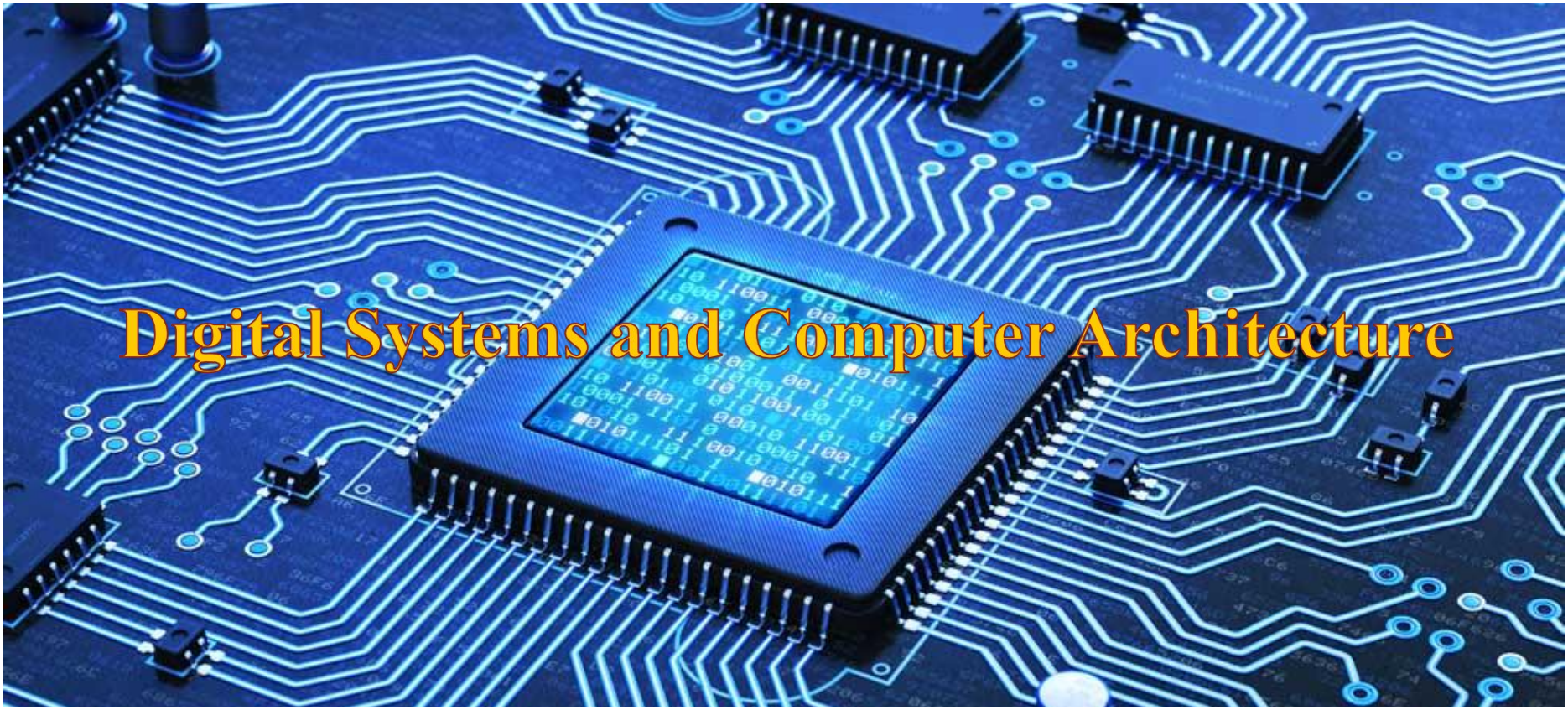


Session 2.6

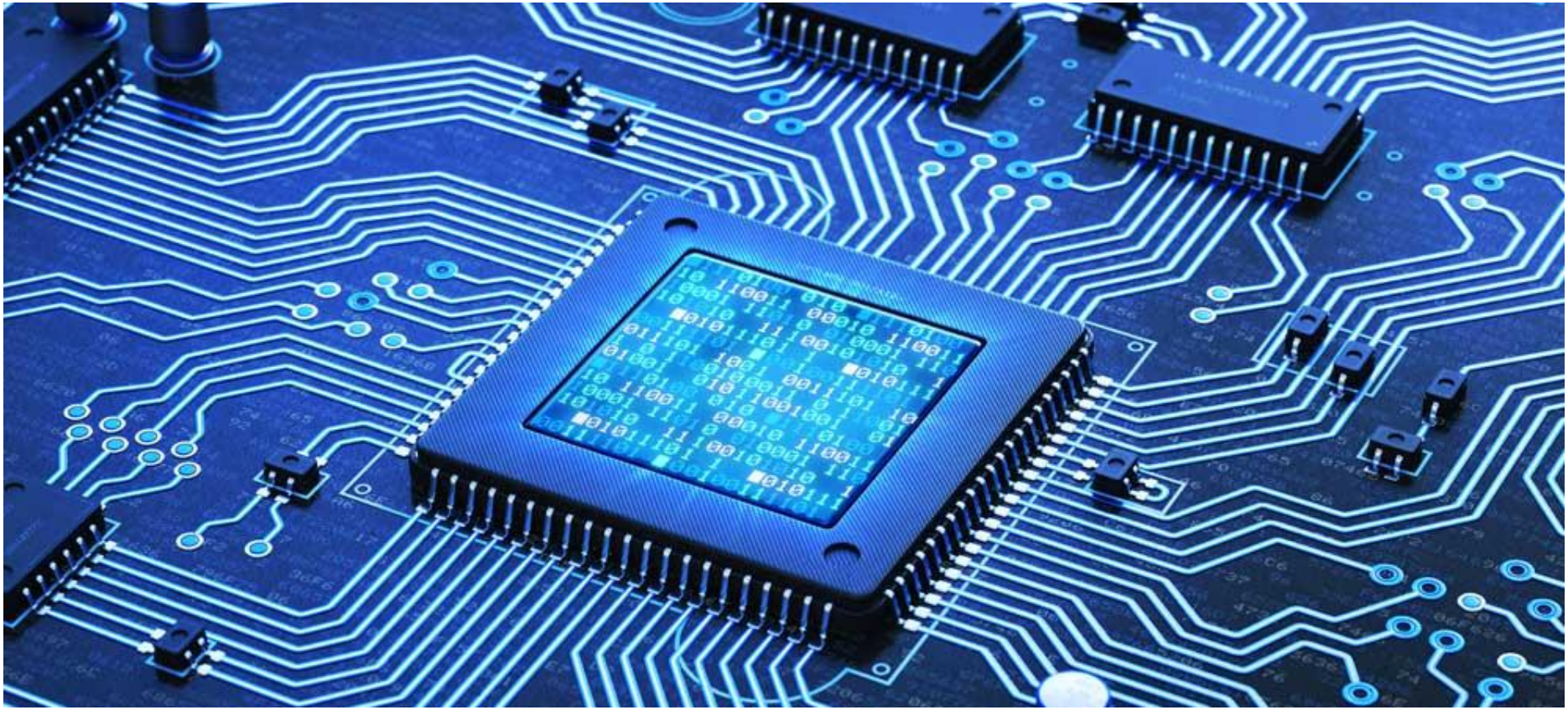
Module 2

Mouli Sankaran

Introduction to Karnaugh map

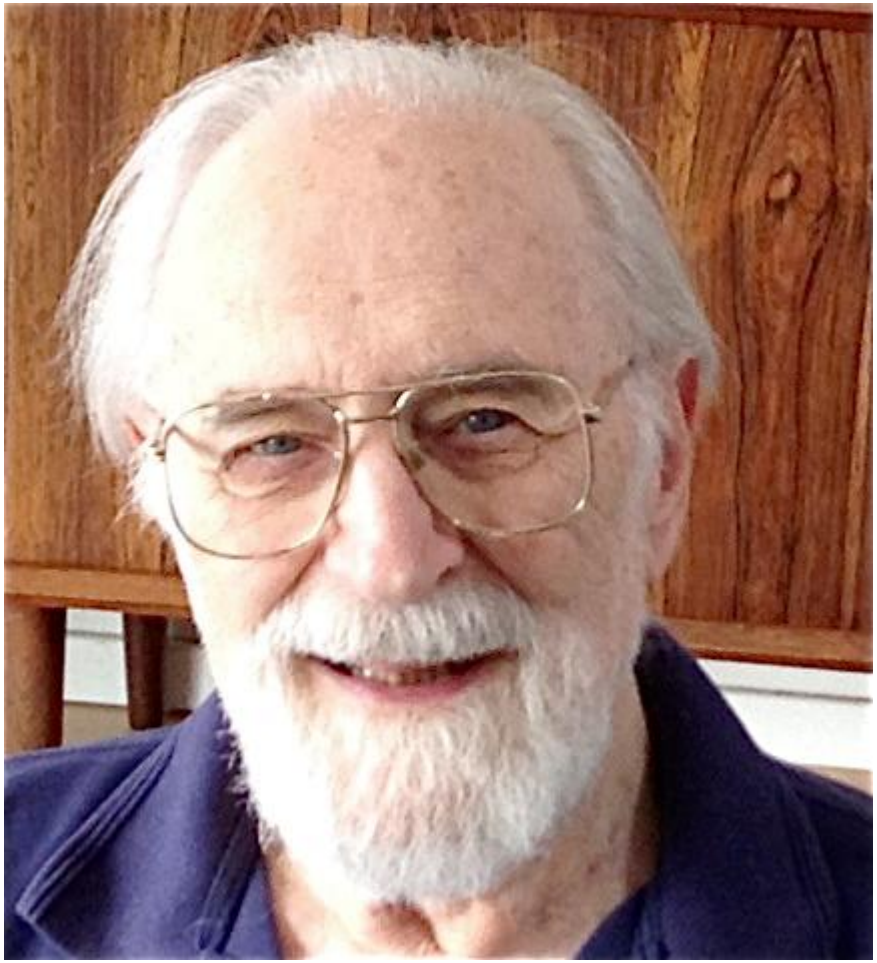
Session 2.6: Focus

- Introduction to Karnaugh map
- Examples
  - SOP form
  - POS form
  - Both SOP and POS
  - Implementation
