PRATHAM

STUDENT SATELLITE INITIATIVE, IITBOMBAY

TITLE:

Attitude Determination and Control System for Pratham

GROUP-MEMBERS:

Shashank Tamaskar

Mallesh Bommanahal

Varzavand Batliwala

Chandrika Bhardwaj

Jaideep Joshi

SUPERVISORS:

Prof. K. Sudhakar

Prof. H. Arya

Prof. Madhu Belur

Prof. Sanjay Bhat

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Chapter 1

Objectives of ADCS and Requirements Capture

1.1 Objectives:

Position determination - To determine the position of the satellite in space

Attitude Determination - To determine the attitude of the body frame* of the satellite with respect to orbit frame*

Attitude control - To bring the satellite into earth pointing orientation after ejection and to maintain this attitude throughout the period of operation

1.2 Requirements from other subsystems:

Position Requirement:

Onboard	
Communication	TBD (Less Stringent than payload)
Controls	TBD(Less Stringent than payload)
On Ground	
Payload	1km

- Position requirements are required by the communication subsystem to know when to begin data transmission when over a ground station.
- Requirements of Control subsystem arise due to need for obtaining the components of earth's magnetic field in the earth fixed frame. This is required for attitude determination.
- Requirements of the Payload subsystem arise due to the need for correlating the position of satellite with the measured value of TEC. This data is required on ground for post processing of the TEC data. Hence, either must be calculated on ground using an orbit estimator or must be transmitted to ground from the satellite.

1.3 Attitude determination:

Onboard		
Controls	Roll Pitch Yaw	TBD TBD TBD

- This is required to achieve attitude control of the satellite. This will also be essential in case of an emergency due to antenna failure.

1.4 Attitude control

Payload	Yaw Stabilization required
Communication	3deg cone about the nadir pointing direction. There are no requirements for jitter control

- Attitude control is required by the communication subsystem to ensure that proper signal reception from the satellite. This is not a stringent requirement and can be further relaxed by designing a more complicated ground station
- Yaw stabilization is required by the payload.

Chapter 2

Literature Survey

2.1 Control strategies employed by other student satellites:

First Batch Launched on June 30,2003

CUTE-I by Tokyo Institute of Technology, Japan	
Deployment Mechanism:	Tokyo Pico-satellite Orbital Deployers (T-POD)
Type:	Single cubesat
Mission:	Test platform based on COTS components. Deployable solar cells, piezoelectric vibrating gyroscope (4 pcs), dual axis accelerometers (4 pcs) and CMOS camera used as sun sensor. The camera pictures could not be transmitted to the ground.
AODC:	Piezoelectric vibrating gyroscope to measure angular velocity, dual axis acelerometer (used to compare to gyros), and CMOS sun sensor
Status:	Nominal operations. The satellite is still operating nominally in mid January 2007 (3 years, 6 months after launch)
Link:	http://lss.mes.titech.ac.jp/ssp/cubesat/index_e.html
XI-IV by University of Tokyo, Japan	
Deployment Mechanism:	Tokyo Pico-satellite Orbital Deployer (T-POD)
Type:	Single cubesat
Mission:	Test platform based on COTS components. Included a camera to take pictures of the earth.
AODC:	Permanent magnet and hysteresis rods
Status:	Nominal operations. The satellite is still operating nominally in mid January 2007 (3 years, 6 months after launch). Latest telemetry analysis dated September 20th, 2007.
Links:	http://www.space.t.u-tokyo.ac.jp/cubesat/index-e.html http://www.space.t.u-tokyo.ac.jp/gs/en/index.aspx
CanX-1 by University of Toronto, Canada	
Deployment Mechanism:	P-POD NLS-1 #1
Type:	Single cubesat
Mission:	Space-testing key technologies for future missions: Low-cost CMOS horizon sensor and star-tracker, GPS receiver.
AODC:	CMOS horizon sensor and star-tracker, GPS receiver, magnetometer, and magnetorquers

Status:	Radio contact never established
Link:	http://www.utias-sfl.net/nanosatellites/CanX1/
DTUsat by Technical University of Denmark	
Deployment Mechanism:	P-POD NLS-1 #2
Type:	Single cubesat
Mission:	MEMS sun sensors and a 600 m tether used to change the orbit. A color CCD camera and electron emitter were not ready on time for launch
AODC:	MEMS sun sensors, magnetometer, and magnetorquers
Status:	Radio contact never established
Link:	http://dtusat1.dtusat.dtu.dk
AAU Cubesat by Ålborg University, Denmark	
Deployment Mechanism:	P-POD NLS-1 #3
Type:	Single cubesat
Mission:	Color CMOS camera
AODC:	Sun sensors, magnetometer, and magnetorquers
Status:	The antennas short-circuited resulting in poor communication performance so only weak beacon signals were received. Also simple two-way communication was established (pinging). Batteries slowly died beginning after slightly more than one month in orbit due to poor packaging (punch-pack).
Link:	http://www.cubesat.auc.dk
Lessons Learned:	http://www.studentspace.aau.dk/publications/AAUCubesatProject.pdf
QuakeSat by Stanford University and Quakesat LLC, USA	
Deployment Mechanism:	P-POD NLS-2
Type:	Triple Cubesat
Mission:	Detect ELF radio emission of seismic activity during earthquakes. Had deployable solar panels, and a magnetometer mounted on a 60 cm boom. The s/c was designed using COTS components.
AODC:	Permanent magnet and hysteresis rods
Status:	Designed for 6 months, it worked flawlessly until at least June 6th, 2004 (more than 11 months). Other source (http://showcase.netins.net/web/wallio/CubeSat.htm) states nominal operations in ultimo December 2006 (3 years, 6 months after launch), and beacon heard on October 7th, 2007
Links:	http://www.quakefinder.com/quakesat.htm and http://www.quakefinder.com/quakesat-ssite/index.htm
Lessons Learned:	http://www.quakefinder.com/fppt/lessons.htm

Second batch launched on October 27,2005

NCube2 by Norwegian University of Science and Technology	
Deployment Mechanism:	Tokyo Pico-satellite Orbital Deployer (T-POD) (deployment time not known)
Type:	Single cubesat
Mission:	Similar to NCube1, the payload consists of an Automatic Identification System. AIS is a mandatory system on all larger ships, which transmits identification and position data messages. The satellite will redirect these messages along with messages from Norwegean reindeer collars.
AODC:	Magnetometer and magnetorquers
Status:	Radio contact never established May have been deployed around December 20th, 2005, when NORAD began tracking a new small object moving away from SSETI-Express. It is believed that outgassing could have allowed the late deployment, however contact has still not been established
Link:	http://www.ncube.no/
UWE-1 by University of Würzburg, Germany	
Deployment Mechanism:	Tokyo Pico-satellite Orbital Deployer (T-POD) (deployment time not known)
Type:	Single cubesat
Mission:	Testing a communication protocol, test of GaAs cells in space, running micro Linux
AODC:	Information not available
Status:	Nominal operations until November 17th, 2005, when it was last heard. Since then contact has been lost completely.
Link:	http://www7.informatik.uni-wuerzburg.de/cubesat
XI-V by University of Tokyo, Japan	
Deployment Mechanism:	Tokyo Pico-satellite Orbital Deployer (T-POD) (deployment time not known)
Type:	Single cubesat
Mission:	Original a backup for XI-IV. The following changes have been added: Test of CIGS and GaAs solar cells, increased resolution of camera and an introduction of rapid shooting mode for estimating attitude motion. A morse message transmission service for radio amateurs has been added
AODC:	Permanent magnet, libration damper
Status:	Nominal operations, first image received November 22nd, 2005. From December 2005, the images are showing some problems. The satellite is still operating nominally ultimo November 2007 (2 years after launch)
Links:	http://www.space.t.u-tokyo.ac.jp/cubesat/index-e.html http://www.space.t.u-tokyo.ac.jp/gs/en/index.aspx

Single cubesat launched on Dec 16,2006

GeneSat-1 by Center for Robotic Exploration and Space Technologies, CA, USA	
Deployment Mechanism:	P-POD
Type:	Triple cubesat (4.6 kg)
Mission:	Perform experiment on E. Coli bacteria in space, first cubesat to carry a biological experiment.
AODC:	Permanent magnets, hysteresis rods
Status:	Nominal operations. 96-hour experiment completed succesfully on December 22nd, 2006
Links:	http://www.crestnrp.org/genesat1/ (Excellent website) http://genesat1.engr.scu.edu/log/opslog.htm (Mission status) http://directory.eoportal.org/pres_GeneSat1.html (Additional information)

Fourth Batch Launched on April 17,2007

CP4 by California Polytechnic Institute, USA	
Deployment Mechanism:	P-POD A#1
Type:	Single cubesat
Mission:	Second flight unit of CP2, which was destroyed during the previous DNEPR launch
AODC:	Same as CP2
Status:	CP4 has been heard from numerous ground stations, but is not responding to tele commands
Link:	http://polysat.calpoly.edu/ (General CalPoly Cubesat website)
AeroCube-2 by the Aerospace Corporation, USA	
Deployment Mechanism:	P-POD A#2
Type:	Single cubesat
Mission:	Similar to AeroCube-1, except added charging system for the Lithium batteries. Mission is to test a communication system and the system bus plus a suite of CMOS cameras done by Harvey Mudd College. The satellite has no deployables. Instead an omnidirectional patch antenna is used.
AODC:	None
Status:	Solar upconverter failed shortly after launch. Batteries dead.
Link:	http://www.aero.org (this is just a link to the company. No mission information available)
CSTB-1 (Cubesat Testbed 1) by The Boeing Company, USA	
Deployment Mechanism:	P-POD A#3

Type:	Single cubesat
Mission:	Testbed for components for future Boeing small-sat missions. Redundant radios, deployable antenna, various non-disclosed sensors
AODC:	Sun sensors, magnetometer and magnetorquers
Status:	Nominal operations
Links:	http://www.boeing.com (this is just a link to the company. No mission information available) http://www.boeing.com/news/frontiers/archive/2006/october/i_ids02.pdf

MAST by Tethers Unlimited, USA

Deployment Mechanism:	P-POD C
Type:	Triple cubesat (actually 3 tethered cubesats)
Mission:	Tether experiment. ~1 million USD for the entire program
AODC:	Unknown + GPS
Status:	Only have contact to one of the two modules
Links:	http://www.tethers.com/Missions.html (Mission info) http://www.tethers.com/MAST_Blog.html (Mission status)

CP3 by California Polytechnic Institute, USA

Deployment Mechanism:	P-POD B#1
Type:	Single cubesat
Mission:	Three-axis magnetorquing experiment
AODC:	Magnetometers and magnetorquers
Status:	Radio contact not yet established
Link:	http://polysat.calpoly.edu/ (General CalPoly Cubesat website)

CAPE-1 by University of Louisiana, USA

Deployment Mechanism:	P-POD B#2
Type:	Single cubesat
Mission:	Camera(?)
AODC:	Permanent magnet and hysteresis rods
Status:	Has power system problems and is semi-operational (battery appears to be dead; currently only operates in the sun). CW has been received by several ground stations. 9600 bps TM packets have not been received by any station
Links:	http://cape.louisiana.edu/ http://jonathanwagner.net/ Lots of pictures

Libertad-1 (Freedom 1) by University of Sergio Arboleda, Columbia

Deployment Mechanism:	P-POD B#3
Type:	Single cubesat
Mission:	Camera and transmission of one stanza of the Colombian national anthem. Note: Powered by primary batteries only. They will last for about 52 days. This is the first Colombian satellite
AODC:	GPS receiver, attitude determination/control unknown

Status:	Nominal operations
Links:	http://www.usergioarboleda.edu.co/proyecto_espacial/index.htm (mostly spanish) and http://www.universia.net.co/galeriadecientificos/noticiasdelacienciaencolombia/libertad1,primersatelitecolombiano4.html (spanish)

Other Student satellite missions (have a major contribution from students)

SEDSAT-1 (OSCAR-33) by University of Huntsville, AL, USA	
Launch:	October 24th, 1998 on a Delte II LV from Vandenberg, CA, USA into a 1054x543 km, low Earth orbit (inclination = 31.44°)
Type:	36 kg microsatellite, 35x35x30 cm3 with several protuberances (antennas etc).
Mission:	Provide multi-spectral remote sensing with a resolution of 200 m from an 800 km orbit. Data provided to the public over the internet. Demonstrate a new ADCS. Reprogrammability of software components.
AODC:	Image processing, AshTech 12-ch GPS receiver, and magnetorquers
Status:	Problems with receivers meant that the imaging system was never used.
Link:	http://www.seds.org/sedsat
SSETI Express by many European universities, supported by ESA	
Launch:	October 27th, 2005 on a Kosmos 3M LV from Plesetsk, Russia into a 686x686, sun synchronous orbit (inclination = 98°) with a local time of 10:30
Type:	Satellite build by students from 10 European universities, supported by ESA
Mission:	MEMS sun sensors (made for DTUsat), color camera (made for AAUsat), propulsion system, Cubesat deployment
AODC:	Sun-sensors, magnetometer, semi-passive magnetic stabilization and a cold-gas payload propulsion system (Nitrogen)
Status:	Contact established Problems with the excess power dissipation system causes insufficient power for battery charging. The cause seems to be a short-circuited MOSFET. Ground tests have shown that it will open-circuit (2nd fail) after about 100 cycles, which should have happend in the middle of November 2005. Contact lost within 12 hours after launch Check http://www.express.space.aau.dk/?language=en&page=news for most up-to-date status
Links:	http://www.esa.int/ssetiexpress http://sseti.gte.tuwien.ac.at/express/mop/ (follow related site links from here)
HIT-SAT by Hokkaido Institute of Technology, Japan	
Launch:	Launched September 22nd, 2006 on a M-V-7 from Uchinoura Space

