

UNIVERSITY OF SCIENCE AND TECHNOLOGY OF HANOI

Vietnam Academy of Science and Technology

Master Program: Space & Earth Observation

COMPREHENSIVE LITERATURE REVIEW

Reliability Improvement of Satellite-Based Quantum Key Distribution Systems Using Retransmission Scheme

Primary Reference:

Nguyen et al. (2021) – Photonic Network Communications
Posts and Telecommunications Institute of Technology (PTIT), Vietnam

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Course: Research Methodology

Literature Coverage:

85+ Papers | 1984–2025 | Foundational to State-of-the-Art
QKD Protocols | Satellite Experiments | Channel Models | Detection Schemes

Hanoi, December 2025

Abstract

This comprehensive literature review examines the field of satellite-based Quantum Key Distribution (QKD), with particular focus on reliability improvement techniques. The primary reference is the work by Nguyen et al. (2021) from the Posts and Telecommunications Institute of Technology (PTIT), Vietnam, which proposes a novel combination of QPSK-based modulation, dual-threshold/heterodyne detection, and key retransmission for enhancing satellite QKD reliability.

Scope: This review covers 85+ research papers spanning from the foundational BB84 protocol (1984) to state-of-the-art developments in 2025, organized into four thematic parts:

1. **Introduction and Paper Analysis:** Detailed examination of Nguyen et al. (2021), including system architecture, mathematical framework, and key contributions
2. **Theoretical Foundations:** Foundational QKD protocols (BB84, E91, CV-QKD) and landmark satellite experiments (Micius, integrated networks)
3. **Technical Aspects:** Atmospheric channel models, detection schemes, error correction methods, and security analysis approaches
4. **Analysis and Synthesis:** Comparative evaluation, research gap identification, and future research directions

Key Findings:

- Nguyen et al. (2021) achieves 20 dB power improvement over conventional schemes and $>1000\times$ Key Loss Rate reduction with 4 retransmissions
- The 3-D Markov chain model provides a novel analytical framework for link-layer reliability analysis
- Integration of physical layer optimization (DT/HD) with link layer mechanisms (ARQ) represents a unique cross-layer approach
- Remaining gaps include experimental validation, finite-key security analysis, and LEO constellation integration

Keywords: Quantum Key Distribution, Satellite Communication, Free-Space Optics, Atmospheric Turbulence, QPSK Modulation, Heterodyne Detection, Dual-Threshold, Key Retransmission, Markov Chain, Reliability, PTIT Vietnam

Papers Reviewed: 85+
Time Span: 1984–2025
Primary Categories: 11
High-Citation Papers: 15+ (>1000 citations each)

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List of Acronyms

ACK	Acknowledgment
APD	Avalanche Photodiode
ARQ	Automatic Repeat Request
BB84	Bennett-Brassard 1984 Protocol
BER	Bit Error Rate
BPF	Bandpass Filter
BPSK	Binary Phase-Shift Keying
CDMA	Code Division Multiple Access
CV-QKD	Continuous-Variable Quantum Key Distribution
CW	Continuous Wave
DD	Direct Detection
DT	Dual Threshold
DTMC	Discrete-Time Markov Chain
DV-QKD	Discrete-Variable Quantum Key Distribution
E91	Ekert 1991 Protocol
FEC	Forward Error Correction
FSO	Free-Space Optical
HD	Heterodyne Detection
H-V	Hufnagel-Valley
IF	Intermediate Frequency
InGaAs	Indium Gallium Arsenide
KLR	Key Loss Rate
LDPC	Low-Density Parity-Check

LEO	Low Earth Orbit
LO	Local Oscillator
LPF	Lowpass Filter
MZM	Mach-Zehnder Modulator
NACK	Negative Acknowledgment
PDF	Probability Density Function
PTIT	Posts and Telecommunications Institute of Technology
QA-DTMC	Queue-Associated Discrete-Time Markov Chain
QBER	Quantum Bit Error Rate
QKD	Quantum Key Distribution
QKER	Quantum Key Error Rate
QPSK	Quadrature Phase-Shift Keying
RF	Radio Frequency
SIM	Subcarrier Intensity Modulation
SNR	Signal-to-Noise Ratio
SNSPD	Superconducting Nanowire Single-Photon Detector
URA	Unauthorized Receiver Attack
USTH	University of Science and Technology of Hanoi
VAST	Vietnam Academy of Science and Technology
VNSC	Vietnam National Space Center

Part I

Introduction and Paper Analysis

Chapter 1

Introduction

This literature review provides a comprehensive analysis of reliability improvement techniques for satellite-based Quantum Key Distribution (QKD) systems, with particular focus on the retransmission scheme proposed by Nguyen et al. (2021) from the Posts and Telecommunications Institute of Technology (PTIT), Vietnam.

1.1 Paper	Under	Review
1.1.1 Bibliographic	Information	

Table 1.1: Paper Identification

Field	Information
Title	Reliability improvement of satellite-based quantum key distribution systems using retransmission scheme
Authors	Nam D. Nguyen, Hang T. T. Phan, Hien T. T. Pham, Vuong V. Mai, Ngoc T. Dang
Journal	Photonic Network Communications
Publisher	Springer
Year	2021
DOI	10.1007/s11107-021-00934-y
Institution	Posts and Telecommunications Institute of Technology (PTIT)
Country	Vietnam

1.1.2 Abstract	Summary
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The paper addresses the design and performance analysis of reliable satellite-based QKD over free-space optics (FSO) channels. The key contributions include:

1. **QPSK-based QKD Protocol:** Optical quadrature phase-shift keying modulation adapted for quantum key distribution
2. **Dual-Threshold/Heterodyne Detection (DT/HD):** Advanced receiver design that reduces QBER and improves sensitivity

3. **Key Retransmission Scheme:** ARQ-based protocol at the link layer to enhance reliability
4. **3-D Markov Chain Model:** Novel analytical framework for Key Loss Rate (KLR) analysis

1.2 Research Context and Motivation

1.2.1 The Need for Quantum-Secure Communication

The development of quantum computing poses fundamental threats to classical cryptographic systems. Shor's algorithm, when implemented on a sufficiently powerful quantum computer, can efficiently factor large integers, thereby breaking RSA and elliptic curve cryptography—the foundations of modern secure communication [1].

Timeline of Quantum Computing Threat:

- **Current (2025):** NISQ-era quantum computers with ~ 1000 qubits
- **Near-term (2030):** Potential for cryptographically-relevant quantum computers
- **Harvest now, decrypt later:** Adversaries may store encrypted data today for future decryption

Quantum Key Distribution (QKD) offers a solution with information-theoretic security—security guaranteed by the laws of physics rather than computational assumptions.

1.2.2 Why Satellite-Based QKD?

While fiber-based QKD has achieved commercial deployment, fundamental limitations restrict its range:

Table 1.2: Comparison of QKD Transmission Media

Medium	Maximum Distance	Limitation
Optical Fiber	~ 400 km	Exponential attenuation (~ 0.2 dB/km)
Terrestrial FSO	~ 10 km	Atmospheric turbulence, weather
Satellite FSO	>1000 km	Lower atmospheric path length

Key advantages of satellite QKD:

- Free-space loss scales as $1/R^2$ (better than exponential fiber loss for long distances)
- Vacuum of space has negligible absorption
- Single satellite can serve multiple ground stations
- Global coverage possible with constellation

1.2.3 The Reliability Challenge

Despite its promise, satellite QKD faces significant reliability challenges:

1. **Atmospheric Turbulence:** Random intensity fluctuations (scintillation) cause signal fading
2. **Free-Space Path Loss:** >40 dB loss for LEO satellites at 600 km altitude
3. **Weather Dependence:** Clouds, rain, and aerosols increase attenuation
4. **Beam Spreading:** Diffraction causes power dilution at receiver
5. **Pointing Errors:** Misalignment between satellite and ground station
6. **Background Noise:** Solar radiation during daytime operation

These factors lead to high Quantum Bit Error Rate (QBER) and potential key transmission failures, motivating the need for reliability improvement techniques.

1.3 Vietnamese Research Context

1.3.1 PTIT Research Group

The Posts and Telecommunications Institute of Technology (PTIT) in Hanoi has established itself as a leading center for optical wireless communication research in Vietnam. Key achievements include:

- **Dual-threshold detection analysis:** Trinh et al. (2018) [2]
- **Reliability improvement schemes:** Nguyen et al. (2021) — the paper under review
- **CV-QKD optimization:** Nguyen et al. (2023) [3]
- **International collaborations:** University of Aizu (Japan), KAIST (Korea)

1.3.2 Regional Significance

Vietnam's strategic location and growing technological capabilities position it well for quantum communication development:

- Vietnam National Space Center (VNSC) under VAST
- USTH graduate programs in space technology
- Regional cooperation within ASEAN
- Tropical atmosphere conditions requiring specific modeling

1.4 Literature Review Objectives

This literature review aims to:

1. **Comprehensive Analysis:** Provide in-depth understanding of Nguyen et al. (2021) contributions
2. **Theoretical Foundation:** Connect the paper to foundational QKD and FSO literature
3. **Technical Deep-Dive:** Analyze the QPSK-based protocol, DT/HD detection, and retransmission scheme
4. **Mathematical Framework:** Review the channel model, QBER derivation, and 3-D Markov chain analysis
5. **Performance Evaluation:** Understand numerical results and their implications
6. **Critical Assessment:** Identify strengths, limitations, and future research directions
7. **Comparative Context:** Position the work within the broader satellite QKD literature

1.5 Review Structure

This literature review is organized as follows:

Chapter 2: Detailed Paper Analysis presents the complete technical analysis of Nguyen et al. (2021), including system architecture, key innovations, and main results.

Chapter 3: QKD Protocol Design examines the QPSK-based QKD protocol, its relationship to BB84, and the dual-threshold/heterodyne detection scheme.

Chapter 4: Channel Model Analysis provides detailed analysis of the FSO channel model including free-space loss, atmospheric attenuation, beam spreading, and Gamma-Gamma turbulence fading.

Chapter 5: Retransmission Scheme analyzes the key retransmission protocol, the 3-D Markov chain model, and Key Loss Rate derivation.

Chapter 6: Performance Analysis reviews numerical results for QBER, P_{sift} , and KLR under various conditions.

Chapter 7: Comparative Study positions the work within the broader literature and compares with alternative approaches.

Chapter 8: Conclusion synthesizes key findings and outlines future research directions.

Chapter 2

Detailed Paper Analysis

This chapter provides a comprehensive technical analysis of Nguyen et al. (2021), examining the system architecture, key innovations, mathematical framework, and main contributions to satellite-based QKD reliability improvement.

2.1 System Architecture Overview

2.1.1 Two-Layer Design

The proposed system operates across two layers:

1. **Physical Layer:** Responsible for quantum key transmission over FSO channel

- QPSK modulator with Mach-Zehnder modulators (MZMs)
- FSO channel with combined loss mechanisms
- Dual-threshold/heterodyne detection (DT/HD) receiver

2. **Link Layer:** Manages key retransmission for reliability

- Buffer management at Alice (satellite)
- ACK/NACK feedback via classical RF channel
- Retransmission protocol with maximum M attempts

2.1.2 Link Configuration

Table 2.1: System Configuration

Component	Location	Function
Alice (Transmitter)	LEO Satellite (600 km)	Key generation, QPSK modulation
Bob (Receiver)	Ground Station (5 m)	DT/HD detection, key recovery
Forward Channel	FSO (Downlink)	Quantum key transmission
Feedback Channel	RF (Classical)	ACK/NACK signaling

2.2 Key Innovations

2.2.1 Innovation 1: QPSK-Based QKD Protocol

The paper adapts Quadrature Phase-Shift Keying (QPSK) for quantum key distribution:
Advantages over alternatives:

- Compared to SIM/BPSK: No RF subcarrier required, simpler implementation
- Compared to polarization encoding: Compatible with coherent detection
- Direct mapping of BB84 four-state structure to four QPSK phase states

Phase State Mapping:

$$\phi_A \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, -\frac{\pi}{4} \right\} \quad (2.2.1)$$

2.2.2 Innovation 2: Dual-Threshold/Heterodyne Detection

The DT/HD receiver combines two techniques:

Heterodyne Detection:

- Signal mixed with strong local oscillator
- Improved receiver sensitivity compared to direct detection
- Both quadratures accessible (though only one used for key)

Dual-Threshold Decision:

$$\text{Decision} = \begin{cases} 0 & \text{if } i \geq d_0 \\ 1 & \text{if } i \leq d_1 \\ X & \text{otherwise (erasure)} \end{cases} \quad (2.2.2)$$

Quantified Improvement: 20 dB reduction in required transmitted power compared to SIM/BPSK-DT.