  - 5-variable K-map
- Don’t cares in K-map
  - Example



Karnaugh map
Simplification Method

Maurice Karnaugh

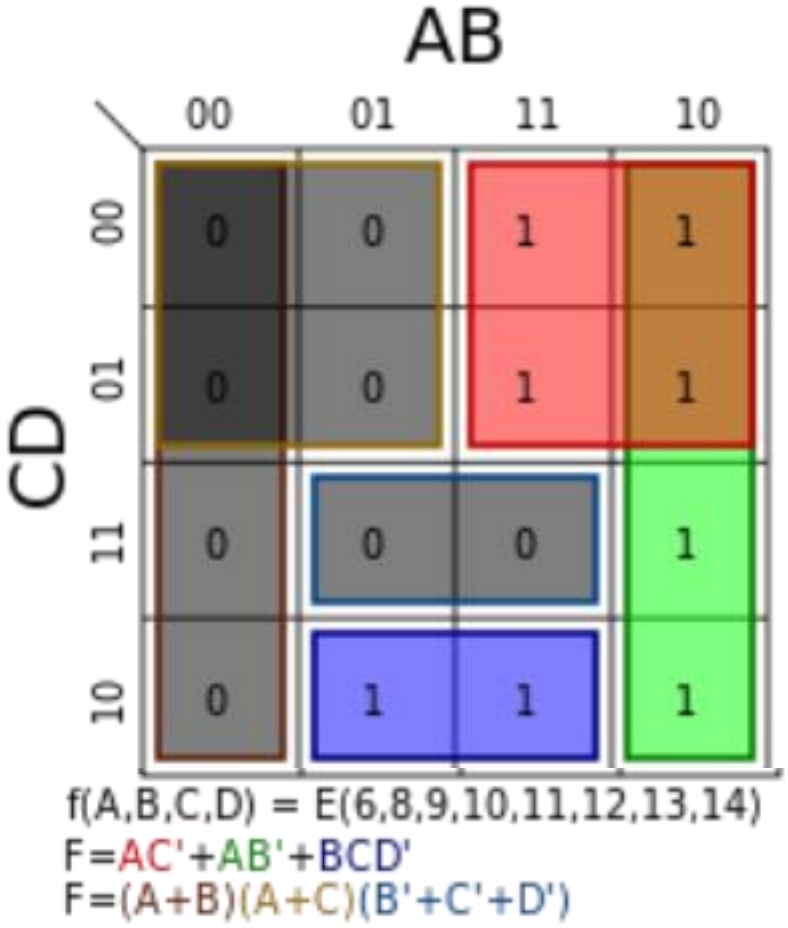


Maurice Karnaugh (**October 4, 1924 – Nov 8, 2022**) is an American physicist and mathematician known for the Karnaugh map used in Boolean algebra.

