

Charotar University of Science and Technology [CHARUSAT]
Faculty of Technology and Engineering
Department of Mathematical Sciences
MA 201.01 Discrete Mathematics
First Internal Exam

Semester: 3rd Sem B.Tech. CE

Date: 16/08/2016 (Tuesday)

Maximum Marks: 30

Time: 09:45am to 10:45am

Instructions:

- (i) All questions are compulsory.
- (ii) Figures to the right indicate **full** marks.
- (iii) Draw figure where it is required.

Section-I**Q.1 Do as directed.**

- (a) Out of 250 candidates who got distinction in an examination, it was revealed that 128 got distinction in Mathematics, 87 in Physics, and 134 in Chemistry. 31 got distinction in Mathematics and Physics, 54 got distinction in Chemistry and Mathematics, 30 got distinction in Chemistry and Physics. Find how many candidates got distinction in Chemistry but not in Mathematics. **01**
- (b) Write given statement into an equivalent logical expression by using predicate logic. **01**
 "Every even number greater than 2 can be expressed as the sum of two primes."

Q.2 Do as Directed.

- (a) State Principle of Mathematical Induction and Principle of Strong Induction. **04**
 If x and y denote any pair of real numbers for which $0 < x < y$, prove by using mathematical induction $0 < x^n < y^n$ for all natural numbers n .

(b) Attempt any three.

1. Show that $(A - B) = A - (A \cap B) = (A \cup B) - B$ **03**

2. Prove that the set of all 3×3 matrices with real entries of the form $\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$ is a **03**

group. Where Multiplication is defined by

$$\begin{bmatrix} 1 & a_1 & b_1 \\ 0 & 1 & c_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a_2 & b_2 \\ 0 & 1 & c_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a_1 + a_2 & b_1 + a_1 c_2 + b_2 \\ 0 & 1 & c_1 + c_2 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Define principal conjunctive normal form and obtain the principal conjunctive normal form without using truth tables for **03**

$$(\neg p \Rightarrow r) \wedge (q \Leftrightarrow p)$$

4. Check the validity of the following arguments. **03**

1) "If I drive to work, then I will arrive in time. I will not arrive in time. Therefore, I do not drive to work."

2) "If today is Tuesday, then yesterday was Monday. Yesterday was Monday. Therefore, today is Tuesday."

Section-II

Q.3 Do as directed.

(a) **True/False:** “ $K_{3,3}$ is planar graph.”

01

(b) **Define:** “Nodebase”

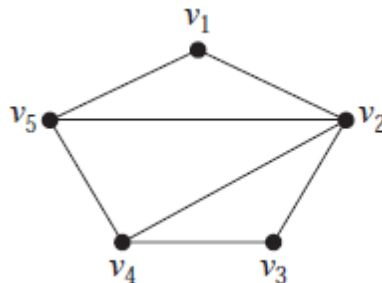
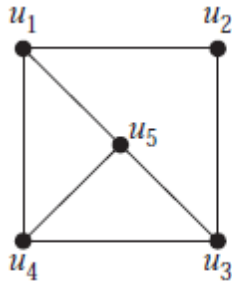
01

Q.4 Do as Directed.

(a) Define “Graph Isomorphism”

05

Following pair of graphs is isomorphic or not. Justify your answer.



(b) **Attempt any two.**

1. Define: “Adjacency Matrix”. Draw a graph from the given Matrix

04

$$A = \begin{pmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \\ 1 & -1 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix}$$

2. A connected simple graph has seven vertices with degrees 3, 3, 2, 3, 3, 2 and 4. How many edges are there? How many regions are there? Draw graph.

04

3. **Define:** Tree, spanning tree, m-ary tree, height of tree

04

Charotar University of Science and Technology [CHARUSAT]
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MA 201.01 Discrete Mathematics
Second Internal Exam

Semester: 3rd Sem B.Tech. CE

Date: 21/10/2016 (Friday)

Maximum Marks: 30

Time: 09:45 am to 10:45 am

Instructions:

- (i) All questions are compulsory.
- (ii) Figures to the right indicate **full** marks.
- (iii) Draw figure where it is required.

