B. Tech Project Report

Rendezvous of Dubins cars in short range

Change

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February 1, 2019

[When starting each section write a line or two stating the things that you will describe and the motivation for the same.]

[Make proper sections: Follow the standard format. Abstract, Introduction, Problem statement and preliminaries, Solution Methods, Conclusion]

1 Introduction

Motion planning is one of the fundamental problem in robotics. Broadly it is a problem of selecting a set of actions which, on execution will take a robot from an initial state to a given final state. The aim of project is to provide feedback based solutions for Dubins vehicle's strategies. → Randevous Focus is on situations when the vehicles are located at a distance in the range of minimum radius of turning. Feedback solutions are aimed to be closed form expressions which can be evaluated in minimal time by the vehicle on-board.

To find feedback solns which can be implemented in negligible time

negligible

2 Dubins Vehicle

The Dubins car can move only in forward direction with a maximum speed upto v_s. The minimum turning radius can be $r_0\,(= 1/u_{max})$.The motion for such a vehicle is governed by following dynamics:

what are x, y, θ ?

$v(t)$ $v \to$ forward velocity

$$
\begin{aligned}
\dot{x}(t) &= v\sin(\theta(t)),\\
\dot{y}(t) &= v\cos(\theta(t)),\\
\dot{\theta}(t) &= uv
\end{aligned}
\tag{1}
$$

$u(t)\,v(t)$ $u \to$ change of orientation

where $(x(t), y(t)) \in \mathbb{R}^2$ is the position of vehicle at time t and $\theta(t) \in [-\pi, \pi]$ is the orientation. Control variable, u is chosen from the interval $[-u_{max}, u_{max}]$. Control variable v lies in interval $[0, v_s]$. Any position of the car in a 2-D plane can be described by a 3-D vector $(x(t), y(t), \theta(t))$. $u(t) \in [-u_m, u_m]$

3 Trajectories → change title to something meaningful.

Let the starting (initial) configuration be defined as (x_0, y_0, θ_0) and final configuration as (x_f, y_f, θ_f). Let C denote an arc of circle of minimum radius (not defined.), r_0, and S denote a straight line path. which path? The circular path C can be of types, L and R, corresponding to vehicle turning with minimum radius of curvature to left and right, respectively. Let C_L denote the circle of minimum radius on left side of initial orientation and C_R denote the circle on right. It is proved in [1] that shortest path between any two configurations for such a vehicle can be expressed as a combination of at most three parts, which can be arcs of C_L or C_R or straight line (S). This leads to six possible cases: $\{RSR, LSL, RSL, LSR, RLR, LRL\}$. Further, if the constraint on angle of arrival at the final destination is relaxed, the shortest paths are changed. In [2], it is proved that in such a case they are reduced to type CS or CC. This generates four possible trajectories: $\{RS, LS, RL, LR\}$. We focus here on the latter case only. What problem are we solving?

define all the terms before using them.

Any person reading for the first time must be able to follow all the arguments.

4 Reachability Sets

Reachability set, R_T, is defined as the set of points a vehicle can reach in time less than or equal to T. Boundary of reachability set can be used to identify the time optimal trajectories. For time much greater than r_0/v_s, the reachability set can be approximated as a circle for Dubins vehicle. But for smaller time intervals it's shape is dependent on the orientation of the vehicle and amount of time. The sets of CL and CC are plotted separately in figure 1 and figure 2. Not defined

