Newton Method for Root Finding

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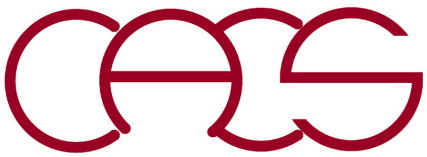


Chemical Potential

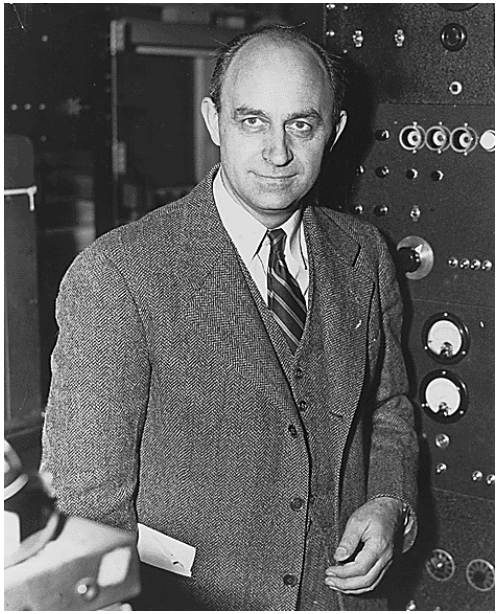
- **Fermi distribution**

$$N_\nu = f(\varepsilon_\nu) = \frac{2}{\exp((\varepsilon_\nu - \mu)/k_B T)+1}$$

- **Determination of chemical potential μ**

$$\sum_\nu N_\nu = \sum_\nu \frac{2}{\exp((\varepsilon_\nu - \mu)/k_B T)+1} = M$$

Total # of electrons
$M = 4n_{Atom}$ for Si
valence electrons



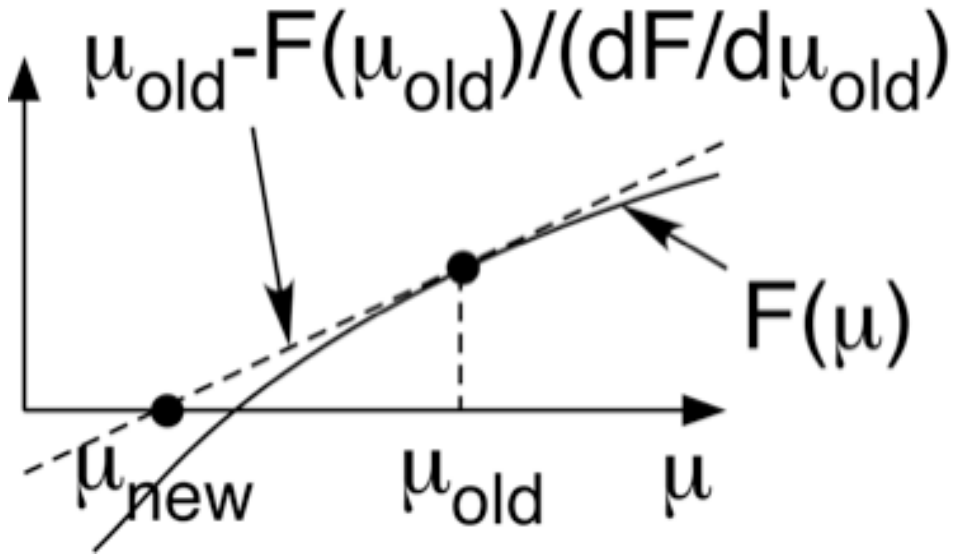
- **Root finding**

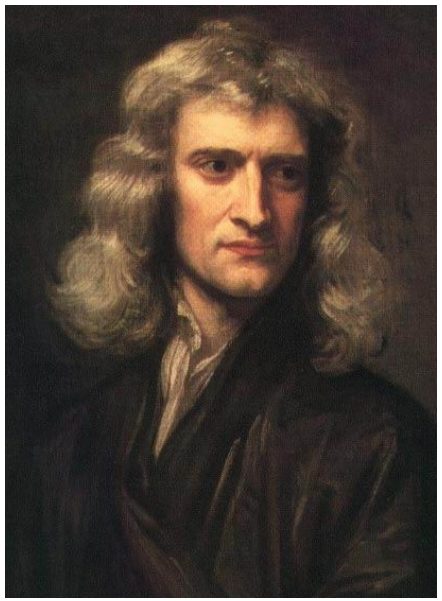
$$F(\mu) = \sum_\nu \frac{2}{\exp((\varepsilon_\nu - \mu)/k_B T)+1} - M = 0$$

Newton Method

- **Repeated linear approximation**

$$F(\mu) \cong F(\mu_{\text{old}}) + \left.\frac{dF}{d\mu}\right|_{\mu=\mu_{\text{old}}} (\mu - \mu_{\text{old}}) = 0 \rightarrow \mu_{\text{new}} = \mu_{\text{old}} - \frac{F(\mu_{\text{old}})}{dF/d\mu|_{\mu=\mu_{\text{old}}}}$$





