Tilburg University

Life Insurance Case Study

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Expected Survival Rate of Pension Fund candidates

$$\mu_x(t)^{31x57} = \begin{bmatrix} 0.01235 & 0.01227 & \dots & 0.00781 & 0.00698 \\ 0.01353 & 0.01393 & \dots & 0.00853 & 0.00849 \\ \vdots & \vdots & \ddots & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.27429 & \dots & \dots & \dots & 0.16351 \\ 0.28523 & \dots & \dots & \dots & 0.18317 \end{bmatrix}$$

Figure 1 presents the relation between the estimated alpha and the age. As we see from the graph, the plot is for sub-population older than 60 and younger than 90, therefore the graph is linear.

In Lee Carter model we have the log of force of mortality instead of using just $\mu_x(t)$, and this causes the linear pattern of the graph which is usually exponentially increasing as we have seen during the lectures if we plot not the $\log(\mu_x(t))$ but the $\mu_x(t)$ dependent on age. For any age group, from LC model of $\log(\mu_x(t)) = \alpha_x + \beta_x\kappa_t$ follows that:

$$\alpha_x = \frac{1}{n}\sum_{t+t_1}^{t_n} log(\hat{\mu}_x(t)) \tag{1}$$

Which solves the optimization problem of LC model and gives as the estimate of the alpha, which is age-group specific. This graphs verifies the rightness of the model and that we are on the right track.

Figure 2 presents the relation between the estimated beta and the age and we observe a non-linear decreasing function of age, but not for all age-groups. For ages between 60 and 75 we observe that the function increases slightly, especially for the age-group 70-75, but from 75 it is decreasing. As we know, $\hat{\beta}$ tells us how certain age-groups are effected on average of these 60 years.

So, all these individuals aged 60-90 are positively effected, (we could also have negative betas but we don't) and the age group 75-80 are the mostly positively effected ones. Negative beta's would indicate an increase in force of mortality but that is not the case, which was expected for a country in EU. How older individuals are, less effected they are(the effect decreases). Finally, we don't interpret the level or the scale of the graph since it is irrelevant and is determined using the constraint that $\sum_{x=x_1}^{x_m} = 1$

Figure 3 presents the relation between the estimated kappa's and the time, which are estimated using as in case of estimated betas, the singular value decomposition(SVD). The κ_t are the only components in the Lee Carter model which capture the development of the force of mortality over time.

We again observe roughly linear but not perfectly linear pattern in the graph. We observe decreasing function of kappas during the period of 1950-2006. Mortality at each age group changes at each own exponential rate, the graph can be considered as linear. The first half of the period has more short term fluctuations compared to the second half, which is also because of the logarithmic transformation. Notice that also in this case we don't observe the scale or the level of the graph since they are dependent on the choice of c and the constraints used that we use in LC model. As expected the graph is centered over zero, because of the used shifting constraint: $\sum_{t=t_1}^{t_n} = 0$.

$$\hat{\sigma}_{\epsilon}^2 = \frac{1}{mn} \sum_{x=x_1}^{x_m} \sum_{t=t_1}^{t_n} \big(log(\hat{\mu}_x(t)) - \hat{\alpha}_x - \hat{\beta}_x \hat{\kappa}_t\big)^2 = 0.0016 \tag{2}$$

Percentage of Variance explained by the model = 0.9426

$$\begin{aligned} \hat{C} &= -0.3318 \\ \hat{\sigma}_k^2 &= 0.5801 \end{aligned} \tag{3}$$

The 95% prediction interval for $\mu_{80}(2086)$ is:

$$exp\big(\hat{\alpha}_{80} + \hat{\beta}_{80}\hat{\kappa}_{2006}\big) exp\big(\hat{\beta}_{80} k \hat{C} + / - 1.96 \hat{\beta}_{80} \sqrt{k \hat{\sigma}_k^2}\big) = (0.0141, 0.0352) \tag{4}$$

The 95% prediction interval for $p_{80}(2086)$ we determine using the previous values from 1.f and using the formula $p_x(t) = \exp(-\mu_x(t))$:

$$(0.9860; 0.9654) \tag{5}$$

The probability of a person aged 60 in 2040 to become strictly older than 75 is the probability that he will survive for at least 15 years:

$$= \bar{G}_{60,2040}(15) = exp\big(-\sum_{j=0}^{s-1} \mu_{x+j}(t+j)\big) = 0.8565 \tag{6}$$

