Language & Logic - Assignment IV

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1 Question 1

1.1 Brief

$\forall_x[F_x \rightarrow (G_x \rightarrow H_x)], \exists_x[F_x \wedge G_x] : \exists[H_x]$

1.2 Answer

1	$\forall_x[F_x \rightarrow (G_x \rightarrow H_x)]$	Premise	$\{\mathbf{1}\}$
2	$F_a \rightarrow (G_a \rightarrow H_a)$	$\forall$-Elimination (1)	$\{\mathbf{1}\}$
3	$\exists_x[F_x \wedge G_x]$	Premise	$\{\mathbf{3}\}$
4	$F_a \wedge G_a$	Hypothesis	$\{\mathbf{4}\}$
5	F_a	$\wedge$-Elimination (4)	$\{\mathbf{4}\}$
6	$(G_a -> H_a)$	$\rightarrow$ Elimination (2,5)	$\{\mathbf{1}, \mathbf{4}\}$
7	G_a	$\wedge$-Elimination (4)	$\{\mathbf{4}\}$
8	H_a	$\rightarrow$ Elimination (6,7)	$\{\mathbf{1}, \mathbf{4}\}$
9	$\exists_x[H_x]$	$\exists$-Introduction (8)	$\{\mathbf{1}, \mathbf{4}\}$
10	$\exists_x[H_x]$	$\exists$-Elimination (3,4,9)	$\{\mathbf{1}, \mathbf{3}\}$

2 Question 2

2.1 Brief

: $\forall_x[((P_x \to Q_x \to P_x) \to P_x]$

2.2 Answer

1	$(P_x \to Q_x) \to P_x$	Hypothesis	$\{\mathbf{1}\}$
2	$\neg P_x$	Hypothesis	$\{\mathbf{1}, \mathbf{2}\}$
3	P_x	Hypothesis	$\{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$
4	$\neg Q_x$	Hypothesis	$\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$
5	P_x	Iteration (3)	$\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$
6	$\neg P_x$	Iteration (2)	$\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$
7	Q_x	Reductio Ad Absurdum (5, 6)	$\{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$
8	$P_x \to Q_x$	→Introduction (3, 7)	$\{\mathbf{1}, \mathbf{2}\}$
9	P_x	→Elimination (1, 8)	$\{\mathbf{1}\}$
10	$\neg P_x$	Iteration (2)	$\{\mathbf{1}, \mathbf{2}\}$
11	P_x	Reductio Ad Absurdum (9, 10)	$\{\mathbf{1}\}$
12	$((P_x \to Q_x) \to P_x) \to P_x$	→ Introduction (1, 11)	$\{\}$
13	$\forall_x[((P_x \to Q_x) \to P_x) \to P_x]$	∀-Introduction (12)	$\{\}$