



Digital Systems and Computer Architecture

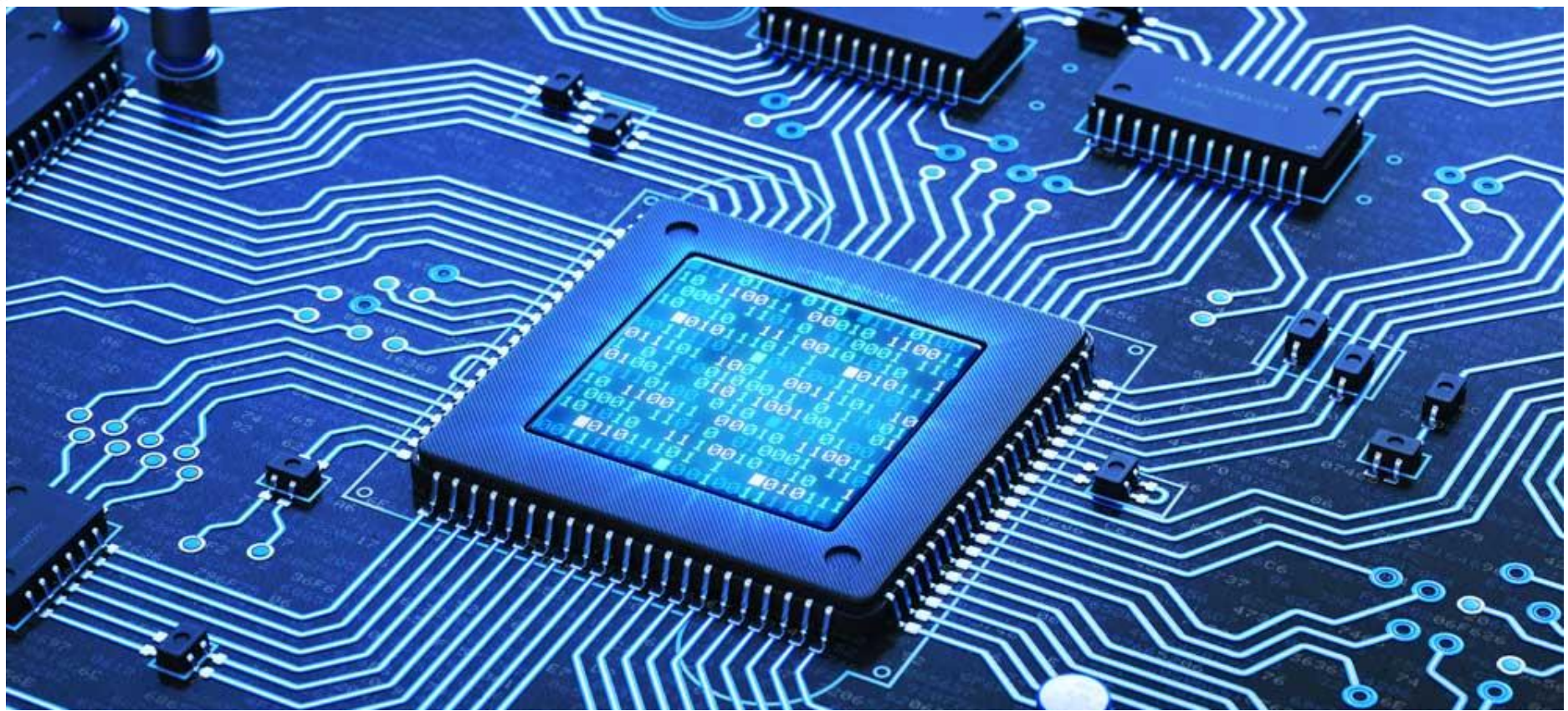
Session 1.5

Module 1b

AC Signals an Introduction

Session 1.5: Focus

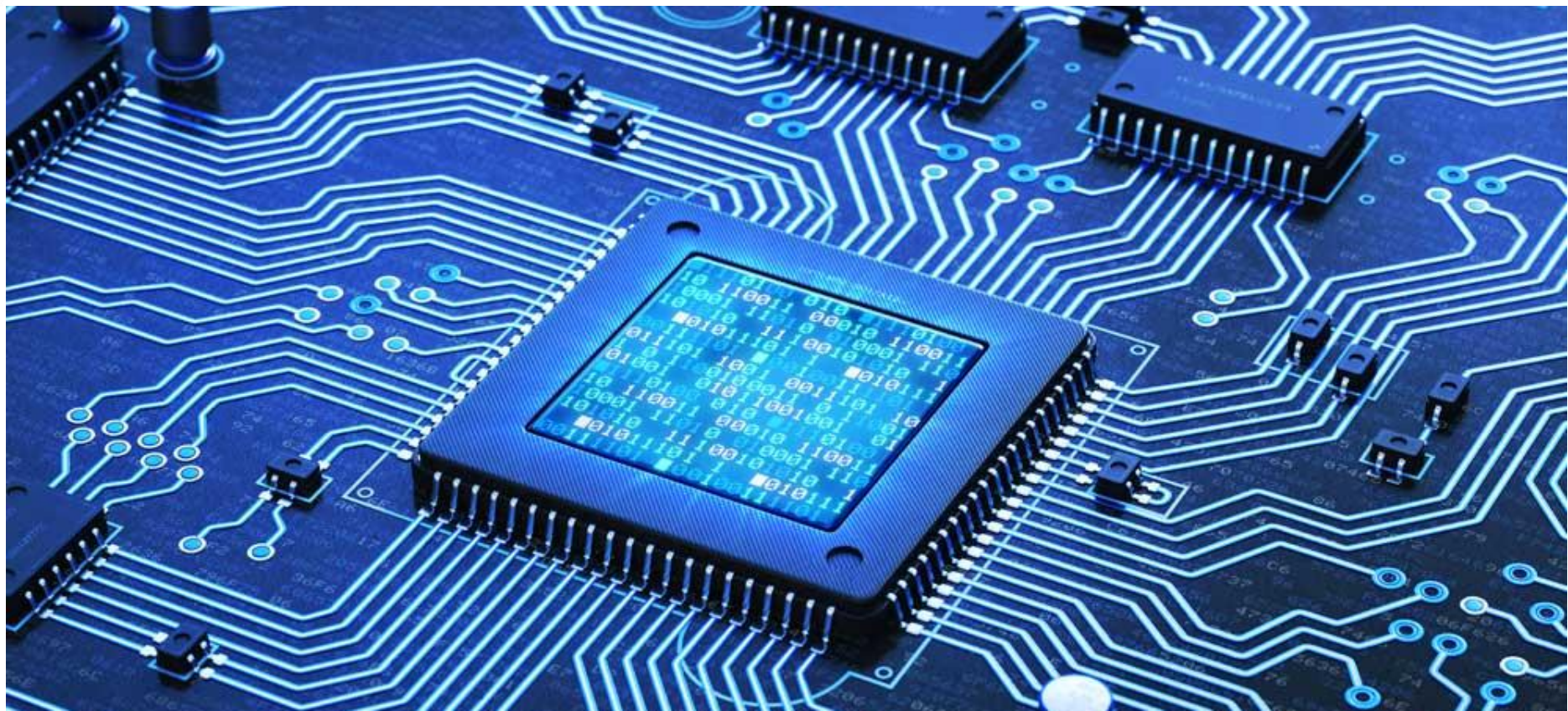
- AC Signals
 - Square Waves
 - Triangular Waves
 - Sinusoids
 - Why Sinusoidal Signals?
- Generation of Sinusoids
 - Attributes of Sinusoids
 - Angular frequency (ω)
 - Phase shift (ϕ)
- Parameters of AC Voltage Signals
 - Peak, Peak-to-peak and Average Voltages
 - Root-mean-square (RMS) Voltage



