_i . This statistic can be called within the routine by the value: WilksRJ. A better solution is proposed by Satterhwaite (1946) defining $W''=\nu W/(q'\bar{\lambda})$, where $\nu=(\sum_i\lambda)^2/\sum_i\lambda_i^2$ for $i=1\ldots,q'$, is the effective number of the degree of freedom. Note that in this case the distribution of the likelihood ratio statistic is a chi-square random variable with ν degree of freedom. This statistic can be called from the routine assigning the value: WilksS. For the adjustments suggested by Chandler and Bate (2007) and Pace, Salvan and Sartori (2011) we refere to the articles (see **References**), these versions can be called from the routine assigning respectively the values: WilksCB and WilksPSS.

Value

An object of class c("data.frame"). The object contain a table with the results of the tested models. The rows represent the responses for each model and the columns the following results:

Num.Par The number of the model's parameters.

Diff.Par The difference between the number of parameters of the model in the previous

row and those in the actual row.

Df The effective number of degree of freedom of the chi-square distribution.

Chisq The observed value of the statistic.

Pr(>chisq) The p-value of the quantile Chisq computed using a chi-squared distribution

with Df degrees of freedom.

Author(s)

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Pace, L., Salvan, A. and Sartori, N. (2011). Adjusting Composite Likelihood Ratio Statistics. *Statistica Sinica*, **21**, 129–148.

Rotnitzky, A. and Jewell, N. P. (1990). Hypothesis Testing of Regression Parameters in Semiparametric Generalized Linear Models for Cluster Correlated Data. *Biometrika*, 77, 485–497.

Satterthwaite, F. E. (1946). An Approximate Distribution of Estimates of Variance Components. *Biometrics Bulletin*, **2**, 110–114.

Varin, C., Reid, N. and Firth, D. (2011). An Overview of Composite Likelihood Methods. *Statistica Sinica*, **21**, 5–42.

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See Also

GeoFit.

Examples

```
library(GeoModels)
### Example 1. Parametric test of Gaussianity
### using SinhAsinh random fields using standard likelihood
###
set.seed(314)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
NN=500
x = runif(NN, 0, 1)
y = runif(NN, 0, 1)
coords = cbind(x,y)
# Parameters
mean=0; nugget=0; sill=1
### skew and tail parameters
skew=0;tail=1 ## H0 is Gaussianity
# underlying model correlation
corrmodel="Wend0"
power2=4;c_supp=0.2
# simulation
param=list(power2=power2, skew=skew, tail=tail,
           mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model, param=param)$data
##### H1
fixed=list(power2=power2,nugget=nugget,mean=mean)
start=list(scale=c_supp,skew=skew,tail=tail,sill=sill)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH1 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                 likelihood="Full",type="Standard",varest=TRUE,
                  optimizer="BFGS",start=start,fixed=fixed)
fitH1$param
##### H0: Gaussianity (i.e tail1=1, skew=0 fixed)
fixed=list(power2=power2,nugget=nugget,mean=mean,tail=1,skew=0)
start=list(scale=c_supp, sill=sill)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH0 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                  likelihood="Full",type="Standard",varest=TRUE,
                  optimizer="BFGS",start=start,fixed=fixed)
# Wald statistic test
GeoTests(fitH1, fitH0 ,statistic='Wald')
```

