Portfolio Optimization using Reinforcement Learning

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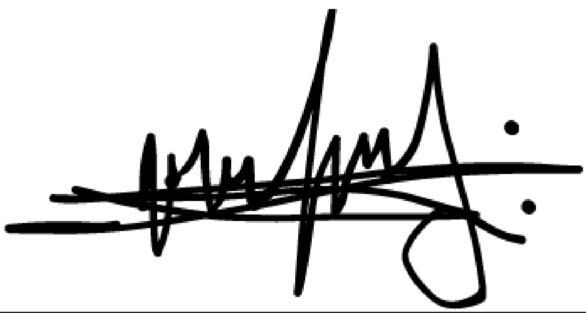
Abstract

In this work, we provide an in-depth overview of the RL framework and elucidate how its fundamental concepts translate to the financial domain of portfolio optimization. It underscores the key advantages of RL over traditional methods and sets the stage for a case study illustrating its effectiveness. This dissertation explores the transformative potential of Reinforcement Learning (RL) techniques within the realm of portfolio optimization in finance. It presents a comprehensive analysis of RL's efficacy vis-á-vis traditional methodologies such as Mean-Variance Optimization (MVO). RL exhibits an intrinsic ability to autonomously adapt to dynamic market conditions, marking a significant advancement over classical approaches. It addresses critical limitations inherent in classical machine learning algorithms like support vector machines and decision trees, notably their incapacity to capture non-linear relationships and adaptability shortcomings. Through a real-world case study, this research showcases RL's capacity to attain superior risk-adjusted returns by dynamically adjusting portfolio strategies in response to evolving market dynamics. By harnessing RL's adaptability and learning capabilities, investors can navigate intricate financial landscapes more effectively, maximizing returns while managing risk.

Keywords:portfolio optimization; reinforcement learning; mean-variance optimization; Black-Litterman model; supervised machine learning methods; deep learning; policy-gradient; Q-learning.

Declaration

I, the undersigned, hereby declare that the work contained in this essay is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.



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Case Studies and Empirical Results

Having explored a diverse range of portfolio optimization methodologies in the previous chapters, from Mean-Variance Optimization to reinforcement learning approaches, this chapter presents a comparative analysis. We will evaluate the performance of each method when applied to a specific case. Metrics such as return, risk, and drawdown will be used to assess the effectiveness of these methods in this context. This comparison aims to identify the strengths and weaknesses of each method. Ultimately, we derive an overall out-performance of RL algorithms.

0.1 Data description

We use this list of tickers, representing a diverse set of assets for portfolio construction and analysis:

$$\begin{aligned} tickers = & [\text{'AAPL', 'MSFT', 'GOOG', 'AMZN', 'BRK-A', 'NVDA', 'V', 'JPM', 'UNH', 'JNJ'} \\ & \text{'BAC', 'WMT', 'PG', 'HD', 'MA', 'XOM', 'PFE', 'DIS', 'CVX','KO', 'AVGO', 'PEP'} \\ & \text{'CSCO', 'WFC', 'COST', 'LLY', 'ADBE'}] \end{aligned}$$

The list contains tickers for 27 widely traded companies across various sectors (Technology, Financials, Healthcare, Industrials, etc.) of the US stock market. We chose to work with the adjusted closing price[1] of these assets from 2010 to 2021.

0.2 Traditional Methods: Comparison

As mentioned in section **??**, the Black Litterman method (BL-method) highly depends on the views given, to visualize it, we present in this section the performances of the BL-method, corresponding to two different investors' views, and the MVO method.

The following table represents view variances (diagonal of the matrix Ω) for both investors.

Tickers	AAPL	MSFT	GOOG	AMZN	BRK-A	LLY	V	JPM	UNH	JNJ
Views1	0.0156	0.0144	0.0156	0.0056	0.0025	0.0025	0.0025	0.0004	0.1600	0.0225
Views2	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025

Tickers	NVDA	PG	HD	XOM	PFE	DIS	CVX	KO	WMT	ADBE
Views1	0.0156	0.0225	0.0156	0.0056	0.0025	0.0025	0.0025	0.1600	0.1600	0.0225
Views2	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600

Figures 1 and 2 illustrate the impact of varying views on the Black-Litterman (BL) method's performance. As shown, well-defined views can lead to the BL method potentially outperforming Mean-Variance Optimization (MVO).

This suggests that incorporating investor judgment through the BL method might be advantageous, particularly when based on credible expert insights.

[1]because it provides a smooth and continuous price series, making it easier to perform analysis and quantitative calculations.

