BTP Presentation

Rendezvous of Dubins cars in short range

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The Task

Finding time optimal trajectories for rendezvous of Dubins vehicles

Problem has been studied before in the case of large distance separation

We look at the case of short distances particularly

Dubins Car

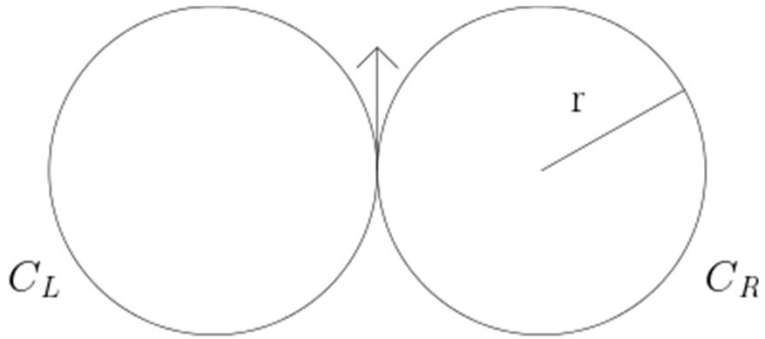
Moves only in forward direction

Maximum speed of v_s.

Minimum turning radius is r (= 1/umax).

Dynamics:

$$\dot{x}(t) = v\sin(\theta(t))$$
$$\dot{y}(t) = v\cos(\theta(t))$$
$$\dot{\theta}(t) = uv$$



Reachability Set

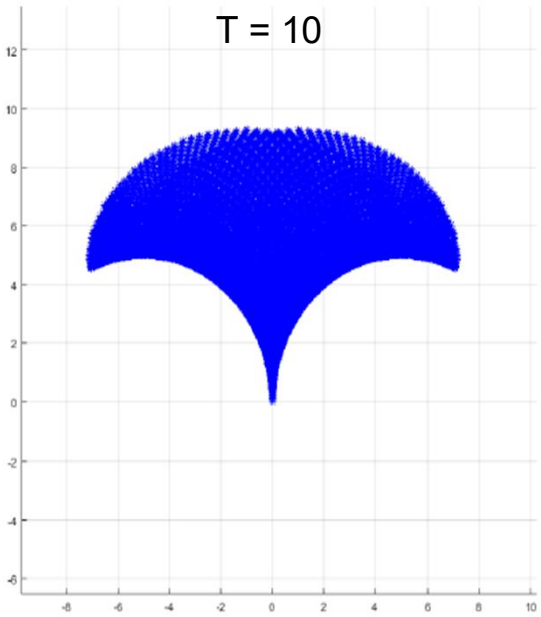
Used to visualize shortest paths to any destination

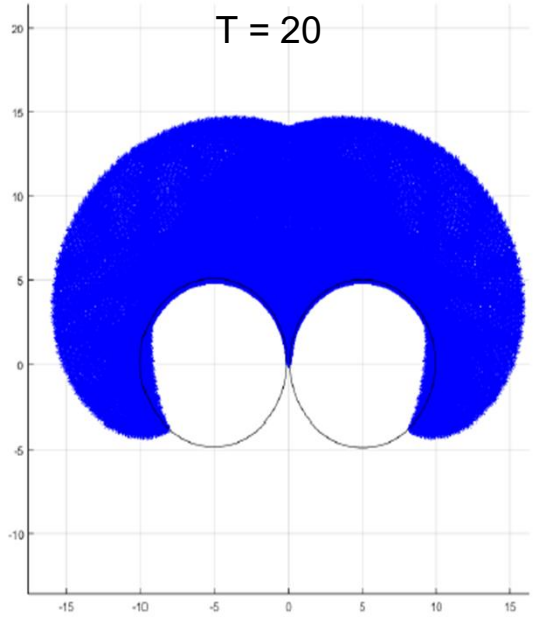
Set of points a vehicle can reach in time ≤ T.