2.2.3 Innovation 3: Key Retransmission Scheme

The ARQ-based retransmission scheme operates as follows:

1. Alice generates key sequence, stores in buffer
2. Transmits via FSO channel to Bob
3. Bob checks received sequence for errors
4. If successful: Send ACK, Alice removes from buffer
5. If failed: Send NACK, Alice retransmits (up to M times)

6. After M failures: Discard sequence, count as key loss

Advantage over FEC:

- No computational overhead for encoding/decoding
- Adapts naturally to channel variations
- Simple implementation

2.2.4 Innovation 4: 3-D Markov Chain Model

Novel analytical framework with three-dimensional state space:

$$\text{State: } (n, s, m) \text{ where } \begin{cases} n \in [0, C] & \text{Buffer queue length} \\ s \in \{B, G\} & \text{Channel state (Bad/Good)} \\ m \in [1, M] & \text{Retransmission attempt number} \end{cases} \quad (2.2.3)$$

This model enables analytical calculation of Key Loss Rate (KLR).

2.3 Main Contributions Summary

Table 2.2: Paper Contributions and Impact

Contribution	Type	Impact
QPSK-based QKD with DT/HD	System Design	20 dB power improvement
Key retransmission scheme	Protocol Innovation	$>1000\times$ KLR reduction
3-D Markov chain model	Analytical Framework	Enables KLR prediction
Comprehensive channel model	Mathematical Derivation	Realistic performance analysis
Parameter optimization	Numerical Results	Practical design guidelines

2.4 Key Results Summary

2.4.1 Physical Layer Results

QBER Performance:

- Achieves $\text{QBER} < 10^{-3}$ with proper DT coefficient selection
- Optimal ς range: 0.7–2.4 (weak turbulence), 1.4–2.8 (strong turbulence)
- $P_{shift} \geq 10^{-2}$ maintained for sufficient key rate

Power Comparison:

Table 2.3: Required Transmitted Power for QBER $\leq 10^{-3}$

Scheme	Required P_T
SIM/BPSK-DT	45 dBm
QPSK-DT/DD	35 dBm
QPSK-DT/HD (Proposed)	25 dBm

2.4.2 Link Layer Results

KLR Improvement with Retransmissions:

Table 2.4: Key Loss Rate vs. Number of Retransmissions

Retransmissions (M)	KLR	Improvement
0 (Conventional)	3×10^{-2}	Baseline
1	$10^{-3} - 10^{-2}$	$10\times$
2	$10^{-4} - 10^{-3}$	$100\times$
4	$< 10^{-4}$	$>1000\times$

Key Finding: Diminishing returns beyond $M = 4$; only 0.5 dB additional power gain from $M = 4$ to $M = 7$.

2.4.3 Security Analysis

Unauthorized Receiver Attack (URA):

- Eve's QBER increases with distance from Bob
- Minimum secure distance: $D_{E-B} > 30$ m (both weak and strong turbulence)
- Security maintained when Eve's QBER $> 10^{-2}$

2.5 System Parameters

Table 2.5: Complete System Parameters

Category	Parameter	Symbol	Value
Physical Constants	Electron charge	q	1.6×10^{-19} C
	Boltzmann constant	k_B	1.38×10^{-23} W/K/Hz
	Planck's constant	\tilde{h}	6.63×10^{-34} J·s
Receiver	Bit rate	R_b	10 Gbps
	Load resistor	R_L	50 Ω
	Excess noise factor	x	0.8
	Avalanche multiplication	\bar{g}	10
	Responsivity	\mathfrak{R}	0.8
	Temperature	T	298 K
	Dark current	I_d	3 nA
Channel	Wavelength	λ	1550 nm
	Satellite altitude	H_S	600 km
	Ground station height	H_G	5 m
	Atmospheric altitude	H_β	20 km
	Zenith angle	ζ	50°
	Wind speed	w	21 m/s
	Beam width	ω_D	50 m
	Detection aperture	a	0.31 m
	Attenuation coefficient	γ	0.43 dB/km
Telescope	Tx gain	G_T	120 dB
	Rx gain	G_R	121 dB
Link Layer	Flow throughput	H	185 seq/s
	Bit sequence length	l_{bs}	3×10^6 bits

Part II

Literature Review: Theoretical Foundations

Chapter 3

Foundational QKD Literature

This chapter reviews the foundational literature in Quantum Key Distribution, from the original BB84 protocol to modern continuous-variable approaches, establishing the theoretical basis for satellite-based QKD systems.

3.1 The BB84 Protocol

3.1.1 Historical Context

The BB84 protocol, proposed by Charles Bennett and Gilles Brassard in 1984 [4], represents the foundational breakthrough in quantum cryptography. Published at the IEEE International Conference on Computers, Systems and Signal Processing in Bangalore, India, this work established the principles that underpin all subsequent QKD protocols.

Key Innovation: BB84 was the first protocol to demonstrate that quantum mechanical principles—specifically the no-cloning theorem and the disturbance caused by measurement—could be exploited to achieve information-theoretically secure key distribution.

3.1.2 Protocol Description

The BB84 protocol operates using four quantum states organized into two conjugate bases:

Table 3.1: BB84 Quantum States

Basis	Bit 0	Bit 1	Representation
Rectilinear (+)	$ 0\rangle$	$ 1\rangle$	Horizontal/Vertical
Diagonal (\times)	$ +\rangle$	$ -\rangle$	$\pm 45^\circ$ Diagonal

Protocol Steps:

1. **Preparation:** Alice randomly chooses a bit value and a basis, prepares the corresponding quantum state
2. **Transmission:** State is sent through quantum channel to Bob

3. **Measurement:** Bob randomly chooses a measurement basis
4. **Sifting:** Alice and Bob publicly compare bases; keep only matching-basis results
5. **Error Estimation:** Sample subset to estimate QBER
6. **Error Correction:** Correct remaining errors using classical protocols
7. **Privacy Amplification:** Reduce Eve's potential information

3.1.3 Security Foundation

The security of BB84 rests on fundamental quantum mechanical principles:

1. **No-Cloning Theorem:** An unknown quantum state cannot be perfectly copied
2. **Measurement Disturbance:** Measuring a quantum state in the wrong basis causes irreversible disturbance
3. **Uncertainty Principle:** Conjugate observables cannot be simultaneously known with arbitrary precision

QBER Threshold: The protocol remains secure when $\text{QBER} < 11\%$ for individual attacks, or $\text{QBER} < 7.1\%$ for coherent attacks.

3.1.4 Relevance to Nguyen et al. (2021)

Nguyen et al. maps the BB84 four-state structure to QPSK phase states:

$$\text{BB84 States} \rightarrow \text{QPSK Phases: } \phi_A \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, -\frac{\pi}{4} \right\} \quad (3.1.1)$$

This mapping preserves the fundamental security properties while enabling coherent detection.

3.2 The E91 Protocol Protocol

3.2.1 Entanglement-Based QKD

Artur Ekert proposed the E91 protocol in 1991 [5], introducing entanglement as the basis for QKD security. This protocol uses maximally entangled photon pairs (Bell states):

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad (3.2.1)$$

3.2.2 Security via Bell Inequality Inequality

E91's security is verified through violation of Bell's inequality:

$$S = |E(a, b) - E(a, b') + E(a', b) + E(a', b')| \leq 2 \text{ (Classical)} \quad (3.2.2)$$

For maximally entangled states: $S = 2\sqrt{2} \approx 2.83$, proving quantum correlations.

3.2.3 Advantages and Challenges

Table 3.2: E91 vs. BB84 Comparison

Aspect	BB84	E91
Source	Single photon/WCP	Entangled pairs
Security verification	QBER estimation	Bell inequality
Implementation	Simpler	More complex
Device-independence	No	Possible
Satellite demonstration	Micius (2017)	Micius (2017)

3.3 Continuous-Variable QKD

3.3.1 Paradigm Shift

Continuous-Variable QKD (CV-QKD), pioneered by Grosshans et al. (2003) [6], represents a fundamental departure from discrete-variable approaches:

Table 3.3: DV-QKD vs. CV-QKD

Aspect	DV-QKD	CV-QKD
Information carrier	Single photons	Coherent states
Detection	Single-photon detectors	Homodyne/Heterodyne
Modulation	Discrete (2/4 states)	Continuous (Gaussian)
Detector technology	APD/SNSPD	Standard photodiodes
Key rate (short range)	Lower	Higher
Maximum distance	~400 km	~200 km
Telecom compatibility	Limited	High

3.3.2 Gaussian Modulation Protocol

The GG02 protocol uses Gaussian-modulated coherent states:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad (3.3.1)$$

where $\alpha = x + ip$ with x, p drawn from Gaussian distributions with variance V_A .

3.3.3 Security Proofs

Leverrier (2015) [7] established composable security for CV-QKD with coherent states, proving security against general attacks in the asymptotic limit. Key developments include:

- **Collective attacks:** Security proven for arbitrary attack strategies with i.i.d. assumption

- **Finite-key analysis:** Composable security with practical key lengths
- **Trusted noise model:** Excess noise bounded by device characterization

3.3.4 Relationship to Nguyen et al. (2021)

Nguyen et al.'s QPSK-based approach bridges DV and CV paradigms:

- Uses **discrete modulation** (4 phase states) like DV-QKD
- Employs **coherent detection** (heterodyne) like CV-QKD
- Security analysis based on **BB84 mapping**

This hybrid approach offers implementation simplicity while maintaining BB84-equivalent security.

3.4 Decoy State Method

3.4.1 Addressing Practical Source Limitations

Practical QKD implementations use weak coherent pulses (WCP) instead of true single photons, introducing vulnerabilities to photon-number-splitting (PNS) attacks. The decoy state method, proposed by Hwang (2003) and refined by Lo, Ma, and Chen (2005), addresses this limitation.

3.4.2 Principle

Alice randomly varies the mean photon number μ of transmitted pulses:

- **Signal state:** $\mu_s \approx 0.5$ (key generation)
- **Decoy state:** $\mu_d \approx 0.1$ (parameter estimation)
- **Vacuum state:** $\mu_v = 0$ (dark count estimation)

3.4.3 Security Enhancement

The decoy state method enables tight bounds on single-photon contribution:

$$Y_1^L \leq Y_1 \leq Y_1^U \quad (3.4.1)$$

where Y_1 is the single-photon yield, enabling security equivalent to ideal single-photon sources.

3.4.4 Implementation in Satellite QKD

The Micius satellite [8] demonstrated decoy-state BB84 for satellite QKD:

- Three-intensity protocol (μ, ν , vacuum)
- QBER $\sim 1.1\%$ achieved
- Key rate up to 40.2 kbps at 530 km distance

3.5 Landmark Review Papers

3.5.1 Gisin et al. (2002)

“Quantum Cryptography” in Reviews of Modern Physics [9] provided the first comprehensive review of QKD, covering:

- Theoretical foundations and security proofs
- Experimental implementations
- Practical considerations and limitations

Citations: >5000 (foundational reference for the field)

3.5.2 Scarani et al. (2009)

“The Security of Practical Quantum Key Distribution” [1] in Reviews of Modern Physics established the framework for analyzing practical QKD security:

Key Contributions:

- Rigorous treatment of practical device imperfections
- Comprehensive attack classification
- Security parameter optimization

Relevance to Nguyen et al.: Provides the security framework for QBER analysis and threshold determination.

3.5.3 Pirandola et al. (2020)

“Advances in Quantum Cryptography” [10] in Advances in Optics and Photonics (225 pages) represents the most comprehensive recent review:

Coverage:

- DV-QKD and CV-QKD protocols
- Satellite and free-space implementations
- Quantum networks and repeaters
- Post-quantum considerations

3.5.4 Xu et al. (2020)

“Secure Quantum Key Distribution with Realistic Devices” [11] in *Reviews of Modern Physics* addresses practical security:

- Device imperfection modeling
- Side-channel attacks and countermeasures
- Measurement-device-independent QKD
- Twin-field QKD for extended range

3.6 Historical Timeline

Table 3.4: QKD Development Timeline

Year	Milestone	Reference
1984	BB84 protocol proposed	Bennett & Brassard
1991	E91 entanglement protocol	Ekert
1992	First experimental BB84 (32 cm)	Bennett et al.
2002	Comprehensive QKD review	Gisin et al.
2003	CV-QKD with coherent states	Grosshans et al.
2005	Decoy state method	Lo, Ma, Chen
2007	200 km fiber QKD	Schmitt-Manderbach
2009	Practical security framework	Scarani et al.
2012	Finite-key analysis	Tomamichel et al.
2015	CV-QKD composable security	Leverrier
2016	Micius satellite launch	USTC/CAS
2017	First satellite QKD	Liao et al.
2020	MDI-QKD over 500 km	Chen et al.
2021	4600 km integrated network	Chen et al.

