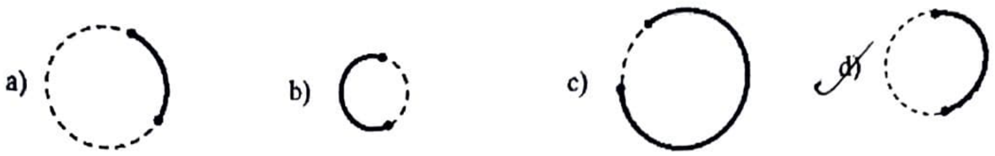
Tatsat Thakker

WORKBOOK: UNIT 1: THE UNIT CIRCLE (TRIG)

UNDERSTANDING RADIANS

1) Consider the arc drawn on each circle. Which arc measure is closest to 3 radians?

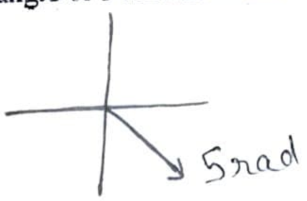


2) Draw the angle $-\frac{7\pi}{8}$ in standard position.

3) Which one of the following angles terminates in Quadrant III?

a) 3 radians   b) $\frac{7\pi}{5}$ radians   c) -210° d) 500°

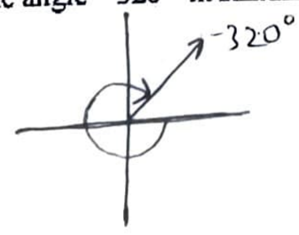
4) Sketch the angle of 5 radians in standard position.



5) Convert $-\frac{13\pi}{5}$ to degrees.

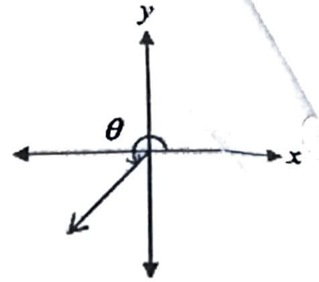
$-\frac{13\cancel{\pi}}{\cancel{5}} \times \frac{\cancel{180}\,36}{\cancel{\pi}} = \boxed{-468^\circ}$

6) Sketch the angle -320° in standard position.



7) The angle 2.95 radians, in standard position, terminates in quadrant: 2nd quadrant

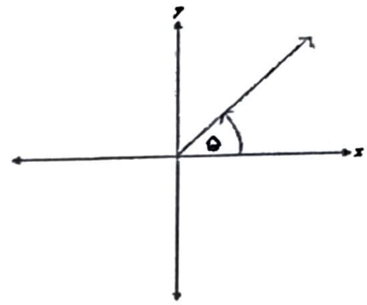
8) Identify a possible value for the angle θ sketched in standard position.



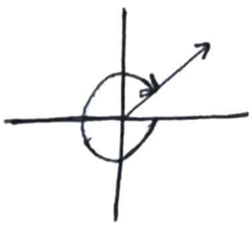
a) 2 b) 3 c) 4 d) 5

9) Tyler incorrectly sketched the angle $\theta = -\frac{7\pi}{4}$ in standard position.

Describe his error.



Rotated in wrong direction.



10) An angle in standard position measures $\frac{3\pi}{4}$.

Determine in which quadrant the terminal arm of this angle is located after a rotation of 3 radians.

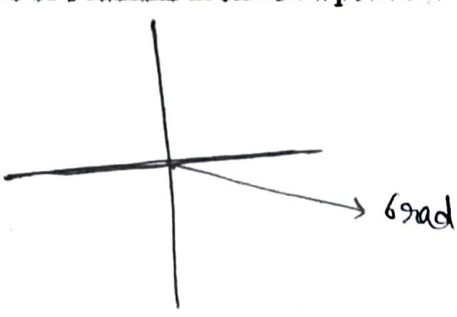
Justify your answer.

4th quadrant. Initially it is in 2nd quadrant and now it will be in 4th as nearlly 180° has ben rotated.

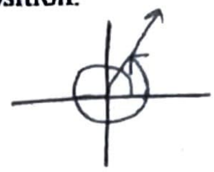
11) Identify 10° in radians.

a) $\frac{1800}{\pi}$ b) $\frac{\pi}{1800}$ c) $\frac{18}{\pi}$ d) $\frac{\pi}{18}$

12) Sketch the angle of 6 radians in standard position.



Sketch the angle $\frac{7\pi}{3}$ in standard position.

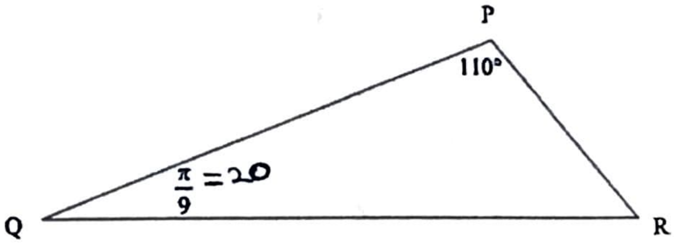


14) Identify the measure of the angle $-\frac{2\pi}{9}$ in degrees.

a) -400° ~~b) -40°~~ c) 40° d) 320°

$\frac{-2}{\cancel{9}} \times \cancel{180}^{20}$

15) Determine the measure of angle R, in radians.



