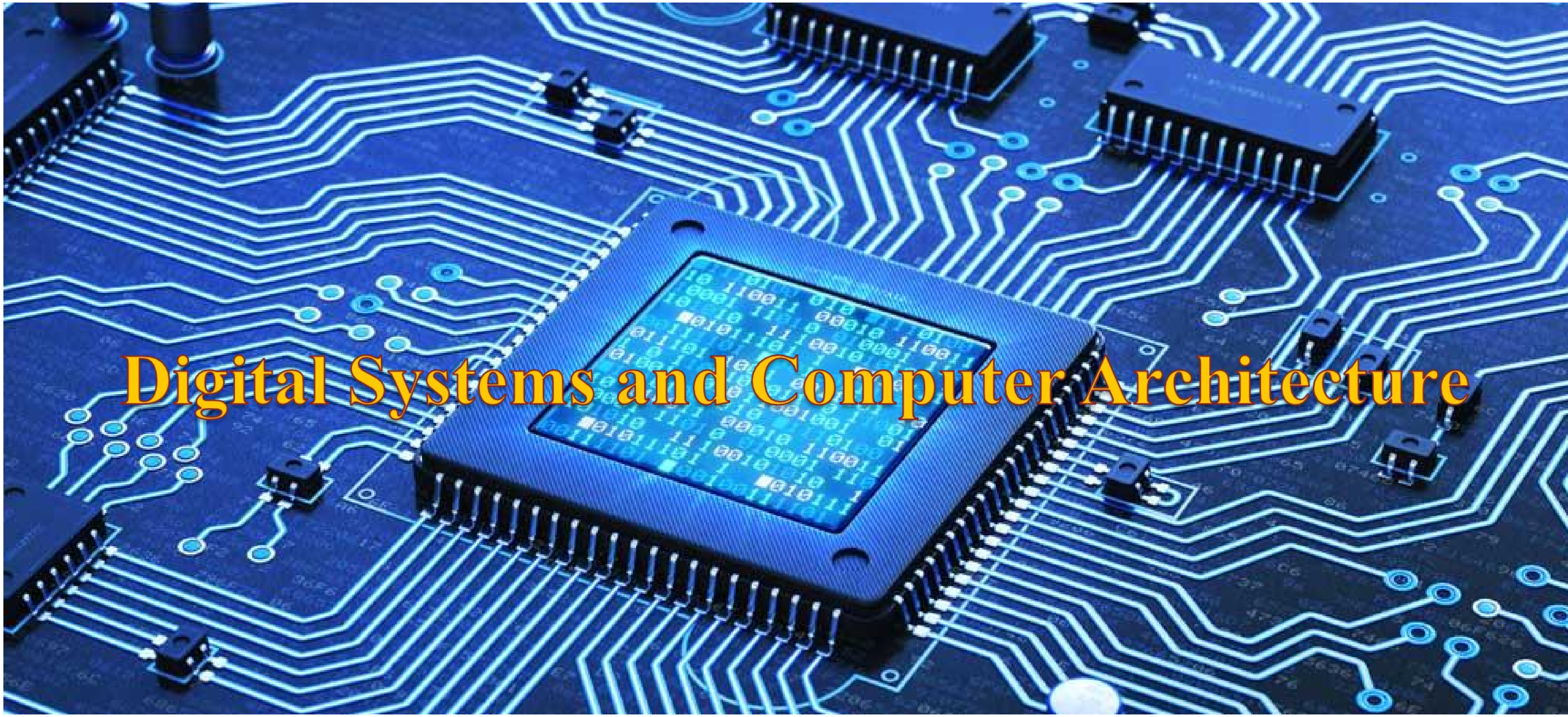


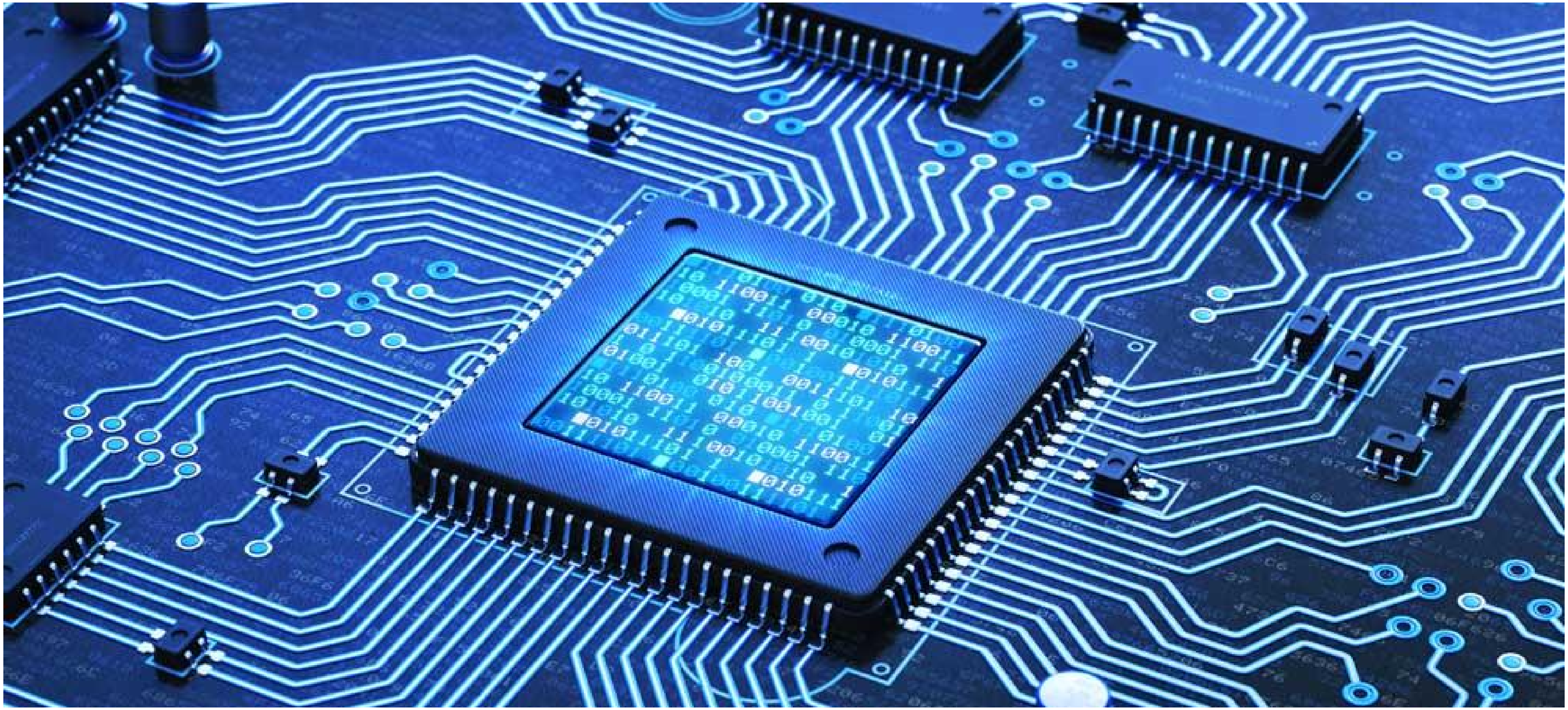
Digital Systems and Computer Architecture

Session 1.3

Module 1a: Kirchhoff’s Laws, Resistors in Series and Parallel

Session 1.3: Focus

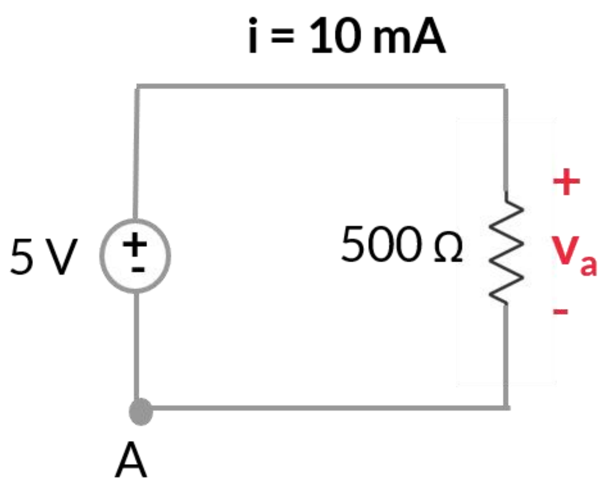
- Conservation of Energy
- Kirchhoff's Current Law
- Kirchhoff's Current Law
- Resistors in Series and Parallel



