Language & Logic - Assignment III

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1 Question 1

1.1 Brief

$$(P \land Q) \to \neg R : R \to (P \to \neg Q)$$

1.2 Answer

1	R	Hypothesis	$\{\mathbf{1}\}$
2	$\quad P$	Hypothesis	$\{\mathbf{1},\mathbf{2}\}$
3	$\quad\quad Q$	Hypothesis	$\{\mathbf{1},\mathbf{2},\mathbf{3}\}$
4	$\quad\quad P \land Q$	$\land$-Introduction$(2,3)$	$\{\mathbf{1},\mathbf{2},\mathbf{3}\}$
5	$\quad\quad (P \land Q) \to \neg R$	Premise	$\{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{5}\}$
6	$\quad\quad \neg R$	$\to$ Elimination$(4,5)$	$\{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{5}\}$
7	$\quad\quad R$	Iteration	$\{\mathbf{1},\mathbf{2},\mathbf{3}\}$
8	$\quad \neg Q$	Reductio Ad Absurdum$(3,6,7)$	$\{\mathbf{1},\mathbf{2},\mathbf{5}\}$
9	$P \to \neg Q$	$\to$ Introduction$(2,8)$	$\{\mathbf{1},\mathbf{5}\}$
10	$R \to (P \to \neg Q)$	$\to$ Introduction$(1,9)$	$\{\mathbf{5}\}$

2 Question 2

2.1 Brief

$$\neg R,\ P \to \neg Q,\ R \to Q,\ P \lor R : \neg Q$$

2.2 Answer

1	P	Hypothesis	$\{\mathbf{1}\}$
2	$P \to \neg Q$	Premise	$\{\mathbf{1},\mathbf{2}\}$
3	$\neg Q$	$\to$ Elimination(1,2)	$\{\mathbf{1},\mathbf{2}\}$
4	R	Hypothesis	$\{\mathbf{4}\}$
5	$\quad Q$	Hypothesis	$\{\mathbf{4},\mathbf{5}\}$
6	$\quad \neg R$	Premise	$\{\mathbf{4},\mathbf{5},\mathbf{6}\}$
7	$\quad R$	Iteration	$\{\mathbf{4},\mathbf{5}\}$
8	$\neg Q$	Reductio Ad Absurdum(5,6,7)	$\{\mathbf{4},\mathbf{6}\}$
9	$P \lor R$	Premise	$\{\mathbf{4},\mathbf{9}\}$
10	$\neg Q$	$\lor$-Elimination(1,4,9)	$\{\mathbf{9}\}$

3 Question 3

3.1 Brief

$$: ((A \vee B) \wedge (\neg B)) \rightarrow A$$

3.2 Answer

1	$((A \vee B) \wedge (\neg B))$	Hypothesis	$\{\mathbf{1}\}$
2	$A \vee B$	$\wedge$ - Elimination(1)	$\{\mathbf{1}\}$
3	$\quad A$	Hypothesis	$\{\mathbf{1}, \mathbf{3}\}$
4	$\quad A$	Iteration(3)	$\{\mathbf{1}, \mathbf{3}\}$
5	$\quad B$	Hypothesis	$\{\mathbf{1}, \mathbf{5}\}$
6	$\quad\quad \neg A$	Hypothesis	$\{\mathbf{1}, \mathbf{5}, \mathbf{6}\}$
7	$\quad\quad B$	Iteration(5)	$\{\mathbf{1}, \mathbf{5}, \mathbf{6}\}$
8	$\quad\quad \neg B$	$\wedge$-Elimination(1)	$\{\mathbf{1}, \mathbf{5}, \mathbf{6}\}$
9	$\quad A$	Reductio Ad Absurdum(5,6,7,8)	$\{\mathbf{1}, \mathbf{5}, \mathbf{6}\}$
10	A	$\vee$-Elimination(2,3,4,5,9)	$\{\mathbf{1}\}$
11	$((A \vee B) \wedge (\neg B))$	$\rightarrow$ Introduction(1,10)	$\{\}$