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```
# likelihood ratio statistic test
GeoTests(fitH1, fitH0 , statistic='Wilks')
### Example 2. Composite likelihood-based hypothesis testing.
### Testing significance of a regression parameter
set.seed(31)
# Define the spatial-coordinates of the points:
N=1500
x = runif(N, 0, 1)
y = runif(N, 0, 1)
X=cbind(rep(1,N),runif(N))
coords=cbind(x,y)
# Set the model's parameters:
corrmodel = "Exp"
mean = 1; mean1=0.3 ## H0: mean1=0
sill = 1
nugget = 0
scale = 0.15/3
# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,mean1=mean1,
             sill=sill,nugget=nugget,scale=scale),X=X)$data
# Pairwise-likelihood fitting of the random field, full model:
start=list(mean=mean, mean1=mean1, sill=sill, scale=scale)
fixed=list(nugget=nugget)
fitH1 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.01,
                  varest=TRUE,likelihood="Marginal",type="Pairwise",
                  optimizer="BFGS",fixed=fixed,start=start,X=X)
# Pairwise-likelihood fitting of the random field, with a nested model:
start=list(mean=mean, sill=sill, scale=scale)
fixed=list(nugget=nugget,mean1=0) # setting H0 restriction
fitH0 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.01,
                  varest=TRUE,likelihood="Marginal",type="Pairwise",
                  optimizer="BFGS",fixed=fixed,start=start,X=X)
# Hypothesis testing results:
# composite Wald statistic test:
GeoTests(fitH1, fitH0 ,statistic='Wald')
\mbox{\#} composite likelihood ratio statistic test with PSS adjustment:
GeoTests(fitH1, fitH0, statistic='WilksPSS')
```

GeoVarestbootstrap *Update a* GeoFit *object using parametric bootstrap for std error esti*mation

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Description

The procedure update a GeoFit object estimating stderr estimation using parametric bootstrap.

Usage

```
\label{local} GeoVarestbootstrap(fit, K=100, sparse=FALSE, GPU=NULL, local=c(1,1), optimizer="Nelder-Mead", lower=NULL, upper=NULL, memdist=TRUE, seed=1)
```

Arguments

fit	A fitted object obtained from the GeoFit.
K	The number of simulations in the parametric bootstrap.
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake).
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed
local	Numeric; number of local work-items of the OpenCL setup
optimizer	The type of optimization algorithm. See GeoFit for details.
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize.
memdist	Logical; if TRUE then the distances in the composite likelihood are computed before the optimization.
seed	Numeric; The seed used in the n-fold kriging cross-validation. Default is 1.

Value

Returns an object of class GeoFit.

Author(s)

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See Also

GeoFit.

GeoVariogram Empirical Variogram estimation

Description

The function returns an empirical estimate of the variogram for spatio (temporal) and bivariate random fields.

Usage

Arguments

data	A d -dimensional vector (a single spatial realisation) or a $(n \times d)$ -matrix $(n \text{ iid spatial realisations})$ or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array $(n \text{ iid spatial realisations})$ or an $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$ -array $(n \text{ iid spatial-temporal realisations})$ or or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid) or an $(d \times d \times t \times n)$ -array $(n \text{ iid spatial-temporal realisations})$ or regular grid). See GeoFit for details.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
cloud	Logical; if TRUE the variogram cloud is computed, otherwise if FALSE (the default) the empirical (binned) variogram is returned.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details of GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.
maxdist	A numeric value denoting the spatial maximum distance, see the Section Details .
maxtime	A numeric value denoting the temporal maximum distance, see the Section Details .
numbins	A numeric value denoting the numbers of bins, see the Section Details .

radius Numeric; a value indicating the radius of the sphere when using the great circle

distance. Default value is the radius of the earth in Km (i.e. 6371)

type A String denoting the type of variogram. The option available is : variogram.

bivariate Logical; if FALSE (the default) the data are interpreted as univariate spatial or

spatial-temporal realisations. Otherwise they are intrepreted as a realization

from a bivariate field.

Details

We briefly report the definitions of semi-variogram used in this function. In the case of a spatial Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h) = 0.5 \sum_{x_i, x_j \in N(h)} (Z(x_i) - Z(x_j))^2 / |N(h)|$$

where N(h) is the set of all the sample pairs whose distances fall into a tolerance region with size h (equispaced intervalls are considered).

In the case of a spatio-temporal Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h, u) = 0.5 \sum_{(x_i, l), (x_j, k) \in N(h, u)} (Z(x_i, l) - Z(x_j, k))^2 / |N(h, u)|$$

where N(h,u) is the set of all the sample pairs whose spatial distances fall into a tolerance region with size h and |k-l|=u. Note, that $Z(x_i,l)$ is the observation at site x_i and time l.

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum spatial distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

The maxtime parameter indicates the maximum temporal distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

Value

Returns an object of class Variogram. An object of class Variogram is a list containing at most the following components:

	bins	Adjacent intervals of grouped spatial distances if cloud=FALSE. Otherwise if
--	------	--

cloud=TRUE all the spatial pairwise distances;

bint Adjacent intervals of grouped temporal distances if cloud=FALSE. Otherwise if

cloud=TRUE all the temporal pairwise distances;

cloud If the variogram cloud is returned (TRUE) or the empirical variogram (FALSE);

centers The centers of the spatial bins; distance The type of spatial distance;

lenbins The number of pairs in each spatial bin;

lenbinst The number of pairs in each spatial-temporal bin;

lenbint The number of pairs in each temporal bin;

maxdist The maximum spatial distance used for the calculation of the variogram. If no

spatial distance is specified then it is NULL;

maxtime The maximum temporal distance used for the calculation of the variogram. If

no temporal distance is specified then it is NULL;

is(TRUE).

variograms The empirical spatial variogram;

variogramst The empirical spatial-temporal variogram;

variogramt The empirical temporal variogram; type The type of estimated variogram

Author(s)

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References

Cressie, N. A. C. (1993) Statistics for Spatial Data. New York: Wiley.

Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. Spring Verlang, New York.

See Also

GeoFit

library(GeoModels)

Examples

