

Language & Logic - Assignment IV

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1 Question 1

1.1 Brief

$\forall_x[F_x \rightarrow (G_x \rightarrow H_x)], \exists_x[F_x \wedge G_x] : \exists[H_x]$

1.2 Answer

1	$\forall_x[F_x \rightarrow (G_x \rightarrow H_x)]$	Premise	{1}
2	$F_a \rightarrow (G_a \rightarrow H_a)$	\forall -Elimination (1)	{1}
3	$\exists_x[F_x \wedge G_x]$	Premise	{3}
4	$F_a \wedge G_a$	\exists -Elimination (3)	{3}
5	F_a	\wedge -Elimination (4)	{3}
6	$(G_a \rightarrow H_a)$	\rightarrow Elimination (2,5)	{1, 3}
7	G_a	\wedge -Elimination (4)	{3}
8	H_a	\rightarrow Elimination (6,7)	{1, 3}
9	$\exists_x[H_x]$	\exists -Introduction (8)	{1, 3}

2 Question 2

2.1 Brief

$: \forall_x[((P_x \rightarrow Q_x \rightarrow P_x) \rightarrow P_x]$

2.2 Answer

1	$(P_x \rightarrow Q_x) \rightarrow P_x$	Hypothesis	{1}
2	$\neg P_x$	Hypothesis	{1, 2}
3	P_x	Hypothesis	{1, 2, 3}
4	$\neg Q_x$	Hypothesis	{1, 2, 3, 4}
5	P_x	Iteration (3)	{1, 2, 3, 4}
6	$\neg P_x$	Iteration (2)	{1, 2, 3, 4}
7	Q_x	Reductio Ad Absurdum (5, 6)	{1, 2, 3}
8	$P_x \rightarrow Q_x$	\rightarrow Introduction (3, 7)	{1, 2}
9	P_x	\rightarrow Elimination (1, 8)	{1}
10	$\neg P_x$	Iteration (2)	{1, 2}
11	P_x	Reductio Ad Absurdum (9, 10)	{1}
12	$((P_x \rightarrow Q_x) \rightarrow P_x) \rightarrow P_x$	\rightarrow Introduction (1, 11)	{}
13	$\forall_x[((P_x \rightarrow Q_x) \rightarrow P_x) \rightarrow P_x]$	\forall -Introduction (12)	{}