

# Package ‘GeoModels’

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**Title** A Package for Geostatistical Gaussian and non Gaussian Data Analysis

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**Depends** R (>= 2.12.0)

**Description** This package provides a set of procedures for a) simulation and estimation of some spatial and spatio-temporal random fields using standard likelihood and a likelihood approximation method called composite likelihood and b) prediction using best linear unbiased prediction. Spatio (temporal) bivariate data estimation involves estimation of both regression and covariance parameters.

Gaussian and some non Gaussian Random fields can be analyzed using the GeoModels package. Among them, Weibull, logGaussian, skewGaussian, T, binomial, negative binomial and circular random fields can be analyzed.

**Imports** methods,

**Suggests** spam, scatterplot3d, dfoptim, dotCall64, fields, mapproj,  
gpuR, gsl, optimParallel, parallel, pbivnorm, plot3D, shape,  
numDeriv, hypergeo, sphereplot, ucminf, VGAM

**License** GPL (>= 2)

**URL** <https://vmoprojs.github.io/GeoModels-page/>

**Repository** GitHub

**Encoding** UTF-8

**BugReports** <https://github.com/vmoprojs/GeoModels/issues>

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anomalies

*Annual precipitation anomalies in U.S.*

---

## Description

A (7252x3)-matrix containing lon/lat and yearly total precipitation anomalies registered at 7.352 location sites in USA. For more details see [http://www.image.ucar.edu/Data/precip\\_tapering/](http://www.image.ucar.edu/Data/precip_tapering/).

## Usage

```
data(anomalies)
```

## Format

A numerical matrix of dimension 7252x3.

## Source

Kaufman, C.G., Schervish, M.J., Nychka, D.W. (2008) Covariance tapering for likelihood-based estimation in large spatial data sets. *Journal of the American Statistical Association, Theory & Methods*, **103**, 1545–1555.

---

CheckBiv*Checking Bivariate covariance models*

---

**Description**

The procedure control if the correlation model is bivariate.

**Usage**

```
CheckBiv(numbermodel)
```

**Arguments**

numbermodel      numeric; the number associated to a given correlation model.

**Value**

Returns TRUE or FALSE depending if the correlation model is bivariate or not.

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**Examples**

```
library(GeoModels)
CheckBiv(CkCorrModel("Bi_matern_sep"))
```

---

CheckDistance*Checking Distance*

---

**Description**

The procedure controls the type of distance.

**Usage**

```
CheckDistance(distance)
```

**Arguments**

distance      String; the type of distance, for the description see [GeoCovmatrix](#). Default is Eucl. Other possible values are Geod and Chor that is euclidean, geodesic and chordal distance.

**Value**

Returns 0,1,2 for euclidean,geodesic, chordal distances respectively. Otherwise returns NULL.

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**See Also**

[GeoFit](#)

---

CheckSph

*Checking if a covariance is valid only on sphere*

---

**Description**

Subroutine called by InitParam. The procedure controls if a covariance model is valid only on the sphere.

**Usage**

CheckSph(numbermodel)

**Arguments**

numbermodel      Numeric; the code number for the covariance model.

**Value**

Returns TRUE or FALSE

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

---

CheckST

*Checking SpaceTime covariance models*

---

**Description**

The procedure control if the correlation model is spacetime.

**Usage**

CheckST(numbermodel)

**Arguments**

numbermodel      numeric; the number associated to a given correlation model.

**Value**

Returns TRUE or FALSE depending if the correlation model is spatiotemporal or not.

**Author(s)**

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**Examples**

```
library(GeoModels)
CheckST(CkCorrModel("gneiting"))
```

---

CkCorrModel	<i>Checking Correlation Model</i>
-------------	-----------------------------------

---

**Description**

The procedure controls if the correlation model inserted is correct.

**Usage**

```
CkCorrModel(corrmodel)
```

**Arguments**

corrmodel          String; the name of a correlation model, for the description see [GeoCovmatrix](#).

**Value**

Return a number associated to a given correlation model if the model is considered in the package.  
Otherwise return NULL.

**Author(s)**

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---

CkInput	<i>Checking Input</i>
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---

**Description**

Subroutine called by the fitting procedures. The procedure controls the the validity of the input inserted by the users.

**Usage**

```
CkInput(coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance,
        fcall, fixed, grid, likelihood, maxdist, maxtime,
        model, n, optimizer, param, radius,
        start, taper, tapsep, type, varest, vartype,
        weighted, X)
```

**Arguments**

coordx	A numeric ( $d \times 2$ )-matrix (where $d$ is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
corrmodel	String; the name of a correlation model, for the description see <a href="#">GeoFit</a> .
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fcall	String; Fitting to call the fitting procedure and simulation to call the simulation.
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if <code>list(nugget=0)</code> the nugget effect is ignored.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the composite likelihood. Marginal is the default.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag separation in the composite-likelihood.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
model	String; the density associated to the likelihood objects. Gaussian is the default.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
optimizer	String; the optimization algorithm (see <a href="#">optim</a> for details). 'Nelder-Mead' is the default.
param	A numeric vector of parameters, needed only in simulation. See <a href="#">GeoSim</a> .
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default.
taper	String; the name of the tapered correlation function.
tapsep	Numeric; an optional value indicating the separable parameter in the space time quasi taper (see <b>Details</b> ).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see <a href="#">GeoFit</a> .
weighted	Logical; if TRUE the likelihood objects are weighted. If FALSE (the default) the composite likelihood is not weighted.
X	Numeric; Matrix of space-time covariates in the linear mean specification.

**Author(s)**

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**See Also**

[GeoFit](#)

---

CkLikelihood

*Checking Composite-likelihood Type*


---

**Description**

Subroutine called by InitParam. The procedure controls the type of the composite-likelihood inserted by the users.

**Usage**

CkLikelihood(likelihood)

**Arguments**

likelihood      String; the configuration of the composite likelihood. Marginal is the default.

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**See Also**

[GeoFit](#)

---

CkModel

*Checking Random Field type*


---

**Description**

Subroutine called by InitParam. The procedure controls the type of random field inserted by the users.

**Usage**

CkModel(model)

**Arguments**

model      String; the density associated to the likelihood objects. Gaussian is the default.

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**See Also**

[GeoFit](#)

---

CkType

*Checking Likelihood Objects*


---

**Description**

Subroutine called by InitParam. The procedure controls the type of likelihood objects inserted by the users.

**Usage**

CkType(type)

**Arguments**

type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
------	--

**Author(s)**

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**See Also**

[GeoFit](#)

---

CkVarType

*Checking Variance Estimates Type*


---

**Description**

Subroutine called by InitParam. The procedure controls the method used to compute the estimates' variances.

**Usage**

CkVarType(type)

**Arguments**

type	String; the method used to compute the estimates' variances. If SubSamp the estimates' variances are computed by the sub-sampling method, see <a href="#">GeoFit</a> .
------	--



**Author(s)**

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**See Also**

[GeoFit](#)

---

CompLik

*Optimizes the Composite log-likelihood*


---

**Description**

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

**Usage**

```
CompLik(bivariate, coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance,
        flagcorr, flagnuis, fixed, GPU, grid, likelihood, local, lower,
        model, n, namescorr, namesnuis, namesparam,
        numparam, numparamcorr, optimizer, onlyvar, parallel, param,
        spacetime, type, upper, varest, vartype,
        weigthed, winconst, winstp, winconst_t, winstp_t, ns, X, sensitivity)
```

**Arguments**

bivariate	Logical; if TRUE then the data come from a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric ( $d \times 2$ )-matrix (where $d$ is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
flagcorr	A numeric vector of binary values denoting which parameters of the correlation function will be estimated.
flagnuis	A numeric vector of binary values denoting which nuisance parameters will be estimated.
fixed	A numeric vector of parameters that will be considered as known values.
GPU	Numeric; if NULL (the default) no GPU computation is performed.

grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the compositelikelihood, see <a href="#">GeoFit</a> .
local	Numeric; number of local work-items of the GPU
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
model	Numeric; the id value of the density associated to the likelihood objects.
n	Numeric; number of trials in a binomial random fields.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numparam	Numeric; the number of parameters to be maximised.
numparamcorr	Numeric; the number of correlation parameters.
optimizer	String; the optimization algorithm (see <a href="#">optim</a> for details). Nelder-Mead is the default. Other possible choices are ucminf, nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
parallel	Logical; if TRUE optimization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.
param	A numeric vector of parameters' values.
spacetime	Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see <a href="#">GeoFit</a> .
weigthed	Logical; if TRUE then decreasing weights coming from a compactly supported correlation function with compact support maxdist (maxtime) are used.
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
ns	Numeric; Number of (dynamical) temporal instants.
X	Numeric; Matrix of space-time covariates in the linear mean specification.
sensitivity	Logical; if TRUE then the sensitivity matrix is computed

**Author(s)**

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**See Also**

[GeoFit](#)

---

CorrelationPar

*Lists the Parameters of a Correlation Model*

---

**Description**

Subroutine called by InitParam and other procedures. The procedure returns a list with the parameters of a given correlation model.

**Usage**

CorrelationPar(corrmodel)

**Arguments**

corrmodel      Integer; an integer associated to a given correlation model.

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**See Also**

[GeoFit](#)

---

CorrParam

*Lists the Parameters of a Correlation Model*

---

**Description**

The procedure returns a list with the parameters of a given correlation model.

**Usage**

CorrParam(corrmodel)

**Arguments**

corrmodel      String; the name of a correlation model.

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**See Also**

[GeoCovmatrix](#)

**Examples**

```
require(GeoModels)
#####
###
### Example 1. Parameters of the Matern model
###
#####

CorrParam("Matern")

#####
###
### Example 2. Parameters of the Generalized Wendland model
###
#####

CorrParam("GenWend")

#####
###
### Example 3. Parameters of the Generalized Wendland model
###
#####

CorrParam("GenCauchy")

#####
###
### Example 4. Parameters of the space time Gneiting model
###
#####

CorrParam("Gneiting")

#####
###
### Example 5. Parameters of the bi-Matern separable model
###
#####

CorrParam("Bi_Matern_sep")
```

---

 DeviceInfo

*Prints Device Information*


---

### Description

Prints the device details available in your computer. Device name, Max compute units, whether it supports double precision, among others.

### Usage

```
DeviceInfo()
```

### Details

The user can take this information into account so that the local parameter is set up in GeoFit when GPU computation is chosen.

### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

### Examples

```
library(GeoModels)
DeviceInfo()
```

---

 GeoCovariogram

*Computes fitted covariance and/or variogram*


---

### Description

The procedure computes and plots covariance or variogram estimated fitting a Gaussian, and non Gaussian spatio (temporal) bivariate random fields. Allows to add the empirical estimates in order to compare them with the fitted model.

### Usage

```
GeoCovariogram(fitted, distance="Eucl", answer.cov=FALSE,
               answer.vario=FALSE, answer.range=FALSE, fix.lags=NULL,
               fix.lagt=NULL, show.cov=FALSE, show.vario=FALSE,
               show.range=FALSE, add.cov=FALSE, add.vario=FALSE,
               pract.range=95, vario, ...)
```

**Arguments**

<code>fitted</code>	A fitted object obtained from the <a href="#">GeoFit</a> or <a href="#">GeoWLS</a> procedures.
<code>distance</code>	String; the name of the spatial distance. The default is <code>Eucl</code> , the euclidean distance. See <a href="#">GeoFit</a> .
<code>answer.cov</code>	Logical; if TRUE a vector with the estimated covariance function is returned; if FALSE (the default) the covariance is not returned.
<code>answer.vario</code>	Logical; if TRUE a vector with the estimated variogram is returned; if FALSE (the default) the variogram is not returned.
<code>answer.range</code>	Logical; if TRUE the estimated practical range is returned; if FALSE (the default) the practical range is not returned.
<code>fix.lags</code>	Integer; a positive value denoting the spatial lag to consider for the plot of the temporal profile.
<code>fix.lagt</code>	Integer; a positive value denoting the temporal lag to consider for the plot of the spatial profile.
<code>show.cov</code>	Logical; if TRUE the estimated covariance function is plotted; if FALSE (the default) the covariance function is not plotted.
<code>show.vario</code>	Logical; if TRUE the estimated variogram is plotted; if FALSE (the default) the variogram is not plotted.
<code>show.range</code>	Logical; if TRUE the estimated practical range is added on the plot; if FALSE (the default) the practical range is not added.
<code>add.cov</code>	Logical; if TRUE the vector of the estimated covariance function is added on the current plot; if FALSE (the default) the covariance is not added.
<code>add.vario</code>	Logical; if TRUE the vector with the estimated variogram is added on the current plot; if FALSE (the default) the correlation is not added.
<code>pract.range</code>	Numeric; the percent of the sill to be reached.
<code>vario</code>	A Variogram object obtained from the <a href="#">GeoVariogram</a> procedure.
<code>...</code>	other optional parameters which are passed to plot functions.

**Value**

The returned object is eventually a list with:

<code>covariance</code>	The vector of the estimated covariance function;
<code>variogram</code>	The vector of the estimated variogram function;

**Author(s)**

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**References**

- Cressie, N. A. C. (1993) *Statistics for Spatial Data*. New York: Wiley.
- Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Springer Verlag, New York.

**See Also**

[GeoFit](#).

**Examples**

```

library(GeoModels)
library(scatterplot3d)

#####
###
### Example 1. Plot of fitted covariance and fitted
### and empirical variograms from a Gaussian RF
### with Matern correlation.
###
#####
set.seed(21)
# Set the coordinates of the points:
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel = "Matern"
model = "Gaussian"
mean = 0
sill = 1
nugget = 0
scale = 0.2/3
smooth=0.5

param=list(mean=mean,sill=sill, nugget=nugget, scale=scale, smooth=smooth)
# Simulation of the Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data

start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget,smooth=smooth)
# Maximum composite-likelihood fitting of the Gaussian random field:
fit = GeoFit(data=data,coordx=coords, corrmodel=corrmodel,model=model,
             likelihood="Marginal",type='Pairwise',start=start,
             optimizer="BFGS", fixed=fixed,maxdist=0.05)

# Empirical estimation of the variogram:
vario = GeoVariogram(data=data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE,show.vario=TRUE, vario=vario,pch=20)

#####
###
### Example 2. Plot of fitted covariance and fitted
### and empirical variograms from a Binomial
### RF with exponential correlation.
###
#####
set.seed(2111)

model="Binomial";n=20
# Set the coordinates of the points:
x = runif(500, 0, 1)
y = runif(500, 0, 1)

```

```

coords=cbind(x,y)

# Set the model's parameters:
corrmodel = "exponential"
mean = 0
sill = 1
nugget = 0
scale = 0.2/3

param=list(mean=mean,sill=sill, nugget=nugget, scale=scale)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param,n=n)$data

start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget)
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
             likelihood="Marginal",type='Pairwise',start=start,n=n,
             optimizer="BFGS", fixed=fixed,maxdist=0.03)

# Empirical estimation of the variogram:
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE,show.vario=TRUE, vario=vario,pch=20)

#####
###
### Example 3. Plot of fitted covariance and fitted
### and empirical variograms from a RF
### RF with Wend0 correlation.
###
#####
set.seed(211)

model="Gamma";shape=4
# Set the coordinates of the points:
x = runif(700, 0, 1)
y = runif(700, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel = "Wend0"
mean = 0
sill = 1
nugget = 0
scale = 0.3
power2=4

param=list(mean=mean,sill=sill, nugget=nugget, scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data

start=list(mean=0,scale=scale,shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)

```



```

# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
             likelihood="Marginal",type='Pairwise',start=start,
             optimizer="BFGS", fixed=fixed,maxdist=0.03)

# Empirical estimation of the variogram:
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE,show.vario=TRUE, vario=vario,pch=20)

#####
###
### Example 4. Plot of fitted and empirical variograms
### from a space time Gaussian random fields
### with double exponential correlation.
###
#####
set.seed(92)
# Define the spatial-coordinates of the points:
x = runif(50, 0, 1)
y = runif(50, 0, 1)
coords=cbind(x,y)
# Define the temporal sequence:
time = seq(0, 15, 1)

# Simulation of the spatio-temporal Gaussian random field:
data = GeoSim(coordx=coords, coordt=time, corrmodel="Exp_Exp",param=list(mean=mean,
                                nugget=nugget,scale_s=0.5/3,scale_t=2/2,sill=sill))$data

fixed=list(nugget=0, mean=0)
start=list(scale_s=0.2, scale_t=0.5, sill=1)
# Maximum composite-likelihood fitting of the space-time Gaussian random field:
fit = GeoFit(data, coordx=coords, coordt=time, corrmodel="Exp_Exp", maxtime=2,
             maxdist=0.1, likelihood="Marginal", type="Pairwise",
             optimizer="BFGS",fixed=fixed, start=start)

# Empirical estimation of spatio-temporal covariance:
vario = GeoVariogram(data,coordx=coords, coordt=time, maxtime=5,maxdist=0.5)

# Plot of the fitted space-time variogram
GeoCovariogram(fit,vario=vario,show.vario=TRUE)

# Plot of covariance, variogram and spatio and temporal profiles:
GeoCovariogram(fit,vario=vario,fix.lagt=1,fix.lags=1,show.vario=TRUE,pch=20)

#####
###
### Example 5. Plot of parametric and empirical variograms
### estimated from a Bivariate Gaussian random fields with
### Matern correlation.
###
#####