Conservation of Energy

Conservation of Energy

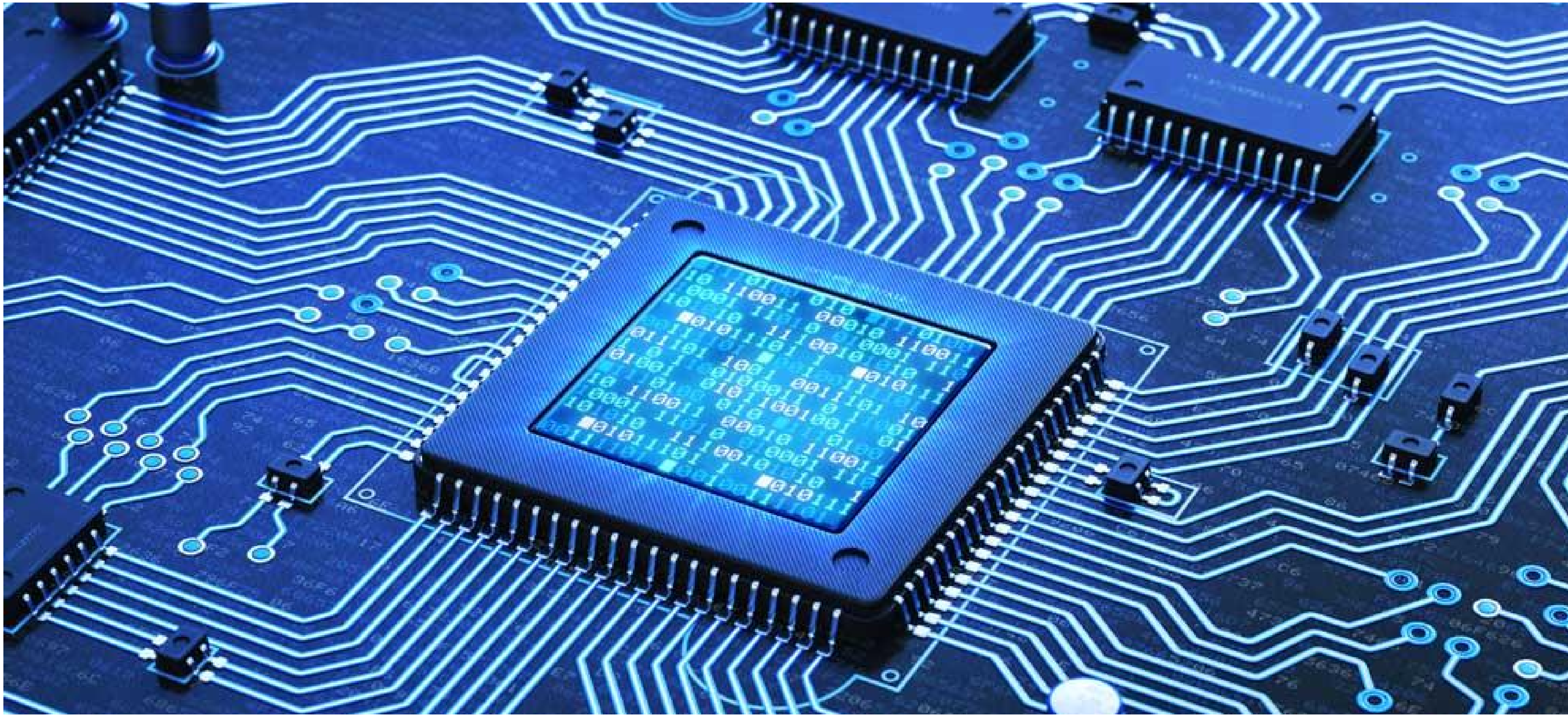
- The law of conservation of energy states that the total energy of an isolated system remains constant; it is said to be conserved over time.
  - Total power supplied + Total power absorbed = 0
  - PS + PA = 0



$$P_A = (i^2 * R) = ((10 * 10^{-3})^2 * 500) = 50\ mW$$

$$P_S = -(v * i) = -(5 * 10 * 10^{-3}) = -50\ mW$$

$$P_S + P_A = -50\ mW + 50\ mW = 0$$



Kirchhoff’s Voltage Law (KVL)

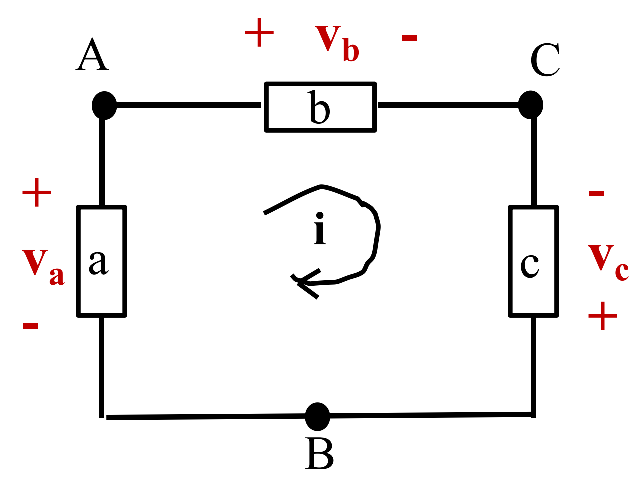
Kirchhoff's Voltage Law

- **The algebraic sum of the voltages around any closed path is zero.**
- If we **trace** out a **closed path**, the **algebraic sum** of the **voltages** across the individual **elements around it** must be **zero**.

$$\boxed{v_1 + v_2 + v_3 + \ldots + v_n = 0}$$

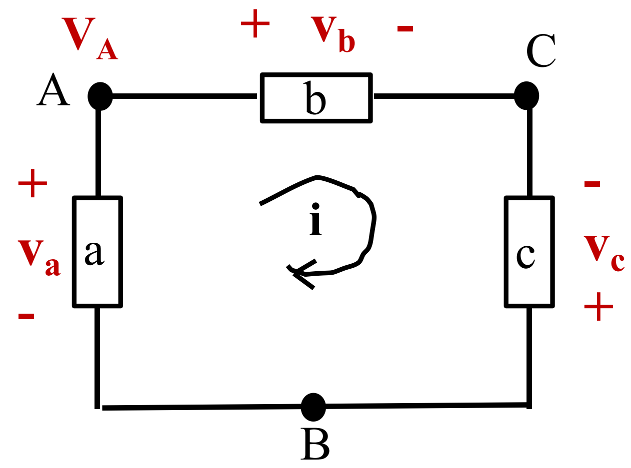
$$\boxed{\sum_{n=1}^{N} v_n = 0}$$

- **Algebraic sum** means that the polarity of the voltages seen while traversing the path is taken care of while adding them up.
- For **example**, the algebraic sum of the voltages of the given circuit, starting from B, in the clockwise direction, is written as:
  - $-v_a + v_b - v_c = 0$
  - **Thus,** $v_a = v_b - v_c$



Proof: KVL

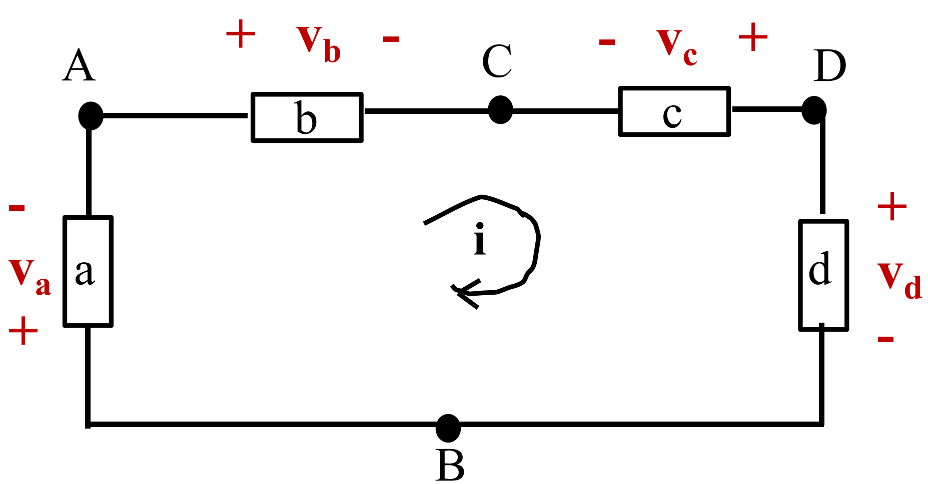
- There is a single **unique value** for any **voltage** in a **circuit**.
- Thus, the **energy** required to **move** an **unit charge** from any point **B** to any other point **A** in a circuit, must have a value **independent** of the **path chosen** to move from **B** to **A**.



- Potential at A with respect to B through element a = $\mathbf{V_A = -v_a}$
- Potential at A with respect to B through elements c and b = $\mathbf{V_A}$
- $\mathbf{= v_c - v_b}$
- If we equate them, we get: $\mathbf{-v_a = v_c - v_b} \rightarrow \mathbf{v_a = v_b - v_c}$
  - Thus, we get the same value that we got when we did algebraic sum of voltages around the closed path.