```
###
### Example 1. Empirical estimation of the semi-variogram from a
### spatial Gaussian random field with exponential correlation.
set.seed(514)
# Set the coordinates of the sites:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords = cbind(x,y)
# Set the model's parameters:
corrmodel = "exponential"
mean = 0
sill = 1
nugget = 0
scale = 0.3/3
# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,
            sill=sill, nugget=nugget, scale=scale))$data
# Empirical spatial semi-variogram estimation:
fit = GeoVariogram(coordx=coords,data=data,maxdist=0.6)
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
    ylim=c(0, max(fit$variograms)), pch=20,
    main="Semi-variogram")
```

```
###
### Example 2. Empirical estimation of the variogram from a
### spatio-temporal Gaussian random fields with Gneiting
### correlation function.
set.seed(331)
# Define the temporal sequence:
# Set the coordinates of the sites:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords = cbind(x,y)
times = seq(1,10,1)
# Simulation of a spatio-temporal Gaussian random field:
data = GeoSim(coordx=coords, coordt=times, corrmodel="gneiting",
             param=list(mean=0,scale_s=0.08,scale_t=0.4,sill=1,
             nugget=0,power_s=1,power_t=1,sep=0.5))$data
# Empirical spatio-temporal semi-variogram estimation:
fit = GeoVariogram(data=data, coordx=coords, coordt=times, maxtime=7,maxdist=0.5)
# Results: Marginal spatial empirical semi-variogram
par(mfrow=c(2,2), mai=c(.5,.5,.3,.3), mgp=c(1.4,.5, 0))
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
    ylim=c(0, max(fit$variograms)), xlim=c(0, max(fit$centers)),
    pch=20,main="Marginal spatial semi-variogram",cex.axis=.8)
# Results: Marginal temporal empirical semi-variogram
plot(fit$bint, fit$variogramt, xlab='t', ylab=expression(gamma(t)),
    ylim=c(0, max(fit$variogramt)),xlim=c(0,max(fit$bint)),
    pch=20,main="Marginal temporal semi-variogram",cex.axis=.8)
# Building space-time semi-variogram
st.vario = matrix(fit$variogramst,length(fit$centers),length(fit$bint))
st.vario = cbind(c(0,fit$variograms), rbind(fit$variogramt,st.vario))
# Results: 3d Spatio-temporal semi-variogram
require(scatterplot3d)
st.grid = expand.grid(c(0,fit$centers),c(0,fit$bint))
scatterplot3d(st.grid[,1], st.grid[,2], c(st.vario),
             highlight.3d=TRUE, xlab="h",ylab="t",
             zlab=expression(gamma(h,t)), pch=20,
             main="Space-time semi-variogram",cex.axis=.7,
             mar=c(2,2,2,2), mgp=c(0,0,0),
             cex.lab=.7)
# A smoothed version
par(mai=c(.2,.2,.2), mgp=c(1,.3, 0))
persp(c(0,fit$centers), c(0,fit$bint), st.vario,
     xlab="h", ylab="u", zlab=expression(gamma(h,u)),
     ltheta=90, shade=0.75, ticktype="detailed", phi=30,
     theta=30, main="Space-time semi-variogram", cex.axis=.8,
```

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```
### Example 3. Empirical estimation of the (cross) semivariograms
### from a bivariate Gaussian random fields with Matern
### correlation function.
# Simulation of a bivariate spatial Gaussian random field:
set.seed(293)
# Define the spatial-coordinates of the points:
x = runif(400, 0, 1)
y = runif(400, 0, 1)
coords=cbind(x,y)
# Simulation of a bivariate Gaussian Random field
# with matern (cross) covariance function
param=list(mean_1=0, mean_2=0, scale_1=0.1/3, scale_2=0.15/3, scale_12=0.15/3,
          sill_1=1, sill_2=1, nugget_1=0, nugget_2=0,
          smooth_1=0.5, smooth_12=0.5, smooth_2=0.5, pcol=0.3)
data = GeoSim(coordx=coords, corrmodel="Bi_matern", param=param)$data
# Empirical semi-(cross)variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=c(0.5,0.5,0.5))
# Variograms plots
par(mfrow=c(2,2))
plot(biv_vario$centers,biv_vario$variograms[1,],pch=20,xlab="h",ylim=c(0,1.2),
                                   ylab="",main=expression(gamma[11](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",
                                   ylab="",main=expression(gamma[12](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",ylab="",
                                   main=expression(gamma[21](h)))
plot(biv_vario$centers,biv_vario$variograms[2,],pch=20,xlab="h",,ylim=c(0,1.2),
                                   ylab="",main=expression(gamma[22](h)))
```