AC Signals

AC Signals and Systems

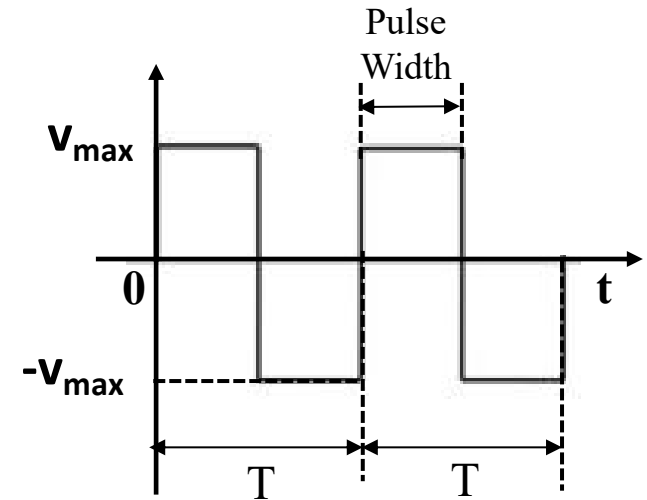
- So far we have seen circuits with constant voltage and current sources.
 - They are called DC (Direct Current) systems or DC circuits.
- We can't do much, only with DC circuits other than delivering DC power.
- Practical circuits that we need are the ones to sample the world, capture or play videos or music, manipulate and control information, etc.
- What we need is AC (Alternating Current) signals and circuits that work with AC signals.



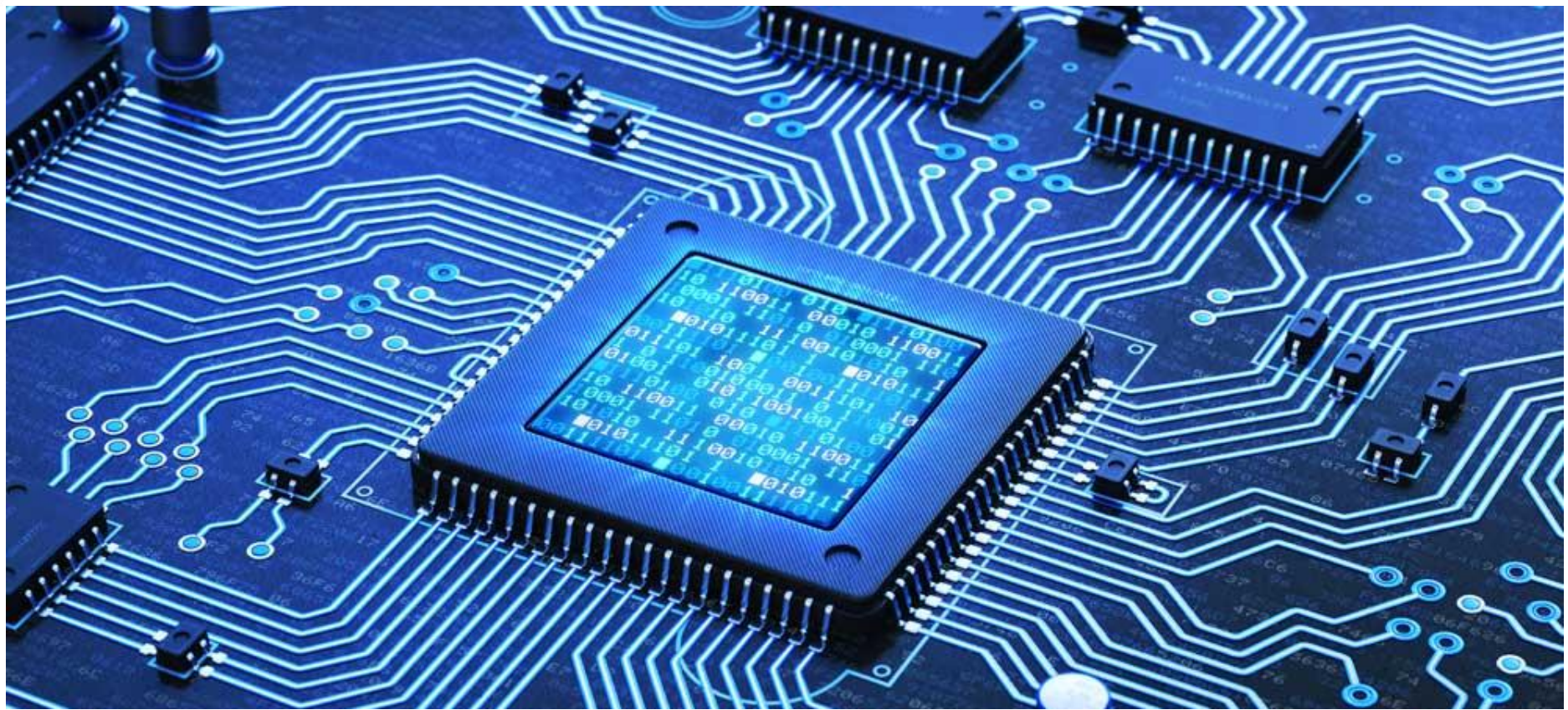
Square Waves

Square Waves

- A square wave is a non-sinusoidal periodic waveform in which the amplitude alternates at a steady frequency between fixed **minimum** and **maximum** values.
- In an ideal square wave, the transitions between minimum and maximum are instantaneous.
- The **square wave** is a special case of a **pulse wave** which allows arbitrary durations at minimum and maximum.
- The ratio of the high period to the total period of a pulse wave is called the **duty cycle**.
 - A true square wave has a 50% duty cycle (equal high and low periods)
- Square waves are often encountered in electronics and signal processing, particularly digital electronics and digital signal processing.