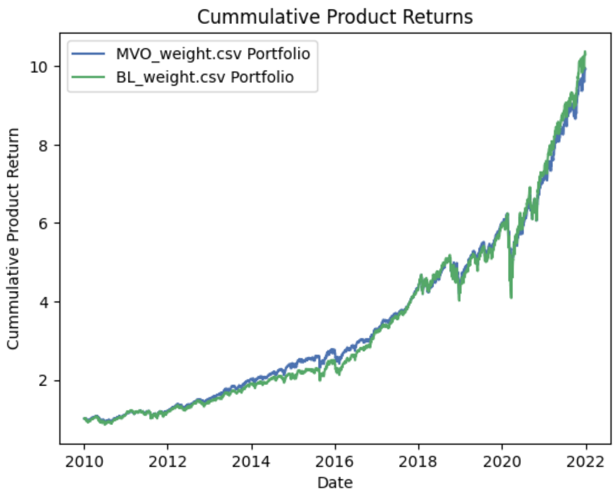

Figure 1: MVO-BL-views1

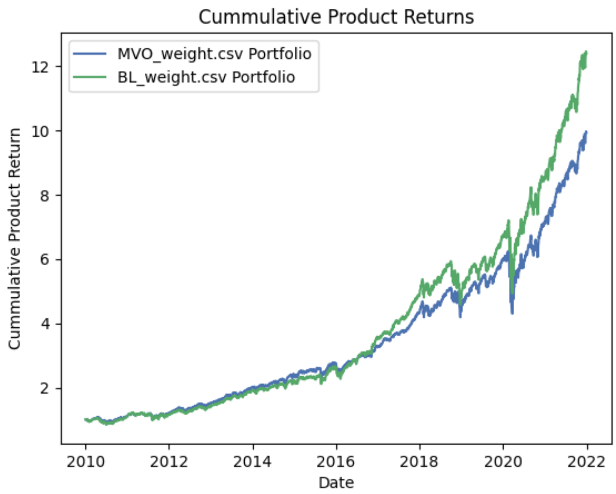

Figure 2: MVO-BL-views2

Figures 3 and 4 present pie charts visualizing the portfolio weights derived from the MVO and BL methods, respectively. Both approaches encourage diversification, but with key differences.

- MVO: The MVO approach tends to allocate weights more equally across all assets, promoting a balanced portfolio distribution.
- BL: The BL method, in this specific case, allocates a slightly higher weight to assets like 'NVDA' and 'BRK-A' compared to the MVO allocation.