Karnaugh Map

- The **Karnaugh map**, also known as the **K-map**, is a method to simplify Boolean algebra expressions.
- **Maurice Karnaugh** introduced it in **1953**
- It **reduces** the need for **extensive calculations** by taking advantage of humans' **pattern recognition** capability
- The required Boolean results are **transferred from** a **truth table** onto **a two-dimensional grid**
  - Where the cells are **ordered** in **Gray code**, and
  - **Each cell** position represents **one combination of input conditions**
  - while each **cell value** represents the corresponding **output** value

Sample: Karnaugh Map - corrected



This image actually shows **Two Karnaugh maps**: for the **function f,** Using **minterms (colored rectangles)** and for its **complement f'**, using **maxterms (gray rectangles)**.

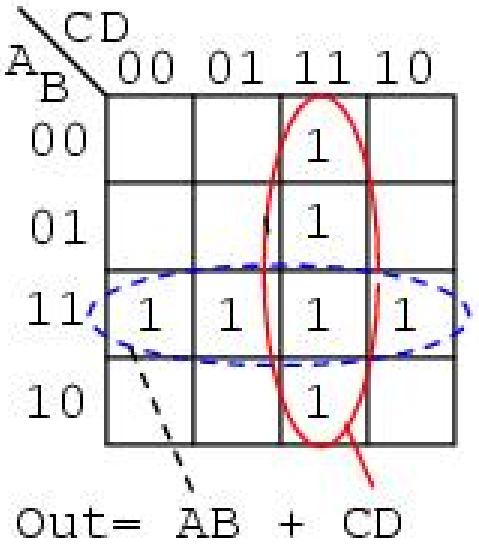
Courtasy: Wikipidea

Why Gray Code in K-map?

- The variable combinations in the K-kap in the adjacent cells **differ only by one bit**, by ordering them using **Gray codes**
- This helps in simplifying them by combing the adjacent elements if both output **one** (for minterms) or **zero** (for maxterms) since they differ by only one value

Example 1 (SOP)

$$Out = \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + ABCD + A\bar{B}CD + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D}$$



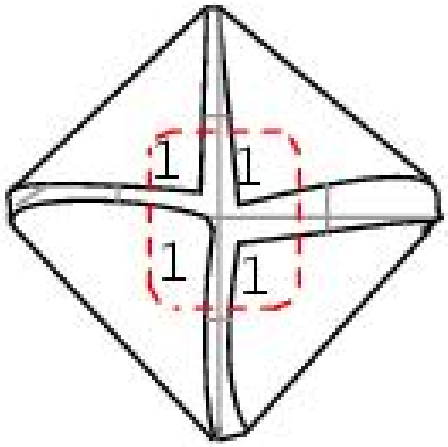
Groups must be a power of 2.

Example 2 (SOP)

$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\,\overline{D} + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D}$$

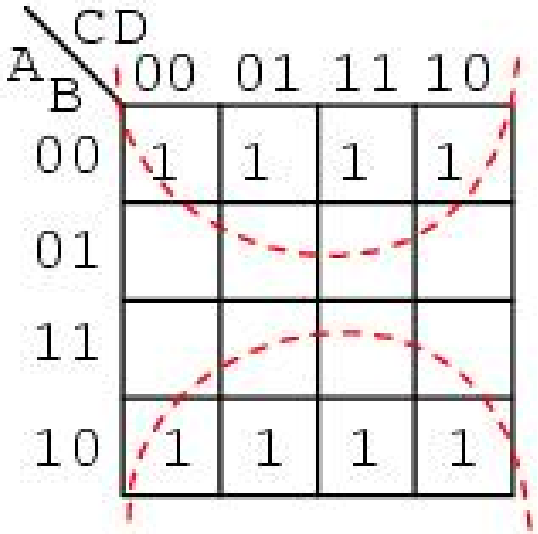
A B \ C D	00	01	11	10
00	1			1
01				
11				
10	1			1

$$Out = \overline{B}\,\overline{D}$$



Example 3 (SOP)

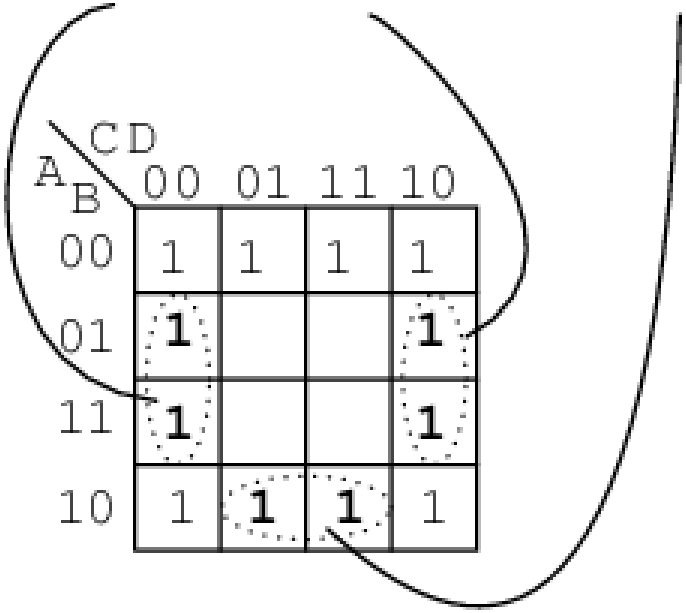
$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\,\overline{D} + \overline{A}\,\overline{B}CD + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,\overline{C}D + A\,\overline{B}C\,\overline{D} + A\,\overline{B}CD$$