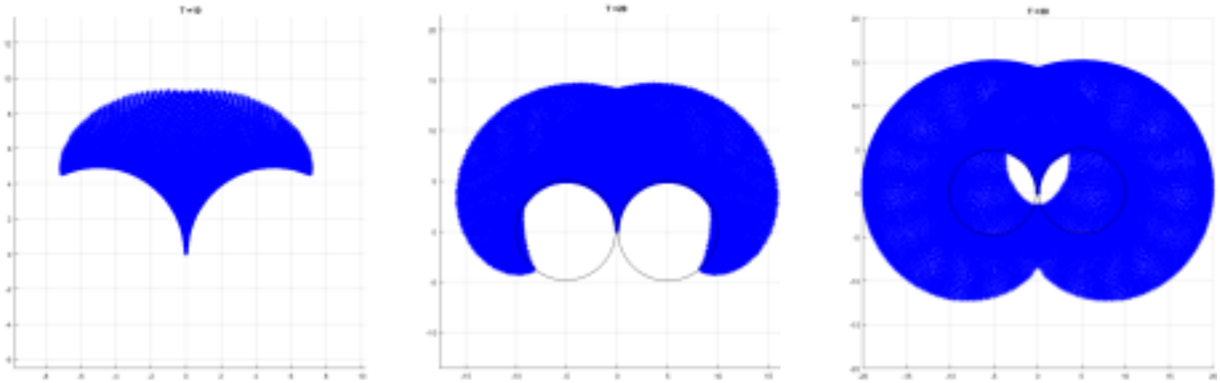

Figure 1: Reachability set for CC trajectories

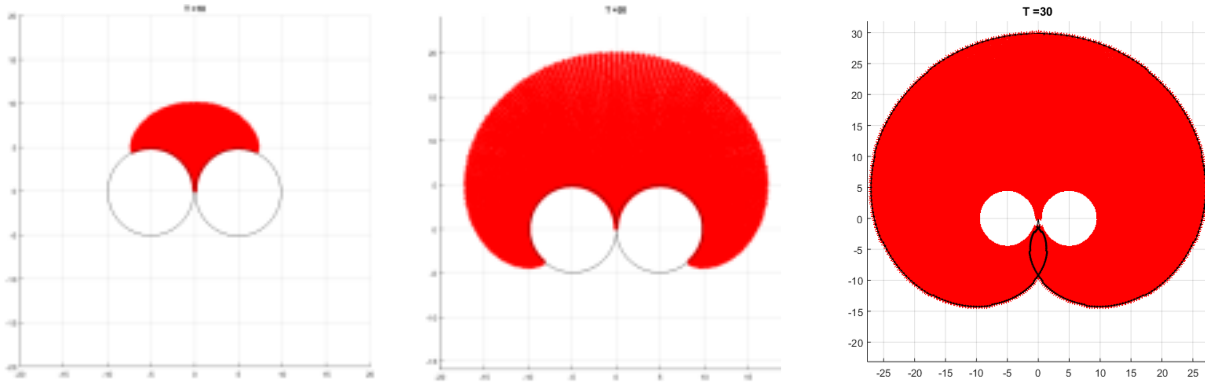

Figure 2: Reachability set for CL trajectories

4.1 and 4.2 are known results. Move them to preliminaries.

4.1 Single agent case

The car was initially programmed to reach the origin, starting from any initial position (x_0, y_0, θ_0), without constraints on angle of arrival. The initial position was taken such that its distance from origin is sufficiently larger than minimum radius of turning. In this case, according to [2], the shortest path will be a CS trajectory. This can be described as a combination of an arc and a tangent to one of the circles of minimum radius. This ~~is~~ was implemented using the feedback strategy:

Algorithm 1 Reach (0,0)

while $\sqrt{x^2 + y^2} \geq \epsilon$ **do**
  if $\theta - \tan^{-1}(y/x) \leq \rho$ **then**
    $u = 0$
  else if $\theta - \tan^{-1}(y/x) \leq 0$ **then**
    $u = -u_{max}$
  else
    $u = u_{max}$
  end if
  update θ, x and y
end while

But this feedback law fails in the situation when final position lie within C_R or C_L. Thus to look at the nearby points we need to modify our solution.

4.2 Characterization of 2-D plane

The strategy for reaching any point in the 2-D plane can be described as follows. Any point outside C_R and C_L can be reached time optimally by a CS trajectory. Whereas, any point within them can be reached optimally by a CC trajectory. This can be proven as follows: Proof not necessary Cite reference

I Take a point outside the circles. CS and CC trajectory are common till point A. Beyond that the shortest path from A to P is a straight line.