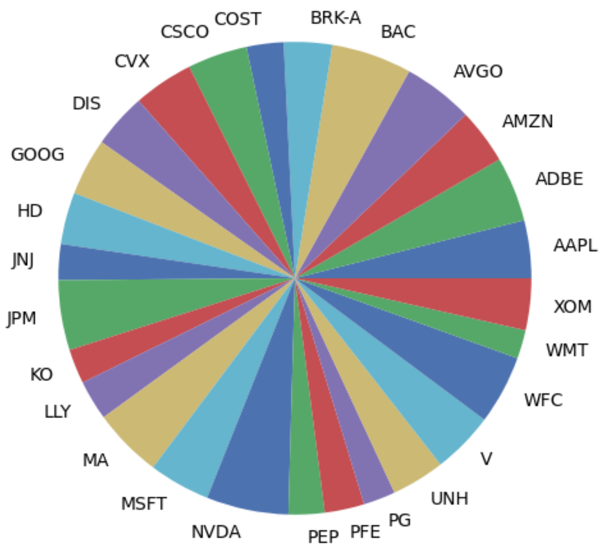

Figure 3: MVO-portfolio

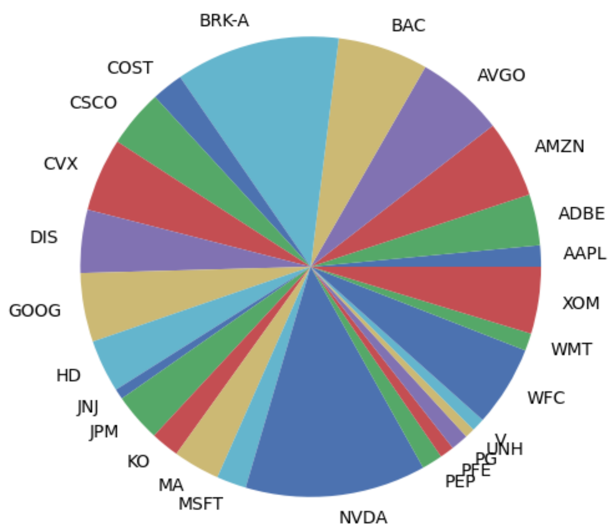

Figure 4: BL-views2-portfolio

0.3 Standard Machine Learning Methods: SVM and Decision tree, comparison

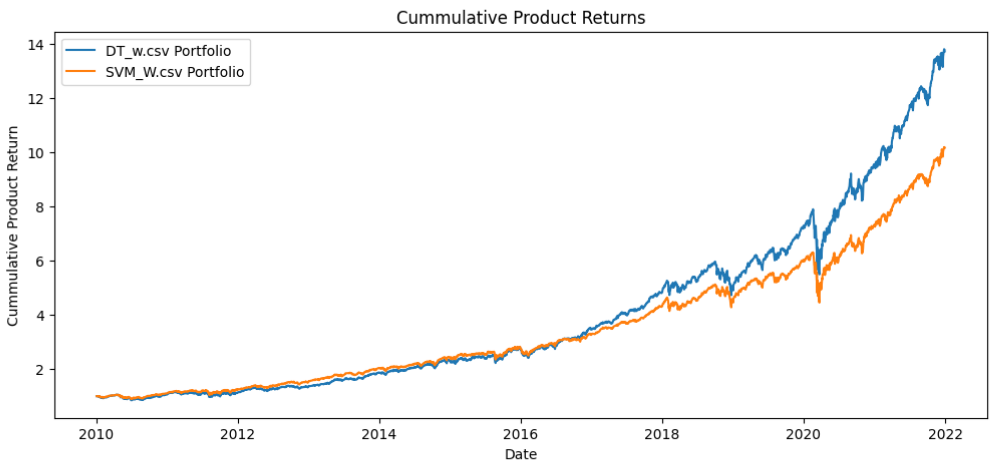

Figure 5: Machine learning methods

Figure 5 indicates that the Decision Tree method potentially outperforms the Support Vector Machine (SVM) for portfolio optimization in this specific case. Furthermore, analyzing the pie charts Figures 6 and 7 reveals distinct investment strategies. Decision Tree Allocates a significant portion of the investment to 'MSFT' assets, followed by 'NVDA'. while SVM Tends towards a more equal distribution of weights across all assets

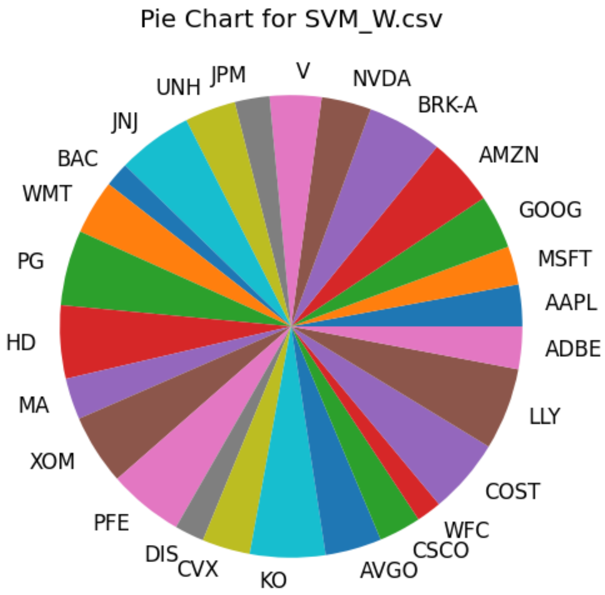

Figure 6: SVM-portfolio

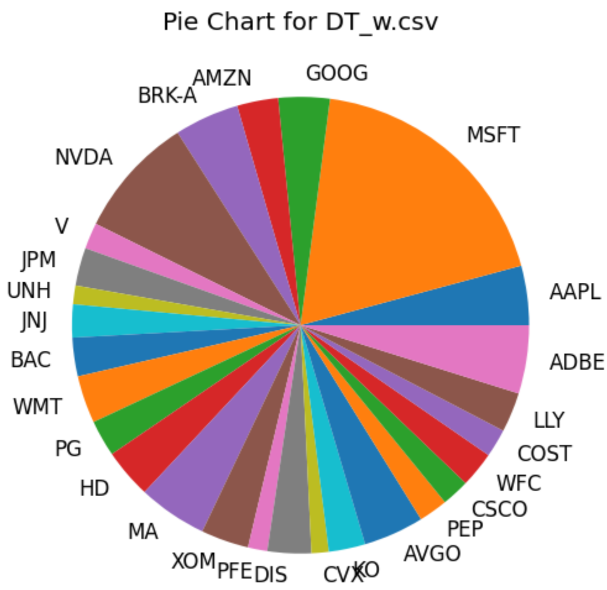

Figure 7: DT-portfolio

Having explored the performances of both traditional and machine learning methods, a comparative analysis based on Figure 8 is presented. It highlights key observations:

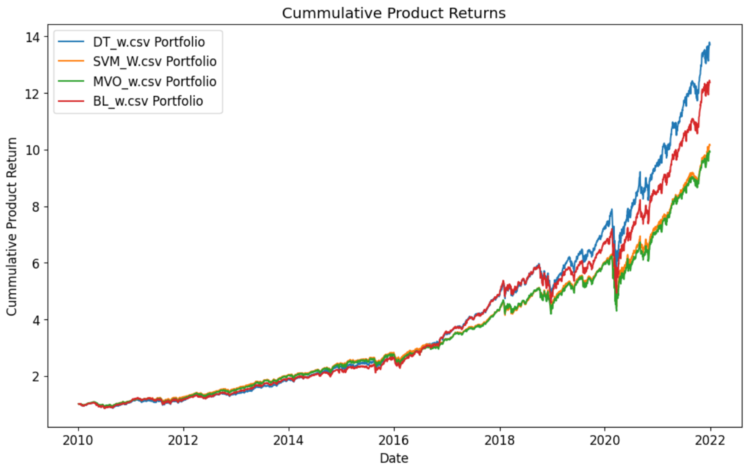

Figure 8: Traditional and Machine Learning Methods

- The Decision Tree method emerges as the leader in terms of returns in this scenario.
- The Black-Litterman approach follows closely, potentially indicating the benefit of incorporating investor views.
- Mean-Variance Optimization(MVO) and Support Vector Machine(SVM) exhibit similar performance in terms of returns.

0.4 Comparison with deep learning methods: LSTM

We now have an idea of the performance of machine learning for portfolio optimization, let's explore the usefulness of the deep learning method in portfolio optimization.

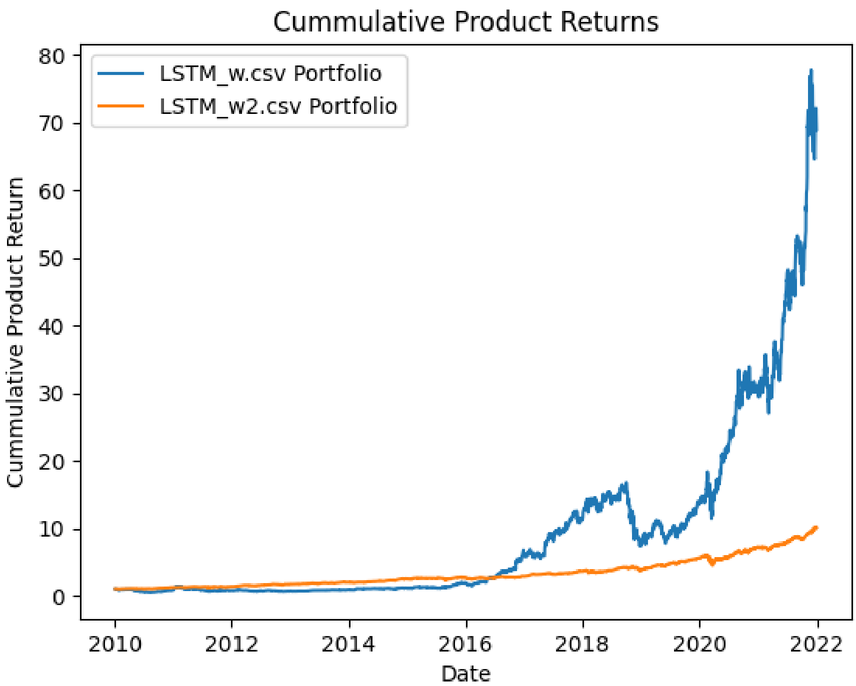

Figure 9: LSTM methods

As deep learning method, we investigate the application of Long Short-Term Memory (LSTM) networks for portfolio optimization. LSTMs have the ability to learn and adapt their weight distribution based on the chosen objective function.

Figure 9 illustrates this concept. It presents two scenarios with different objective functions for the LSTM:

- Scenario 1 (Maximizing Return): The objective function here is designed to prioritize maximizing the portfolio's return. In the resulting weight distribution, as shown in the pie chart 10, the LSTM favors the asset with higher expected returns which is the 'NVDA'.
- Scenario 2 (Return-Risk Trade-off): This scenario incorporates a more balanced objective function that aims to not only maximize return but also minimize risk. The LSTM's weight distribution, as shown in the pie chart 11 reflects this balance, by allocating weights to assets that offer a balance between return potential and risk.