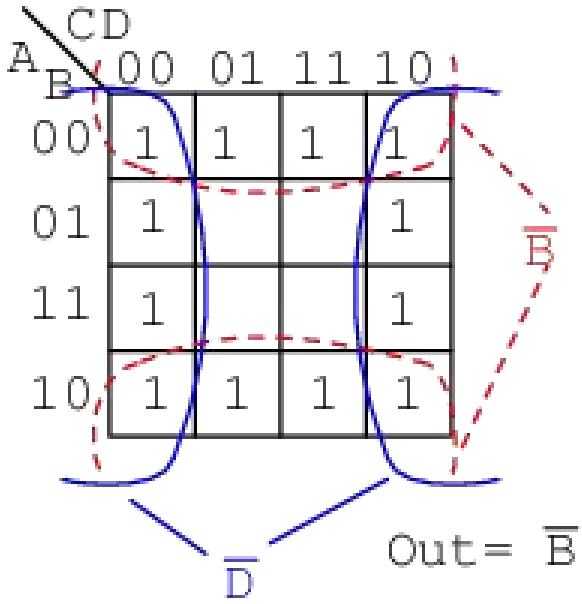


$$Out = \overline{B}$$

Example 4 (SOP)

$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,\overline{B}\,C\,\overline{D} + B\,\overline{C}\,\overline{D} + B\,C\,\overline{D} + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,D + A\,\overline{B}\,C\,\overline{D}$$

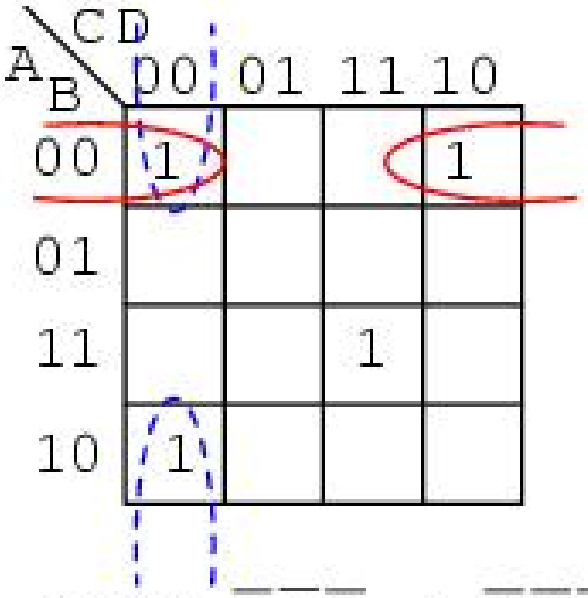




$$Out = \overline{B} + \overline{D}$$

Example 5 (SOP)

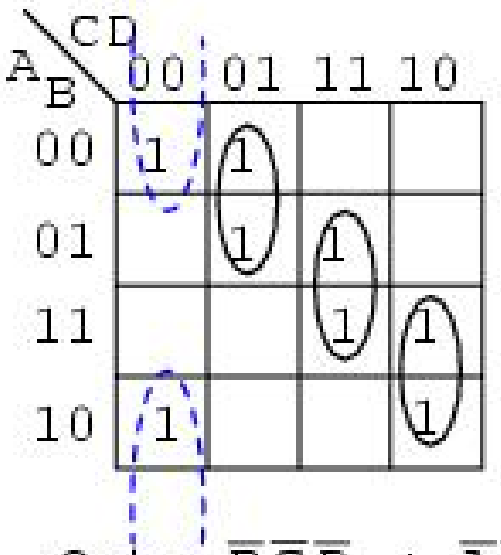
$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\,\overline{D} + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,B\,C\,D$$

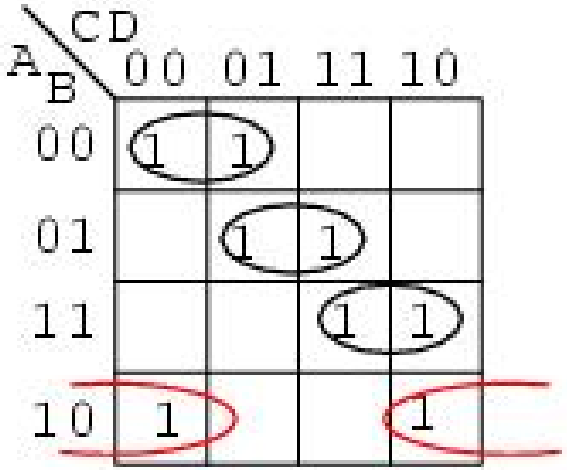


$$Out = \overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{D} + A\,B\,C\,D$$

Example 6 (SOP)

$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}D + \overline{A}BCD + ABCD + ABC\overline{D} + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}C\overline{D}$$



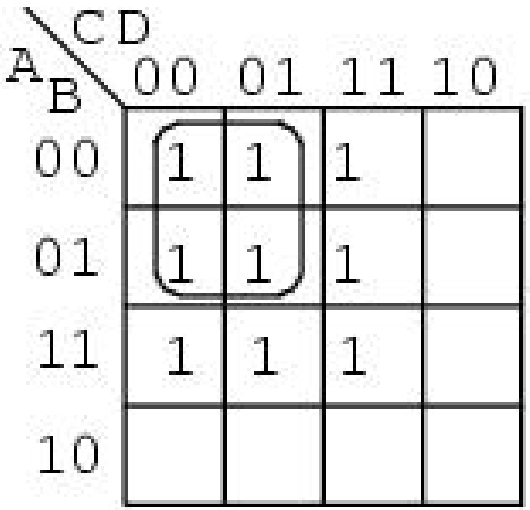


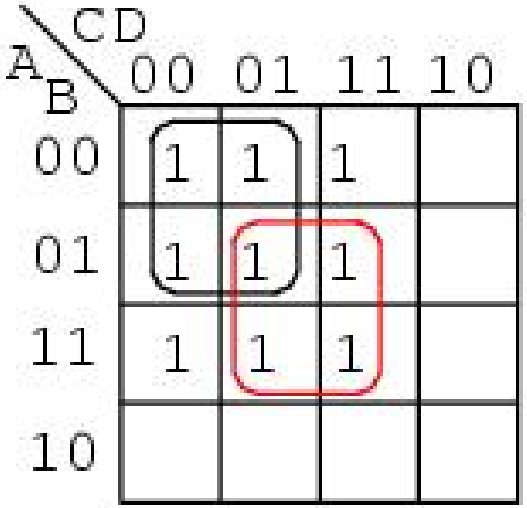
$$Out = \overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{C}D + BCD + AC\overline{D}$$