II Take a point inside C_R. Draw a circle of minimum radius passing through B and tangent to C_L. This intersects C_L at P. Thus, CS and CC trajectory are common till point P (Figure 3). To prove that CC is optimal we need to show

$$\begin{aligned} R(2\pi - \phi) + l &\geq r(2\pi - \theta)) \\ \text{or } l &\geq R(\phi - \theta) = R\alpha \\ \text{Since } \phi \geq \theta, \text{we have } l &\geq R\,tan\alpha \\ \text{As } \alpha \in [0, \pi/2], R\,tan\alpha &\geq R\alpha \end{aligned} \tag{2}$$

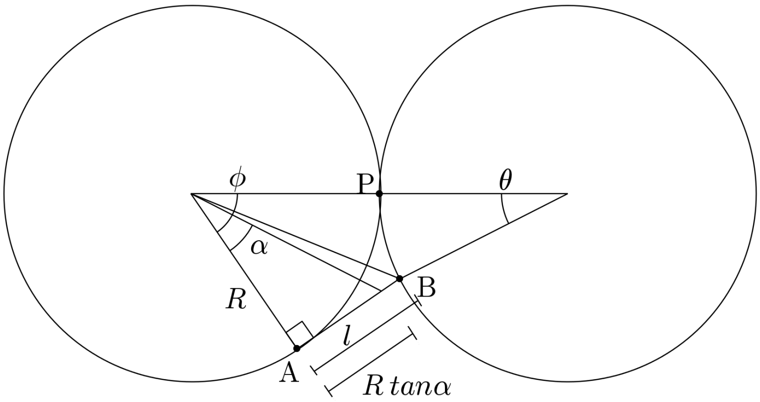

Figure 3: Proof of CC case

5 Two vehicle case

(time optimal for what) State the problem mathematically $\min_{u_1, u_2} T$, $x(t) = (\cdot)$ ⋮

Following a similar approach as for one vehicle case, the rendezvous problem of two vehicles can be looked at by dividing the plane in different sections based on their initial positions and searching for time optimal trajectories of both the vehicles. Let d be the distance between the two vehicles. A rectangular coordinate system is chosen such that its origin is at position of vehicle A, $P_A = (0, 0)$ and the positive direction of x-axis is towards vehicle B, $P_B = (d, 0)$. Let α and β be the angles their initial orientations make with the positive x-axis when measured counter-clockwise.

5.1 Equivalency Groups

(No motivation as to why this is done?)

The classification of plane can be proceeded by first dividing the range of possible orientations α and β in four quadrants. Quadrant 1 corresponds to the range $[0, \pi/2]$, quadrant 2 to $[\pi/2, \pi]$, quadrant 3 to $[\pi/3, \pi/2]$, and quadrant 4 to the range $[3\pi/2, 2\pi]$. Since α and β ?

can be in any of the four quadrants, there are 16 different possible combinations of quadrants. We represent those 16 by a 4×4 matrix, $\{a_{ij}\}$, where index i corresponds to the quadrant number of vehicle A, and index j of vehicle B. Q_i^A ; Q_j^B not defined.

Element a_{ij} therefore describes the class of all paths (not defined) whose initial and final orientation angles (α, β) belong to the quadrants Q_i^A i and Q_j^B j, respectively. For example, the case $\alpha \in [0, \pi/2]$, $\beta \in [\pi/2, \pi]$ corresponds to the element a_{12} and covers all those paths whose orientation angles belong to the first and second quadrant, respectively. Not clear.

It will be shown below that these 16 classes can be reduced to six independent clusters, called equivalency groups. We consider two trajectories to be equivalent if one can be converted into another by taking a reflection along x or y axis. trajectories of A or B?

Proposition: Let $T_{\alpha,\beta}$ be the optimum trajectories for vehicle A and B together, then following trajectories are equivalent: unclear

$T_{\alpha,\beta} \simeq T_{\pi-\beta,\pi-\alpha}$ I did not understand!

Explain slowly with Figures and Examples.

$T_{\alpha,\beta} \simeq T_{-\alpha,-\beta}$. How?