```

```

# Simulation of a bivariate spatial Gaussian random field:
set.seed(892)
# Define the spatial-coordinates of the points:
x = runif(200, -1, 1)
y = runif(200, -1, 1)
coords=cbind(x,y)

# Simulation of a bivariate Gaussian Random field
# with matern (cross) covariance function
scale_1 = 0.25/3
scale_2 = 0.2/3
scale_12 = 0.15/3
sill_1=1
sill_2=1
smooth=0.5
pcol=0.3
param=list(mean_1=0,mean_2=0,scale_1=scale_1,scale_2=scale_2,scale_12=scale_12,
            sill_1=sill_1,sill_2=sill_2,nugget_1=0,nugget_2=0,
            smooth_1=smooth,smooth_12=smooth,smooth_2=smooth,pcol=pcol)
data = GeoSim(coordx=coords, corrmodel="Bi-Matern", param=param)$data

# Empirical bivariate variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=c(1,1,1))

# selecting fixed and estimating parameters
fixed=list(mean_1=0,mean_2=0,nugget_1=0,nugget_2=0,
            smooth_1=smooth,smooth_12=smooth,smooth_2=smooth)
start=list(sill_1=var(data[,1]),sill_2=var(data[,2]),
            scale_1=scale_1,scale_2=scale_2,scale_12=scale_12,
            pcol=cor(data[,1],data[,2]))

# Maximum likelihood fitting of the bivariate random field:
fit= GeoFit(data, coordx=coords, corrmodel="Bi-Matern",likelihood="Marginal",
            optimizer="BFGS", type="Pairwise",
            start=start,fixed=fixed,maxdist=c(0.1,0.1,0.1))

GeoCovariogram(fit, vario=biv_vario,show.vario=TRUE,pch=20)

```

## Description

The function computes the covariance matrix associated to a spatial or spatio-temporal or a bivariate spatial Gaussian or non Gaussian random field with given covariance model and a set of spatial location sites and temporal instants.

## Usage

```

GeoCovmatrix(coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel,
             distance="Eucl", grid=FALSE, maxdist=NULL, maxtime=NULL,

```

```
model="Gaussian", n=1, param, radius=6371, sparse=FALSE,
taper=NULL, tapsep=NULL, type="Standard", X=NULL)
```

### Arguments

coordx	A numeric ( $d \times 2$ )-matrix (where $d$ is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a ( $d \times 2$ )-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. At the moment implemented only for the Gaussian case. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $T$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See <a href="#">GeoFit</a> .
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid). See <a href="#">GeoFit</a> .
maxdist	Numeric; an optional positive value indicating the marginal spatial compact support in the case of tapered covariance matrix. See <a href="#">GeoFit</a> .
maxtime	Numeric; an optional positive value indicating the marginal temporal compact support in the case of spacetime tapered covariance matrix. See <a href="#">GeoFit</a> .
n	Numeric; the number of trials in a binomial random fields. Default is 1.
model	String; the type of RF. See <a href="#">GeoFit</a> .
param	A list of parameter values required for the covariance model.
radius	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE the function return an object of class spam. This option should be used when a parametric compactly supporte covariance is used. Default is FALSE.
taper	String; the name of the taper correlation function if type is Tapering, see the Section <b>Details</b> .
tapsep	Numeric; an optional value indicating the separabe parameter in the space-time non separable taper or the colocated correlation parameter in a bivariate spatial taper (see <b>Details</b> ).
type	String; the type of covariance matrix Standard (the default) or Tapering for tapered covariance matrix.
X	Numeric; Matrix of space-time covariates.

## Details

In the spatial case, the covariance matrix of the random vector

$$[Z(s_1), \dots, Z(s_n)]^T$$

with a specific spatial covariance model is computed. Here  $n$  is the number of the spatial location sites.

In the space-time case, the covariance matrix of the random vector

$$[Z(s_1, t_1), Z(s_2, t_1), \dots, Z(s_n, t_1), \dots, Z(s_n, t_m)]^T$$

with a specific space time covariance model is computed. Here  $m$  is the number of temporal instants.

In the bivariate case, the covariance matrix of the random vector

$$[Z_1(s_1), Z_2(s_1), \dots, Z_1(s_n), Z_2(s_n)]^T$$

with a specific spatial bivariate covariance model is computed.

The location site  $s_i$  can be a point in the  $d$ -dimensional euclidean space with  $d = 2$  or a point (given in lon/lat degree format) on a sphere of arbitrary radius.

Here there is the list of all the implemented space and space-time and bivariate correlation models. The argument `param` is a list including all the parameters of a given correlation model specified by the argument `corrmodel`. For each correlation model one can check the associated correlation parameters using `CorrParam`. In what follows  $\kappa > 0$ ,  $\beta > 0$ ,  $\alpha, \alpha_s, \alpha_t \in (0, 2]$ , and  $\gamma \in [0, 1]$ . The associated parameters in the argument `param` are `smooth`, `power2`, `power`, `power_s`, `power_t` and `sep` respectively. Moreover let  $1(A) = 1$  when  $A$  is true and 0 otherwise.

- Spatial correlation models:

1. *Cauchy* defined as:

$$R(h) = (1 + h^2)^{-\beta/2}$$

It is a special case of the Gencauchy model.

2. *Exp* defined as:

$$R(h) = e^{-h}$$

This model is a special case of the Matern and the Stable model.

3. *GenCauchy* (generalised *Cauchy*) defined as:

$$R(h) = (1 + h^\alpha)^{-\beta/\alpha}$$

If  $h$  is the geodesic distance then  $\alpha \in (0, 1]$ .

4. *Matern* defined as:

$$R(h) = 2^{1-\kappa} \Gamma(\kappa)^{-1} h^\kappa K_\kappa(h)$$

If  $h$  is the geodesic distance then  $\kappa \in (0, 0.5]$

5. *Stable* defined as:

$$R(h) = e^{-h^\alpha}$$

If  $h$  is the geodesic distance then  $\alpha \in (0, 1]$ .

6. *Wave* defined as:

$$R(h) = \sin(h)/h$$

This model is valid only for dimensions less than or equal to 3.

7. *Wend0* defined as:

$$R(h) = (1 - h)^\mu 1(h \in [0, 1])$$

where  $\mu \geq 0.5(d + 1)$ . If  $h$  is the geodesic distance then  $\mu \geq 2$ .

8. *Wend1* defined as:

$$R(h) = (1 - h)^{\mu+1} (1 + (\mu + 1)h) 1(h \in [0, 1])$$

where  $\mu \geq 0.5(d + 1) + 1$ . If  $h$  is the geodesic distance then  $\mu \geq 4$ .

9. *Wend2* defined as:

$$R(h) = (1 - h)^{\mu+2} (1 + (\mu + 2)h + (1/3)((\mu + 1)^2 - 1)h^2) 1(h \in [0, 1])$$

where  $\mu \geq 0.5(d + 1) + 2$ . If  $h$  is the geodesic distance then  $\mu \geq 6$ .

10. *GenWend* (Generalized Wendland) defined as:

$$R(h) = \int_h^1 [(1 - x)^{\mu-1} (x^2 - h^2)^{\kappa-1} 1(h \in [0, 1])] dx / B(2\kappa + 1, \mu)$$

where  $\mu \geq 0.5(d + 1) + \kappa$ . The cases  $\kappa = 0, 1, 2$  correspond to the *Wend0*, *Wend1* and *Wend2* respectively.

11. *Multiquadric* defined as:

$$R(h) = (1 - \alpha 0.5)^{2\beta} / (1 + (\alpha 0.5)^2 - \alpha \cos(h))^\beta, \quad h \in [0, \pi]$$

This model is valid on the unit sphere and  $h$  is the geodesic distance.

12. *Sinpower* defined as:

$$R(h) = 1 - (\sin(h/2))^\alpha, \quad h \in [0, \pi]$$

This model is valid on the unit sphere and  $h$  is the geodesic distance.

13. *Smoke* defined as:

$$R(h) = K * 1F2(1/\alpha, 1/\alpha + 0.5, 2/\alpha + 0.5 + \kappa), \quad h \in [0, \pi]$$

where  $K = (\Gamma(a)\Gamma(i))/\Gamma(i)\Gamma(o)$ . This model is valid on the unit sphere and  $h$  is the geodesic distance.

- Spatio-temporal correlation models.

- Non-separable models:

1. *Gneiting* defined as:

$$R(h, u) = e^{-h^{\alpha_s} / ((1 + u^{\alpha_t})^{0.5\gamma\alpha_s})} / (1 + u^{\alpha_t})$$

2. *Gneiting\_GC*

$$R(h, u) = e^{-u^{\alpha_t} / ((1 + h^{\alpha_s})^{0.5\gamma\alpha_t})} / (1 + h^{\alpha_s})$$

where  $h$  can be both the euclidean and the geodesic distance

3. *Iacocesare*

$$R(h, u) = (1 + h^{\alpha_s} + u_t^\alpha)^{-\beta}$$

4. *Porcu*

$$R(h, u) = (0.5(1 + h^{\alpha_s})^\gamma + 0.5(1 + u^{\alpha_t})^\gamma)^{-\gamma^{-1}}$$

5. *Porcu1*

$$R(h, u) = (e^{-h^{\alpha_s} (1 + u^{\alpha_t})^{0.5\gamma\alpha_s}}) / ((1 + u^{\alpha_t})^{1.5})$$

6. *Stein*

$$R(h, u) = (h^{\psi(u)} K_{\psi(u)}(h)) / (2^{\psi(u)} \Gamma(\psi(u) + 1))$$

where  $\psi(u) = \nu + u^{0.5\alpha_t}$

7. *Wenx\_space*,  $x = 0, 1, 2$  defined as:

$$R(h, u) = \phi(u)^{3.5+2x} \text{Wenx}(h/\phi(u), \mu_s), \quad x = 0, 1, 2$$

where  $\phi(u) = (1 + u^{0.5\alpha_t})^{-\gamma}$ ,  $0 < \gamma \leq \alpha_t/2$ ,  $\mu_s \geq 0.5(d+5) + x$ .

8. *Wenx\_time*,  $x = 0, 1, 2$  defined as:

$$R(h, u) = \phi(h)^{3.5+2x} \text{Wenx}(u/\phi(h); \mu_t), \quad x = 0, 1, 2$$

where  $\phi(h) = (1 + h^{0.5\alpha_s})^{-\gamma}$ ,  $0 < \gamma \leq \alpha_s/2$ ,  $\mu_t \geq 0.5(d+5) + x$ .

9. *Multiquadric\_st* defined as:

$$R(h, u) = ((1 - 0.5\alpha_s)^2 / (1 + (0.5\alpha_s)^2 - \alpha_s\psi(u)\cos(h)))^{\alpha_s}, \quad h \in [0, \pi]$$

where  $\psi(u) = (1 + (u/a_t)^{\alpha_t})^{-1}$ . This model is valid on the unit sphere and  $h$  is the geodesic distance.

10. *Sinpower\_st* defined as:

$$R(h, u) = (e^{\alpha_s \cos(h)\psi(u)/a_s} (1 + \alpha_s \cos(h)\psi(u)/a_s)) / k$$

where  $\psi(u) = (1 + (u/a_t)^{\alpha_t})^{-1}$  and  $k = (1 + \alpha_s/a_s) \exp(\alpha_s/a_s)$ ,  $h \in [0, \pi]$  This model is valid on the unit sphere and  $h$  is the geodesic distance.

## – Separable models.

Space-time separable correlation models are easily obtained as the product of a spatial and a temporal correlation model, that is

$$R(h, u) = R(h)R(u)$$

Several combinations are possible:

1. *Exp\_Exp* defined as:

$$R(h, u) = \text{Exp}(h)\text{Exp}(u)$$

2. *Matern\_Matern* defined as:

$$R(h, u) = \text{Matern}(h; \kappa_s) \text{Matern}(u; \kappa_t)$$

3. *Stable\_Stable* defined as:

$$R(h, u) = \text{Stable}(h; \alpha_s) \text{Stable}(u; \alpha_t)$$

4. *Wendx\_Wendy* defined as

$$R(h, u) = \text{Wendx}(h; \mu_s) \text{Wendy}(u; \mu_t), \quad x, y = 0, 1, 2$$

Note that some models are nested. (The *Exp\_Exp* with *Matern\_Matern* for instance.)

## • Spatial bivariate correlation models (see below):

1. *Bi\_Matern* (Bivariate full Matern model)
2. *Bi\_Matern\_contr* (Bivariate Matern model with constraints)
3. *Bi\_Matern\_sep* (Bivariate separable Matern model)

4. *Bi\_LMC* (Bivariate linear model of coregionalization)
  5. *Bi\_LMC\_contr* (Bivariate linear model of coregionalization with constraints )
  6. *Bi\_Wendx* (Bivariate full Wendland model)
  7. *Bi\_Wendx\_contr* (Bivariate Wendland model with constraints)
  8. *Bi\_Wendx\_sep* (Bivariate separable Wendland model)
  9. *Bi\_Smoke* (Bivariate full Smoke model on the unit sphere)
- Spatial taper.  
For spatial covariance tapering the taper functions are:
    1. *Bohman* defined as:

$$T(h) = (1 - h)(\sin(2\pi h)/(2\pi h)) + (1 - \cos(2\pi h))/(2\pi^2 h)1_{[0,1]}(h)$$

2. *Wendlandx*,  $x = 0, 1, 2$  defined as:

$$T(h) = \text{Wendx}(h; x + 2), x = 0, 1, 2$$

- Spatio-temporal tapers.  
For spacetime covariance tapering the taper functions are:
  1. *Wendlandx\_Wendlandy* (Separable tapers)  $x, y = 0, 1, 2$  defined as:

$$T(h, u) = \text{Wendx}(h; x + 2)\text{Wendy}(h; y + 2), x, y = 0, 1, 2.$$

2. *Wendlandx\_time* (Non separable temporal taper)  $x = 0, 1, 2$  defined as: *Wenx\_time*,  $x = 0, 1, 2$  assuming  $\alpha_t = 2$ ,  $\mu_s = 3.5 + x$  and  $\gamma \in [0, 1]$  to be fixed using *tapsep*.
3. *Wendlandx\_space* (Non separable spatial taper)  $x = 0, 1, 2$  defined as: *Wenx\_space*,  $x = 0, 1, 2$  assuming  $\alpha_s = 2$ ,  $\mu_t = 3.5 + x$  and  $\gamma \in [0, 1]$  to be fixed using *tapsep*.

- Spatial bivariate taper (see below).

1. *Bi\_Wendlandx*,  $x = 0, 1, 2$

#### Remarks:

In what follows we assume  $\sigma^2, \sigma_1^2, \sigma_2^2, \tau^2, \tau_1^2, \tau_2^2, a, a_s, a_t, a_{11}, a_{22}, a_{12}, \kappa_{11}, \kappa_{22}, \kappa_{12}, f_{11}, f_{12}, f_{21}, f_{22}$  positive.

The associated parameters in *param* are *sill*, *sill\_1*, *sill\_2*, *nugget*, *nugget\_1*, *nugget\_2*, *scale*, *scale\_s*, *scale\_t*, *scale\_1*, *scale\_2*, *scale\_12*, *smooth\_1*, *smooth\_2*, *smooth\_12*, *a\_1*, *a\_12*, *a\_21*, *a\_2* respectively.

Let  $R(h)$  be a spatial correlation model given in standard notation. Then the covariance model applied with arbitrary variance, nugget and scale equals to:

$$C(h) = (\sigma^2 + \tau^2 1(h = 0))R(h/a, \dots), \quad h \geq 0$$

Similarly if  $R(h, u)$  is a spatio-temporal correlation model given in standard notation, then the covariance model is:

$$C(h, u) = (\sigma^2 + \tau^2 1(h = 0, u = 0))R(h/a_s, u/a_t, \dots) \quad h \geq 0, u \geq 0$$

Here ‘...’ stands for additional parameters.

