First La Serena School for Data Science, 2013.

Image Synthesis in Radio Interferometry

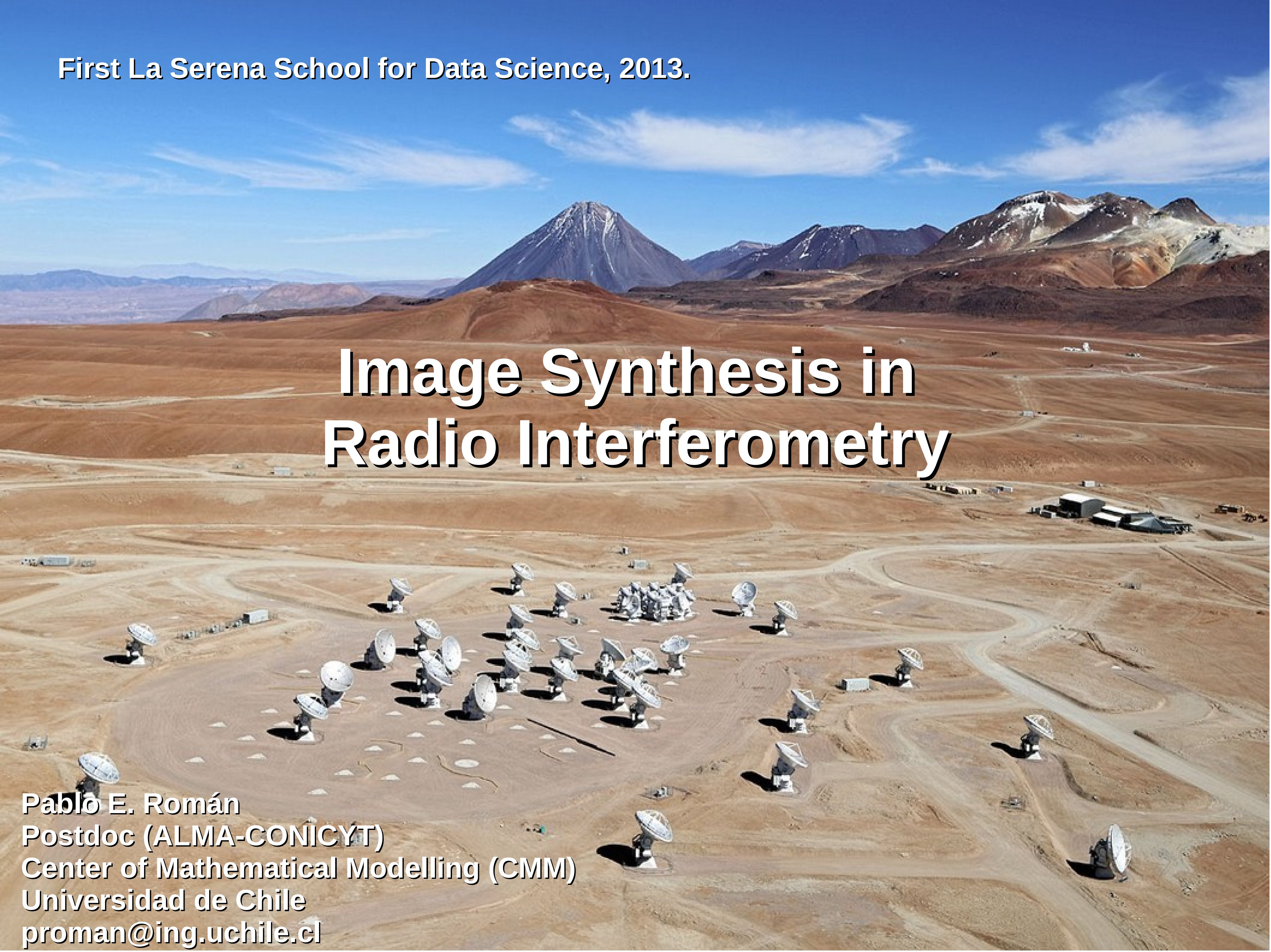

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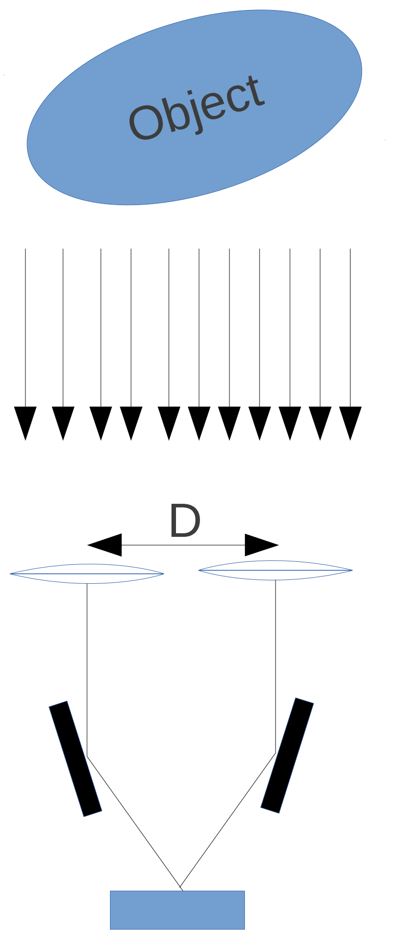
Image Synthesis in Radio Interferometry

- **Today morning**:
  - What is interferometry about
  - Why interferometry is cool.
  - Image Synthesis: How to recover an image.
  - Current trends

- **Afternoon**:
  - Practical Image Synthesis with CASA: Science verification data.