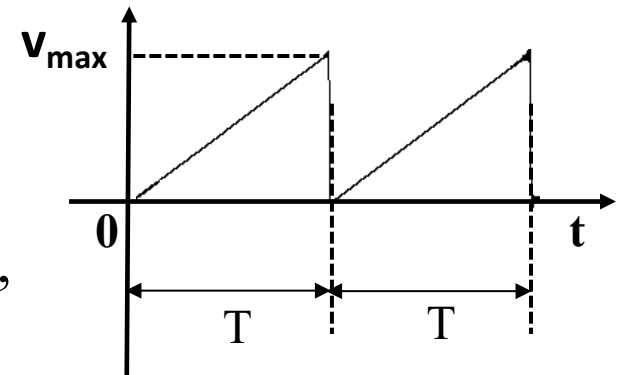
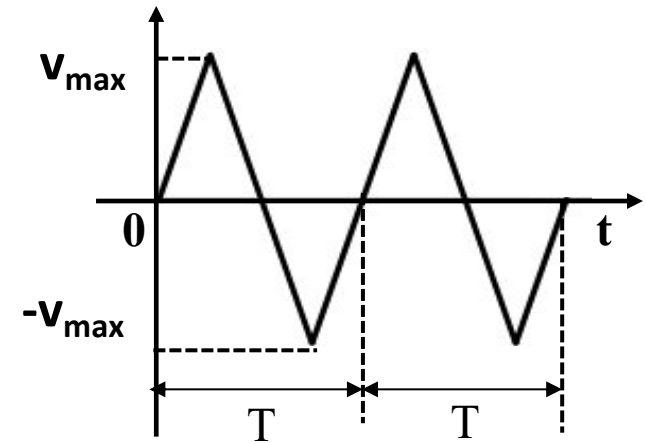
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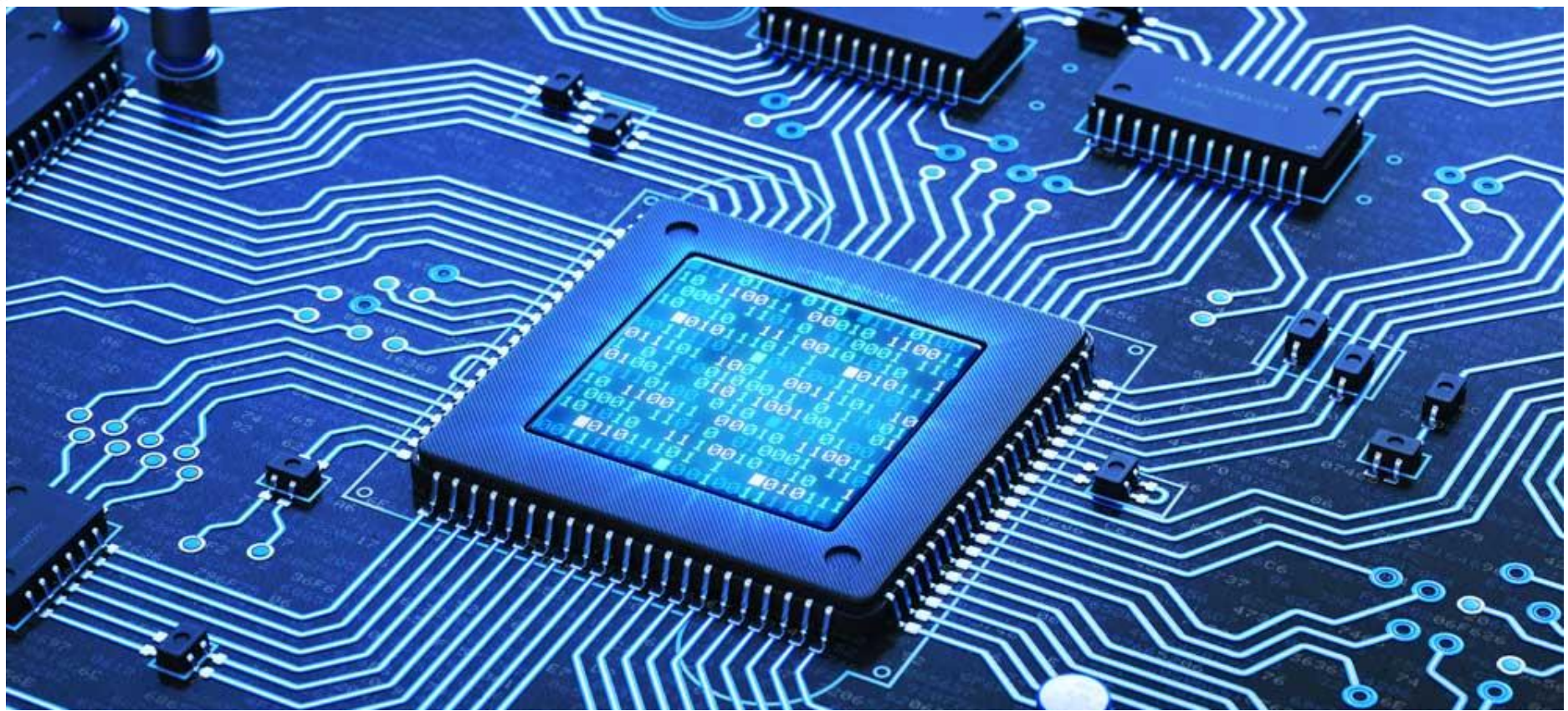
Triangle Waves

Triangle Waves

- A **triangular** or **triangle wave** is a non-sinusoidal waveform named for its triangular shape.
- It is periodic and also piecewise linear.
- The individual segments of this signal is a linearly changing signal, either increasing or decreasing.
- Triangle waves have equal rise and fall times.
- Waves with unequal rise and fall times are called **sawtooth waves**.
- Triangular waves are used in sampling circuits, tone generation circuits, frequency generator circuits, etc.