GeoWLS

cex.lab=.8)

WLS of Random Fields

Description

the function returns the parameters' estimates and the estimates' variances of a random field obtained by the weighted least squares estimator.

Usage

GeoWLS GeoWLS

Arguments

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data	A d -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array (n iid spatial realisations on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid). See GeoFit for details.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d -dimensional vector giving 1-dimension of spatial coordinates.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description (see GeoFit).
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details of GeoFit.
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
maxdist	A numeric value denoting the maximum distance, see Details and GeoFit.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation (see GeoFit.
model	String; the type of random field. Gaussian is the default, see GeoFit for the different types.
optimizer	String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the default.
numbins	A numeric value denoting the numbers of bins, see the Section Details
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see GeoFit).
weighted	Logical; if TRUE then the weighted least square estimator is considered. If FALSE (the default) then the classic least square is used.

Details

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

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Value

Returns an object of class WLS. An object of class WLS is a list containing at most the following components:

bins Adjacent intervals of grouped distances;

bint Adjacent intervals of grouped temporal separations

centers The centers of the bins;

coordx The vector or matrix of spatial coordinates;

coordy The vector of spatial coordinates; coordt The vector of temporal coordinates;

convergence A string that denotes if convergence is reached;

corrmodel The correlation model;

data The vector or matrix of data;
distance The type of spatial distance;
fixed The vector of fixed parameters;

iterations The number of iteration used by the numerical routine;

maxdist The maximum spatial distance used for the calculation of the variogram used in

least square estimation. If no spatial distance is specified then it is NULL;

maxtime The maximum temporal distance used for the calculation of the variogram used

in least square estimation. If no temporal distance is specified then it is NULL;

message Extra message passed from the numerical routines;

model The type of random fields;

numcoord The number of spatial coordinates;

numtime The number the temporal realisations of the random field;

param The vector of parameters' estimates; variograms The empirical spatial variogram; variogramt The empirical temporal variogram;

variogramst The empirical spatial-temporal variogram;

weighted A logical value indicating if its the weighted method;

wls The value of the least squares at the minimum.