$\angle R = 180 - 130$

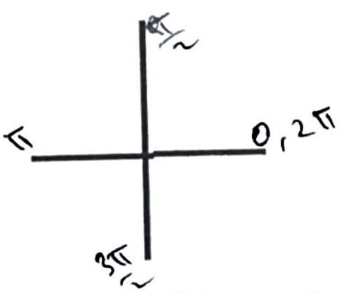
$\angle R = 50$

$\angle R = \frac{\cancel{50}^{5}\pi}{\cancel{180}_{18}} \Rightarrow \boxed{\angle R = \frac{5\pi}{18}}$

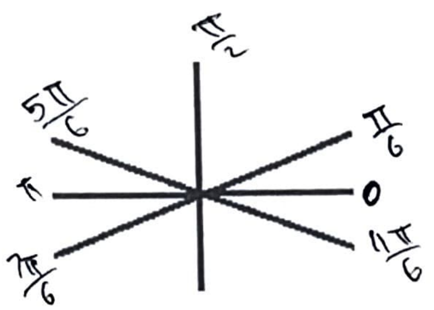
~~$\boxed{\angle R = \frac{5\pi}{6}}$~~

UNIT CIRCLE

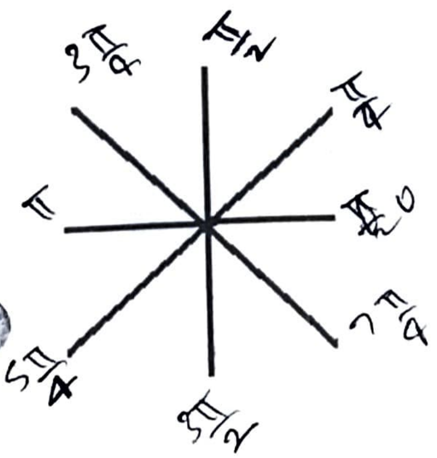
1) Place all the 90° increments on the circle in radians



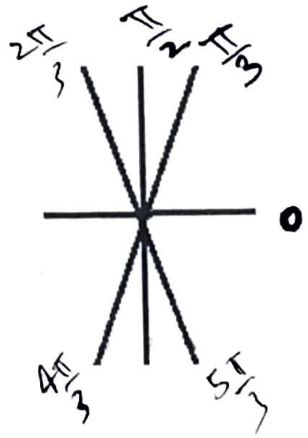
2) Place all the 30° increments on the circle in radians



3) Place all the 45° increments on the circle in radians



4) Place all the 60° increments on the circle in radians



6) Determine the coordinates of a point (x, y) on the unit circle if you are given $\theta = 30^\circ$ where θ is in standard position.

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

7) The point $P(\theta)$ lies on the unit circle. What are the coordinates of the point if $\theta = 300^\circ$?

a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ b) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ ~~d)~~ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

8) The point $P(\theta)$ lies on the unit circle. What are the coordinates of the point P if $\theta = 120^\circ$?

~~a)~~ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ b) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ c) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

ARC LENGTH

1) A student is using the formula $s = \theta r$ to find an arc length of a circle. Given a central angle measure of 35° and a radius of 6 cm, the student's solution is as follows:

$s = (35)(6)$

$s = 210$ cm

Explain why this solution is incorrect.

Write the correct solution.

The angle must be first converted into radians.

$35 \times \frac{\pi}{180} = \boxed{0.611 \text{ rad}}$

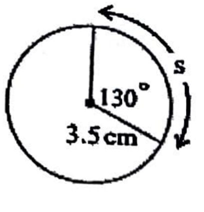
$S = 0.611(6)$

$\boxed{S = 3.666 \text{ cm}}$

2) A central angle of a circle subtends an arc length of 5π cm. Given the circle has a radius of 9 cm, find the measure of the central angle in degrees.

Use the information in the diagram to determine the value of the arc length "s".

3)



$s = \frac{\theta}{180} \pi r$

$= \frac{130}{180} \times 3.14(3.5)$

$= \boxed{7.9387 \text{ cm}}$

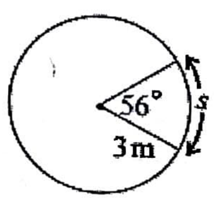
4) Determine the arc length subtended by a central angle if the diameter is 19 cm and the central angle is 1.6 radians.

$S = 19 \times 1.6$

$\boxed{S = 30.4 \text{ cm}}$

Use the information in the diagram to determine the value of the arc length "s", given the central angle of 56°.

5)



$S = \frac{56}{180} 3.14 \times 3$

$\boxed{S = 2.931}$

6) A pizza with a diameter of 15 inches is cut into equal slices, each with a central angle of 36°. Determine the length of the crust on the outer edge of one slice of pizza.

$S = \frac{1}{5}\left(\frac{15}{2}\right) 3.14$

$= \boxed{4.712 \text{ inches}}$

7) A wheel has a diameter of 20 cm and rotates through a central angle of 252°. Determine how far the wheel rolled.

$S = \frac{252}{180} \times 20 \times \pi$

$= \boxed{43.982}$

8) Determine the radius of a circle which has an arc length of 5 cm with a central angle of 3 radians.

$S = 5\text{ cm}$

$\theta = 3\text{ rad}$

$S = \theta r$

$5 = 3r$

$\boxed{r = 1.667\text{ cm}}$

9) Pierre pushes his car into a garage. The radius of a tire on his car is 22 cm. Determine the distance travelled by his car if the tire rotated a total of 1000°.