	Center, Japan into a 600 x 250 km sun-synchronous orbit (inclination 98°)
Type:	2.2 kg, 12 inch cube
Mission:	Test of spacecraft bus
AODC:	Sun-sensors, magnetometer, gyro, and magnetorquers
Status:	Operational, problems with the power subsystem in the end of October 2006, telemetry received October 21st, 2007 by HAM
Links:	http://www.hit.ac.jp/~satori/hitsat/index-e.html http://www.dk3wn.info/sat/afu/sat_hitsat.shtml (German)
RAFT1 and MARScom by US Naval Academy Satellite Lab, USA	
Launch:	The satellites were deployed from the Space Shuttle Discovery during STS-116 mission on December 21st, 2006 Initial altitude of 170 to 185 km, resulting in a lifetime of 75 to 200 days
Type:	Two 5" cubes, 3 (MARScom) and 4 (RAFT1) kg
Mission:	The satellites will be the first of their size with the ability to be tracked by the Navy Space Surveillance (NSSS) radar fence. The satellites will also function as amateur radio transponders
AODC:	Permanent magnet
Status:	RAFT1: Nominal Operations MARScom: No information available
Links:	http://web.ew.usna.edu/~raft/index.htm http://web.usna.navy.mil/~bruninga/raft.html http://web.usna.navy.mil/~bruninga/ande-raft-ops.html

2.2 Compilation of the available data:

Table 1: Sensors

Sr. No	Sensor	No. of Satelliets
1	Sun Sensors and Magnetometer	6
2	GPS receiver	3
3	Magnetometer (only)	2
4	Sun Sensors, Magnetometer and Gyroscope	1
5	Gyroscope and Magnetometer	1
6	Sun sensor and by measuring input power of solar panels	1
7	Piezoelectric vibrating gyroscope, dual axis accelerometer, and CMOS sun sensor	1
8	CMOS horizon sensor, star-tracker, magnetometer; and GPS receiver	1
9	None	1

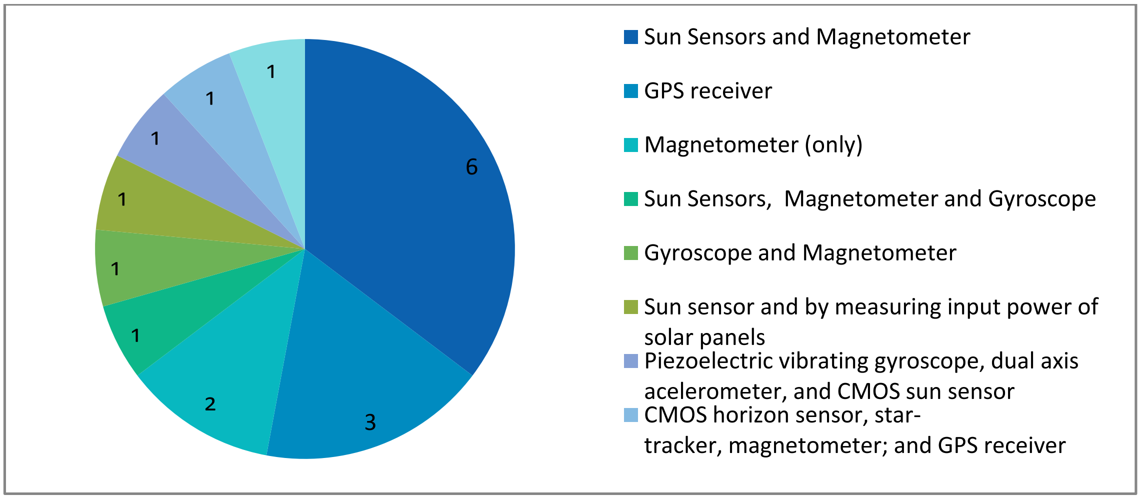
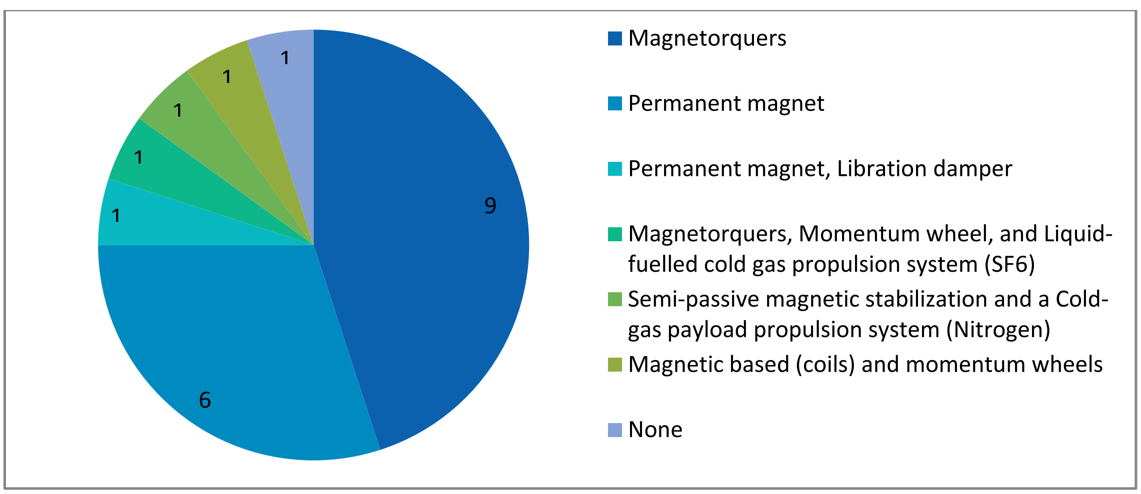


Table 2: Actuators

Sr.No.	Actuator	No. of Satelliets
1	Magnetorquers	9
2	Permanent magnet	6
3	Permanent magnet, Libration damper	1
4	Magnetorquers, Momentum wheel, and Liquid-fuelled cold gas propulsion system (SF6)	1
5	Semi-passive magnetic stabilization and a Cold-gas payload propulsion system (Nitrogen)	1
6	Magnetic based (coils) and momentum wheels	1
7	None	1



2.3 Initial choice of sensors and actuators

Based on the literature survey we finalized upon the following sensors and actuators:

Position Determination	GPS
Attitude Determination	Magnetometer + sun-sensor
Attitude Control	3 axis Magnetorquer

With these as the choice of sensors and actuators we expected to achieve following accuracy-

Position Determination	10 m
Attitude Determination	0.5 deg
Attitude Control	1 deg

Reference: http://mtech.dk/thomsen/space/cubesat.php

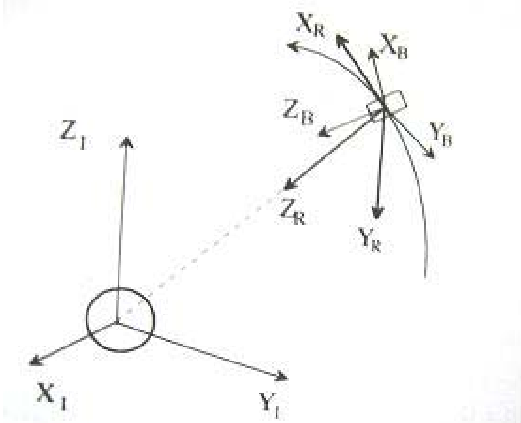
Chapter 3

Modeling the Satellite Dynamics

3.1 Frames of reference

3.1.a A Brief Description:

A brief description of the frames of reference used in the simulation is as given:



a) Inertial Frame (I)
   - X → First point of Aries
   - Z → Axis of rotation of Earth
   - Y → Z × X

b) Orbit Reference Frame (R)
   - X → In the plane of orbit along direction of motion
   - Z → Towards Earth
   - Y → Z × X

c) Body Frame
   - Along the principal body axes

3.1.b Conversion between various frames of reference:

a) Orbit Frame and Body Frame

   Φ, θ, Ψ are Euler angles of body with respect to orbit frame

$$T_{BR} = \begin{bmatrix} \cos\theta\cos\Psi & \cos\theta\sin\Psi & -\sin\theta \\ -\cos\theta\sin\Psi + \sin\Phi\sin\theta\cos\Psi & \cos\Phi\cos\Psi + \sin\Phi\sin\theta\sin\Psi & \sin\Phi\cos\theta \\ \sin\Phi\sin\Psi + \cos\Phi\sin\theta\cos\Psi & -\sin\Phi\cos\Psi + \cos\Phi\sin\theta\sin\Psi & \cos\Phi\cos\theta \end{bmatrix}$$

b) Inertial Frame and Orbit Frame

   Given the orbital elements with
   - Ω = Longitude of the ascending node
   - i = Inclination
   - ω = Argument of periapsis

- θ = True anomaly

Following rotations are required

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = A_X(-\frac{\pi}{2})A_Z(\frac{\pi}{2})A_Z(\omega+\theta)A_X(i)A_Z(\Omega)\begin{bmatrix} X_I \\ Y_I \\ Z_I \end{bmatrix}$$

$$T_{RI} = \begin{bmatrix} -\sin(\omega+\theta)\cos\Omega - \cos i \sin\Omega\cos(\omega+\theta) & -\sin(\omega+\theta)\sin\Omega + \cos(\omega+\theta)\cos i\cos\Omega & \sin(\omega+\theta)\sin i \\ -\sin i\sin\Omega & \sin i\cos\Omega & -\cos i \\ -\cos(\omega+\theta)\cos\Omega + \cos i\sin(\omega+\theta)\sin\Omega & -\cos(\omega+\theta)\sin\Omega - \sin(\omega+\theta)\cos i\cos\Omega & -\sin(\omega+\theta)\sin i \end{bmatrix}$$

3.2 Attitude Dynamics and Kinematics

1) Given a vector A and frames B,I

   $\frac{d\vec{A}}{dt}_I = \frac{d\vec{A}}{dt}_B + \vec{\omega}_{BI} \times \vec{A}$

   $\vec{\omega}_{BI}$ → angular velocity of B wrt. I

2) Angular momentum

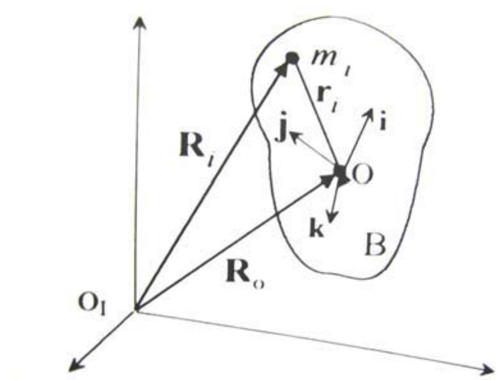


Fig. 1: Body and Inertial frames

In the figure above point O is the center of mass of the body and is also the origin of the body frame

i, j, k → body frame unit vectors

Therefore we have

$\vec{R}_i = \vec{R}_o + \vec{r}_i$

$\dot{\vec{R}}_i|_{IB} = \dot{\vec{R}}_o|_{IB} + \dot{\vec{r}}_I|_{BB} + \vec{\omega}_{BIB} \times \vec{r}_{iB}$

Where $\dot{\vec{R}}_i|_{IB}$ = rate of change of R_i wrt. inertial frame resolved in body frame and so on.

This can also be written as-

$\dot{\vec{R}}_i = \vec{V}_o + \vec{V}_i + \vec{\omega}_{BI} \times \vec{r}_I$

Since satellite is a rigid body, so $\vec{V}_i = 0$

Angular momentum of the body is given by

$dL = dm\,(\vec{r}_i \times \dot{\vec{R}}_i)$

$\vec{L} = \int_V dm\,[\vec{r}_i \times (\vec{V}_o + \vec{\omega}_{BI} \times \vec{r}_i)]$

$= \int_V \vec{r}_i \times (\vec{\omega}_{BI} \times \vec{r}_i)\, dm$ since $\int_V \vec{r}_i\, dm$ = C.M. = 0

Resolving in body frame

$\vec{L} = [I]\,\vec{\omega}_{BIB}$ where, $\vec{\omega}_{BIB}$ is the angular velocity of B wrt I resolved in B and

$$[I] = \begin{bmatrix} I_{XX} & -I_{XY} & -I_{XZ} \\ -I_{YX} & I_{YY} & -I_{YZ} \\ -I_{ZX} & -I_{ZY} & I_{ZZ} \end{bmatrix}$$

Similarly we can also derive the expression for Kinetic energy of the satellite

$E_{KE} = E_{Translational} + E_{Rotational}$

$E_{Translational} = \frac{1}{2} m\, {V_o}^2$

$E_{Rotational} = \frac{1}{2}\, \omega^T [I]\, \omega$

3)Rate of change of Angular Momentum

$\vec{M} = \frac{d\vec{L}}{dt}\,|_I$ $\vec{L}_{IB} = [I]\,\vec{\omega}_{BI}$

$\frac{d\vec{L}}{dt}\,|_I = \frac{d\vec{L}}{dt}\,|_B + \vec{\omega}_{BI} \times \vec{L}$

$\vec{M} = [I]\,\dot{\vec{\omega}} + \vec{\omega} \times [I]\,\vec{\omega}$

If principle axes coincide with body axes then we get

$M_{xB} = I_x\dot{\omega}_x + \omega_y\omega_z(I_z - I_y)$

M_{yB} = $I_y\dot{\omega}_y$ + $\omega_x\omega_z(I_x - I_z)$

M_{zB} = $I_z\dot{\omega}_z$ + $\omega_x\omega_y(I_y - I_x)$

ω_x, ω_y, ω_z are $\vec{\omega}_{BIB}$'s components.