Author(s)

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References

Cressie, N. A. C. (1993) Statistics for Spatial Data. New York: Wiley.

Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. Spring Verlang, New York.

See Also

GeoFit, optim

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Examples

```
library(GeoModels)
# Set the coordinates of the sites:
set.seed(211)
x <- runif(200, 0, 1)
set.seed(98)
y <- runif(200, 0, 1)
coords <- cbind(x,y)</pre>
### Example 1. Least square fitting of a Gaussian random field
### with exponential correlation.
###
# Set the model's parameters:
corrmodel <- "Exponential"</pre>
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.15/3
param <- list(mean=0,sill=sill, nugget=nugget, scale=scale)</pre>
# Simulation of the Gaussian random field:
set.seed(2)
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=param)$data</pre>
fixed=list(nugget=0, mean=mean)
start=list(scale=scale,sill=sill)
# Least square fitting of the random field:
fit <- GeoWLS(data=data,coordx=coords, corrmodel=corrmodel,</pre>
       fixed=fixed,start=start,maxdist=0.5)
# Results:
print(fit)
### Example 3. Least square fitting of a spatio-temporal
### Gaussian random field with double exponential correlation.
# Define the temporal sequence:
time <- seq(1, 10, 1)
mean <- 0
sill <- 1
scale_s <- 0.15/3
scale_t <- 2/3
param <- list(mean=0,scale_s=scale,scale_t=scale_t,sill=sill,nugget=nugget)</pre>
```

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Lik

Optimizes the Log Likelihood

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximization of the log-likelihood.

Usage

Arguments

bivariate	Logical; if TRUE then the data come from a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d -dimensional vector assigning 1-dimension of spatial coordinates.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
flagcor	A numeric vector of flags denoting which correlation parameters have to be estimated.
flagnuis	A numeric verctor of flags denoting which nuisance parameters have to estimated.

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fixed A numeric vector of parameters that will be considered as known values.

grid Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -

matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.

lower An optional named list giving the values for the lower bound of the space pa-

rameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of

the list must be the same of the names in the start list.

model Numeric; the id value of the density associated to the likelihood objects.

namescorr String; the names of the correlation parameters.

String; the names of the nuisance parameters.

namesparam String; the names of the parameters to be maximised.

numcoord Numeric; the number of coordinates.

numpairs Numeric; the number of pairs.

numparamcor Numeric; the number of the correlation parameters.

Numeric; the number of temporal observations.

mdecomp String; the type of matrix decomposition used in the simulation. Default is

cholesky. The other possible choices is svd (Singular values decomposition).

optimizer String; the optimization algorithm (see optim for details). Nelder-Mead is the

default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the

case of one-dimensional optimization, the function optimize is used.

onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is

computed without optimizing. FALSE is the default.

parallel Logical; if TRUE optmization is performed using optimParallel using the maxi-

mum number of cores, when optimizer is L-BFGS-B.FALSE is the default.

param A numeric vector of parameters.

sparse Logical; if TRUE then maximum likelihood is computed using sparse matrices

algorithms. FALSE is the default.

radius Numeric; the radius of the sphere when considering data on a sphere.

Numeric: vector of number of location sites for each temporal instants

setup A List of useful components for the estimation based on the maximum tapered

likelihood.

spacetime Logical; if the random field is spatial (FALSE) or spatio-temporal (TRUE).

variest Logical; if TRUE the estimate' variances and standard errors are returned. FALSE

is the default.

taper String; the name of the taper correlation function.

type String; the type of the likelihood objects. If Pairwise (the default) then the

marginal composite likelihood is formed by pairwise marginal likelihoods.

upper An optional named list giving the values for the upper bound of the space pa-

rameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names

of the list must be the same of the names in the start list.

X Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

Author(s)

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See Also

GeoFit

MatDecomp Matrix decomposition

Description

Matrix decomposition.

Usage

MatDecomp(mtx, method)

Arguments

mtx numeric; a square positive or semipositive definite matrix.

method string; the type of matrix decomposition. Two possible choices: cholesky and

svd.