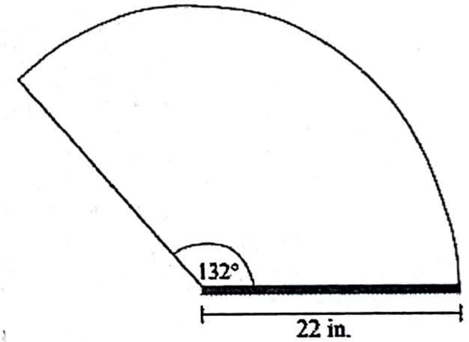
$S = \frac{100\cancel{0}}{18\cancel{0}} \times 22 \times \pi$

$= \boxed{383.972\text{ cm}}$

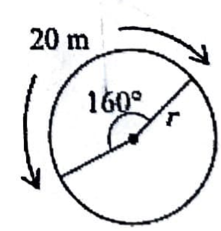
10) A section of a car windshield is cleaned by a wiper as shown in the diagram below. The arm of the wiper is 22 inches, and it rotates through a central angle of 132°. Determine the length of the arc that is created by the tip of the wiper.

$S = \frac{132}{180} 22\pi$

$= \boxed{50.684\text{ inches}}$



11) Determine the length of the radius, r, given an arc length of 20 metres and a central angle of 160°.



$S = 20\text{ m}$

$\theta = 160°$

$= \frac{16\cancel{0}\pi}{18\cancel{0}} = \frac{8\pi}{9}$

$= 2.793$

$r = \frac{S}{\theta} = \boxed{7.161}$

Ariane uses the formula $s = \theta r$ to determine the arc length of a circle that has a central angle of $20°$ and a radius of 15 cm.

Below is Ariane's work:

$s = \theta r$
$s = (20)(15)$
$s = 300$ cm

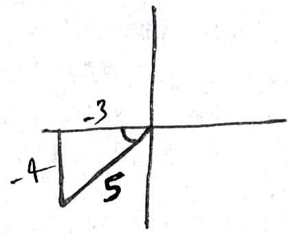
12) Describe her error.

XYR RATIOS

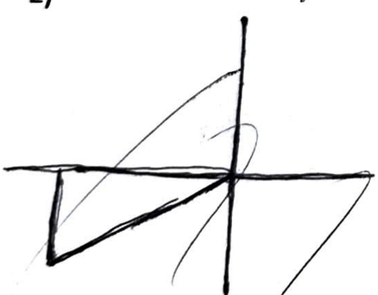
Your classmate, Leo, was absent for one of his math lessons.

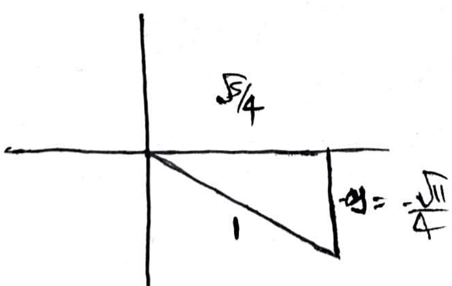
1) Explain to Leo how to determine the cosecant ratio for an angle in standard position given that $P(-3, -4)$ is a point on the terminal arm of the angle.

$\boxed{\csc\theta = \frac{-5}{4}}$



2) The terminal arm of an angle θ, in standard position, intersects the unit circle in Quadrant IV at a point $P\left(\frac{\sqrt{5}}{4}, y\right)$. Determine the value of $\sin\theta$.





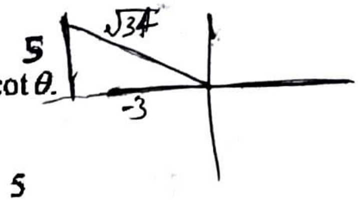
$y = \sqrt{1 - \frac{5}{16}}$

$y = \sqrt{\frac{11}{16}}$

$\boxed{y = \frac{-\sqrt{11}}{4}}$

$\boxed{\sin\theta = -\frac{\sqrt{11}}{4}}$

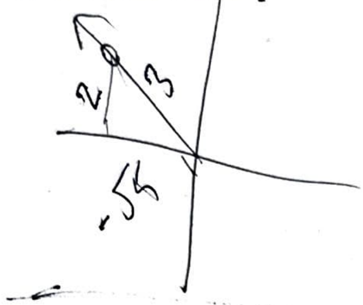
3) Given the point $A(-3, 5)$ on the terminal arm of an angle θ, identify the value of $\cot\theta$.



a) $-\frac{3}{5}$ b) $-\frac{5}{3}$ c) $-\frac{4}{5}$ d) $-\frac{5}{4}$

4) If θ terminates in quadrant II and $\csc\theta = \frac{3}{2}$, determine the exact value of $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$



$\csc^2\theta - \cot^2\theta = 1$

$\frac{9}{4} - \cot^2\theta = \frac{4}{4}$

$\sqrt{\frac{5}{4}}$ $\frac{\sqrt{5}}{2}$

$\tan\theta = -\frac{2}{\sqrt{5}}$

$-\frac{2\sqrt{5}}{5}$

5) Is the point $\left(\frac{3}{4}, -\frac{\sqrt{3}}{4}\right)$ on the unit circle?