Let  $R(h)$  be a spatial taper given in standard notation. Then the taper function applied with an arbitrary compact support ( $d_s$ ) equals to:

$$T(h) = R(h/d_s)$$

Then the tapered covariance function is given by:

$$C^{tap}(h) = T(h)C(h)$$

Similarly if  $R(h, u)$  is a spatio-temporal taper given in standard notation, then the taper function applied with arbitrary compact supports  $(d_s, d_t)^T$  equals to:

$$T(h, u) = R(h/d_s, u/d_t)$$

Then the tapered covariance function is given by:

$$C^{tap}(h, u) = T(h, u)C(h, u)$$

Compact supports  $d_s$  and  $d_t$  can be set by the user with `maxdist` and `maxtime`.

The bivariate models implemented are the following :

1. *Bi\_Matern* defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i \sigma_j + \tau_i^2 1(i = j, h = 0)) \text{Matern}(h/a_{ij}, \kappa_{ij}) \quad i, j = 1, 2. \quad h \geq 0$$

where  $\rho = \rho_{12} = \rho_{21}$  is the correlation colocated parameter and  $\rho_{ii} = 1$ . The model *Bi\_Matern\_sep* (separable matern) is a special case when  $a = a_{11} = a_{12} = a_{22}$  and  $\kappa = \kappa_{11} = \kappa_{12} = \kappa_{22}$ . The model *Bi\_Matern\_contr* (constrained matern) is a special case when  $a_{12} = 0.5(a_{11} + a_{22})$  and  $\kappa_{12} = 0.5(\kappa_{11} + \kappa_{22})$

2. *Bi\_Wendx* ( $x = 0, 1, 2$ ) defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i \sigma_j + \tau_i^2 1(i = j, h = 0)) \text{Wendx}(h/a_{ij}, \nu_{ij} + 1) \quad i, j = 1, 2. \quad h \geq 0$$

where  $\rho = \rho_{12} = \rho_{21}$  is the correlation colocated parameter and  $\rho_{ii} = 1$ . The model *Bi\_Wendx\_sep* (separable wendland) is a special case when  $a = a_{11} = a_{12} = a_{22}$  and  $\mu = \mu_{11} = \mu_{12} = \mu_{22}$ . The model *Bi\_Wendx\_contr* (constrained matern) is a special case when  $a_{12} = 0.5(a_{11} + a_{22})$  and  $\mu_{12} = 0.5(\mu_{11} + \mu_{22})$

3. *Bi\_LMC* defined as:

$$C_{ij}(h) = \sum_{k=1}^2 (f_{ik} f_{jk} + \tau_i^2 1(i = j, h = 0)) R(h/a_k)$$

where  $R(h)$  is a correlation model. The model *Bi\_LMC\_contr* is a special case when  $f = f_{12} = f_{21}$ . Bivariate LMC models, in the current version of the package, is obtained with  $R(h)$  equal to the exponential correlation model.

The bivariate spatial tapers implemented are the following :

1. *Bi\_Wendlandx*,  $x = 0, 1, 2$  defined as:

$$T_{ij}(h) = r_{ij} \text{Wendx}(h/d_{ij}, x), \quad i, j = 1, 2 \quad x = 0, 1, 2 \quad h \geq 0$$

with  $r_{ii} = 1$  and  $r_{12} = r_{21}$  to be fixed using `tapsep`.

If  $T_{ij}(h)$  is a bivariate taper, Then the tapered bivariate covariance function is given by:

$$C_{ij}^{tap}(h) = T_{ij}(h)C_{ij}(h)$$

Compact supports  $d_{11}, d_{12}, d_{22}$  can be set by the user with `maxdist`.



**Value**

Returns an object of class CovMat. An object of class CovMat is a list containing at most the following components:

bivariate	Logical:TRUE if the Gaussian random field is bivariaete otherwise FALSE;
coordx	A $d$ -dimensional vector of spatial coordinates;
coordy	A $d$ -dimensional vector of spatial coordinates;
coor dt	A $t$ -dimensional vector of temporal coordinates;
coordx_dyn	A list of $t$ matrices of spatial coordinates;
covmatrix	The covariance matrix if type isStandard. An object of class spam if type is Tapering or Standard and sparse is TRUE.
corrmodel	String: the correlation model;
distance	String: the type of spatial distance;
grid	Logical:TRUE if the spatial data are in a regular grid, otherwise FALSE;
nozero	In the case of tapered matrix the percentage of non zero values in the covariance matrix. Otherwise is NULL.
maxdist	Numeric: the marginal spatial compact support if type is Tapering;
maxtime	Numeric: the marginal temporal compact support if type is Tapering;
n	The number of trial for Binomial RFs
namescorr	String: The names of the correlation parameters;
numcoord	Numeric: the number of spatial coordinates;
numtime	Numeric: the number the temporal coordinates;
model	The type of RF, see <a href="#">GeoFit</a> .
param	Numeric: The covariance parameters;
tapmod	String: the taper model if type is Tapering. Otherwise is NULL.
spacetime	TRUE if spatio-temporal and FALSE if spatial covariance model;
sparse	Logical: is the returned object of class spam? ;

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**See Also**

[GeoKrig](#), [GeoSim](#), [GeoFit](#)

**Examples**

```
library(GeoModels)
library(spam)

#####
###
### Example 1. Spatial covariance matrix associated to
### a Matern correlation model
###
#####

# Define the spatial-coordinates of the points:
x = runif(500, 0, 1)
y = runif(500, 0, 1)
coords = cbind(x,y)

# Correlation Parameters for Matern model
CorrParam("Matern")

# Matern Parameters
param=list(smooth=0.5,sill=1,scale=0.2,nugget=0)

matrix1 = GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param)
dim(matrix1$covmatrix)

#####
###
### Example 2. Spatial tapered Covariance matrix associated to
### a Matern correlation model
###
#####

matrix2 = GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param,
                      maxdist=0.2,taper="Wendland1",type="Tapering")
# Tapered covariance matrix
as.matrix(matrix2$covmatrix)[1:15,1:15]

# Percentage of no zero values in the tapered matrix
matrix2$nozero

#####
###
### Example 3. Spatial covariance matrix associated to
### a Generalized Wendland correlation model
###
#####

# Gen Wendland Parameters
param=list(sill=1,scale=0.2,nugget=0,smooth=0,power2=4)
```

```

matrix3 = GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=param,sparse=TRUE)

# Percentage of no zero values in the tapered matrix
matrix3$nozero

#####
###
### Example 4. Spatial covariance matrix associated to
### a Generalized Cauchy correlation model
###
#####

# Gen Cauchy Parameters
param=list(sill=1,scale=0.2,nugget=0,power1=1,power2=1)

# Correlation Parameters for Gen Cauchy model
CorrParam("GenCauchy")

matrix4 = GeoCovmatrix(coordx=coords, corrmodel="GenCauchy", param=param)

matrix4$covmatrix[1:4,1:4]

#####
###
### Example 5. Covariance matrix associated to
### a space-time double exponential correlation model
###
#####

# Define the temporal-coordinates:
times = seq(1, 4, 1)

# Define covariance parameters
param=list(scale_s=0.3,scale_t=0.5,sill=1)

# Correlation Parameters for double exp model
CorrParam("Exp_Exp")

# Simulation of a spatial Gaussian random field:
matrix5 = GeoCovmatrix(coordx=coords, coordt=times, corrmodel="Exp_Exp",
                        param=param)

dim(matrix5$covmatrix)

#####
###
### Example 6. Covariance matrix associated to
### a skew gaussian RF with Exp correlation model
###
#####

param=list(sill=1,scale=0.3/3,nugget=0,skew=4)
# Simulation of a spatial Gaussian random field:

```

```

matrix6 = GeoCovmatrix(coordx=coords, corrmodel="Exp", param=param,
                      model="SkewGaussian")

# covariance matrix
matrix6$covmatrix[1:10,1:10]

#####
###
### Example 7. Covariance matrix associated to
### a Weibull RF with Genwend correlation model
###
#####

param=list(sill=1,scale=0.3,nugget=0,shape=4,mean=0,smooth=1,power2=5)
# Simulation of a spatial Gaussian random field:
matrix7 = GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=param,
                      sparse=TRUE,model="Weibull")

# covariance matrix
matrix7$nozero

#####
###
### Example 8. Covariance matrix associated to
### a binomial gaussian RF with Wendland correlation model
###
#####

param=list(sill=1,scale=0.2,nugget=0,power2=4)
# Simulation of a spatial Gaussian random field:
matrix8 = GeoCovmatrix(coordx=coords, corrmodel="Wend0", param=param,n=5,
                      model="Binomial")

# covariance matrix
matrix8$covmatrix[1:10,1:10]

#####
###
### Example 9. Covariance matrix associated to
### a bivariate Matern exponential correlation model
###
#####

set.seed(8)
# Define the spatial-coordinates of the points:
x = runif(10, -1, 1)
y = runif(10, -1, 1)
coords = cbind(x,y)

# Parameters
param=list(scale=0.3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
          nugget_1=0,nugget_2=0,smooth=0.5,pcol=-0.25)

# Covariance matrix

```

```
matrix9 = GeoCovmatrix(coordx=coords, corrmodel="Bi_matern_sep", param=param)$covmatrix
head(matrix8)
```

GeoFit

*Max-Likelihood-Based Fitting of Gaussian and non Gaussian RFs.***Description**

Maximum weighted composite-likelihood fitting for Gaussian and some Non-Gaussian univariate spatial, spatio-temporal and bivariate spatial RFs. The function returns the model parameters' estimates and the estimates' variances. Moreover the function allows to fix any of the parameters and setting upper/lower bound in the optimization.

**Usage**

```
GeoFit(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel, distance="Eucl",
       fixed=NULL, GPU=NULL, grid=FALSE, likelihood='Marginal', local=c(1,1), lower=NULL,
       maxdist=NULL, maxtime=NULL, method="cholesky", model='Gaussian',
       n=1, onlyvar=FALSE, optimizer='Nelder-Mead', parallel=FALSE, radius=6371, sensitivity=F,
       sparse=FALSE, start=NULL, taper=NULL, tapsep=NULL, type='Pairwise',
       upper=NULL, varest=FALSE, vartype='SubSamp', weighted=FALSE, winconst=NULL,
       winstp=NULL, winconst_t=NULL, winstp_t=NULL, X=NULL)
```

**Arguments**

data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid). For the description see the Section <b>Details</b> .
coordx	A numeric $(d \times 2)$ -matrix (where $d$ is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .

distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fixed	An optional named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated.
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
likelihood	String; the configuration of the composite likelihood. Marginal is the default, see the Section <b>Details</b> .
local	Numeric; number of local work-items of the OpenCL setup
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite or tapered likelihood computation. See the Section <b>Details</b> for more information.
maxtime	Numeric; an optional positive value indicating the maximum temporal separation considered in the composite or tapered likelihood computation (see <b>Details</b> ).
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section <b>Details</b> .
n	Numeric; number of trials in a binomial RF; number of successes in a negative binomial RF
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
optimizer	String; the optimization algorithm (see <a href="#">optim</a> for details). Nelder-Mead is the default. Other possible choices are ucminf, nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
parallel	Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
sensitivity	Logical; if TRUE then the sensitivity matrix is computed
sparse	Logical; if TRUE then maximum likelihood is computed using sparse matrices algorithms (spam packake).It should be used with compactly supported covariance models.FALSE is the default.
start	An optional named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see <b>Details</b> ).
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.

tapsep	Numeric; an optional value indicating the separable parameter in the space time adaptive taper (see <b>Details</b> ).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods (see <b>Details</b> ).
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; (SubSamp the default) the type of method used for computing the estimates' variances, see the Section <b>Details</b> .
weighted	Logical; if TRUE the likelihood objects are weighted, see the Section <b>Details</b> . If FALSE (the default) the composite likelihood is not weighted.
winconst	Numeric; a bivariate positive vector for computing the spatial sub-window in the sub-sampling procedure. See <b>Details</b> for more information.
winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure. The case 1 correspond to no overlapping. See <b>Details</b> for more information.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure. See <b>Details</b> for more information.
winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure. The case 1 correspond to no overlapping. See <b>Details</b> for more information.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

## Details

Note, that the standard likelihood may be seen as particular case of the composite likelihood. In this respect GeoFit provides maximum likelihood fitting. Only composite likelihood estimation based on pairs are considered. Specifically marginal pairwise likelihood is considered for each type of random field (Gaussian and not Gaussian). In the Gaussian case other types of estimation are available: conditional and difference pairwise likelihood, covariance tapering and cross-validation method.

The optimization method is specified using `optimizer`. The default method is Nelder-mead and other available methods are `ucminf`, `nlm`, `BFGS`, `L-BFGS-B` and `nlminb`. In the last two cases upper and lower bounds constraints in the optimization can be specified using `lower` and `upper`.

Depending on the dimension of data and on the name of the correlation RF model, the observations are assumed as a realization of a spatial, spatio-temporal or bivariate RF. Specifically, with data, `coordx`, `coordy`, `coordt` parameters:

- If data is a numeric  $d$ -dimensional vector, `coordx` and `coordy` are two numeric  $d$ -dimensional vectors (or `coordx` is  $(d \times 2)$ -matrix and `coordy`=NULL), then the data are interpreted as a single spatial realisation observed on  $d$  spatial sites;
- If data is a numeric  $(t \times d)$ -matrix, `coordx` and `coordy` are two numeric  $d$ -dimensional vectors (or `coordx` is  $(d \times 2)$ -matrix and `coordy`=NULL), `coordt` is a numeric  $t$ -dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on  $d$  spatial sites and for  $t$  times.

- If data is a numeric  $(2 \times d)$ -matrix, coordx and coordy are two numeric  $d$ -dimensional vectors (or coordx is  $(d \times 2)$ -matrix and coordy=NULL), then the data are interpreted as a single spatial realisation of a bivariate RF observed on  $d$  spatial sites.
- If data is a list, coordxdyn is a list and coordt is a numeric  $t$ -dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on dynamical spatial sites (different locations sites for each temporal instants) and for  $t$  times.

It is possible consider data on a regular grid using option grid=TRUE. Specifically:

- If data is a numeric  $(d \times d)$ -matrix, coordx and coordy are two numeric  $d$ -dimensional vectors, grid=TRUE, then the data are interpreted as a single spatial RF realisation observed on  $d$  equispaced spatial sites (named regular grid).
- If data is a numeric  $(d \times d \times t)$ -array, coordx and coordy are two numeric  $d$ -dimensional vectors, coordt is a numeric  $t$ -dimensional vector, grid=TRUE, then the data are interpreted as a single spatial-temporal realisation of a RF observed on  $d$  equispaced spatial sites and for  $t$  times.
- If data is a numeric  $(d \times d \times 2)$ -array, coordx and coordy are two numeric  $d$ -dimensional vectors, grid=TRUE, then the data are interpreted as a single realisation of a bivariate RF observed on  $d$  equispaced spatial sites.

When grid=FALSE is possible to specify a matrix of covariates using X. Specifically:

- In the spatial case X must be a  $(d \times k)$  covariates matrix associated to data a numeric  $d$ -dimensional vector;
- In the spatiotemporal case X must be a  $(N \times k)$  covariates matrix associated to data a numeric  $(t \times d)$ -matrix, where  $N = t \times d$ ;
- In the spatiotemporal case X must be a  $(N \times k)$  covariates matrix associated to data a numeric  $(t \times d)$ -matrix, where  $N = 2 \times d$ ;

The corrmmodel parameter allows to select a specific correlation function for the RF. (See [GeoCovmatrix](#) ).

The distance parameter allows to consider different kinds of spatial distances. The settings alternatives are:

1. Eucl, the euclidean distance (default value);
2. Chor, the chordal distance;
3. Geod, the geodesic distance;

The likelihood parameter represents the composite-likelihood configurations. The settings alternatives are:

1. Conditional, the composite-likelihood is formed by conditionals likelihoods;
2. Marginal, the composite-likelihood is formed by marginals likelihoods;
3. Full, the composite-likelihood turns out to be the standard likelihood;

The model parameter indicates the type of RF considered. The available options are:

RF with marginal symmetric distribution:

- Gaussian, for a Gaussian RF
- Logistic, for a Logistic RF (see Bevilacqua, M. Caamano C., Gaetan , C. 2018)
- StudentT, for a StudentT RF (see Bevilacqua, M. Caamano C., 2018)



RF with positive right skewed marginal distribution:

- Gamma for a Gamma RF (see Bevilacqua, M. Caamano C., 2018)
- Gamma for a Weibull RF (see Bevilacqua, M. Caamano C., 2018)
- LogGaussian for a LogGaussian RF
- LogLogistics for a LogLogistic RF (see Bevilacqua, M. Caamano C., Gaetan , C. 2018)

RF with with possibly asymmetric marginal distribution:

- SkewGaussian for a skew Gaussian RF (see Hao Zhang, H. and El-Shaarawi A. (2010))
- SinhAsinh for Sinh-arcsinh RF (see Bevilacqua, M. Caamano C., 2018)

RF with for directional data

- Wrapped for a wrapped Gaussian RF (see Alegria A., Bevilacqua, M., Porcu, E. (2016))

Rf with marginal counts data

- Binomial for a Binomial RF (see Bevilacqua, M. Caamano C., Gaetan C. (2018))
- BinomialNeg for a negative Binomial RF(see Bevilacqua, M. Caamano C., Gaetan C. (2018))

For a given model the associated parameters are given by nuisance and covariance parameters. In order to obtain the nuisance parameter use [NuisParam](#). In order to obtain the covariance parameter associated to a given covariance model use [CorrParam](#).