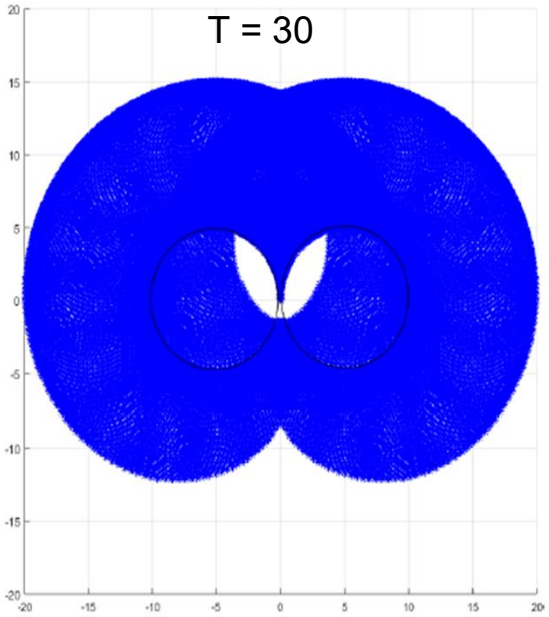
Shortest path has been proved to be CL or CC[1]

1. G. J. Cockayne, E & W. C. Hall, “Plane motion of a particle subject to curvature constraints”

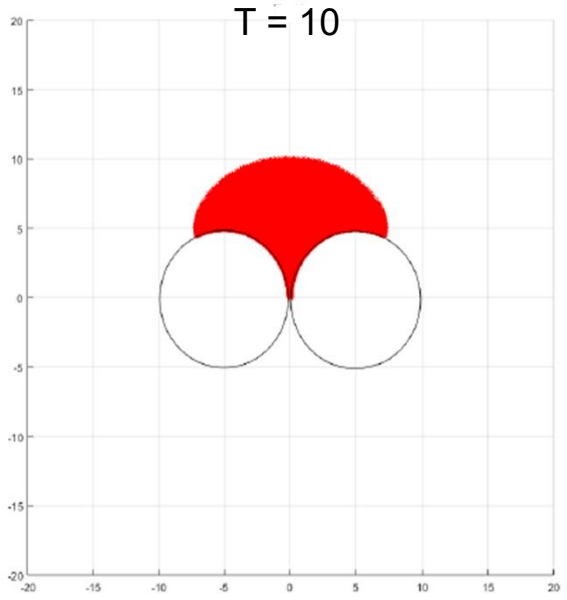
CC reachability sets