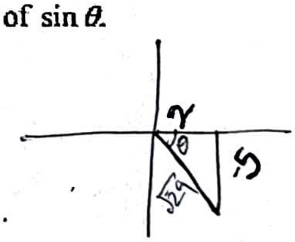
Justify your answer.

$\frac{9}{16} + \frac{3}{16} \neq 1$

no

6) Given that $\cot\theta = -\frac{2}{5}$, where θ is in Quadrant IV, determine the exact value of $\sin\theta$.

$\boxed{\sin\theta = \frac{-5}{\sqrt{29}}}$

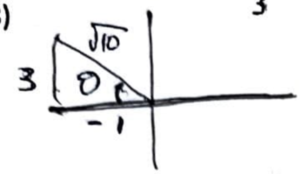


7) If θ terminates in quadrant III and $\cos\theta = -\frac{6}{7}$, determine the exact value of $\tan\theta$.

$\sin\theta = -\frac{\sqrt{13}}{7}$

$\boxed{\tan\theta = \frac{\sqrt{13}}{6}}$

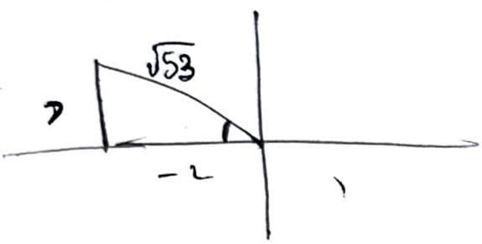
8) Given $\cot\theta = -\frac{1}{3}$, where θ is in quadrant II, determine the exact value of $\sin\theta$.



$\boxed{\sin\theta = \frac{3}{\sqrt{16}}}$

9) The point $(-2,7)$ is on the terminal arm of an angle in standard position.

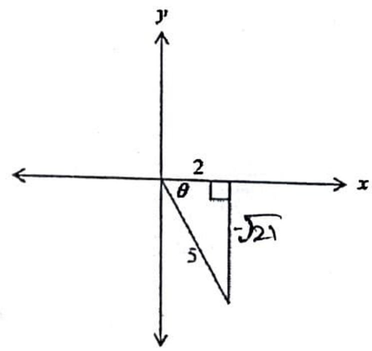
Determine the coordinates of the corresponding point, $P(\theta)$, on the unit circle.



$\boxed{P(\theta) = \frac{7}{\sqrt{53}}, \frac{-2}{\sqrt{53}}}$

10) Given the following triangle, determine $\csc\theta$.

$\boxed{\csc\theta = -\frac{5}{\sqrt{21}}}$



11) Determine if the point $\left(-\frac{\sqrt{7}}{5}, \frac{2}{5}\right)$ is on the unit circle.

Justify your answer.

Sum of squares of x & y = 1

$\left(\frac{-\sqrt{7}}{5}\right)^2 + \left(\frac{2}{5}\right)^2$

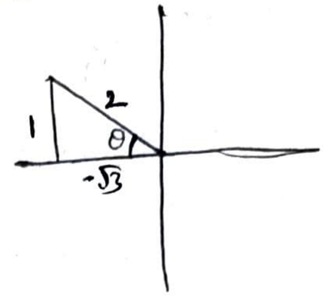
$= \frac{7+4}{25} = \frac{11}{25} \neq 1$

not on unit circle

12) The point $(-\sqrt{3}, 1)$ is on the terminal arm of an angle θ, in standard position.

a) Determine $\tan\theta$. $\frac{5\pi}{6}$ $\frac{-1}{\sqrt{3}}$

b) Determine a possible value of θ, in radians. $\frac{3\pi}{10}$ $\frac{5\pi}{6}$



13) The point $\left(-\frac{5}{6}, b\right)$ is on the unit circle and is in quadrant III.

Determine the exact value of b.

$$\left(-\frac{5}{6}\right)^2 + b^2 = 1$$

$$\frac{25}{36} + 36b^2 = 36$$

$$36b^2 = 11$$

$$\boxed{b = -\frac{\sqrt{11}}{6}}$$

14) Given $\csc\theta = -4$ and θ is in quadrant IV,

a) determine the exact value of $\cos\theta$.

b) determine the exact value of $\cot\theta$.

$$\sin\theta = \frac{-1}{4}$$

$$\cos\theta = \frac{\sqrt{15}}{4}$$

$$\boxed{\cot\theta = -\sqrt{15}}$$

15) Given $\sec\theta = -\frac{5}{4}$ and $\tan\theta > 0$, state the quadrant in which θ terminates.

Justify your answer.

$$\sec\theta = \frac{-5}{4}$$

This implies that $\cos\theta$ is negative & $\tan\theta$ is positive.

III[rd] quad

16) Given that $\csc\theta = -\frac{4}{\sqrt{7}}$ and $\cos\theta > 0$, determine the exact value of $\tan\theta$.

$\sin\theta = \frac{-\sqrt{7}}{4}$ $\cos\theta = \frac{3}{4}$

$\boxed{\tan\theta = \frac{-\sqrt{7}}{3}}$

17) Identify the quadrant in which θ terminates if $\sec\theta = -\frac{4}{3}$ and $\sin\theta > 0$.

IInd quadrant.

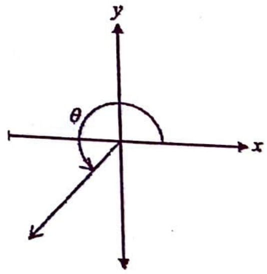
18) Determine the exact value of $\cot\theta$ if $\cos\theta = -\frac{4}{7}$ and $\sin\theta$ is positive.