3.7 Chapter Summary

This chapter established the theoretical foundations underlying Nguyen et al. (2021):

1. **BB84 Protocol:** Provides the four-state structure mapped to QPSK phases
2. **CV-QKD:** Introduces coherent detection applicable to heterodyne receivers
3. **Decoy States:** Addresses practical source limitations in satellite implementations
4. **Security Framework:** Establishes QBER thresholds and analysis methodology

Key Insight: Nguyen et al.’s QPSK-DT/HD approach synthesizes concepts from multiple foundational works, creating a practical system that leverages coherent detection advantages while maintaining BB84-equivalent security structure.

Chapter 4

Satellite QKD Experiments

This chapter reviews the landmark experimental demonstrations of satellite-based QKD, from the Micius satellite missions to the integrated space-ground quantum network, providing the experimental context for Nguyen et al.'s theoretical contributions.

4.1 The Micius Satellite (QUSS) Overview

4.1.1 Mission

The Quantum Experiments at Space Scale (QUSS) mission, featuring the Micius satellite, represents humanity's first dedicated quantum science satellite. Named after the ancient Chinese philosopher Mozi (), this \$100 million mission was launched on August 16, 2016 [8].

Table 4.1: Micius Satellite Specifications

Parameter	Specification
Launch Date	August 16, 2016
Launch Vehicle	Long March 2D
Orbit	Sun-synchronous LEO
Altitude	~500 km
Inclination	97.4°
Mass	631 kg
Design Life	2 years (exceeded)
Operating Institution	CAS/USTC
Lead Scientist	Prof. Jian-Wei Pan

4.1.2 Scientific Payload

The satellite carries three main experimental payloads:

1. **QKD Transmitter:** Decoy-state BB84 source

- Wavelength: 850 nm

- Repetition rate: 100 MHz
- Decoy intensities: μ, ν , vacuum

2. **Entanglement Source:** Spontaneous parametric down-conversion

- Entangled photon pairs at 810 nm
- >5.9 million pairs/second
- Fidelity >90%

3. **Quantum Teleportation Payload:** Bell-state measurement capability

4.2 Micius Experimental Results

4.2.1 Satellite-to-Ground QKD (2017)

Liao et al. [8] demonstrated the first satellite-to-ground QKD, published in Nature:
Experimental Configuration:

- Ground station: Xinglong, near Beijing
- Distance range: 507 km to 1034 km
- Measurement duration: 273 seconds per pass

Key Results:

Table 4.2: Micius QKD Performance Results

Metric	530 km	1034 km
Sifted key rate	40.2 kbps	1.2 kbps
Secure key rate	12.0 kbps	0.4 kbps
QBER	1.1%	3.2%
Channel loss	21.5 dB	41.5 dB

Significance: Demonstrated that satellite QKD can exceed ground-based fiber performance for distances >400 km.

4.2.2 Entanglement Distribution (2017)

Yin et al. [12] achieved record-breaking entanglement distribution, published in Science:
Configuration:

- Two ground stations: Delingha and Lijiang
- Separation: 1203 km
- Simultaneous detection of entangled pairs

Results:

- Bell inequality violation: $S = 2.37 \pm 0.09 > 2$
- Detection rate: 1.1 pairs/second
- Fidelity: 87.4%

Significance: Demonstrated entanglement preservation over unprecedented distances, opening possibility for device-independent QKD.

4.2.3 Intercontinental QKD (2018)

Liao et al. [13] demonstrated China-Austria QKD, published in Physical Review Letters:
Configuration:

- Ground stations: China (multiple) and Austria (Graz)
- Total distance: 7600 km
- Satellite as trusted relay node

Demonstration:

- 75-minute encrypted videoconference between Beijing and Vienna
- 128-bit AES keys exchanged via QKD
- First intercontinental quantum-secured communication

4.2.4 Full Quantum Teleportation (2021)

Pan's team demonstrated full quantum state teleportation over 1200 km ground distance using satellite-distributed entanglement.

4.3 Integrated Space-Ground Network (2021)

4.3.1 Chen et al. (2021) - Nature

Chen et al. [14] published “An integrated space-to-ground quantum communication network over 4,600 kilometres” in Nature, representing the most comprehensive quantum network demonstration.

Network Architecture:

- **Fiber backbone:** 2000 km Beijing-Shanghai link
- **Satellite links:** 2600 km via Micius
- **Metropolitan networks:** 4 QMANS (Shanghai, Hefei, Jinan, Beijing)
- **Trusted nodes:** 32 fiber relay stations
- **Users:** 150+ across government, banking, grid

Table 4.3: Integrated Network Performance

Link Type	Key Rate	Improvement
Satellite-ground	47.8 kbps	40× vs. previous
Fiber backbone	Variable	Continuous operation
Metropolitan	High rate	Local applications

Performance:

Applications Demonstrated:

- Banking transactions (ICBC, Bank of China)
- Power grid communications (State Grid)
- Government e-services
- Encrypted voice/video conferencing

4.4 Other Satellite QKD Missions

4.4.1 Jinan-1 Micro-Satellite (2022)

China launched the Jinan-1 micro-nano satellite in July 2022, demonstrating cost-effective QKD:

- Smaller form factor than Micius
- Compact ground stations
- 2025: Achieved QKD with South Africa (12,900 km)

4.4.2 Planned Missions (2025-2027)

Table 4.4: Upcoming Satellite QKD Missions

Mission	Country	Launch	Objective
Eagle-1	ESA	2025-2026	Operational QKD service
QUBE-II	Germany	2025	CubeSat BB84 demonstration
QEYSSat	Canada	2025	LEO constellation study
Next-gen Micius	China	2025	LEO constellation (2-3 sats)
MEO satellite	China	2027	Extended coverage

4.4.3 Japanese NICT Experiments

Japan's NICT has conducted ground-to-LEO experiments using the SOCRATES satellite, demonstrating:

- Uplink QKD feasibility
- Pointing acquisition and tracking
- Photon transmission through atmosphere

4.5 Comparison with Nguyen et al. (2021)

4.5.1 System Parameter Comparison

Table 4.5: Nguyen et al. vs. Micius Systems

Parameter	Nguyen (2021)	Micius (2017)
<i>System Configuration</i>		
Satellite altitude	600 km	500 km
Protocol	QPSK-based	Decoy-state BB84
Wavelength	1550 nm	850 nm
Modulation	Phase (QPSK)	Polarization
Detection	Heterodyne (APD)	Single-photon (Si-APD)
<i>Error Handling</i>		
Method	ARQ retransmission	LDPC FEC
Complexity	Low	Medium
Adaptability	Channel-adaptive	Fixed rate
<i>Validation</i>		
Status	Simulation	Experimental

4.5.2 Complementary Contributions

While Micius demonstrated experimental feasibility, Nguyen et al. addresses:

1. **Alternative Detection:** Coherent detection vs. single-photon
2. **Reliability Mechanism:** ARQ vs. FEC approach
3. **Analytical Framework:** 3-D Markov model for link-layer analysis
4. **Wavelength Choice:** Telecom-band (1550 nm) for compatibility

4.6 Lessons from Satellite Experiments

4.6.1 Technical Insights

1. **Downlink Preferred:** Satellite-to-ground links experience less turbulence impact than uplinks
2. **Night Operation:** Current systems operate primarily at night to avoid solar background noise
3. **Pointing Critical:** Sub-microradian pointing accuracy essential for stable links
4. **LEO Optimal:** Low Earth orbit provides best balance of loss and pass duration
5. **Trusted Nodes:** Practical networks currently require trusted relay satellites

4.6.2 Relevance to Nguyen et al.

The experimental lessons inform Nguyen et al.'s design choices:

- **LEO Configuration:** 600 km altitude consistent with optimal range
- **Downlink Scenario:** Satellite-to-ground transmission
- **Atmospheric Modeling:** Gamma-Gamma distribution validated by experiments
- **Error Handling:** Retransmission addresses practical channel variability

4.7 Chapter Summary

This chapter reviewed satellite QKD experimental achievements:

1. **Micius Satellite:** First dedicated quantum satellite, demonstrating QKD, entanglement distribution, and teleportation
2. **Performance Benchmarks:** 40.2 kbps sifted key rate, 1.1% QBER, 1200 km entanglement
3. **Integrated Network:** 4600 km space-ground network with 150+ users
4. **Future Missions:** Eagle-1, QEYSSat, and expanded Chinese constellation
5. **Context for Nguyen et al.:** Experimental validation supports theoretical assumptions; novel contributions address detection and reliability challenges

Gap Identification: Experimental work has focused on DV-QKD with single-photon detection. Nguyen et al.'s coherent detection and retransmission approach offers an alternative paradigm requiring future experimental validation.

Part III

Literature Review: Technical Aspects

Chapter 5

Atmospheric Channel Models

This chapter reviews the literature on atmospheric channel modeling for free-space optical communications, with emphasis on turbulence models, the Gamma-Gamma distribution, and their application to satellite-based QKD systems.

5.1 Free-Space Optical Channel Characteristics

5.1.1 Loss Mechanisms

The satellite-to-ground FSO channel introduces multiple loss mechanisms that affect QKD performance:

$$h_{total} = h_l \cdot h_a \cdot h_s \cdot h_p \cdot h_t \quad (5.1.1)$$

where:

- h_l : Free-space path loss
- h_a : Atmospheric attenuation
- h_s : Beam spreading loss
- h_p : Pointing error loss
- h_t : Atmospheric turbulence fading

5.1.2 Literature Foundation

Kaushal and Kaddoum (2017) [15] provide a comprehensive survey of space optical communication challenges:

Key Topics Covered:

- Atmospheric effects (absorption, scattering, turbulence)
- Mitigation techniques (adaptive optics, diversity)
- Link budget considerations
- Pointing, acquisition, and tracking

5.2 Free-Space Path Loss

5.2.1 Inverse Square Law

The geometric spreading of optical beams follows:

$$h_l = \left(\frac{\lambda}{4\pi D_{SG}} \right)^2 \quad (5.2.1)$$

where D_{SG} is the satellite-to-ground distance:

$$D_{SG} = \sqrt{(H_S - H_G)^2 \cdot \sec^2(\zeta) + 2R_E(H_S - H_G) \cdot \sec(\zeta)} \quad (5.2.2)$$

Typical Values (Nguyen et al. parameters):

- $H_S = 600$ km, $\zeta = 50^\circ$: $D_{SG} \approx 783$ km
- Path loss: ~ 260 dB (before telescope gains)

5.3 Atmospheric Attenuation

5.3.1 Beer-Lambert Law

Atmospheric attenuation follows exponential decay:

$$h_a = \exp \left(-\gamma \cdot \frac{H_\beta - H_G}{\cos(\zeta)} \right) \quad (5.3.1)$$

where γ is the attenuation coefficient (dB/km) and H_β is the effective atmospheric height.

5.3.2 Weather Dependence

Table 5.1: Atmospheric Attenuation Coefficients

Weather Condition	γ (dB/km)	Visibility (km)
Very clear	0.0 – 0.2	>50
Clear	0.2 – 0.5	23 – 50
Light haze	0.5 – 1.0	10 – 23
Haze	1.0 – 2.0	4 – 10
Light rain	2.0 – 4.0	2 – 4
Heavy rain	>10	<1

Relevance: Nguyen et al. uses $\gamma = 0.43$ dB/km (clear conditions) as baseline.

5.4 Atmospheric Turbulence

5.4.1 Physical Origin

Atmospheric turbulence arises from temperature variations causing refractive index fluctuations. These fluctuations are characterized by the refractive index structure parameter C_n^2 , which varies with altitude, time, and location.

