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YOU MAY NOT USE A CALCULATOR.

1 Write appropriate indices in the boxes to make the following statements true. The first has been completed for you.

e.g $10^{\boxed{3}} = 1000$

a $5^{\boxed{-2}} = \frac{1}{25}$ ✓

b $10^{\boxed{0}} - 1$ ✓

c $7^{8} - \frac{7^{5}}{7^{\boxed{-3}}}$ ✓

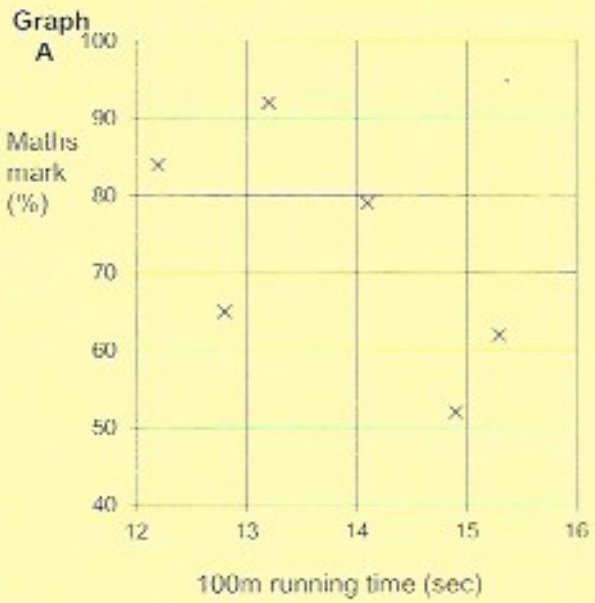
d $2^{\boxed{1}} + 2^{-4} - 2^{-3}$ ✗

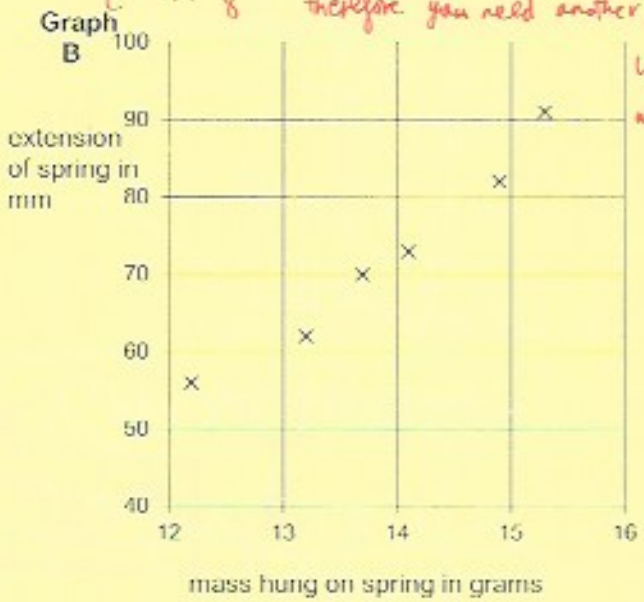
$2^{-4} = \frac{1}{16}$ $\frac{1}{16} = \frac{1}{8} \div 2$

$2^{-3} = \frac{1}{8}$ therefore you need another $\frac{1}{16}$ (−1) which is another 2^{-4}

[4] 3/4

2





a Describe the sign and strength of correlation shown by each graph.

Graph A has a very weak negative correlation, so people who run 100m faster have better math results on average in that class, and vice versa.

Graph B has a very strong positive correlation, so the more mass in g is hung on the spring, the longer the spring extends and vice versa.

[2] 2/2

b Graph A shows the Maths marks and 100 m running times of John's friends.
John says:
"among my friends, the ones who are better at Maths have shorter 100 m running times".

Is this a reasonable thing for John to say? Explain your answer.

Yes. The correlation, however, is still very weak and is only true for that class, supposed. John could say "In general", and "tend to have", and "vice versa".

