Linear Algebra Project

Data masking

Table of Contents

Goals

- Develop a data transforming algorithm for the Sure Tomorrow insurance company that would make it hard to recover personal information from the transformed data;
- Prove that the algorithm works correctly;
- The data should be protected in such a way that the quality of machine learning models doesn't suffer.

Data description

- **Features:** insured person's gender, age, salary, and number of family members.
- **Target:** number of insurance benefits received by the insured person over the last five years.

Imports

In [18]:

```
import pandas as pd
import matplotlib
import numpy as np
import seaborn as sns
import re

from sklearn import linear_model
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split

from sklearn.metrics import r2_score

import matplotlib.pyplot as plt
%matplotlib inline

import sys
import warnings
if not sys.warnoptions:
       warnings.simplefilter("ignore")

pd.set_option('display.max_rows', None, 'display.max_columns', None)

print("Setup Complete")
```

```
Setup Complete
```

Input data

In [3]:

```
try:
    df = pd.read_csv('insurance_us.csv')

except:
    df = pd.read_csv('/datasets/insurance_us.csv')
```

Descriptive statistics

In [4]:

```
df.head()
```

Out[4]:

	Gender	Age	Salary	Family members	Insurance benefits
0	1	41.0	49600.0	1	0
1	0	46.0	38000.0	1	1
2	0	29.0	21000.0	0	0
3	0	21.0	41700.0	2	0
4	1	28.0	26100.0	0	0

Notes for preprocessing:

- Convert `Age` column to integers;
- The target is categorical, it's a classification task in essence but we will use linear regression for the task's sake (despite the quality of such a model).

In [5]:

```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 5000 entries, 0 to 4999
Data columns (total 5 columns):
 #   Column              Non-Null Count  Dtype  
---  ------              --------------  -----  
 0   Gender              5000 non-null   int64  
 1   Age                 5000 non-null   float64
 2   Salary              5000 non-null   float64
 3   Family members      5000 non-null   int64  
 4   Insurance benefits  5000 non-null   int64  
dtypes: float64(2), int64(3)
memory usage: 195.4 KB
```

Notes for preprocessing:

- There are 5000 observations with 4 features and 1 target variables;
- Column names should be converted to lower case;
- No missing values.

In [6]:

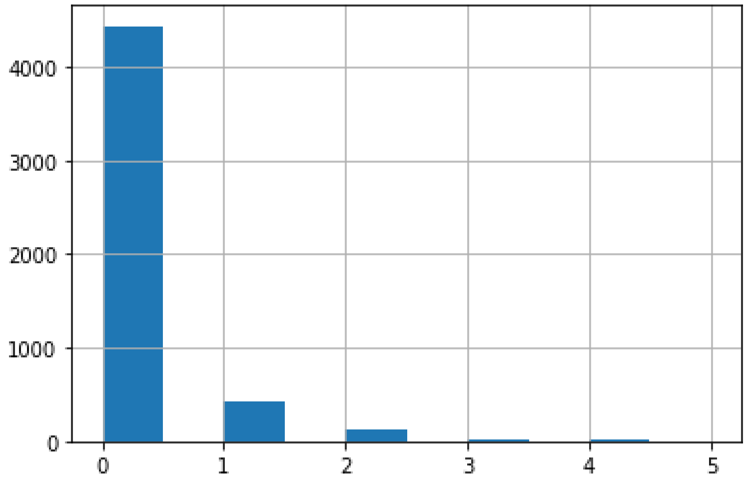
```
df.describe()
```

Out[6]:

	Gender	Age	Salary	Family members	Insurance benefits
count	5000.000000	5000.000000	5000.000000	5000.000000	5000.000000
mean	0.499000	30.952800	39916.360000	1.194200	0.148000
std	0.500049	8.440807	9900.083569	1.091387	0.463183
min	0.000000	18.000000	5300.000000	0.000000	0.000000
25%	0.000000	24.000000	33300.000000	0.000000	0.000000
50%	0.000000	30.000000	40200.000000	1.000000	0.000000
75%	1.000000	37.000000	46600.000000	2.000000	0.000000
max	1.000000	65.000000	79000.000000	6.000000	5.000000

In [7]:

```
df['Insurance benefits'].hist();
```



Notes for preprocessing:

- Both values in the `Gender` column are equally represented;
- `Age` and `Salary` columns are normally distributed (mean close to median, max/min values within mean +/- 3 std). Typical age of a client is 30 years old, typical salary is around 40k USD;
- `Family members` number ranges from 0 to 6, also close to normal distribution;
- Most people didn't receive insurance benefits over the last five years. The target ranges from 0 to 5, classes are imbalanced.

Data preprocessing

Column names

In [8]:

```
columns = []
for name in df.columns.values:
    name = re.sub('([A-Z])', r' \1', name).lower().replace(' ', '_')[1:]
    columns.append(name)
```

In [9]:

```
df.columns = columns
```

In [10]:

```
df.head()
```

Out[10]:

	gender	age	salary	family_members	insurance_benefits
0	1	41.0	49600.0	1	0
1	0	46.0	38000.0	1	1
2	0	29.0	21000.0	0	0
3	0	21.0	41700.0	2	0
4	1	28.0	26100.0	0	0

Data type change

As mentioned above, let's convert the `age` column into the integer type.