This leads to following equivalency groups:

$$
\begin{array}{ll}
a_{11} \simeq a_{22} \simeq a_{33} \simeq a_{44} & a_{14} \simeq a_{23} \simeq a_{32} \simeq a_{41} \\
a_{12} \simeq a_{43} & a_{21} \simeq a_{34} \\
a_{13} \simeq a_{42} & a_{24} \simeq a_{31}
\end{array}
$$

Each of these six equivalent groups are discussed by varying the distance between the vehicles. Why only three? You must justify each claim that you make.

There are three possible scenarios for rendezvous. One is both vehicles move for the same duration till they meet at a point. Another is when one vehicle moves for a longer duration than another vehicle. We call this trajectory to be degenerate in second vehicle. Last possibility is for one vehicle to reach the initial position of the other directly. The first two cases are denoted by T[CS CS] and third one is denoted by T[CS 0] or T[0 CS].

5.2 a_{11}

How do you know such d_1, d_2, d_3 exist?

Let d_1 be the minimum distance at which direct path to initial position of B by vehicle A is optimal. Similarly let d_2 be the minimum distance at which T[LS LS] is the optimal trajectory and d_3 be the minimum distance at which T[RS LS] is the optimal trajectory. If the distance between the two vehicles, d, is less than d_1 then the path followed by A and B is RS and LS respectively. (Again, why??)

Now if d_2 is less than d_3, following sequence of optimal trajectories is followed:

- T[RS 0] for $d_1 \leq d \leq d_2$ (why?)
- T[LS LS] for $d_2 \leq d \leq d_3$
- T[RS LS] for $d_3 \leq d \leq d_4$

Else if $d_3 \geq d_4$, then sequence is: (why?)