Author(s)

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MatSqrt, MatInv, MatLogDet

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

Description

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

Usage

MatSqrt(mat.decomp,method)
MatInv(mat.decomp,method)
MatLogDet(mat.decomp,method)

Arguments

mat.decomp numeric; a matrix decomposition.

method string; the type of matrix decomposition. Two possible choices: cholesky and

svd.

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Author(s)

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See Also

MatDecomp

Examples

```
library(GeoModels)
### Example 1. Inverse of Covariance matrix associated to
### a Matern correlation model
# Define the spatial-coordinates of the points:
x <- runif(15, 0, 1)
y <- runif(15, 0, 1)
coords <- cbind(x,y)</pre>
# Matern Parameters
param=list(smooth=0.5, sill=1, scale=0.2, nugget=0)
a=matrix <- GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param)</pre>
## decomposition with cholesky method
b=MatDecomp(a$covmat,method="cholesky")
## inverse of covariance matrix
inverse=MatInv(b,method="cholesky")
```

NuisParam

Lists the Nuisance Parameters of a Random Field

Description

Subroutine called by InitParam and other procedures. The procedure returns a list with the nuisance parameters of a given random field model.

Usage

```
NuisParam(model, bivariate=FALSE,num_betas=c(1,1))
```

Arguments

model	String; the name of a random field.
bivariate	Logical; if FALSE (the default) the correlation model is univariate spatial or spatial-temporal. Otherwise is bivariate.
num_betas	Numerical; the number of mean parameters in the linear specification (default is 1)

Prscores 93

Author(s)

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See Also

GeoFit

Examples

```
library(GeoModels)
NuisParam("Gaussian")
NuisParam("Binomial")
NuisParam("Weibull",num_betas=2)
NuisParam("SkewGaussian", num_betas=3)
NuisParam("SinhAsinh")
NuisParam("StudentT")
## note that in the bivariate case sill and nugget are considered as correlation parameteres
NuisParam("Gaussian", bivariate=TRUE)
```

Prscores

Computation of three predictive scores: RMSE, LSCORE, CRPS for spatial, spatiotemporal and bivariate Gaussian RF.

Description

The function computes RMSE, LSCORE, CRPS predictive scores.

Usage

```
Prscores(data, method="cholesky", matrix)
```

Arguments

data A d-dimensional vector (a single spatial realisation) or a $\mathbf{a}(t \times d)$ -matrix (a single spatial-temporal realisation). or a $\mathbf{a}(2 \times d)$ -matrix (a single bivariate realisation). method String; the type of matrix decomposition used in the computation of the predictive scores. Default is cholesky. The other possible choices is svd.

An object of class matrix. See the Section **Details**.

Details

matrix

For a given covariance matrix object (GeoCovmatrix) and a given spatial, spatiotemporal or bivariare realization from a Gaussian random field, the function computes three predictive scores.

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Value

Returns a list containing the following informations:

RMSE Root-mean-square error predictive score

LSCORE Logarithmic predictive score

CRPS Continuous ranked probability predictive score

Author(s)

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References

Zhang H. and Wang Y. (2010). *Kriging and cross-validation for massive spatial data*. Environmetrics, **21**, 290–304.

Gneiting T. and Raftery A. Strictly Proper Scoring Rules, Prediction, and Estimation. Journal of the American Statistical Association, 102

See Also

GeoCovmatrix

Examples

StartParam 95

StartParam	Initializes the Parameters for Estimation Procedures	

Description

Subroutine called by the fitting procedures. The procedure initializes the parameters for the fitting procedure.

Usage

Arguments

coordx	A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section Details .
fcall	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
fixed	A named list giving the values of the parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the composite likelihood.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
model	String; the density associated to the likelihood objects. Gaussian is the default.
n	Numeric; number of trials for binomial random fields.
param	A numeric vector of parameter values required in the simulation procedure of random fields.