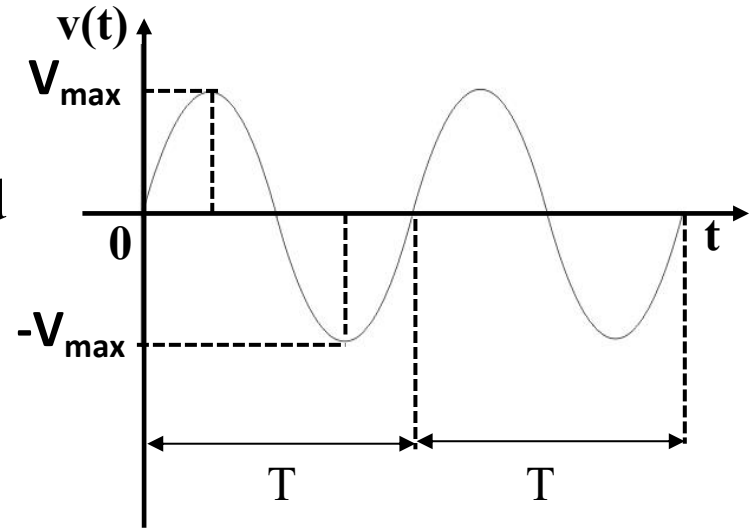
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Sinusoidal Signals

Sinusoidal Signals

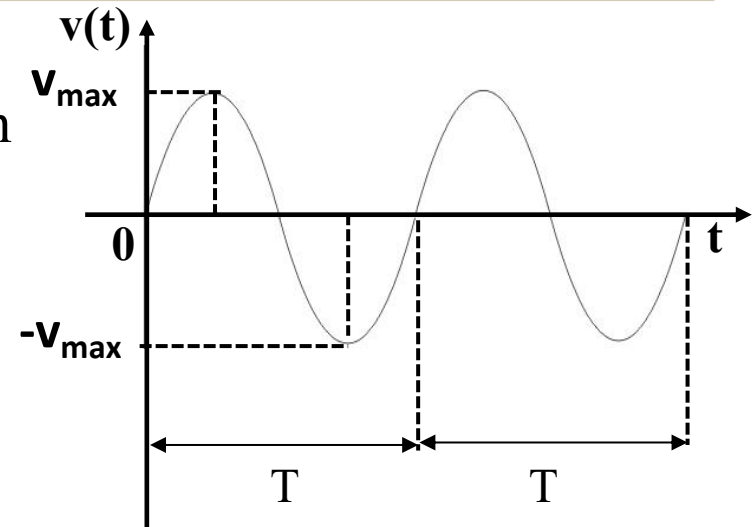
- Alternating voltages and currents are sinusoidal waves.
- Simple **sine waves** are normally referred to as **sinusoids**.
- A sinusoidal voltage (current) fluctuates periodically both in **polarity** and **direction**.