Example 1: Applying KVL to a Circuit

- One **method** that leads to **least error** while writing the **KVL** equation is the following:
- **Moving** mentally around the **closed path** in a **clockwise direction** and writing down directly the **voltages** of each element whose **(+) terminal** is entered, and
- Writing down the **negative** of every **voltage** first met at the **(−) terminal** is entered.



Start from B and move in the clockwise direction

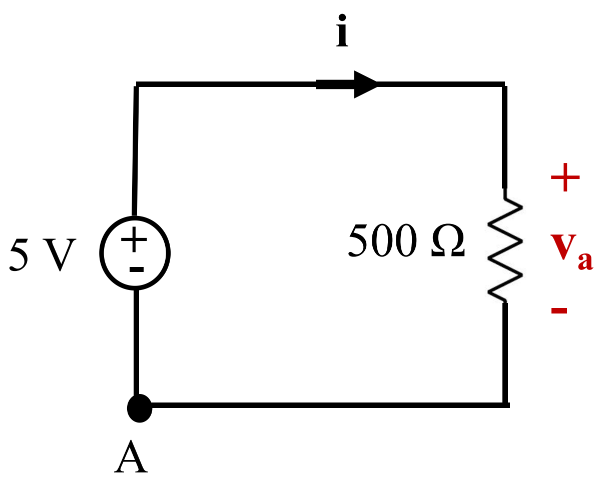
$$v_a + v_b - v_c + v_d = 0$$

It can be written as:

$$v_c = v_a + v_b + v_d$$

Problem 1: KVL

- Find **i** and **Power Supplied** (P_s) by **voltage source**:



Start from the node A and move in
the clockwise direction till reaching A again.

$- 5 + (i * 500) = 0$ **: KVL** and **Ohm's law**

i = 5 / 500 = 10 mA

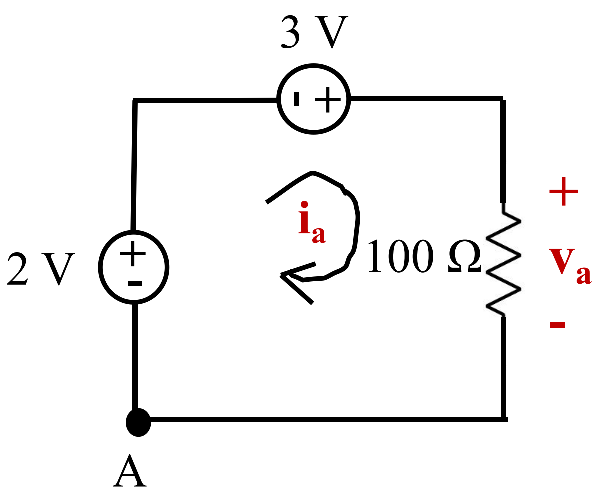
Note: By convention power supplied is a negative quantity.

$P_S = - (v * i) = - (5 * 10 * 10^{-3}) = - 50\ mW$

Ans: 10 mA and - 50 mW

Problem 2: KVL

- Find $\mathbf{v_a}$ and $\mathbf{i_a}$:



Start from the node A and move in the clockwise direction till reaching A again.

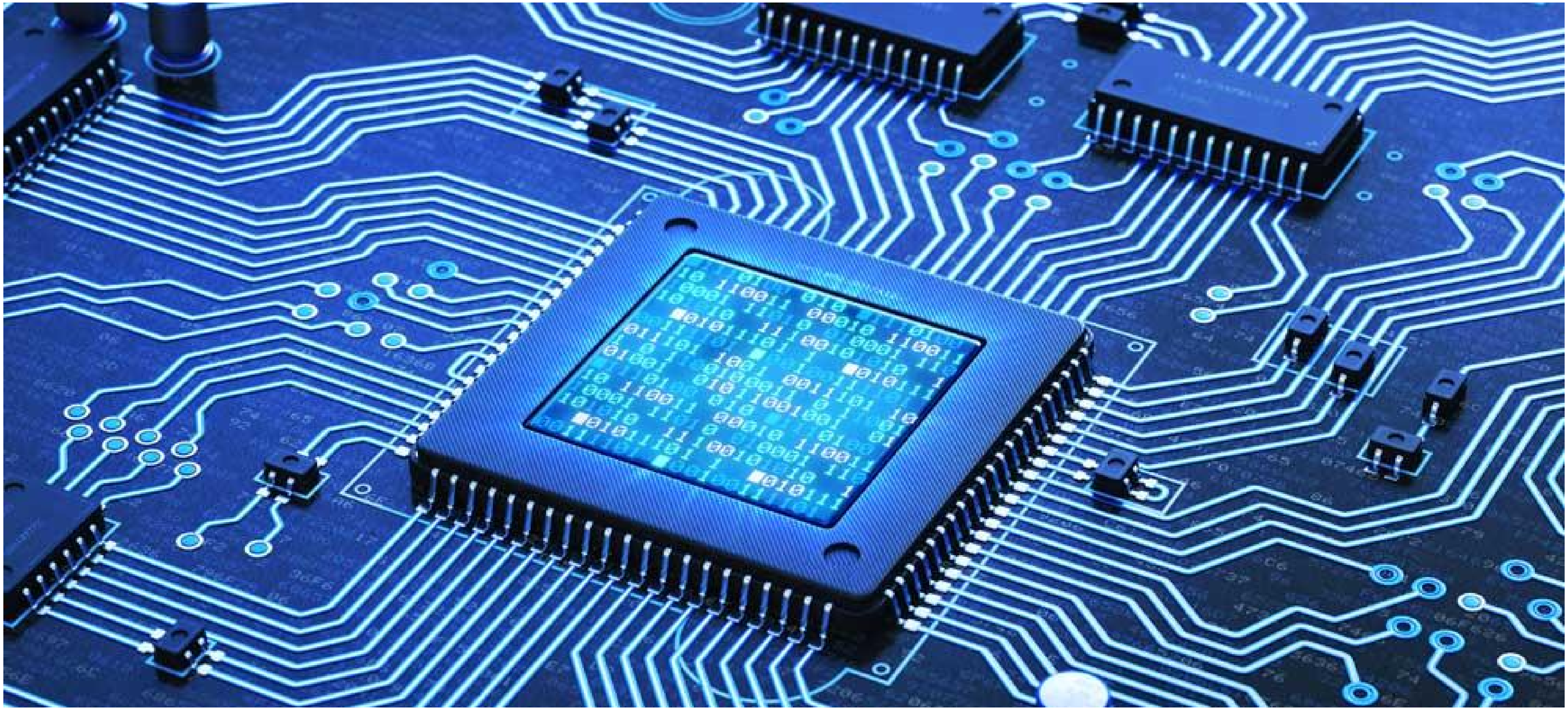
$$-2 - 3 + v_a = 0 \quad \text{:Applying KVL}$$

$$v_a = 5\ V$$

$$i_a = v_a / 100 = 5 / 100$$

$$i_a = 50\ mA$$

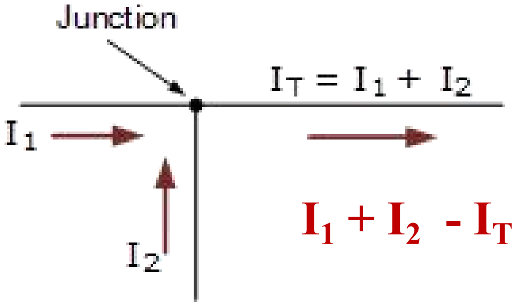
Ans: 5 V and 50 mA



Kirchhoff’s Current Law (KCL)

Kirchhoff's Current Law (KCL)

- **KCL** states that **the algebraic sum of the currents entering any node is zero.**



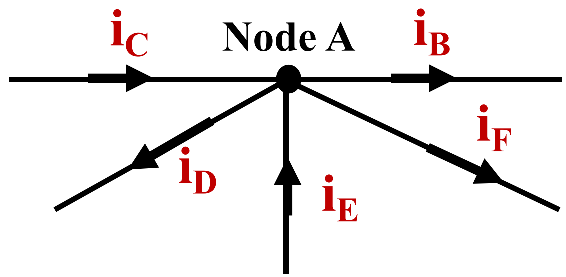
$$I_1 + I_2 - I_T = 0$$

$$\boxed{i_1 + i_2 + i_3 + \ldots + i_n = 0}$$

$$\boxed{\sum_{n=1}^{N} i_n = 0}$$

- This mathematical statement proves the fact that **charges cannot accumulate at a node**.
- A **node** is **not** a **circuit element**, and it certainly **cannot store, destroy, or generate charge**.
- Hence, the currents must sum to zero at any node in the circuit