Section-I**Q-1 Do as directed.**

- a. Is there any Boolean algebra with odd number of elements? Justify your answer. **01**
- b. Define a POSET. **01**

Q-2 Let $(L, *, \oplus)$ be a lattice, then prove $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b, \forall a, b \in L$ **05**

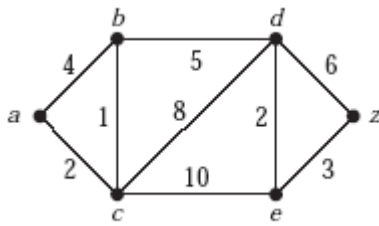
Q-3 Attempt any two.

- a. Define sub-lattice and discuss the set $S = \{1, 5, 10, 70\}$ is a sub-lattice of (S_{70}, D) or not. **04**
- b. Find value of a Boolean function $f : (P(X))^3 \rightarrow P(X)$ defined by **04**
 $f(a, b, c) = (ab)' + cb + a'$ for $a = \{l, m\}, b = \{m, n\}, c = \emptyset$, where $X = \{l, m, n\}$.
- c. Define meet irreducible elements and join irreducible elements of a lattice and find out meet irreducible and join irreducible elements for $(S_{10}, *, \oplus)$. **04**
- d. Obtain partitions of the set \mathbb{N} with respect to the relation defined as **04**
 $R := \{(x, y) : x \in \mathbb{N}, y \in \mathbb{N}, x - y \text{ is multiple of } 2\}$.

Section-II**Q.4 Do as directed.**

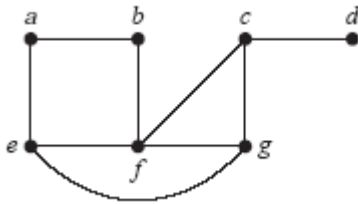
- a. How many distinct spanning trees can be obtained from the graph K_4 ? **01**
- b. Is recurrence relation $a_n = a_{n-1} - (a_{n-2})^2$ linear? Justify your answer. **01**

- Q-5** Apply Dijkstra's algorithm to find shortest path from vertex a to vertex z in following weighted graph. **05**

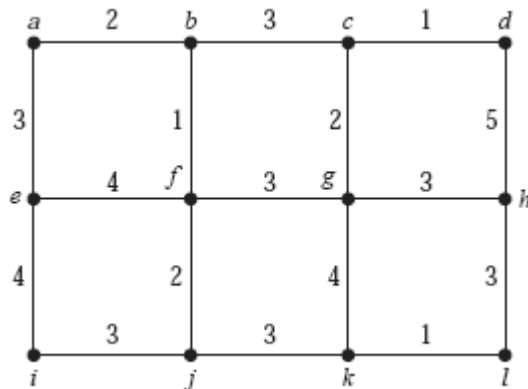


- Q-6** Attempt any two.

- a. Find the solution to the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$, with initial conditions $a_0 = 1, a_1 = -2$ and $a_2 = -1$. **04**
- b. Find all spanning tree of the following simple graph . **04**



- c. Solve the recurrence relation $a_k = 8a_{k-1}$ for $k = 1, 2, 3, \dots$ with given initial condition $a_0 = 9$ using generating function. **04**
- d. Use Kruskal's algorithm to find a minimum spanning tree in the following graph. **04**



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Faculty of Technology and Engineering
Department of Mathematical Sciences
MA201.01 Discrete Mathematics
Internal Exam (D2D Students)

Semester: 3rd Sem B. Tech. CE

Maximum Marks: 50

Date: 10/11/2016 (Thursday)

Time: 01:10 pm to 03:10 pm

Instructions:

- (i) All questions are compulsory.
- (ii) Figures to the right indicate **full** marks.
- (iii) Draw figure where it is required

Q-1 Do as directed.

