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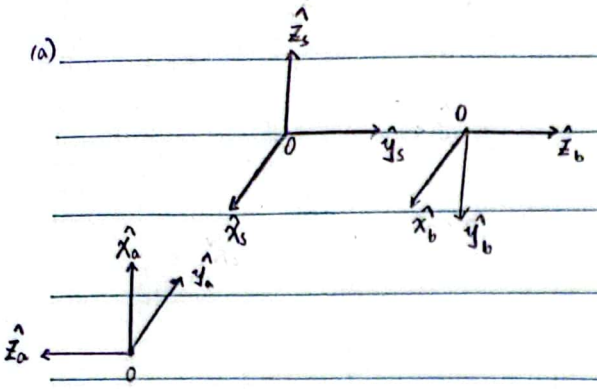
(a)



$$P_{sa} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \qquad R_{sa} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P_{sb} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \qquad R_{sb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$P_{ab} = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix} \qquad R_{ab} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(b) 由 (a)，知 $R_{sa} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$ $R_{sb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

$$T_{sa} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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c) $T_{sb}^{-1} = \begin{bmatrix} R_{sb} & P_{sb} \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R_{sb}^T & -R_{sb}^T P_{sb} \\ 0 & 1 \end{bmatrix}$

$R_{sb}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad R_{sb}^T P_{sb} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$T_{sb}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad T_{sb}^{-1} = T_{bs}$

$\therefore R_{bs} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad P_{bs} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$

(b)

$R_{sb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$ $\quad R_{sb} R_{bs} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $T_{ab} = T_{as} T_{sb} = T_{sa}^{-1} T_{sb} = \begin{bmatrix} R_{sa}^T & -R_{sa}^T P_{sa} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{sb} & R_{sb} P_{sb} \\ 0 & 1 \end{bmatrix}$

$R_{sa} = \begin{bmatrix} 0 & -1 & 0 \\ & & \end{bmatrix}$ $\quad R_{sa}^T = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ $\quad R_{sa}^T P_{sa} = \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix}$

$T_{ab} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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同(a)其中 R_{ab}, P_{ab} 相同, 坐标系正确.

(e) $T_1 = T_{sa} T_{sb} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

T_1 对应的是 T_{sa} 相对物体坐标系的变换.

$T_2 = T_{sb} T_{sa} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

T_2 对应的是 T_{sa} 相对固定坐标系的变换

(f) ~~$P_s = T_s$~~

$\begin{bmatrix} P_s \\ 1 \end{bmatrix} = T_{sb} \begin{bmatrix} P_b \\ 1 \end{bmatrix}$ ~~$P_s = R_{sb} P_b =$~~

$\begin{bmatrix} P_s' \\ 1 \end{bmatrix} = T_{sb} \begin{bmatrix} P_b^T \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \\ 1 \end{bmatrix}$

$P_s = (1, 5, -2)$

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$$v' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \\ 1 \end{bmatrix}$$

$p' = (1, 5, -2)$

$$v'' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$p'' = (1, -3, 0)$

b) $V_a = [Ad_{T_{as}}] V_s = \begin{bmatrix} R_{as}^T & 0 \\ -R_{as}^T [P_{as}] & R_{as}^T \end{bmatrix} V_s$

=

$$V_a = [Ad_{T_{as}}] V_s = \begin{bmatrix} R_{sa}^T & 0 \\ -R_{sa}^T [P_{sa}] & R_{sa}^T \end{bmatrix} V_s^T$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \\ -2 \\ -3 \end{bmatrix}$$

$= (1 \ -3 \ -2 \ 3 \ 1 \ 5)$

$V_a = (1, -3, -2, 3, 1, 5)$

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(i) $w = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\theta = 90°$ $p = (3, 0, 0)$

$\hat{w} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}$ $[\hat{w}] = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$

$G^{-1}(\theta) = \frac{2}{\pi} I - \frac{1}{2}[\hat{w}] + \left(\frac{2}{\pi} - \frac{1}{2} \times 1\right)[\hat{w}]^2$

$v = G^{-1}(\theta) p = \begin{pmatrix} \frac{3}{4} + \frac{3}{\pi} \\ -\frac{3}{4}\sqrt{2} \\ \frac{3}{\pi} - \frac{3}{4} \end{pmatrix}$

$[S]\theta = \begin{bmatrix} 0 & -1.1107 & 0 & 2.618 \\ 1.1107 & 0 & -1.1107 & -1.666 \\ 0 & 1.1107 & 0 & 0.322 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$S = (-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, \frac{3}{4} + \frac{3}{\pi}, -\frac{3}{4}\sqrt{2}, \frac{3}{\pi} - \frac{3}{4})$

$q = (\frac{3}{4} + \frac{3}{\pi}, -\frac{3}{4}\sqrt{2}, \frac{3}{\pi} - \frac{3}{4})$

~~$h = \frac{3}{4}\sqrt{2}$~~ $h = (0, -\frac{3}{4}\sqrt{2}, 0)$

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(i) $s\theta = (0, 1, 2, 3, 0, 0)$

$\|w\| = 1$

$s = (0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{3}{\sqrt{5}}, 0, 0)$

$\hat{w} = (0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})^T$

$v = (\frac{3}{\sqrt{5}}, 0, 0)^T$

$\theta = \sqrt{5}$

$$e^{[w]\theta} = I + \sin\theta[\hat{w}] + (1-\cos\theta)[\hat{w}]^2$$

$$= I + 0.787[\hat{w}] + 1.617[\hat{w}]^2$$

$$= \begin{pmatrix} 1 & 0.296 & 0.447 \\ 0.894 & 1 & 0 \\ -0.447 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -1.617 & 0 & 0 \\ 0 & -1.2936 & 0.6468 \\ 0 & 0.6468 & 0.3234 \end{pmatrix}$$

$$= \begin{pmatrix} 0.617 & 0.296 & 0.447 \\ 0.894 & -0.2936 & 0.6468 \\ -0.447 & 0.6468 & 1.3234 \end{pmatrix}$$

$$(I\theta + (1-\cos\theta)[\hat{w}] + (\theta - \sin\theta)[\hat{w}]^2)v$$

$$= (I\theta + 1.617[\hat{w}] + 1.449[\hat{w}]^2)v$$

$$= \begin{pmatrix} 1.056 \\ 1.941 \\ -0.97 \end{pmatrix}$$

$$e^{[s]\theta} = \begin{pmatrix} 0.617 & 0.296 & 0.447 & 1.056 \\ 0.894 & -0.2936 & 0.6468 & 1.941 \\ -0.447 & 0.6468 & 1.3234 & -0.97 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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