5.4.2 Hufnagel-Valley Model

The Hufnagel-Valley (H-V) turbulence profile models C_n^2 as a function of altitude [16]:

$$C_n^2(h) = 0.00594 \left(\frac{w}{27} \right)^2 (10^{-5}h)^{10} e^{-h/1000} + 2.7 \times 10^{-16} e^{-h/1500} + C_n^2(0) e^{-h/100} \quad (5.4.1)$$

where:

- w : RMS wind speed (typically 21 m/s)
- h : Altitude in meters
- $C_n^2(0)$: Ground-level turbulence strength

Turbulence Regimes:

- Weak: $C_n^2(0) = 5 \times 10^{-15} \text{ m}^{-2/3}$
- Strong: $C_n^2(0) = 7 \times 10^{-12} \text{ m}^{-2/3}$

5.4.3 Scintillation Index

The Rytov variance characterizes scintillation strength:

$$\sigma_R^2 = 2.25k^{7/6} \sec^{11/6}(\zeta) \int_{H_G}^{H_S} C_n^2(h) \left(1 - \frac{h - H_G}{H_S - H_G}\right)^{5/6} (h - H_G)^{5/6} dh \quad (5.4.2)$$

5.5 Gamma-Gamma Turbulence Model

5.5.1 Al-Habash et al. (2001)

Al-Habash, Andrews, and Phillips [16] developed the Gamma-Gamma (GG) distribution for modeling irradiance fluctuations in moderate-to-strong turbulence:

$$f_{h_t}(h_t) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h_t^{(\alpha+\beta)/2-1} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta h_t} \right) \quad (5.5.1)$$

where:

- α, β : Large-scale and small-scale scintillation parameters
- $K_\nu(\cdot)$: Modified Bessel function of the second kind
- $\Gamma(\cdot)$: Gamma function

5.5.2 Scintillation Parameters

For spherical wave propagation (satellite downlink):

$$\alpha = \left[\exp \left(\frac{0.49\sigma_R^2}{(1 + 1.11\sigma_R^{12/5})^{7/6}} \right) - 1 \right]^{-1} \quad (5.5.2)$$

$$\beta = \left[\exp \left(\frac{0.51\sigma_R^2}{(1 + 0.69\sigma_R^{12/5})^{5/6}} \right) - 1 \right]^{-1} \quad (5.5.3)$$

5.5.3 Physical Interpretation

The GG model treats turbulence as a multiplicative process:

$$h_t = h_X \cdot h_Y \quad (5.5.4)$$

where $h_X \sim \text{Gamma}(\alpha, 1/\alpha)$ (large eddies) and $h_Y \sim \text{Gamma}(\beta, 1/\beta)$ (small eddies).

5.5.4 Vasylyev et al. (2016)

Vasylyev, Semenov, and Vogel [17] extended atmospheric channel modeling specifically for quantum communications:

Key Contributions:

- Quantum-specific treatment of channel loss
- Entanglement preservation analysis through turbulent channels
- Elliptic beam approximation for realistic beam shapes

5.6 Beam Spreading and Wandering

5.6.1 Liorni et al. (2019)

Liorni, Kampermann, and Bruß [18] analyzed beam effects on satellite QKD:

Topics Covered:

- Diffraction-limited beam spreading
- Turbulence-induced beam wandering
- Combined pointing and tracking effects
- Weather-dependent performance

5.6.2 Effective Beam Width

The beam width at the receiver includes diffraction and turbulence contributions:

$$\omega_{eq}^2 = \omega_D^2 \left(1 + \frac{D_{SG}^2}{k^2 \omega_0^4} + 1.33 \sigma_R^2 \Lambda^{5/6} \right) \quad (5.6.1)$$

5.6.3 Ma et al. (2015)

Ma et al. [19] analyzed satellite-to-ground coherent optical communications with spatial diversity:

Relevance to Nguyen et al.:

- Validated GG distribution for satellite downlinks
- Provided framework for heterodyne detection analysis
- Demonstrated spatial diversity benefits

5.7 Channel Model Implementation in Nguyen et al.

5.7.1 Combined Channel Coefficient

Nguyen et al. combines all effects into a single channel coefficient:

$$h = h_l \cdot h_a \cdot h_s \cdot h_t \quad (5.7.1)$$

5.7.2 Specific Parameter Values

5.7.3 Integration with QBER Analysis

The turbulence-affected channel coefficient determines received power:

$$P_R = P_T \cdot G_T \cdot G_R \cdot h \quad (5.7.2)$$

This feeds into SNR calculation and subsequently QBER analysis.

5.8 Recent Developments (2020-2025)

5.8.1 Fisher-Snedecor F Distribution

Recent experimental data suggest the F distribution may provide better fit across weak-to-strong turbulence than Gamma-Gamma in some scenarios.

Table 5.2: Channel Parameters in Nguyen et al. (2021)

Parameter	Symbol	Value
Wavelength	λ	1550 nm
Satellite altitude	H_S	600 km
Ground station height	H_G	5 m
Atmospheric height	H_β	20 km
Zenith angle	ζ	50°
Wind speed	w	21 m/s
Beam width at ground	ω_D	50 m
Attenuation coefficient	γ	0.43 dB/km
<i>Weak Turbulence</i>		
$C_n^2(0)$		$5 \times 10^{-15} \text{ m}^{-2/3}$
α		Calculated via Eq. 5.5.2
β		Calculated via Eq. 5.5.3
<i>Strong Turbulence</i>		
$C_n^2(0)$		$7 \times 10^{-12} \text{ m}^{-2/3}$

5.8.2 Machine Learning Approaches

Emerging research applies ML for:

- Channel state prediction
- Adaptive parameter optimization
- Turbulence mitigation

5.8.3 Tropical Atmosphere Considerations

For Vietnam and similar regions, specialized modeling may be needed:

- Higher humidity effects
- Monsoon season variability
- Aerosol loading differences

5.9 Chapter Summary

This chapter reviewed atmospheric channel modeling literature relevant to Nguyen et al. (2021):

1. **Hufnagel-Valley Model:** Standard altitude-dependent turbulence profile
2. **Gamma-Gamma Distribution:** Primary model for moderate-to-strong turbulence
3. **Scintillation Parameters:** α, β derived from Rytov variance
4. **Beam Effects:** Spreading and wandering impact on received power

5. Weather Dependence: Attenuation varies significantly with conditions

Key Insight: Nguyen et al. adopts well-established atmospheric models (GG distribution, H-V profile) providing solid foundation for QBER and KLR analysis.

Chapter 6

Detection Schemes and Modulation

This chapter reviews detection techniques and modulation schemes for optical QKD, with emphasis on coherent detection methods and the dual-threshold approach proposed by PTIT researchers.

6.1 Detection Paradigms in QKD

6.1.1 Single-Photon Detection

Traditional DV-QKD relies on single-photon detectors:

Table 6.1: Single-Photon Detector Technologies

Technology	Wavelength	Efficiency	Dark Count
Si-APD	850 nm	50-70%	<100 Hz
InGaAs APD	1550 nm	10-25%	1-10 kHz
SNSPD	Broadband	>90%	<10 Hz

Micius Implementation: Silicon APDs at 850 nm, providing good efficiency but limiting wavelength choice.

6.1.2 Coherent Detection

Coherent detection offers an alternative paradigm:

- **Homodyne:** Measures single quadrature (X or P)
- **Heterodyne:** Measures both quadratures simultaneously

Advantages:

- Uses standard telecom photodiodes
- Works at 1550 nm (fiber-compatible)
- Higher sensitivity with local oscillator gain
- Compatible with existing coherent communication infrastructure

6.2 Heterodyne Detection for QKD

6.2.1 Principle of Operation

In heterodyne detection, the signal is mixed with a strong local oscillator (LO) at a slightly different frequency:

$$E_{total} = E_s e^{i\omega_s t + i\phi_s} + E_{LO} e^{i\omega_{LO} t} \quad (6.2.1)$$

The photocurrent contains the beat signal:

$$i(t) \propto 2\sqrt{P_s P_{LO}} \cos((\omega_s - \omega_{LO})t + \phi_s) \quad (6.2.2)$$

6.2.2 SNR Enhancement

The LO provides effective amplification:

$$\text{SNR}_{het} = \frac{2\Re^2 P_s P_{LO}}{2q\Re P_{LO} B + \sigma_{th}^2} \quad (6.2.3)$$

For strong LO ($P_{LO} \gg P_s$), shot noise limited operation is achieved.

6.2.3 Application to QKD

Research on coherent detection for QKD includes:

- **CV-QKD:** Standard detection method for continuous-variable protocols
- **Discrete-Modulated CV-QKD:** Enables security with QPSK/8PSK modulation
- **PTIT Approach:** Heterodyne detection with dual-threshold decision

6.3 Dual-Threshold Detection

6.3.1 Trinh et al. (2018)

Trinh et al. [2] introduced dual-threshold detection for QKD over FSO:

Key Innovation: Instead of a single decision threshold, two thresholds define three decision regions:

$$\text{Decision} = \begin{cases} 0 & \text{if } i \geq d_0 \\ 1 & \text{if } i \leq d_1 \\ X & \text{if } d_1 < i < d_0 \text{ (erasure)} \end{cases} \quad (6.3.1)$$

6.3.2 Threshold Configuration

The thresholds are defined relative to the decision point d :

$$d_0 = d + \varsigma \cdot \sigma \quad (6.3.2)$$

$$d_1 = d - \varsigma \cdot \sigma \quad (6.3.3)$$

where ς is the dual-threshold coefficient and σ is the noise standard deviation.

6.3.3 Trade-off Analysis

Table 6.2: DT Coefficient Trade-offs

ς Value	QBER	P_{sift}
Small (<0.5)	Higher	Higher
Optimal (0.7–2.8)	Low	Acceptable
Large (>3.0)	Very low	Very low

6.3.4 Evolution: DT/DD to DT/HD

Table 6.3: Detection Scheme Evolution at PTIT

Paper	Year	Detection	Improvement
Trinh et al.	2018	DT/DD	Baseline
Nguyen et al.	2021	DT/HD	+20 dB sensitivity
Nguyen et al.	2023	DT/HD + CV-QKD	Extended to CV

6.4 Modulation Schemes for QKD

6.4.1 Polarization Encoding

Traditional BB84 uses polarization states:

- Rectilinear: $|H\rangle, |V\rangle$
- Diagonal: $|D\rangle, |A\rangle$

Advantages: Direct mapping to BB84 states **Challenges:** Polarization alignment, fiber birefringence

6.4.2 Phase Encoding

Phase-encoded QKD maps information to optical phase:

$$|\psi\rangle = |\alpha e^{i\phi}\rangle \quad (6.4.1)$$

QPSK for QKD:

$$\phi \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\} \quad (6.4.2)$$

6.4.3 Higher-Order Modulation

Research has explored higher-order modulation for QKD:

- **8-PSK:** 3 bits per symbol, higher spectral efficiency
- **16-QAM:** 4 bits per symbol, requires amplitude discrimination
- **Gaussian:** Continuous modulation for CV-QKD

6.4.4 QPSK vs. Gaussian

Table 6.4: QPSK vs. Gaussian Modulation for QKD

Aspect	QPSK (Nguyen)	Gaussian (CV-QKD)
Alphabet	4 discrete	Continuous
Preparation	Digital	Analog
Security proof	Via BB84	Dedicated CV proofs
Implementation	Simpler	More complex
Key rate	Moderate	Higher (short dist.)

6.5 Receiver Architecture

6.5.1 Nguyen et al. Receiver Design

The proposed DT/HD receiver consists of:

1. Optical Front-End:

- 90° optical hybrid
- Local oscillator generation
- Balanced photodetector pair

2. Electrical Processing:

- Transimpedance amplifier
- Low-pass filter
- Dual-threshold comparator

3. Decision Logic:

- Three-level output (0, 1, X)
- Erasure handling
- Sifting coordination

6.5.2 Noise Sources

The receiver noise model includes:

$$\sigma_{total}^2 = \sigma_{shot}^2 + \sigma_{thermal}^2 + \sigma_{dark}^2 \quad (6.5.1)$$

Shot Noise:

$$\sigma_{shot}^2 = 2q(\Re P_{LO} + I_d)B \cdot \bar{g}^2 F \quad (6.5.2)$$

Thermal Noise:

$$\sigma_{thermal}^2 = \frac{4k_B T B}{R_L} \quad (6.5.3)$$

6.6 QBER Analysis

6.6.1 Error Probability Derivation

For heterodyne detection with QPSK, the bit error probability:

$$P_e = \int_0^\infty Q\left(\sqrt{\frac{2h \cdot \text{SNR}}{1 + h \cdot \text{SNR}}}\right) f_{h_t}(h) dh \quad (6.6.1)$$

where $Q(\cdot)$ is the Q-function.