[2] 2/2

3 a Write in standard form $\frac{23}{10^6}$

$= 23 \times \frac{1}{10^6}$ $= 2.3 \times 10^{-5}$

$= 23 \times 10^{-6}$

[1] 1/1

b Simplify as far as possible $(3 \times 6^9) \times (2 \times 6^5)$

$= (30000 \times 6^5) \times (2 \times 6^5)$

$= 30002 \times 6^5$ $= 6 \times 6^{14}$

$= 3.0002 \times 6^9$ $= 6^{15}$

[2] 0/2 (-2)

4 Calculate, giving your answers in standard form.

a $(3 \times 10^9) \div (4 \times 10^{-3})$

$= \frac{3 \times 10^9}{4 \times 10^{-3}}$

$= 0.75 \times 10^{12}$

$= 7.5 \times 10^{11}$

[2] 2/2

b $(1.2 \times 10^{15}) - (8.0 \times 10^{14})$

$= (12 \times 10^{14}) - (8 \times 10^{14})$

$= 20 \times 10^{14}$ $= 4 \times 10^{14}$

$= 2 \times 10^{15}$

[2] 1/2 (-1)

c $(5 \times 10^8)^3$

$= 125 \times 10^{24}$

$= 1.25 \times 10^{26}$

[2] 2/2

d $\sqrt{6.4 \times 10^{-11}}$

$= \sqrt{64 \times 10^{-10}}$ $= \sqrt{64 \times 10^{-12}}$

$= 8 \times 10^{-5}$ $= 8 \times 10^{-6}$

[2] 1/2 (-1)

5 For the locus given by the relation $3y + x = 5$:

a give the co-ordinates of the points on the locus with

i x co-ordinate -25,

(-25, 10)

ii y co-ordinate $\frac{1}{2}$,

(3.5, 0.5)

iii equal x and y co-ordinates;

(1.25, 1.25)

Think: what if y is 1? what if y is 0?

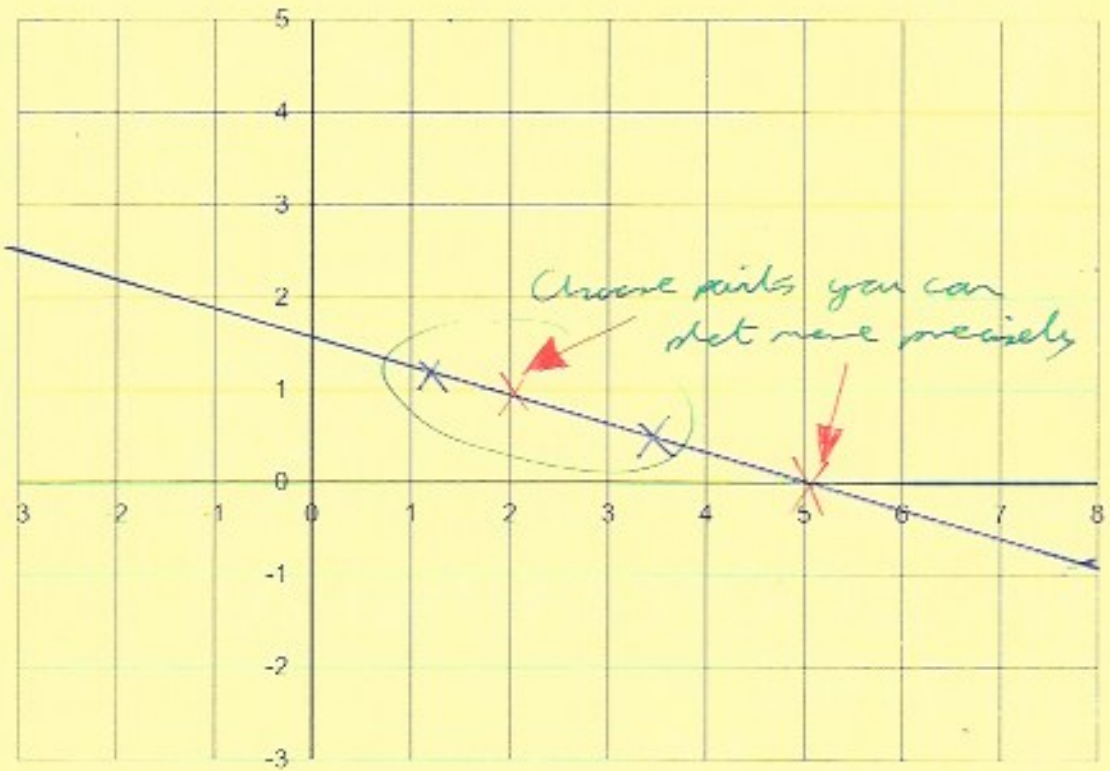
$y = 0,\ x = 5$

$y = 1,\ x = 2$

3/3

[3]

b show the locus given by the relation $3y + x = 5$ on the axes provided below.