KCL: Algebraic Sums



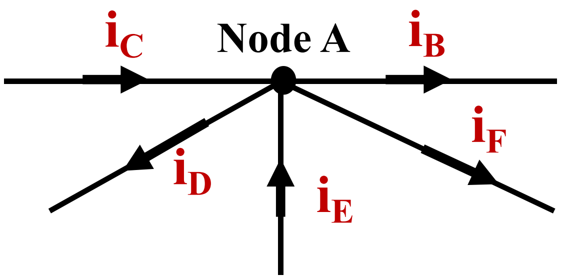
- The **algebraic sum** of the four **currents entering** the **node** must be **zero**:

$$\boxed{i_C + i_E + (-\,i_D) + (-\,i_F) + (-\,i_B) = 0}$$

- Similarly, this law could equally be well applied to the **algebraic sum** of the **currents leaving** the **node:**

$$\boxed{i_D + i_F + i_B + (-\,i_C) + (-\,i_E) = 0}$$

Example: KCL

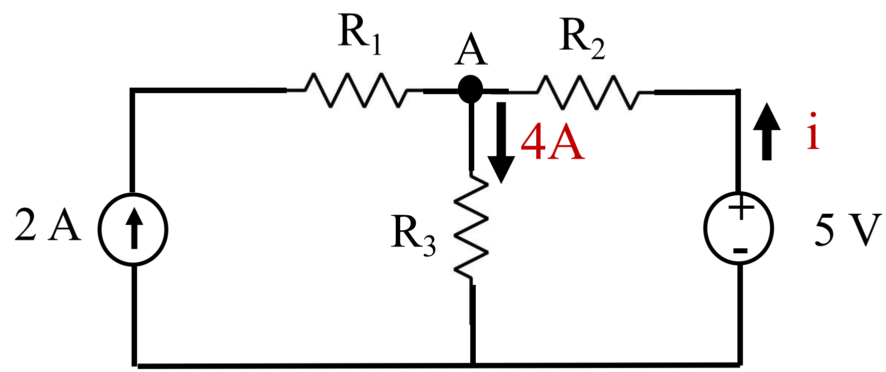


Current entering the node A = Current leaving the node A

$$i_C + i_E = i_B + i_D + i_F$$

Problem 3: KCL

- Find the current **i**:

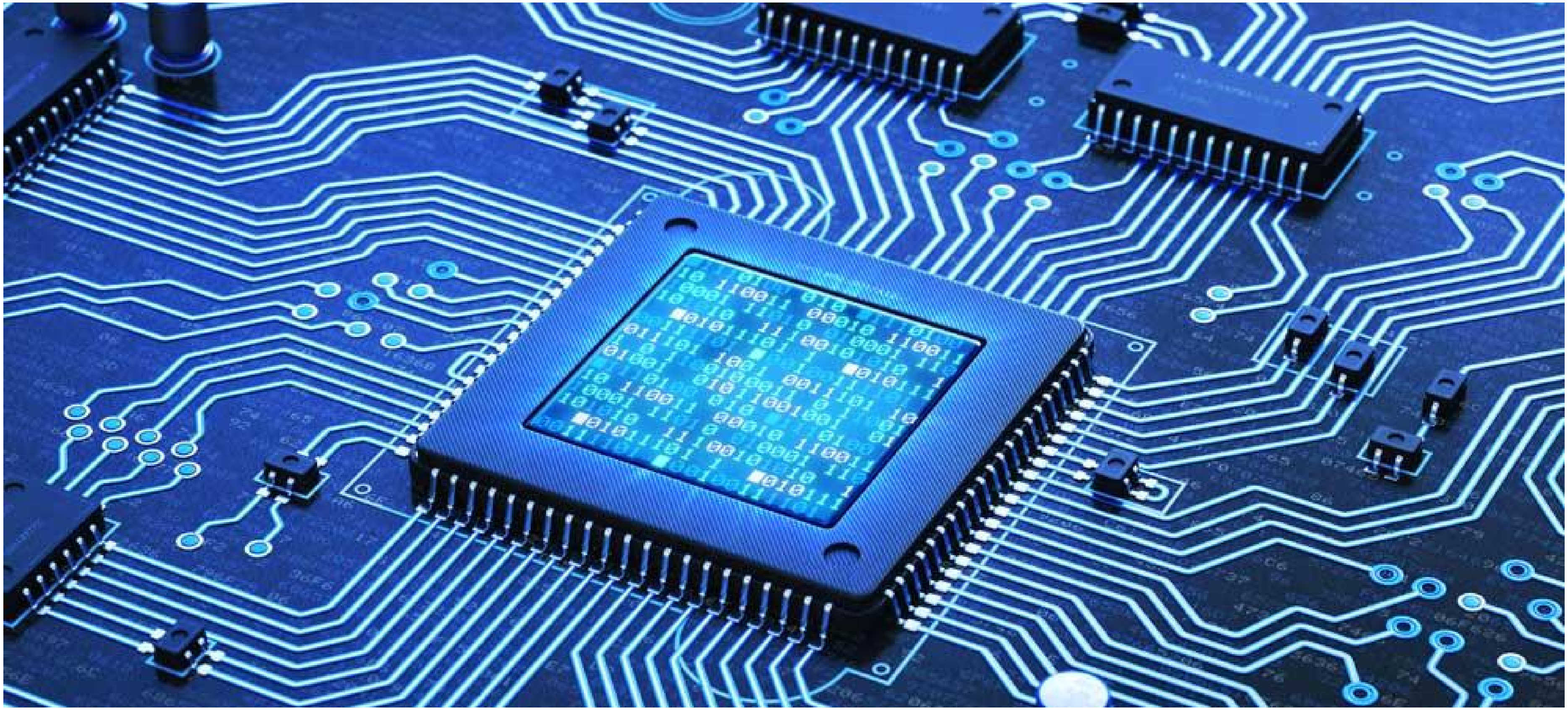


- The **total current entering** the **node A** = **2 + i**
- The **total current leaving** the **node A** = **4 A**
- As per KCL, total current entering a node is equal to the total current leaving the node.
- Thus, 2 + i = 4, then **i = 2 A**

Give V_A in terms of circuit elements

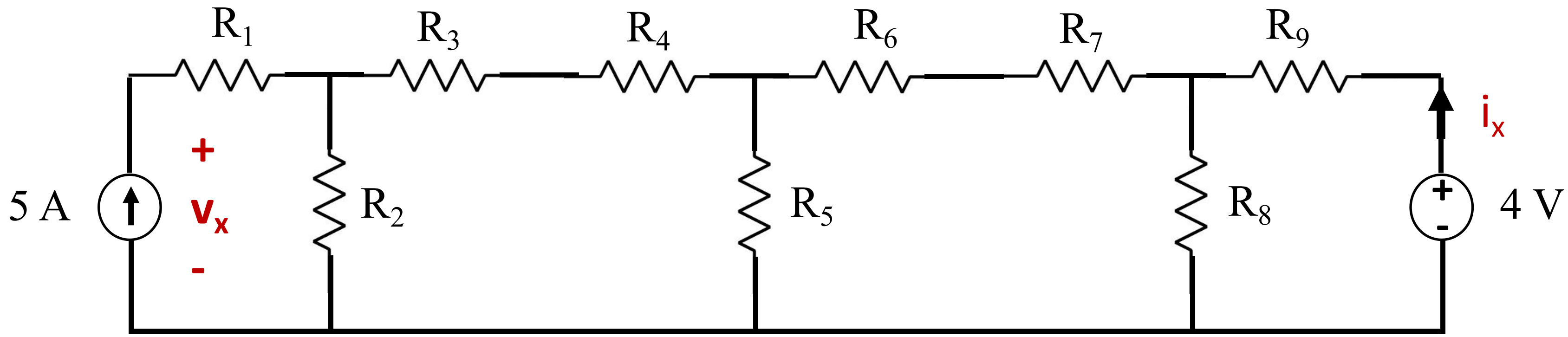
ANS: i = 2 A

$V_A = 4 * R_3$ $\quad$ $V_A = -i R_2 + 5$



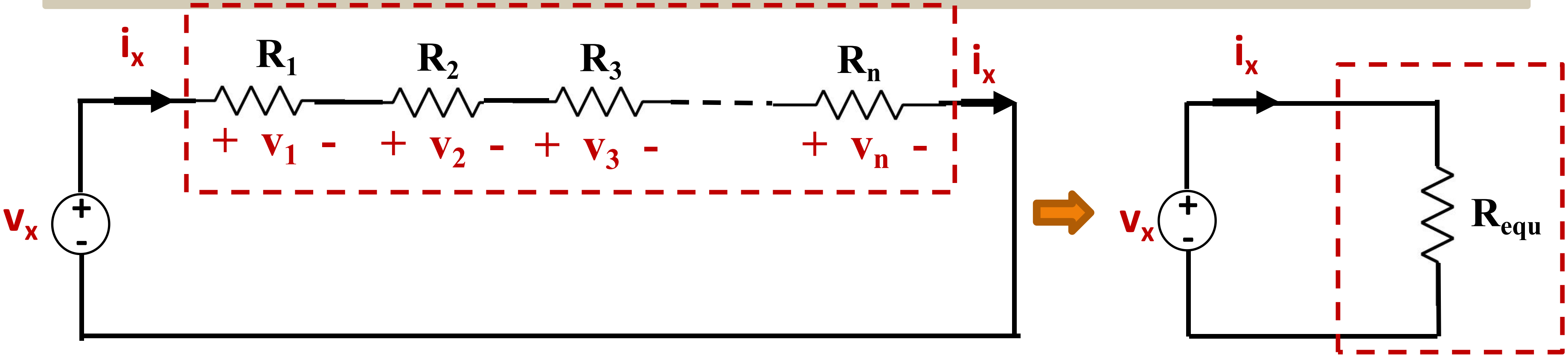
Resistors in Series

Combining of Resistors



- It is often possible to **replace** relatively complicated **resistor combinations** with a single equivalent resistor.
- This is useful when we are not specifically interested in the current, voltage, or power associated with any of the individual resistors in the combinations.
- All the current, voltage, and power relationships in the remainder of the circuit will be unchanged.