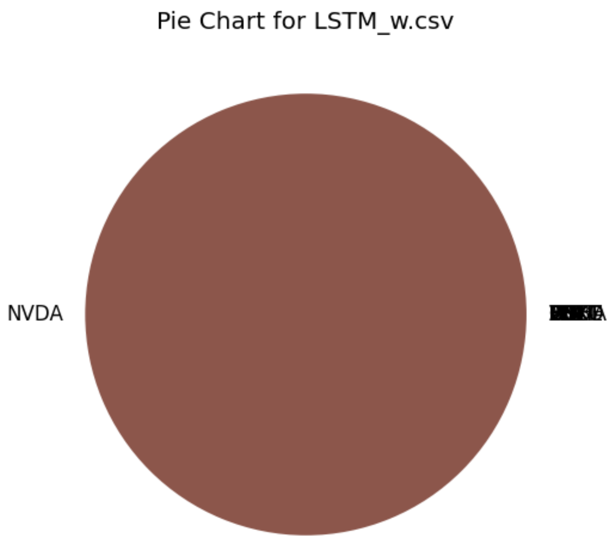

Figure 10: LSTM-portfolio

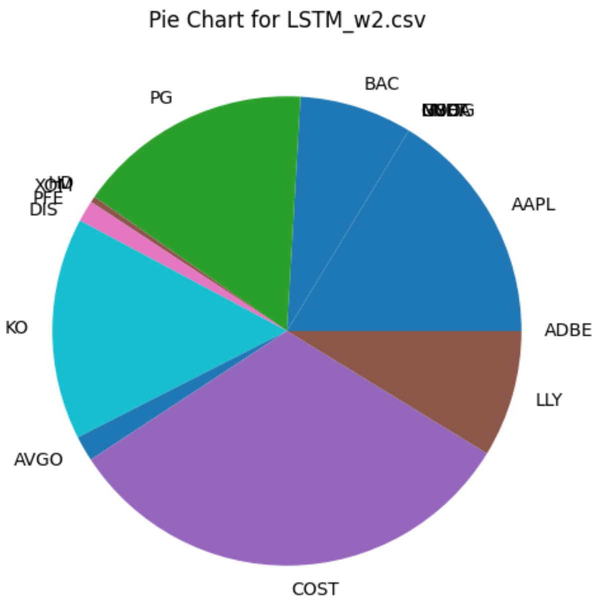

Figure 11: LSTM-2 portfolio

After exploring how LSTMs perform in portfolio optimization, let's compare this method to the previously mentioned approaches. Figure 12 shows the returns achieved by the LSTM strategy alongside the two machine learning methods and the traditional approaches.

We can see that the LSTM strategy in the first scenario outperforms all others, followed by the decision tree and the Black litterman method. The SVM and MVO methods have the lower returns. However, the LSTM's performance in the second scenario is significantly lower. This is because the second scenario prioritizes minimizing risk, which may limit potential returns.

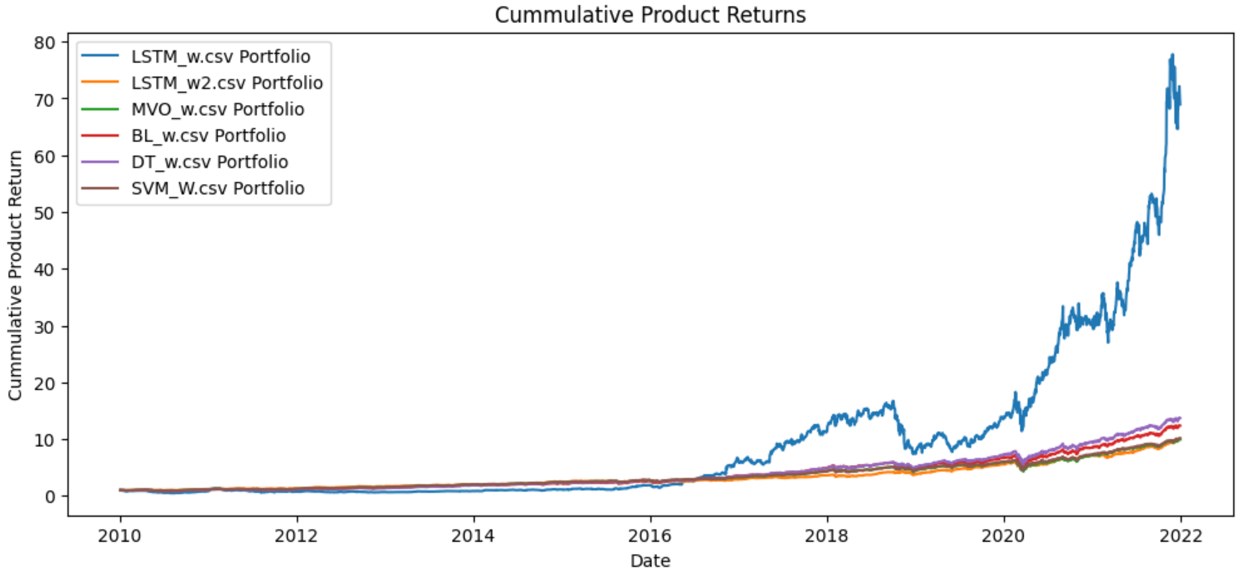

Figure 12: LSTM & other methods

0.5 Reinforcement learning methods

We now examine how reinforcement learning can be applied to portfolio optimization. We'll utilize the policy gradient and Q-learning approaches previously introduced in sections **??** and **??**, respectively.