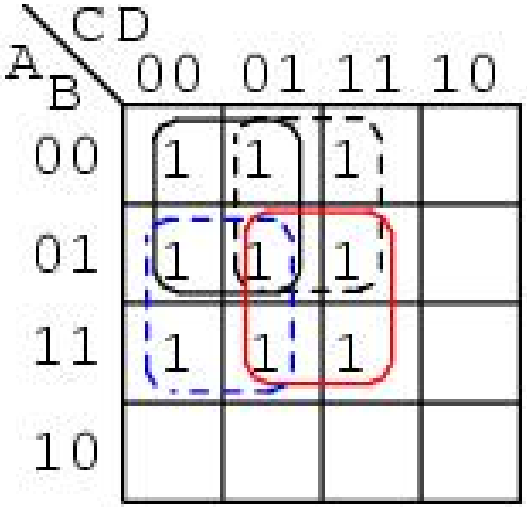
$$Out = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BD + ABC + A\overline{B}\,\overline{D}$$

Example 7 (SOP)

$$Out = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}B\,\overline{C}\,\overline{D} + \overline{A}B\,\overline{C}D + \overline{A}BC\,\overline{D} + \overline{A}BCD + AB\,\overline{C}\,\overline{D} + AB\,\overline{C}D + ABC\,\overline{D} + ABCD$$



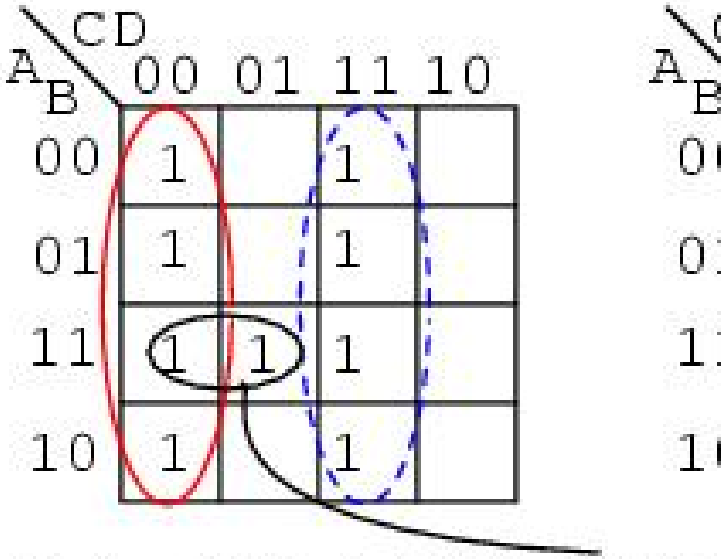




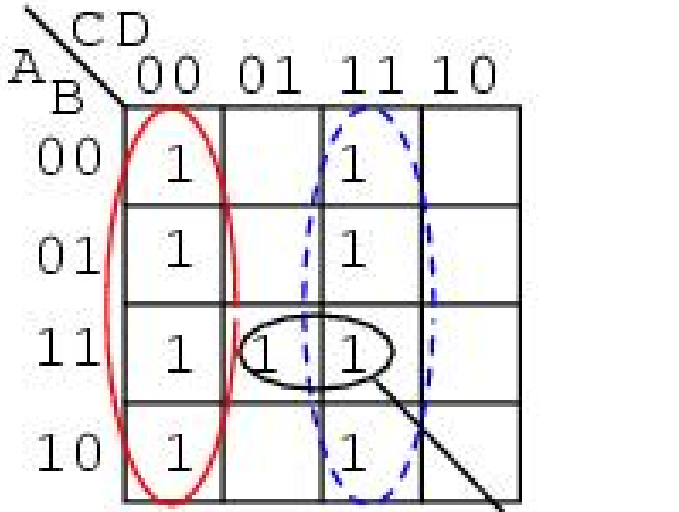
$$Out = \overline{A}\,\overline{C} + \overline{A}D + B\,\overline{C} + BD$$

Example 8 (SOP)

$$Out = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD$$



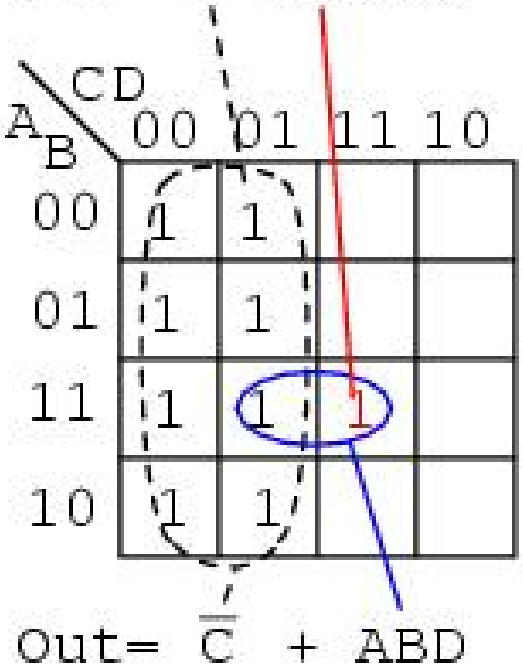
$$Out = \bar{C}\bar{D} + CD + A\bar{B}\bar{C}$$



$$Out = \bar{C}\bar{D} + CD + ABD$$

Example 9 (SOP)