6.6.2 Conditional QBER

Given successful sifting (non-erasure):

$$\text{QBER} = \frac{P_e}{P_{sift}} \quad (6.6.2)$$

6.6.3 Optimal DT Coefficient Ranges

Table 6.5: Optimal ς Ranges from Nguyen et al.

Condition	ς Range	Criterion
Weak turbulence	0.7 – 2.4	$\text{QBER} \leq 10^{-3}$, $P_{sift} \geq 10^{-2}$
Strong turbulence	1.4 – 2.8	$\text{QBER} \leq 10^{-3}$, $P_{sift} \geq 10^{-2}$

6.7 Chapter Summary

This chapter reviewed detection and modulation techniques for QKD:

1. **Detection Paradigms:** Single-photon vs. coherent detection trade-offs
2. **Heterodyne Detection:** LO gain provides 20 dB sensitivity improvement
3. **Dual-Threshold:** Erasure region reduces QBER at cost of P_{sift}
4. **QPSK Modulation:** Maps BB84 structure to phase states
5. **PTIT Contribution:** DT/HD combination novel for satellite QKD

Key Innovation: Nguyen et al.'s DT/HD approach bridges DV and CV paradigms, offering practical implementation advantages while maintaining security based on BB84 structure.

Chapter 7

Error Handling and Reliability

This chapter reviews error handling approaches in QKD, comparing Forward Error Correction (FEC), Automatic Repeat reQuest (ARQ), and hybrid methods, with focus on the retransmission scheme proposed by Nguyen et al.

7.1 Error Correction in QKD

7.1.1 The Reconciliation Problem

After quantum transmission, Alice and Bob share correlated but not identical bit strings.
Error correction (reconciliation) must:

1. Correct errors between Alice's and Bob's strings
2. Minimize information leakage to Eve
3. Achieve efficiency close to Shannon limit

Efficiency Metric:

$$f = \frac{H(A|B)_{actual}}{H(A|B)_{Shannon}} \geq 1 \quad (7.1.1)$$

where $f = 1$ represents Shannon-limited performance.

7.1.2 Error Correction Paradigms

Table 7.1: Error Correction Paradigms for QKD

Paradigm	Direction	Interaction	Example
FEC	One-way	None	LDPC, Polar
Interactive	Two-way	Multiple rounds	CASCADE, Winnow
Hybrid	Both	Adaptive	LDPC + verification
ARQ	Retransmission	ACK/NACK	Nguyen et al.

7.2 CASCADE Protocol

7.2.1 Historical Significance

CASCADE, proposed by Brassard and Salvail (1994), was the first practical error correction protocol for QKD.

Algorithm:

1. Divide key into blocks of size k_1
2. Exchange parities for each block
3. Binary search to locate errors in mismatched blocks
4. Double block size and repeat with shuffling
5. Continue for multiple passes

7.2.2 Performance Characteristics

- **Efficiency:** $f \approx 1.16$ (practical implementations)
- **Latency:** High due to interactive nature
- **Throughput:** Limited by round-trip communication

7.2.3 Recent Revival

Mueller et al. (2025) [?] demonstrated that optimized CASCADE implementations can achieve:

- Competitive throughput with modern hardware
- Lower latency than previously assumed
- Robustness across varying QBER

7.3 LDPC Codes for QKD

7.3.1 Low-Density Parity-Check Codes

LDPC codes offer near-Shannon-limit performance:

$$\mathbf{H} \cdot \mathbf{c}^T = \mathbf{0} \quad (7.3.1)$$

where \mathbf{H} is a sparse parity-check matrix.

7.3.2 Application to QKD

Milicevic et al. (2018) [?] developed quasi-cyclic multi-edge LDPC codes for long-distance QKD:

Key Results:

- 142 km fiber transmission achieved
- Secret key rate: 6.64×10^{-8} bits/pulse
- Information throughput: 7.16 kbit/s
- GPU-accelerated decoding

7.3.3 Challenges

- **Error Floor:** Performance degradation at low QBER
- **Rate Sensitivity:** Codes optimized for narrow QBER range
- **Complexity:** Encoding/decoding computational overhead

7.4 Polar Codes

7.4.1 Channel Polarization

Polar codes, invented by Arikan (2009), achieve Shannon capacity through channel polarization:

$$W_N^{(i)} \rightarrow \begin{cases} \text{Perfect channel} & \text{as } N \rightarrow \infty \\ \text{Useless channel} & \end{cases} \quad (7.4.1)$$

7.4.2 Application to QKD

Polar codes for QKD reconciliation offer:

- Theoretical capacity achievement
- Successive cancellation decoding
- Lower latency than LDPC in some regimes

7.4.3 RC-LDPC-Polar Codes (2024)

Recent work combines LDPC and polar coding advantages:

- Rate-compatible design
- Adaptive to varying channel conditions
- Improved performance over pure approaches

7.5 ARQ-Based Reliability: Nguyen et al.

7.5.1 Paradigm

Shift

Nguyen et al. (2021) introduces a fundamentally different approach—using retransmission rather than error correction:

Philosophy:

- Accept transmission failures as inherent to channel
- Retransmit failed sequences rather than correct errors
- Trade latency for simplicity and reliability

7.5.2 Protocol

Operation

1. Alice generates key sequence, stores in buffer
2. Transmit sequence via FSO channel
3. Bob performs error checking (e.g., CRC or hash)
4. **Success:** Bob sends ACK, Alice discards from buffer
5. **Failure:** Bob sends NACK, Alice retransmits
6. After M failures, sequence is discarded (key loss)

7.5.3 Comparison

with

FEC

Table 7.2: ARQ vs. FEC Comparison

Aspect	ARQ (Nguyen)	FEC (LDPC)
Computational overhead	Low	High
Latency	Variable	Fixed
Adaptability	Channel-adaptive	Rate-fixed
Implementation	Simple	Complex
Reliability	Controllable via M	Fixed by code
Throughput	Reduced	Near-constant

7.5.4 Advantages

1. **Simplicity:** No complex encoder/decoder required
2. **Adaptivity:** Natural adaptation to channel conditions
3. **Flexibility:** Reliability tunable via M parameter
4. **Compatibility:** Works with any modulation scheme

7.6 3-D Markov Chain Analysis

7.6.1 State Space Definition

Nguyen et al. develops a three-dimensional Markov chain model:

$$\text{State: } (n, s, m) \quad (7.6.1)$$

where:

- $n \in [0, C]$: Buffer queue length ($C = \text{capacity}$)
- $s \in \{B, G\}$: Channel state (Bad, Good)
- $m \in [1, M]$: Retransmission attempt number

7.6.2 Transition Probabilities

Key transition probabilities:

- p_{GB} : Good \rightarrow Bad transition
- p_{BG} : Bad \rightarrow Good transition
- P_{sift}^G, P_{sift}^B : Sifting probabilities in each state

7.6.3 Key Loss Rate Derivation

The KLR is derived from steady-state analysis:

$$\text{KLR} = \sum_{s \in \{B, G\}} \pi_{C,s,M} \cdot P_{loss}^s \quad (7.6.2)$$

where $\pi_{n,s,m}$ is the steady-state probability of state (n, s, m) .

7.6.4 Novel Contribution

This 3-D Markov model is a unique contribution:

- First analytical framework for ARQ in satellite QKD
- Enables closed-form KLR calculation
- Provides optimization insights without extensive simulation

7.7 Key Loss Rate Performance

7.7.1 KLR vs. Retransmissions

7.7.2 Diminishing Returns

Optimal Choice: $M = 4$ provides best trade-off between reliability and latency.

Table 7.3: KLR Improvement with Retransmissions

<i>M</i>	KLR (Weak)	KLR (Strong)	Improvement
0	3×10^{-2}	5×10^{-2}	Baseline
1	10^{-3}	10^{-2}	$30\times$
2	10^{-4}	10^{-3}	$300\times$
4	$< 10^{-5}$	$< 10^{-4}$	$>1000\times$
7	$< 10^{-6}$	$< 10^{-5}$	$>10000\times$

Table 7.4: Power Gain vs. Retransmission Count

<i>M</i>	Increase	Power Gain	Recommendation
	$1 \rightarrow 2$	1.0 dB	Significant
	$2 \rightarrow 3$	0.7 dB	Worthwhile
	$3 \rightarrow 4$	0.5 dB	Marginal
	$4 \rightarrow 7$	0.5 dB total	Not recommended

7.8 Chapter Summary

This chapter reviewed error handling approaches for QKD:

1. **CASCADE:** Interactive protocol, high efficiency, high latency
2. **LDPC:** Near-Shannon performance, complex, rate-sensitive
3. **Polar Codes:** Capacity-achieving, emerging for QKD
4. **ARQ (Nguyen):** Simple, adaptive, controllable reliability
5. **3-D Markov Model:** Novel analytical framework for KLR

Key Contribution: Nguyen et al.'s ARQ approach offers a fundamentally different reliability paradigm that trades computational complexity for implementation simplicity and channel adaptivity.

Chapter 8

Security Analysis Literature

This chapter reviews the security analysis literature for practical QKD systems, covering attack models, security proofs, finite-key analysis, and device imperfections.

8.1 QKD Security Framework

8.1.1 Information-Theoretic Security

QKD provides security based on physical laws rather than computational assumptions:

- **No-Cloning:** Quantum states cannot be perfectly copied
- **Measurement Disturbance:** Eavesdropping creates detectable errors
- **Unconditional Security:** Secure against unlimited computational power

8.1.2 Security Hierarchy

Table 8.1: Attack Classification Hierarchy

Attack Type	Power	QBER Threshold
Individual attacks	Weakest	14.6%
Collective attacks	Medium	11.0%
Coherent attacks	Strongest	7.1%

8.2 Practical Security Framework

8.2.1 Scarani et al. (2009)

“The Security of Practical Quantum Key Distribution” [1] established the definitive framework for analyzing practical QKD security:

Key Contributions:

1. Device Imperfection Modeling:

- Non-ideal single-photon sources
- Detector inefficiencies and dark counts
- Channel losses and noise

2. Attack Analysis:

- Photon-number-splitting attacks
- Intercept-resend strategies
- Trojan horse attacks

3. Security Parameter Calculation:

- QBER threshold derivation
- Privacy amplification requirements
- Finite-key corrections

8.2.2 Relevance to Nguyen et al.

Scarani's framework provides:

- QBER threshold (10^{-3}) justification
- Security analysis methodology
- Foundation for URA attack analysis

8.3 Finite-Key Security

8.3.1 The Finite-Key Challenge

Asymptotic security proofs assume infinite key length. Practical systems require finite-key analysis:

$$l_{\text{secure}} = l_{\text{sifted}} - \text{leak}_{EC} - \text{leak}_{PE} - PA \quad (8.3.1)$$

where:

- leak_{EC} : Information leaked during error correction
- leak_{PE} : Information leaked during parameter estimation
- PA: Privacy amplification compression

8.3.2 Tomamichel et al. (2012)

“Tight Finite-Key Analysis for Quantum Cryptography” [20] in Nature Communications:
Key Contributions:

- Composable security definitions
- Tight bounds on finite-size effects
- Practical parameter optimization

8.3.3 Impact on Key Rate

Finite-key effects become significant for:

- Short satellite passes (limited transmission time)
- Low repetition rate systems
- High-loss channels

Gap in Nguyen et al.: The paper uses asymptotic analysis; finite-key effects not incorporated.

8.4 Realistic Device Security

8.4.1 Xu et al. (2020)

“Secure Quantum Key Distribution with Realistic Devices” [11] in Reviews of Modern Physics:

Coverage:

1. Source Imperfections:

- Multi-photon emission
- State preparation flaws
- Side-channel leakage

2. Detector Vulnerabilities:

- Blinding attacks
- Time-shift attacks
- Efficiency mismatch

3. Countermeasures:

- Decoy states for source issues
- MDI-QKD for detector immunity
- Device characterization protocols

8.4.2 Measurement-Device-Independent QKD

MDI-QKD removes all detector side-channel attacks:

- Bell state measurement at untrusted node
- Security independent of detector imperfections
- Demonstrated over 500 km fiber

8.5 CV-QKD Security

8.5.1 Leverrier (2015)

“Composable Security Proof for CV-QKD” [7]:

Contribution: First composable security proof for Gaussian-modulated CV-QKD with coherent states against collective attacks.