Figure 13 illustrates that the policy gradient method outperforms the Q-learning method in this context.

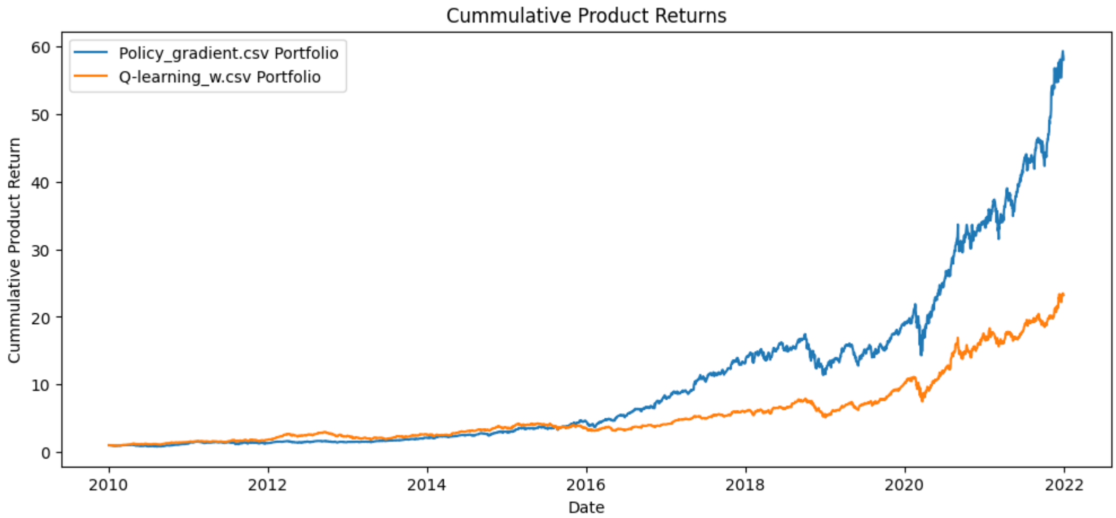

Figure 13: RL-methods

Let's analyze the weight distribution of each method using pie charts (Figures 14 and 15). These charts reveal a clear distinction in portfolio diversification between the Q-learning and policy-gradient approaches.

From the charts, the Q-learning portfolio exhibits a less diversified allocation compared to the policy-gradient portfolio, in the pie chart 14, a significant portion of the capital is invested in a single asset, 'AAPL'. This heavy concentration in one asset exposes the portfolio to greater risk if the performance of 'AAPL' deviates significantly from expectations.

In contrast, the policy-gradient portfolio (Figure 15) demonstrates a more balanced allocation across assets. While it prioritizes 'NVDA' and 'AVGO', it also invests in 'AMZN' and other assets. This diversification helps mitigate risk by reducing the portfolio's dependence on the performance of any single asset.