$\cos\theta = \frac{-4}{7}$ $\sin\theta = \frac{\sqrt{35}}{7}$

$\boxed{\cot\theta = \frac{-4}{\sqrt{35}}}$

COTERMINAL ANGLES

1) Angle θ, measuring $\frac{5\pi}{4}$, is drawn in standard position as shown below.
Determine the measures of all angles in the interval $[-4\pi, 2\pi]$ that are coterminal with θ.



$\left[\frac{-16\pi}{4}, \frac{8\pi}{4}\right]$

$\boxed{\frac{-3\pi}{4}, \frac{-11\pi}{4}}$

2) Find the coterminal angle to $\frac{27\pi}{5}$ over the interval $[-360°, 0°)$.

$[-2\pi, 0]$ $\left[\frac{-10\pi}{5}, 0\right]$

$\boxed{\frac{-3\pi}{5}}$

Determine one positive and one negative coterminal angle with the angle $\frac{5\pi}{6}$.

$$\boxed{\frac{-7\pi}{6}, \frac{17\pi}{6}}$$

4) A co-terminal angle for $\theta = \frac{11\pi}{3}$ in the domain $-2\pi \le \theta \le 0$ would be:

a) $-\frac{5\pi}{3}$ b) $-\frac{\pi}{3}$ c) $\frac{\pi}{3}$ d) $\frac{5\pi}{3}$

5) Determine the coterminal angles with $\frac{2\pi}{3}$ over the interval $[-2\pi, 4\pi]$. $[-2\pi, 4\pi] = \left[-\frac{6\pi}{3}, \frac{12\pi}{3}\right]$

$-\frac{4\pi}{3}, \frac{8\pi}{3}$

6) State a coterminal angle for $\theta = \frac{9\pi}{4}$.

coterminal for $\theta = \frac{9\pi}{4} \Rightarrow \frac{\pi}{4}$

7) Given $\theta = 40^\circ$,

a) convert θ to radians. $\frac{40\pi}{180} = \boxed{\frac{2\pi}{9}}$

b) determine the coterminal angles of θ where $\theta \in \mathbb{R}$. ($\theta \in \mathbb{R}$ means the angle can be any real number)

$2n\pi + \theta$

8) Identify a coterminal angle for $\theta = -\frac{\pi}{3}$.

a) $\frac{\pi}{3}$ b) $\frac{4\pi}{3}$ c) $\frac{7\pi}{3}$ d) $\frac{11\pi}{3}$

9) Given the angle $\frac{25\pi}{7}$, identify the coterminal angle on the interval $[-2\pi, 0]$.

a) $\frac{18\pi}{7}$ b) $\frac{11\pi}{7}$ c) $-\frac{3\pi}{7}$ d) $-\frac{10\pi}{7}$

10) Identify the coterminal angle of $\frac{\pi}{5}$ over the interval $-\pi \leq \theta \leq 4\pi$.

$\left[\frac{-5\pi}{5}, \frac{20\pi}{5}\right]$

a) $-\frac{9\pi}{5}$ b) $-\frac{\pi}{5}$ c) $\frac{3\pi}{5}$ d) $\frac{11\pi}{5}$

11) Determine the coterminal angle of $\frac{\pi}{5}$ over the interval $[-2\pi, 0]$.

$\boxed{\frac{-9\pi}{5}}$

$\left[\frac{-10\pi}{5}, 0\right]$

12) Identify the expression that represents all angles that are coterminal with $\frac{\pi}{3}$.

a) $\frac{\pi}{3} + \pi k,\ k \in I$ b) $\frac{\pi}{3} + \pi k,\ k \in \mathbb{R}$ c) $\frac{\pi}{3} + 2\pi k,\ k \in I$ d) $\frac{\pi}{3} + 2\pi k,\ k \in \mathbb{R}$

$\sin\theta = "y"$ $\cos\theta = "x"$

Unit circle

EXACT VALUE

$\tan\theta = \frac{y}{x}$

1) Find the exact value of the following expression:

$\csc\theta = y$

$$\sin\left(\frac{11\pi}{3}\right) \cdot \sec\left(\frac{4\pi}{3}\right) \cdot \tan\left(-\frac{5\pi}{6}\right)$$

$-\frac{\sqrt{3}}{2} \times (-2) \times \left(\frac{-1}{\sqrt{3}}\right) = -1$

2) Explain how to find the exact value of $\sec\left(\frac{19\pi}{6}\right)$.

3rd quadrant

$\sec\left(\frac{19\pi}{6}\right) = \sec\left(\frac{18\pi}{6} + \frac{\pi}{6}\right)$

$= \sec\left(2\pi + \pi + \frac{\pi}{6}\right) = \sec\left(\pi + \frac{\pi}{6}\right)$

$= -\sec\frac{\pi}{6} = \boxed{\frac{-2}{\sqrt{3}}}$

3) Evaluate:

$$\csc\left(\frac{11\pi}{6}\right) + \sin^2\left(-\frac{3\pi}{4}\right) + \cos\left(\frac{23\pi}{3}\right)$$

$-2 + \frac{1}{2} + \left(\frac{-1}{2}\right) = \boxed{\frac{-1}{2}}$ -1

-2

4) Evaluate:

$$\left(\sin\frac{11\pi}{3}\right)\left(\sec\frac{11\pi}{6}\right)$$

$\cancel{\left(\frac{\sqrt{3}}{2}\right)\left(2\right)}$ $= \sqrt{3}$

$\frac{-\sqrt{3}}{2}\left(\frac{2}{\sqrt{3}}\right) = -1$

5) Evaluate and simplify $\sec\left(\frac{5\pi}{6}\right) \cdot \tan\left(-\frac{\pi}{6}\right)$.

