Hybrid MPI+OpenMP Parallel MD

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Objective: Hands-on experience in default programming language (MPI+OpenMP) for hybrid parallel computing on a cluster of multicore computing nodes

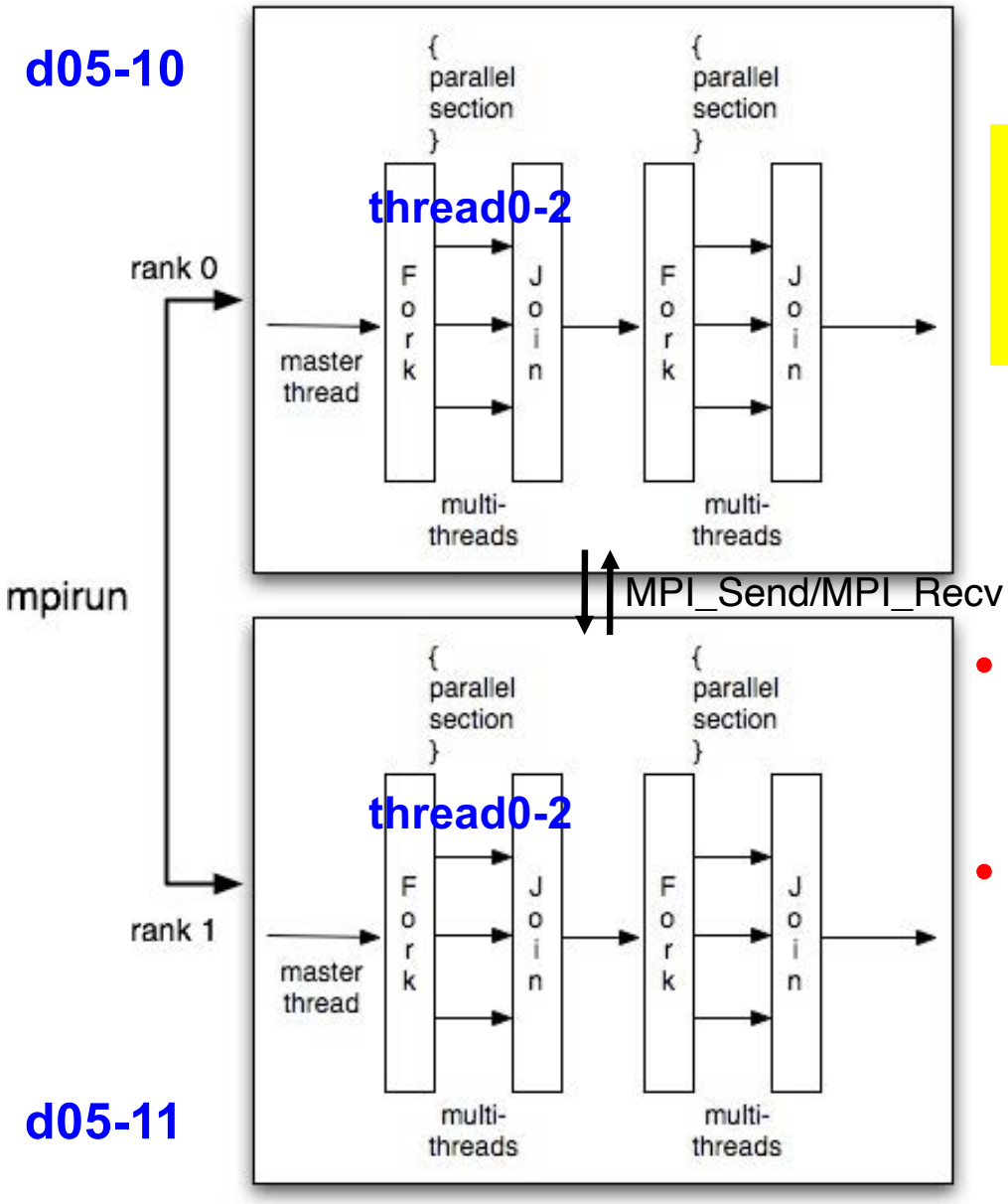
Alternative to MPI-only: million ssh's & management of million processes by MPI daemon