8.5.2 Discrete Modulation Security

For QPSK-like discrete modulation:

- Security proofs developed post-2018
- No longer requires linear channel assumption
- Bounded via uncertainty principle methods

Relevance: Nguyen et al.’s QPSK approach benefits from these developments.

8.6 Security Analysis in Nguyen et al.

8.6.1 Unauthorized Receiver Attack (URA)

Nguyen et al. analyzes security against an eavesdropper (Eve) positioned near Bob:

Attack Model:

- Eve places receiver within satellite beam footprint
- Bob at beam center ($r = 0$)
- Eve at distance D_{E-B} from Bob

8.6.2 Eve’s Received Power

Eve’s power decreases with distance from beam center:

$$P_{R,Eve} \propto A_0 \exp\left(-\frac{2D_{E-B}^2}{\omega_{Deq}^2}\right) \quad (8.6.1)$$

Table 8.2: Eve's QBER vs. Distance

D_{E-B} (m)	Weak Turb.	Strong Turb.	Security
0	Same as Bob	Same as Bob	Compromised
10	$\sim 10^{-3}$	$\sim 10^{-2}$	Marginal
20	$\sim 10^{-2}$	$\sim 10^{-2}$	Marginal
30	$> 10^{-2}$	$> 10^{-2}$	Secure
50+	$> 10^{-1}$	$> 10^{-1}$	Secure

8.6.3 Security Boundary

Conclusion: $D_{E-B} > 30$ m ensures Eve cannot obtain usable key.

8.6.4 Limitations of Analysis

1. **URA Only:** Does not consider intercept-resend or other attacks
2. **Asymptotic:** Finite-key effects not analyzed
3. **Collective Attacks:** General attack security not proven

8.7 Recent Security Developments (2022-2025)

8.7.1 Numerical Security Proofs

Physical Review Research (2022) presents numerical security proofs for:

- Decoy-state BB84 with basis misalignment
- MDI-QKD practical implementations
- Fine-grained statistics utilization

8.7.2 Finite-Key for Heterodyne (2025)

arXiv:2501.10278 addresses:

- Phase imbalance in heterodyne detection
- Practical finite-key bounds
- System optimization under realistic conditions

8.7.3 Post-Quantum Considerations

While QKD is quantum-safe, practical considerations include:

- Authentication channel security
- Key management infrastructure
- Hybrid classical-quantum systems

8.8 Chapter

Summary

This chapter reviewed security analysis literature:

1. **Foundational Framework:** Scarani et al. established practical security analysis
2. **Finite-Key:** Tomamichel et al. provided composable security with finite resources
3. **Device Security:** Xu et al. addressed realistic device imperfections
4. **CV-QKD:** Leverrier proved composable security for coherent detection
5. **Nguyen et al.:** Analyzed URA scenario; gaps remain in finite-key and general attacks

Key Gap: Nguyen et al. provides important URA analysis but would benefit from finite-key analysis and security proof against general attacks.

Part IV

Analysis and Synthesis

Chapter 9

Comparative Analysis

This chapter synthesizes the literature reviewed in previous chapters, positioning Nguyen et al. (2021) within the broader context of satellite QKD research and identifying its unique contributions.

9.1 Methodology	Comparison	
9.1.1 Detection	Approach	Comparison

Table 9.1: Detection Approaches in Satellite QKD Literature

System	Detection	Wavelength	Sensitivity	Complexity
Micius (2017)	SPD (Si-APD)	850 nm	High	High
CV-QKD (Dequal)	Coherent	1550 nm	Medium	Medium
DT/DD (Trinh)	Direct	1550 nm	Low	Low
DT/HD (Nguyen)	Heterodyne	1550 nm	Very High	Medium

9.1.2 Protocol	Comparison
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Table 9.2: Protocol Approaches Comparison

Approach	Modulation	States	Security Basis
BB84 (Micius)	Polarization	4	BB84 proof
Gaussian CV-QKD	Gaussian	∞	CV proof
Discrete CV-QKD	QPSK/8PSK	4/8	CV proof
QPSK-BB84 (Nguyen)	Phase	4	BB84 mapping

Table 9.3: Error Handling Approaches

Method	Type	Overhead	Adaptivity	Implementation
CASCADE	Interactive	High	Low	Complex
LDPC	FEC	Medium	Low	Complex
Polar	FEC	Medium	Low	Medium
ARQ (Nguyen)	Retransmit	Low	High	Simple

Table 9.4: Transmitted Power Comparison ($\text{QBER} \leq 10^{-3}$)

Scheme	Required P_T	Relative Gain
SIM/BPSK-DT (Baseline)	45 dBm	0 dB
QPSK-DT/DD (Trinh 2018)	35 dBm	10 dB
QPSK-DT/HD (Nguyen 2021)	25 dBm	20 dB

9.1.3 Error Handling Comparison

9.2 Performance Comparison

9.2.1 Power Requirements

9.2.2 Key Rate Comparison

Table 9.5: Key Rate Performance Comparison

System	Distance	Key Rate	Status
Micius (2017)	530 km	40.2 kbps sifted	Experimental
Micius (2017)	1034 km	1.2 kbps sifted	Experimental
Chen (2021)	4600 km	47.8 kbps	Experimental
Nguyen (2021)	600 km (LEO)	Simulation	Theoretical

9.2.3 Reliability Comparison

9.3 Unique Contributions of Nguyen et al.

9.3.1 Novel Elements

1. First ARQ for Satellite QKD

- No prior literature applies retransmission to quantum key distribution
- Paradigm shift from error correction to error avoidance
- Enabled by classical feedback channel availability

2. 3-D Markov Chain Model

- Novel state space: (buffer, channel, retransmission)
- Analytical KLR calculation capability

Table 9.6: Reliability Metrics Comparison

System	Error Handling	Reliability Metric	Value
Micius	LDPC FEC	QBER	1.1–3.2%
Chen Network Nguyen	Trusted relays ARQ ($M = 4$)	Network availability KLR	>99% $< 10^{-4}$

- Optimization framework without extensive simulation

3. DT/HD Integration

- First combination of dual-threshold with heterodyne for QKD
- 20 dB improvement over previous PTIT work
- Bridges DV and CV detection paradigms

4. Cross-Layer Design

- Physical layer: QPSK + DT/HD
- Link layer: ARQ retransmission
- Integrated optimization approach

9.3.2 Research Gaps Addressed

Table 9.7: Research Gaps Addressed

Gap in Literature	Addressed?	How
Reliability without FEC overhead	✓	ARQ scheme
Link layer QKD analysis	✓	3-D Markov model
Coherent detection for BB84-like	✓	QPSK + heterodyne
KLR metric formalization	✓	Analytical derivation
Vietnamese satellite QKD research	✓	PTIT contribution

9.4 PTIT Research Evolution

9.4.1 Research Timeline

Table 9.8: PTIT Satellite QKD Research Timeline

Year	Paper	Contribution	Advancement
2018	Trinh et al.	DT/DD for QKD	Introduced DT concept
2019	Vu et al.	HAP-aided relay	Extended to HAP
2021	Nguyen et al.	DT/HD + ARQ	+20 dB + reliability
2023	Nguyen et al.	CV-QKD extension	Extended to CV-QKD
2023	Vu et al.	Network coding	Multi-satellite

9.4.2 Cumulative Contributions

The PTIT research group has systematically:

1. Introduced dual-threshold detection for QKD
2. Improved sensitivity through heterodyne detection
3. Added reliability through retransmission
4. Extended to CV-QKD protocols
5. Explored network-level optimizations

9.5 International Context

9.5.1 Geographic Distribution of Research

Table 9.9: Satellite QKD Research by Region

Region	Focus	Key Institutions
China	Experimental demonstration	USTC, CAS
Europe	Mission planning	ESA, DLR, CNES
Japan	Ground experiments	NICT, JAXA
Canada	LEO development	Waterloo, CSA
Singapore	CubeSat QKD	NUS, CQT
Vietnam	Theoretical analysis	PTIT, VAST

9.5.2 Vietnamese Position

Vietnam's contribution through PTIT:

- Theoretical foundations for practical systems
- Novel detection and reliability approaches
- Regional capacity building
- Potential foundation for future missions

9.6 Critical Assessment

9.6.1 Strengths

1. **Novel Integration:** First combination of QPSK, DT/HD, and ARQ
2. **Analytical Rigor:** Comprehensive mathematical framework
3. **Practical Focus:** Realistic system parameters
4. **Significant Improvement:** Quantifiable 20 dB and $>1000\times$ gains
5. **Regional Contribution:** Advances Vietnamese research capability

9.6.2 Limitations

1. **Simulation Only:** No experimental validation
2. **Idealized Assumptions:**
 - Perfect pointing and tracking
 - No phase noise in local oscillator
 - Ideal modulator/demodulator
3. **Security Gaps:**
 - Asymptotic analysis only
 - Limited attack model (URA)
 - No composable security proof
4. **System Gaps:**
 - Single satellite link
 - No handover consideration
 - No network integration

9.7 Chapter Summary

This comparative analysis establishes:

1. **Unique Position:** Nguyen et al. occupies a unique niche combining coherent detection with retransmission reliability
2. **Performance Gains:** 20 dB power improvement and $>1000\times$ KLR reduction are significant
3. **Research Gap Filling:** Addresses previously unexplored link-layer reliability for satellite QKD
4. **Foundation Building:** Provides theoretical foundation for future Vietnamese quantum satellite missions
5. **Remaining Work:** Experimental validation and security analysis extensions needed

Chapter 10

Research Gaps and Future Directions

This chapter identifies remaining research gaps in satellite-based QKD reliability improvement and outlines promising future research directions based on the literature synthesis.

10.1 Theoretical Gaps

10.1.1 Finite-Key Security Analysis

Gap: Nguyen et al. uses asymptotic security analysis. Practical implementations require finite-key bounds.

Required Work:

1. Security bounds for practical key lengths (10^6 – 10^9 bits)
2. Minimum block size determination for target security
3. Composable security proof incorporating retransmission

Related Literature:

- Tomamichel et al. (2012): Finite-key framework
- arXiv:2501.10278 (2025): Finite-key for imperfect heterodyne

10.1.2 Advanced Eavesdropper Models

Gap: Only Unauthorized Receiver Attack (URA) analyzed. More sophisticated attacks not considered.

Required Work:

1. Collective Attacks:

- Eve performs identical operation on each signal
- Stores quantum memory for later measurement

2. Coherent Attacks:

- Most general attack strategy
- Joint operation on entire transmission

3. Side-Channel Attacks:

- Timing information leakage
- Modulator imperfections
- Detector vulnerabilities

10.1.3 Hybrid ARQ-FEC Analysis

Gap: Pure ARQ approach may not be optimal for all conditions.

Research Questions:

- When does hybrid ARQ+FEC outperform pure ARQ?
- Optimal FEC code rate for retransmission scenarios
- Security implications of hybrid approaches

10.2 Practical Implementation Gaps

10.2.1 Pointing and Tracking

Gap: Perfect beam tracking assumed. Realistic pointing errors not modeled.

Required Work:

1. Pointing Error Model:

$$h_p = A_0 \exp\left(-\frac{2r_p^2}{\omega_{eq}^2}\right) \quad (10.2.1)$$

where r_p is pointing jitter radius

2. Acquisition Protocol:

- Initial beam acquisition time
- Tracking loop bandwidth requirements
- Re-acquisition after interruption

3. Combined Effects:

- Pointing + turbulence interaction
- Impact on QBER and P_{sift}
- Adaptation of DT coefficient

10.2.2 Phase Noise and Synchronization

Gap: Ideal local oscillator and perfect phase synchronization assumed.

Challenges:

- LO phase noise impact on QPSK detection
- Doppler shift compensation for LEO satellite
- Carrier frequency offset estimation

10.2.3 Experimental

Validation

Gap: All results are simulation-based.

Validation Pathway:

1. Component Level:

- QPSK modulator characterization
- DT/HD receiver implementation
- APD performance verification

2. Subsystem Level:

- End-to-end link demonstration
- ARQ protocol implementation
- Buffer management testing

3. System Level:

- Ground testbed with emulated satellite channel
- Turbulence chamber testing
- Field trials (ground-to-ground)

10.3 System-Level

Gaps

10.3.1 LEO Constellation

Integration

Gap: Single satellite link analyzed. Constellation operation not considered.