3/4

[4] (-1)

c A point lies on the locus defined by the relation $3y + x = 5$.
If the y coordinate increases by 1, how does the x coordinate change?
(The new point must also lie on the locus.)

Decreases by 2

Decreases by 3! (-3)!

[2] 0/2

(-2)

6

a Write down $\sqrt{196}$

14

[1] 1/1

Simplify the following:

b $(3x^2)^2$

$= 9x^4$

[1] 1/1

c $\left(\frac{3a}{2}\right)^{-2}$

$= \left(\left(\frac{3a}{2}\right)^2\right)^{-1}$

$= \frac{4}{9a^2}$

[1] 1/1

d $\sqrt{9x^{64}}$

$= 3x^{32}$

[1] 1/1

7 Work out $\sqrt{226^2 - 224^2}$

$226^2 - 224^2 = (226-224)(226+224)$

$(226-224) = 2$

$(226+224) = 450$

$450 \times 2 = 900$

$\sqrt{900} = 30$

[3] 3/3

8 a In the expression $5x - 2y - 7$, substitute $y = 3x - 4$ and simplify the result.

$y = 3x - 4$

$2y = 6x - 8$

~~$-2y + 6x + 8$~~

$5x - 6x + 8 - 7 = -x + 1$ ✓

[2] 2

b Solve the simultaneous equations

$y = 5 + x$

$3y + 2x = 5$

using a method which involves substitution.

$y = 5 + x$

$3y = 15 + 3x$

$15 + 3x + 2x = 5$

$5x = -10$

$x = -2$ ✓

$y = 5 + x$

$y = 5 - 2$

$y = 3$ ✓

[3] 3

BONUS QUESTIONS

9 Write the following in increasing order of size. Explain your reasoning clearly.

2^{31} 4^{15} 8^{11} 16^{8} 32^{6}

Increasing order of size: 32^6 and 4^{15}, 2^{31}; 16^8, 8^{11} Great!

Because $4^{15} = (2^2)^{15}$

$4^{15} = 2^{30}$

$2^{31} > 4^{15}$

$8^{11} = (2^3)^{11}$

$8^{11} = 2^{33}$

$8^{11} > 2^{31}$

$16^8 = (2^4)^8$

$16^8 = 2^{32}$

$16^8 > 2^{31}$

$32^6 = (2^5)^6$

$32^6 = 2^{30}$

$2^{31} > 32^6$

$32^6 = 4^{15}$ they are equal!

3/3

10 [32] Given that $4^x + 4^x + 4^x + 4^x = 4^{16}$, what is x?

$4^x + 4^x = 4^x \times$

$4^x \times 4^x \times 4^x \times 4^x = 4^x \times 4^x \times 4^x \times 4^x = 4^{x+1}$

$4^{x+4} = 4^{16}$

$x = 2x$

$x^4 = 16$

11 When $n = 81$, what is the value of $\frac{n^{20}}{3^n}$?

$\frac{81^{20}}{3^{81}}$ → Try $\frac{(3^4)^{20}}{3^{81}}$

$= \frac{3^{80}}{3^{81}} = \frac{3}{3^2} = \frac{3}{9} = \frac{1}{3}$

12 A class of twelve students is given two Latin tests. Each test is marked out of 20.
Explain how it is possible for the following statements both to be true.

i The back-to-back stem and leaf diagrams showing the marks in the two tests are identical.

ii All but one of the pupils did better in the second test than in the first.

In the first test, here are the results in terms of x:

Student:	1	2	3	4	5	6	7	8	9	10	11	12
Test 1 Mark:	x	$x+1$	$x+2$	$x+3$	$x+4$	$x+5$	$x+6$	$x+7$	$x+8$	$x+9$	$x+10$	$x+11$
Test 2 Mark:	$x+1$	$x+2$	$x+3$	$x+4$	$x+5$	$x+6$	$x+7$	$x+8$	$x+9$	$x+10$	$x+11$	x

Great! nice explanation

Therefore the marks stayed the same, but shifted, so that all but one of the students gained 1 mark on their test.

3/3

13 What are the co-ordinates of the point where the line which passes through (2, 5) and (7, 10) meets the line which passes through (11, 32) and (39, 4)?

Ask for more paper if you need it