https://aiichironakano.github.io/cs596/Kunaseth-HTM-PDSEC13.pdf

MPI+X: https://www.hpcwire.com/2014/07/16/compilers-mpix

Hybrid MPI+OpenMP Programming

Each MPI process spawns multiple OpenMP threads



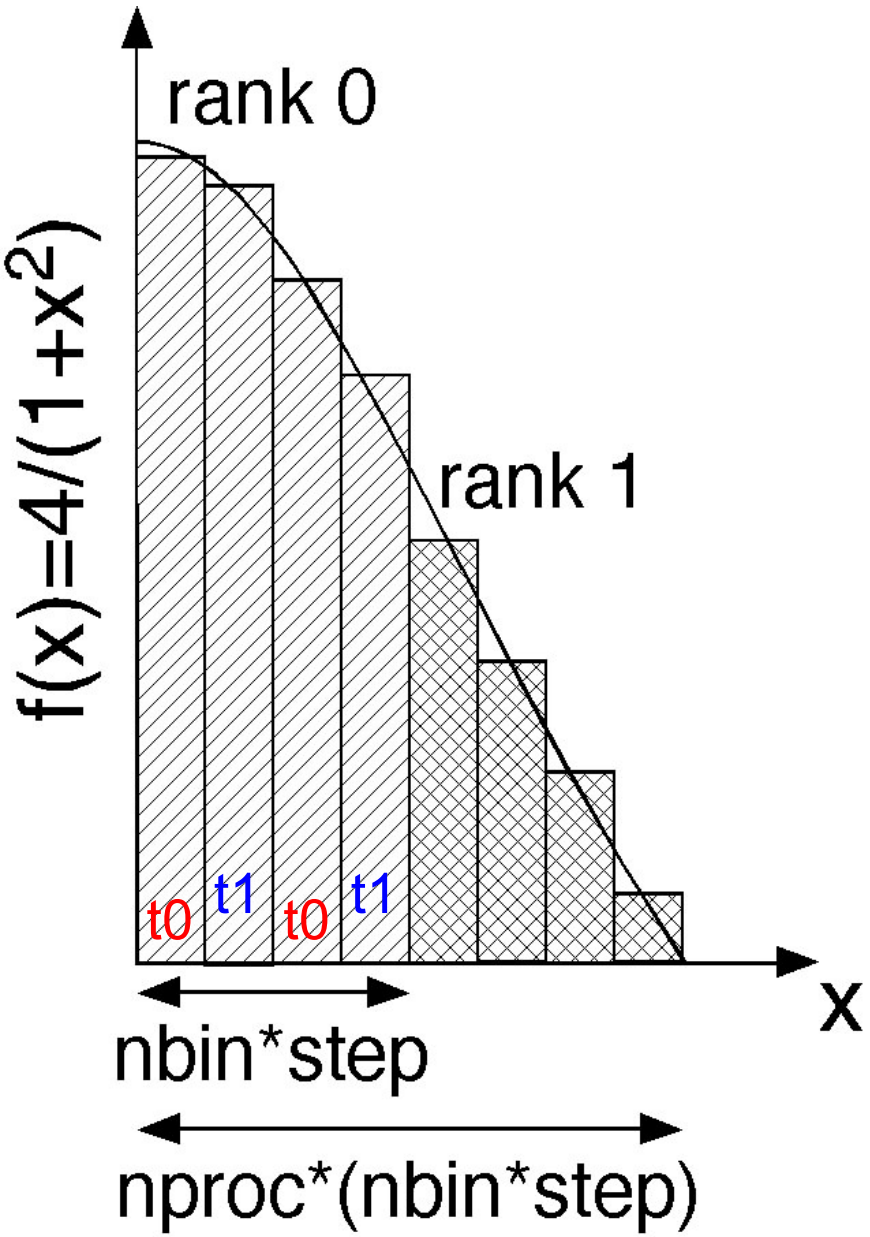

```
In a Slurm script:
  mpirun –n 2
In the code:
  omp_set_num_threads(3);
```

- MPI processes communicate by sending/receiving messages
- OpenMP threads communicate by writing to/reading from shared variables

MPI+OpenMP Calculation of π

- **Spatial decomposition: Each MPI process integrates over a range of width `1/nproc`, as a discrete sum of `nbin` bins each of width `step`**
- **Interleaving: Within each MPI process, `nthreads` OpenMP threads perform part of the sum as in `omp_pi.c`**

$$\pi = \int_0^1 \frac{4}{1+x^2}\,dx \cong \Delta \sum_{i=0}^{N-1} \frac{4}{1+x_i^2}$$