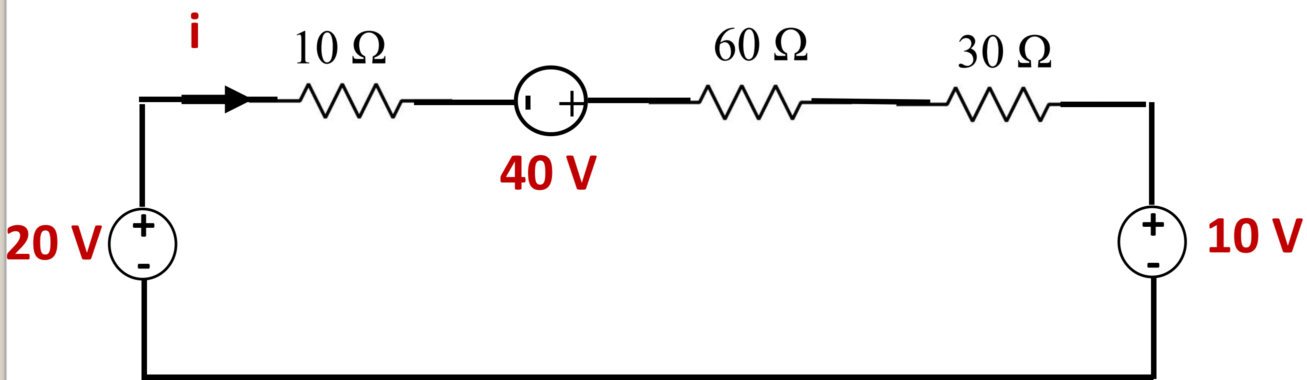
Derivation: Resistors in Series



- The current $\mathbf{i_x}$ flowing through all the resistors in series is the **same**.
- Applying **KVL** we get:
  - $\mathbf{v_x = v_1 + v_2 + v_3 \cdots + v_n}$
- Using Ohm's law:
  - $\mathbf{v_x = i_x R_1 + i_x R_2 + i_x R_3 \cdots + i_x R_n = i_x (R_1 + R_2 + R_3 \cdots + R_n)}$
  - $\mathbf{v_x = i_x R_{equ}}$
- $\boxed{\mathbf{R_{equ} = R_1 + R_2 + R_3 \cdots + R_n}}$

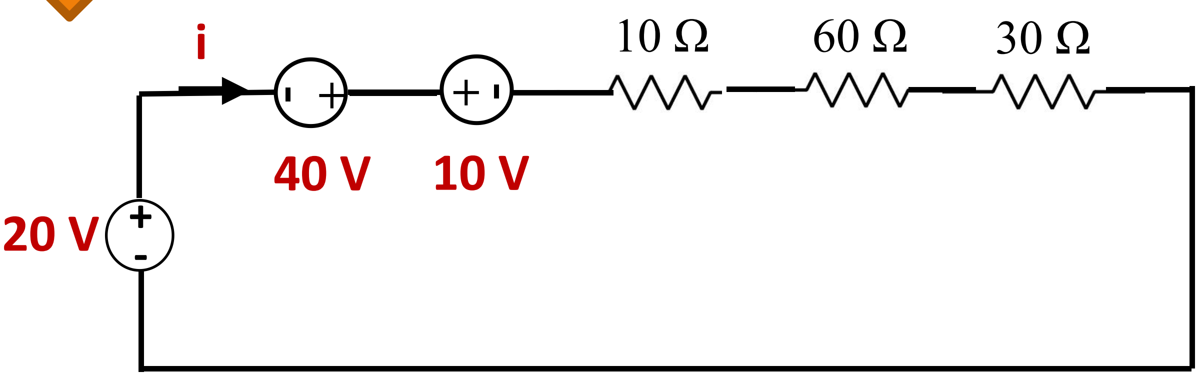
Problem 4: Resistors in Series

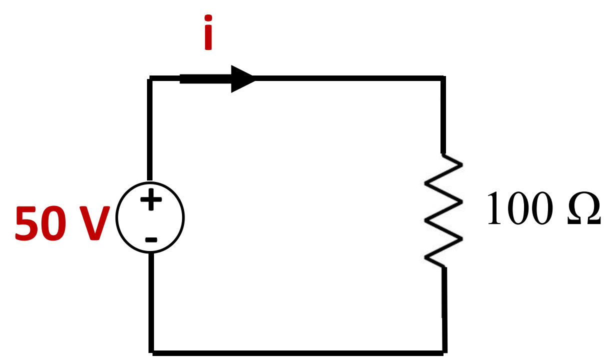
- Use **source** and **resistor combinations** to find **i**: **Ans: 500 mA**