Attitude Kinematics

Euler angles are defined wrt reference frame (R)

$\vec{\omega}_{BI} = \vec{\omega}_{BR} + \vec{\omega}_{RI}$

$\vec{\omega}_{BRB} = [\, p\ q\ r]$

$\vec{\omega}_{BIB} = [\, \omega_x\ \omega_y\ \omega_z]$

Following 3-2-1 system we get the following transformation matrix

$$T_{BR} = \begin{bmatrix} \cos\theta\cos\Psi & \cos\theta\sin\Psi & -\sin\theta \\ -\cos\Phi\sin\Psi + \sin\Phi\sin\theta\cos\Psi & \cos\Phi\cos\Psi + \sin\Phi\sin\theta\sin\Psi & \sin\Phi\cos\theta \\ \sin\Phi\sin\Psi + \cos\Phi\sin\theta\cos\Psi & -\sin\Phi\cos\Psi + \cos\Phi\sin\theta\sin\Psi & \cos\Phi\cos\theta \end{bmatrix}$$

Relation between $\vec{\omega}_{BRB}$ and $\dot{\Phi}, \dot{\Psi}, \dot{\theta}$

$$\vec{\omega}_{BRB} = A_\Psi A_\vartheta A_\Phi \begin{bmatrix} 0 \\ 0 \\ \dot{\Psi} \end{bmatrix} + A_\Phi A_\vartheta \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + A_\Phi \begin{bmatrix} \dot{\Phi} \\ 0 \\ 0 \end{bmatrix}$$

Solving we get

$\mathrm{p} = \dot{\Phi} - \dot{\Psi}\sin\theta$

$\mathrm{q} = \dot{\theta}\cos\Phi + \dot{\Psi}\sin\Phi\cos\theta$

$\mathrm{r} = \dot{\Psi}\cos\Phi\cos\theta - \dot{\theta}\sin\Phi$

Solving for Euler angles we get

$\dot{\Phi}$ = p + (q$\sin\Phi + \mathrm{r}\cos\Phi$) $\tan\theta$

$\dot{\theta}$ = q$\cos\Phi - r\sin\Phi$

$\dot{\Psi}$ = (q$\sin\Phi + r\cos\Phi$) $\sec\theta$

To find $\vec{\omega}_{RIB}$

$\hat{k}_R = \frac{-\vec{r}}{|\vec{r}|}$

$\hat{j}_R$ = - $\frac{(\vec{r}\times\vec{v})}{|(\vec{r}\times\vec{v})|}$

$\hat{\iota}_R$ = $\hat{j}_R \times \hat{k}_R$

ω_j= - $\frac{di_R}{dt}$. $\hat{k}$

Assuming a circular orbit we get

$\omega_{RI\hat{\iota}}$ = - $\frac{d\hat{k}_R}{dt}$. $\hat{j}_R$ = - $\frac{1}{|\vec{r}|}(\frac{\vec{v}.\vec{v}\times\vec{r}}{|\vec{v}\times\vec{r}|})$ = o

$\omega_{RI\hat{j}}$ = - $\frac{d\hat{k}_R}{dt}$. $\hat{\iota}_R$ = - $\frac{\vec{v}}{|\vec{r}|}$. $(\frac{(\vec{v}\times\vec{r})\times(-\vec{r})}{|\vec{v}\times\vec{r}||\vec{r}|})$ = - $\frac{v}{r}$ = - ω_o

$\omega_{RI\hat{k}}$ = - $\frac{d\hat{j}_R}{dt}$. $\hat{\iota}_R$ = o

ω_{RI} = [o - ω_o o]^T

Therefore the equations of Kinematics are

$$\overrightarrow{\boldsymbol{\omega_{RIB}}} = \boldsymbol{T_{BR}}\overrightarrow{\boldsymbol{\omega_{RI}}}$$

$$=> \omega_{RIB} = \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix}\mathrm{p}\\ \mathrm{q}\\ \mathrm{r}\end{bmatrix} + T_{BR}\begin{bmatrix}0\\ -\omega_0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} + \omega_0\begin{bmatrix}\mathrm{c_\Theta s_\varphi}\\ \mathrm{c_\varphi c_{\psi+} s_\varphi s_\Theta s_\psi}\\ \mathrm{-s_\varphi c_{\psi+} c_\varphi s_\Theta s_\psi}\end{bmatrix}$$

Where,

c_Θ: $cos\Theta$

s_Θ: $sin\Theta$

Solving we get

p =ω_x + ω_0(cosΘ sin φ)

q=ω_y+ω_0 $(\cos\varphi\cos\psi + sin\varphi sin\Theta sin\,\psi)$

r=$\omega_z + \omega_0(-sin\,\varphi cos\,\psi + cos\varphi sin\,\Theta sin\,\psi)$

3.3 Disturbances

Gravity gradient torque is the most dominant of the torques acting on the satellite and hence for an initial modeling of the disturbances only gravity gradient has been modeled

Gravity gradient torque

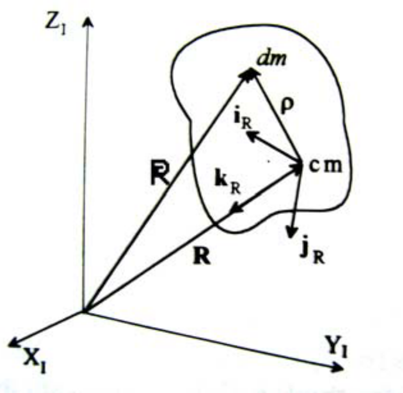


Fig. 1: Gravity Gradient Torque

$\vec{R} = \vec{R}_o + \vec{\rho}$

$$\overrightarrow{dF} = \frac{-\mu dm}{|\vec{R}|^3}\,\vec{R}$$

$$\overrightarrow{d\tau} = \frac{-\mu dm}{|\vec{R}|^3}\,\vec{\rho} \times \vec{R}$$

Now,

$$\frac{1}{|\vec{R}|^3} = \frac{1}{|\vec{R}_o + \vec{\rho}|^3} \approx \frac{1}{|\vec{R}|^3}[1 - \frac{3\overrightarrow{R_0} \cdot \vec{\rho}}{|\vec{R}_o|^2}]$$

Therefore,

$$\overrightarrow{d\tau} = \frac{-\mu dm}{\left|\vec{R}_o\right|^3}\,(\,\vec{\rho} \times \vec{R}_o - \frac{3\overrightarrow{R_0} \cdot \vec{\rho}}{\left|\vec{R}_o\right|^2}\vec{\rho} \times \vec{R}_o)$$

$$\therefore \tau = \frac{3\mu}{\left|\vec{R}_o\right|^5}\int (\overrightarrow{R_0} \cdot \vec{\rho})(\vec{\rho} \times \vec{R}_o)\,dm$$

Solving in body frame and integrating, we get

$$J_x = \frac{3\mu}{2{R_o}^3}\,(I_z - I_y)\sin 2\varphi\cos^2\theta$$

$$J_y = \frac{3\mu}{2{R_o}^3}\,(I_z - I_x)\sin 2\theta\; cos\,\varphi$$

$$J_z = \frac{3\mu}{2{R_o}^3}\,(I_x - I_y)\sin 2\theta\; sin\,\varphi$$

3.4 Control Torques

Magnetic control torque

$$\vec{\tau} = \vec{m} \times \vec{B}$$

m = magnetic moment

B = magnetic field

If B_{x_I}, B_{y_I}, B_{z_I} are the field values in I then

$$\begin{bmatrix} B_{x_R} \\ B_{y_R} \\ B_{z_R} \end{bmatrix} = T_{RI} \begin{bmatrix} B_{x_I} \\ B_{y_I} \\ B_{z_I} \end{bmatrix}$$

Also,

$$\begin{bmatrix} B_{x_B} \\ B_{y_B} \\ B_{z_B} \end{bmatrix} = T_{BR} \begin{bmatrix} B_{x_R} \\ B_{y_R} \\ B_{z_R} \end{bmatrix}$$

$$\vec{\tau} = \vec{m} \times \vec{B} = -\vec{B} \times \vec{m} = -\tilde{B}m$$

$$\begin{bmatrix} T_{xB} \\ T_{yB} \\ T_{zB} \end{bmatrix} = \begin{bmatrix} 0 & B_{z_B} & -B_{y_B} \\ -B_{z_B} & 0 & B_{x_B} \\ B_{y_B} & -B_{x_B} & 0 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

3.5 Control Law

The following control laws are being used on the satellite

Nominal Operational

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} -k1 & 0 & 0 \\ 0 & -k2 & 0 \\ 0 & 0 & -k3 \end{bmatrix} \begin{bmatrix} 0 & B_{z_B} & -B_{y_B} \\ -B_{z_B} & 0 & B_{x_B} \\ B_{y_B} & -B_{x_B} & 0 \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} -K1 & 0 & 0 \\ 0 & -K2 & 0 \\ 0 & 0 & -K3 \end{bmatrix} \begin{bmatrix} 0 & B_{z_B} & -B_{y_B} \\ -B_{z_B} & 0 & B_{x_B} \\ B_{y_B} & -B_{x_B} & 0 \end{bmatrix} \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix}$$

Detumbling

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} -k1 & 0 & 0 \\ 0 & -k2 & 0 \\ 0 & 0 & -k3 \end{bmatrix} \begin{bmatrix} \dot{B_{x_B}} \\ \dot{B_{y_B}} \\ \dot{B_{z_B}} \end{bmatrix}$$

3.6 Linearised Equations

1) J → ω
   $M_{xB} = I_x\dot{\omega}_x + \omega_y\omega_z(I_z - I_y)$
   $M_{yB} = I_y\dot{\omega}_y + \omega_x\omega_z(I_x - I_z)$
   $M_{zB} = I_z\dot{\omega}_z + \omega_x\omega_y(I_y - I_x)$

2) ω → pqr
   $p = \omega_x + \omega_o(\cos\theta\sin\Psi)$
   $q = \omega_y + \omega_o\,(\cos\Phi\cos\Psi + \sin\Phi\sin\theta\sin\Psi)$
   $r = \omega_z + \omega_o\,(-\sin\Phi\cos\Psi + \cos\Phi\sin\theta\sin\Psi)$

   $therefore$
   $p \approx \omega_x + \omega_o\psi$
   $q \approx \omega_y + \omega_o$
   $r \approx \omega_z - \omega_o\phi$

3) pqr → $\theta\psi\phi$

$\dot{\Phi}$ = p + (q$\sin\Phi + \mathrm{r}\cos\Phi)\tan\theta$

$\dot{\theta}$ = q$\cos\Phi - r\sin\Phi$

$\dot{\Psi}$ = (q$\sin\Phi + r\cos\Phi)\sec\theta$

$\dot{\Phi} \approx$ p + rθ

$\dot{\theta} \approx$ q$-r\varphi$

$\dot{\Psi} \approx$ q$\varphi + r$

Solving equations in (3) we get approximately,

$$\dot{\Phi} \approx p$$

$$\dot{\theta} \approx q$$

$$\dot{\Psi} \approx r$$

Substituting in equations of (2) we get,

$$\omega_x \approx \dot{\Phi} - \omega_o\psi$$

$$\omega_y \approx \dot{\theta} - \omega_o$$

$$\omega_z \approx \dot{\Psi} + \omega_o \phi$$

Substituting in equations of (1) we get,

$$M_{xB} = I_x \ddot{\varphi} + \omega_o (I_y - I_z - I_x)\dot{\Psi} + \omega_o{}^2 (I_y - I_z)\varphi$$
$$M_{yB} = I_y \ddot{\theta}$$
$$M_{zB} = I_z \ddot{\Psi} + \omega_o (I_x + I_z - I_y)\dot{\Psi} + \omega_o{}^2 (I_y - I_x)\Psi$$

Linearized equation of gravity gradient torque

$$G_x = 3\omega_o{}^2 (I_z - I_y)\varphi$$

$$G_y = 3\omega_o{}^2 (I_z - I_x)\theta$$

$$G_z = 0$$

Linearised equation for magnetic torque

$$\begin{bmatrix} T_{xB} \\ T_{yB} \\ T_{zB} \end{bmatrix} = \begin{bmatrix} 0 & B_{z_B} & -B_{y_B} \\ -B_{z_B} & 0 & B_{x_B} \\ B_{y_B} & -B_{x_B} & 0 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

$$\begin{bmatrix} B_{xB} \\ B_{yB} \\ B_{zB} \end{bmatrix} = \begin{bmatrix} 0 & \Psi & -\theta \\ -\Psi & 0 & \varphi \\ \theta & -\varphi & 0 \end{bmatrix} \begin{bmatrix} B_{xR} \\ B_{yR} \\ B_{zR} \end{bmatrix}$$

Substituting we get:

$$\tau_{xB} = m_y B_{zR} - m_z B_{yR} + m_y B_{xR}\Theta - m_z B_{zR}\varphi - m_y B_{yR}\varphi + m_z B_{xR}\psi$$

$$\tau_{yB} = m_z B_{xR} - m_x B_{zR} + m_z B_{yR}\psi - m_x B_{xR}\Theta - m_z B_{zR}\Theta + m_x B_{yR}\varphi$$

$$\tau_{zB} = m_x B_{yR} - m_y B_{xR} + m_x B_{zR}\varphi - m_y B_{yR}\psi - m_x B_{xR}\psi + m_y B_{zR}\Theta$$

Since, we are linearizing about zero attitude and zero control moment therefore terms which have a multiplication of attitude and magnetic moment can be assumed to be zero.