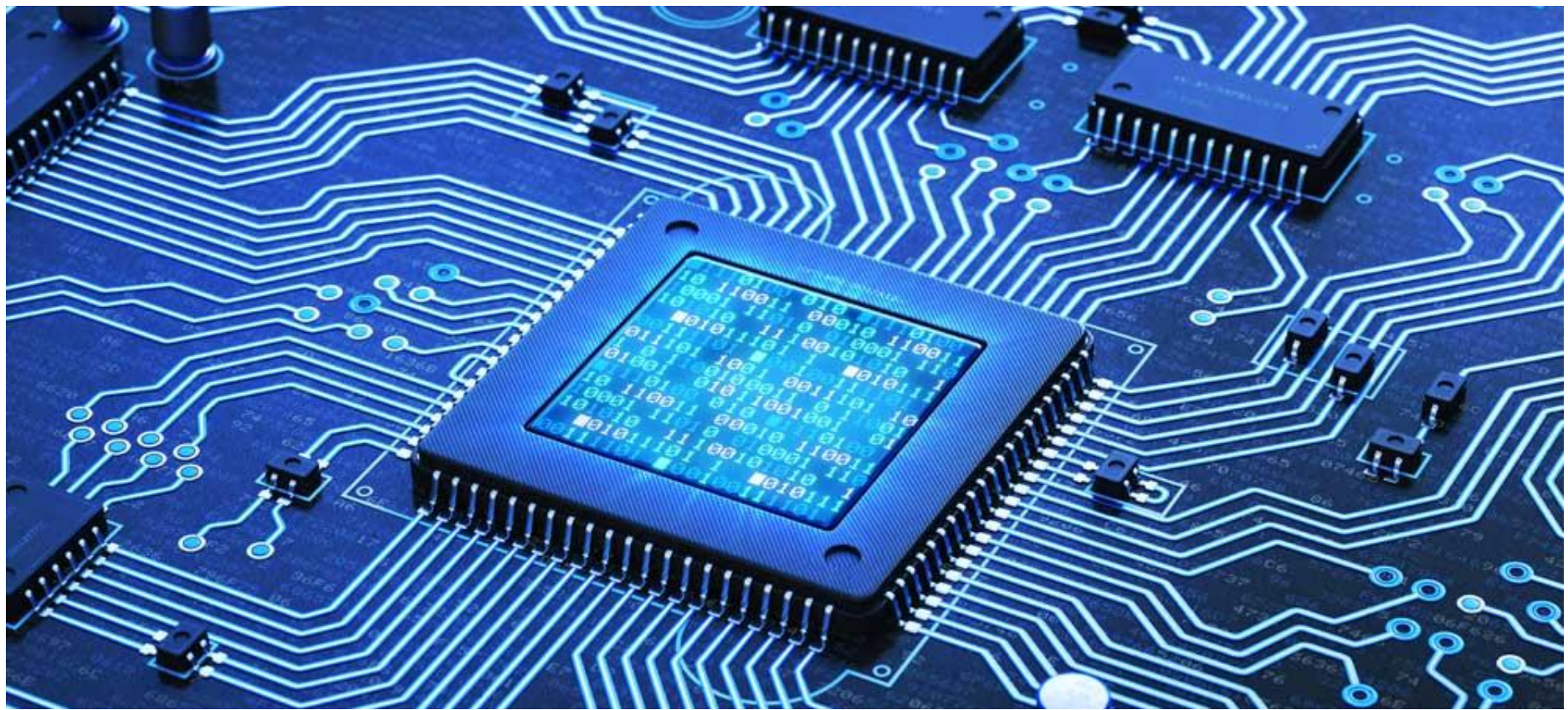


- In the signal here, the amplitude alternates between $+V_{\max}$ and $-V_{\max}$
- The time taken for this sinusoidal wave to complete one full cycle is called the **Period (T)** and it is measured in **seconds**.
- **Frequency (f)** of this signal is measured in **Hertz (Hz)**.
 - f is given as $\frac{1}{T}$
- If the **frequency** of the signal is **50 Hz**, then $T = \frac{1}{f} = \frac{1}{50} = 0.02 = \mathbf{20 \text{ msecs}}$

Why Sinusoidal Signals?

1. AC generators produce sinusoidal voltages when rotors are made to rotate in the presence of magnetic field.
2. Any periodic waveforms can also be written in terms of sinusoidal functions using the Fourier theorem.
3. As you are aware, derivatives and integrals of sinusoids are also sinusoids.
4. Moreover, generation, transmission and consumption are also easier with sinusoidal signals.
5. Sinusoidal signals are also easy to analyze.

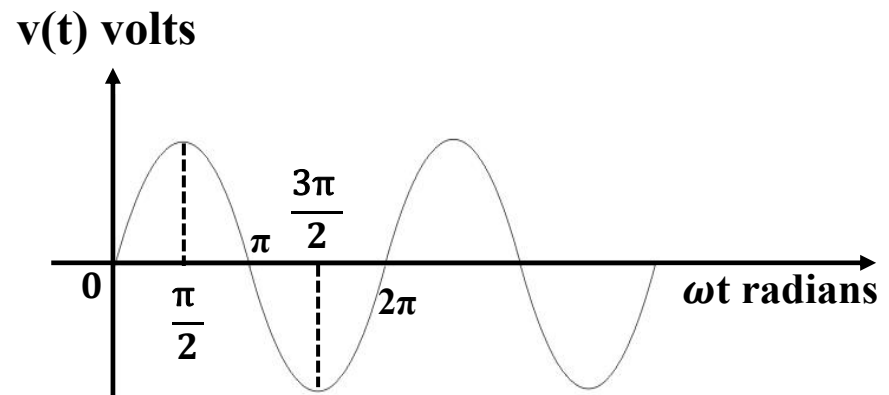
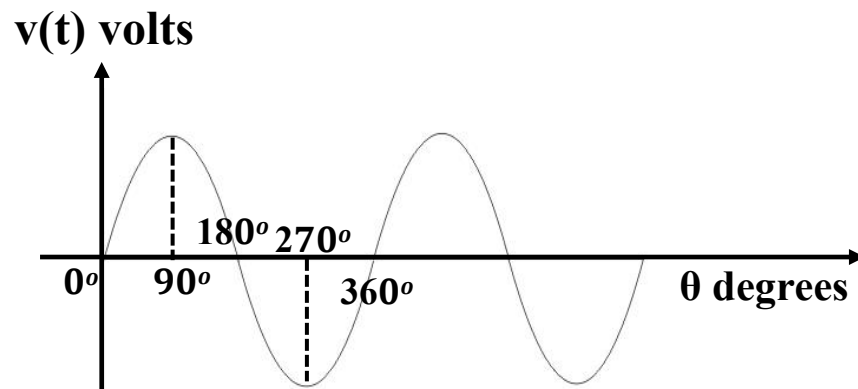
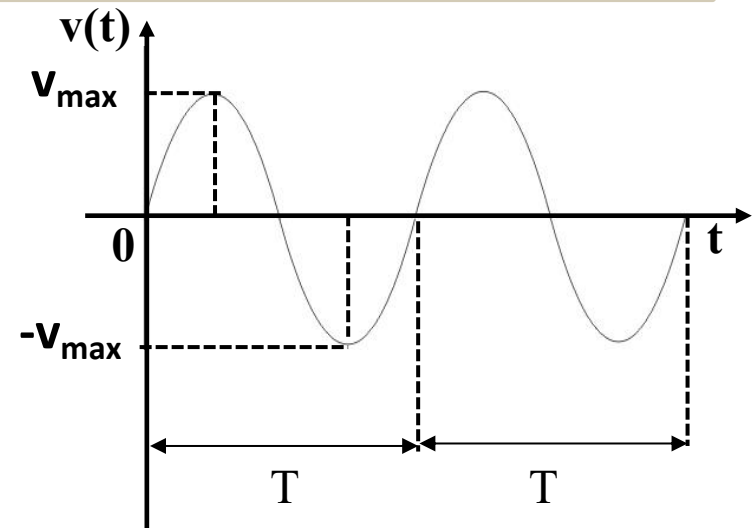




Sinusoids

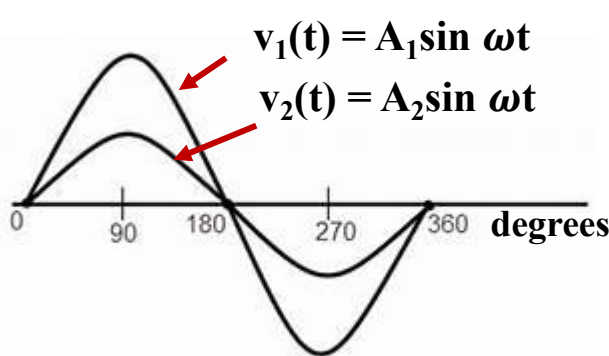
Angular Frequency (ω)

- Sinusoidal signal with an angular frequency of ω is **$\sin \omega t$**
- Unit of ω is **radians per seconds**.
- Angular frequency: **$\omega = 2\pi f$**
 - Where **f** is frequency in **Hz**.
- $\sin \omega t = \sin 2\pi f t$ which is a function of time.