All the nuisance and covariance parameters must be specified by the user using the `start` and the `fixed` parameter. Specifically:

The `start` parameter allows to specify (as starting values for the optimization) the parameters to be estimated. The `fixed` parameter allows to fix some of the parameters.

Regression parameters in the linear specification must be specified as `mean`, `mean1`, . . . `meank` (see [NuisParam](#)). In this case a matrix of covariates with suitable dimension can be specified using the parameter `X`. In the case of a single mean then `X` should not be specified and it is interpreted as a vector of ones.

The `taper` parameter, optional in case that `type=Tapering`, indicates the type of taper correlation model. (See [GeoCovmatrix](#))

The `type` parameter represents the type of likelihood used in the composite-likelihood definition. The possible alternatives are listed in the following scheme.

If a Gaussian or (any) non Gaussian RF is considered then the possible combination is marginal pairwise likelihoods (`likelihood=Marginal`) and `type="Pairwise"`).

If a Gaussian RF is considered (`model=Gaussian`) then:

- If the composite is formed by marginal likelihoods (`likelihood=Marginal`):
  - Pairwise, the composite-likelihood is defined by the pairwise likelihoods;
  - Difference, the composite-likelihood is defined by likelihoods which are obtained as difference of the pairwise likelihoods.
- If the composite is formed by conditional likelihoods (`likelihood=Conditional`)
  - Pairwise, the composite-likelihood is defined by the pairwise conditional likelihoods.
- If the composite is formed by a full likelihood (`likelihood=Full`):
  - Standard, the objective function is the classical multivariate likelihood;
  - Restricted, the objective function is the restricted version of the full likelihood (e.g. Harville 1977, see **References**);

- Tapering, the objective function is the tapered 2 (unbiased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);
- Tapering1, the objective function is the tapered 1 (biased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);
- CV, the objective function is the cross validation estimation method ;

The `maxdist` parameter set the maximum spatial distance below which pairs of sites with inferior distances are considered in the composite-likelihood. This can be lower of the maximum spatial distance. **Note** that this corresponds to use a weighted composite-likelihood with binary weights. Pairs with distance less than `maxdist` have weight 1 and are included in the likelihood computation, instead those with greater distance have weight 0 and then excluded. The default is `NULL`, in this case the effective maximum spatial distance between sites is considered.

The same arguments of `maxdist` are valid for `maxtime` but here the weighed composite-likelihood regards the case of spatial-temporal field. At the moment is implemented only for Gaussian RFs. The default is `NULL`, in this case the effective maximum temporal lag between pairs of observations is considered.

In the case of tapering likelihood `maxdist` and `maxtime` describes the spatial and temporal compact support of the taper (see [GeoCovmatrix](#)). If they are not specified then the maximum spatial and temporal distances are considered. In the case of space time adaptive taper the `tapsep` parameter allows to specify the spatio temporal compact support (see [GeoCovmatrix](#)).

The `varest` parameter specifies if the standard error estimation of the estimated parameters must be computed. For Gaussian RF and standard (restricted) likelihood estimation, standard errors are computed as square root of the diagonal elements of the Fisher Information matrix (asymptotic covariance matrix of the estimates under increasing domain). For Gaussian RF and tapered and composite likelihood estimation, standard errors estimate are computed as square root of the diagonal elements of the Godambe Information matrix. (asymptotic covariance matrix of the estimates under increasing domain (see Shaby, B. and D. Ruppert (2012) for tapering and Bevilacqua et. al. (2012) , Bevilacqua and Gaetan (2013) for weighted composite likelihood)). The `vartype` parameter specifies the method used to compute the estimates' variances in the composite likelihood case. In particular for estimating the variability matrix  $J$  in the Godambe expression matrix. This parameter is considered if `varest=TRUE`. The options are:

- `SubSamp` (the default), indicates the Sub-Sampling method;

The `weighted` parameter specifies if the likelihoods forming the composite-likelihood must be weighted. If `TRUE` the weights are selected by opportune procedures that improve the efficient of the maximum composite-likelihood estimator (not implemented yet). If `FALSE` the efficient improvement procedure is not used.

For computing the standard errors by the sub-sampling procedure, `winconst` and `winstp` parameters represent respectively a positive constant used to determine the sub-window size and the step with which the sub-window moves.

In the spatial case (subset of  $R^2$ ), the domain is seen as a rectangle  $B \times H$ , therefore the size of the sub-window side  $b$  is given by  $b = \text{winconst} \times \sqrt{B}$  (similar is of  $h$ ). For a complete description see Lee and Lahiri (2002). By default `winconst` is set  $B/(4 \times \sqrt{B})$ . The `winstp` parameter is used to determine the sub-window step. The latter is given by the proportion of the sub-window size, so that when `winstp=1` there is not overlapping between contiguous sub-windows. In the spatial case by default `winstp=0.5`. The sub-window is moved by successive steps in order to cover the entire spatial domain. Observations, that fall in disjoint or overlapping windows are considered independent samples.

In the spatio-temporal case `winconst_t` represents the lenght of the temporal sub-window. By default the size of the sub-window is computed following the rule established in Li et al. (2007). By default `winstp` is the time step.

**Value**

Returns an object of class `GeoFit`. An object of class `GeoFit` is a list containing at most the following components:

<code>bivariate</code>	Logical: TRUE if the Gaussian RF is bivariate, otherwise FALSE;
<code>clic</code>	The composite information criterion, if the full likelihood is considered then it coincides with the Akaike information criterion;
<code>coordx</code>	A $d$ -dimensional vector of spatial coordinates;
<code>coordy</code>	A $d$ -dimensional vector of spatial coordinates;
<code>coordt</code>	A $t$ -dimensional vector of temporal coordinates;
<code>coordx_dyn</code>	A list of dynamical (in time) spatial coordinates;
<code>convergence</code>	A string that denotes if convergence is reached;
<code>corrmodel</code>	The correlation model;
<code>data</code>	The vector or matrix or array of data;
<code>distance</code>	The type of spatial distance;
<code>fixed</code>	The vector of fixed parameters;
<code>iterations</code>	The number of iteration used by the numerical routine;
<code>likelihood</code>	The configuration of the composite likelihood;
<code>logComplik</code>	The value of the log composite-likelihood at the maximum;
<code>maxdist</code>	The maximum spatial distance used in the weighed composite likelihood. If no spatial distance is specified then it is NULL;
<code>maxtime</code>	The maximum temporal distance used in the weighed composite likelihood. If no spatial distance is specified then it is NULL;
<code>message</code>	Extra message passed from the numerical routines;
<code>model</code>	The density associated to the likelihood objects;
<code>n</code>	The number of trials in a binominal RF; the number of successes in a negative Binomial RFs;
<code>nozero</code>	In the case of tapered likelihood the percentage of non zero values in the covariance matrix. Otherwise is NULL.
<code>numcoord</code>	The number of spatial coordinates;
<code>numtime</code>	The number of the temporal realisations of the RF;
<code>param</code>	The vector of parameters' estimates;
<code>param</code>	The radius of the sphere in the case of great circle distance;
<code>stderr</code>	The vector of standard errors;
<code>sensmat</code>	The sensitivity matrix;
<code>varcov</code>	The matrix of the variance-covariance of the estimates;
<code>varimat</code>	The variability matrix;
<code>vartype</code>	The method used to compute the variance of the estimates;
<code>type</code>	The type of the likelihood objects.
<code>winconst</code>	The constant used to compute the window size in the spatial sub-sampling;
<code>winstp</code>	The step used for moving the window in the spatial sub-sampling;
<code>winconst_t</code>	The constant used to compute the window size in the spatio-temporal sub-sampling;
<code>winstp_</code>	The step used for moving the window in the spatio-temporal sub-sampling;
<code>X</code>	The matrix of covariates;

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## Examples

```

library(GeoModels)
library(fields)

#####
##### Examples of spatial Gaussian RFs #####
#####

#####
###
### Example 1, 2 : Estimation of a spatial Gaussian RF with
### Matern correlation using pairwise likelihood and
### maximum likelihood with BFGS and nlminb optimization
###
#####

# Define the spatial-coordinates of the points:
set.seed(3)
N=400 # number of location sites
x <- runif(N, 0, 1)
set.seed(6)
y <- runif(N, 0, 1)
coords <- cbind(x,y)

# Define spatial matrix covariates
X=cbind(rep(1,N),runif(N))

# Set the covariance model's parameters:
corrmodel <- "Matern"
mean <- 0.2
mean1 <- -0.5
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5
param<-list(mean=mean,mean1=mean1,sill=sill,nugget=nugget,scale=scale,smooth=smooth)

# Simulation of the spatial Gaussian RF:
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param,X=X)$data

fixed<-list(nugget=nugget,smooth=smooth)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)

#####
###
### Example 1. Maximum pairwise likelihood fitting of
### Gaussian RFs with exponential correlation.
###
#####
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
               optimizer="BFGS",maxdist=0.05,likelihood="Marginal",
               type="Pairwise", start=start,fixed=fixed,X=X)

print(fit1)

```

```
#####
###
### Example 2. Standard Maximum likelihood fitting of
### Gaussian RFs with exponential correlation.
###
#####
I=Inf
lower<-list(mean=-I,mean1=-I,scale=0,sill=0)
upper<-list(mean=I,mean1=I,scale=I,sill=I)
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
               optimizer="nllminb",upper=upper,lower=lower,
               likelihood="Full",type="Standard",
               start=start,fixed=fixed,X=X)

print(fit2)

#####
##### Examples of spatial non-Gaussian RFs #####
#####

#####
###
### Example 3. Maximum pairwise likelihood fitting of spatial
### Gamma and Weibull RFs with Generalized Wendland correlation
### using Nelder-Mead and BFGS
#####
set.seed(524)
# Define the spatial-coordinates of the points:
N=500
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
shape=2
scale=0.2
smooth=0

model="Weibull"
corrmodel="GenWend"
param=list(mean=mean,mean1=mean1,sill=1-nugget,scale=scale,
           shape=shape,nugget=nugget,power2=4,smooth=smooth)
# Simulation of a non stationary weibull RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,
               param=param)$data

fixed<-list(nugget=nugget,power2=4,sill=1-nugget,smooth=smooth)
start<-list(mean=mean,mean1=mean1,scale=scale,shape=shape)

# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
               maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
               optimizer="Nelder-Mead",
```

```

        start=start,fixed=fixed)
print(fit$param)

model="Gamma"
start<-list(mean=mean,mean1=mean1,scale=scale)
fixed<-list(nugget=nugget,power2=4,sill=1-nugget,shape=6)
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel="Wend0", model=model,
              maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
              optimizer="BFGS",
              start=start,fixed=fixed)

print(fit$param)

#####
###
### Example 4. Maximum pairwise likelihood fitting of
### StudentT spatial RFs with Wendland correlation
###
#####

set.seed(15274)
# Define the spatial-coordinates of the points:
N=300
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
sill=0.5
scale=0.2
df=4    ## degrees of freedom
model="StudentT"
corrmodel="Wend0"

# Simulation of a studentT RF:
param=list(mean=mean,mean1=mean1,sill=sill,scale=scale,df=1/df,nugget=nugget,power2=4)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,
              param=param)$data
## estimation assuming df unknown
fixed<-list(nugget=nugget,power2=4)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill,df=1/df)

I=Inf
lower<-list(mean=-I,mean1=-I,scale=0,sill=0,df=0)
upper<-list(mean=I,mean1=I,scale=I,sill=I,df=0.5)

# Maximum pairwise composite-likelihood fitting of the RF:

fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
              maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
              lower=lower,upper=upper,optimizer="nlnminb",start=start,fixed=fixed)
print(fit1$param)

```

```

## df must be rounded and fixed
df=round(1/(as.numeric(fit1$param['df'])))
fixed<-list(nugget=nugget,power2=4,df=1/df)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)

lower<-list(mean=-I,mean1=-I,scale=0,sill=0)
upper<-list(mean=I,mean1=I,scale=I,sill=I)

# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
              maxdist=.05,likelihood="Marginal",type="Pairwise",X=X,
              lower=lower,upper=upper, optimizer="nlminb" , start=start,fixed=fixed)
print(fit$param)

#####
###
### Example 5. Maximum pairwise likelihood fitting of
### SinhAsinh-Gaussian spatial RFs with Wendland correlation
###
#####
set.seed(261)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
x <- runif(500, 0, 1)
y <- runif(500, 0, 1)
coords <- cbind(x,y)

corrmodel="Wend0"
mean=0;nugget=0
sill=1
skew=-0.5
tail=1.5
power2=4
c_supp=0.2

# model parameters
param=list(power2=power2,skew=skew,tail=tail,
           mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param)$data

plot(density(data))
fixed=list(power2=power2,nugget=nugget)
start=list(scale=c_supp,skew=skew,tail=tail,mean=mean,sill=sill)
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
              maxdist=0.05,likelihood="Marginal",type="Pairwise",
              optimizer="BFGS" , start=start,fixed=fixed)
print(fit1$param)
# Maximum likelihood:
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
              likelihood="Full",type="Standard",
              optimizer="BFGS" , start=start,fixed=fixed)

```



```

print(fit2$param)

#####
###
### Example 6. Maximum pairwise likelihood fitting of
### Binomial and negative Binomial RFs
### with exponential correlation
###
#####

set.seed(422)
N=350
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
mean=0.1; mean1=0.8; mean2=-0.5 # regression parameters
X=cbind(rep(1,N),runif(N),runif(N)) # matrix covariates
corrmodel <- "Wend0"
param=list(mean=mean,mean1=mean1,mean2=mean2,sill=1,nugget=0,scale=0.2,power2=4)
# Simulation of the spatial Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="Binomial", n=10,X=X,
               param=param)$data

fixed <- list(nugget=nugget,power2=4,sill=1)
start <- list(scale=0.2,mean=mean,mean1=mean1,mean2=mean2)
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel,n=10, X=X, start=start,
               maxdist=0.05,model="Binomial", fixed=fixed, optimizer="BFGS")

print(fit1)

set.seed(220)
# Simulation of the spatial Negative Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="BinomialNeg", n=5,X=X,
               param=param)$data
# Maximum pairwise likelihood:
fit2 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel, n=5,X=X, start=start,
               maxdist=0.05, model="BinomialNeg", fixed=fixed, optimizer="BFGS")

print(fit2)

#####
##### Examples of spatio-temporal RFs #####
#####
set.seed(52)
# Define the temporal sequence:
time <- seq(1, 10, 1)

# Define the spatial-coordinates of the points:
x <- runif(20, 0, 1)
set.seed(42)
y <- runif(20, 0, 1)
coords=cbind(x,y)

```

```

# Set the covariance model's parameters:
corrmodel="Exp_Exp"
scale_s=0.2/3
scale_t=1
sill=1
nugget=0
mean=0

param<-list(mean=0,scale_s=scale_s,scale_t=scale_t,
            sill=sill,nugget=nugget)

# Simulation of the spatial-temporal Gaussian RF:
data <- GeoSim(coordx=coords,coordt=time,corrmodel=corrmodel,
              param=param)$data

#####
###
### Example 7. Maximum pairwise likelihood fitting of a
### space time Gaussian RF with double-exponential correlation
### using BFGS
###
#####
# Fixed parameters
fixed<-list(nugget=nugget)
# Starting value for the estimated parameters
start<-list(mean=mean,scale_s=scale_s,scale_t=scale_t,sill=sill)

# Maximum composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,coordt=time,
              corrmodel="Exp_Exp",maxtime=1,maxdist=0.3,
              likelihood="Marginal",type="Pairwise",
              optimizer="BFGS" , start=start,fixed=fixed)

print(fit)

#####
###
### Example 8. Maximum standard likelihood fitting of a
### space time Gaussian RF observed on dynamical spatial coordinates
### with double-exponential correlation using BFGS
###
#####

maxN=50
coordx_dyn=list()
set.seed(31)
for(k in 1:length(time))
{
  NN=sample(1:maxN,size=1)
  x <- runif(NN, 0, 1)
  y <- runif(NN, 0, 1)
  coordx_dyn[[k]]=cbind(x,y)
}

data <- GeoSim(coordx_dyn=coordx_dyn, coordt=time, corrmodel="Exp_Exp",
              param=param)$data

I=Inf