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} 0 & B_{z_R} & -B_{y_R} \\ -B_{z_R} & 0 & B_{x_R} \\ B_{y_R} & -B_{x_R} & 0 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

Here B refers to Magnetic field resolved in orbit frame with B_x, B_y, B_z as its components.
Substituting the value of magnetic moment from the expression of control torque we get and writing them in state space form we get,

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ \ddot{\phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix}=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\-4{\omega_o}^2\sigma_x&0&0&0&0&\omega_o(1-\sigma_x)\\0&-3{\omega_o}^2\sigma_y&0&0&0&0\\0&0&-{\omega_o}^2\sigma_z&-\omega_o(1-\sigma_z)&0&0\end{bmatrix}\begin{bmatrix}\phi\\ \theta\\ \psi\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\0&\frac{B_{zB}}{I_x}&\frac{-B_{yB}}{I_x}\\ \frac{-B_{zB}}{I_y}&0&\frac{B_{xB}}{I_y}\\ \frac{B_{yB}}{I_z}&\frac{-B_{xB}}{I_z}&0\end{bmatrix}\begin{bmatrix}m_x\\ m_y\\ m_z\end{bmatrix}$$

This is of the form,

$$\dot{X}=AX+BU$$

$$X=\begin{bmatrix}\phi\\ \theta\\ \psi\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}$$

$$A=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\-4{\omega_o}^2\sigma_x&0&0&0&0&\omega_o(1-\sigma_x)\\0&-3{\omega_o}^2\sigma_y&0&0&0&0\\0&0&-{\omega_o}^2\sigma_z&-\omega_o(1-\sigma_z)&0&0\end{bmatrix}$$

$$B=\begin{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\\ \begin{bmatrix}0&\frac{B_{zB}}{I_x}&\frac{-B_{yB}}{I_x}\\ \frac{-B_{zB}}{I_y}&0&\frac{B_{xB}}{I_y}\\ \frac{B_{yB}}{I_z}&\frac{-B_{xB}}{I_z}&0\end{bmatrix}\end{bmatrix}$$

$$\sigma_x=\frac{I_y-I_z}{I_x}$$

$$\sigma_y=\frac{I_x-I_z}{I_y}$$

$$\sigma_z=\frac{I_y-I_x}{I_z}$$

$$\omega_o=\sqrt{\frac{GM}{R}}$$

These equations have been derived from the following linearized equations

$$\ddot{\phi}=-4{\omega_o}^2\sigma_x\phi+\omega_o(1-\sigma_x)\dot{\psi}+\frac{T_{dx}}{I_x}$$

$$\ddot{\theta}=-3{\omega_o}^2\sigma_y\theta+\frac{T_{dy}}{I_y},\qquad \ddot{\psi}=-{\omega_o}^2\sigma_z\psi-\omega_o(1-\sigma_z)\dot{\phi}+\frac{T_{dx}}{I_x}$$

Chapter 4

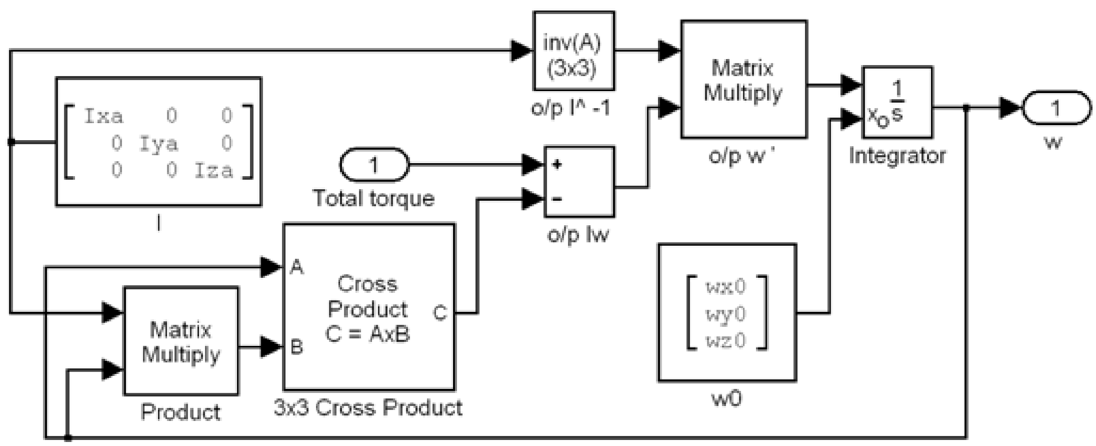
Simulation

4.1 System dynamics and Kinematics

The following subsystems model the dynamics and the kinematics of the satellite.

Torque to $\vec{\omega}_{BIB}$

This model uses the total torque acting on the system and gives use the angular velocity of the Body frame w.r.t. Inertial frame.



Inputs

- Total Torque (in body frame); $\vec{\tau} = [\, \tau_x \; \tau_y \; \tau_z]^T$ torques in body frame

Output

- Angular velocity of Body frame w.r.t. Inertial frame resolved in Body Frame; $\vec{\omega}_{BIB} = [\, \omega_x \; \omega_y \; \omega_z]^T$

Parameters

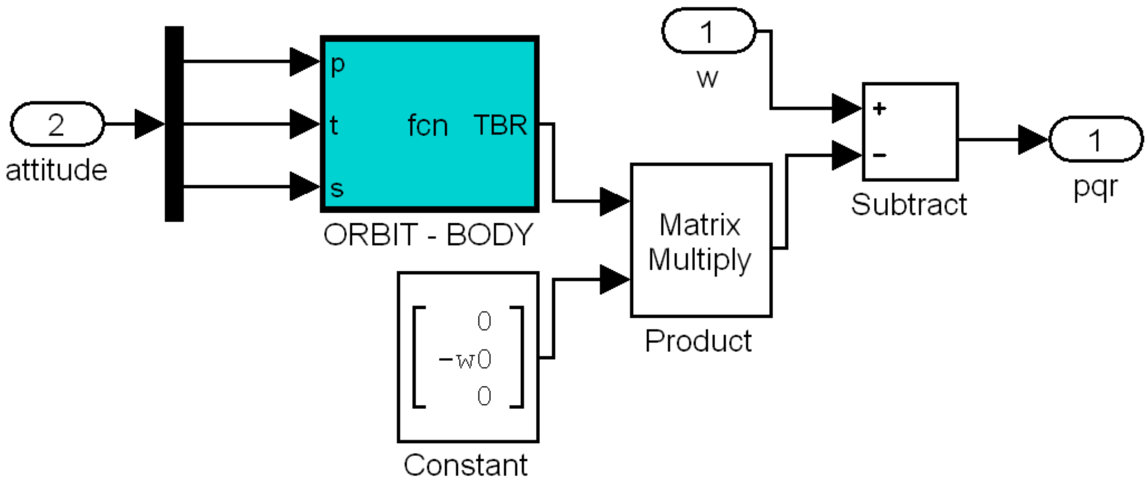
- Moment of inertia (Principal) $[I]$ = $\begin{bmatrix} I_{XX} & 0 & 0 \\ 0 & I_{YY} & 0 \\ 0 & 0 & I_{ZZ} \end{bmatrix}$

Equations

- $d\,\vec{\omega}_{BIB}/_{dt} = I^{-1}\big[\tau - [\vec{\omega}_{BIB} \times I\vec{\omega}_{BIB}]\big]$

$\vec{\omega}_{BIB}$ to $\vec{\omega}_{BRB}$

This subsystem converts the angular velocity of the Body Frame w.r.t. Inertial Frame to angular velocity of the Body Frame w.r.t. Reference (Orbit) Frame.



Inputs

- Angular velocity of Body Frame w.r.t. Inertial Frame resolved in Body Frame; $\vec{\omega}_{BIB} = [\ \omega_x\ \omega_y\ \omega_z]^T$
- Attitude; $[\phi\ \theta\ \psi]^T$

Output

- Angular velocity of Body Frame w.r.t. Orbit Frame resolved in Body Frame; $\vec{\omega}_{BRB} = [p\ q\ r]^T$

Parameters

- Orbital angular velocity; ω_o

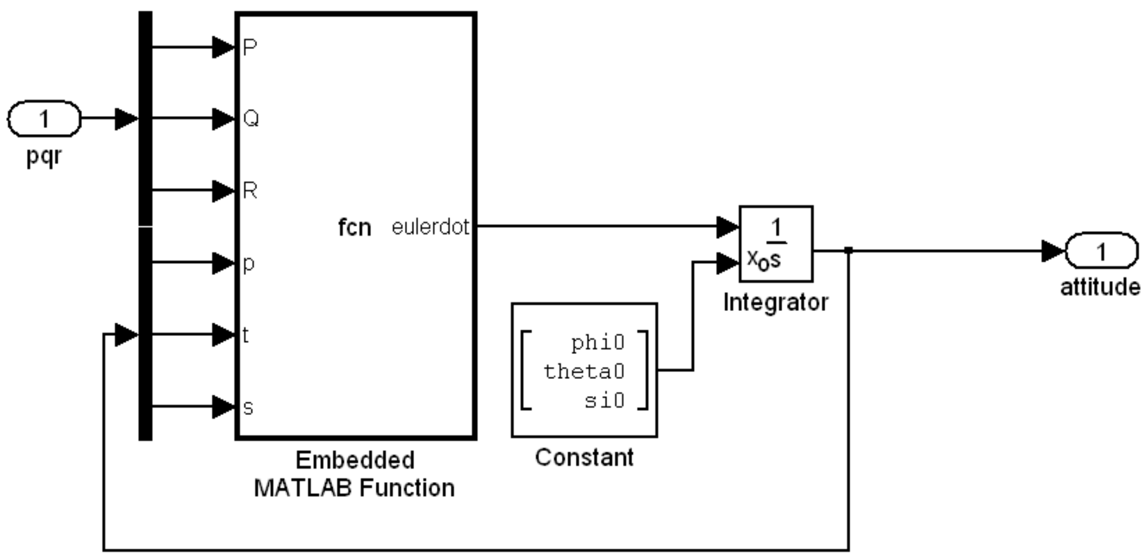
Equations

- $$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \omega_0 \begin{bmatrix} \cos\theta \sin\phi \\ \cos\phi \cos\psi + \sin\phi \sin\theta \sin\psi \\ -\sin\phi \cos\psi + \cos\phi \sin\theta \sin\psi \end{bmatrix}$$

4.2 Modeling the attitude dynamics in absence of external perturbations

$\vec{\omega}_{BRB}$ to Attitude

This subsystem first converts the angular velocity of the Body Frame w.r.t. Reference (Orbit) Frame to the time rate of the attitude which is then integrated to get the attitude.



Inputs

- Angular velocity of Body Frame w.r.t. Orbit Frame resolved in Body Frame; $\vec{\omega}_{BRB} = [p\ q\ r]^T$

Output

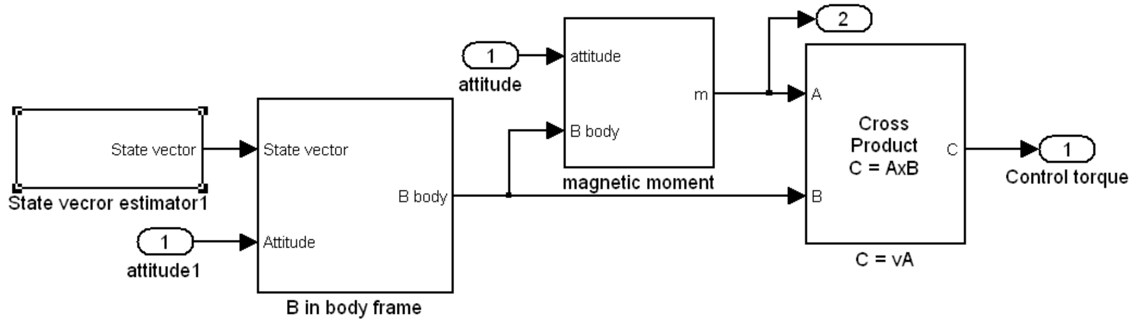
- Attitude; $[\phi\ \theta\ \psi]^T$

Equations

- $$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Control Torque

This subsystem models the magnetic torquers, giving the control torque.