$\cancel{-2 \cdot \frac{-1}{\sqrt{3}} = \boxed{\frac{2}{\sqrt{3}}}}$

$\frac{-2}{\sqrt{3}} \cdot \frac{-1}{\sqrt{3}} = \boxed{\frac{2}{3}}$

Evaluate:

6) $\left(\cos\frac{11\pi}{3}\right)\left(\csc\frac{11\pi}{6}\right)$

$\left(\frac{1}{2}\right)(-2) = -1$

7) Evaluate $\cos\left(\cos\left(\frac{3\pi}{2}\right)\right)$. 1

a) 1 b) ½ c) 0 d) -1

Evaluate:

8) $\sec^2\left(\frac{\pi}{6}\right) + \tan\left(\frac{7\pi}{6}\right)\csc\left(-\frac{2\pi}{3}\right)$

$\frac{4}{3} + \left(\frac{-1}{\sqrt{3}}\right)\left(\frac{-2}{\sqrt{3}}\right)$

$\frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

Evaluate.

9) $\dfrac{\cot\left(-\frac{5\pi}{6}\right)}{\sin\left(\frac{17\pi}{3}\right)}$ $\qquad \dfrac{\sqrt{3}}{\frac{-\sqrt{3}}{2}} = -2$

Evaluat

10)

Evaluate the following expression.

10)
$$\tan\left(\frac{2\pi}{3}\right)\csc\left(\frac{-2\pi}{3}\right)+\cos(3\pi)$$

$-\sqrt{3}\left(\frac{-2}{\sqrt{3}}\right) + (-1)$

$= \cancel{(3)}$ 1

11) Identify the exact value of $\sec\left(-\frac{7\pi}{3}\right)$.

$+\sec\left(\frac{7\pi}{3}\right) = +\sec\left(2\pi + \frac{\pi}{3}\right)$

$+2$

~~a) −2~~

b) $-\frac{2}{\sqrt{3}}$

c) $\frac{2}{\sqrt{3}}$

~~d)~~ 2

Evaluate.

$$\frac{\cot\left(\frac{11\pi}{6}\right)\sin\left(-\frac{4\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)}$$

$\frac{(-\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)}{\frac{-1}{2}} = 3$

12)

Evaluate.

$$\sin^2\left(-\frac{\pi}{3}\right)+\cos\left(\frac{17\pi}{6}\right)\sec\left(\frac{\pi}{6}\right)$$

$\frac{3}{4} + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right)$

$= \boxed{\frac{-1}{4}}$

13)

Evaluate.

14) $\cos\left(\pi \bullet \sin\left(-\frac{\pi}{6}\right)\right)$ $\quad \cos\left(-\frac{\pi}{2}\right)$

$= \boxed{0}$

SOLVING TRIG EQUATIONS

Gina correctly started to answer the following question. Complete her solution.

Question: Solve the following equation for all real values of θ.
Express your answer in radians correct to 3 decimal places.

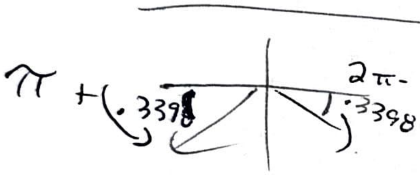
$$3\sin^2\theta - 14\sin\theta - 5 = 0$$

Gina's solution: $3\sin^2\theta - 14\sin\theta - 5 = 0$

1) $(3\sin\theta + 1)(\sin\theta - 5) = 0$

$\sin\theta = -\frac{1}{3}$ | $\sin\theta = 5$

NO SOL



$\sin^{-1}(1 \div 3)$

$\theta = 5.943 + 2\pi k \quad k \in I$

$\theta = 3.481 + 2\pi k \quad k \in I$

NO SOL

2) Which of the following represents the general solution to the equation $\tan\theta = -1$?

a) $\theta = \frac{\pi}{4} + 2k\pi, k \in I$ b) $\theta = \frac{\pi}{4} + k\pi, k \in I$ c) $\theta = \frac{3\pi}{4} + 2k\pi, k \in I$ d) $\theta = \frac{3\pi}{4} + k\pi, k \in I$

Solve the equation $\csc^2\theta + 3\csc\theta - 4 = 0$ over the interval $[0, 2\pi]$.
Express your answers as exact values or correct to 3 decimal places.

$(\csc\theta + 4)(\csc\theta - 1) = 0$

$\csc\theta = -4$ or $\csc\theta = 1$

$\sin\theta = \frac{-1}{4}$ $\quad\quad$ $\sin\theta = 1$

$= \frac{\pi}{2}$

$\theta = 3.394$ rad

$\theta = 6.539$ rad

4) Solve the following equation over the interval $0 \le \theta < 2\pi$.