- T[RS 0] for $d_1 \leq d \leq d_3$
- T[RS LS] for $d_3 \leq d \leq d_4$

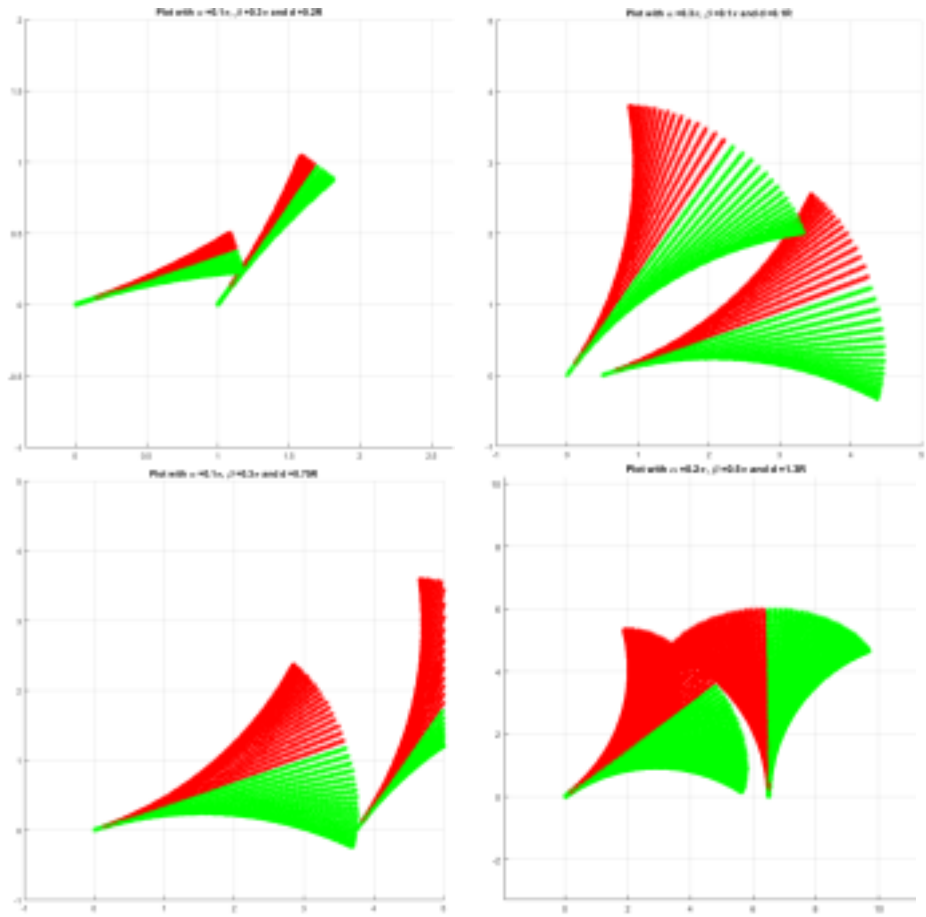

Figure 4: Trajectories for a_{11}

For all values of d, T[RS,LS] $\leq$ T[RS,RS] as any point on the right of line passing through B and at angle β is reached by A later than the corresponding point on the line. Not clear.
Similarly we can prove T[RS,LS] $\leq$ T[LS,RS].

5.3 a_{12}

Let d_1 be the distance at which straight line path by one vehicle and CS by other is optimal. Then we can write the sequence of optimal trajectories as:

- For $0 < d \leq d_1$
  - For $\alpha \leq \pi - \beta$ T[LS LS]
  - For $\alpha \geq \pi - \beta$ T[RS RS]
- For $d_1 \leq d$ T[RS LS]

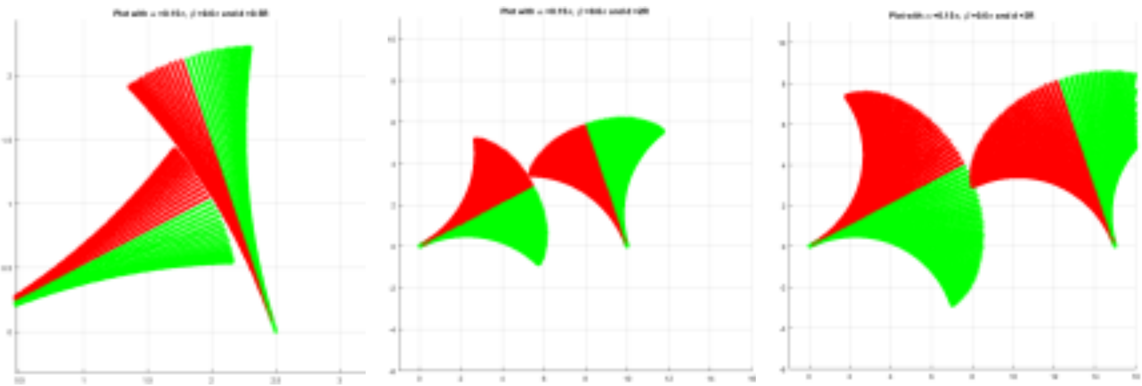

Figure 5: A12

5.4 a_{13}

- If both are inside each other's circles of minimum radius:
  - For $\alpha \leq -\pi + \beta$ T[LS RS]
  - For $\alpha \geq -\pi + \beta$ T[RS LS]
- If both are outside each other's circles:
  - T[RS RS]
- If one is inside and other is outside:
  - For B inside A's circle: T[RS RS] or T[0 RS]
  - For A inside B's circle: T[LS L] or T[LS 0]