Inputs

- Attitude; $[\phi\ \theta\ \psi]^T$

Output

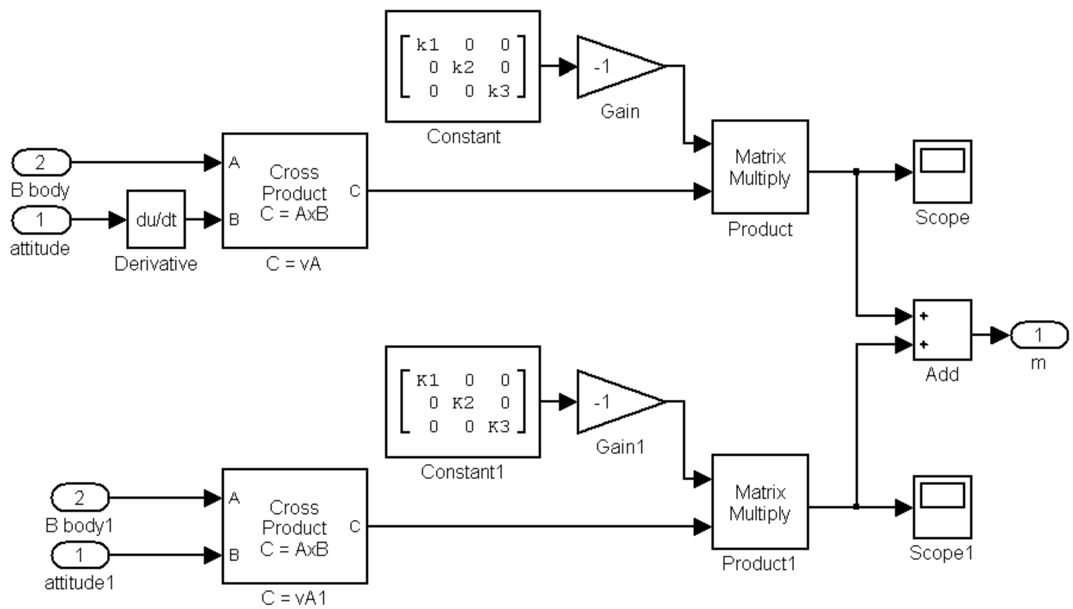
- Control Torques; $\overrightarrow{\tau_B} = [\ \tau_{xB}\ \tau_{yB}\ \tau_{zB}]^T$

Equations

- $$\begin{bmatrix}\tau_{xB}\\ \tau_{yB}\\ \tau_{zB}\end{bmatrix} = \begin{bmatrix}0 & B_{z_B} & -B_{y_B}\\ -B_{z_B} & 0 & B_{x_B}\\ B_{y_B} & -B_{x_B} & 0\end{bmatrix}\begin{bmatrix}m_x\\ m_y\\ m_z\end{bmatrix}$$

Control Law

The control law controls the satellite by changing magnetic moments of the Torquers. The first part of the control law is used for momentum dumping whereas the second part applies a restoring torque getting the satellite to the desired orientation.



Inputs

- Attitude; $[\phi\ \theta\ \psi]^T$
- Magnetic Field in Body Frame; B_B = $[B_{xB}\ B_{yB}\ B_{zB}]^T$

Output

- Torquer Moments; m = $[\ m_x\ m_y\ m_z]^T$

Parameters

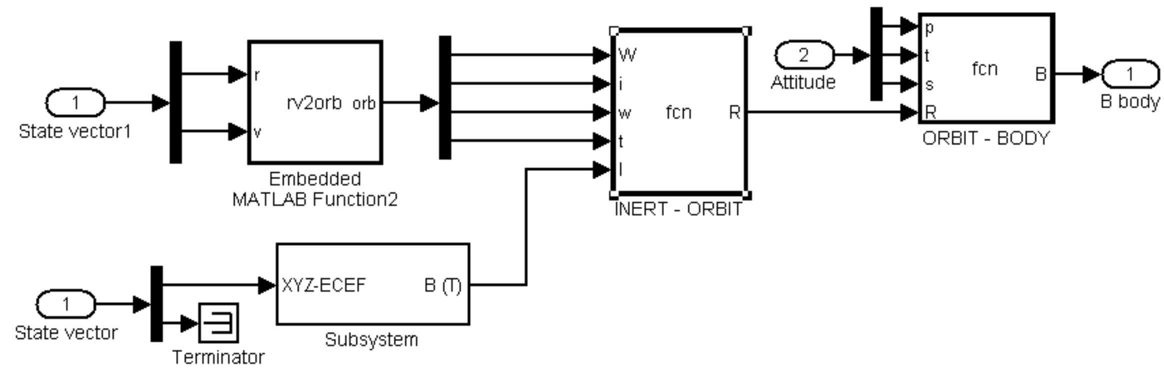
- Damping parameter; k = $\begin{bmatrix} k1 & 0 & 0 \\ 0 & k2 & 0 \\ 0 & 0 & k3 \end{bmatrix}$
- Stiffness Parameter; K = $\begin{bmatrix} K1 & 0 & 0 \\ 0 & K2 & 0 \\ 0 & 0 & K3 \end{bmatrix}$

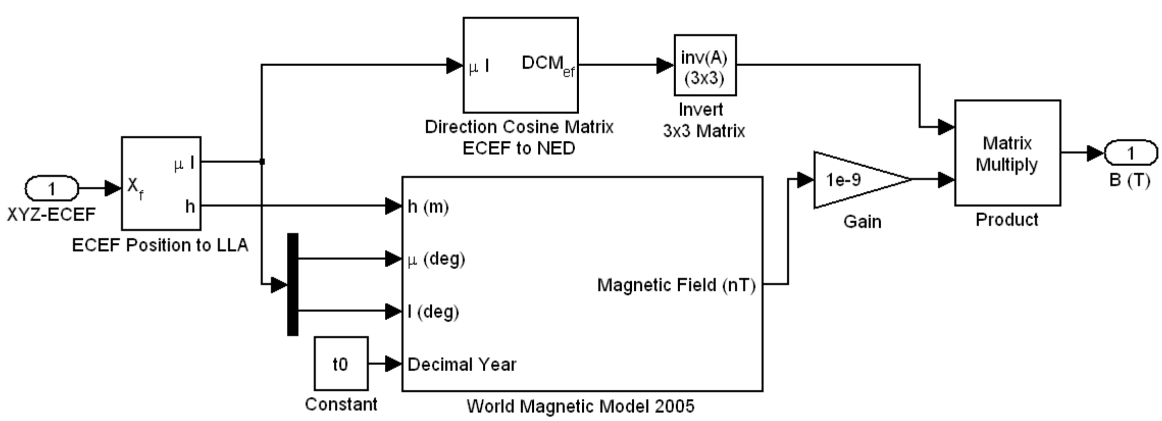
Equation

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = -\begin{bmatrix} k1 & 0 & 0 \\ 0 & k2 & 0 \\ 0 & 0 & k3 \end{bmatrix} \begin{bmatrix} 0 & B_{z_B} & -B_{y_B} \\ -B_{z_B} & 0 & B_{x_B} \\ B_{y_B} & -B_{x_B} & 0 \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} K1 & 0 & 0 \\ 0 & K2 & 0 \\ 0 & 0 & K3 \end{bmatrix} \begin{bmatrix} 0 & B_{z_B} & -B_{y_B} \\ -B_{z_B} & 0 & B_{x_B} \\ B_{y_B} & -B_{x_B} & 0 \end{bmatrix} \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix}$$

Magnetic Field in Body Frame

This subsystem gives the local magnetic field in the body co-ordinates. This is required to calculate the magnetic moment in the previous block.





Using Matlab®'s "World Magnetic Model 2005" model for Earth's magnetic field we get B_{xyz} in the North-East down (NED) coordinate system, which we need to convert it into inertial coordinate using Matlab®'s "Direction Cosine Matrix ECEF to NED" block.

Two frame changes (being Inertial frame to Orbit frame and Orbit frame to Body Frame) follow, giving magnetic field projected in body frame.

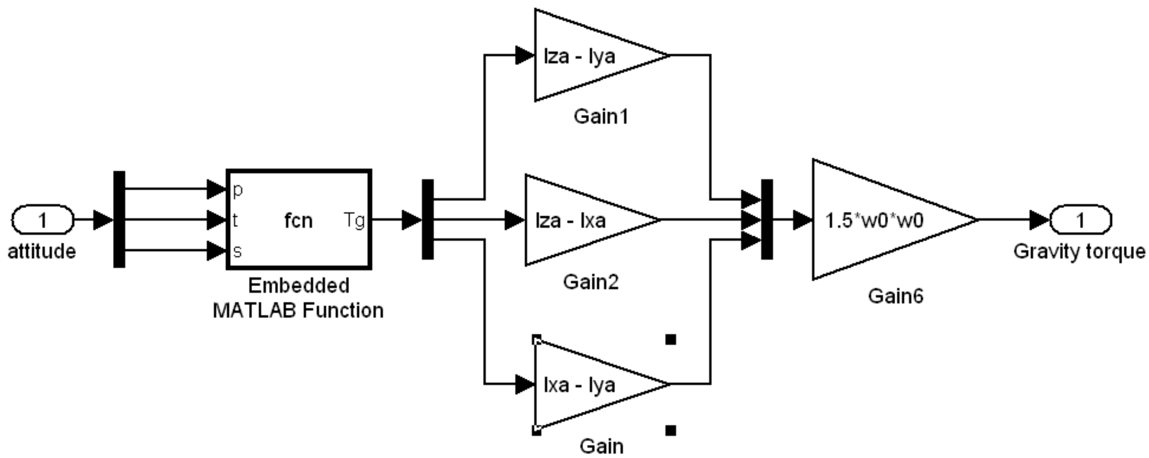
The World Magnetic Model 2005 take the Geodetic LLA (Latitude, Longitude and Altitude) and Decimal Year; and gives the Local magnetic field in the NED (North-East-Down) co-ordinate system.

4.3 Modeling the Environmental perturbations

Following is the model of the environment of the Satellite orbit.

Gravity gradient torques

This subsystem gives us the gravity gradient torques acting on the satellite.



Inputs

- Attitude; $[\phi\ \theta\ \psi]^T$

Output

- Gravity Gradient Torques; $\overrightarrow{\tau_G} = [\ \tau_{xG}\ \tau_{yG}\ \tau_{zG}]^T$

Parameters

- Moment of inertia (Principal) $[I]$ = $\begin{bmatrix} I_{XX} & 0 & 0 \\ 0 & I_{YY} & 0 \\ 0 & 0 & I_{ZZ} \end{bmatrix}$
- Orbital angular velocity, ω_o

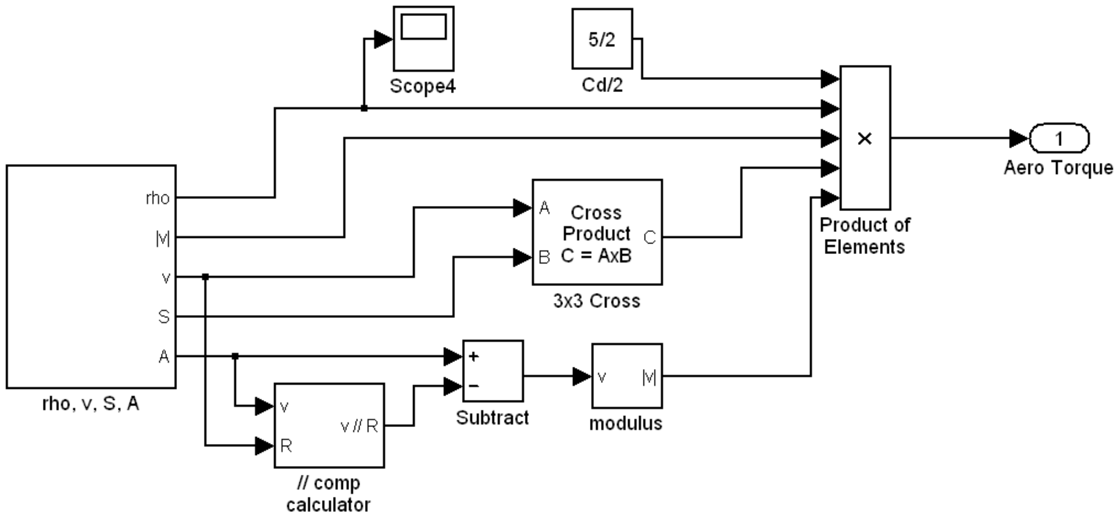
Equation

- $J_x = \frac{3}{2}\ \omega_o^2 (I_{ZZ} - I_{YY}) \sin 2\varphi \cos^2 \theta$
- $J_y = \frac{3}{2} \omega_o^2\ (I_{ZZ} - I_{XX}) \sin 2\theta\ cos\ \varphi$
- $J_z = \frac{3}{2}\ \omega_o^2 (I_{XX} - I_{YY}) \sin 2\theta\ sin\ \varphi$

4.4 Modeling with the environmental perturbations applied to the system

Aerodynamic Torques

This subsystem gives the torques acting on the satellite due to the rarified gases in the LEO (Low Earth Orbit).