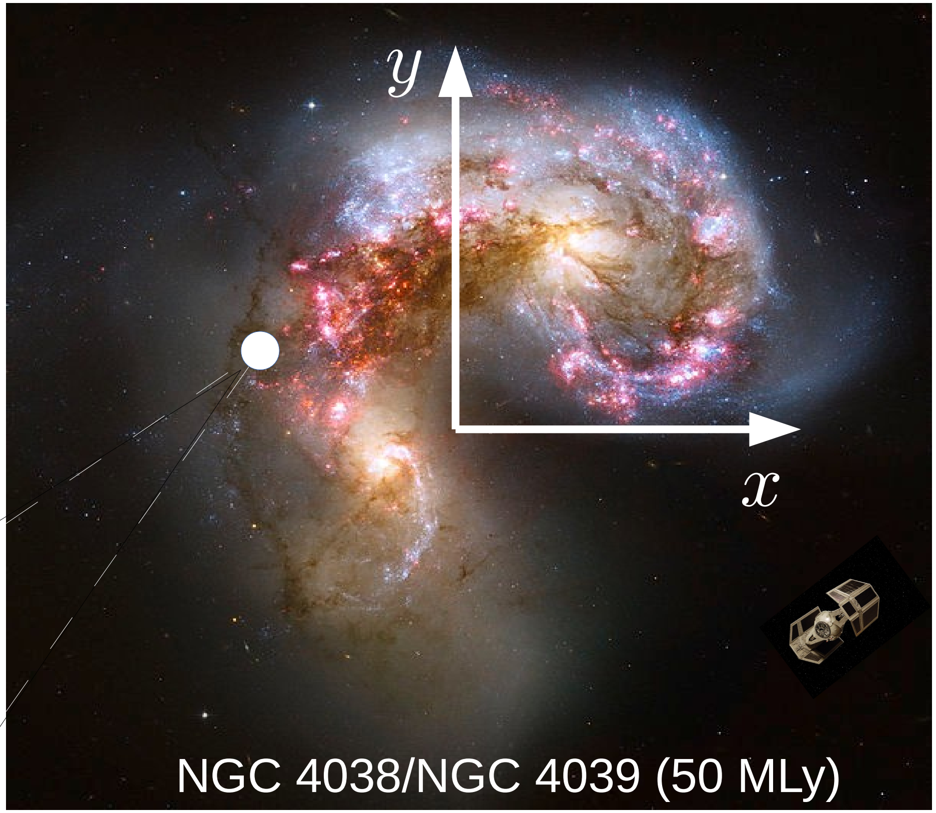
Interferometry

- Double Slit experiment.
- Michelson stellar interferometer.
- Resolution: λ/D
- Instead of having a HUGE telescope use a large D!



Radio Interferometry

A long time ago
in a galaxy far, far away...

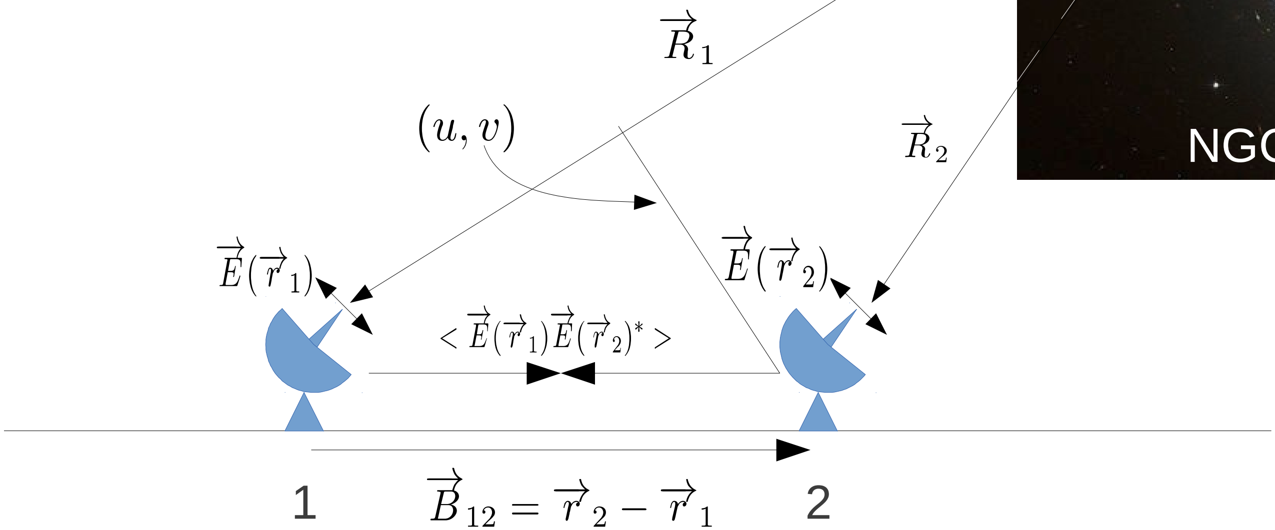


Van Cittert–Zernike theorem

$$< \vec{E}(\vec{r}_1)\vec{E}(\vec{r}_2)^* > \propto \int \frac{dxdy}{\sqrt{1-x^2-y^2}} I(x,y)A(x,y)e^{-2\pi i(xu+yv)}$$

