# Tilburg University

## Life Insurance Case Study

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## Expected Premium Analysis for a Pension Fund

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## Deriving an expression for $\prod$ Premium for Pension

As it is introduced in our widow's pension , the premiums are paid by both (x) and (y) as long as they are alive and not retired with an standardized amount of 1.
Moreover, in case they are both alive and not retired they as a household they never pay more than 1.5 (standardized) unit. So, they get discount of 0.5 unit than they had to. Therefore, we get following expression for the present value of the Premium Payments:

$$\prod = \sum_{k\geq 0} v^k \mathbf{1}\{T_x \geq k\}\mathbf{1}\{k \leq ret_x - x - 1\} + \sum_{k\geq 0} v^k \mathbf{1}\{T_y \geq k\}\mathbf{1}\{k \leq ret_y - y - 1\}$$

$$- 0.5 * \sum_{k\geq 0} v^k \mathbf{1}\{T_x \geq k\}\mathbf{1}\{T_y \geq k\}\mathbf{1}\{k \leq ret_x - x - 1\}\mathbf{1}\{k \leq ret_y - y - 1\}$$

$$= \sum_{k=0}^{retx-x-1} v^k \mathbf{1}\{T_x \geq k\} + \sum_{k=0}^{rety-y-1} v^k \mathbf{1}\{T_y \geq k\} - 0.5 * \sum_{k=0}^{min(ret_x-x-1, ret_y-y-1)} v^k \mathbf{1}\{T_x \geq k\}\mathbf{1}\{T_y \geq k\} \tag{1}$$

## Give an expression for the expectation of $\prod$

$$E(\Pi) = \sum_{k=0}^{retx-x-1} v^k *_k p_x + \sum_{k=0}^{rety-y-1} v^k *_k p_y - 0.5 * \sum_{k=0}^{min(ret_x-x-1, ret_y-y-1)} v^k *_k p_x *_k p_y \tag{2}$$

So,we simplify the expression of the present value and we obtain that the expectation of the present value of the premium payments is equal to the discounted sum of the premium payments of the female aged(22) as long as she is alive(multiplied with her surviving probability) and not retired, the same holds for the male aged(37). Moreover, we subtract then the discount of 0.5 unit over this payments in case both female and male are alive and not retired (multiplied with their survival probabilities, until one of them dies).

## Give an expression for the net premium

In order to find the expression for the net premium we first determine the expression of the loss function of this pension which is:

$$\begin{aligned} L &= C * Y - \pi * \Pi \\ E(L) &= C * E(Y) - \pi * E(\Pi) \end{aligned} \tag{3}$$

In case of the Net Premium the expectation of the loss function is equal to 0. The $\pi$ must then satisfy the net premium principle formula.

$$\begin{aligned} E(L) &= 0 \\ C * E(Y) &= \pi * E(\Pi) \\ \pi &= C * \frac{E(Y)}{E(\Pi)} \end{aligned} \tag{4}$$

Where C is the amount of pension payments equal to 50.000.According to Assignment 1:

$$E(Y) = \sum_{k \geq ret_x - x} v^k *_k p_x * (1 -_k p_y) + \sum_{k \geq ret_y - y} v^k *_k p_y * (1 -_k p_x) \tag{5}$$

Using the expressions (2) , (4) and (5) we obtain:

$$\pi = 50.000 * \frac{\sum\limits_{k \geq ret_x - x} v^k *_k p_x * (1 -_k p_y) + \sum\limits_{k \geq ret_y - y} v^k *_k p_y * (1 -_k p_x)}{\sum\limits_{k=0}^{retx-x-1} v^k *_k p_x + \sum\limits_{k=0}^{rety-y-1} v^k *_k p_y - 0.5 * \sum\limits_{k=0}^{min(ret_x - x-1, ret_y - y-1)} v^k *_k p_x *_k p_y} \tag{6}$$

## Calculating Expected Net Premium

$$ret_x = 75,\ ret_y = 70,\ i = 2\%,\ (x) = 22,\ (y) = 37,\ v = \frac{1}{1+i} = 0.9804 \tag{7}$$

$$E(Y) = \sum_{k \geq 53} 0.9804^k *_k p_x * (1 -_k p_y) + \sum_{k \geq 33} 0.9804^k *_k p_y * (1 -_k p_x) \tag{8}$$

We get:

E($T_x$) = 68.9097
E($T_y$) = 49.06
E(Y) = 156430.46