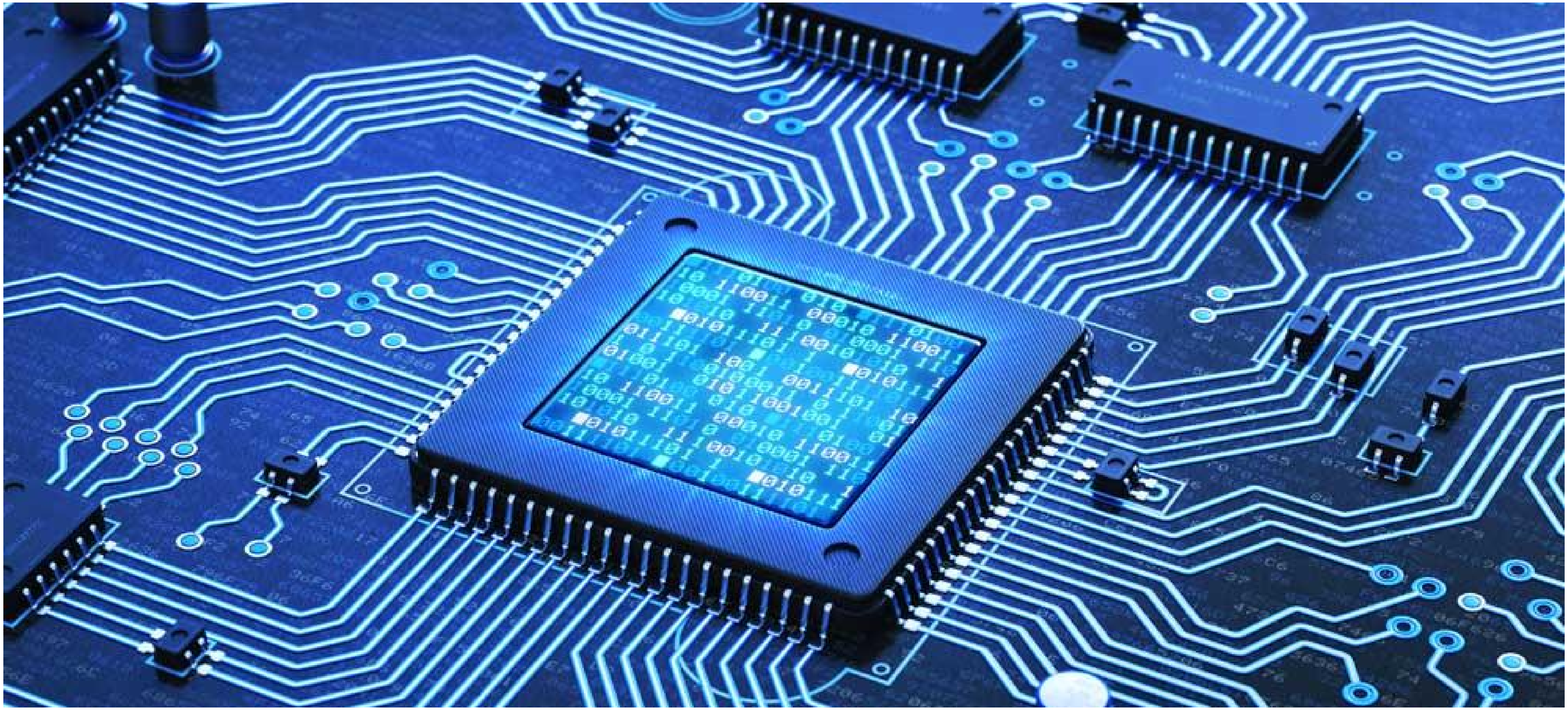


$$50 = i * 100$$

$$i = \frac{50}{100} = 500 \text{ mA}$$

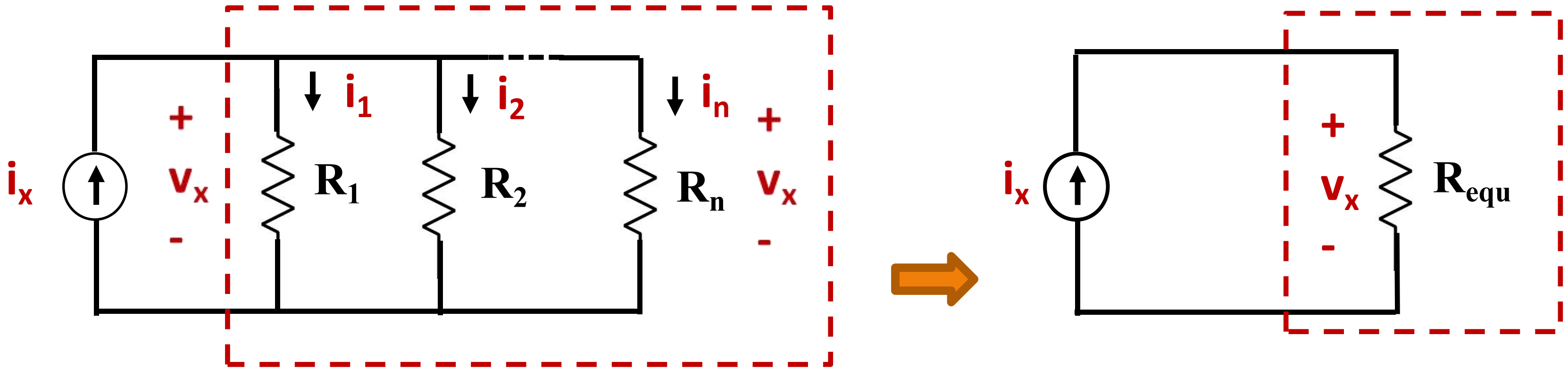






Resistors in Parallel

Resistors in Parallel



- The voltage (v_x) across all the resistors in parallel is the same.
- Applying **KCL** we get:
  - $i_x = i_1 + i_2 \cdots + i_n$
- Using Ohm's law:
  - $i_x = \frac{v_x}{R_1} + \frac{v_x}{R_2} + \cdots + \frac{v_x}{R_n} \quad = \quad v_x \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \right)$
  - $i_x = \frac{v_x}{R_{equ}}$

$$\boxed{\frac{1}{R_{equ}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

Resistors in Parallel

$$\boxed{\frac{1}{R_{equ}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

Conductance (G)

Conductance is the reciprocal of Resistance $= \frac{1}{Resistance}$

$\mathbf{G} = \frac{1}{R}$

Unit of **Conductance** is **mho (℧)**

- The $\mathbf{R_{equ}}$ can be written as well:

$$R_{equ}^{-1} = R_1^{-1} + R_2^{-1} + \cdots + R_n^{-1}$$

- In terms of **Conductance**:

$$\mathbf{G_{equ} = G_1 + G_2 + \ldots + G_n}$$

- A parallel combination is also indicated by the following shorthand notation:

$$\mathbf{R_{equ} = R_1 \,||\, R_2 \,||\, \cdots \,||\, R_n}$$

Two Resistors in Parallel

- The special case of only two parallel resistors is encountered fairly often, and is given by

$$R_{equ} = R_1 \parallel R_2 \Rightarrow R_{equ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

- Or, more simply,

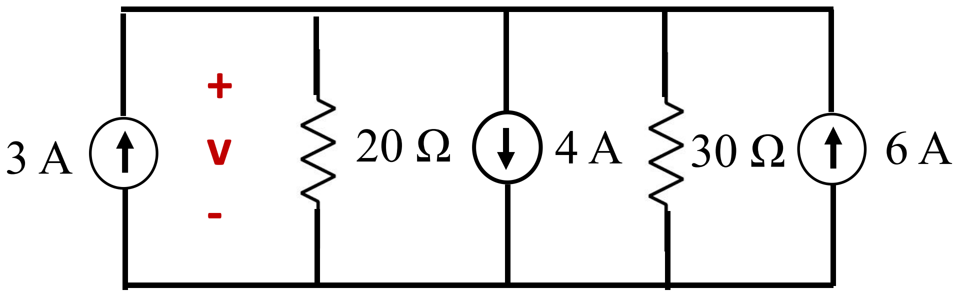
$$\boxed{R_{equ} = \frac{R_1 R_2}{R_1 + R_2}}$$

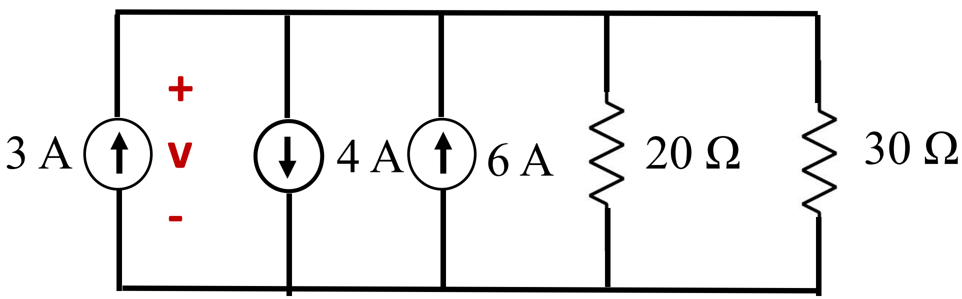
- The above equation is worth remembering, although it is a common error to attempt to generalize this to more than two resistors, which is incorrect.

$$R_{equ} \neq \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}$$

Problem 5: Resistors in Parallel

- Find **v** : **60 V**





Combine the current sources:

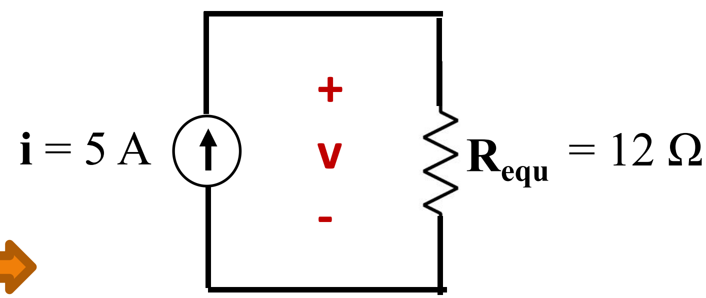
$$\mathbf{i = 3 - 4 + 6 = 5\ A}$$

Combine the parallel resistors:

$$R_{equ} = \frac{20 * 30}{20 + 30} = 600 / 50$$

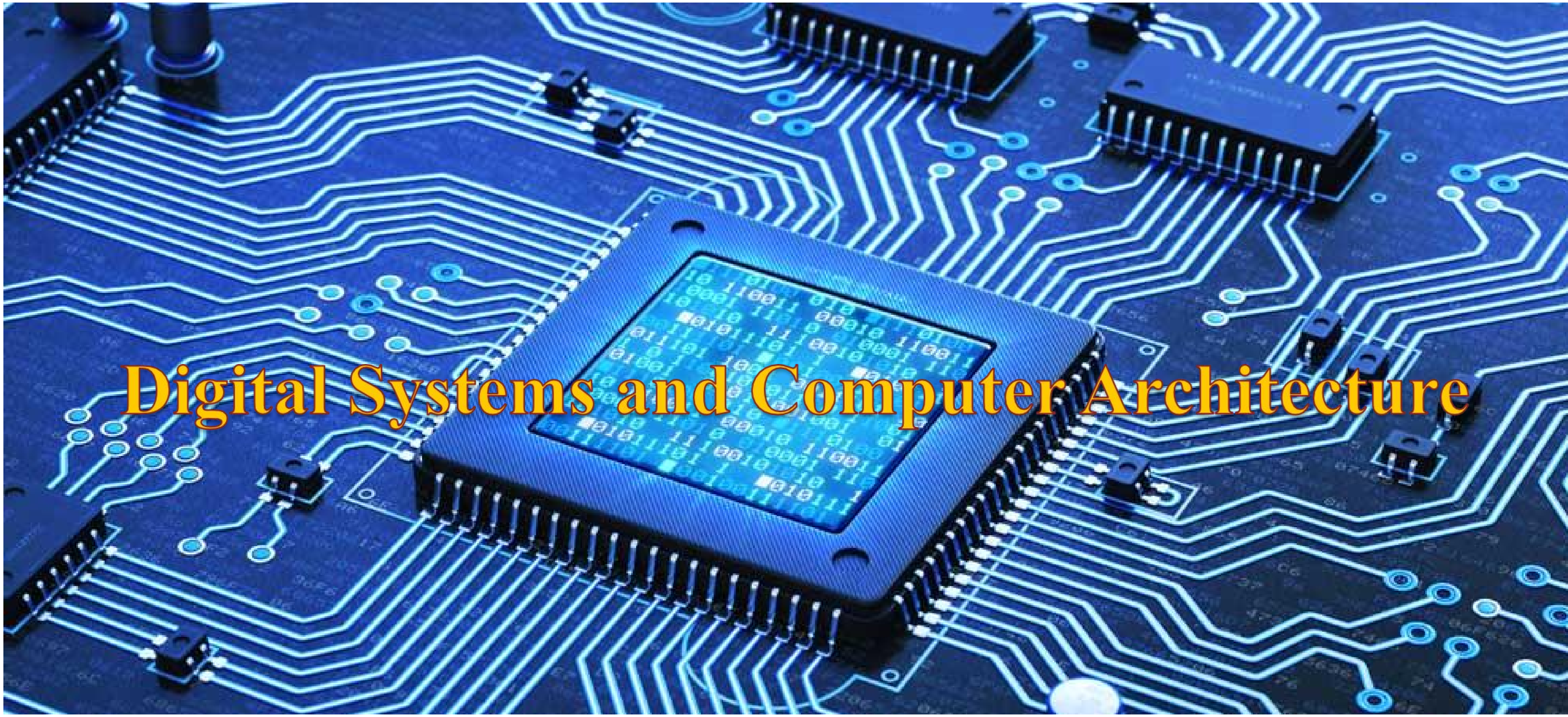
$$R_{equ} = 12\ \Omega$$

$$v = 5 * 12 = 60\ V$$



Session 1.3: Summary

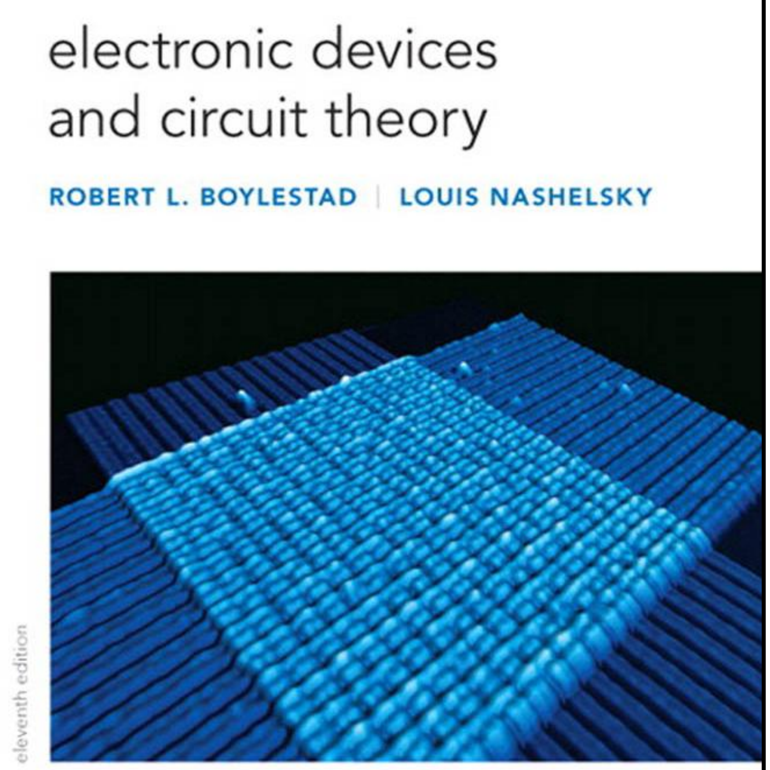
- Conservation of Energy
- Kirchhoff's Current Law
- Kirchhoff's Current Law
- Resistors in Series and Parallel



