1. Prove there are no wffs of length 2, 3, or 6 but all other numbers work

Solution. We define the formula building operations

$$\varepsilon_{\neg}(\alpha) = (\neg\alpha) \tag{1}$$
$$\varepsilon_{\wedge}(\alpha, \beta) = (\alpha \wedge \beta) \tag{2}$$
$$\varepsilon_{\vee}(\alpha, \beta) = (\alpha \vee \beta) \tag{3}$$
$$\varepsilon_{\rightarrow}(\alpha, \beta) = (\alpha \rightarrow \beta) \tag{4}$$
$$\varepsilon_{\leftrightarrow}(\alpha, \beta) = (\alpha \leftrightarrow \beta) \tag{5}$$

We notice that the result of the operations use 4, 5, 5, 5, and 5 symbols respectively. If we add to our list any single symbol α this allows for a wff of length 1. We can see clearly then that we cannot have a wff of length 2 or 3. Since the smallest we can have is of length 1, and the next smallest is of length 4. Each of these operations add 3 new symbols to a wff, so the question becomes how can we split 6 into these additions. If we start with either 1, 4, or 5 symbols (which are all possible), we see that we cannot add multiples of 3, or 4 to any of these numbers to get a wff of length 6. For 7 we see that we can add 3 to a wff of length 4 and get a wff of length 7 (this can be done by applying $\varepsilon_{\neg}$ on α twice).