Research Directions:

1. Multi-Satellite Coverage:

- Optimal constellation design for Vietnam
- Coverage overlap analysis
- Handover frequency estimation

2. Handover Protocols:

- Key continuity during handover
- Buffer management across satellites
- Retransmission state transfer

3. Key Routing:

- Inter-satellite key relay
- Trusted node requirements
- Network key rate optimization

10.3.2 Hybrid FSO/RF Architecture

Gap: Pure FSO system assumed. Backup RF channel not integrated.

Opportunities:

- RF backup during weather outages
- Classical channel for ACK/NACK (already assumed)
- Hybrid key management protocols

10.3.3 Network Integration

Gap: Stand-alone QKD system. Integration with existing networks not addressed.

Considerations:

- Key management system integration
- Classical encryption interoperability
- Network protocol stack placement

10.4 Future Research Directions

10.4.1 Near-Term (1–2 Years)

1. Finite-Key Analysis Extension

- Incorporate finite-size corrections into QBER analysis
- Determine minimum key length for target security level
- Optimize block size for satellite pass duration

2. Pointing Error Integration

- Add realistic pointing jitter model
- Analyze combined pointing and turbulence effects
- Develop adaptive DT coefficient adjustment

3. Ground Testbed Development

- Implement QPSK modulator and DT/HD receiver
- Validate with emulated satellite channel
- Demonstrate ARQ protocol operation

10.4.2 Medium-Term (3–5 Years)

1. Machine Learning Integration

- Channel state prediction for proactive parameter adjustment
- Optimal DT coefficient selection via reinforcement learning
- Anomaly detection for security monitoring

2. LEO Constellation Analysis

- Multi-satellite coverage optimization for Vietnam
- Handover protocol development
- Key routing algorithm design

3. Tropical Atmosphere Modeling

- Vietnam-specific turbulence profiles
- Monsoon season characterization
- Optimal ground station site selection

10.4.3 Long-Term (5+ Years)

1. Vietnamese Quantum Satellite Mission

- Payload design based on PTIT research
- Ground station network development
- International collaboration framework

2. Regional Quantum Network

- ASEAN quantum connectivity
- Cross-border secure communication
- Regional standards development

3. Quantum Internet Integration

- Entanglement-based protocols
- Quantum repeater integration
- Global quantum network participation

Table 10.1: Proposed Research Roadmap

Activity	Timeline	Priority	Dependencies
Finite-key analysis	Year 1	High	None
Pointing error model	Year 1	High	None
Component testbed	Year 1–2	High	Funding
ML integration study	Year 2–3	Medium	Testbed
Tropical atmosphere	Year 2–3	Medium	Field data
Constellation design	Year 3–4	Medium	Analysis tools
Satellite mission design	Year 4–5	Low	All above
Regional network	Year 5+	Low	Mission

10.5 Research Roadmap

10.6 Chapter Summary

This chapter identified research gaps and future directions:

Critical Gaps:

1. Finite-key security analysis
2. Pointing and tracking effects
3. Experimental validation

Important Gaps:

1. Advanced attack models
2. LEO constellation integration
3. Hybrid ARQ-FEC optimization

Future Opportunities:

1. Machine learning for adaptive optimization
2. Vietnamese quantum satellite mission
3. Regional quantum network development

The research by Nguyen et al. provides a solid foundation for continued development toward practical satellite QKD systems.

Chapter 11

Conclusion

This chapter synthesizes the key findings from this comprehensive literature review, summarizes the contributions of Nguyen et al. (2021), and provides final recommendations for future research.

11.1 Literature Review Summary

This literature review examined 85+ papers spanning four decades of quantum key distribution research, organized into four thematic parts:

11.1.1 Part I: Introduction and Paper Analysis

- Established the context for satellite-based QKD research
- Provided detailed technical analysis of Nguyen et al. (2021)
- Identified system architecture, innovations, and key results

11.1.2 Part II: Theoretical Foundations

- **Foundational Protocols:** BB84, E91, CV-QKD, and decoy states
- **Satellite Experiments:** Micius achievements and integrated networks
- **Key Insight:** Nguyen et al. builds upon established foundations while introducing novel reliability mechanisms

11.1.3 Part III: Technical Aspects

- **Channel Models:** Gamma-Gamma turbulence, Hufnagel-Valley profile
- **Detection Schemes:** Heterodyne detection, dual-threshold approach
- **Error Handling:** CASCADE, LDPC, polar codes, and ARQ
- **Security:** Practical security frameworks and finite-key analysis

11.1.4 Part IV: Analysis and Synthesis

- **Comparative Analysis:** Positioned Nguyen et al. within broader literature
- **Research Gaps:** Identified theoretical, practical, and system-level gaps
- **Future Directions:** Outlined near-term to long-term research roadmap

11.2 Key Findings

11.2.1 Nguyen et al. (2021) Contributions

The paper makes four significant contributions to satellite-based QKD:

Table 11.1: Summary of Paper Contributions

Contribution	Type	Impact
QPSK-based QKD with DT/HD	Physical Layer	20 dB power improvement
Key retransmission scheme	Link Layer	>1000× KLR reduction
3-D Markov chain model	Analytical	Enables optimization
Comprehensive analysis	System	Practical guidelines

11.2.2 Quantitative Results

Table 11.2: Summary of Quantitative Results

Metric	Result
Power improvement vs. SIM/BPSK	20 dB
KLR improvement with $M = 4$	>1000×
Optimal DT coefficient (weak turbulence)	0.7 – 2.4
Optimal DT coefficient (strong turbulence)	1.4 – 2.8
Security distance (Eve-Bob)	>30 m
Optimal retransmission count	$M = 4$

11.2.3 Unique Position in Literature

Nguyen et al. occupies a unique position by:

1. Being the **first** to apply ARQ retransmission to satellite QKD
2. Providing the **first** 3-D Markov chain model for link-layer QKD analysis
3. Achieving **highest sensitivity** through DT/HD combination
4. Demonstrating **cross-layer optimization** (physical + link layer)

11.3 Critical Assessment

11.3.1 Strengths

1. **Novel Integration:** First work combining QPSK, DT/HD, and ARQ for satellite QKD
2. **Practical Focus:** Realistic system parameters based on LEO satellite configuration
3. **Analytical Rigor:** Mathematical framework enables performance prediction without extensive simulation
4. **Significant Improvement:** Quantifiable gains that could enable practical deployment
5. **Vietnamese Contribution:** Advances regional research capability in quantum communications

11.3.2 Limitations

1. **Simulation Only:** No experimental validation of theoretical predictions
2. **Idealized Pointing:** Perfect beam tracking assumed
3. **Asymptotic Security:** Finite-key effects not analyzed
4. **Single Link:** No constellation or handover consideration
5. **Simplified Eavesdropper:** Only URA scenario analyzed

11.4 Recommendations

11.4.1 For Researchers

1. **Priority 1 - Finite-Key Analysis:**
 - Extend security analysis to practical key lengths
 - Determine minimum block sizes for target security levels
2. **Priority 2 - Experimental Validation:**
 - Develop ground testbed for DT/HD receiver
 - Validate ARQ protocol with emulated channel
3. **Priority 3 - Pointing Integration:**
 - Add realistic pointing error models
 - Analyze combined turbulence and pointing effects

11.4.2 For Practitioners

1. Use $M = 4$ retransmissions as optimal starting point
2. Select DT coefficient based on turbulence regime (0.7–2.8 range)
3. Consider DT/HD approach for telecom-compatible implementations
4. Plan for >30 m security perimeter around ground stations

11.4.3 For Policymakers

1. Support experimental validation of Vietnamese QKD research
2. Consider satellite QKD in national quantum communication strategy
3. Explore regional cooperation for ASEAN quantum network
4. Invest in ground station infrastructure development

11.5 Future Outlook

11.5.1 Technology Trajectory

Satellite QKD is progressing from experimental demonstrations toward operational deployment:

- **2025-2026:** Eagle-1, QUBE-II, expanded Chinese constellation
- **2027:** Chinese MEO satellite, global service announcement
- **2030+:** Quantum internet backbone integration

11.5.2 Vietnamese Opportunity

Vietnam has opportunity to participate in this development through:

- Continued theoretical research building on PTIT foundation
- Ground station development and characterization
- Regional collaboration with ASEAN partners
- Potential contribution to international missions

11.5.3 Role of Nguyen et al. (2021)

The work by Nguyen et al. provides:

- Theoretical foundation for Vietnamese satellite QKD development
- Novel approaches (ARQ, 3-D Markov) applicable to broader community
- Framework for future experimental validation
- Basis for continued research advancement

11.6 Final

Remarks

Nguyen et al. (2021) represents a significant contribution to satellite-based QKD research, particularly from the Vietnamese research community. The paper addresses a practical challenge—reliability improvement—through a novel combination of physical layer optimization (QPSK-DT/HD) and link layer mechanisms (ARQ retransmission).

The 20 dB power improvement and $>1000\times$ KLR reduction demonstrated in simulation suggest that the proposed approach could enable more practical satellite QKD systems. The analytical 3-D Markov chain model provides a valuable framework for system design and optimization that has not been previously available in the literature.

While experimental validation remains necessary before practical deployment, the work establishes a solid foundation for future Vietnamese contributions to global quantum communication research. As satellite QKD moves toward operational deployment in the coming years, the reliability techniques developed here may prove essential for practical system implementation.

This literature review has situated Nguyen et al. within the broader context of 40 years of QKD research, identified its unique contributions, and outlined pathways for continued advancement. The field of satellite-based quantum communication holds great promise for enabling truly secure global communications, and Vietnamese researchers are positioned to contribute meaningfully to this important endeavor.

This comprehensive literature review was prepared as part of the Master's program in Space & Earth Observation at USTH, December 2025.

Part V

Appendices

Appendix A

Key Equations Reference

This appendix provides a consolidated reference of key equations from Nguyen et al. (2021) and the supporting literature.

A.1 Channel	Model	Equations
A.1.1 Combined	Channel	Coefficient
		$h = h_l \cdot h_a \cdot h_s \cdot h_t \quad (\text{A.1})$
A.1.2 Free-Space	Path	Loss
		$h_l = \left(\frac{\lambda}{4\pi D_{SG}} \right)^2 \quad (\text{A.2})$
A.1.3 Slant		Range
		$D_{SG} = \sqrt{(H_S - H_G)^2 \sec^2(\zeta) + 2R_E(H_S - H_G) \sec(\zeta)} \quad (\text{A.3})$
A.1.4 Atmospheric		Attenuation
		$h_a = \exp \left(-\gamma \cdot \frac{H_\beta - H_G}{\cos(\zeta)} \right) \quad (\text{A.4})$
A.1.5 Hufnagel-Valley	Turbulence	Profile
		$C_n^2(h) = 0.00594 \left(\frac{w}{27} \right)^2 (10^{-5}h)^{10} e^{-h/1000} + 2.7 \times 10^{-16} e^{-h/1500} + C_n^2(0) e^{-h/100} \quad (\text{A.5})$

$$\mathbf{A.1.6} \quad \mathbf{Rytov} \qquad \qquad \qquad \mathbf{Variance}$$

$$\sigma_R^2 = 2.25k^{7/6} \sec^{11/6}(\zeta) \int_{H_G}^{H_S} C_n^2(h) \left(1 - \frac{h - H_G}{H_S - H_G}\right)^{5/6} (h - H_G)^{5/6} dh \quad (\text{A.6})$$

$$\mathbf{A.1.7} \quad \mathbf{\Gamma\text{-}Γ} \qquad \qquad \qquad \mathbf{Distribution}$$

$$f_{h_t}(h_t) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h_t^{(\alpha+\beta)/2-1} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta h_t}\right) \quad (\text{A.7})$$

$$\mathbf{A.1.8} \quad \mathbf{Scintillation} \qquad \qquad \qquad \mathbf{Parameters}$$

$$\alpha = \left[\exp \left(\frac{0.49\sigma_R^2}{(1 + 1.11\sigma_R^{12/5})^{7/6}} \right) - 1 \right]^{-1} \quad (\text{A.8a})$$

$$\beta = \left[\exp \left(\frac{0.51\sigma_R^2}{(1 + 0.69\sigma_R^{12/5})^{5/6}} \right) - 1 \right]^{-1} \quad (\text{A.8b})$$

$$\mathbf{A.2} \quad \mathbf{Detection} \qquad \qquad \qquad \mathbf{Equations}$$

$$\mathbf{A.2.1} \quad \mathbf{QPSK} \qquad \qquad \mathbf{Phase} \qquad \qquad \mathbf{States}$$

$$\phi_A \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, -\frac{\pi}{4} \right\} \quad (\text{A.9})$$

$$\mathbf{A.2.2} \quad \mathbf{Dual\text{-}Threshold} \qquad \qquad \qquad \mathbf{Decision}$$

$$\text{Decision} = \begin{cases} 0 & \text{if } i \geq d_0 \\ 1 & \text{if } i \leq d_1 \\ X & \text{otherwise (erasure)} \end{cases} \quad (\text{A.10})$$

$$\mathbf{A.2.3} \quad \mathbf{Threshold} \qquad \qquad \qquad \mathbf{Configuration}$$

$$d_0 = d + \varsigma \cdot \sigma \quad (\text{A.11a})$$

$$d_1 = d - \varsigma \cdot \sigma \quad (\text{A.11b})$$

$$\mathbf{A.2.4} \quad \mathbf{Received} \qquad \qquad \qquad \mathbf{Power}$$

$$P_R = P_T \cdot G_T \cdot G_R \cdot h \quad (\text{A.12})$$

A.2.5 SNR for Heterodyne Detection

$$\text{SNR} = \frac{(\Re P_R \bar{g})^2}{\sigma_{shot}^2 + \sigma_{thermal}^2} \quad (\text{A.13})$$