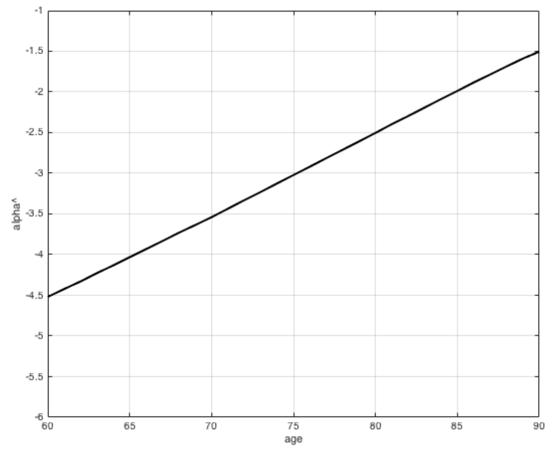

Figure 1: The relation between α_x and age

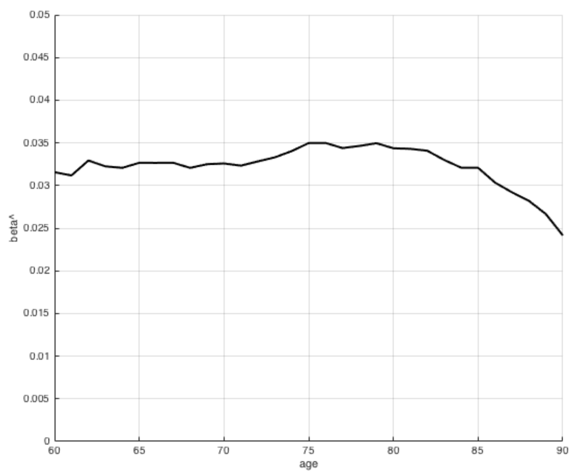

Figure 2: The relation between β_x and age

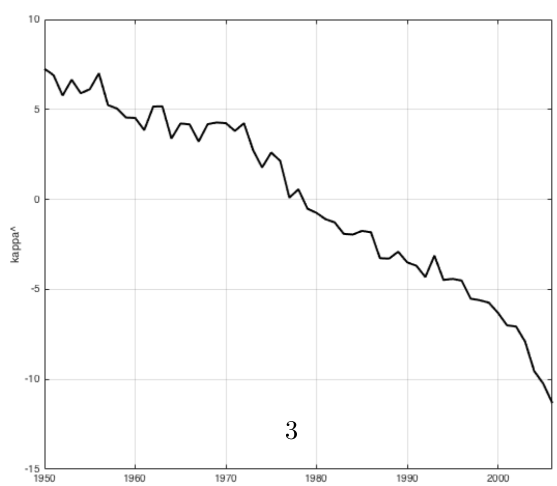

Figure 3: The relation between κ_t and time

The probability of a person aged 60 in 2040 to pass away aged 80 is the probability that this person aged 60 will survive for 20 years and will die within 1 year which is:

$$
\begin{aligned}
&P[T_{60}(2040) < 21 | T_{60}(2040) \geq 20] = \frac{P[T_{60}(2040) < 21, T_{60}(2040) \geq 20]}{P[T_{60}(2040) \geq 20]} \\
&= \frac{P[20 \leq T_{60}(2040) < 21]}{P[T_{60}(2040) \geq 20]} = \frac{P[0 \leq T_{80}(2060) < 1]}{P[T_{60}(2040) \geq 20]} \\
&= \frac{q_{80}(2060)}{\bar{G}_{60,2040}(20)} = \frac{1 - exp(\mu_{80}(2060))}{exp\big(-\sum_{j=0}^{19} \mu_{60+j}(2040+j)\big)} = 0.0386
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
C_{2040} &= -0.4039 \\
C_{2050} &= -0.3318 \\
C_{2060} &= -0.3417 \\
C_{2070} &= -0.4281 \\
C_{2080} &= -0.3963
\end{aligned} \tag{8}
$$