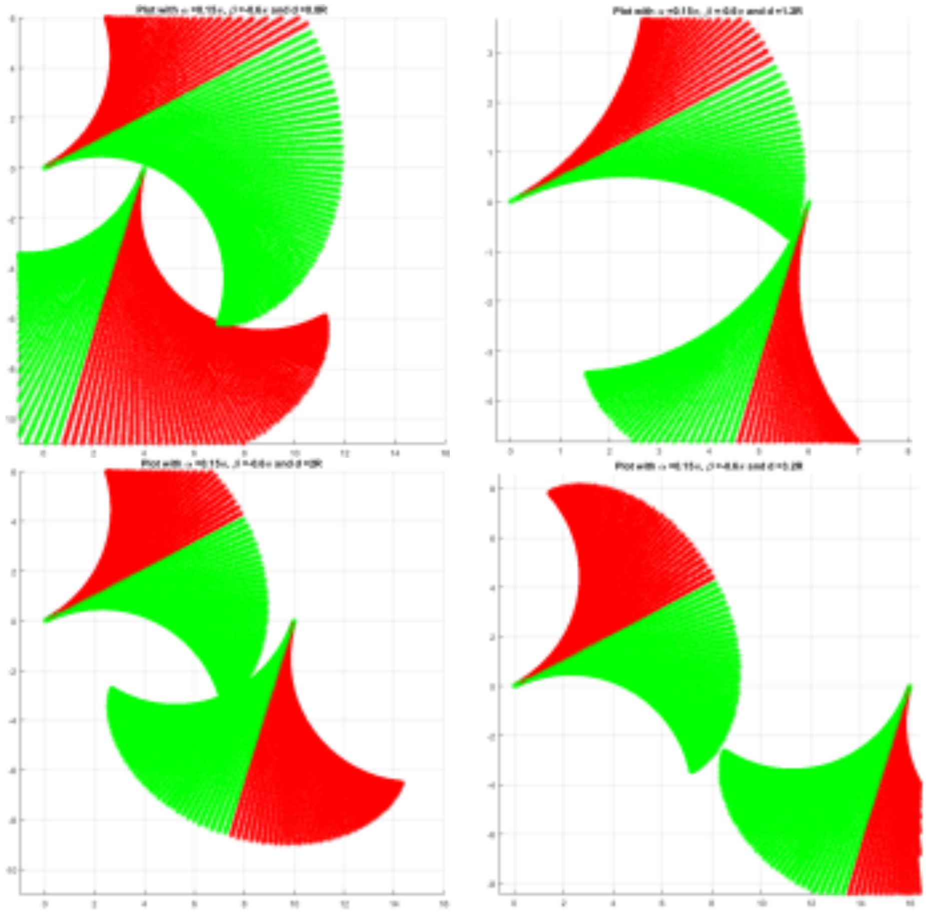

Figure 6: A13

5.5 a_{14}

- If B lies in the circle of A:
  - T[RS L]
- If B lies outside the circle of A, in increasing order of distance:
  - T[RS 0]
  - T[RS R]
  - T[RS RS]

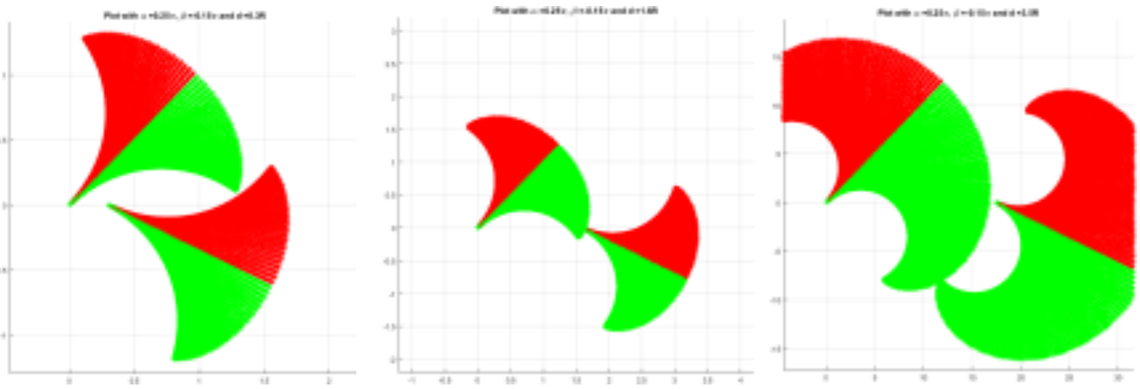

Figure 7: A14

5.6 a_{21}

The optimal trajectory is T[RS LS] for all distances.
This can be proved as any CS curve starting from A will intersect the straight line passing through B (S_B) at angle β before any point on the right of S_B. Thus T[RS LS] $\leq$ T[RS RS]. Similarly T[RS LS] $\leq$ T[LS RS] and T[RS LS] $\leq$ T[LS RS].