Inputs

- Orbital State Vector; v = $[\, v_x\ v_y\ v_z]^T$, r = $[\, x\ y\ z]^T$

Output

- Aerodynamic Torques; $\overrightarrow{\tau_A} = [\, \tau_{xA}\ \tau_{yA}\ \tau_{zA}]^T$

Parameters

- Co- efficient of Drag; C_d
- Position of Center of pressure w.r.t. Center of mass; S_{CP}

Equation

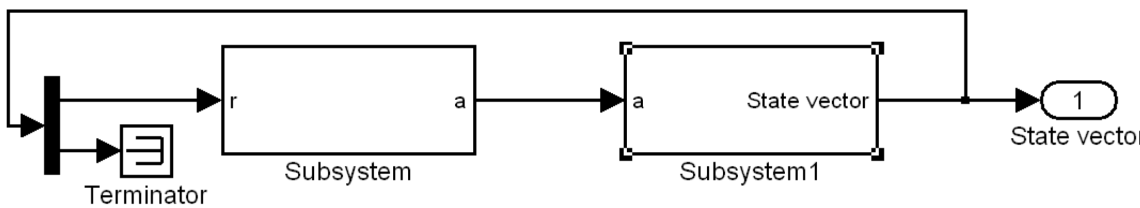
- Using Matlab®'s "NRLMSISE-00", an empirical, global model of the Earth's atmosphere, and $\vec{r}$ we get the local air density; ρ.
- $\vec{\tau}_A = \frac{1}{2}\rho\vec{v}^2\left|\vec{A} - \vec{A}\cdot\hat{v}\right|\hat{v} \times \vec{S}_{CP}$

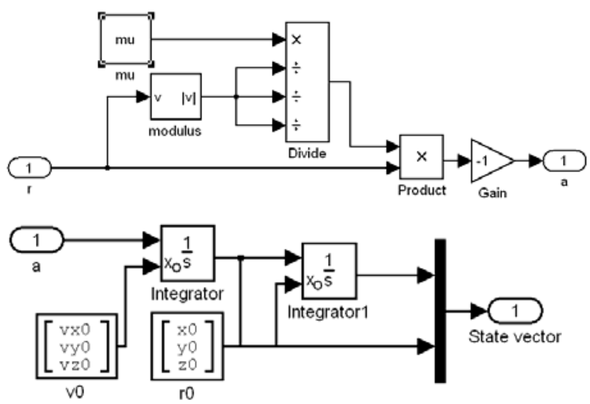
Chapter 5

Orbital State Vector Estimator

5.1 Orbit Estimation-need

This subsystem estimates the orbital state vector which is required for estimating the position and velocity of the satellite for other blocks such as Matlab®'s "World Magnetic Model 2005", Aerodynamic Torques etc.





Output

- Orbital State Vector; v = $[\, v_x \; v_y \; v_z]^T$, r = $[\, x \; y \; z]^T$

Parameters

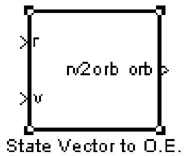
- Standard gravitational parameter; μ = $3.986004418 \times 10^{14}$

Equations

- $\ddot{r} = \frac{\mu}{|r|^2}\hat{r}$
- $v = \dot{r}$

5.2 Orbital State Vector to Orbital Elements

This subsystem converts the Orbital State Vector to the Orbital Elements required for the conversion of the Inertial frame to the Orbit frame.



Inputs

- Orbital State Vector; v = $[\ v_x\ v_y\ v_z]^T$, r = $[\ x\ y\ z]^T$

Output

- Orbital Elements; $[a\ e\ \Omega\ i\ \omega\ \nu]$

Parameters

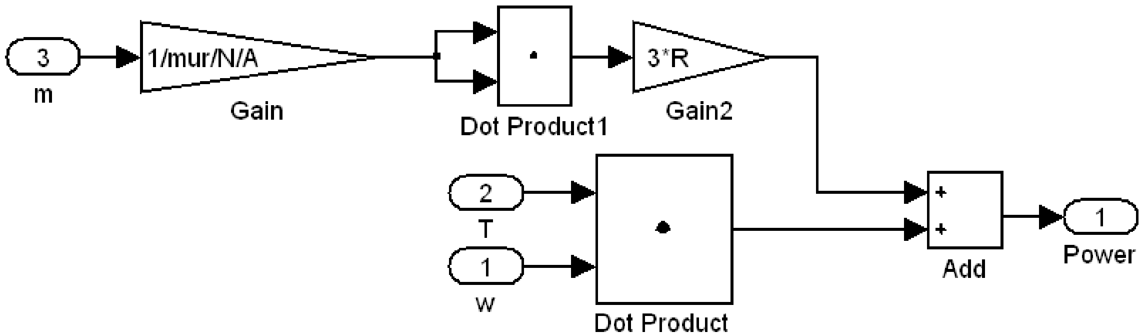
- Standard gravitational parameter; μ = 3.986004418×10^{14}

Equations

- $\vec{h} = \vec{r} \times \vec{v}$
- $a = 1 \Big/ \left(\frac{2}{\hat{r}} - \frac{\hat{v}^2}{\mu}\right)$
- $\vec{e} = \frac{\vec{v} \times \vec{h}}{\mu} - \hat{r}$
- $e = |\vec{e}|$
- $\Omega = atan2(h_x, h_y)$
- $i = \cos^{-1}\left(h_z \Big/ |\vec{h}| \right)$
- $\omega = atan2\left(e_z \Big/ \sin i , \left(\frac{e_x}{\cos \Omega} + \frac{\tan \Omega}{\tan i} \right) \right)$
- $\nu = \cos^{-1}\left(\vec{e} \cdot \vec{r} \Big/ |e||r| \right)$

5.3 Estimation of power consumption of Torquers

This subsystem estimates the power consumed by the Torquers.



Inputs

- Angular velocity of Body Frame w.r.t. Inertial Frame resolved in Body Frame; $\vec{\omega}_{BIB} = [\ \omega_x\ \omega_y\ \omega_z]^T$
- Control Torques; $\overrightarrow{\tau_B} = [\ \tau_{xB}\ \tau_{yB}\ \tau_{zB}]^T$
- Torquer Moments; m = $[\ m_x\ m_y\ m_z]^T$

Output

- Power consumed by the Torquer; P

Parameters

- Electrical resistance of the Torquer Coils; R
- Loop area of Torquer Coil; A_C
- Number of turns in the Torquer Coil; N_C
- Electrical Efficiency of the Torquer; η

Equations

- m = i.N_C.A_C
- $P = \left({i^2R}/{\eta}\right) + \tau \cdot \omega$

Chapter 6

Choice of Sensors and Actuators

6.1 Initial choice of sensors and actuators

1. Sun Sensor

a) Design of the Sun position model

Frames of reference:

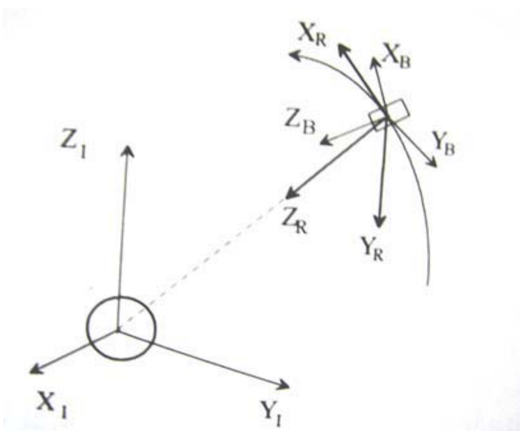


Fig. 1: Frames of reference

1) Inertial Frame (I)
   X → First point of Aries
   Z → Axis of rotation of Earth
   Y → Z × X

2) Orbit Reference Frame (R)
   X → In the plane of orbit along direction of motion
   Z → Towards Earth
   Y → Positive to plane of the orbit

Calculations

On 21 March x- axes of both frames coincide.

M = mean anomaly

E = inclination

Sun vector in SO = $\begin{bmatrix}1\\0\\0\end{bmatrix}$

To transform,

$$X_I = T_{ISO}X_{SO}$$

$$= A_x(-t)A_z(-M)S_{SO}$$

$$X_I = \begin{bmatrix}\cos M\\ \cos t \sin M\\ \sin t \sin M\end{bmatrix}$$

$$M = \omega t$$

b) Given attitude of satellite, sun vector and position in ECI, to find the intensity of light on each face

$\vec{A}_i = A\hat{\imath}$ where i = x, -x, y, -y, z, -z

Let $\hat{S}$ be sun vector in body;

$$A_{i_\perp} = A(\hat{\imath} - \hat{\imath} \cdot \hat{s})$$

$$I_i = I_0 A_{i_\perp} = I_0 A(\hat{\imath} - \hat{\imath} \cdot \hat{s})$$

I_o = solar constant

The sun sensor SLSD-71N8

Specifications can be seen in detail in the datasheet.

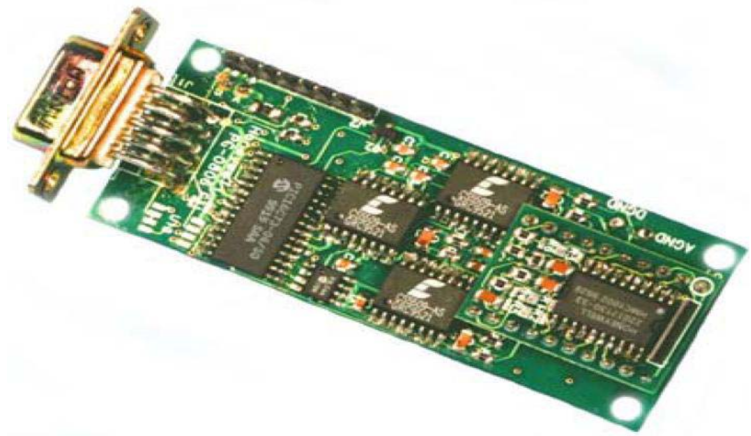
Brief Specs:

1) Short circuit current: 170 μA
2) Open circuit voltage: 0.4 V
3) Junction capacitance: 100pF
4) Reverse dark current: 1.7 μA
5) Spectral sensitivity: 0.55 A/W
6) Acceptance half angle: 60 deg

2. Magnetometer

Section 1: Introduction

A **magnetometer** measures the strength and/or direction of the magnetic field in its vicinity. We have planned to use a 3-axis magnetometer to measure the direction of the Earth's magnetic field with respect to the satellite. Distortions of the magnetic field may occur from the satellite structure and other satellite components, and this must be taken into account.



Section 2: HMR2300

2.1 Process of selection:

2.1.1 Comparison:

	HMC 1001/1002	HMC2003	HMR2300
Output	digital	analog	Digital output(RS-232 or RS-485)
Sensitivity	3mV/V/Gauss	1V/Gauss	
Field Resolution	40 μGauss	40 μGauss	70 μGauss
Field Range	$\mp$ 2 Gauss	$\mp$ 2 Gauss	$\mp$ 2 Gauss
Space Heritage	AAUsat	considered by	Cute 1.7

2.1.2 Conclusion:

Digital Output makes the data handling simpler. The protocol RS-232 is also easy to implement for data communications. The above reasons favour the use of HMR2300 and HMC 1001/1002.

HMC2300 has a better field resolution as compared to HMC 1001/1002. HMR2300 has been used by Cute 1.7 student satellite made by Tokyo Institute of Technology, Japan which was launched successfully recently (28 April 2008).

Hence, the choice is HMR2300.

2.2 General Description:

The Honeywell HMR2300 is a three-axis smart digital magnetometer to detect the strength and direction of the incident magnetic field. The three of Honeywell's magneto-resistive sensors are oriented in orthogonal directions to measure the X, Y and Z vector components of a magnetic field. These sensor outputs are converted to 16-bit digital values using an internal delta-sigma A/D converter. An onboard EEPROM stores the magnetometer's configuration for consistent operation. The data output is serial full duplex RS-232 or half-duplex RS-485 with 9600 or 19,200 rates.