$(\tan\theta - 3)(\tan\theta + 1) = 0$

$\tan\theta = 3$ or $\tan\theta = -1$

$\theta = 1.250$ $\quad\quad$ $\theta = \frac{3\pi}{4}$

$\theta = 4.391$ $\quad\quad$ $\theta = \frac{7\pi}{4}$

5) The general solution to the equation $\cos\theta = -\frac{1}{2}$ is:

a) $\left.\begin{array}{l} \theta = \frac{\pi}{3} + 2\pi k \\ \theta = \frac{5\pi}{3} + 2\pi k \end{array}\right\}$ where $k \in I$

b) $\left.\begin{array}{l} \theta = \frac{\pi}{3} + \pi k \\ \theta = \frac{5\pi}{3} + \pi k \end{array}\right\}$ where $k \in I$

c) $\left.\begin{array}{l} \theta = \frac{2\pi}{3} + 2\pi k \\ \theta = \frac{4\pi}{3} + 2\pi k \end{array}\right\}$ where $k \in I$

d) $\left.\begin{array}{l} \theta = \frac{2\pi}{3} + \pi k \\ \theta = \frac{4\pi}{3} + \pi k \end{array}\right\}$ where $k \in I$

Solve the following equation over the interval $[0, 2\pi)$.

$\tan^2\theta + 2.8\tan\theta + 1.96 = 0$

Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$.

6)

$\tan\theta = \frac{-2.8 \pm \sqrt{0}}{2a}$

$= -1.4$

$\theta = 2.191$

$\theta = 5.333$

Given the equation $2\sin^2\theta - 3\sin\theta + 1 = 0$, verify that $\theta = \frac{\pi}{2}$ is a solution.

7)

For $\theta = \frac{\pi}{2}$,

$2\sin^2\left(\frac{\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right) + 1$

$= 2 - 3 + 1 = 0$ $\boxed{LHS = RHS}$

Explain why the equation $\sec\theta = \frac{1}{4}$ has no solution.

8)

For $\sec\theta = 1/4$

$\cos\theta = 4$

no sol.

9) Talla incorrectly solved the following trigonometric equation:

Solve: $2\sec x - 5 = 0;\ 0^\circ \le x \le 360^\circ$.

Talla's work:

$2\sec x - 5 = 0$

$\sec x = \frac{5}{2}$

No solution, sec x cannot be greater than 1.

a) Explain her error.

There is some solution that exists.

$\sec x = \frac{5}{2}$

$\cos x = \frac{2}{5}$

b) Determine the correct solution.

$\cos x = \frac{2}{5}$

$\boxed{x = 66.422 \quad x = 293.578}$

10) Solve $\tan^2\theta - 5\tan\theta + 4 = 0$ where $\theta \in \mathbb{R}$.

$(\tan\theta - 4)(\tan\theta - 1) = 0$

$\tan\theta = 4$ $\quad$ $\tan\theta = +1$

$\boxed{1.326 + \pi k}$

$\cancel{\theta = \frac{\pi}{4} + 2k\pi}$

$\cancel{\theta = \frac{5\pi}{4} + 2k\pi}$

$\boxed{\theta = \frac{\pi}{4} + \pi k}$

11) Explain why there is no solution for the equation $\csc\theta = -\frac{1}{2}$.

$\csc\theta = \frac{-1}{2}$

$\sin\theta = -2$

not possible

Solve the following equation over the interval $[0, 2\pi]$:

12) $\sin^2\theta + 6\sin\theta - 2 = 0$

$\sin\theta = \frac{-6 \pm \sqrt{44}}{2}$

$= \boxed{-3 \pm \sqrt{11}}$

$\sin\theta = -0.317$

$\boxed{\theta = 3.464 \text{ rad}}$

$\boxed{\theta = 5.960 \text{ rad}}$

13) Solve $(2\sin\theta - 1)(\sin\theta + 1) = 0$ where $\theta \in \mathbb{R}$.

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\theta = \frac{\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$.

$\theta = \frac{5\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$

$\theta = \frac{3\pi}{2} + 2k\pi$, $k \in \mathbb{Z}$.

Solve the following equation over the interval $[0, 2\pi]$:

14) $3\sin^2\theta - 10\sin\theta - 8 = 0$

$\sin\theta = y$

$3y^2 - 12y + 2y - 8 = 0$

$3y(y-4) + 2(y-4) = 0$

$(3y+2)(y-4) = 0$

$y = \frac{-2}{3}$

~~$y = 3.871$~~ $\theta = 3.871$ &

$\theta = 5.553$

15) Identify the equation that has a general solution of $\left.\begin{array}{l}\theta=\frac{\pi}{6}+2\pi k \\ \theta=\frac{5\pi}{6}+2\pi k\end{array}\right\}$ where $k \in I$

a) $\sin\theta=\frac{1}{2}$ ✓ b) $\cos\theta=\frac{1}{2}$ c) $\sin\theta=\frac{\sqrt{3}}{2}$ d) $\cos\theta=\frac{\sqrt{3}}{2}$

16) Describe the error that was made when solving the following equation:

$\sin^2\theta+\sin\theta-2=1$

$\sin^2\theta+\sin\theta=3$

$\sin\theta(\sin\theta+1)=3$

$\sin\theta=3$ $\sin\theta+1=3$

$\sin\theta=2$

$\therefore$ No solution $\therefore$ No solution

The solve tried to isolate but was expected was that to make equation $=0$,

17) Solve the following equation algebraically over the interval $0 \le \theta \le 2\pi$.