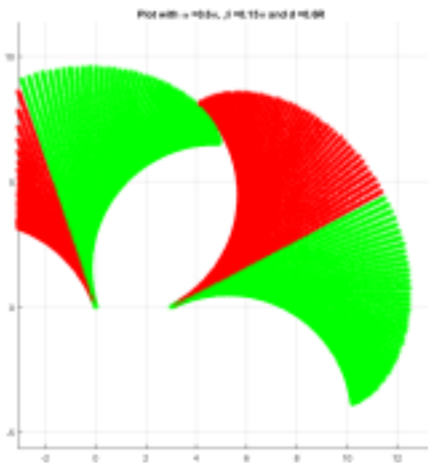

Figure 8: A21

5.7 a_{24}

For $\pi - \alpha \leq 2\pi - \beta$ T[LS RS]
For $\pi - \alpha \geq 2\pi - \beta$ T[RS LS]

6 Curvature

Meeting of two vehicles can be seen as the intersection of their rechability sets. Radius of curvature of the reachability set can be used to find intersection them.
Radius of curvature decreases uniformly as turning time increases from 0 to T. It reduces to 0 at t = T, implying that is a corner point.

7 Types of paths

The meeting of two vehicles can occur by one of the four possible paths:

- Along the common tangent
- At the intersection of two circles of minimum radius
- At the initial position of one of the vehicles : direct
- At a point on minimum radius circle of one, reached by a CS path by another, with nonzero S: CSC

Proposition: Time optimal point of intersection for two vehicles cannot lie within any of the four circles of minimum radius. In other words trajectotry for anyone of them cannot be CC.

Proposition: Intersection of CS paths by both vehicles is along common tangent.

Proof: For a time optimal meeting, intersection of paths occurs as the the boundaries of reachability set of both vehicles. Let the boundary made by CS paths be called outer boundary and that made by C alone be called inner boundary. Thus intersection of CS paths by both vehicles happens on intersection of outer boundaries of both at one point.

The above four paths can be seen from these two propositions. Since CC is ruled out, we have four possible trajectories for each vehicle: CS (with both C and L of non-zero length), C, S and zero (both C and S are zero). This gives following cases:

	CS	**C**	**S**	**zero**
CS	common tangent	CSC	common tangent	direct
C	CSC	circle intersection	CSC	direct
S	common tangent	CSC	common tangent	direct
zero	direct	direct	direct	-

- If both the vehicles are in each other's circles of minimum radius, only intersection of circles and CSC are possible
- If the distance between vehicles is greater that 4R the intersection is always along common tangent
- a_{12} and a_{21} do not have a direct path for any value of d

8 Rendezvous point calculation

Optimal path can be determined by calculating the path length for the four possible cases given in above section and choosing the minimum time taking one. For the case of common tangent, total path length along circles and tangent need to be calculated and divided in two to give time of one vehicle. For the case of intersection of circles is intersection point is computed and length along the arc for both is calculated separately. Maximum of the two is used for calculating final time. Length along direct path can be calculated by treating as a single vehicle programmed to reach a predetermined point.

CSC requires a minimization routine. Here the outer boundary intersects with a point on inner boundary or at corner point. Since the outer boundary is always the time limiting path, we focus on the CS path only. Thus we need to minimize the time taken by CS path to intersect a point on the arc of the circle reached by other vehicle in lesser time.

9 Future Work

The observed sequences for each of the six cases remains to be proved. Next step would be to calculate the exact rendezvous point from knowledge of the initial state alone. Building on these, then further feedback based laws for the two vehicle case can be stated. We would further investigate the ways to generalize these results and methods for more than two vehicles.

References

[1] L. Dubins, "On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents," *American Journal of Mathematics*, vol. 79, pp. 497 – 516, 1957.

[2] G. J. Cockayne, E & W. C. Hall, "Plane motion of a particle subject to curvature constraints," *Siam Journal on Control*, vol. 13, p. 13. 10.1137/0313012, 1975.