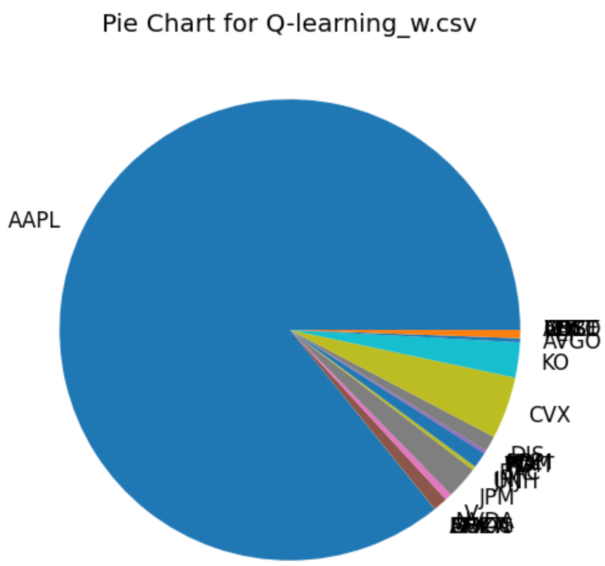

Figure 14: Q-learning-portfolio

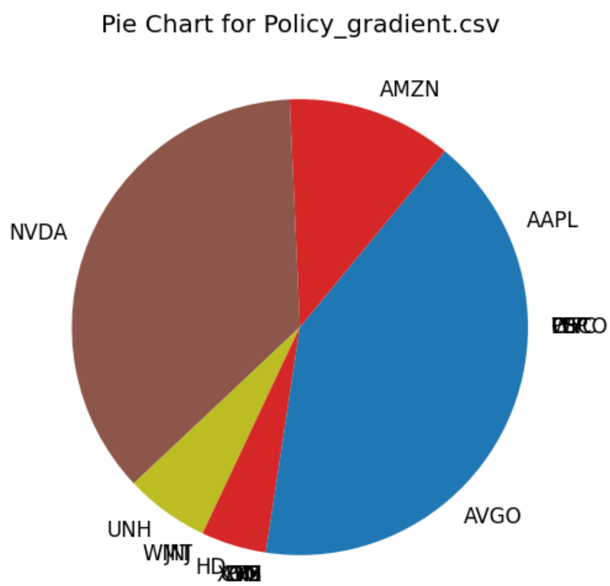

Figure 15: policy-gradient-portfolio

To acquire in-depth knowledge, let's compare the performance of reinforcement learning methods against the previously explored approaches (traditional and machine learning). Figure 16 reveals some key observations:

- The LSTM strategy in scenario 1 emerges as the top performer in terms of returns. However, this comes at a significant cost: a highly concentrated portfolio heavily invested in a single asset. This concentration exposes the portfolio to substantial risk if the asset 'NVDA' underperforms.
- In contrast, both reinforcement learning algorithms (policy gradient and Q-learning) demonstrate a compelling balance between performance and risk. They achieve high returns while maintaining a more diversified allocation across assets. This diversification mitigates risk by reducing the portfolio's dependence on any single asset's performance.
- The remaining methods, including traditional and machine learning approaches, generally exhibit lower performance compared to the reinforcement learning algorithms. While their diversification is moderate, their overall return potential appears limited.