Visibility Projected coordinates

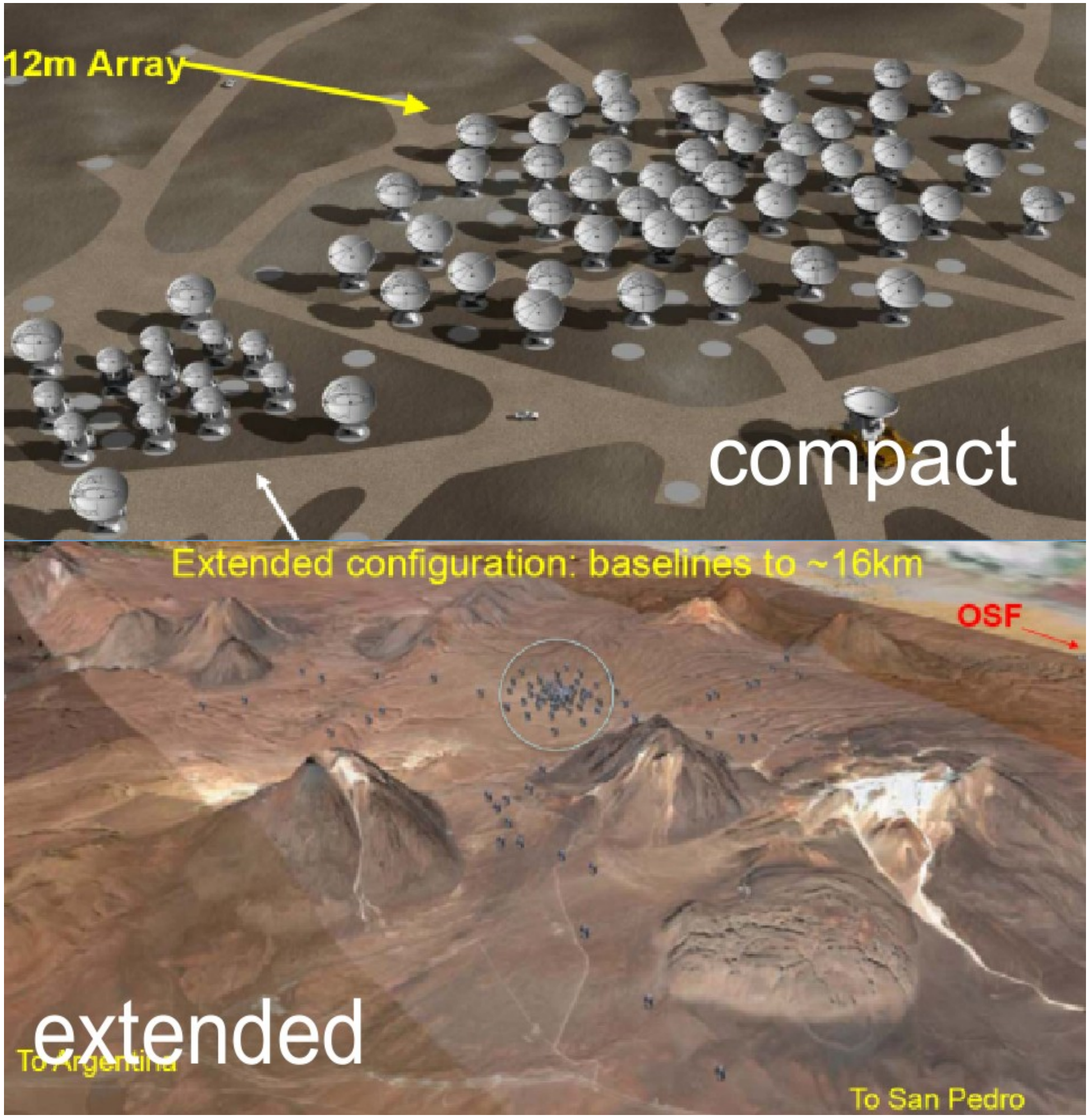
- $(u,v) \sim B/\lambda$
- Theoretical resolution: $\lambda/\mathrm{Max}(B)$
- Fourier Transform $(B^2/(R\lambda) \sim 10^{-14})$