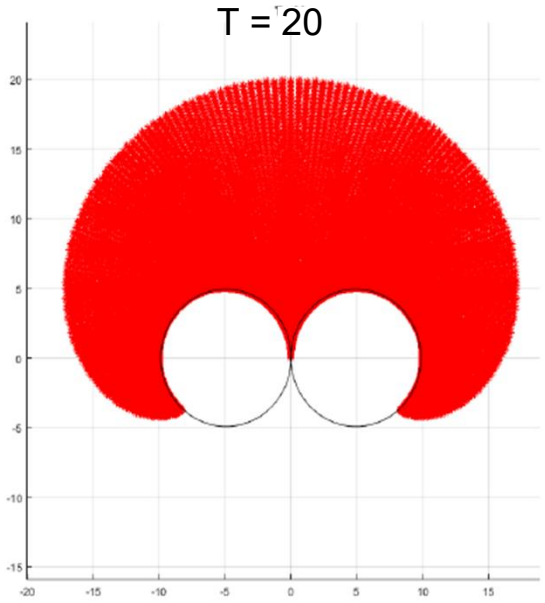


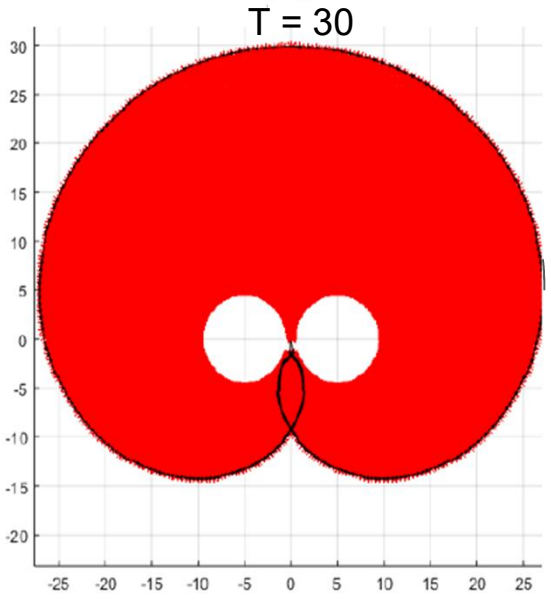




CL reachability sets



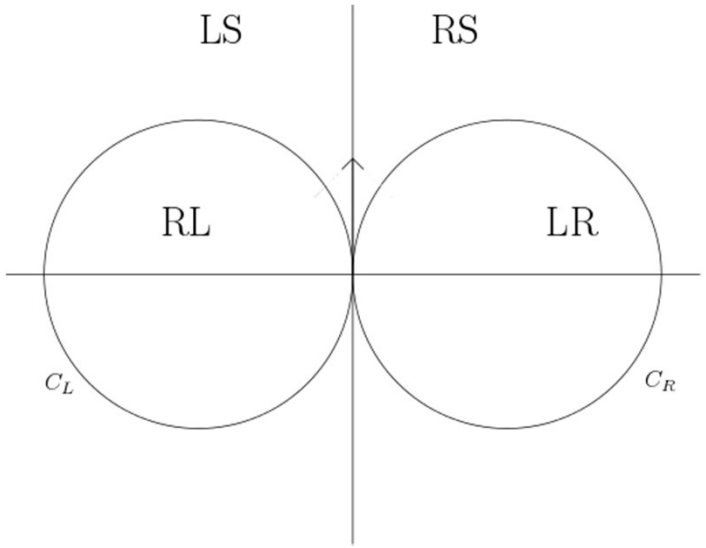




Characterization of 2-D plane

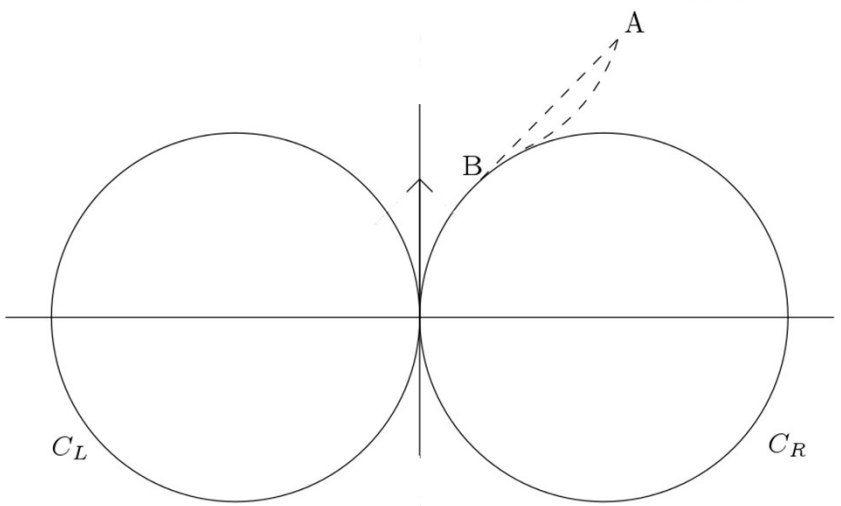
Any point outside the circles is reached fastest by CL curve