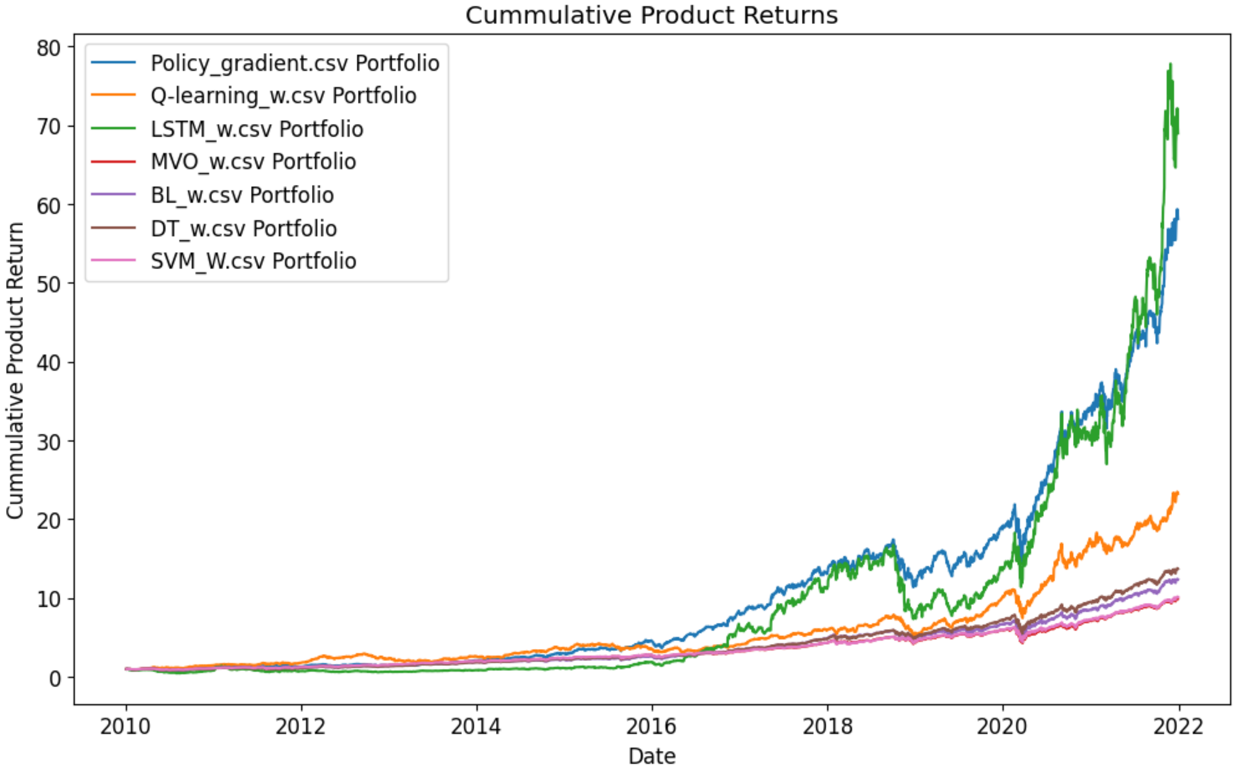

Figure 16: Comparison all methods

0.6 Performance Metrics and Evaluation Criteria

We end this chapter with a comparative analysis based on evaluation criteria such as the maximum Drawdown, the Value at Risk, the Conditional value at risk defined in sections **??**, **??**, and **??**, respectively.

Model	**Expected Return**	**-Maximum Drawdown**	**-VaR_95**	**-CVaR_95**	**Standard Dev**
MVO	0.000820	31.16%	1.62%	2.64%	0.010924
BL	0.000902	31.55%	1.72%	2.81%	0.011703
SVM	0.000822	29.36%	1.52%	2.49%	0.010346
DT	0.000937	30.39%	1.71%	2.82%	0.011808
LSTM 1	0.001761	56.04%	3.83%	5.81%	0.026867
LSTM 2	0.000813	24.78%	1.38%	2.27%	0.009715
Policy Gradient	0.001498	34.76%	2.55%	4.00%	0.017413
Q-learning	0.001173	37.92%	2.42%	3.70%	0.016187

Figure 17: Table of performance metrics

The table 17 compares the performance of various portfolio optimization models based on several risk-return metrics. From it we can extract the following interpretation:

1. Expected Return column shows the average return each model expects to generate. LSTM of the has the highest expected return (0.001761), followed by Policy gradient (0.001498) and Q-learnig (0.001173).

2. Maximum Drawdown: This measures the largest peak-to-trough decline in portfolio value. While LSTM-1 boasts the highest return, it also has the highest maximum drawdown (56.04%), indicating a much riskier strategy. SVM and DT have the lowest maximum drawdown (around 24-31$\%$).

3. $VaR - 95$ & $CVaR - 95$: $VaR - 95$ (Value at Risk) tells us the potential maximum loss we could experience with 95% confidence level. $CVaR - 95$ (Conditional VaR) is the expected loss given that a loss exceeding the VaR threshold occurs. Here, SVM and LSTM 2 have the lowest VaR and $CVaR$ values, while the reinforcement learning methods appear a bit more risky, but acceptable.

4. Standard Deviation: This reflects the overall volatility of the portfolio's returns. LSTM-1 again has the highest standard deviation (0.026867), further highlighting its volatility. SVM and LSTM-2 have the lowest standard deviations, while the reinforcement learning methods present an acceptable standard deviation.

Overall, although LSTM-1 presents the possibility of achieving the highest returns, it also carries the greatest risk in terms of volatility and other risk metrics. Black litterman and Decision Tree take a more balanced approach, offering moderate returns and lower risk. SVM also demonstrates potential with lower risk indicators. Additionally, reinforcement learning methods present a promising portfolio option, with high returns and an acceptable level of risk.

To clearly visualize the performance of each method, we present the table 18, analyzing their performance, using the Sharpe ratio as a key metric, with a risk-free rate of $r_f = 0.00045$.

Models	MVO	BL	SVM	DT	LSTM-1	LSTM-2	PG	Q-learning
Sharpe $(\times 10^{-2})$	3.3882	3.8626	3.5920	4.1264	0.1183	3.7392	6.0185	4.4690

Figure 18: Sharpe ratio of each method

Key findings:

- **Reinforcement Learning Dominance:** The table reveals that the two reinforcement learning (policy gradient (PG), Q-learning) methods achieve the highest Sharpe ratios. This attests that RL techniques can outperform other methods in portfolio optimization.

- **Deep Learning Potential**: The contrast result of Scenario one and two of the LSTM highlights the crucial role of effective training for deep learning methods in achieving optimal performance.

Overall, the analysis emphasizes the effectiveness of reinforcement learning for portfolio optimization. Additionally, it suggests the potential of deep learning with proper training. Further investigation into deep learning techniques appears warranted, this motivates our study of the deep deterministic policy gradient in section **??**.

Acknowledgements

> "It is not that we think we are qualified to do anything on our own. Our qualification comes from God." 2 Corinthians 3:5

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Appendix

This repository contains the Python code of the algorithms studied in this essay.

Python implementations