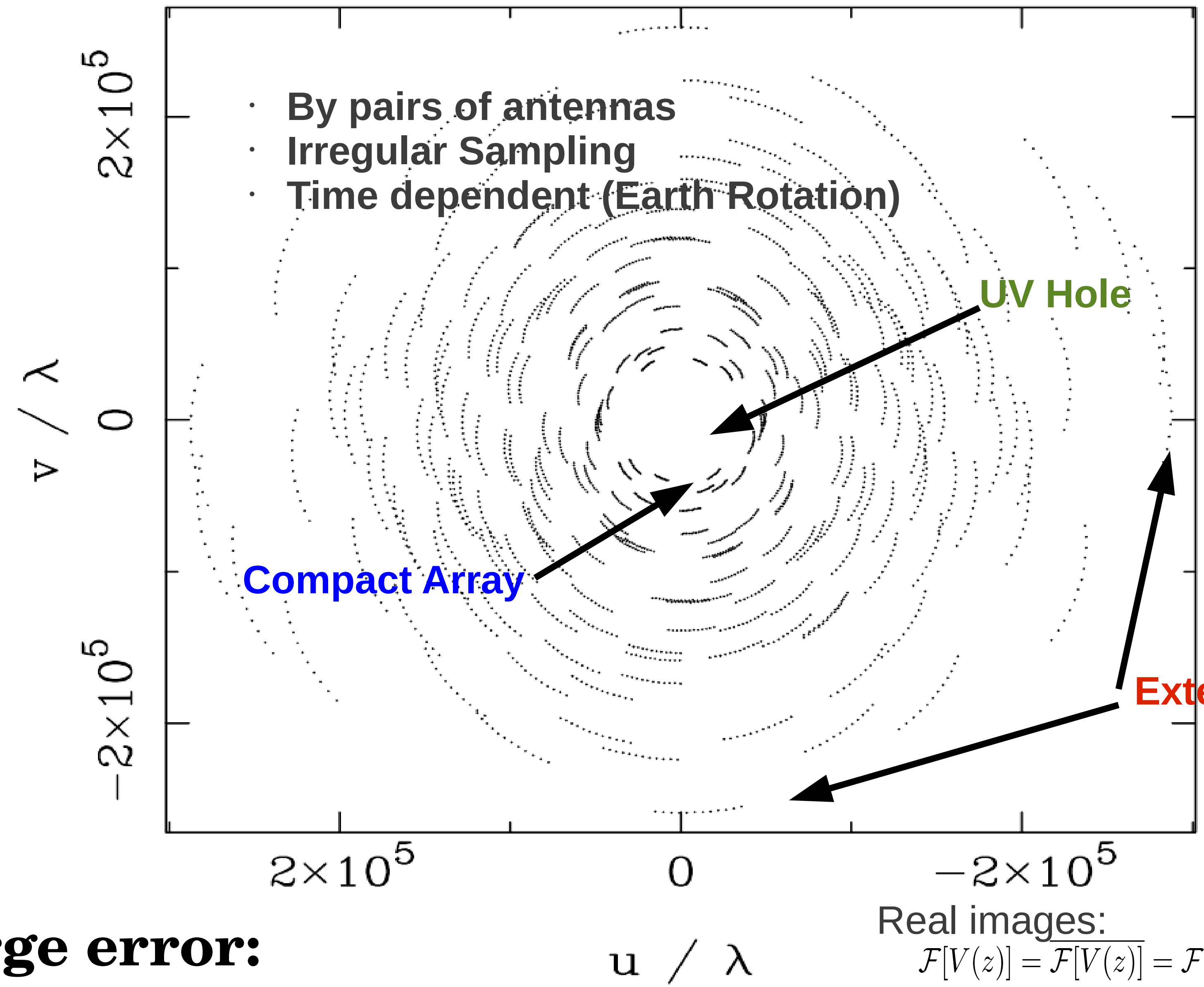
vs. GBT (100 mts)

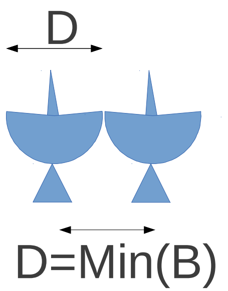
ALMA Telescope

- Array of 66 antennas. = =2145 pairs.
- Different baselines densities: 12 mt to 16 km.
- .$\lambda \sim 10^{-3} - 10^{-4}$ (milimeter/submilimeter)