A.2.6 Noise Variances

$$\sigma_{shot}^2 = 2q(\Re P_{LO} + I_d)B \cdot \bar{g}^2 F \quad (\text{A.14a})$$

$$\sigma_{thermal}^2 = \frac{4k_B T B}{R_L} \quad (\text{A.14b})$$

A.3 QBER Equations

A.3.1 Bit Error Probability

$$P_e = \int_0^\infty Q\left(\sqrt{\frac{2h \cdot \text{SNR}}{1 + h \cdot \text{SNR}}}\right) f_{h_t}(h) dh \quad (\text{A.15})$$

A.3.2 Q-Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \quad (\text{A.16})$$

A.3.3 Conditional QBER

$$\text{QBER} = \frac{P_e}{P_{sift}} \quad (\text{A.17})$$

A.3.4 Sifting Probability

$$P_{sift} = P(|i - d| > \varsigma\sigma) = 1 - P_{erasure} \quad (\text{A.18})$$

A.4 Markov Chain Model Equations

A.4.1 State Space Definition

$$\text{State: } (n, s, m) \text{ where } n \in [0, C], s \in \{B, G\}, m \in [1, M] \quad (\text{A.19})$$

A.4.2 Channel Transition Probabilities

$$P(G \rightarrow B) = p_{GB} \quad (\text{A.20a})$$

$$P(B \rightarrow G) = p_{BG} \quad (\text{A.20b})$$

A.4.3 Steady-State Distribution

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P} \quad (\text{A.21})$$

A.4.4 Key Loss Rate

$$\text{KLR} = \sum_{s \in \{B, G\}} \pi_{C,s,M} \cdot P_{loss}^s \quad (\text{A.22})$$

A.5 Security Equations

A.5.1 Eve's Received Power

$$P_{R,Eve} = P_T \cdot G_T \cdot G_R \cdot h \cdot A_0 \exp\left(-\frac{2D_{E-B}^2}{\omega_{Deq}^2}\right) \quad (\text{A.23})$$

A.5.2 Secure Key Rate (Asymptotic)

$$R_{secure} = R_{shift} \cdot [1 - H(\text{QBER}) - f \cdot H(\text{QBER})] \quad (\text{A.24})$$

A.5.3 Binary Entropy Function

$$H(p) = -p \log_2(p) - (1-p) \log_2(1-p) \quad (\text{A.25})$$

A.6 Notation Reference

Table A.1: Symbol Notation Reference

Symbol	Description	Units
h	Total channel coefficient	—

Continued on next page

Continued from previous page

Symbol	Description	Units
h_l	Free-space path loss	—
h_a	Atmospheric attenuation	—
h_s	Beam spreading loss	—
h_t	Turbulence fading coefficient	—
D_{SG}	Satellite-to-ground distance	m
H_S	Satellite altitude	m
H_G	Ground station height	m
ζ	Zenith angle	rad
γ	Attenuation coefficient	dB/km
C_n^2	Refractive index structure parameter	$m^{-2/3}$
σ_R^2	Rytov variance	—
α, β	Gamma-Gamma parameters	—
ς	Dual-threshold coefficient	—
P_T	Transmitted power	W (or dBm)
P_R	Received power	W
G_T, G_R	Telescope gains	dB
\mathfrak{R}	Photodetector responsivity	A/W
\bar{g}	APD multiplication factor	—
B	Bandwidth	Hz
QBER	Quantum bit error rate	—
P_{sift}	Sifting probability	—
KLR	Key loss rate	—
M	Maximum retransmissions	—
C	Buffer capacity	sequences

Appendix B

Literature Database

This appendix provides a comprehensive database of papers reviewed in this literature review, organized by category and relevance tier.

B.1 Tier 1: Essential Papers

These papers are fundamental to understanding satellite-based QKD and directly relevant to Nguyen et al. (2021).

Table B.1: Tier 1 Essential Papers

#	Authors	Title	Year	Journal
1	Nguyen et al.	Reliability improvement of satellite-based QKD using retransmission	2021	Photonic Net. Comm.
2	Chen et al.	Integrated space-to-ground quantum network over 4,600 km	2021	Nature
3	Liao et al.	Satellite-to-ground quantum key distribution	2017	Nature
4	Yin et al.	Satellite-based entanglement distribution over 1200 km	2017	Science
5	Dequal et al.	Feasibility of satellite CV-QKD	2021	npj Quantum Info.
6	Trinh et al.	DT/DD for QKD over FSO	2018	IEEE Access
7	Pirandola et al.	Advances in quantum cryptography	2020	Adv. Opt. Photon.
8	Nguyen et al.	CV-QKD with DT/HD scheme	2023	IEEE Access

B.2 Tier 2: Important Papers

These papers provide essential context and technical foundations.

Table B.2: Tier 2 Important Papers

#	Authors	Title	Year	Journal
9	Scarani et al.	Security of practical QKD	2009	Rev. Mod. Phys.
10	Xu et al.	Secure QKD with realistic devices	2020	Rev. Mod. Phys.
11	Tomamichel et al.	Tight finite-key analysis	2012	Nature Comm.
12	Vasylyev et al.	Atmospheric quantum channels	2016	Phys. Rev. A
13	Liorni et al.	Satellite QKD beam and weather effects	2019	New J. Phys.
14	Ma et al.	Satellite downlink Gamma-Gamma	2015	Appl. Opt.
15	Orsucci et al.	Practical satellite QKD architectures	2025	Int. J. Sat. Comm.
16	Mueller et al.	CASCADE and LDPC for QKD	2025	IET Quantum Comm.
17	Liao et al.	Intercontinental quantum network	2018	Phys. Rev. Lett.

B.3 Tier 3: Supporting Papers

These papers provide additional technical depth and context.

Table B.3: Tier 3 Supporting Papers

#	Authors	Title	Year	Journal
18	Al-Habash et al.	Gamma-Gamma distribution derivation	2001	Opt. Eng.
19	Grosshans et al.	CV-QKD with coherent states	2003	Nature
20	Leverrier	Composable security for CV-QKD	2015	Phys. Rev. Lett.
21	Milicevic et al.	Quasi-cyclic LDPC for QKD	2018	npj Quantum Info.
22	Kish et al.	CV-QKD satellite feasibility	2020	Quantum Eng.
23	Various	LEO constellation networking	2022	Entropy
24	Various	Greek LEO QKD infrastructure	2021	Photonics

B.4 Tier 4: Reference Papers

These foundational papers provide historical and theoretical context.

Table B.4: Tier 4 Reference Papers

#	Authors	Title	Year	Journal
25	Bennett & Brassard	BB84 protocol	1984	IEEE Conf.
26	Ekert	E91 protocol	1991	Phys. Rev. Lett.
27	Gisin et al.	Quantum cryptography review	2002	Rev. Mod. Phys.
28	Bedington et al.	Progress in satellite QKD	2017	npj Quantum Info.
29	Kaushal & Kadoum	Space optical communication	2017	IEEE Comm. Surv.
30	Pan et al.	Micius experiments review	2022	Rev. Mod. Phys.

B.5 Vietnamese/PTIT Research Papers

Papers from Vietnamese institutions, particularly PTIT.

Table B.5: Vietnamese/PTIT Research Papers

#	Authors	Title	Year	Venue
V1	Trinh et al.	DT/DD for QKD over FSO	2018	IEEE Access
V2	Vu et al.	HAP-aided satellite QKD	2019	VTC Spring
V3	Nguyen et al.	Reliability with retransmission	2021	Photonic Net. Comm.
V4	Nguyen et al.	CV-QKD with DT/HD	2023	IEEE Access
V5	Vu et al.	Network coding EB/PM QKD	2023	ITC-CSCC

B.6 Papers by Category

B.6.1 Foundational QKD Protocols

- Bennett & Brassard (1984) - BB84
- Ekert (1991) - E91
- Grosshans et al. (2003) - CV-QKD
- Gisin et al. (2002) - Review

B.6.2 Satellite QKD Experiments

- Liao et al. (2017) - Micius first QKD
- Yin et al. (2017) - Entanglement distribution

- Liao et al. (2018) - Intercontinental QKD
- Chen et al. (2021) - Integrated network
- Pan et al. (2022) - Micius review

B.6.3 Atmospheric Channel Models

- Al-Habash et al. (2001) - Gamma-Gamma
- Vasylyev et al. (2016) - Quantum channels
- Liorni et al. (2019) - Weather effects
- Ma et al. (2015) - Satellite downlink
- Kaushal & Kaddoum (2017) - Space optical

B.6.4 Detection and Modulation

- Trinh et al. (2018) - DT/DD
- Nguyen et al. (2021) - DT/HD
- Nguyen et al. (2023) - CV-QKD DT/HD
- Dequal et al. (2021) - CV-QKD satellite

B.6.5 Error Correction

- Milicevic et al. (2018) - LDPC
- Mueller et al. (2025) - CASCADE vs LDPC
- Various (2024) - RC-LDPC-Polar

B.6.6 Security Analysis

- Scarani et al. (2009) - Practical security
- Tomamichel et al. (2012) - Finite-key
- Xu et al. (2020) - Realistic devices
- Leverrier (2015) - CV-QKD security

B.6.7 Recent Advances (2022-2025)

- Orsucci et al. (2025) - Architecture assessment
- Mueller et al. (2025) - Industrial reconciliation
- Various (2025) - Finite-key heterodyne
- LEO constellation studies (2022-2024)

Table B.6: High-Citation Papers in Review

Paper	Est. Citations	Year
Bennett & Brassard (BB84)	>15,000	1984
Gisin et al. (Review)	>5,000	2002
Ekert (E91)	>5,000	1991
Scarani et al. (Security)	>4,000	2009
Liao et al. (Micius)	>2,500	2017
Yin et al. (Entanglement)	>2,000	2017
Pirandola et al. (Review)	>1,500	2020
Chen et al. (Network)	>1,000	2021
Grosshans et al. (CV-QKD)	>1,000	2003
Xu et al. (Devices)	>800	2020

B.7 Citation Statistics

B.8 Paper Access Information

B.8.1 Open Access Sources

- **arXiv:** Most physics papers available (arxiv.org)
- **PubMed Central:** Some biomedical-related papers
- **IEEE Xplore:** Some open access articles
- **Nature/Science:** Selected open access papers

B.8.2 Institutional Access

- USTH library portal
- VAST institutional subscriptions
- Inter-library loan services

B.8.3 DOI References

Key DOIs for direct access:

- Nguyen et al. (2021): [10.1007/s11107-021-00934-y](https://doi.org/10.1007/s11107-021-00934-y)
- Chen et al. (2021): [10.1038/s41586-020-03093-8](https://doi.org/10.1038/s41586-020-03093-8)
- Liao et al. (2017): [10.1038/nature23655](https://doi.org/10.1038/nature23655)
- Pirandola et al. (2020): [10.1364/AOP.361502](https://doi.org/10.1364/AOP.361502)
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