$2\cos^2\theta+9\cos\theta-5=0$

$\cos\theta = u$

$2u^2+9u-5=0$

$2u^2+10u - u-5=0$

$(2u-1)(u+5)=0$

$u=\frac{1}{2}$

Solve the following equation algebraically over the interval $[0,2\pi]$.

18) $6\sin^2\theta+\sin\theta-1=0$

$6\sin^2\theta + 3\sin\theta - 2\sin\theta - 1 = 0$

$3\sin\theta(2\sin\theta+1) - 1(2\sin\theta+1) = 0$

$(3\sin\theta-1)(2\sin\theta+1) = 0$

$\sin\theta = \frac{1}{3}$, $\sin\theta = \frac{-1}{2}$

$\theta = 0.340$ rad | $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta = 2.802$ rad

Maurice incorrectly solved the equation, $\sin\theta+1=0$, over the interval $[0°,360°]$.

$\sin\theta + 1 = 0$

$\sin\theta = -1$

$\sin\theta = 270°$

19) Describe his error.

On the last step she accidently wrote 270° as $\sin\theta$ which is instead the value of θ.

20) Solve $\sec\theta+2=0$ over the interval $[0, 2\pi]$.

~~sin~~ $\cos\theta = \frac{-1}{2}$

$\frac{2\pi}{3}, \frac{4\pi}{3}$

21) Solve $\sin\theta=-\frac{\sqrt{3}}{2}$, where $\theta\in\mathbb{R}$.

$\theta = \frac{4\pi}{3} + 2\pi k$, $k \in I$ & $\theta = \frac{5\pi}{3} + 2\pi k$, $k \in I$

Verify that $\theta = \frac{4\pi}{3}$ is a solution of the equation $4\cos^2\theta - 1 = 0$.

For $\theta = \frac{4\pi}{3}$ to be the solution, by plugging $\theta = \frac{4\pi}{3}$, the equation must satisfy.

$4\cos^2\theta - 1 = 0 \Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}$

For $\theta = \frac{4\pi}{3}$

$\cos\theta = \frac{-1}{2}$

which satisfies the eq.

Solve for θ, algebraically, over the interval $[0, 2\pi]$.

$\csc^2\theta + 2\csc\theta - 8 = 0$

23)

$\cancel{\csc^2}\ (\csc\theta + 4)(\csc\theta - 2) = 0$

$\csc\theta = -4, 2$

$\sin\theta = \frac{-1}{4}, -\frac{1}{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6}$ | $3.395, 6.030$

Solve, algebraically, over the interval $[0, 2\pi)$.

$\sin x(\sec x + 3) = 0$

24)

$\sin x(\sec x + 3) = 0$

$\sin x = 0$ or $\sec x = -3$

$\cos x = \frac{-1}{3}$

$x = 0$

$x = 1.911, 4.373$

25) Verify that the equation $2\cos^2 x = \sin x + 1$ is true for $x = \frac{\pi}{6}$.

For $x = \frac{\pi}{6}$

LHS (Left hand side) $= 2\cos^2\frac{\pi}{6}$

$= 2\left(\frac{\sqrt{3}}{2}\right)^2$

$= 2\left(\frac{3}{4}\right)$

$= \boxed{\frac{3}{2}}$

RHS (Right hand side) $= \sin x + 1$

$= \frac{1}{2} + 1$

$= \boxed{\frac{3}{2}}$

LHS = RHS, Hence proved.

26) Given $\sin\theta = \frac{1}{2}$, determine all possible values of θ over the interval $[-2\pi, 2\pi]$.

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-7\pi}{6}, \frac{-11\pi}{6}$

27) Solve $2\sin^2\theta - 7\sin\theta - 4 = 0$ where $\theta \in \mathbb{R}$.

Let $\sin\theta = x$

$2x^2 - 7x - 4 = 0$

$2x^2 - 8x + x - 4 = 0$

$2x(x-4) + 1(x-4) = 0$

$(2x+1)(x-4) = 0$

$x = 4$, $x = \frac{-1}{2}$

$\downarrow$ not possible

$\downarrow$ $\sin\theta = \frac{-1}{2}$

$\boxed{\theta = \frac{7\pi}{6} + 2\pi k \quad k \in \mathbb{Z}}$ $\boxed{\theta = \frac{11\pi}{6} + 2\pi k \quad k \in \mathbb{Z}}$

Kennedy was asked to solve the equation $\tan\theta = 1$ over all real numbers.

Below is Kennedy's solution:

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

28) Describe her error.

Kennedy was asked to answer all the real numbered angles for the equation. In order to do so, she has to add $+2\pi k, k \in I$ to her answers.

Solve for θ, algebraically, over the interval $[0,2\pi]$.

29) $3\sin^2\theta + 6\sin\theta + 2 = 0$

$$3\sin^2\theta + 6\sin\theta + 2 = 0$$

$$\sin\theta = \frac{-6 \pm \sqrt{12}}{6}$$

$$= \frac{-6 \pm 2\sqrt{3}}{6}$$

$$= \frac{-3 \pm \sqrt{3}}{3}$$

$$\sin\theta = -0.422$$

$$\theta = 3.578, 5.847$$