Any point inside the circles is reached fastest by CC curve

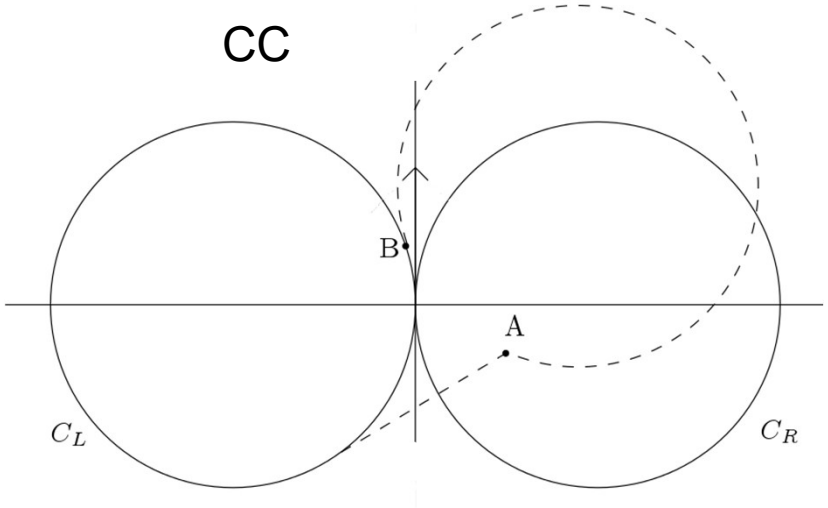


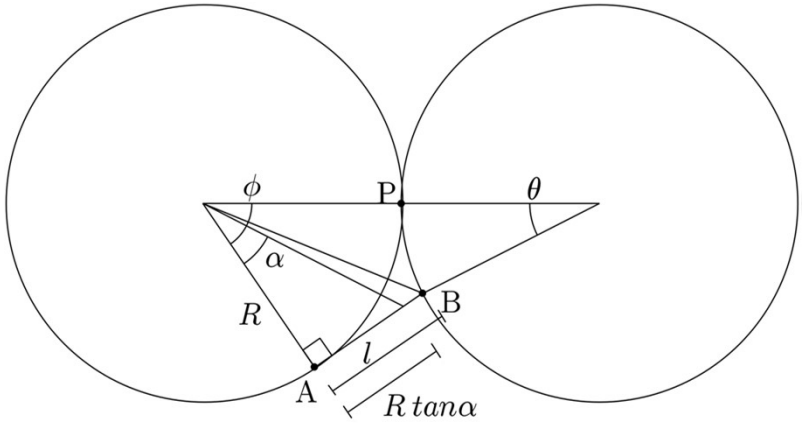
Sketch of Proof

CL



CC

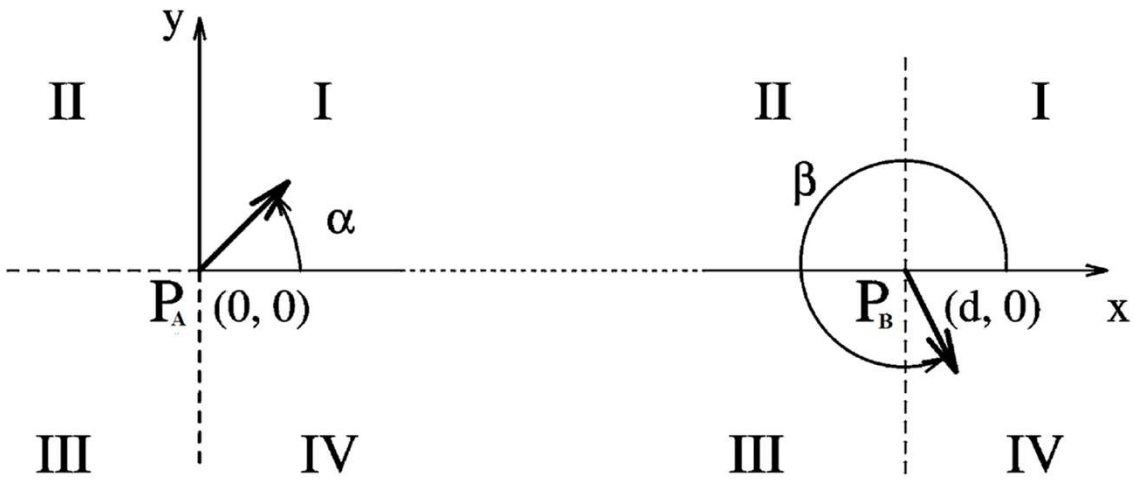




Two vehicle case

Categorizing the 2-D plane for two vehicles

Coordinate basis and study 16 combinations of quadrants



Equivalence groups

Symmetry along x and y axis

$$T(\alpha,\beta) \simeq T(\pi-\beta,\pi-\alpha) \text{ and } T(\alpha,\beta) \simeq T(-\alpha,-\beta)$$