Radio telescope uv-coverage





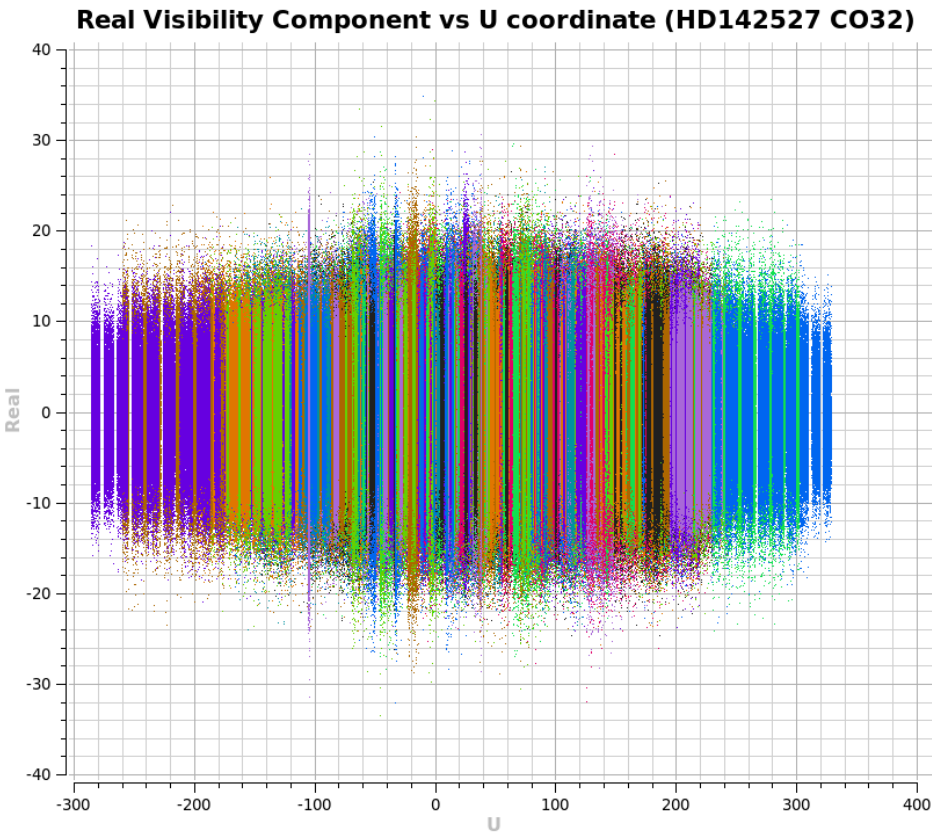
Real images:

$$\mathcal{F}[V(z)] = \overline{\mathcal{F}[\overline{V(z)}]} = \mathcal{F}[\bar{V}(-z)] \Rightarrow \bar{V}(-z) = V(z)$$

Large error:

$V(u_k,v_k) \pm \sigma_k$!!!

Visibilities

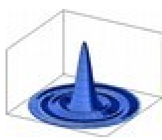


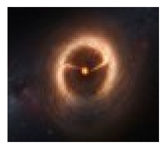
- $V(u_k,v_k) \pm \sigma_k$
- $.\sigma_k \sim V$!!!
- $.\sigma_k \sim N(0,s)$

Interferometer Measurement & the Fourier Synthesis Problem

[Briggs et al., Synthesis Imaging in Radio Astronomy, ASP, Chpt 7, 180, 1999.]

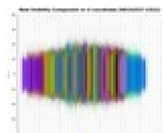
$$\mathcal{A}(x,y)I(x,y) = \int_{\mathbb{R}^2} V(u,v)e^{2\pi i(ux+vy)}dudv$$

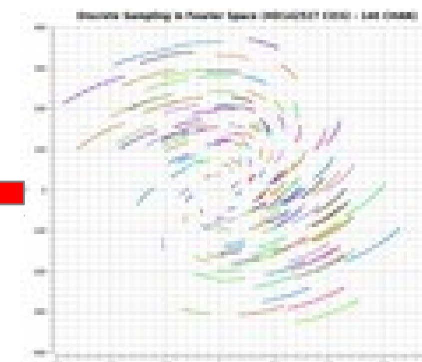

Primary Beam $\star$



Image

= F


Visibility



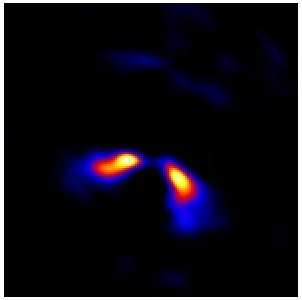
- It is an Inverse Problem
- Given an **IRREGULAR** sampling $\{V(u_k,v_k) \pm \sigma_k\}_{k=1,\ldots,N}$
- Find a “good approximation” for $\{I(x_j,y_j)\}_{j=1,\ldots,M}$
- Typical order of Magnitude: $M=10^6$-10^7, $N=10^4$-10^6
- Typically an Ill-posed problem in the Hadamard sense.

[P. Marechal et al, MATH COMPUT MODEL, 49, 11, 2008]

- Canonical digital processing use FFT but in a **REGULAR** mesh

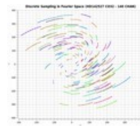
Naive solution: The Dirty Image

HD142527
345,8GHz
HCO
ALMA
Cycle 0

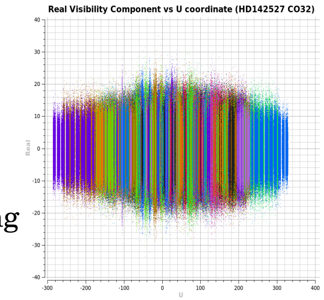


Image

uv-Sampling

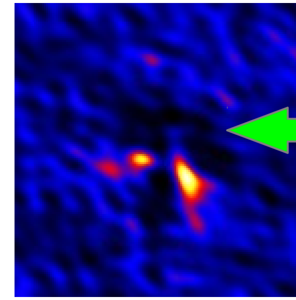


$\{(u_k, v_k)\}_{k=1,\dots,N}$



Visibility

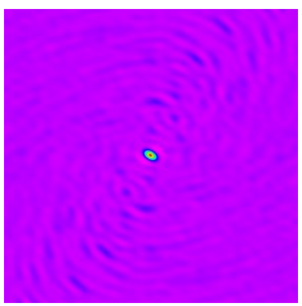
Sum



Dirty Image I^D

Plenty of
artifacts
(negatives,
ripples, ...)