- a. Let $A = \{1, 2, 3, 4\}$ then give directed graph and matrix representation of the relation R on set A defined as $R := \{(x, y) : x \in A, y \in A, x \leq y\}$. **01**
- b. Whether the statement " $K_{3,3}$ is a planar graph." is true or false? **01**
- c. Whether the statement "All the entries of undirected graph in a incidence matrix are either zero or one." is true or false? **01**
- d. Out of 80 students in a class, 60 play football, 53 play hockey and 35 play both games. How many students do not play any of these games? **01**
- e. Define: Semi group. **01**
- f. Is the operation $*$ defined on the set of integers \mathbb{Z} as $x * y = x \div y \forall x, y \in \mathbb{Z}$ a binary operation? Justify. **01**
- g. State Stone's representation theorem. **01**
- h. Convert the statement "79 is not a perfect square." into its logical equivalent expression containing appropriate predicate and quantifier. **01**

Q-2

- a. State principle of mathematical induction and principle of strong mathematical induction. Use appropriate principle to prove $1 + 2 + \dots + n = \frac{n(n+1)}{2}$. **05**
- b. **Attempt any two.**
 - 1. Use truth table to obtain principle conjunctive normal form of the logical expression $(\neg p \Rightarrow r) \wedge (q \Leftrightarrow p)$. **04**
 - 2. Apply inference rules to check validity of the following argument. **04**
 "If I drive to work then I will arrive in time. I do not drive to work. Therefore, I will not arrive in time."
 - 3. Check whether the set of logical connectives $\{\neg, \wedge\}$ is a minimal functionally complete set of logical connectives or not. **04**
 - 4. If $B \subseteq A$ then show that $B = A \setminus (A \setminus B)$. **04**

Q-3

- a. State and prove isotonicity property for a lattice. **04**
- b. Consider Boolean algebra $(S_{70}, *, \oplus, ', 1, 0)$ with $a * b = \gcd\{a, b\}$ & $a \oplus b = \text{lcm}\{a, b\}$ then find image of $(1, 10, 70)$ under the Boolean function $f : (S_{70})^3 \rightarrow S_{70}$ defined as $f(x, y, z) = (xy)' + zy + x'$. **04**

OR

Q-3

- a. Check whether $L = \{1, 5, 10\}$ is a sub - lattice of (S_{30}, D) . **04**
- b. Show that there is no Boolean algebra with three elements. **04**

Q-4

- a. Define: Group, Ring. **04**
- b. Let $H = \{0, \pm 4, \pm 8, \dots\}$ be a subgroup of $(\mathbb{Z}, +)$ then find all left cosets of H in $(\mathbb{Z}, +)$. **04**

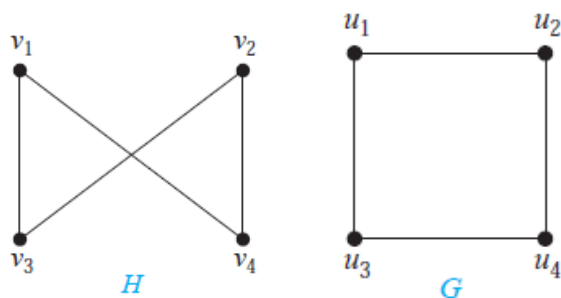
OR

Q-4

- a. Check whether $\{0, 1, 2, 3\}$ with respect to addition modulo 4 is a group or not. **04**
- b. Prove that the set $G = \{1, -1, i, -i\}$ with respect to usual complex multiplication forms a cyclic group with generator i. **04**

Q-5

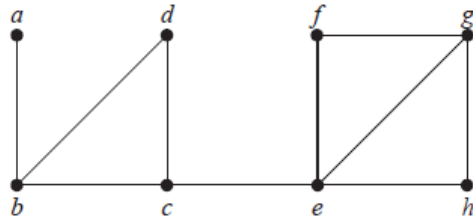
- a. Define “Graph Isomorphism” and check whether graphs $G = (V, E)$ and $H = (W, F)$, displayed in figure are isomorphic or not? **05**



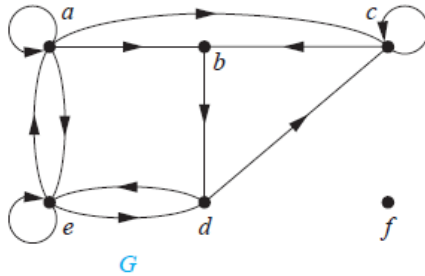
- b. Attempt any two.
1. Define: “Adjacency Matrix and Incidence Matrix”. Draw a graph from the given Matrix. **04**
(ordering of vertices is a, b, c, d)