Out= $\overline{C}$ + ABCD



Out= $\overline{C}$ + ABD

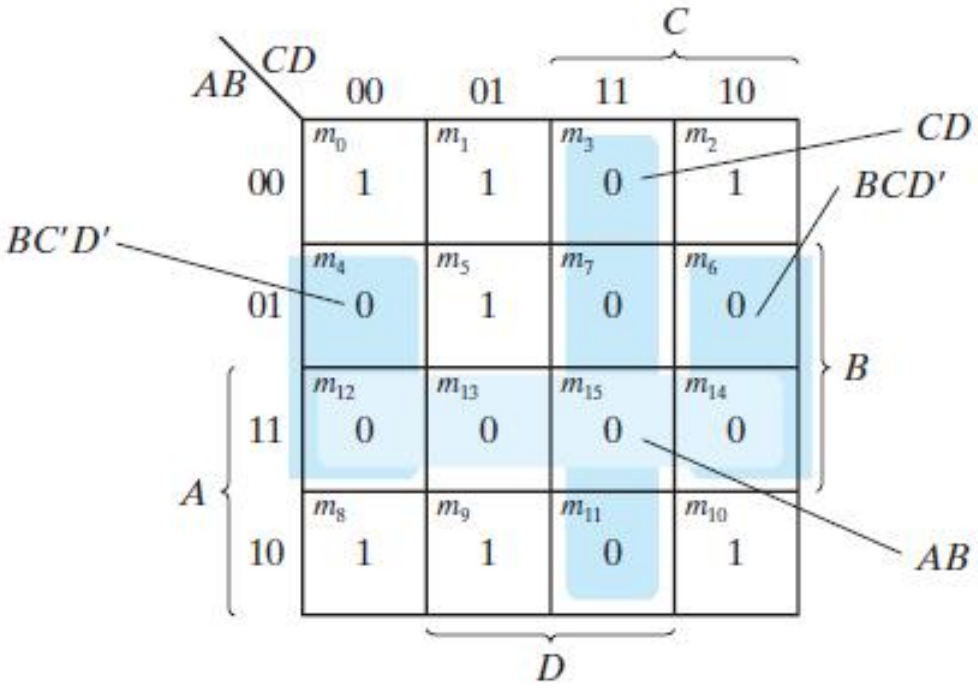
Simplification by Boolean Algebra

Out= $\overline{C}$ + ABCD

Applying rule **A + $\overline{A}$B = A + B** to the $\overline{C}$ + ABCD term

Out= $\overline{C}$ + ABD

SOP and POS Forms



SOP

$$F = B'D' + B'C' + A'C'D$$

Combing 0 terms yields:

$$F' = AB + CD + BD'$$

Applying DeMorgan's theorem

Complement of F' is F

POS

$$F = (A' + B')(C' + D')(B' + D)$$

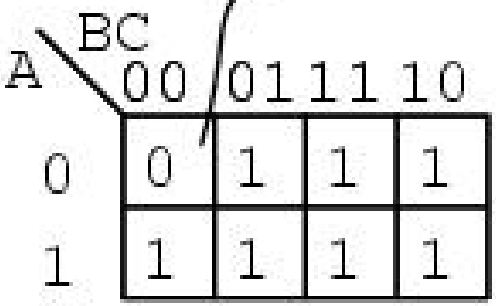
Example 1 (POS)

Out = (A + B + C)

Maxterm = A + B + C

Numeric = 1 1 1

Complement = 0 0 0



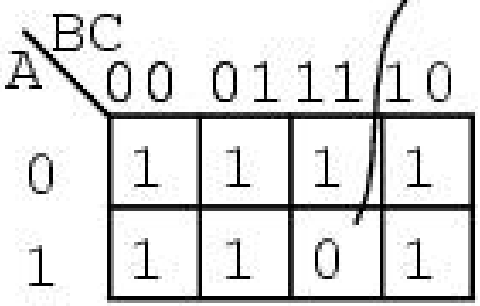
Example 2 (POS)

$$\text{Out} = (\overline{A} + \overline{B} + \overline{C})$$

$$\text{Maxterm} = \overline{A} + \overline{B} + \overline{C}$$

Numeric = 0 0 0

Complement = 1 1 1



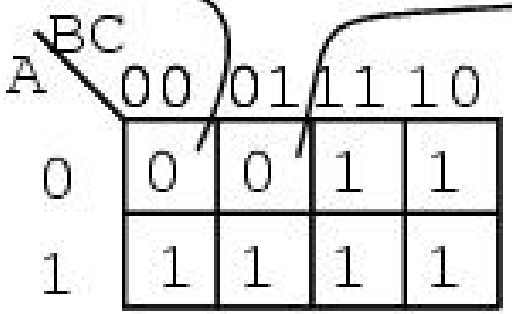
Example 3 (POS) … contd in the next slide

$$Out = (A+B+C)(A+B+\overline{C})$$

Maxterm= (A+B+C) Maxterm= $(A+B+\overline{C})$

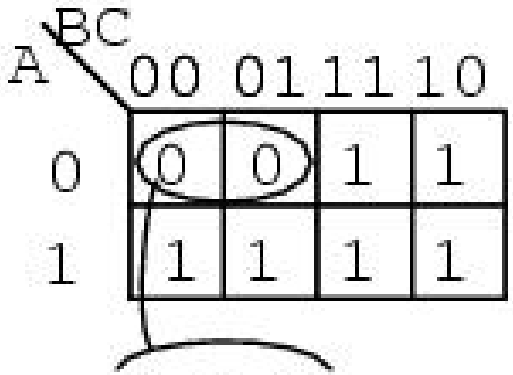
Numeric= 1 1 1 Numeric= 1 1 0

Complement= 0 0 0 Complement= 0 0 1



Example 3 (POS) - contd

$$Out = (A+B+C)(A+B+\overline{C})$$



A B C = 0 0 X

Complement = 1 1 X

Sum-term = (A + B)