$$\mathcal{A}(x_j, y_j) I^D(x_j, y_j) = \sum_{k=1}^{N} V(u_k, v_k) e^{2\pi i (x_j u_k + y_j v_k)}$$
$$= B^D * \mathcal{F}[V]$$



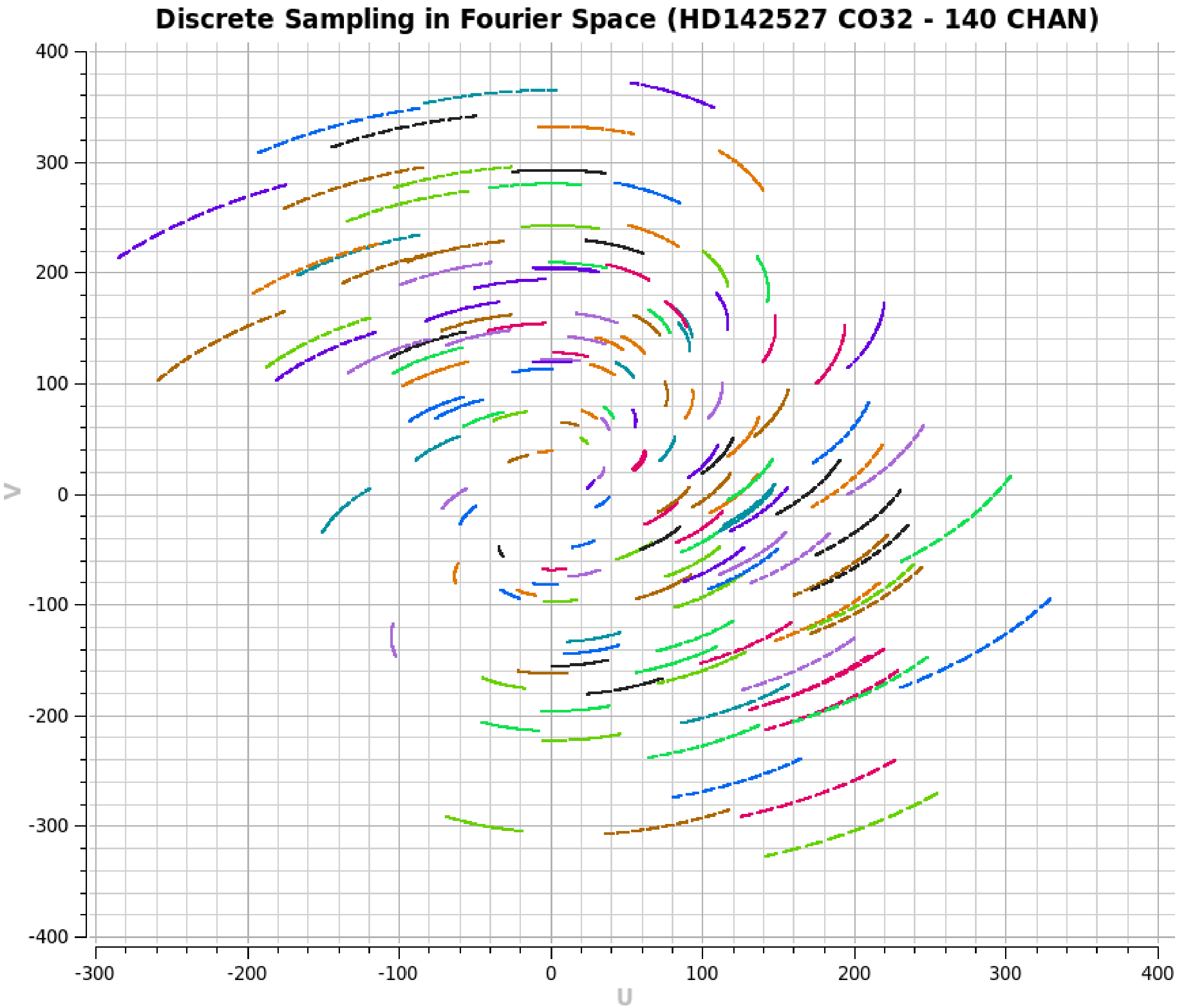
Dirty Beam B^D

$$B^D(x, y) = \sum_{k=1}^{N} w_k \cos\left(2\pi (x u_k + y v_k)\right)$$

If a 2d-regular sampling and
V has compact support then
the sum have close approx. (Shanon-Whitacker)

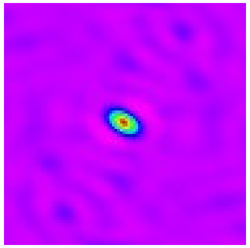
It works even when the sampling is slightly close (less than a 0.25 period to the grid) to regular, <u>which is not the case of radio interferometry</u>.
[Marvasti, Nonuniform Sampling, Kluwer, 2001]

… not the case of interferometry



Challenging problems

1. Applying the inverse Fast Fourier Transform DO NOT perform a good job.
2. The transformation is in fact NON-INVERTIBLE: **infinite possible solutions**.
3. Solution do not depend continuously from data.
4. Data is noisy (stochastic sampling): $\sigma_k \sim V_k$
5. Data volumes: **Tera bytes and Megapixel images(millions of variables)**.
6. Measurement in physics should have error notion: Today **Informally Estimated**!
7. Sky object never seen before at this wavelength and resolution!