```

```

lower<-list(mean=-I,scale_s=0,scale_t=0,sill=0)
upper<-list(mean=-I,scale_s=I,scale_t=I,sill=I)
fit <- GeoFit(data=data,coordx_dyn=coordx_dyn,coordt=time,
              corrmodel="Exp_Exp",likelihood="Full",type="Standard",
              optimizer="BFGS", lower=lower,upper=upper,
              start=start,fixed=fixed)
print(fit)

#####
##### Examples of spatial bivariate RFs #####
#####

#####
###
### Example 9. Maximum, and pairwise likelihood fitting of a
### bivariate Gaussian RF with separable Bivariate matern
### (cross) correlation model using BFGS and a parallel
### implementation of L-BFGS-B
#####

# Define the spatial-coordinates of the points:
set.seed(5)
x <- runif(250, 0, 1)
y <- runif(250, 0, 1)
coords=cbind(x,y)
# parameters
param=list(mean_1=0,mean_2=0,scale=0.1,smooth=0.5,sill_1=1,sill_2=1,
           nugget_1=0,nugget_2=0,pcol=0.2)
# Simulation of a spatial Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel="Bi_Matern_sep",
              param=param)$data

# selecting fixed and estimated parameters
fixed=list(nugget_1=0,nugget_2=0,smooth=0.5)
start=list(mean_1=0,mean_2=0,sill_1=var(data[1,]),sill_2=var(data[2,]),
           scale=0.1,pcol=cor(data[1,],data[2,]))

# Maximum pairwise likelihood

fitcl<- GeoFit(data=data, coordx=coords, corrmodel="Bi_Matern_sep",
              likelihood="Marginal",type="Pairwise",
              optimizer="BFGS" , start=start,fixed=fixed,
              maxdist=c(0.05,0.05,0.05))
print(fitcl)

I=Inf
lower=list(mean_1=-I,mean_2=-I,sill_1=0,sill_2=0,
           scale=0,pcol=-1)
upper=list(mean_1=I,mean_2=I,sill_1=I,sill_2=I,
           scale=I,pcol=1)
# Maximum likelihood :
require(optimParallel)

```

```
fitml<- GeoFit(data=data, coordx=coords, likelihood="Full",
               corrmmodel="Bi_Matern_sep", type="Standard",
               upper=upper, lower=lower, optimizer="L-BFGS-B",
               parallel=TRUE,
               start=start, fixed=fixed)
print(fitml)
```

GeoKrig

*Spatial (bivariate) and spatio temporal optimal linear prediction for Gaussian and non Gaussian RFs.*

## Description

For a given set of spatial location sites and temporal instants, the function computes optimal linear prediction and associated mean square error for the Gaussian and non Gaussian case.

## Usage

```
GeoKrig(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmmodel, distance="Eucl",
        grid=FALSE, loc, maxdist=NULL, maxtime=NULL, method="cholesky",
        model="Gaussian", n=1, nloc=NULL, mse=FALSE, lin_opt=TRUE,
        param, radius=6371, sparse=FALSE, taper=NULL, tapsep=NULL,
        time=NULL, type="Standard", type_mse=NULL,
        type_krig="Simple", weighed=TRUE, which=1, X=NULL, Xloc=NULL)
```

## Arguments

data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.
coordx	A numeric $(d \times 2)$ -matrix (where $d$ is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates used for prediction. $qndd$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius <code>radius</code> are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates used for prediction; <code>coordy</code> is interpreted only if <code>coordx</code> is a numeric vector or <code>grid=TRUE</code> otherwise it will be ignored. Optional argument, the default is <code>NULL</code> then <code>coordx</code> is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is <code>NULL</code> then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is <code>NULL</code>
corrmmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is <code>Eucl</code> , the euclidean distance. See the Section <b>Details</b> of <a href="#">GeoFit</a> .

grid	Logical; if FALSE (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
lin_opt	Logical; If TRUE (default) then optimal (pairwise) linear kriging is computed. Otherwise optimal (pairwise) kriging is computed in the mean square sense.
loc	A numeric ( $n \times 2$ )-matrix (where n is the number of spatial sites) giving 2-dimensions of spatial coordinates to be predicted.
maxdist	Numeric; an optional positive value indicating the maximum spatial compact support in the case of covariance tapering kriging.
maxtime	Numeric; an optional positive value indicating the maximum temporal compact support in the case of covasriance tapering kriging.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
nloc	Numeric; the number of trials of the locations sites to be predicted in a binomial random fields type II. Default is 1.
mse	Logical; if TRUE (the default) MSE of the kriging predictor is computed
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section <b>Details</b> .
param	A list of parameter values required for the correlation model. See the Section <b>Details</b> .
radius	Numeric; the radius of the sphere if coordinates are passed in lon/lat format;
sparse	Logical; if TRUE kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
taper	String; the name of the taper correlation function, see the Section <b>Details</b> .
tapsep	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see <b>Details</b> ).
time	A numeric ( $m \times 1$ ) vector (where m is the number of temporal instants) giving the temporal instants to be predicted; the default is NULL then only spatial prediction is performed.
type	String; if Standard then standard kriging is performed; if Tapering then kriging with covariance tapering is performed; if Pairwise then pairwise kriging is performed
type_mse	String; if Theoretical then theoretical MSE pairwise kriging is computed. If SubSamp then an estimation based on subsampling is computed.
type_krig	String; the type of kriging. If Simple (the default) then simple kriging is performed. (See the Section <b>Details</b> ).
weighthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime) are used in the pairwise kriging.
which	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
Xloc	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.

## Details

Best linear unbiased predictor and associated mean square error is computed for Gaussian and some non Gaussian cases. Specifically, for a spatial or spatio-temporal or spatial bivariate dataset, given a set of spatial locations and temporal instants and a correlation model `corrmodel` with some fixed parameters and given the type of RF (`model`) the function computes simple kriging, for the specified spatial locations `loc` and temporal instants `time`, providing also the respective mean square error. For the choice of the spatial or spatio temporal correlation model see details in [GeoCovmatrix](#) function. The list `param` specifies mean and covariance parameters, see [CorrParam](#) and [GeoCovmatrix](#) for details. The `type_krig` parameter indicates the type of kriging. In the case of simple kriging, the known mean can be specified by the parameter `mean` in the list `param` (See examples). In the Gaussian case, it is possible to perform kriging based on covariance tapering for simple kriging (Furrer et. al, 2008). In this case, space or space-time tapered function and spatial or spatio-temporal compact support must be specified. For the choice of a space or space-time tapered function see [GeoCovmatrix](#). When performing kriging with covariance tapering, sparse matrix algorithms are exploited using the package `spam`.

## Value

Returns an object of class `Kg`. An object of class `Kg` is a list containing at most the following components:

<code>bivariate</code>	TRUE if spatial bivariate cokriging is performed, otherwise FALSE;
<code>coordx</code>	A $d$ -dimensional vector of spatial coordinates used for prediction;
<code>coordy</code>	A $d$ -dimensional vector of spatial coordinates used for prediction;
<code>coor dt</code>	A $t$ -dimensional vector of temporal coordinates used for prediction;
<code>corrmodel</code>	String: the correlation model;
<code>covmatrix</code>	The covariance matrix if type is Standard. An object of class <code>spam</code> if type is Tapering
<code>data</code>	The vector or matrix or array of data used for prediction
<code>distance</code>	String: the type of spatial distance;
<code>grid</code>	TRUE if the spatial data used for prediction are observed in a regular grid, otherwise FALSE;
<code>loc</code>	A $(n \times 2)$ -matrix of spatial locations to be predicted.
<code>n</code>	The number of trial for Binomial RFs
<code>nozero</code>	In the case of tapered simple kriging the percentage of non zero values in the covariance matrix. Otherwise is NULL.
<code>numcoord</code>	Numeric: the number $d$ of spatial coordinates used for prediction;
<code>numloc</code>	Numeric: the number $n$ of spatial coordinates to be predicted;
<code>numtime</code>	Numeric: the number $d$ of the temporal instants used for prediction;
<code>numt</code>	Numeric: the number $m$ of the temporal instants to be predicted;
<code>model</code>	The type of RF, see <a href="#">GeoFit</a> .
<code>param</code>	Numeric: The covariance parameters;
<code>pred</code>	A $(m \times n)$ -matrix of spatio or spatio temporal kriging prediction;
<code>radius</code>	Numeric: the radius of the sphere if coordinates are passed in lon/lat format;
<code>spacetime</code>	TRUE if spatio-temporal kriging and FALSE if spatial kriging;
<code>tapmod</code>	String: the taper model if type is Tapering. Otherwise is NULL.

time	A $m$ -dimensional vector of temporal coordinates to be predicted;
type	String: the type of kriging (Standard or Tapering).
type_krig	String: the type of kriging.
mse	A $(m \times n)$ -matrix of spatio or spatio temporal mean square error kriging prediction;

### Author(s)

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### References

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlag, New York.  
 Furrer R., Genton, M.G. and Nychka D. (2006). *Covariance Tapering for Interpolation of Large Spatial Datasets*. Journal of Computational and Graphical Statistics, **15-3**, 502–523.

### See Also

[GeoCovmatrix](#)

### Examples

```
library(GeoModels)
library(fields)

#####
##### Examples of Spatial kriging #####
#####

# Define the spatial-coordinates of the points:
set.seed(79)
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel_1 = "exponential"
mean=0
sill=1
nugget=0
scale=0.3/3
param=list(mean=mean,sill=sill,nugget=nugget,scale=scale)

# Set the wendland parameters (two compatible correlations):
corrmodel_2 = "Wend0"
mean=0
sill=1
nugget=0
power2=3
c_supp=0.3
param_wen=list(mean=mean,sill=sill,nugget=nugget,scale=c_supp,power2=power2)

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel_1,
```

```

param=param)$data

# locations to predict
xx=seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))

#####
###
### Example 1. Spatial simple kriging of n sites of a
### Gaussian random fields with exponential correlation.
###
#####
pr=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_1,
           param= param, data=data,mse=TRUE)

#####
###
### Example 2. Spatial tapered simple kriging of n sites of a
### Gaussian random fields with exponential correlation.
###
#####
pr_tap=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_1,data=data,
               param= param,type="Tapering",maxdist=0.2,taper="Wendland1",mse=TRUE)
#####
###
### Example 3. Spatial simple kriging of n sites of a
### Gaussian random fields using a compatible Wendland model
###
#####
param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,power2=power2)
pr_wen=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_2,data=data,
               param=param_wen,sparse=TRUE,mse=TRUE)

colour = rainbow(100)
par(mfrow=c(3,2))
zlim=c(-2.6,2.6)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",
           main="Simple Kriging with exponential model ")

# simple kriging map prediction variance
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")

# simple tapered kriging map prediction
image.plot(xx, xx, matrix(pr_tap$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",main="Simple Tapered Kriging")

# simple tapered kriging map prediction variance
image.plot(xx, xx, matrix(pr_tap$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")

# simple tapered kriging map prediction
image.plot(xx, xx, matrix(pr_wen$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",main="Simple Kriging with Wendland model")

```



```

# simple kriging map prediction variance
image.plot(xx, xx, matrix(pr_wen$mse,ncol=length(xx)),col=colour,
             xlab="",ylab="",main="Std error")

#####
###
### Example 4. Spatial simple kriging of a binomial
### random field
###
#####
set.seed(312)
model="Binomial";n=6

# Define the spatial-coordinates of the points:
x = runif(800)
y = runif(800)
coords=cbind(x,y)
#### mean and covariance parameters ####
mean=0
sill=1
nugget=0
scale=0.2
#####
param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,power2=4)
# Simulation of the Binomial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel="Wend0",model=model,n=n,
              sparse=TRUE,param=param)$data

par(mfrow=c(1,2))

#### map of simulated data
quilt.plot(x, y, data,nlevel=n+1,col=rainbow(n+1),zlim=c(0,n), main="Data")

## estimation with pairwise likelihood
fixed=list(nugget=nugget,power2=4,sill=1)
start=list(mean=0,scale=scale)
# Maximum pairwise likelihood fitting :
fit = GeoFit(data, coordx=coords, corrmodel="Wend0",model=model,n=n,
             likelihood='Marginal', type='Pairwise',maxdist=0.03,
             optimizer="BFGS", start=start,fixed=fixed)

# locations to predict
xx=seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))

## simple kriging
pr=GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,n=n,
           sparse=TRUE,param= as.list(c(fit$param,fixed)))

#standard binomial kriging
map_binom=matrix(round(pr$pred),ncol=length(xx))
image.plot(xx, xx, map_binom,nlevel=n+1,col=rainbow(n+1),zlim=c(0,n),
           xlab="",ylab="",main="Simple Kriging ")

```

```
#####
###
### Example 5. Spatial simple kriging of the residuals of a
###          Weibull random field
###
#####
set.seed(312)

model="Weibull"

corrmodel = "GenWend"
# Define the spatial-coordinates of the points:
NN=800
coords=cbind(runif(NN),runif(NN))
## matrix covariates
a0=rep(1,NN)
a1=runif(NN,-1,1)
X=cbind(a0,a1)
# Set model parameters
shape=2
## regression parameters
mean = 1
mean1= -0.2
# correlation parameters
sill = 1
nugget = 0
power2=4
scale = 0.3
smooth=0

## simulation
param=list(shape=shape,nugget=nugget,mean=mean,mean1=mean1,
           scale=scale,sill=sill,power2=power2,smooth=smooth)

data = GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
              model=model,sparse=TRUE,X=X)$data

#####starting and fixed parameters
fixed=list(nugget=nugget,power2=power2,smooth=smooth,sill=sill)
start=list(mean=mean,mean1=mean1, scale=scale,shape=shape)

## estimation with pairwise likelihood
maxdist=0.1
fit2 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel,X=X,
              maxdist=maxdist,likelihood="Marginal",type="Pairwise",
              optimizer="BFGS",start=start,fixed=fixed, model = model)
## obtaining residuals
res=GeoResiduals(fit2)

# locations to predict
xx=seq(0,1,0.025)
loc_to_pred=as.matrix(expand.grid(xx,xx))

#optimal linear kriging for residuals
pr=GeoKrig(data=res$data, coordx=coords,loc=loc_to_pred,corrmodel=corrmodel,model=model,mse=TRUE,
```

```

        sparse=TRUE,param=as.list(c(res$param,res$fixed)))

## map of residuals
par(mfrow=c(1,3))
quilt.plot(coords,res$data,main="Residuals")
map=matrix(pr$pred,ncol=length(xx))
mapmse=matrix(pr$mse,ncol=length(xx))
image.plot(xx, xx, map,
            xlab="",ylab="",main="Residuals Kriging ")

image.plot(xx, xx, mapmse,
            xlab="",ylab="",main="Simple Kriging ")

#####
###
### Example 5. Spatial simple kriging of a t
### random field
###
#####

#####
model="StudentT"
df=7
corrmodel = "Wend0"
nset=700
coords=cbind(runif(nset),runif(nset))

mean = 0
sill = 1
nugget = 0
power2=4
scale = 0.2

# Starting value for the estimated parameters
set.seed(3132)
param=list(nugget=nugget,mean=mean, scale=scale,sill=sill,df=1/df,power2=power2)
data = GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
              model=model,sparse=TRUE)$data

fixed=list(nugget=nugget,power2=4,df=1/df)
start=list(mean=mean, scale=scale,sill=sill)
I=Inf
upper=list(mean=I, scale=I,sill=I)
lower=list(mean=-I, scale=0,sill=0)
# Maximum pairwise composite-likelihood fitting of the RF:
fit = GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
             maxdist=0.03,likelihood="Marginal",type="Pairwise",
             lower=lower,upper=upper,
             optimizer="nlnmb", start=start,fixed=fixed, model = model)

# locations to predict
xx=seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))

#optimal linear kriging
pr=GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,