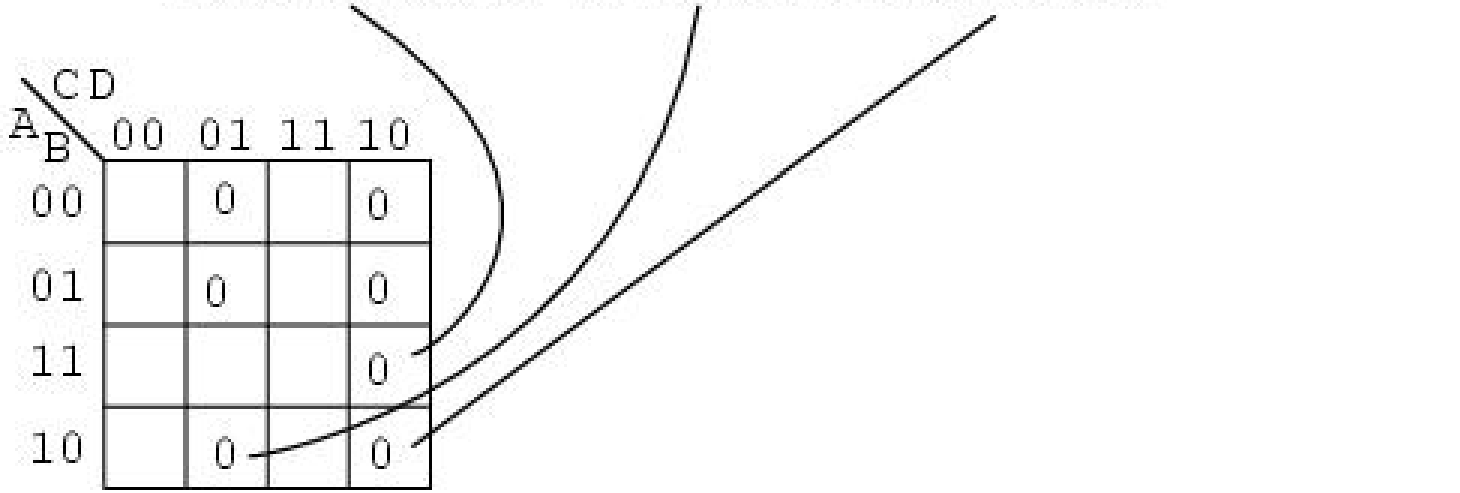
Out = (A + B)

Quiz 1

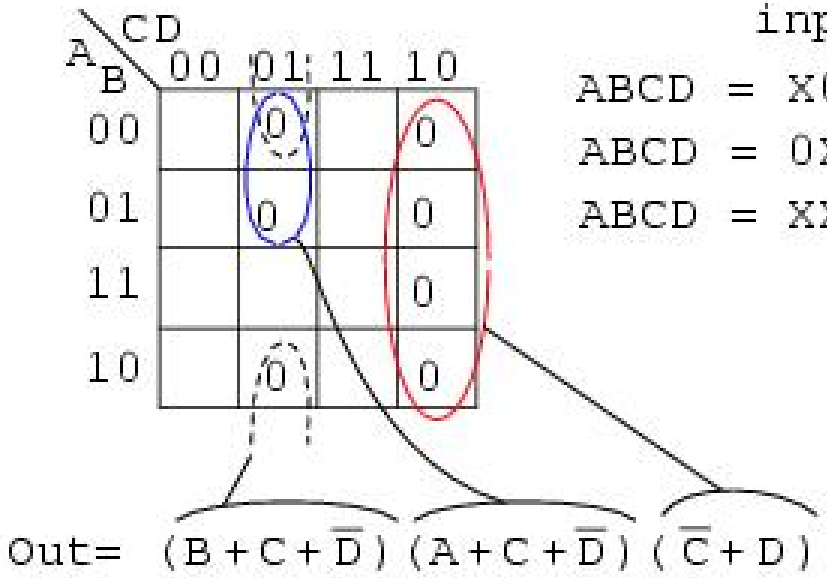
Simplify the Boolean expression using K-map

$$Out = (A+B+C+\overline{D})(A+B+\overline{C}+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+B+C+\overline{D})(\overline{A}+B+\overline{C}+D)$$

$$Out = (A+B+C+\overline{D})(A+B+\overline{C}+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+B+C+\overline{D})(\overline{A}+B+\overline{C}+D)$$



Quiz 1: Solution

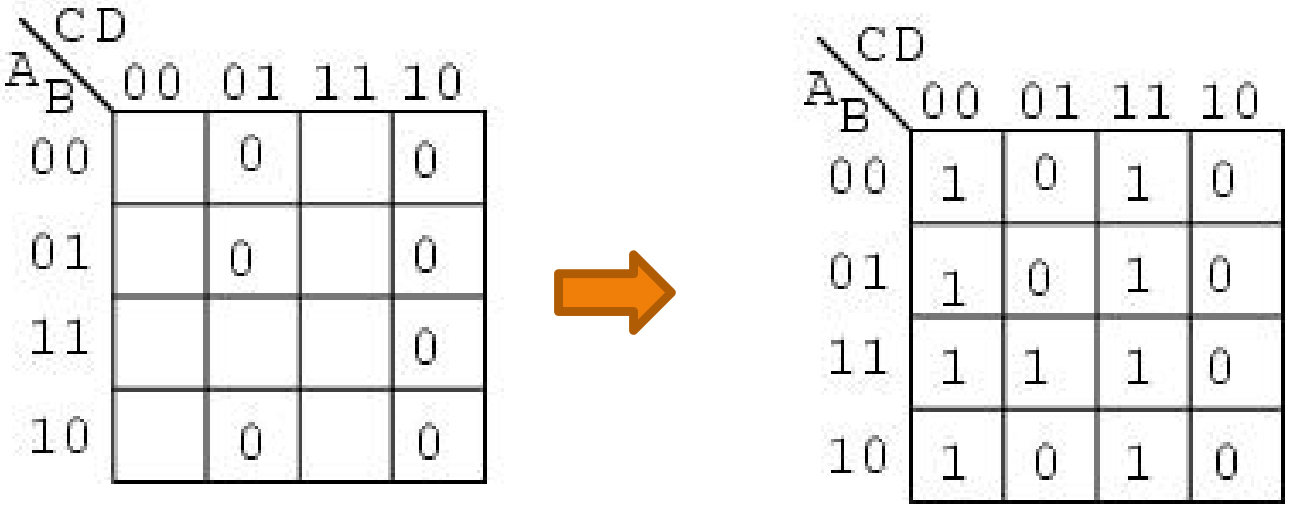


	input		complement		Sum-term
ABCD =	X001	>	X110	>	(B + C + $\overline{D}$)
ABCD =	0X01	>	1X10	>	(A + C + $\overline{D}$)
ABCD =	XX10	>	XX01	>	($\overline{C}$ + D)

Quiz 2:

Simplify the **Product-Of-Sums** Boolean expression below, providing a **result** in **SOP** form