Dirty Beam B^D

A 1974 approach: The CLEAN Algorithm

[HOGBOM, A&A Supl., 15, 417-426, 1974]

(0) Initialize $I^R = I^D$, $I^M = 0$, $k = 1$.

(1) Find the maximum peak's position (x_k, y_k) in image I^R.

(2) $I^M \leftarrow I^M + \lambda_k \, \delta(x - x_k, y - y_k)$

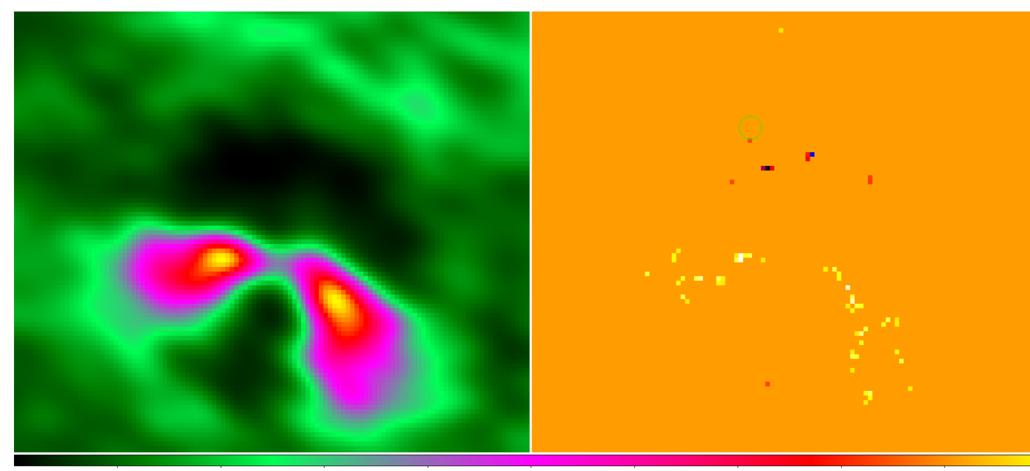
$I^R \leftarrow I^R - B^D * \lambda_k \, \delta(x - x_k, y - y_k)$

(3) If $|I^R|_\infty < \varepsilon$. then stop,

else $k \leftarrow k+1$ goto (1)

(4) Return $I^R + B^C * I^M$ (Restored Image)

- Each component $\delta(x - x_k, y - y_k)$ is attenuated by λ_k .
- A first version considers:
  - $\lambda_k = \gamma \, |I^R|$
  - δ is a Dirac delta
  - B^C is a fitted gaussian of B^D



Restored Image Model Image

The CLEAN Algorithm

- BASIS of STATE-OF-THE-ART DECONVOLVING ALGORITHMS:
- SOFTWARE CASA NRAO (http://casa.nrao.edu/)
- DIFMAP (ftp://ftp.astro.caltech.edu/pub/difmap/difmap.html)
- MIRIAD

  (http://www.atnf.csiro.au/computing/software/miriad/)

ATCA

Difmap Caltech

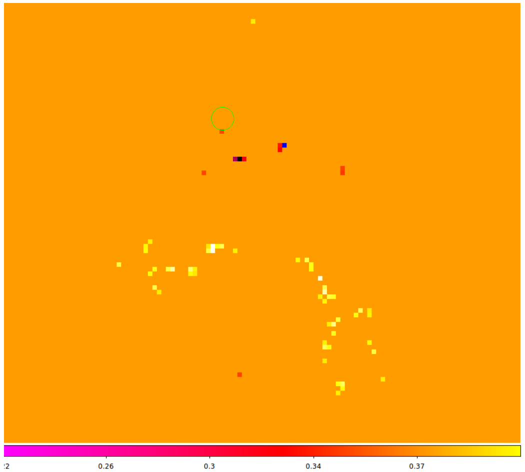
The CLEAN Algorithm

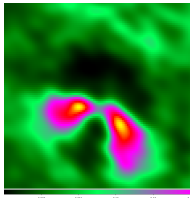
- **Matching pursuit**: **Greedy** algorithm minimizing the number of image's component summing maximal sources.

[Lannes et al, A&A Suppl. Ser., 123,1, 1997]

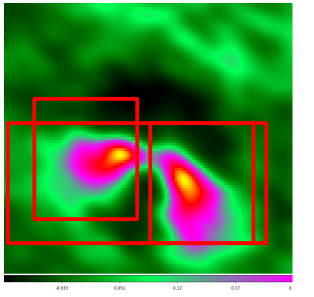
- Few components describes main sources → **Sparse Representation!**

$$\min_{(x_1,y_1),\ldots,(x_L,x_L)} ||(\lambda_1,\ldots,\lambda_L)||_0$$
$$s.t.$$
$$I^D - \sum_k^l B^D * \lambda_l \delta(x-x_k, y-y_k) = I^R(l)$$
$$\lambda_{l+1} = \gamma ||I^R(l)||_\infty$$