In [11]:

```
df['age'] = df['age'].astype(int)
```

Duplicates

Let's check if any rows are duplicated.

In [12]:

```
df.duplicated().sum()
```

Out[12]:

```
153
```

In [13]:

```
df.drop_duplicates(inplace=True, ignore_index=True)
```

In [16]:

```
df.shape
```

Out[16]:

```
(4847, 5)
```

Theoretical proof

In this section we will provide a theoretical proof based on the equation of linear regression that the quality of a model doesn't change if we mask features. The features are multiplied by an invertible matrix P.

Denote:

- X — feature matrix (zero column consists of unities)
- y — target vector
- P — matrix by which the features are multiplied
- w — linear regression weight vector (zero element is equal to the shift)

Predictions:

$$a = Xw$$

Training objective:

$$\min_w d_2(Xw, y)$$

Training formula:

$$w = (X^T X)^{-1} X^T y$$

The new weight vector after multiplying our features by P will look like this:

$$w_P = ((XP)^T XP)^{-1} (XP)^T y = (P^T X^T XP)^{-1} P^T X^T y = (P^T \cdot (X^T X) \cdot P)^{-1} P^T X^T y = P^{-1} \cdot (X^T X$$
$$= P^{-1} \cdot (X^T X)^{-1} \cdot E \cdot X^T y = P^{-1} \cdot (X^T X)^{-1} \cdot X^T y$$

In the above formula we can identify the training formula for w:

$$w = (X^T X)^{-1} X^T y$$

Let's transform it:

$$w_P = P^{-1} w$$

Now we'll make predictions using the formula for the new feature matrix and the weight vector

$$a_P = X_P w_P = XPP^{-1} w = Xw = Xw$$

That is

$$a_P = a$$

QED

Linear regression implementation

In [14]:

```
class LinearRegressionImpl:
    def fit(self, X, y):
        n_samples = len(y)
        X = np.hstack((np.ones((n_samples, 1)), X))
        w = np.linalg.inv(X.T @ X) @ X.T @ y
        self.w0 = w[0]
        self.w = w[1:]
        return self

    def predict(self, X):
        return X @ self.w + self.w0
```

Data preparation

Let's separate out features and the target and create a random masking matrix (P). Its shape must be $4 * 4$ because when we multiply our features matrix (X) of shape $4847 * 4$ by P, the masked features matrix (X_masked) will have the same shape as X.

In [15]:

```
X = df.drop('insurance_benefits', axis=1)
y = df['insurance_benefits']
P = np.random.rand(4,4) # Obfuscation/masking matrix
X_masked = X@P
X_masked.shape
```

Out[15]:

```
(4847, 4)
```

Let's test whether P is invertible.

In [22]:

```
np.linalg.inv(P)
```

Out[22]:

```
array([[-2.40267075,  0.03028028,  1.97512819,  0.15679336],
       [-0.27535015,  1.08181464, -0.36962234,  0.4562798 ],
       [ 1.38313115,  0.52462742, -0.09425914, -1.8109074 ],
       [ 1.26081924, -1.67409134, -0.34236605,  1.88690607]])
```

The inverse of P exists, so it's not a singular matrix and can be used for the masking purpose.

Algorithm testing

Original features LR implementation

In [25]:

```
model_1 = LinearRegressionImpl()
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=12345)
model_1.fit(X_train, y_train)
predictions_1 = model_1.predict(X_test)
r2_score(y_test, predictions_1)
```

Out[25]:

```
0.4230772749214826
```

Original features LR sklearn

In [26]:

```
model_2 = LinearRegression()
model_2.fit(X_train, y_train)
predictions_2 = model_2.predict(X_test)
r2_score(y_test, predictions_2)
```

Out[26]:

```
0.42307727492147296
```

Masked features LR implementation

In [27]:

```
model_3 = LinearRegressionImpl()
X_train, X_test, y_train, y_test = train_test_split(X_masked, y, random_state=12
345)
model_3.fit(X_train, y_train)
predictions_3 = model_3.predict(X_test)
r2_score(y_test, predictions_3)
```

Out[27]:

```
0.4230772729159781
```

Masked features LR sklearn

In [28]:

```
model_4 = LinearRegression()
model_4.fit(X_train, y_train)
predictions_4 = model_4.predict(X_test)
r2_score(y_test, predictions_4)
```

Out[28]:

```
0.4230772749215429
```

All 4 models resulted in approximately the same R2 metric value. It means that our data masking technique and our implementation of the linear regression didn't change the quality of the model.

Conclusion

In this project we have developed a data transforming algorithm for the Sure Tomorrow insurance company that makes it hard to recover personal information from the transformed data. Steps:

1. First of all, we have familiarized ourselves with the data by performing the descriptive statistics;
2. In the data preprocessing step we have converted column names to lower case, converted the `age` column into the integer type and removed duplicates;
3. Next, we have provided a theoretical proof based on the equation of linear regression that the quality of a model doesn't change if we mask features;
4. We have then implemented our own linear regression class;
5. Next, data was prepared for testing and we have created a random P matrix and made sure it was invertible and finally multiplied our features X by P;
6. We have tested the algorithm and came to the following conclusion: all 4 models resulted in approximately the same R2 metric value. It means that **our data masking technique and our implementation of the linear regression didn't change the quality of the model.**