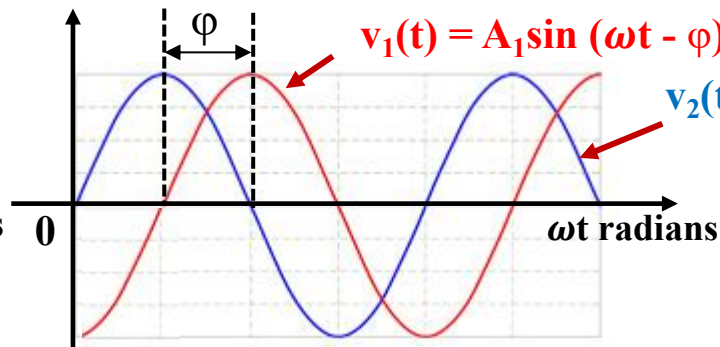


$$2\pi \text{ radians} = 360 \text{ degrees}$$

Phase Shift (ϕ)



Two signals are **in-phase**



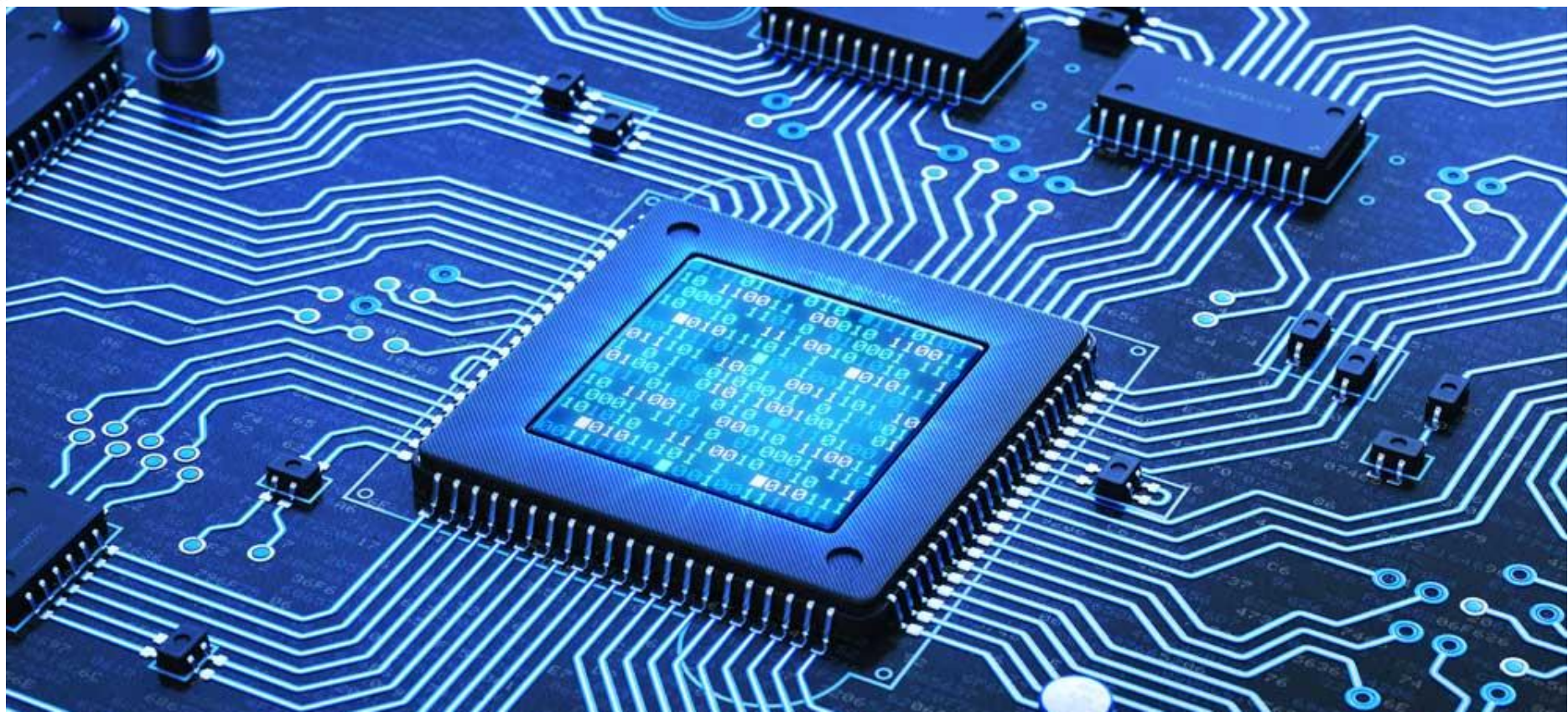
Two signals are
out-of-phase by ϕ

Here, $A_1 = A_2$

How much is ϕ ?

$\frac{\pi}{2}$ radians

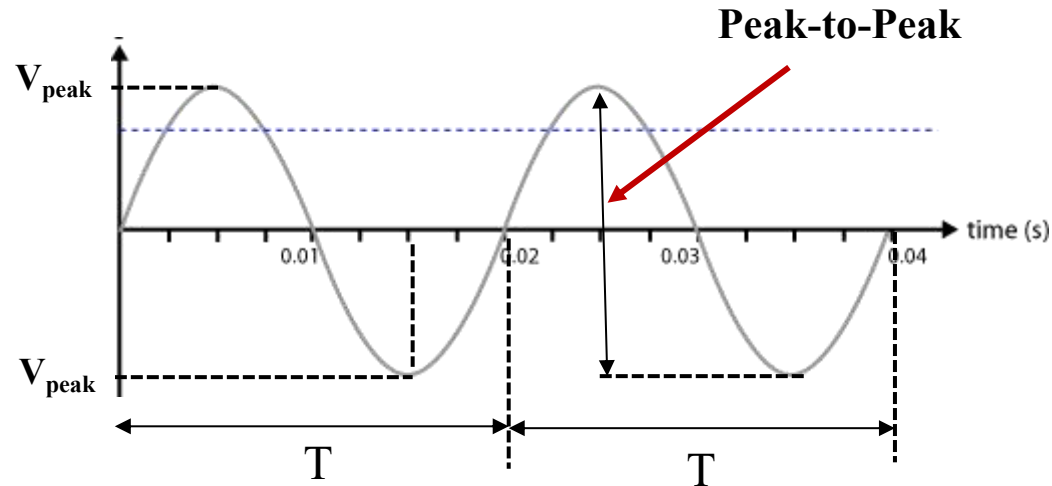
- Here, the **red** sinusoid is said to be **lagging** behind **blue** by $\frac{\pi}{2}$ radians.
- It can also be stated as **blue** sinusoid is **leading** the **red** by $\frac{\pi}{2}$ radians.
- The **phase** of a sine wave is an angular measurement that specifies the position of the wave relative to a reference wave.
- Any signal that does not pass through zero at $t = 0$ is said to be having a phase shift.
- Here, the blue sinusoid is the reference signal and the red sinusoid is said to be lagging behind the reference signal (blue) by $\frac{\pi}{2}$ radians or **90 degrees**.