$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}.$$

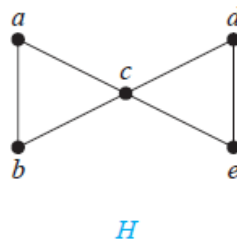
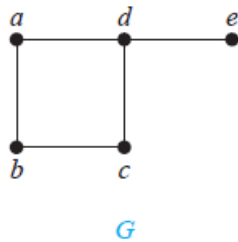
2. Find the cut vertices, cut edges, edge connectivity and vertex connectivity in the following graph. 04



3. Define Degree of a vertex of Directed Graph. Find the in-degree and out-degree of each vertex in the following shown graph G with directed edges. 04



4. Define Euler path and Euler circuit of Graph. Following graphs have Hamiltonian circuit or not. 04



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MA201.01 Discrete Mathematics
Internal Improvement Test

Semester: 3rd Semester B. Tech. (CE)

Maximum Marks: 30

Date: 02/05/2017 (Tuesday)

Time: 09:15 am to 10:15 am

Instructions:

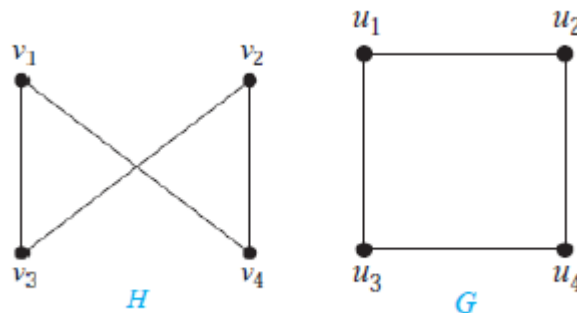
- (i) Figures to the right indicate *full* marks.
- (ii) Draw figure where it is required.
- (iii) Use of scientific calculator is allowed.

Q-1 Do as directed.

- (a) Draw all trees with 4 vertices. [01]
- (b) Check whether the statement " $K_{m,n}$ is regular if $m \neq n$ " is true or false.. [01]
- (c) In propositional logic a proposition which is always false is known as _____. [01]
- (d) Convert the statement "79 is not a perfect square." into its logical equivalent expression containing appropriate predicate and quantifier. [01]
- (e) State converse of the statement "If a quadrilateral is square then it must be parallelogram." [01]

Q-2 Attempt any five.**[25]**

- (a) Prove the set of logical connectives $\{ \neg, \wedge \}$ is a minimal functionally complete set of logical connectives.
- (b) Define "Graph Isomorphism" and check whether graphs $G = (V, E)$ and $H = (W, F)$, displayed in figure are isomorphic or not?

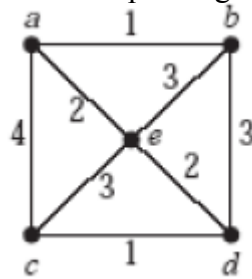


- (c) Define: "Adjacency Matrix and Incidence Matrix". Draw a graph from the given Matrix. (ordering of vertices is a, b, c, d)

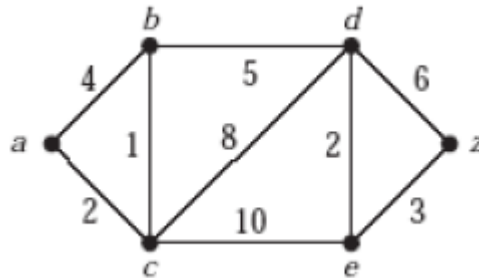
$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

- (d) Define: Tree, spanning tree, m-ary tree, height of a tree, minimal spanning trees.

- (e) Apply Prim's algorithm to find minimum spanning tree for following graph.



- (f) Apply Dijkstra's algorithm to find shortest path from vertex a to vertex z in following weighted graph.



- (g) State Principle of Mathematical Induction and Principle of Strong Induction. Prove by using mathematical induction $5^k - 4k - 1$ is divisible by 16 for all natural numbers k .
- (h) Define principal conjunctive normal form and obtain the principal conjunctive normal form without using truth tables for $(\neg p \Rightarrow r) \wedge (p \Leftrightarrow q)$.