References

Reference 1: DS & CA

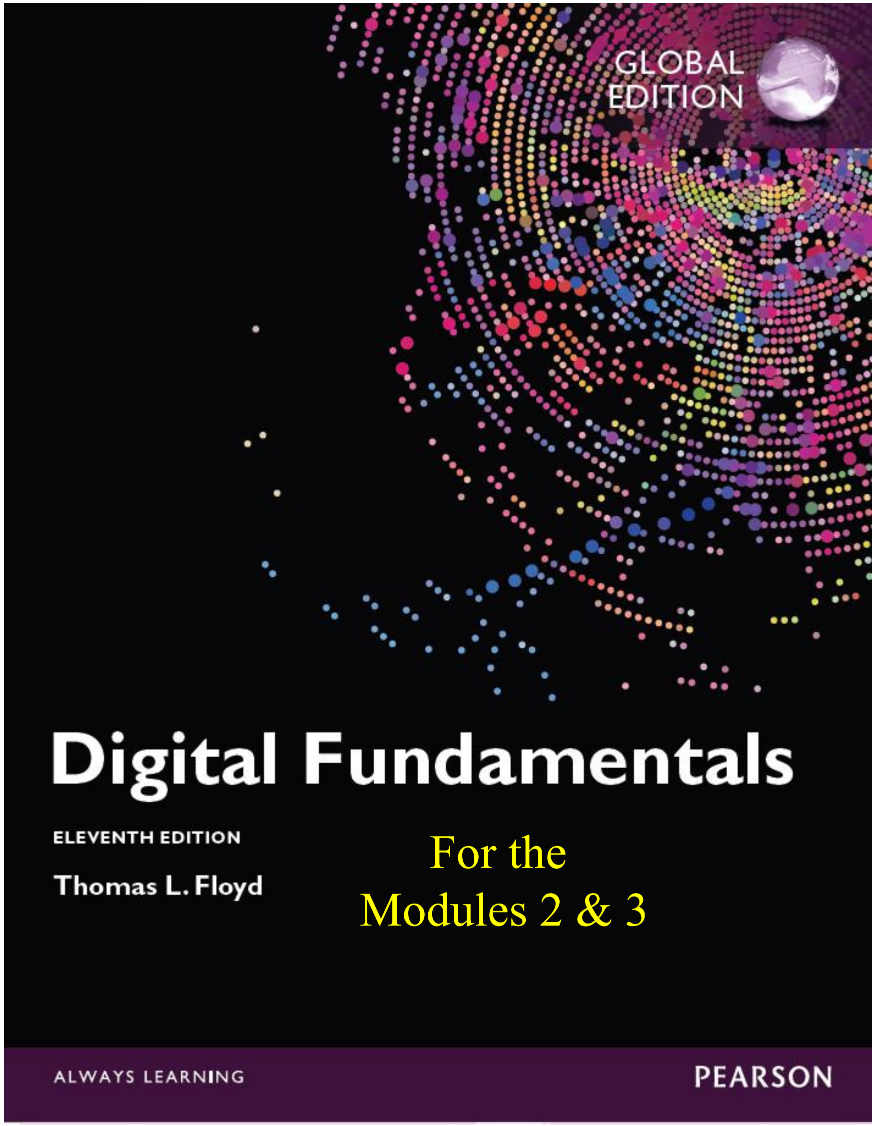
Ref 1



For the
Module 1

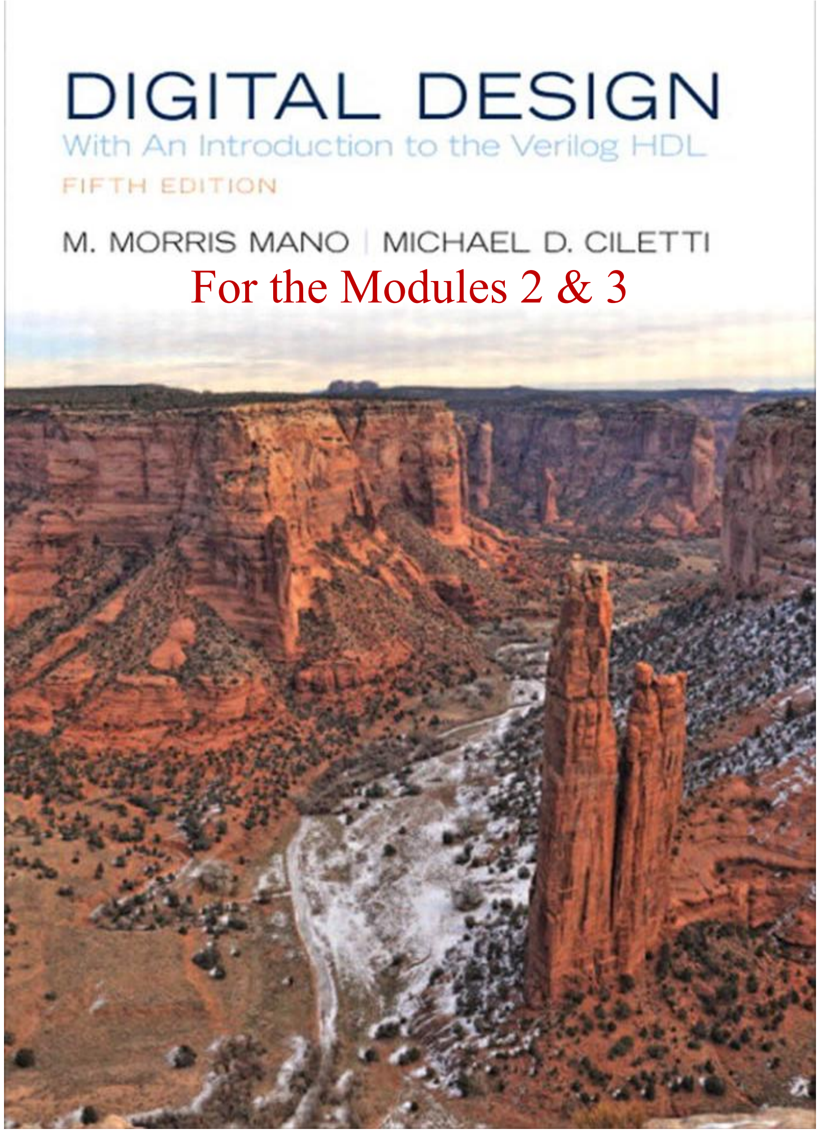
References 2 & 3: DS & CA

Ref 2



For the Modules 2 & 3

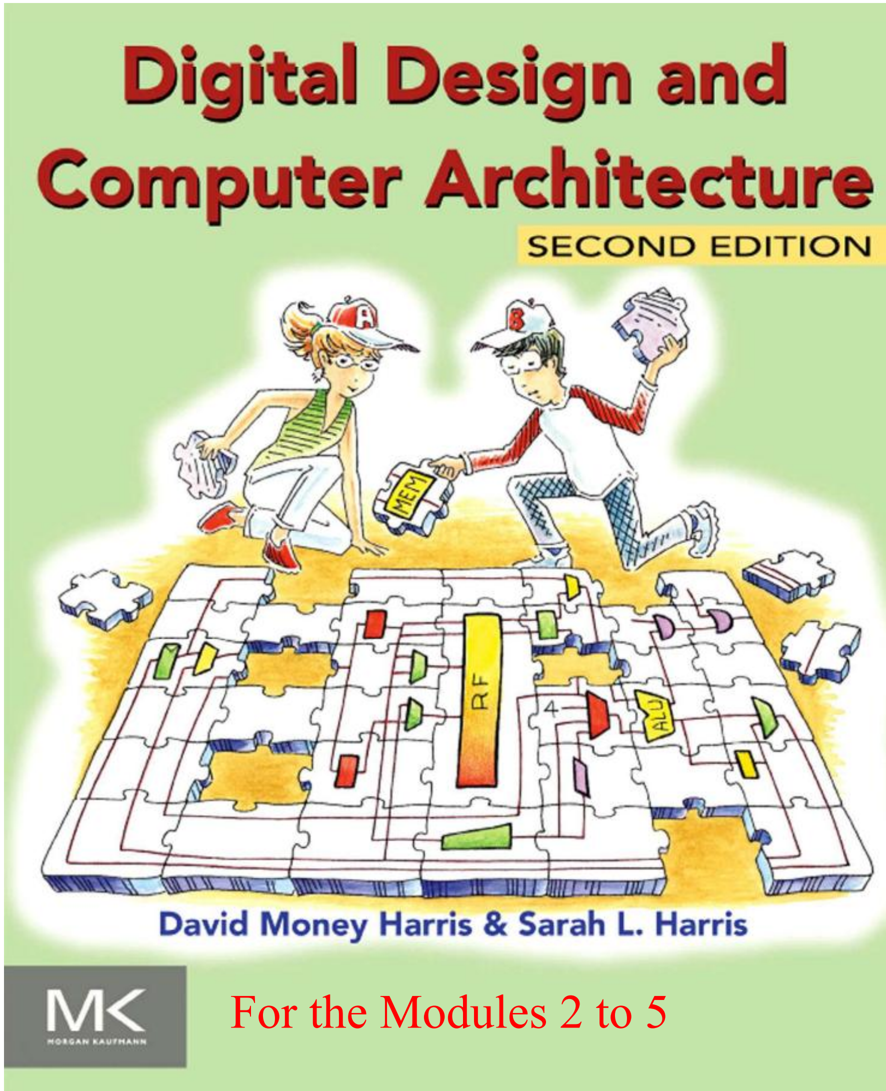
Ref 3



For the Modules 2 & 3

References 4 & 5: DS and CA

Ref 4



For the Modules 2 to 5

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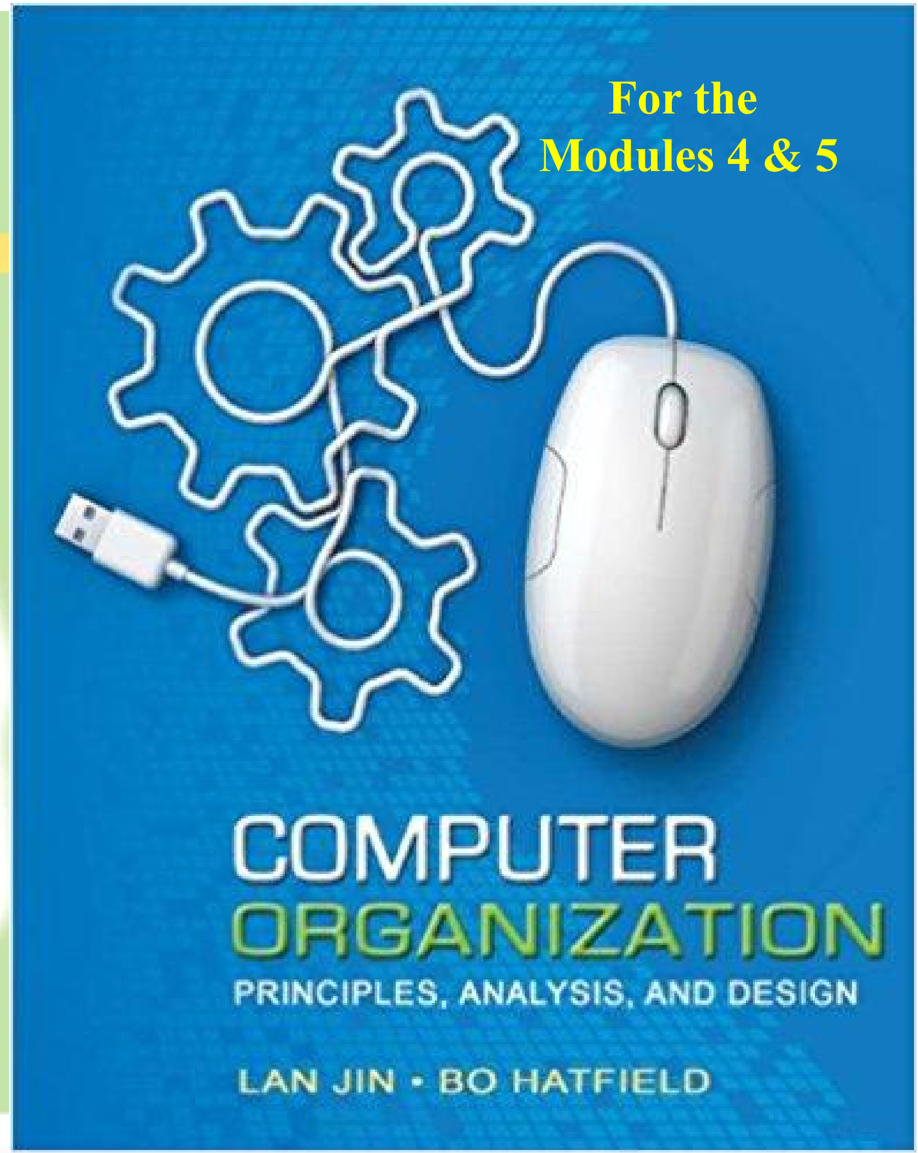

For the Modules 4 & 5