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parscale	A numeric vector of scaling factor to improve the maximizing procedure, see optim.
paramrange	A numeric vector of parameters ranges, see optim.
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure.
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see Details).
type	String; the type of likelihood objects. Temporary value set to be "WLeast-Square" (weighted least-square) in order to compute the starting values.
typereal	String; the real type of likelihood objects. See GeoFit.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section Details .
weighted	Logical; if TRUE the likelihood objects are weighted, see GeoFit.
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
Χ	Numeric; Matrix of space-time covariates.
memdist	Logical; if TRUE then the distances in the composite likelihood are computed before the optmization.

Author(s)

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See Also

GeoFit

Description

A matrix containing daily wind speeds, in kilometers per hour, from 1961 to 1978 at 12 sites in Ireland.

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Usage

```
data(irishwinds)
```

Format

A (6574×11) -matrix containing wind speed observations.

Source

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

winds.coords

Weather Stations of the Irish Daily Wind Speeds

Description

A data frame containing information about the weather stations where the data are recorded in Ireland.

Usage

```
data(irishwinds)
```

Format

A data frame containing site - the name of the city (character), abbr - the abbrevation (character), elev - the elevation (numeric), lat - latitude (numeric) and lon - longitude.

Source

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

WlsStart

Computes Starting Values based on Weighted Least Squares

Description

Subroutine called by GeoFit. The function returns opportune starting values for the composite-likelihood fitting procedure based on weighted least squares.

Usage

```
WlsStart(coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance, fcall,
    fixed, grid,likelihood, maxdist, maxtime, model, n, param,
    parscale, paramrange, radius,start, taper, tapsep, type, varest,
    vartype, weighted, winconst,winconst_t, winstp_t, winstp,X,memdist)
```

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Arguments

coordx A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions

of coordinates or a numeric vector assigning 1-dimension of coordinates.

coordy A numeric vector assigning 1-dimension of coordinates; coordy is interpreted

only if coordx is a numeric vector otherwise it will be ignored.

coordt A numeric vector assigning 1-dimension of temporal coordinates.

coordx_dyn A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial

coordinates. Optional argument, the default is NULL

corrmodel String; the name of a correlation model, for the description.

data A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.

distance String; the name of the spatial distance. The default is Eucl, the euclidean

distance. See the Section Details.

fcall String; "fitting" to call the fitting procedure and "simulation" to call the simula-

tion procedure.

fixed A named list giving the values of the parameters that will be considered as

known values.

grid Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -

matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.

likelihood String; the configuration of the composite likelihood.

maxdist Numeric; an optional positive value indicating the maximum spatial distance

considered in the composite-likelihood computation.

maxtime Numeric; an optional positive value indicating the maximum temporal separa-

tion considered in the composite-likelihood computation.

model String; the name of the model. Here the default is NULL.

n Numeric; number of trials in a binomial random field.

param A numeric vector of parameter values required in the simulation procedure of

random fields.

parscale A numeric vector with scaling values for improving the maximisation routine.

paramrange A numeric vector with the range of the parameter space.

radius Numeric; a value indicating the radius of the sphere when using the great circle

distance. Default value is the radius of the earth in Km (i.e. 6371)

start A numeric vector with starting values.

taper String; the name of the type of covariance matrix. It can be Standard (the

default value) or Tapering for taperd covariance matrix.

tapsep Numeric; an optional value indicating the separabe parameter in the space time

quasi taper (see Details).

type String; the type of estimation method.

varest Logical; if TRUE the estimates' variances and standard errors are returned. FALSE

is the default.

vartype String; the type of estimation method for computing the estimate variances, see

the Section Details.

weighted Logical; if TRUE the likelihood objects are weighted, see GeoFit.

winconst Numeric; a positive value for computing the spatial sub-window in the sub-

sampling procedure.

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Numeric; a value in (0,1] for defining the the proportion of overlapping in the spatial sub-sampling procedure.

Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.

Numeric; a value in (0,1] for defining the the proportion of overlapping in the temporal sub-sampling procedure.

Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

Memdist Logical; if TRUE then the distances in the composite likelihood are computed before the optmization.

Author(s)

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See Also

GeoFit.

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