```

```

        sparse=TRUE,param= as.list(c(fit$param,fit$fixed)))

par(mfrow=c(1,2))
#### map of simulated data
quilt.plot(coords[,1], coords[,2], data, main="Data")

map_t=matrix(pr$pred,ncol=length(xx))
image.plot(xx, xx, map_t,
           xlab="",ylab="",main="Simple Kriging ")
#####
##### Examples of spatio temporal kriging #####
#####

# Define the spatial-coordinates of the points:
x = runif(80, 0, 1)
y = runif(80, 0, 1)
coords=cbind(x,y)
times=1:8

# Define model correlation and associated parameters
corrmodel="exp_exp"
param=list(nugget=0,mean=0,scale_s=0.2/3,scale_t=1,sill=1)

# Simulation of the space time Gaussian random field:
set.seed(31)
data=GeoSim(coordx=coords,coordt=times,corrmodel=corrmodel,
            param=param)$data

# Maximum pairwise likelihood fitting of the space time random field:
start = list(scale_s=0.2/3,scale_t=1,sill=1,mean=0)
fixed = list(nugget=0)
fit = GeoFit(data, coordx=coords, coordt=times,corrmodel=corrmodel,
            likelihood='Marginal', type='Pairwise',start=start,fixed=fixed,
            optimizer="BFGS", maxdist=0.1,maxtime=1)

#####
###
### Example 6. Spatio temporal simple kriging of n locations
### sites and m temporal instants for a Gaussian random fields
### with estimated double exponential correlation.
###
#####
set.seed(3)
param=as.list(c(fit$param,fit$fixed))

# locations to predict
xx=seq(0,1,0.04)
loc_to_pred=as.matrix(expand.grid(xx,xx))
# Define the times to predict
times_to_pred=1:2

pr=GeoKrig(loc=loc_to_pred,time=times_to_pred,coordx=coords,coordt=times,
          corrmodel=corrmodel, param=param,data=data,mse=TRUE)

par(mfrow=c(2,2))
zlim=c(-2.5,2.5)
colour = rainbow(100)

```

```

for(i in 1:2) {
  image.plot(xx, xx, matrix(pr$pred[i,],ncol=length(xx)),col=colour,
    zlim=zlim, main = paste(" Kriging Time=" , i),ylab="")
  image.plot(xx, xx, matrix(pr$mse[i,],ncol=length(xx)),col=colour,
    main = paste("Std err Time=" , i),ylab="")
}

#####
##### Examples of spatial bivariate cokriging #####
#####

#####
###
### Example 7. Bivariate simple cokriging of n locations
### for a Gaussian random fields with separable Matern correlation
###
#####
# Define the spatial-coordinates of the points:
x = runif(80, 0, 1)
y = runif(80, 0, 1)
coords=cbind(x,y)

# Simulation of a spatial bivariate Gaussian random field
# with Matern separable covariance model

set.seed(12)
param=list(scale=0.3/3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
  nugget_1=0,nugget_2=0,pcol=0.7,smooth=0.5)

data = GeoSim(coordx=coords, corrmodel="Bi_matern_sep", param=param)$data

fixed=list(nugget_1=0,nugget_2=0,smooth=0.5,mean_1=0,mean_2=0)
start=list(sill_1=var(data[,1]),sill_2=var(data[,2]),scale=0.3/3,
  pcol=cor(data[,1],data[,2]))

# Maximum Composite likelihood fitting of the random field:
fitcl= GeoFit(data, coordx=coords, corrmodel="Bi_matern_sep",
  likelihood="Marginal",type="Pairwise",maxdist=0.1,
  optimizer="BFGS", start=start,fixed=fixed)

# locations to predict
xx=seq(0,1,0.04)
loc_to_pred=as.matrix(expand.grid(xx,xx))

pr1=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
  param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=1,mse=TRUE)

pr2=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
  param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=2,mse=TRUE)

par(mfrow=c(2,2))

# simple kriging map prediction of the first variable

```

```

image.plot(xx, xx, matrix(pr1$pred,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="First Simple coKriging")

# simple kriging map prediction variance of the first variable
image.plot(xx, xx, matrix(pr1$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")

# simple kriging map prediction of the second variable
image.plot(xx, xx, matrix(pr2$pred,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Second Simple coKriging")

# simple kriging map prediction variance of the second variable
image.plot(xx, xx, matrix(pr2$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")

```

---

GeoNeighborhood

*Spatio (temporal) neighborhood selection for local kriging.*


---

### Description

The procedure select a spatio (temporal) neighborhood for a given spatial (temporal) location.

### Usage

```

GeoNeighborhood(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, distance="Eucl", grid=FALSE,
               loc, maxdist=NULL,maxtime=NULL, radius=6371, time=NULL, X=NULL)

```

### Arguments

data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid).
coordx	A numeric $(d \times 2)$ -matrix (where $d$ is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of <a href="#">GeoFit</a> .
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).

loc	A $(1 \times 2)$ -matrix giving the spatial coordinate of the location for which a neighborhood is computed .
maxdist	Numeric; a positive value indicating the maximum spatial distance considered in the spatial neighborhood selection.
maxtime	Numeric; a positive value indicating the maximum temporal distance considered in the temporal neighborhood selection.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
time	Numeric; a value giving the temporal instant for which a neighborhood is computed.
X	Numeric; Matrix of space-time covariates.

**Value**

Returns a list containing the following informations:

coordx	A matrix of the coordinates of the computed spatial neighborhood;
coor dt	A vector of the computed temporal neighborhood;
data	The vector of data associated with the spatio (temporal) neighborhood;
distance	The type of spatial distance;
numcoord	The number of location sites of the spatial neighborhood;
numtime	The number of instants of the temporal neighborhood;
radius	The radius of the sphere if coordinates are passed in lon/lat format;
spacetime	TRUE if spatio-temporal and FALSE if spatial RF;
X	The matrix of spatio (temporal) covariates associated with the computed spatio (temporal) neighborhood;

**Author(s)**

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**Examples**

```
library(GeoModels)
#####
#### Example: spatial local kriging ####
#####
set.seed(7)
coords=cbind(runif(100),runif(100))

param=list(nugget=0,mean=0,scale=0.2,sill=1,
           power2=4,smooth=1)

data_all = GeoSim(coordx=coords, corrmodel="GenWend",
                  param=param)$data

##location to predict
loc_to_pred=matrix(c(0.5,0.5),1,2)

loc_kri=GeoNeighborhood(data_all, coordx=coords,
```

```

loc=loc_to_pred,maxdist=0.15)

## global kriging
pr_all=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="GenWend",
               param=param,mse=TRUE, data=data_all)
## local kriging
pr_loc=GeoKrig(loc=loc_to_pred,coordx=loc_kri$coordx,corrmodel="GenWend",
               param=param,mse=TRUE, data=loc_kri$data)

pr_all$pred;pr_loc$pred
pr_all$mse;pr_loc$mse

#####
#### Example: spatio temporal local kriging ####
#####

set.seed(78)
coords=cbind(runif(100),runif(100))
coordt=seq(0,4,0.25)

param=list(nugget=0,mean=0,scale_s=0.2/3,scale_t=0.5/3,sill=1)

data_all = GeoSim(coordx=coords, coordt=coordt,corrmodel="Exp_Exp",
                  param=param)$data
##location to predict
loc_to_pred=matrix(c(0.5,0.5),1,2)
time=2

loc_kri=GeoNeighborhood(data_all, coordx=coords, coordt=coordt,
                        loc=loc_to_pred,maxdist=0.4,maxtime=2.5)
## global kriging
pr_all=GeoKrig(loc=loc_to_pred,time=time,coordx=coords,coordt=coordt,
               corrmodel="Exp_Exp",
               param=param,mse=TRUE, data=data_all)
## local kriging
pr_loc=GeoKrig(loc=loc_to_pred,time=time,coordx=loc_kri$coordx,
               coordt=loc_kri$coordt,corrmodel="Exp_Exp",
               param=param,mse=TRUE, data=loc_kri$data)

pr_all$pred;pr_loc$pred
pr_all$mse;pr_loc$mse

```

---

GeoResiduals

---

*Computes fitted covariance and/or variogram*


---

## Description

The procedure return a GeoFit object associated to the estimated residuals

## Usage

```
GeoResiduals(fit)
```



**Arguments**

`fit` A fitted object obtained from the [GeoFit](#).

**Value**

A GeoFit object with the estimated residuals

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**See Also**

[GeoFit](#).

**Examples**

```
library(GeoModels)
set.seed(211)

model="Weibull";shape=4
N=700 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)

# regression parameters
mean = 5
mean1=0.8

X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
nugget = 0
scale = 0.3
power2=4

param=list(mean=mean,mean1=mean1, sill=sill, nugget=nugget,
           scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data

start=list(mean=mean,mean1=mean1, scale=scale,shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
            likelihood="Marginal",type='Pairwise',start=start,
            fixed=fixed,maxdist=0.1)

res=GeoResiduals(fit)
mean(res$data) # should be 1
```

```
# Empirical estimation of the variogram for the residuals:
vario = GeoVariogram(res$data, coordx=coords, maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(res, show.vario=TRUE, vario=vario, pch=20)
```

GeoSim

*Simulation of Gaussian and non Gaussian Random Fields.*

## Description

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields. The function return a realization of a Random Field for a given covariance model and covariance parameters. Simulation is based on Cholesky decomposition.

## Usage

```
GeoSim(coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmmodel, distance="Eucl",
       GPU=NULL, grid=FALSE, local=c(1,1), method="cholesky", model='Gaussian', n=1, param,
       radius=6371, sparse=FALSE, X=NULL)
```

## Arguments

coordx	A numeric ( $d \times 2$ )-matrix (where $d$ is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a ( $d \times 2$ )-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of <a href="#">GeoFit</a> .
GPU	Numeric; if NULL (the default) no GPU computation is performed.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
local	Numeric; number of local work-items of the GPU
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section <b>Details</b> .

n	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1.
param	A list of parameter values required in the simulation procedure of RFs, see <b>Examples</b> .
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam package). It should be used with compactly supported covariance models.FALSE is the default.
x	Numeric; Matrix of space-time covariates.

### Value

Returns an object of class GeoSim. An object of class GeoSim is a list containing at most the following components:

bivariate	Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;
coordx	A $d$ -dimensional vector of spatial coordinates;
coordy	A $d$ -dimensional vector of spatial coordinates;
coordt	A $t$ -dimensional vector of temporal coordinates;
coordx_dyn	A list of dynamical (in time) spatial coordinates;
corrmodel	The correlation model; see <a href="#">GeoCovmatrix</a> .
data	The vector or matrix or array of data, see <a href="#">GeoFit</a> ;
distance	The type of spatial distance;
model	The type of RF, see <a href="#">GeoFit</a> .
n	The number of trial for Binomial RFs;the number of successes in a negative Binomial RFs;
numcoord	The number of spatial coordinates;
numtime	The number the temporal realisations of the RF;
param	The vector of parameters' estimates;
radius	The radius of the sphere if coordinates are passed in lon/lat format;
randseed	The seed used for the random simulation;
spacetime	TRUE if spatio-temporal and FALSE if spatial RF;

### Author(s)

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### Examples

```
library(GeoModels)
library(mapproj)
library(fields)

#####
###
### Example 1. Simulation of a spatial Gaussian RF on a regular grid
```

```

###
#####

# Define the spatial-coordinates of the points:
x <- seq(0,1,0.045)
y <- seq(0,1,0.045)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Matern", param=list(smooth=0.5,
    mean=0,sill=1,scale=0.4/3,nugget=0))$data

# Simulation of a spatial Gaussian RF with Generalized Wendland correlation function
# using sparse algorithm matrices
set.seed(261)
data2 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend", param=list(smooth=0,
    power2=4,mean=0,sill=1,scale=0.4,nugget=0))$data
par(mfrow=c(1,2))
image.plot(x,y,data1,col=terrain.colors(100),main="Matern",xlab="",ylab="")
image.plot(x,y,data2,col=terrain.colors(100),main="Wendland",xlab="",ylab="")

#####
###
### Example 2. Simulation of a spatial binomial RF
### with underlying exponential correlation
### on a regular grid
###
#####

# Define the spatial-coordinates of the points:
x <- seq(0, 1, 0.022)
y <- seq(0, 1, 0.022)
coords <- cbind(x,y)
set.seed(251)
n=5
# Simulation of a spatial Binomial RF:
sim <- GeoSim(x,y,grid=TRUE, corrmodel="Wend0",
    model="Binomial",n=n,sparse=TRUE,
    param=list(nugget=0,mean=0,scale=.2,sill=1,power2=4))

image.plot(x,y,sim$data,nlevel=n+1,col=terrain.colors(n+1),zlim=c(0,n))

#####
###
### Example 3. Simulation of a spatial Weibull RF
### with underlying exponential correlation
###
#####
# Define the spatial-coordinates of the points:
x <- seq(0,1,0.032)
y <- seq(0,1,0.032)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Exponential",model="Weibull",
    param=list(shape=1.2,mean=0,sill=1,scale=0.3/3,nugget=0))$data

```

```

image.plot(x,y,data1,col=terrain.colors(200),main="Weibull RF",xlab="",ylab="")

#####
###
### Example 4. Simulation of a spatial t RF
### with with underlying exponential correlation
###
#####
# Define the spatial-coordinates of the points:
x <- seq(0,1,0.03)
y <- seq(0,1,0.03)
set.seed(268)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend",model="StudentT", sparse=TRUE,
               param=list(df=1/4,mean=0,sill=1,scale=0.3,nugget=0,smooth=1,power2=5))$data
image.plot(x,y,data1,col=terrain.colors(100),main="Student-t RF",xlab="",ylab="")

#####
###
### Example 5. Simulation of a sinhasinh RF
### with underlying Wend0 correlation.
###
#####

# Define the spatial-coordinates of the points:
x <- runif(800, 0, 2)
y <- runif(800, 0, 2)
coords <- cbind(x,y)
set.seed(261)
corrmodel="Wend0"
# Simulation of a spatial Gaussian RF:
param=list(power2=4,skew=0,tail=1,
           mean=0,sill=1,scale=0.2,nugget=0) ## gaussian case
data0 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
plot(density(data0),xlim=c(-7,7))

param=list(power2=4,skew=0,tail=0.7,
           mean=0,sill=1,scale=0.2,nugget=0) ## heavy tails
data1 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data1),lty=2)

param=list(power2=4,skew=0.5,tail=1,
           mean=0,sill=1,scale=0.2,nugget=0) ## asymmetry
data2 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data2),lty=3)

#####

```

```

###
### Example 6. Simulation of a bivariate Gaussian RF
### with separable bivariate exponential correlation model
### on a regular grid.
###
#####

# Define the spatial-coordinates of the points:
x <- seq(-1,1,0.08)
y <- seq(-1,1,0.08)

# Simulation of a bivariate spatial Gaussian RF:
# with a separable Bivariate Matern
set.seed(12)
param=list(mean_1=0,mean_2=0,scale=0.12,smooth=0.5,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=0.5)
data <- GeoSim(x,y,grid=TRUE,corrmodel="Bi_matern_sep",
              param=param)$data
par(mfrow=c(1,2))

image.plot(x,y,data[,1],col=terrain.colors(100),main="1",xlab="",ylab="")
image.plot(x,y,data[,2],col=terrain.colors(100),main="2",xlab="",ylab="")


#####
###
### Example 7. Simulation of a spatio temporal Gaussian RF.
### observed on dynamical location sites with double exponential correlation
###
#####

# Define the dynamical spatial-coordinates of the points:

coordt=1:5
coordx_dyn=list()
maxN=40
set.seed(8)
for(k in 1:length(coordt))
{
  NN=sample(1:maxN,size=1)
  x <- runif(NN, 0, 1)
  y <- runif(NN, 0, 1)
  coordx_dyn[[k]]=cbind(x,y)
}
coordx_dyn

param<-list(nugget=0,mean=0,scale_s=0.2/3,scale_t=2/3,sill=1)
data <- GeoSim(coordx_dyn=coordx_dyn, coordt=coordt, corrmodel="Exp_Exp",
              param=param)$data
## spatial realization at first temporal instants
data[[1]]
## spatial realization at third temporal instants
data[[3]]

```

```
#####
###
### Example 8. Simulation of a Gaussian RF
### with a Wend0 correlation on the planet earth
###
#####

require(plot3D)
require(sphereplot)
distance="Geod";radius=6371

NN=4500 ## total point on the sphere on lon/lat format
set.seed(80)
coords=pointsphere(NN,c(-180,180),c(-90,90),c(radius,radius))
## Set the wendland parameters
corrmodel <- "Wend0"
param<-list(mean=0,sill=1,nugget=0,scale=radius*0.3,power2=3)
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(coordx=coords,corrmodel=corrmodel,sparse=TRUE,
               distance=distance,radius=radius,param=param)$data
#converting in 3d cartesian coordinates
b=sph2car(coords[,1], coords[,2], radius = radius, deg = TRUE)
x0 = b[,1]; y0 = b[,2]; z0 = b[,3]
# plotting
scatter3D(x0,y0,z0,colvar=data,clim=c(min(data),max(data)),pch=20,cex=0.8,
col=rainbow(200))

#####
###
### Example 9. Simulation of a Gaussian RF
### with Wend0 model on USA
###
#####

distance="Geod";radius=6378.88
NN=40
x=seq(-125,-64,length.out=NN)
y=seq(27,50, length.out =NN)
nrow(expand.grid(x,y))
## Set the wendland parameters
corrmodel <- "Wend0"
param<-list(mean=0,sill=1,nugget=0,scale=radius*0.3,power2=3)
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(x,y,grid=TRUE,corrmodel=corrmodel,sparse=TRUE,
               distance=distance,radius=radius,param=param)$data
image.plot(x,y,data,col=terrain.colors(100),xlab="",ylab="")
map("usa", add = TRUE)
```

```
#####
###
### Example 10. Simulation of a Wrapped RF
### with underlying exponential correlation
### on a regular grid
###
#####

# Define the spatial-coordinates of the points:
x <- runif(200,0, 1)
y <- runif(200,0, 1)
coords <- cbind(x,y)
set.seed(251)

# Simulation of a spatial wrapped RF:
sim <- GeoSim(coordx=coords, corrmodel="Exp",
              model="Wrapped",
              param=list(nugget=0,mean=0,scale=.1,sill=1))$data

long <- 0.08;
x1 <- coords[,1] + long*cos(sim)
y1 <- coords[,2] + long*sin(sim)
eps <- 0.1
plot(0,xlim=c(0-eps,1+eps),ylim=c(0-eps,1+eps));
require(shape)
Arrows(coords[,1], coords[,2], x1, y1, arr.length = 0.2, code = 2, arr.type = "triangle",col=1)
```

---

GeoTests

---

*Statistical Hypothesis Tests for Nested Models*


---

## Description

The function performs statistical hypothesis tests for nested models based on composite or standard likelihood versions of Wald-type and Wilks-type (likelihood ratio) statistics.