Fig: HMR2300

2.3 Specifications:

Characteristics	Conditions	Min	Typ	Max	Units

Power Supply:

Supply Voltage	Pin 9 referenced to pin 5 (Ground)	6.5		15	Volts
Supply Current	Vsupply=15V,with S/R=On		27	35	mA

Mechanical:

Weight	PCB Only PCB and Non-Flanged Enclosure PCB and Flanged Enclosure		28 94 98		grams
Vibration	Operating, 5 to 10Hz for 2 Hours 10 to 2kHz for 30 Minutes		 10 2.0		 mm g

Temperature:

Operating	Ambient	-40		+85	°C
Storage	Ambient, Unbiased	-55		125	°C

Magnetic Field:

Range	Full Scale(FS), Total Field	-2		+2	gauss

	Applied				
Resolution	Applied Field to Change Output	67			micro-gauss
Accuracy	RSS of All Errors @+25°C ∓1 gauss		.01	.52	%FS

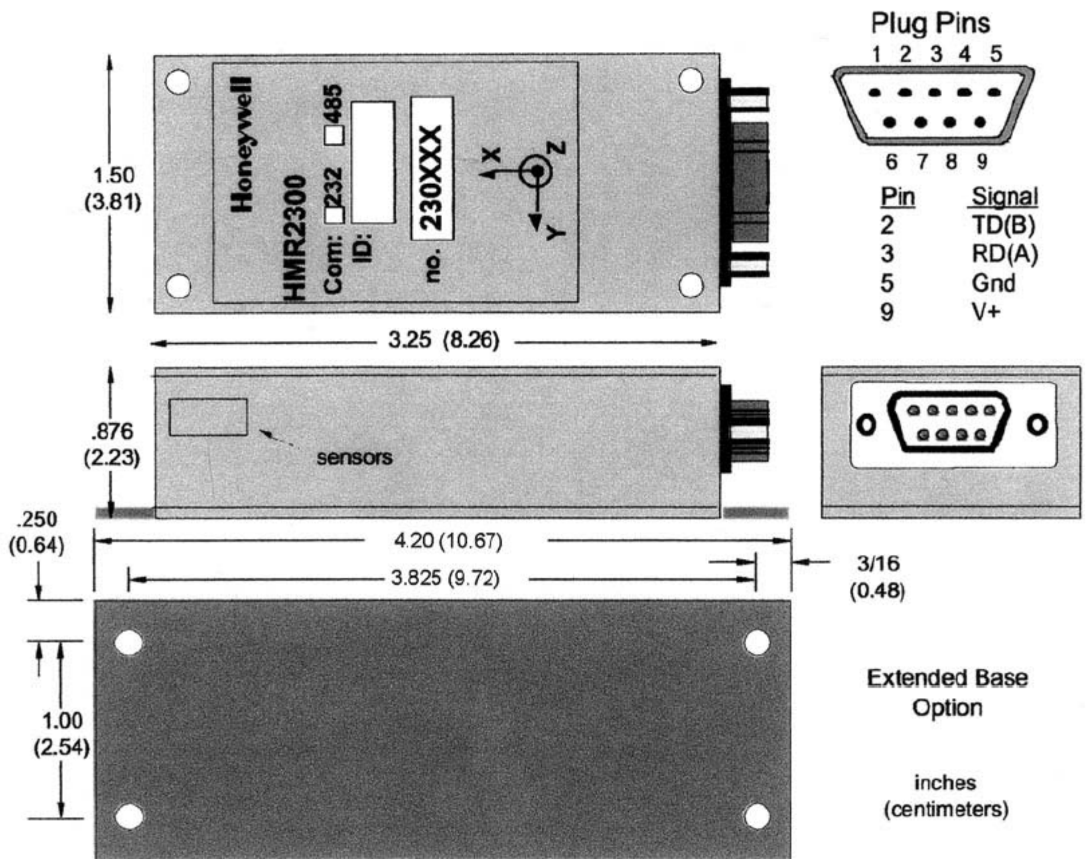


Fig: HMR2300 case dimensions

All the other specifications can be seen in the datasheet of HMR2300.

2.4 Data Communications:

The use of RS-232 is preferred by the On-Board Computer System for data communications.

A RS-232 development kit version is available with HMR2300 that includes a windows compatible demo program, interface cable, AC adapter, and carrying case.

2.5 Application Precautions:

1. The presence of ferrous materials, such as nickel, iron steel, and cobalt near the magnetometer will create disturbances in the earth's magnetic field that will distort the X, Y and Z field measurements. So, it should be avoided.

2. The presence of the earth's magnetic field must be taken into account.

3. The variance of the earth's magnetic field in different parts of the world must be taken into account too.

4. Perming effects on the HMR2300 circuit board need to be considered. If the HMR2300 is exposed to fields greater than 10 gauss, then the enclosure/circuit boards degaussed for highest sensitivity and resolution are recommended. (Degaussing wands are readily available from local electronics tool suppliers and are inexpensive.)

Actuators

Magnetorquer

Magnetorquer is the primary active actuator for our satellite. Three torquers are required for three axis attitude control of the satellite.

Hardware Design

2.1 The concept of hardware design:

The design of the magnetorquers is based on the assumption that all three coils have same properties. This means that the mass of each coil is equal to one third of the total mass.

2.2 Design Constraints (per coil):

Physical constraints:	
Maximum Dimensions	180*180 mm
Maximum Weight	50 g
Maximum Power/Torquer	1 W
Maximum Mag. Moment	0.666 Am^2 *(See Calculations in appendix)
Voltage	3.3V
Environmental conditions:	
Minimum Temp	-100 C
Max Temp	100 C
Normal Temp	15 C

Section 3: Numerical Analysis

3.1 Formulae used:

$\vec{\Gamma} = \vec{m} \times \vec{B}$

$m = \mu_r niA$

$i = V/R$

R=4ρL/πd^2;

A = ab;   L = 2n (a + b);

V=3.3V(as specified by the Power System)

W = (L×πd^2×σ)/4;

Where,

Γ= Torque
P = power
R = resistance
n = number of turns
i = current
m = magnetic moment
B = Earth's magnetic field = 3×10^{-5}T
d = diameter of wire
a = 18 cm
b= 18 cm (assuming a square torque)
ρ = resistivity of wire
σ = density of wire
Θ = angle between B and m
$\mu_r = 1$(Air Core torque)
W = weight of the wire

3.2 Calculations:

1. Magnetic moment:

$m = \mu_r niA$

$m = K_1 d^2$

$$K_1 = \frac{\pi\mu_r Vab}{8\rho(a+b)}$$

2. Weight:

W=(L×πd^2×σ)/4;

W=K_2nd^2

$$K_2 = \frac{\pi}{2}(a+b)\sigma$$

3. Power:

$$P = V^2/R$$

$$P = K_3 \frac{d^2}{n}$$

$$K_3 = \frac{\pi V^2}{8\rho(a+b)}$$

Section 4: Selection of the coil wire

4.1 Variables to be considered:

1. Coil Material
For the design we will be choosing between Al and Cu wires. Advantage of Al wires is light weight where as that of Cu wires is low resistance.

2. Wire diameter
Wire diameter is extremely critical because :

$$m \sim d\hat{}2$$

Therefore thicker the wire diameter more will be the magnetic moment produced.

3. No. of Turns
Even though Magnetic moment is independent of no. of turns it is an important factor when requirements of weight and power need to be satisfied.

4.2 Constraint Analysis:

1. $m_{max} = 0.666\ Am^2$
2. $P_{max} = 1W$
3. $W_{max} = 5 \times 10^{-2}$ m

For Copper(All SI units)
$\rho = 1.70 \times 10^{-8}$
$\sigma = 8900$
$K_1 = 6.8607 \times 10^6$

$K_2 = 5.0328 \times 10^3$
$K_3 = 6.9877 \times 10^8$

To satisfy **1**

$$d_0 = \sqrt{\frac{m_{max}}{K_1}} = 3.1171 \times 10^{-4} m$$

To satisfy **2**

$$n \geq \frac{K_3 {d_0}^2}{P_{max}} = 67.89$$

To satisfy **3**

$$n \leq \frac{W_{max}}{K_2 {d_0}^2} = 102.25$$

4.3 Comparison between Al and Cu as coil wire material:

Since torque will be operating at less than maximum power point for most of the operation cycle therefore weight is the primary criterion for optimization.
From this we get the following specifications for the torque

	Al wire torque		Cu wire torquer	
Geometric				
Wire Diameter	4.78E-01	mm	0.312	mm
No. of turns	70		70	
a	18	cm	18	cm
b	18	cm	18	cm
Power				
Voltage	3.3	V	3.3	V
Current	0.294	A	0.294	A
Power(max)	0.9702	W	0.9702	W
Main Specifications				
Magnetic Moment	6.68E-01	Am2	0.668	Am2
Power(max)	0.972	W	0.972	W
Weight	24.4	g	34.3	g

Section 5: Other parameters

Other parameters which we need to consider during the detailed design of the torque:

5.1 Temperature variation of the various parameters of the torque.

5.2 Time constant of the torque.

Section 6: Coil Driver

6.1 Introduction:

The controller on PRATHAM requires that the magnetic field generated by the coils has to be both positive and negative. Thereby the current that has to run through the coils has to be both positive and negative. Since there is only 3.3 V available(as quoted by the Power Systems), the coils will be driven by a H-Bridge in order to make both positive and negative currents through the coils.

6.2 H-bridge:

In accordance with our requirements, we need an H-bridge with 2 pnp and 2 npn transistors. The H-bridge works by either turn on the Q1 and Q3 transistor to generate a positive current or by turning on the Q2 and Q4 transistor to generate a negative current. This is illustrated in the figure below. The coil is connected between C1,C2 and C3,C4 on the H-bridge.

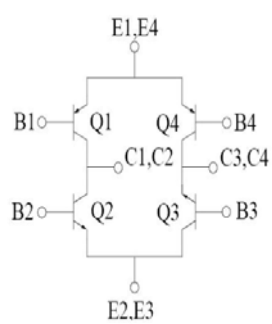


Fig: H-bridge basic configuration

The model chosen as of now is for the observation purpose:

ZHB6718 by Zetex

6.3 Shottky diodes:

Diodes are placed at C1, C2 and C3,C4 on the H-bridge to either Vcc or Gnd, in order to remove peak voltage from the coils. The peak voltage will pass through the H-bridge thereby damaging it if the diodes were not placed on the H-bridge. The H-bridge will be destroyed if the spark voltage is applied for longer periods of time. The figure below shows how the diodes are placed in the circuitry.

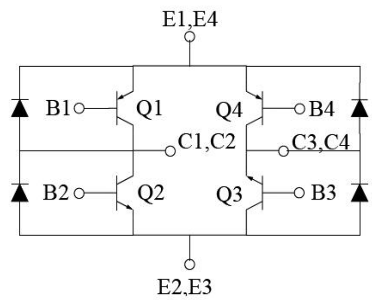


Fig: H-bridge with shottky diodes

6.4 Pulse Width Modulation:

To control the torque provided by the coils one must control the current flowing through the coils. Pulse Width Modulation, PWM, is a well tested method of controlling the current in an inductive circuit.

More study required!!!

Appendix

1. Calculations for maximum torque required:

Max GG Torque:

Torque(Γ)=$(3\mu_g/2R^3)[I_z - I_x]$ sin2θ

Γ_{max}=$(3\mu_g/2R^3)[I_z - I_x]$

Here,

μ_g=GM= $(6.67\times10^{-11})\times(6\times10^{24}) = 4.002\times10^{14}$ (S.I. units)

G=Gravitational constant;

M=Mass of the Earth;

m=mass of satellite;

m_t=tip mass of GGBoom=0.250 kg

l=length of one side=0.30 m

L=Length of tether wire(Boom Length)=10 m

R=Distance of satellite from the centre of the earth=6400+670 km

=7070 km

2. Assumption: Assuming CUBIC SATELLITE

$I_z = ml^2/6$

$I_x = \frac{ml^2}{6} + m_t l^2$

$\Gamma_{max} = -(\frac{3\mu_g}{2R^3})m_t l^2$

$\Gamma_{max} = -(3x4.002x10^{14} \div (2x(7070x10^3)^3))(0.250 \text{ x } 10^2$

$\Gamma_{max} = 4.24\ x\ 10^{-5}$ (S.I.units)

Other torques are smaller

$\boldsymbol{\Gamma_{max} = 10^{-5}}$

For θ =π/2 ; B=3×10^{-5}T

Max torque required=1/3(Γ_{max})

6.2 Simulations to verify/validate the choice

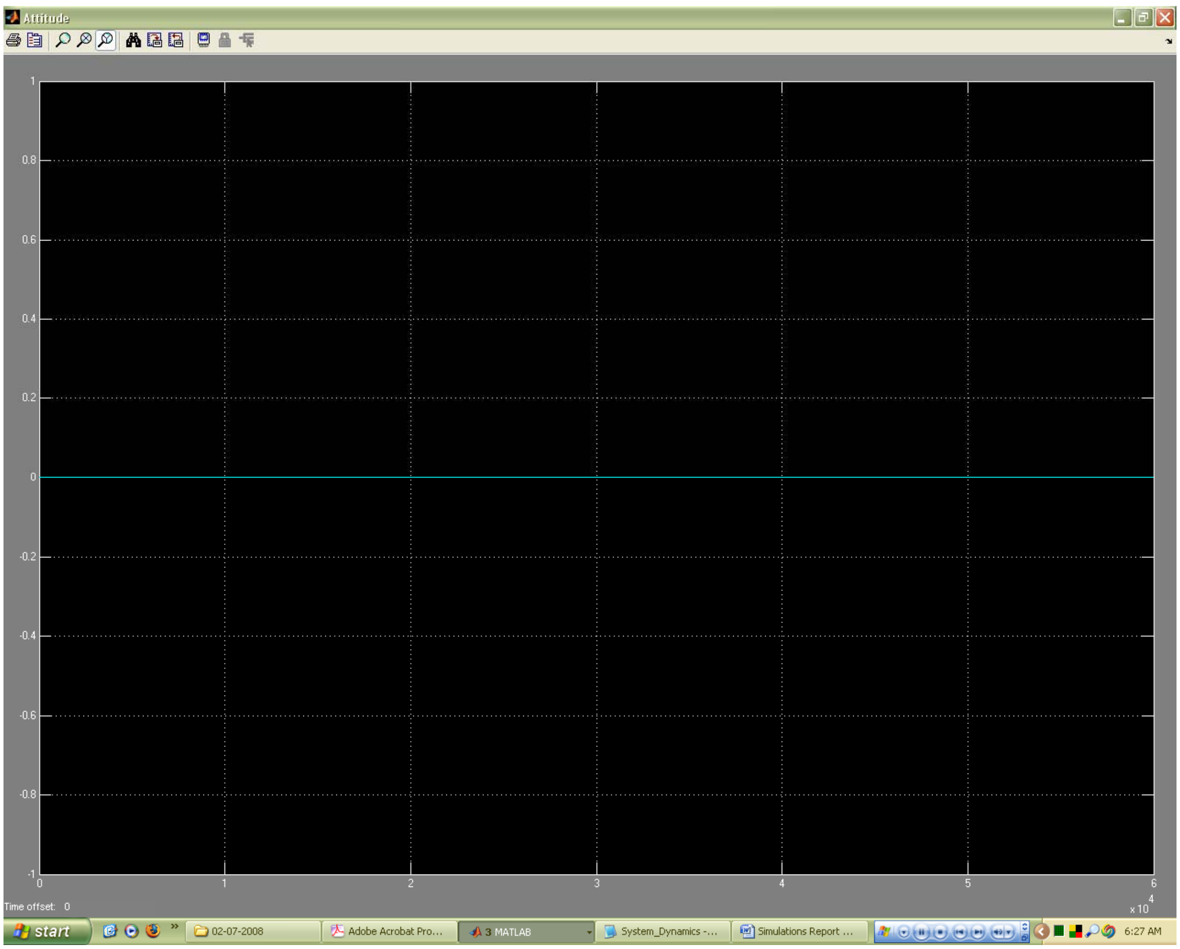
System Dynamics

The following graphs show the simulation of system dynamics of the satellite for the following inputs.