Peak and Peak-to-Peak Voltages

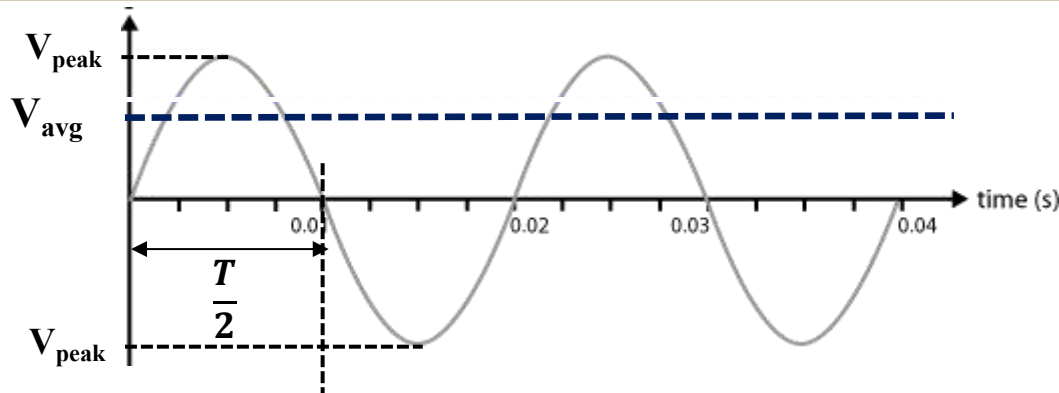
Peak and Peak-to-Peak Voltages

Quiz1: Find the frequency.



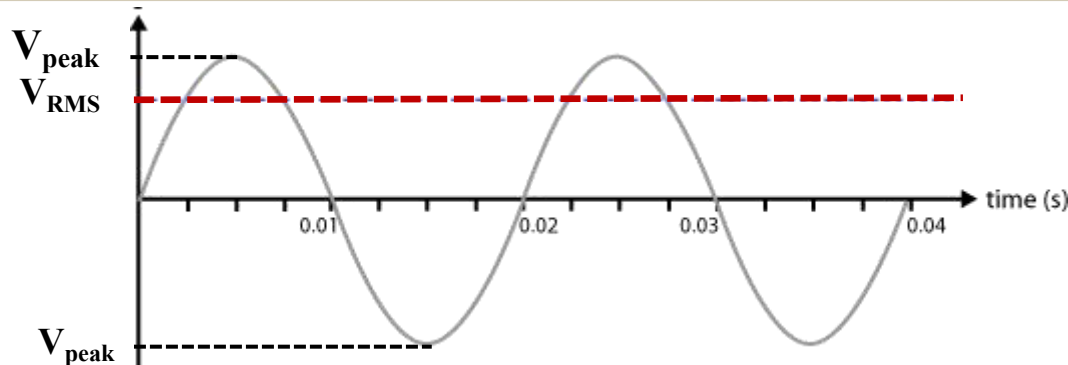
- As all of you are aware, in India the standard AC voltage has the **frequency** is **50 Hz**.
 - Period (T) of this signal = $\frac{1}{\text{Frequency}} = \frac{1}{50} = 0.02 \text{ Seconds} = 20 \text{ msecs}$
- Peak voltage of the AC voltage is shown as V_{peak} .
- Peak-to-Peak voltage is given as the total voltage between the positive and negative peaks of the AC voltage.

Average Voltage



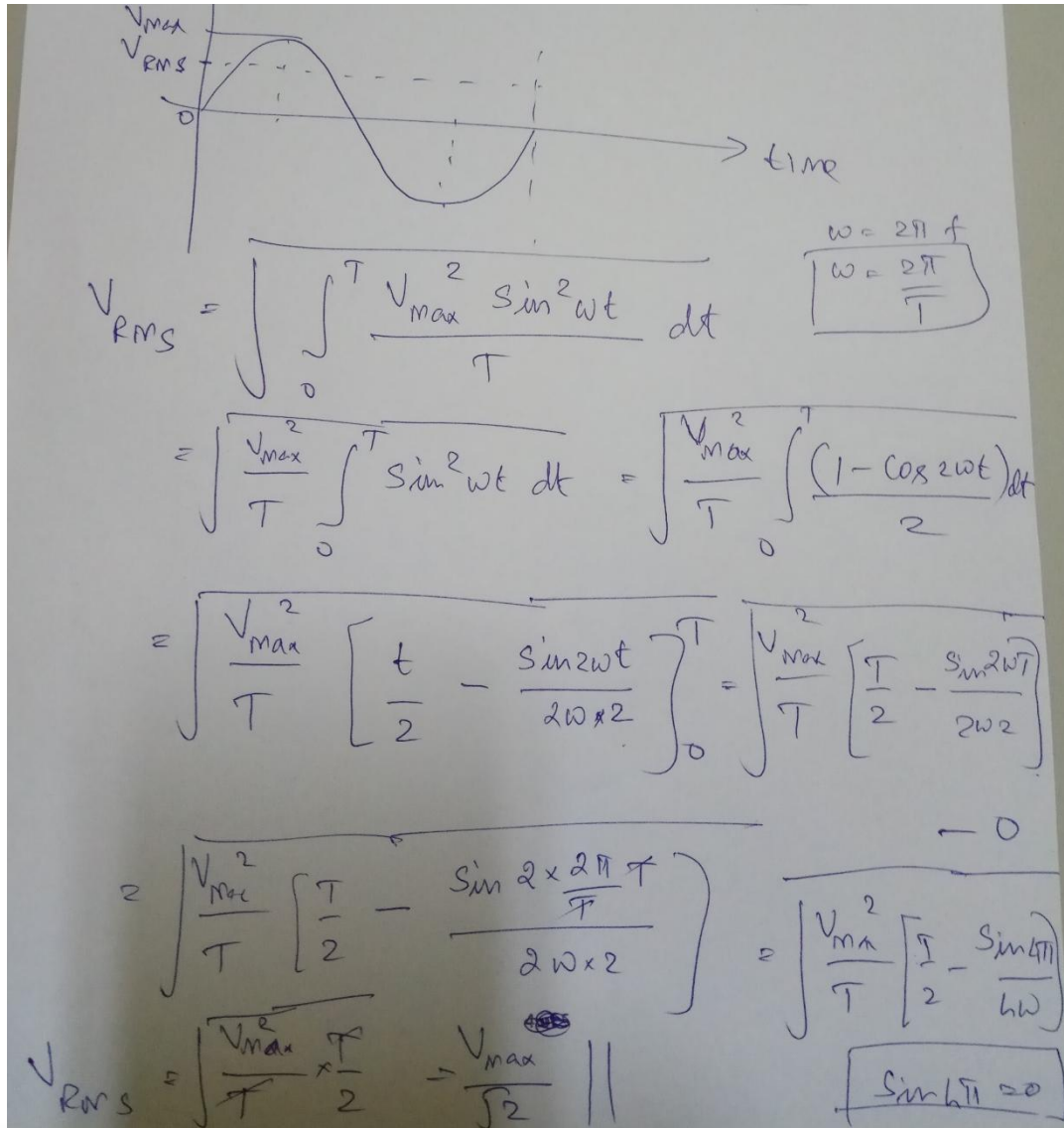
- Since AC voltage is alternating between positive and negative voltages, the total **average value** (V_{avg}) over the full cycle will be equal to zero.
- For this reason, the average voltage is computed by considering only the positive half-cycle of AC signal.
- Thus, average value of positive half-cycle can be computed as:
- $$V_{\text{avg}} = \frac{1}{T/2} \int_0^{T/2} V_{\text{peak}} \sin \omega t \, dt = 0.637 V_{\text{peak}}$$

