Урок 8

1. $u = 3 - 8x + 6y$, $x^2 + y^2 = 36$

$L(x, y, \lambda) = 3 - 8x + 6y + \lambda(x^2 + y^2 - 36)$

$L'_x = -8 + \lambda \cdot 2x = 0 \quad x = \frac{8}{2\lambda}$

$L'_y = 6 + \lambda \cdot 2y = 0 \quad y = -\frac{6}{2\lambda}$

$L'_\lambda = x^2 + y^2 - 36 = 0 \quad \frac{64}{4\lambda^2} + \frac{36}{4\lambda^2} = 36 \Rightarrow$

$4\lambda^2 \cdot 36 = 100$

$\lambda^2 = \frac{100}{4 \cdot 36} = \frac{25}{36}; \quad \lambda = \pm\frac{5}{6}$

$(4.8; -3.6; \frac{5}{6})$; $(-4.8; 3.6; -\frac{5}{6})$

$L''_{xx} = 2\lambda \qquad L''_{xy} = L''_{yx} = 0$

$L''_{yy} = 2\lambda \qquad L''_{x\lambda} = L''_{\lambda x} = 2x$

$L''_{\lambda\lambda} = 0 \qquad L''_{y\lambda} = L''_{\lambda y} = 2y$

Матрица Гессе

$$\begin{bmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{bmatrix} = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} = 0 \begin{vmatrix} 2\lambda & 0 \\ 0 & 2\lambda \end{vmatrix} - 2x \begin{vmatrix} 2x & 0 \\ 2y & 2\lambda \end{vmatrix} + 2y \begin{vmatrix} 2x & 2\lambda \\ 2y & 0 \end{vmatrix} = -8x^2\lambda - 8y^2\lambda = -8\lambda(x^2 + y^2) = -8\lambda \cdot 36 = 288\lambda$$

$4\lambda^2$; $4x\lambda - 0$

$\Rightarrow -288\lambda$

если $\lambda = \frac{5}{6}$, то $\Delta < 0 \Rightarrow (4.8; -3.6; \frac{5}{6})$ - min

если $\lambda = -\frac{5}{6}$, то $\Delta > 0 \Rightarrow (-4.8; 3.6; -\frac{5}{6})$ - max

2. $U = 2x^2 + 12xy + 32y^2 + 15 \quad x^2 + 16y^2 = 64$

$L(x, y, \lambda) = 2x^2 + 12xy + 32y^2 + 15 + \lambda(x^2 + 16y^2 - 64)$

$L'_x = 4x + 12y + 2\lambda x = 0 \quad x = -\frac{6y}{2+\lambda}$

$L'_y = 12x + 64y + 32\lambda y = 0 \quad y = -\frac{3x}{16 + 8\lambda}$

$L'_\lambda = x^2 + 16y^2 - 64 = 0$

$\left(-\frac{6y}{2+\lambda}\right)^2 + 16 \cdot \left(-\frac{3x}{8(2+\lambda)}\right)^2 = 64$

$\frac{64 \cdot 36y^2 + 144x^2}{64(2+\lambda)^2} = 64$

$\frac{144(16y^2 + x^2)}{144} = \frac{64 \cdot 64(2+\lambda)^2}{144}$

$144 = 64(2+\lambda)^2$

$18 = 8(4 + 4\lambda + \lambda^2)$

$9 = 4(4 + 4\lambda + \lambda^2)$

$9 = 16 + 16\lambda + 4\lambda^2$

$4\lambda^2 + 16\lambda + 7 = 0$

$D = 16^2 - 4 \cdot 4 \cdot 7 = 144$

$\lambda_1 = \frac{-16-\sqrt{144}}{2\cdot 4} = -\frac{28}{8} = -3{,}5$

$\lambda_2 = \frac{-16+\sqrt{144}}{8} = -\frac{4}{8} = -\frac{1}{2}$

$x_1 = -\frac{6y}{2-3{,}5} = \frac{6y}{1{,}5} = 4y$

$(4y)^2 + 16y^2 = 64$

$16y^2 + 16y^2 = 64$

$y^2 = 2 \Rightarrow y_1 = \pm\sqrt{2}$

$x_1 = \pm 4\sqrt{2}$

$x_2 = -\frac{6y}{2-0{,}5} = -\frac{6y}{1{,}5} = -4y$

$(-4y)^2 + 16y^2 = 64$

$y_2 = \pm\sqrt{2}$

$x_2 = \pm 4\sqrt{2}$

$L''_{xx} = 4 + 2\lambda$

$L''_{yy} = 64 + 32\lambda$

$L''_{\lambda\lambda} = 0$

$L''_{xy} = L''_{yx} = 12$

$L''_{x\lambda} = L''_{\lambda x} = 2x$

$L''_{y\lambda} = L''_{\lambda y} = 32y$

$$\begin{vmatrix} 0 & 2x & 32y \\ 2x & 4+2\lambda & 12 \\ 32y & 12 & 64+32\lambda \end{vmatrix} = 0 \cdot \begin{vmatrix} 4+2\lambda & 12 \\ 12 & 64+32\lambda \end{vmatrix} - 2x \begin{vmatrix} 2x & 12 \\ 32y & 64+32\lambda \end{vmatrix}$$

$128x + 64x\lambda - 384y$

$$+ 32y \begin{vmatrix} 2x & 4+2\lambda \\ 32y & 12 \end{vmatrix} =$$

$$(24x - 128y - 64y\lambda)$$

$$= -2x(128x + 64x\lambda - 384y) + 32y(24x - 128y - 64y\lambda)$$

$$= -256x^2 + 128x^2\lambda - 768xy + 768xy - 4096y^2 - 2048y^2\lambda$$

$$= -256x^2 + 128x^2\lambda - 4096y^2 - 2048y^2\lambda$$

$$= 128x^2(2 + \lambda) - 2048y^2(2 - \lambda)$$

$$= -128(2+\lambda)(x^2 + 16y^2) = -8192(2+\lambda)$$

Если $\lambda = -3{,}5$, $\Delta > 0$ $(\pm 4\sqrt{2}; \pm\sqrt{2}; -3.5)$ – min

Если $\lambda = -\frac{1}{2}$, $\Delta < 0$ $(\pm 4\sqrt{2}; \pm\sqrt{2}; -\frac{1}{2})$ – max.