Number of possibilities reduced to 6 by establishing equivalence groups

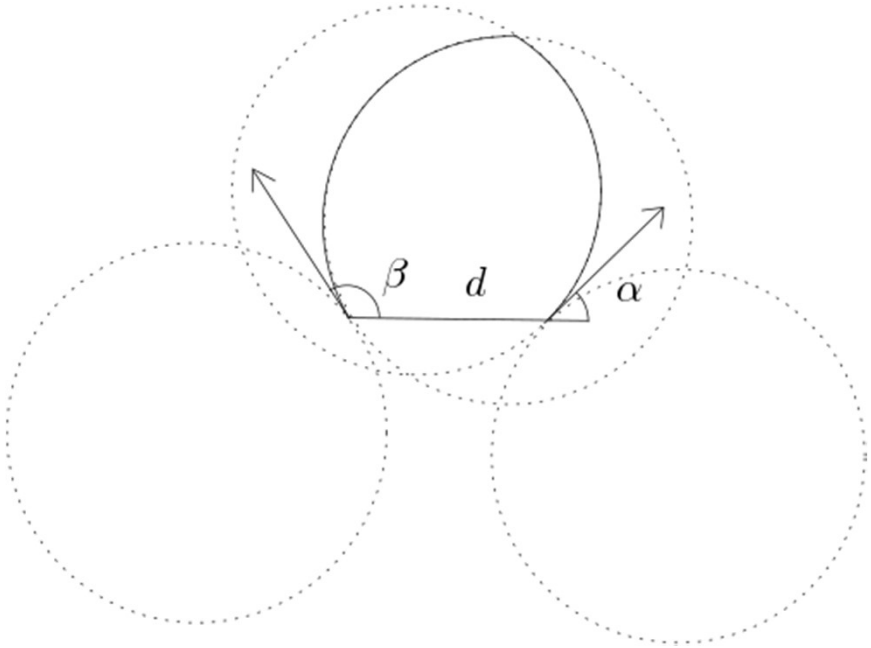
$$a_{11} \simeq a_{22} \simeq a_{33} \simeq a_{44} \qquad a_{14} \simeq a_{23} \simeq a_{32} \simeq a_{41}$$

$$a_{12} \simeq a_{43} \qquad a_{21} \simeq a_{34}$$

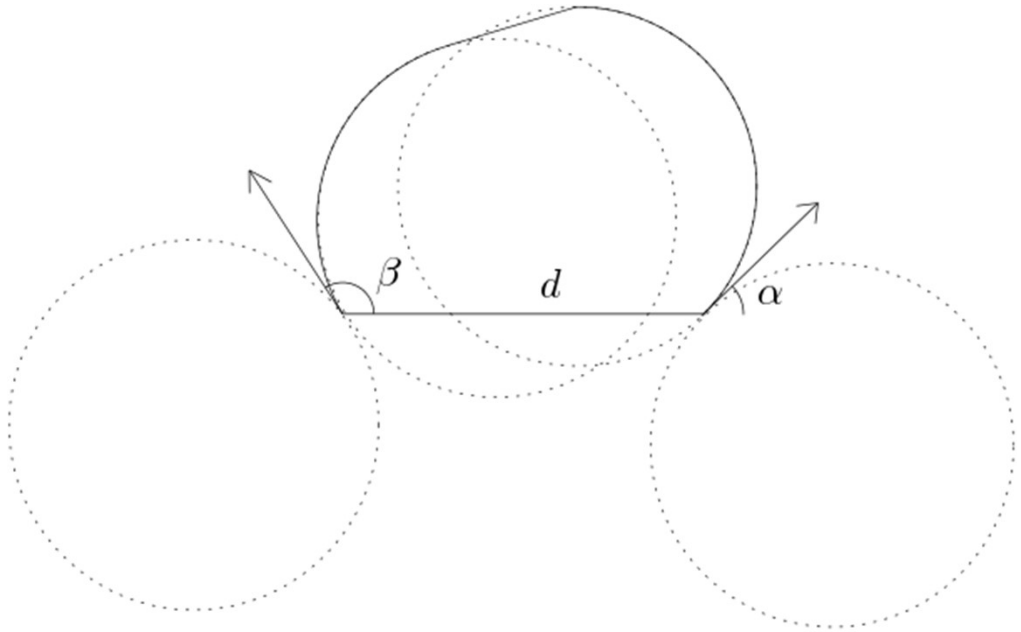
$$a_{13} \simeq a_{42} \qquad a_{24} \simeq a_{31}$$