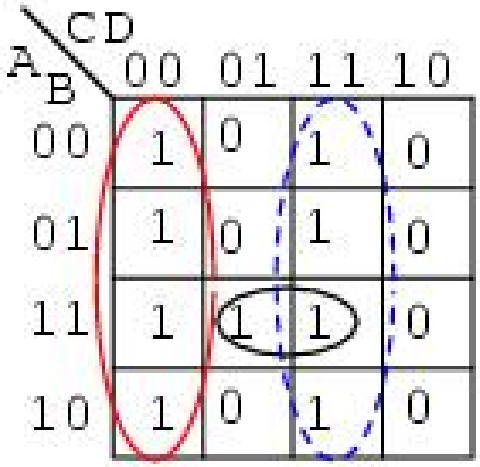
$$Out= (A+B+C+\overline{D})(A+B+\overline{C}+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$
$$(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+B+C+\overline{D})(\overline{A}+B+\overline{C}+D)$$



Quiz 2: Solution

Simplify the Product-Of-Sums Boolean expression below, providing a **result** in **SOP** form

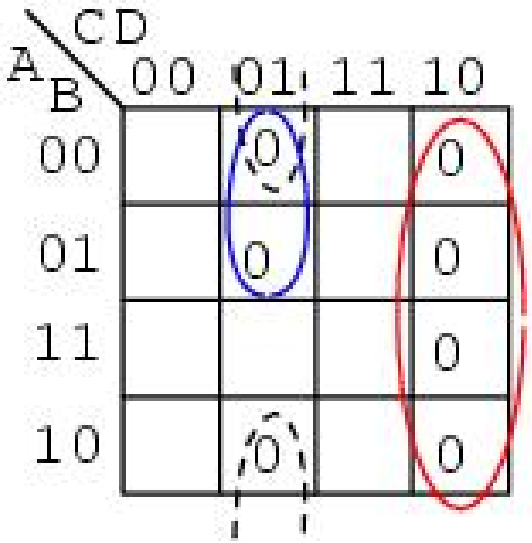
$$Out= (A+B+C+\overline{D})(A+B+\overline{C}+D)\ (A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$
$$(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+B+C+\overline{D})(\overline{A}+B+\overline{C}+D)$$

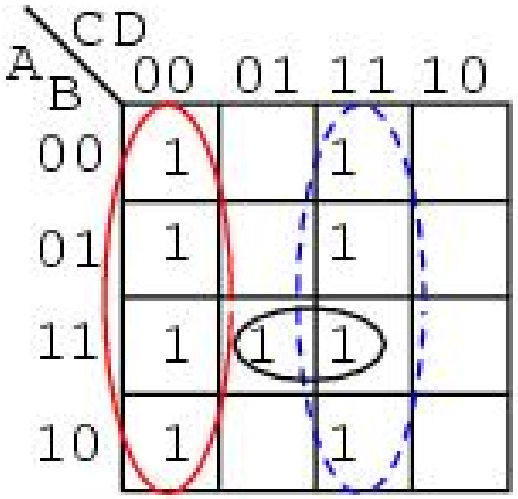


$$Out= \overline{C}\,\overline{D} + C\,D + A\,B\,D$$

Same Example: Both SOP and POS

$$Out = (A+B+C+\overline{D})(A+B+\overline{C}+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$
$$(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+B+C+\overline{D})(\overline{A}+B+\overline{C}+D)$$



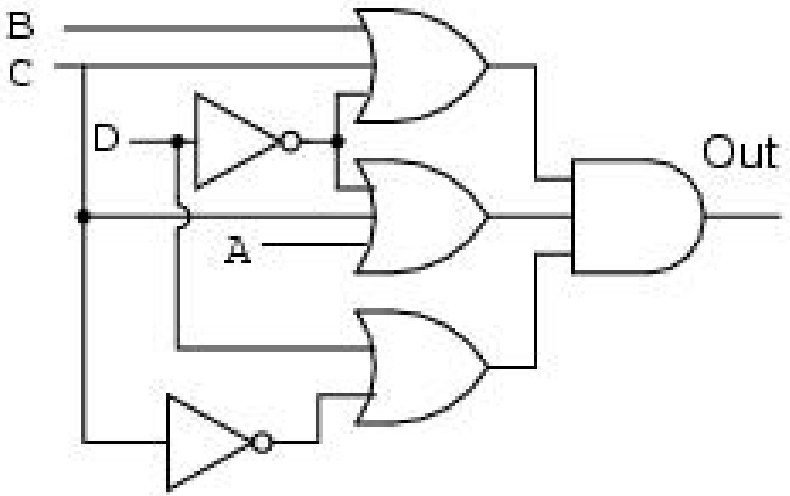


$$Out = \overline{C}\,\overline{D} + CD + ABD$$

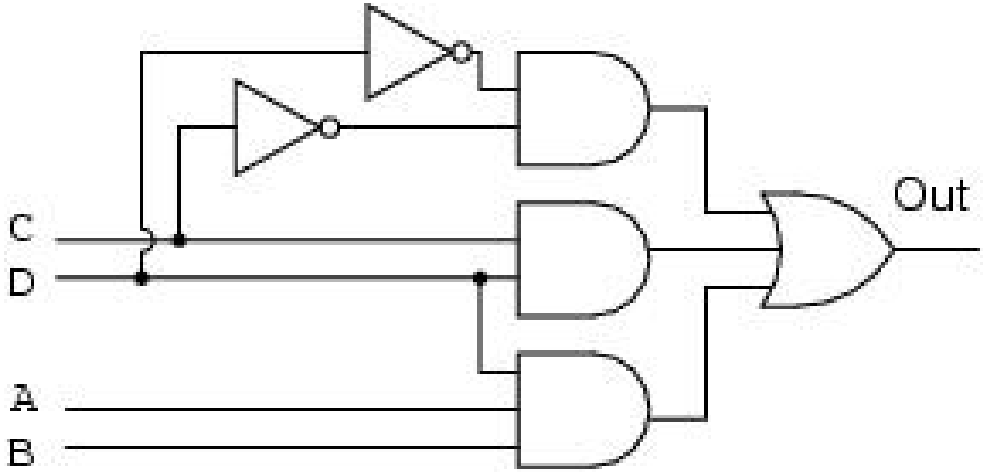
$$Out = (B+C+\overline{D})(A+C+\overline{D})(\overline{C}+D)$$

Implementation

Out= $(B + C + \overline{D})(A + C + \overline{D})(\overline{C} + D)$



Out= $\overline{C}\,\overline{D} + C\,D + A\,B\,D$



Session 2.6: Summary

- Introduction to Karnaugh map
- Examples
  - SOP form
  - POS form
  - Both SOP and POS
  - Implementation