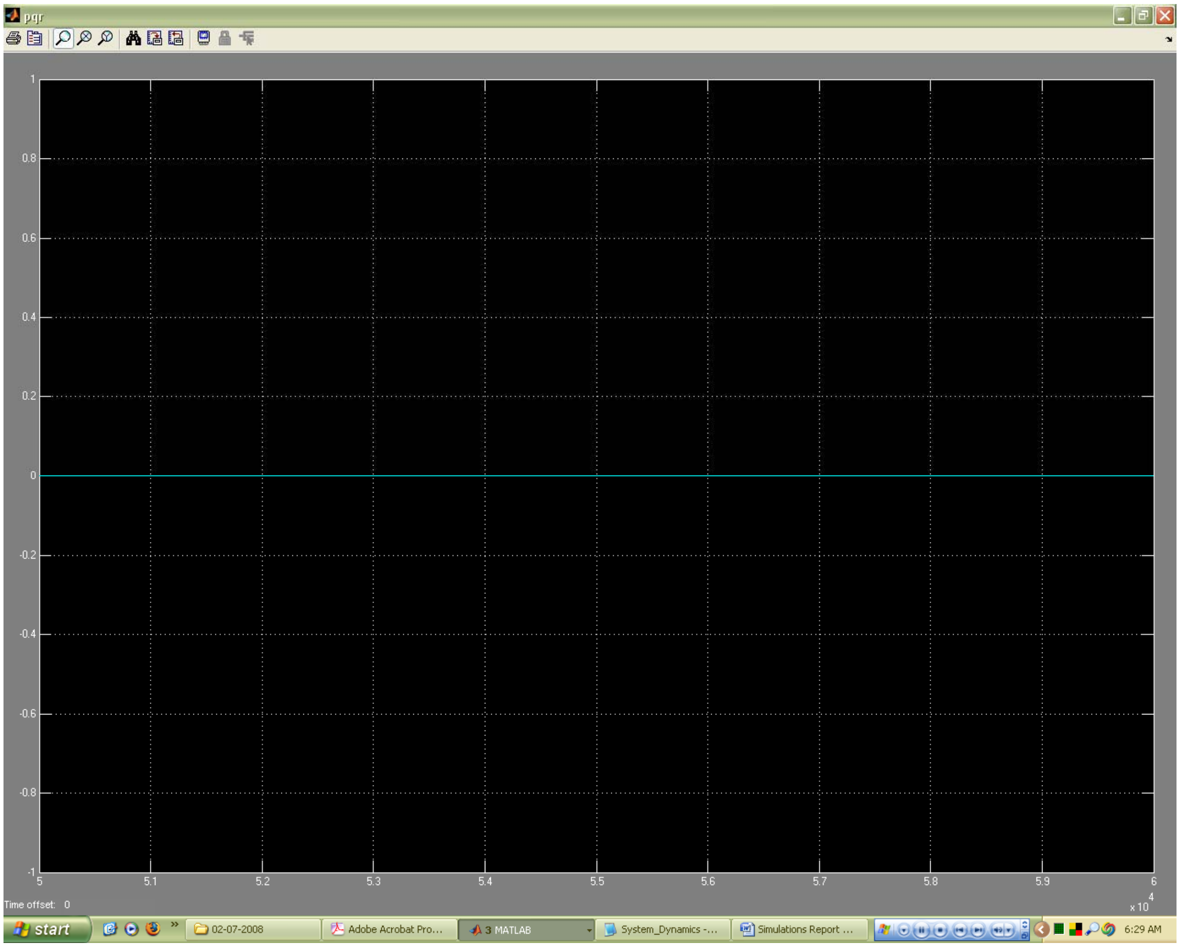
Case 1

- $\phi = 0$
- $\theta = 0$
- $\psi = 0$
- $p = 0$
- $q = 0$
- $r = 0$
- $I_x = 0.1$
- $I_y = 0.1$
- $I_z = 0.1$

2.1.1 Attitude



2.1.2 p, q, r.


pqr
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
5
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
6
x 10^4
Time offset: 0
start
02-07-2008
Adobe Acrobat Pro...
3 MATLAB
System_Dynamics -...
Simulations Report ...
6:29 AM

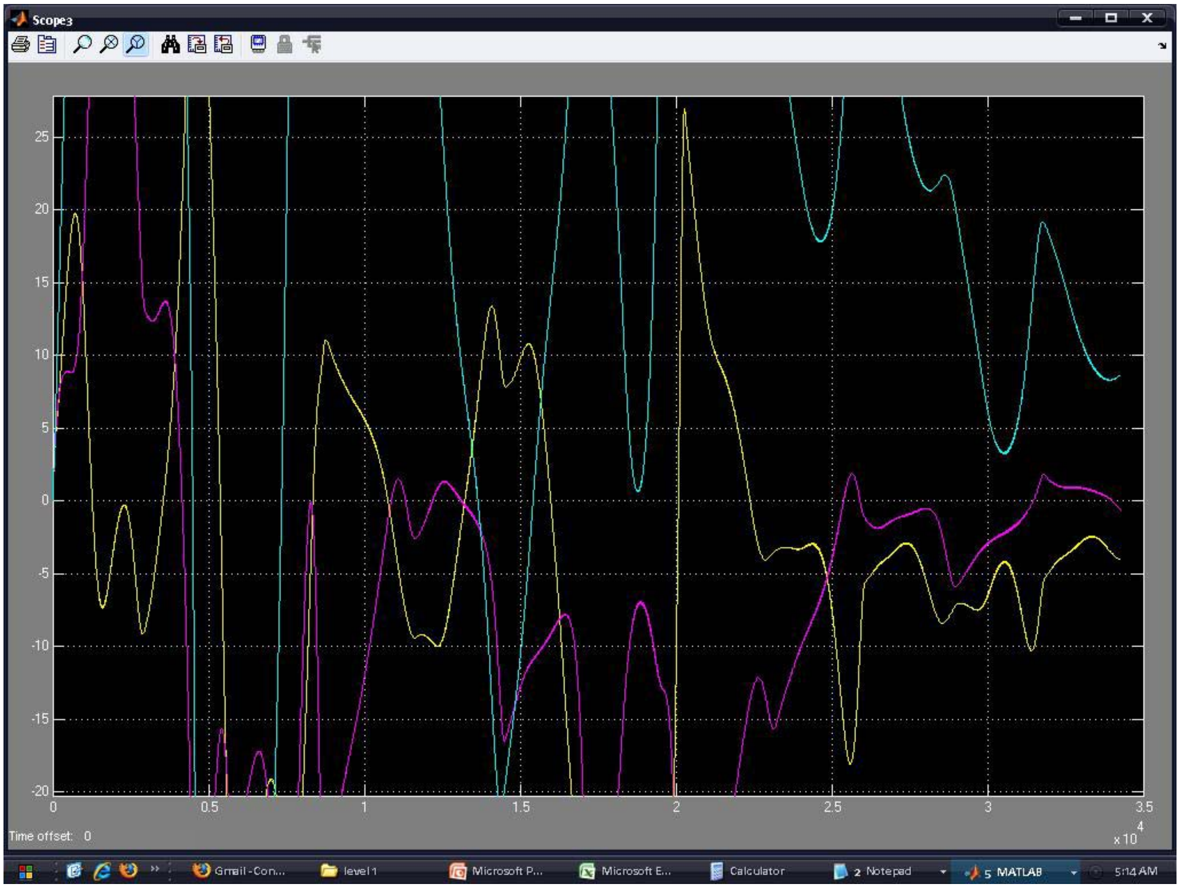
Control Strategies

Case 2

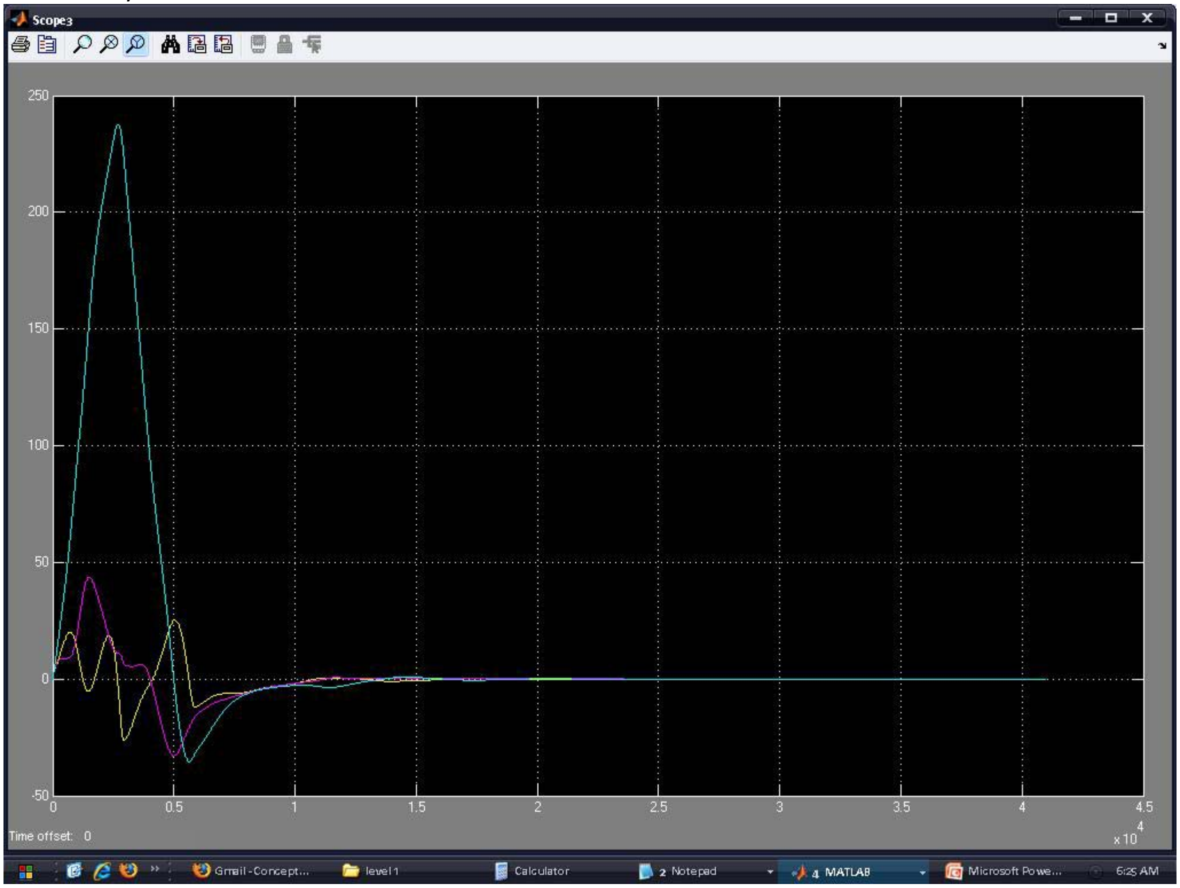
Without Gyro

- $\phi = 0$
- $\theta = 0$
- $\psi = 0$
- p = 0.06
- q = 0.06
- r = 0.06
- I_{xa}=.10
- I_{ya}=.10
- I_{za}=.10
- k1=1e+7
- k2=1e+7
- k3=1e+7
- K1=5000
- K2=5000
- K3=5000
- Tau=100

2.1.1 Attitude



2.1.2 With Gyro



1.1.3 Errors due to Sun Sensor and Manetometer

Tau_Magnetometer=100

Tau_Sunsensor=100

Magnetometer error=

Sunsensor error=5 deg

Magnetometer error=0.52% error