## Usage

```
GeoTests(object1, object2, ..., statistic)
```

## Arguments

<code>object1</code>	An object of class <code>GeoFit</code> .
<code>object2</code>	An object of class <code>GeoFit</code> that is a nested model within <code>object1</code> .
<code>...</code>	Further successively nested objects.
<code>statistic</code>	String; the name of the statistic used within the hypothesis test (see <b>Details</b> ).



## Details

The implemented hypothesis tests for nested models are based on the following statistics:

1. Wald-type (Wald);
2. Likelihood ratio or Wilks-type (Wilks under standard likelihood); For composite likelihood available variants of the basic version are:
  - Rotnitzky and Jewell adjustment (WilksRJ);
  - Satterhwaite adjustment (WilksS);
  - Chandler and Bate adjustment (WilksCB);
  - Pace, Salvan and Sartori adjustment (WilksPSS);

More specifically, consider an  $p$ -dimensional random vector  $\mathbf{Y}$  with probability density function  $f(\mathbf{y}; \theta)$ , where  $\theta \in \Theta$  is a  $q$ -dimensional vector of parameters. Suppose that  $\theta = (\psi, \tau)$  can be partitioned in a  $q'$ -dimensional subvector  $\psi$  and  $q''$ -dimensional subvector  $\tau$ . Assume also to be interested in testing the specific values of the vector  $\psi$ . Then, one can use some statistical hypothesis tests for testing the null hypothesis  $H_0 : \psi = \psi_0$  against the alternative  $H_1 : \psi \neq \psi_0$ . Composite likelihood versions of 'Wald' and 'score' statistics have the usual asymptotic chi-square distribution with  $q'$  degree of freedom. The Wald-type statistic is

$$W = (\hat{\psi} - \psi_0)^T (G^{\psi\psi})^{-1} (\hat{\theta}) (\hat{\psi} - \psi_0),$$

where  $G_{\psi\psi}$  is the  $q' \times q'$  submatrix of the Godambe or Fisher information pertaining to  $\psi$  and  $\hat{\theta}$  is the maximum likelihood estimator from the full model. This statistic can be called from the routine GeoTests assigning at the argument statistic the value: Wald.

Alternatively to the Wald-type statistic one can use the composite version of the Wilks-type or likelihood ratio statistic, given by

$$W = 2[C\ell(\hat{\theta}; \mathbf{y}) - C\ell\{\psi_0, \hat{\tau}(\psi_0); \mathbf{y}\}].$$

In the composite likelihood case, the asymptotic distribution of the composite likelihood ratio statistic is given by

$$W \sim \sum_i \lambda_i \chi^2,$$

for  $i = 1, \dots, q'$ , where  $\chi_i^2$  are  $q'$  iid copies of a chi-square one random variable and  $\lambda_1, \dots, \lambda_{q'}$  are the eigenvalues of the matrix  $(H^{\psi\psi})^{-1} G^{\psi\psi}$ . There exist several adjustments to the composite likelihood ratio statistic in order to get an approximated  $\chi_{q'}^2$ . For example, Rotnitzky and Jewell (1990) proposed the adjustment  $W' = W/\bar{\lambda}$  where  $\bar{\lambda}$  is the average of the eigenvalues  $\lambda_i$ . This statistic can be called within the routine by the value: WilksRJ. A better solution is proposed by Satterhwaite (1946) defining  $W'' = \nu W/(q'\bar{\lambda})$ , where  $\nu = (\sum_i \lambda)^2 / \sum_i \lambda_i^2$  for  $i = 1 \dots, q'$ , is the effective number of the degree of freedom. Note that in this case the distribution of the likelihood ratio statistic is a chi-square random variable with  $\nu$  degree of freedom. This statistic can be called from the routine assigning the value: WilksS. For the adjustments suggested by Chandler and Bate (2007) and Pace, Salvan and Sartori (2011) we refer to the articles (see **References**), these versions can be called from the routine assigning respectively the values: WilksCB and WilksPSS.

## Value

An object of class `c("data.frame")`. The object contain a table with the results of the tested models. The rows represent the responses for each model and the columns the following results:

Num.Par	The number of the model's parameters.
---------	---------------------------------------

Diff.Par	The difference between the number of parameters of the model in the previous row and those in the actual row.
Df	The effective number of degree of freedom of the chi-square distribution.
Chisq	The observed value of the statistic.
Pr(>chisq)	The p-value of the quantile Chisq computed using a chi-squared distribution with Df degrees of freedom.

### Author(s)

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### References

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### See Also

[GeoFit](#).

### Examples

```
library(GeoModels)
#####
###
### Example 1. Parametric test of Gaussianity
### using SinhAsinh random fields using standard likelihood
###
#####
set.seed(314)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
NN=500
x = runif(NN, 0, 1)
y = runif(NN, 0, 1)
coords = cbind(x,y)
# Parameters
mean=0; nugget=0; sill=1
### skew and tail parameters
skew=0;tail=1 ## H0 is Gaussianity
# underlying model correlation
corrmodel="Wend0"
```

```

power2=4;c_supp=0.2

# simulation
param=list(power2=power2,skew=skew,tail=tail,
           mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data = GeoSim(coordx=coords, corrmmodel=corrmmodel,model=model, param=param)$data

##### H1
fixed=list(power2=power2,nugget=nugget,mean=mean)
start=list(scale=c_supp,skew=skew,tail=tail,sill=sill)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH1 = GeoFit(data=data,coordx=coords,corrmmodel=corrmmodel, model=model,
               likelihood="Full",type="Standard",varest=TRUE,
               optimizer="BFGS",start=start,fixed=fixed)

fitH1$param

##### H0: Gaussianity (i.e tail=1, skew=0 fixed)
fixed=list(power2=power2,nugget=nugget,mean=mean,tail=1,skew=0)
start=list(scale=c_supp,sill=sill)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH0 = GeoFit(data=data,coordx=coords,corrmmodel=corrmmodel, model=model,
               likelihood="Full",type="Standard",varest=TRUE,
               optimizer="BFGS",start=start,fixed=fixed)

# Wald statistic test
GeoTests(fitH1, fitH0 ,statistic='Wald')
# likelihood ratio statistic test
GeoTests(fitH1, fitH0 , statistic='Wilks')

#####
###
### Example 2. Composite likelihood-based hypothesis testing.
### Testing significance of a regression parameter
###
#####
set.seed(31)
# Define the spatial-coordinates of the points:
N=1500
x = runif(N, 0, 1)
y = runif(N, 0, 1)
X=cbind(rep(1,N),runif(N))
coords=cbind(x,y)

# Set the model's parameters:
corrmmodel = "Exp"
mean = 1; mean1=0.3 ## H0: mean1=0
sill = 1
nugget = 0
scale = 0.15/3

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmmodel=corrmmodel, param=list(mean=mean,mean1=mean1,
                       sill=sill,nugget=nugget,scale=scale),X=X)$data

```

```
# Pairwise-likelihood fitting of the random field, full model:
start=list(mean=mean,mean1=mean1,sill=sill,scale=scale)
fixed=list(nugget=nugget)
fitH1 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.01,
               varest=TRUE,likelihood="Marginal",type="Pairwise",
               optimizer="BFGS",fixed=fixed,start=start,X=X)

# Pairwise-likelihood fitting of the random field, with a nested model:
start=list(mean=mean,sill=sill,scale=scale)
fixed=list(nugget=nugget,mean1=0) # setting H0 restriction
fitH0 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, maxdist=0.01,
               varest=TRUE,likelihood="Marginal",type="Pairwise",
               optimizer="BFGS",fixed=fixed,start=start,X=X)

# Hypothesis testing results:
# composite Wald statistic test:
GeoTests(fitH1, fitH0 ,statistic='Wald')
# composite likelihood ratio statistic test with PSS adjustment:
GeoTests(fitH1, fitH0, statistic='WilksPSS')
```

---

GeoVarestbootstrap	<i>Update a GeoFit object using parametric bootstrap for std error estimation</i>
--------------------	---

---

## Description

The procedure update a GeoFit object estimating stderr estimation using parametric bootstrap.

## Usage

```
GeoVarestbootstrap(fit,K=100,sparse=FALSE,GPU=NULL,local=c(1,1),optimizer="Nelder-Mead")
```

## Arguments

fit	A fitted object obtained from the <a href="#">GeoFit</a> .
K	The number of simulations in the parametric bootstrap.
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake).
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed
local	Numeric; number of local work-items of the OpenCL setup
optimizer	The type of optimization algorithm. See <a href="#">GeoFit</a> for details.

## Value

Returns an object of class GeoFit.

**Author(s)**

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**See Also**

[GeoFit](#).

---

GeoVariogram	<i>Empirical Variogram estimation</i>
--------------	---------------------------------------

---

**Description**

The function returns an empirical estimate of the variogram for spatio (temporal) and bivariate random fields.

**Usage**

```
GeoVariogram(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, cloud=FALSE,
             distance='Eucl', grid=FALSE, maxdist=NULL,
             maxtime=NULL, numbins=NULL, radius=6371,
             type='variogram', bivariate=FALSE)
```

**Arguments**

data	A $d$ -dimensional vector (a single spatial realisation) or a $(n \times d)$ -matrix ( $n$ iid spatial realisations) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array ( $n$ iid spatial realisations on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$ -array ( $n$ iid spatial-temporal realisations) or or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid) or an $(d \times d \times t \times n)$ -array ( $n$ iid spatial-temporal realisations on regular grid). See <a href="#">GeoFit</a> for details.
coordx	A numeric $(d \times 2)$ -matrix (where $d$ is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
cloud	Logical; if TRUE the variogram cloud is computed, otherwise if FALSE (the default) the empirical (binned) variogram is returned.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of <a href="#">GeoFit</a> .

grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.
maxdist	A numeric value denoting the spatial maximum distance, see the Section <b>Details</b> .
maxtime	A numeric value denoting the temporal maximum distance, see the Section <b>Details</b> .
numbins	A numeric value denoting the numbers of bins, see the Section <b>Details</b> .
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
type	A String denoting the type of variogram. The option available is : variogram.
bivariate	Logical; if FALSE (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are interpreted as a realization from a bivariate field.

### Details

We briefly report the definitions of semi-variogram used in this function. In the case of a spatial Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h) = 0.5 \sum_{x_i, x_j \in N(h)} (Z(x_i) - Z(x_j))^2 / |N(h)|$$

where  $N(h)$  is the set of all the sample pairs whose distances fall into a tolerance region with size  $h$  (equispaced intervals are considered).

In the case of a spatio-temporal Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h, u) = 0.5 \sum_{(x_i, l), (x_j, k) \in N(h, u)} (Z(x_i, l) - Z(x_j, k))^2 / |N(h, u)|$$

where  $N(h, u)$  is the set of all the sample pairs whose spatial distances fall into a tolerance region with size  $h$  and  $|k - l| = u$ . Note, that  $Z(x_i, l)$  is the observation at site  $x_i$  and time  $l$ .

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) least squares.

The maxdist parameter indicates the maximum spatial distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

The maxtime parameter indicates the maximum temporal distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

### Value

Returns an object of class Variogram. An object of class Variogram is a list containing at most the following components:

bins	Adjacent intervals of grouped spatial distances if cloud=FALSE. Otherwise if cloud=TRUE all the spatial pairwise distances;
bint	Adjacent intervals of grouped temporal distances if cloud=FALSE. Otherwise if cloud=TRUE all the temporal pairwise distances;
cloud	If the variogram cloud is returned (TRUE) or the empirical variogram (FALSE);
centers	The centers of the spatial bins;

distance	The type of spatial distance;
lenbins	The number of pairs in each spatial bin;
lenbinst	The number of pairs in each spatial-temporal bin;
lenbint	The number of pairs in each temporal bin;
maxdist	The maximum spatial distance used for the calculation of the variogram. If no spatial distance is specified then it is NULL;
maxtime	The maximum temporal distance used for the calculation of the variogram. If no temporal distance is specified then it is NULL;
spacetime_dyn	If the space-time variogram is obtained using dynamical coordinates then it is(TRUE).
variograms	The empirical spatial variogram;
variogramst	The empirical spatial-temporal variogram;
variogramt	The empirical temporal variogram;
type	The type of estimated variogram

**Author(s)**

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**References**

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- Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Springer Verlag, New York.

**See Also**

[GeoFit](#)

**Examples**

```
library(GeoModels)

#####
###
### Example 1. Empirical estimation of the semi-variogram from a
### spatial Gaussian random field with exponential correlation.
###
#####
set.seed(514)
# Set the coordinates of the sites:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords = cbind(x,y)
# Set the model's parameters:
corrmodel = "exponential"
mean = 0
sill = 1
nugget = 0
scale = 0.3/3

# Simulation of the spatial Gaussian random field:
```

```

data = GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,
  sill=sill, nugget=nugget, scale=scale))$data

# Empirical spatial semi-variogram estimation:
fit = GeoVariogram(coordx=coords,data=data,maxdist=0.6)

# Results:
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
  ylim=c(0, max(fit$variograms)), pch=20,
  main="Semi-variogram")

#####
###
### Example 2. Empirical estimation of the variogram from a
### spatio-temporal Gaussian random fields with Gneiting
### correlation function.
###
#####

set.seed(331)
# Define the temporal sequence:
# Set the coordinates of the sites:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords = cbind(x,y)
times = seq(1,10,1)

# Simulation of a spatio-temporal Gaussian random field:
data = GeoSim(coordx=coords, coordt=times, corrmodel="gneiting",
  param=list(mean=0,scale_s=0.08,scale_t=0.4,sill=1,
  nugget=0,power_s=1,power_t=1,sep=0.5))$data

# Empirical spatio-temporal semi-variogram estimation:
fit = GeoVariogram(data=data, coordx=coords, coordt=times, maxtime=7,maxdist=0.5)

# Results: Marginal spatial empirical semi-variogram
par(mfrow=c(2,2), mai=c(.5,.5,.3,.3), mgp=c(1.4,.5, 0))
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
  ylim=c(0, max(fit$variograms)), xlim=c(0, max(fit$centers)),
  pch=20,main="Marginal spatial semi-variogram",cex.axis=.8)

# Results: Marginal temporal empirical semi-variogram
plot(fit$bint, fit$variogramt, xlab='t', ylab=expression(gamma(t)),
  ylim=c(0, max(fit$variogramt)),xlim=c(0,max(fit$bint)),
  pch=20,main="Marginal temporal semi-variogram",cex.axis=.8)

# Building space-time semi-variogram
st.vario = matrix(fit$variogramst,length(fit$centers),length(fit$bint))
st.vario = cbind(c(0,fit$variograms), rbind(fit$variogramt,st.vario))

# Results: 3d Spatio-temporal semi-variogram
require(scatterplot3d)
st.grid = expand.grid(c(0,fit$centers),c(0,fit$bint))
scatterplot3d(st.grid[,1], st.grid[,2], c(st.vario),
  highlight.3d=TRUE, xlab="h",ylab="t",
  zlab=expression(gamma(h,t)), pch=20,

```



```

main="Space-time semi-variogram",cex.axis=.7,
mar=c(2,2,2,2), mgp=c(0,0,0),
cex.lab=.7)

# A smoothed version
par(mai=c(.2,.2,.2,.2),mgp=c(1,.3, 0))
persp(c(0,fit$centers), c(0,fit$bint), st.vario,
      xlab="h", ylab="u", zlab=expression(gamma(h,u)),
      ltheta=90, shade=0.75, ticktype="detailed", phi=30,
      theta=30,main="Space-time semi-variogram",cex.axis=.8,
      cex.lab=.8)

#####
###
### Example 3. Empirical estimation of the (cross) semivariograms
### from a bivariate Gaussian random fields with Matern
### correlation function.
###
#####
# Simulation of a bivariate spatial Gaussian random field:
set.seed(293)
# Define the spatial-coordinates of the points:
x = runif(400, 0, 1)
y = runif(400, 0, 1)
coords=cbind(x,y)

# Simulation of a bivariate Gaussian Random field
# with matern (cross) covariance function
param=list(mean_1=0,mean_2=0,scale_1=0.1/3,scale_2=0.15/3,scale_12=0.15/3,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,
           smooth_1=0.5,smooth_12=0.5,smooth_2=0.5,pcol=0.3)
data = GeoSim(coordx=coords, corrmodel="Bi_matern", param=param)$data

# Empirical semi-(cross)variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=c(0.5,0.5,0.5))

# Variograms plots
par(mfrow=c(2,2))
plot(biv_vario$centers,biv_vario$variograms[1,],pch=20,xlab="h",ylim=c(0,1.2),
      ylab="",main=expression(gamma[11](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",
      ylab="",main=expression(gamma[12](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",ylab="",
      main=expression(gamma[21](h)))
plot(biv_vario$centers,biv_vario$variograms[2,],pch=20,xlab="h",ylim=c(0,1.2),
      ylab="",main=expression(gamma[22](h)))

```

## Description

the function returns the parameters' estimates and the estimates' variances of a random field obtained by the weighted least squares estimator.