Average and RMS Voltage



- Since AC voltage is alternating between positive and negative voltages, the total **average value** (V_{avg}) over the full cycle will be equal to zero.
 - So, the only average value of positive half-cycle is $0.637V_{peak}$ is considered to be average value of a sinusoidal voltage signals.
- The RMS (Root Mean Square) of AC signal is a measure of AC voltage, which is the heating value of an equivalent DC voltage would create, if it passes through a resistor.
- It is shown as V_{RMS} , which is a DC equivalent of an AC voltage.

RMS Voltage Derivation – Not part of the Syllabus



Handwritten derivation of RMS voltage formula:

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T V_{max}^2 \sin^2 \omega t \, dt}$$

Given: $\omega = 2\pi f$, $\omega = \frac{2\pi}{T}$

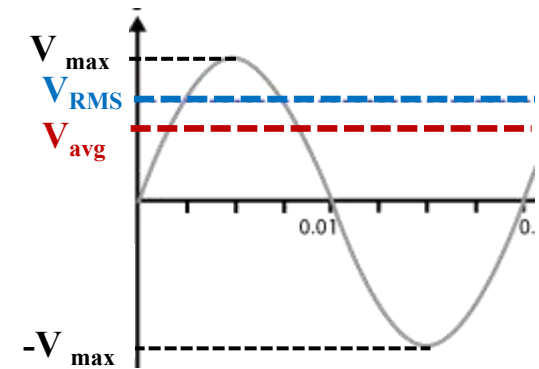
$$= \sqrt{\frac{V_{max}^2}{T} \int_0^T \sin^2 \omega t \, dt} = \sqrt{\frac{V_{max}^2}{T} \int_0^T \frac{(1 - \cos 2\omega t)}{2} \, dt}$$

$$= \sqrt{\frac{V_{max}^2}{T} \left[\frac{t}{2} - \frac{\sin 2\omega t}{2\omega \times 2} \right]_0^T} = \sqrt{\frac{V_{max}^2}{T} \left[\frac{T}{2} - \frac{\sin 4\pi}{4\omega} \right]}$$

Since $\sin 4\pi = 0$:

$$V_{RMS} = \sqrt{\frac{V_{max}^2}{T} \times \frac{T}{2}} = \frac{V_{max}}{\sqrt{2}}$$

- RMS voltage can be derived as shown here.



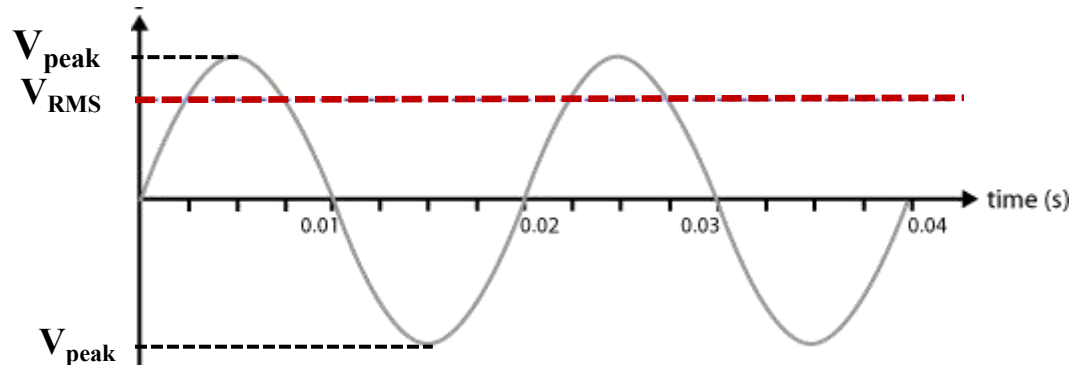
$$V_{RMS} = \frac{V_{max}}{\sqrt{2}}$$

$$V_{RMS} = 0.707 V_{peak}$$

$$V_{avg} = 0.637 V_{peak}$$

Standard AC Voltage in India

Quiz2: What are the values of V_{RMS} and V_{peak} ?



- The standard AC voltage in India has the frequency of 50 Hz and V_{RMS} voltage of **230 V**.
- $V_{\text{peak}} = V_{\text{RMS}} / 0.707 = 230 / 0.707 = \mathbf{325\text{ V}}$

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