MPI+OpenMP Calculation of π: hpi.c

https://aiichironakano.github.io/cs596/src/hybrid/hpi.c

```
#include <stdio.h>
#include <mpi.h>
#include <omp.h>
#define NBIN 100000
#define MAX_THREADS 8
void main(int argc,char **argv) {
  int nbin,myid,nproc,nthreads,tid;
  double step,sum[MAX_THREADS]={0.0},pi=0.0,pig;
  MPI_Init(&argc,&argv);
  MPI_Comm_rank(MPI_COMM_WORLD,&myid);
  MPI_Comm_size(MPI_COMM_WORLD,&nproc);
  nbin = NBIN/nproc; step = 1.0/(nbin*nproc);
  omp_set_num_threads(2);
  #pragma omp parallel private(tid)
  {
    int i;
    double x;
    nthreads = omp_get_num_threads();
    tid = omp_get_thread_num();
    for (i=nbin*myid+tid; i<nbin*(myid+1); i+=nthreads) {
      x = (i+0.5)*step; sum[tid] += 4.0/(1.0+x*x); }
    printf("rank tid sum = %d %d %e\n",myid,tid,sum[tid]);
  }
  for (tid=0; tid<nthreads; tid++) pi += sum[tid]*step;
  MPI_Allreduce(&pi,&pig,1,MPI_DOUBLE,MPI_SUM,MPI_COMM_WORLD);
  if (myid==0) printf("PI = %f\n",pig);
  MPI_Finalize();}
```

Shared variables among all threads

$$NBIN \rightarrow \lfloor NBIN/nproc \rfloor \times nproc = \overbrace{nbin}^{\text{\# of bins per rank}} \times nproc$$

Local variables: Different values needed for different threads

Who does what!

Inter-thread reduction

Inter-rank reduction

MPI+OpenMP Example: `hpi.c`

- **Compilation on `discovery.usc.edu`**

```
mpicc -o hpi hpi.c -fopenmp
```

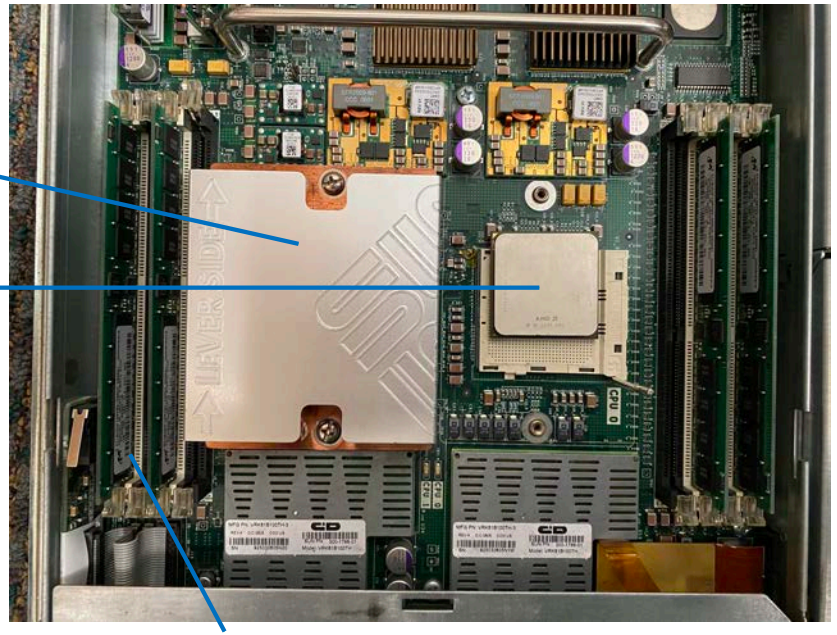
- **Slurm script**

```
#!/bin/bash
#SBATCH --nodes=2
#SBATCH --ntasks-per-node=1
#SBATCH --cpus-per-task=2
#SBATCH --time=00:00:59
#SBATCH --output=hpi.out
#SBATCH -A anakano_429
mpirun -n $SLURM_NNODES ./hpi
```

- **Output**

```
rank tid sum = 1 1 6.434981e+04
rank tid sum = 1 0 6.435041e+04
rank tid sum = 0 0 9.272972e+04
rank tid sum = 0 1 9.272932e+04
PI = 3.141593
```

Number of nodes



- **Find information about Slurm nodes & partitions**

[anakano@discovery ~]$ **sinfo** View information

```
PARTITION AVAIL  TIMELIMIT  NODES  STATE NODELIST
main*        up 2-00:00:00    281    mix d05-[08-15,26-29,31-37,39,42],...
epyc-64      up 2-00:00:00     26  alloc b22-[01-09,11-25,28-29]
...
```