- **Algorithm**

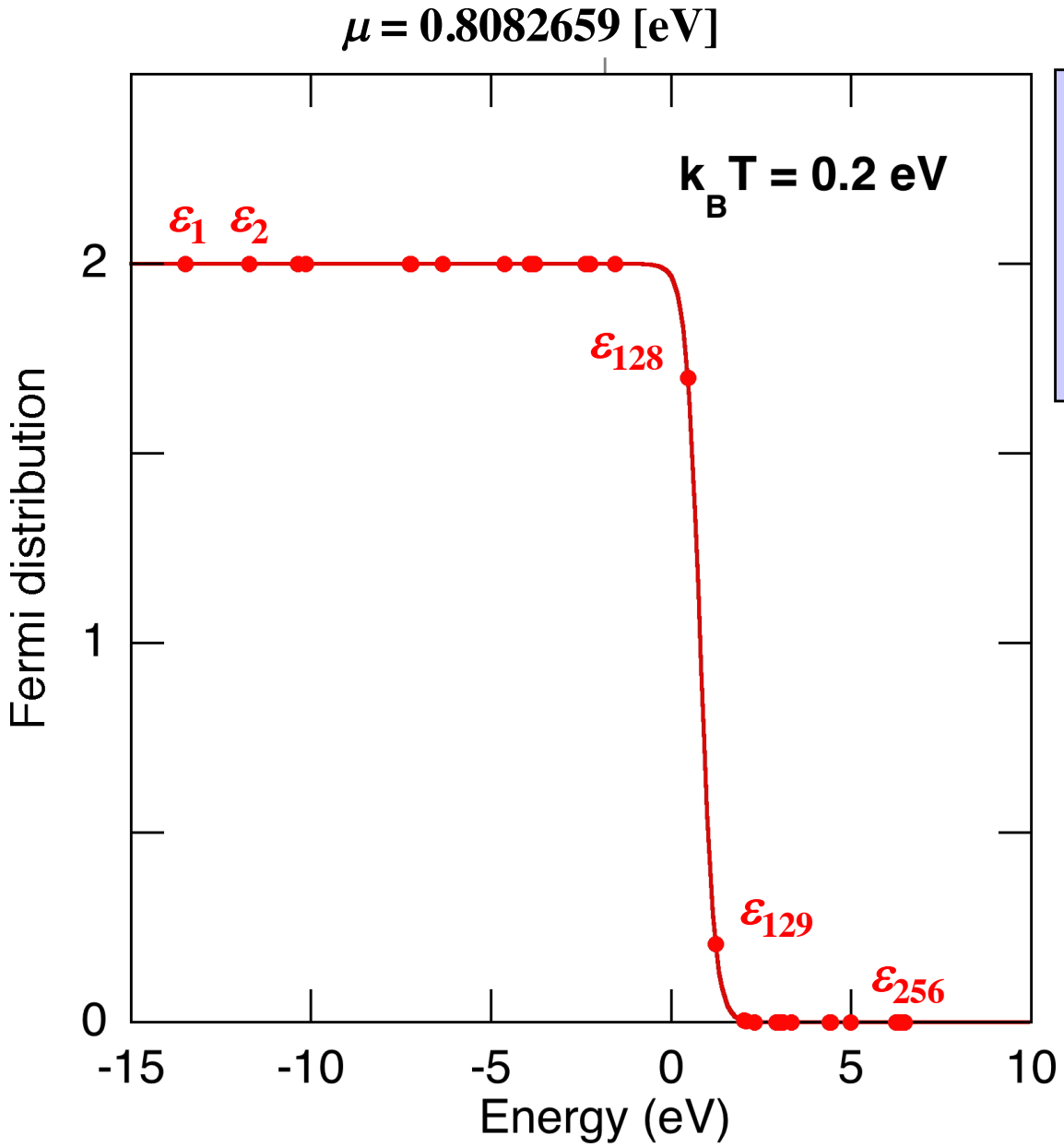
1. Begin with an initial guess, μ, of the root
2. Repeat the recursion

$$\mu \leftarrow \mu - \frac{F(\mu)}{dF/d\mu}$$

until the difference, $|F/(\mathrm{d}F/\mathrm{d}\mu)|$, between successive approximations becomes less than the prescribed error tolerance, μ_{tol}

Example: Silicon Crystal

- **Tight-binding energy eigenvalues for $M = 4\times64 = 256$**



1. $\mu \leftarrow \mu_{\text{guess}} = \varepsilon_{M/2}$
2. Repeat

$$\mu \leftarrow \mu - \frac{F(\mu)}{dF/d\mu}$$

until $|F/(\mathrm{d}F/\mathrm{d}\mu)| < \mu_{\text{tol}} = 10^{-10}$

$$F(\mu) = \sum_{\nu} \frac{2}{\exp((\varepsilon_\nu - \mu)/k_{\mathrm{B}}T)+1} - M$$

$$dF/d\mu = ?$$