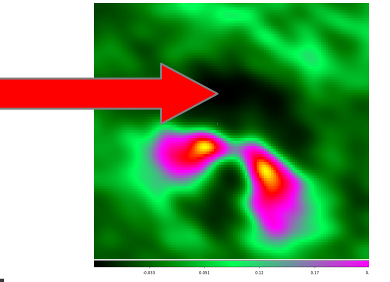




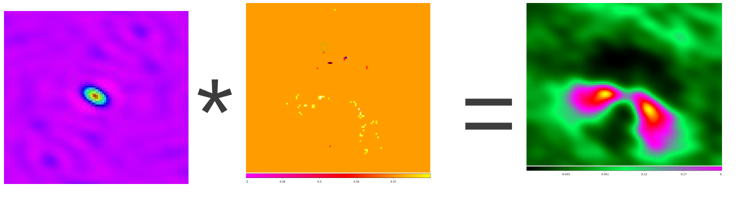
The CLEAN Algorithm



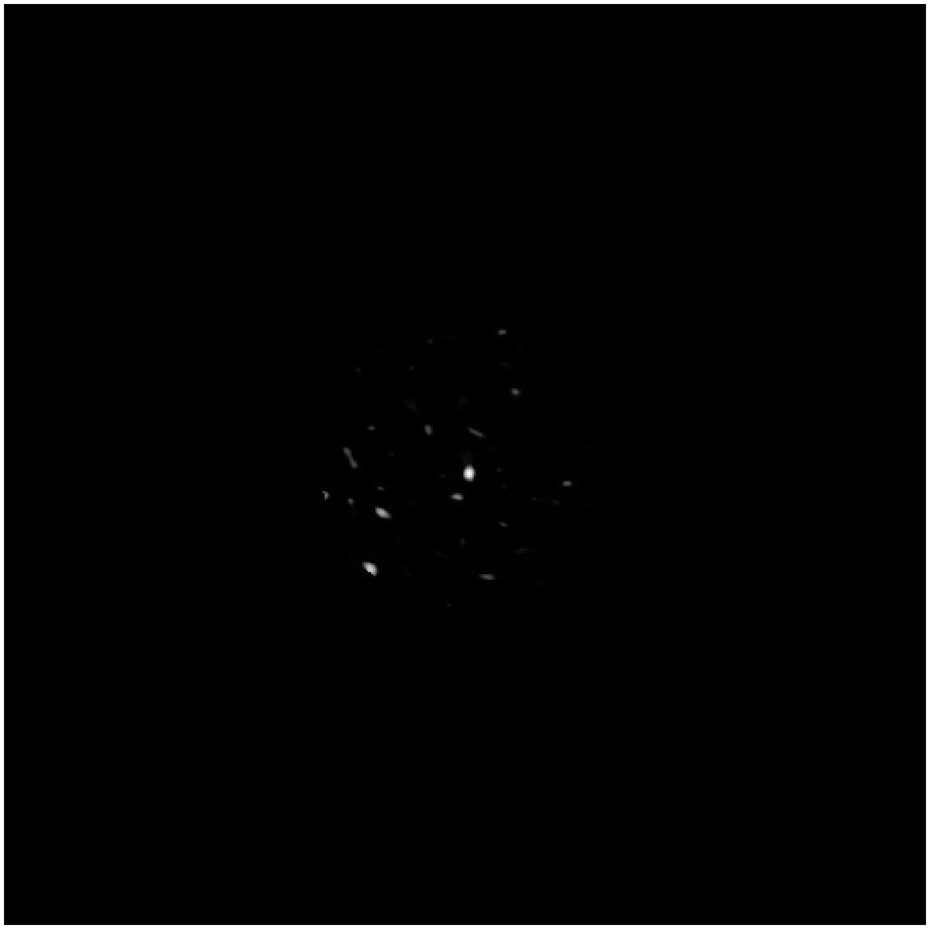
- User dependent: Automatic version of the algorithm does not work with extended sources. User must indicates where to put more components.



- Negative artifact for extended sources. It does not warranty positiveness.
- Work the best for compact sources.
- Resulting model lack statistical basis: Visibility variance are not included or reflected as an error map.
- Astronomers circumvent this problem with image restoration including model residuals and resolution (Clean Beam convolution).



Other approach: The MEM algorithm



- Minimize square errors and an entropic term.
- Positivity restriction of I.
- Equivalent to a Bayesian maximum likelihood.
- Typically a million variable problem: Very Expensive!
- However, solution have an statistical basis!

$$\min_I \sum_k ||V_k^{\mathrm{obs}} - V^{\mathrm{mod}}(u_k, v_k)||^2 - \lambda \sum_I I \log(I/M)$$

Image Synthesis: My personal opinion

- Trend topic in signal analysis.
- CLEAN beautiful algorithm that implement COMPRESSED SENSING, 30 years before this trend topics appears on literature.
- CLEAN optimize total intensity. It Do not have a clear statistical basis.
- Matching Pursuit, an efficient greedy heuristics that find suboptimal images but faster than others.
- Statistical results could obtained from large non-linear program (e.g. MEM).
- I am looking for a way to adapt Matching Pursuit to MEM.
- Medical Tomography.