Groups studied by increasing initial distance between the cars

Shortest paths for a_{21}



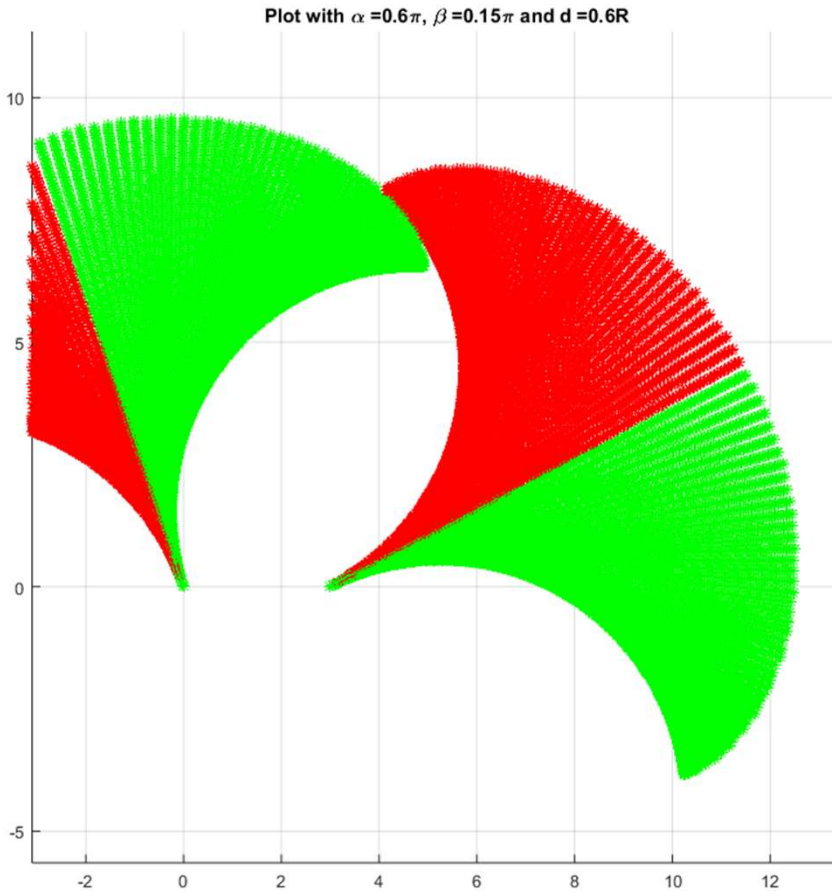
Shortest paths for a_{21}



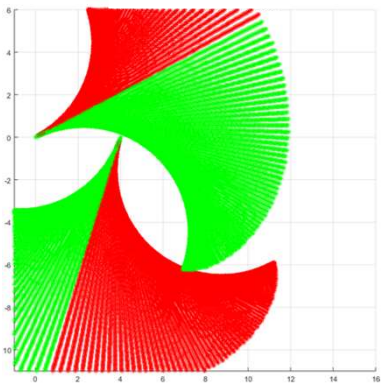
Shortest paths for a_{21}

RS for vehicle A

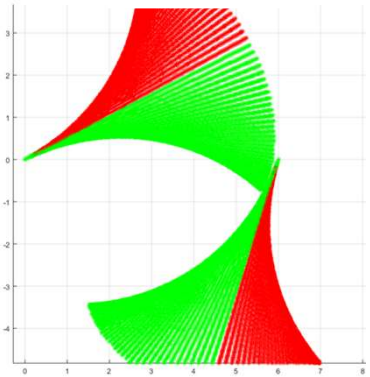
LS for vehicle B



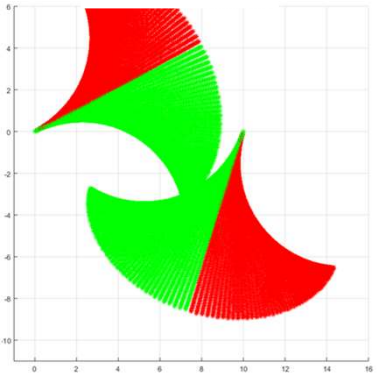
Shortest path for a_{13}



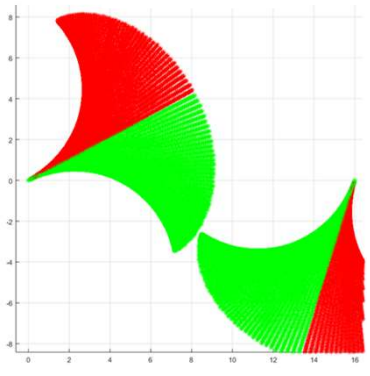
d = 0.8r



d = 1.2r



d = 2r



d = 3.2r

Shortest path for a_{13}

- If both are inside each other's circles of minimum radius:
  - For $\alpha \leq -\pi + \beta$ T[LS RS]
  - For $\alpha \geq -\pi + \beta$ T[RS LS]
- If both are outside each other's circles:
  - T[RS RS]
- If one is inside and other is outside:
  - For B inside A's circle: T[RS RS] or T[0 RS]
  - For A inside B's circle: T[LS L] or T[LS 0]

Future Work

- Geometric proofs for each of the observed cases
- Calculation the exact rendezvous point from knowledge of the initial state
- Formulate feedback based laws

Thank You