## Usage

```
GeoWLS(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmmodel,
        distance="Eucl", fixed=NULL, grid=FALSE, maxdist=NULL,
        maxtime=NULL, model='Gaussian', optimizer='Nelder-Mead',
        numbins=NULL, radius=6371, start=NULL, weighted=FALSE)
```

## Arguments

data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array ( $n$ iid spatial realisations on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid). See <a href="#">GeoFit</a> for details.
coordx	A numeric $(d \times 2)$ -matrix (where $d$ is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmmodel	String; the name of a correlation model, for the description (see <a href="#">GeoFit</a> ).
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of <a href="#">GeoFit</a> .
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
maxdist	A numeric value denoting the maximum distance, see <b>Details</b> and <a href="#">GeoFit</a> .
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation (see <a href="#">GeoFit</a> ).
model	String; the type of random field. Gaussian is the default, see <a href="#">GeoFit</a> for the different types.
optimizer	String; the optimization algorithm (see <a href="#">optim</a> for details). 'Nelder-Mead' is the default.
numbins	A numeric value denoting the numbers of bins, see the Section <b>Details</b>
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see <a href="#">GeoFit</a> ).
weighted	Logical; if TRUE then the weighted least square estimator is considered. If FALSE (the default) then the classic least square is used.

## Details

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum distance below which the shorter distances will be considered in the calculation of the (weighed) least squares.

## Value

Returns an object of class WLS. An object of class WLS is a list containing at most the following components:

bins	Adjacent intervals of grouped distances;
bint	Adjacent intervals of grouped temporal separations
centers	The centers of the bins;
coordx	The vector or matrix of spatial coordinates;
coordy	The vector of spatial coordinates;
coor dt	The vector of temporal coordinates;
convergence	A string that denotes if convergence is reached;
corrmodel	The correlation model;
data	The vector or matrix of data;
distance	The type of spatial distance;
fixed	The vector of fixed parameters;
iterations	The number of iteration used by the numerical routine;
maxdist	The maximum spatial distance used for the calculation of the variogram used in least square estimation. If no spatial distance is specified then it is NULL;
maxtime	The maximum temporal distance used for the calculation of the variogram used in least square estimation. If no temporal distance is specified then it is NULL;
message	Extra message passed from the numerical routines;
model	The type of random fields;
numcoord	The number of spatial coordinates;
numtime	The number the temporal realisations of the random field;
param	The vector of parameters' estimates;
variograms	The empirical spatial variogram;
variogramt	The empirical temporal variogram;
variogramst	The empirical spatial-temporal variogram;
weighted	A logical value indicating if its the weighted method;
wls	The value of the least squares at the minimum.

## Author(s)

Moreno Bevilacqua, <[moreno.bevilacqua@uv.cl](mailto:moreno.bevilacqua@uv.cl)>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <[victor.morales@uv.cl](mailto:victor.morales@uv.cl)>, <https://sites.google.com/site/moralesonatevictor/>

## References

- Cressie, N. A. C. (1993) *Statistics for Spatial Data*. New York: Wiley.
- Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Springer Verlag, New York.

**See Also**

[GeoFit](#), [optim](#)

**Examples**

```
library(GeoModels)

# Set the coordinates of the sites:

set.seed(211)
x <- runif(200, 0, 1)
set.seed(98)
y <- runif(200, 0, 1)
coords <- cbind(x,y)

#####
###
### Example 1. Least square fitting of a Gaussian random field
### with exponential correlation.
###
#####

# Set the model's parameters:
corrmodel <- "Exponential"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.15/3
param <- list(mean=0,sill=sill, nugget=nugget, scale=scale)
# Simulation of the Gaussian random field:
set.seed(2)
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=param)$data

fixed=list(nugget=0,mean=mean)
start=list(scale=scale,sill=sill)
# Least square fitting of the random field:
fit <- GeoWLS(data=data,coordx=coords, corrmodel=corrmodel,
              fixed=fixed,start=start,maxdist=0.5)

# Results:
print(fit)

#####
###
### Example 3. Least square fitting of a spatio-temporal
### Gaussian random field with double exponential correlation.
###
#####

# Define the temporal sequence:
time <- seq(1, 10, 1)
mean <- 0
```

```

sill <- 1
scale_s <- 0.15/3
scale_t <- 2/3
param <- list(mean=0,scale_s=scale,scale_t=scale_t,sill=sill,nugget=nugget)
# Simulation of the Gaussian random field:
set.seed(35)
data <- GeoSim(coordx=coords,coordt=time, corrmodel="exp_exp",
               param=param)$data

fixed<-list(nugget=nugget,mean=0)
start<-list(scale_s=scale_s,scale_t=scale_t,sill=1)
# Weighted least square estimation:
fit <- GeoWLS(data=data, coordx=coords,coordt=time, corrmodel="exp_exp",
               ,maxdist=0.5,maxtime=3,fixed=fixed,start=start)

# Results
print(fit)

```

Lik

*Optimizes the Log Likelihood*

## Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximization of the log-likelihood.

## Usage

```

Lik(bivariate,coordx,coordy,coordt,coordx_dyn,corrmodel,data,fixed,flagcor,flagnuis,
    grid,lower,mdecomp,model,namescorr,namesnuis,namesparam,numcoord,
    numpairs,numparamcor,numtime,optimizer,onlyvar,parallel,param,radius,setup,
    spacetime,sparse,varest,taper,type,upper,ns,X)

```

## Arguments

bivariate	Logical; if TRUE then the data come from a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric $(d \times 2)$ -matrix (where $d$ is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector assigning 1-dimension of spatial coordinates.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.

flagcor	A numeric vector of flags denoting which correlation parameters have to be estimated.
flagnuis	A numeric vector of flags denoting which nuisance parameters have to be estimated.
fixed	A numeric vector of parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
model	Numeric; the id value of the density associated to the likelihood objects.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numcoord	Numeric; the number of coordinates.
numpairs	Numeric; the number of pairs.
numparamcor	Numeric; the number of the correlation parameters.
numtime	Numeric; the number of temporal observations.
mdecomp	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd (Singular values decomposition).
optimizer	String; the optimization algorithm (see <a href="#">optim</a> for details). Nelder-Mead is the default. Other possible choices are ucminf, nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
parallel	Logical; if TRUE optimization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B. FALSE is the default.
param	A numeric vector of parameters.
sparse	Logical; if TRUE then maximum likelihood is computed using sparse matrices algorithms. FALSE is the default.
radius	Numeric; the radius of the sphere when considering data on a sphere.
ns	Numeric; vector of number of location sites for each temporal instants
setup	A List of useful components for the estimation based on the maximum tapered likelihood.
spacetime	Logical; if the random field is spatial (FALSE) or spatio-temporal (TRUE).
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
taper	String; the name of the taper correlation function.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

**Author(s)**

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**See Also**

[GeoFit](#)

---

MatDecomp	<i>Matrix decomposition</i>
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---

**Description**

Matrix decomposition.

**Usage**

MatDecomp(mtx, method)

**Arguments**

mtx	numeric; a square positive or semipositive definite matrix.
method	string; the type of matrix decomposition. Two possible choices: cholesky and svd.

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

---

MatSqrt, MatInv, MatLogDet	<i>Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.</i>
----------------------------	--

---

**Description**

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

**Usage**

MatSqrt(mat.decomp, method)  
 MatInv(mat.decomp, method)  
 MatLogDet(mat.decomp, method)

**Arguments**

mat.decomp	numeric; a matrix decomposition.
method	string; the type of matrix decomposition. Two possible choices: cholesky and svd.

**Author(s)**

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**See Also**

[MatDecomp](#)

**Examples**

```
library(GeoModels)
#####
###
### Example 1. Inverse of Covariance matrix associated to
### a Matern correlation model
###
#####
# Define the spatial-coordinates of the points:
x <- runif(15, 0, 1)
y <- runif(15, 0, 1)
coords <- cbind(x,y)
# Matern Parameters
param=list(smooth=0.5,sill=1,scale=0.2,nugget=0)
a=matrix <- GeoCovmatrix(coordx=coords, corrmmodel="Matern", param=param)

## decomposition with cholesky method
b=MatDecomp(a$covmat,method="cholesky")
## inverse of covariance matrix
inverse=MatInv(b,method="cholesky")
```

---

NuisParam

*Lists the Nuisance Parameters of a Random Field*

---

**Description**

Subroutine called by InitParam and other procedures. The procedure returns a list with the nuisance parameters of a given random field model.

**Usage**

```
NuisParam(model, bivariate=FALSE,num_betas=1)
```



**Arguments**

model	String; the name of a random field.
bivariate	Logical; if FALSE (the default) the correlation model is univariate spatial or spatial-temporal. Otherwise is bivariate.
num_betas	Numerical; the number of mean parameters in the linear specification (default is 1)

**Author(s)**

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

**See Also**

[GeoFit](#)

**Examples**

```
library(GeoModels)

NuisParam("Gaussian")

NuisParam("Binomial")

NuisParam("Weibull", num_betas=2)

NuisParam("SkewGaussian", num_betas=3)

NuisParam("SinhAsinh")

NuisParam("StudentT")
## note that in the bivariate case sill and nugget are considered as correlation parameteres
NuisParam("Gaussian", bivariate=TRUE)
```

---

Prscores

---

*Computation of three predictive scores: RMSE, LSCORE, CRPS for spatial, spatiotemporal and bivariate Gaussian RF.*


---

**Description**

The function computes RMSE, LSCORE, CRPS predictive scores.

**Usage**

```
Prscores(data, method="cholesky", matrix)
```

## Arguments

data	A $d$ -dimensional vector (a single spatial realisation) or a $a(t \times d)$ -matrix (a single spatial-temporal realisation). or a $a(2 \times d)$ -matrix (a single bivariate realisation).
method	String; the type of matrix decomposition used in the computation of the predictive scores. Default is cholesky. The other possible choices is svd.
matrix	An object of class matrix. See the Section <b>Details</b> .

## Details

For a given covariance matrix object ([GeoCovmatrix](#)) and a given spatial, spatiotemporal or bivariate realization from a Gaussian random field, the function computes three predictive scores.

## Value

Returns a list containing the following informations:

RMSE	Root-mean-square error predictive score
LSCORE	Logarithmic predictive score
CRPS	Continuous ranked probability predictive score

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>, <https://sites.google.com/a/uv.cl/moreno-bevilacqua/home>, Víctor Morales Oñate, <victor.morales@uv.cl>, <https://sites.google.com/site/moralesonatevictor/>

## References

- Zhang H. and Wang Y. (2010). *Kriging and cross-validation for massive spatial data*. *Environmetrics*, **21**, 290–304.
- Gneiting T. and Raftery A. *Strictly Proper Scoring Rules, Prediction, and Estimation*. *Journal of the American Statistical Association*, **102**

## See Also

[GeoCovmatrix](#)

## Examples

```
library(GeoModels)
library(fields)

#####
##### Examples of predictive score computation #####
#####

# Define the spatial-coordinates of the points:
x <- runif(500, 0, 2)
y <- runif(500, 0, 2)
coords=cbind(x,y)
matrix1 <- GeoCovmatrix(coordx=coords, corrmatrix="Matern", param=list(smooth=0.5,
sill=1,scale=0.2,nugget=0))
```

```
data <- GeoSim(coordx=coords, corrmodel="Matern", param=list(mean=0,smooth=0.5,
  sill=1,scale=0.2,nugget=0))$data

Pr_scores <- Prscores(data,matrix=matrix1)

Pr_scores
```

---

**StartParam**
*Initializes the Parameters for Estimation Procedures*


---

**Description**

Subroutine called by the fitting procedures. The procedure initializes the parameters for the fitting procedure.

**Usage**

```
StartParam(coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance, fcall,
  fixed, grid, likelihood, maxdist, maxtime, model, n, param,
  parscale, paramrange, radius, start, taper, tapsep,
  type, typereal, varest, vartype, weighted, winconst,
  winstp, winconst_t, winstp_t, X)
```

**Arguments**

coordx	A numeric ( $d \times 2$ )-matrix (where $d$ is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fcall	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
fixed	A named list giving the values of the parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the composite likelihood.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation.

radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
model	String; the density associated to the likelihood objects. Gaussian is the default.
n	Numeric; number of trials for binomial random fields.
param	A numeric vector of parameter values required in the simulation procedure of random fields.
parscale	A numeric vector of scaling factor to improve the maximizing procedure, see <a href="#">optim</a> .
paramrange	A numeric vector of parameters ranges, see <a href="#">optim</a> .
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure.
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separable parameter in the space time adaptive taper (see <b>Details</b> ).
type	String; the type of likelihood objects. Temporary value set to be "WLeast-Square" (weighted least-square) in order to compute the starting values.
typereal	String; the real type of likelihood objects. See <a href="#">GeoFit</a> .
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section <b>Details</b> .
weighted	Logical; if TRUE the likelihood objects are weighted, see <a href="#">GeoFit</a> .
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in (0, 1] for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in (0, 1] for defining the the proportion of overlapping in the temporal sub-sampling procedure.
X	Numeric; Matrix of space-time covariates.

### Author(s)

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### See Also

[GeoFit](#)

winds

*Irish Daily Wind Speeds***Description**

A matrix containing daily wind speeds, in kilometers per hour, from 1961 to 1978 at 12 sites in Ireland.

**Usage**

```
data(irishwinds)
```

**Format**

A  $(6574 \times 11)$ -matrix containing wind speed observations.

**Source**

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

winds.coords

*Weather Stations of the Irish Daily Wind Speeds***Description**

A data frame containing information about the weather stations where the data are recorded in Ireland.

**Usage**

```
data(irishwinds)
```

**Format**

A data frame containing site - the name of the city (character), abbr - the abbreviation (character), elev - the elevation (numeric), lat - latitude (numeric) and lon - longitude.

**Source**

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

---

WlsStart

---

*Computes Starting Values based on Weighted Least Squares*


---

### Description

Subroutine called by GeoFit. The function returns opportune starting values for the composite-likelihood fitting procedure based on weighed least squares.

### Usage

```
WlsStart(coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance, fcall,
         fixed, grid, likelihood, maxdist, maxtime, model, n, param,
         parscale, paramrange, radius, start, taper, tapsep, type, varest,
         vartype, weighted, winconst, winconst_t, winstp_t, winstp, X)
```

### Arguments

coordx	A numeric ( $d \times 2$ )-matrix (where $d$ is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fcall	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
fixed	A named list giving the values of the parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the composite likelihood.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
maxtime	Numeric; an optional positive value indicating the maximum temporal separation considered in the composite-likelihood computation.
model	String; the name of the model. Here the default is NULL.
n	Numeric; number of trials in a binomial random field.
param	A numeric vector of parameter values required in the simulation procedure of random fields.
parscale	A numeric vector with scaling values for improving the maximisation routine.
paramrange	A numeric vector with the range of the parameter space.

radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
start	A numeric vector with starting values.
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for tapered covariance matrix.
tapsep	Numeric; an optional value indicating the separable parameter in the space time quasi taper (see <b>Details</b> ).
type	String; the type of estimation method.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section <b>Details</b> .
weighted	Logical; if TRUE the likelihood objects are weighted, see <a href="#">GeoFit</a> .
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in (0, 1] for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in (0, 1] for defining the the proportion of overlapping in the temporal sub-sampling procedure.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

**Author(s)**

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