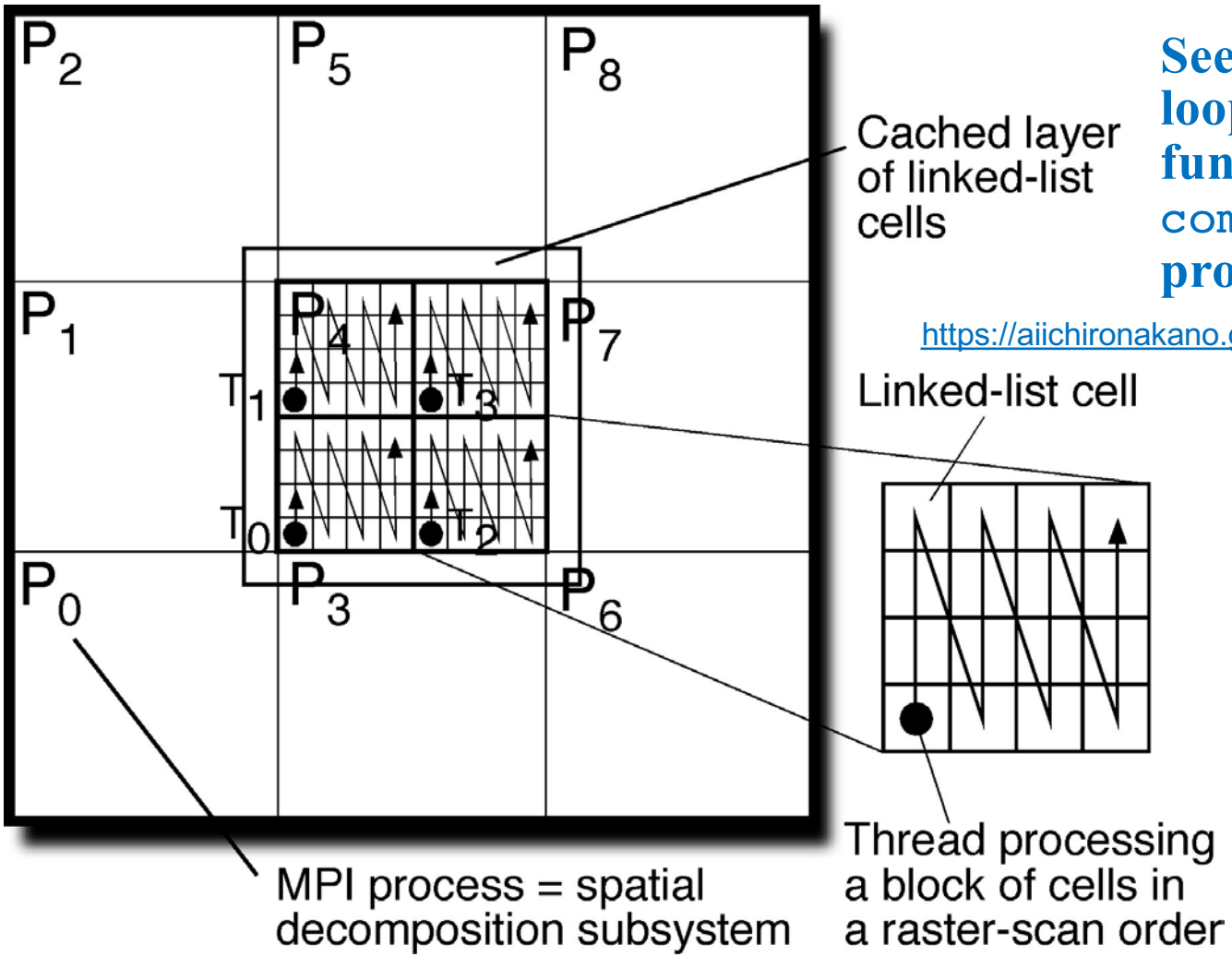
[anakano@discovery ~]$ **sinfo2** View detailed information

```
NODELIST PARTITION  STATE    NODES    SOCKETS    CORES      MEMORY     GRES   ACTIVE_FEATURES
e13-35   main*      down*    1        2          8          63400      (null) xeon-2640v3
...
```

Hybrid MPI+OpenMP Parallel MD

- **OpenMP threads handle blocks of linked-list cells in each MPI process (= spatial-decomposition subsystem)**

Big picture = who does what: loop index $\xrightarrow[\text{map}]{}$ thread



See the outermost loop over `mc[3]` in function `compute_accel()` in program `pmd.c`

https://aiichironakano.github.io/cs596/src/pmd/pmd.c

Linked-List Cell Block

Variables

- `vthrd[0|1|2]` = # of OpenMP threads per MPI process in the x|y|z direction.
- `nthrd` = # of OpenMP threads = `vthrd[0]×vthrd[1]×vthrd[2]`.
- `thbk[3]`: `thbk[0|1|2]` is the # of linked-list cells in the x|y|z direction that each thread is assigned.

In main():

```
omp_set_num_threads(nthrd);
```

In hmd.h:

```
int vthrd[3]={2,2,1},nthrd=4;
int thbk[3];
```

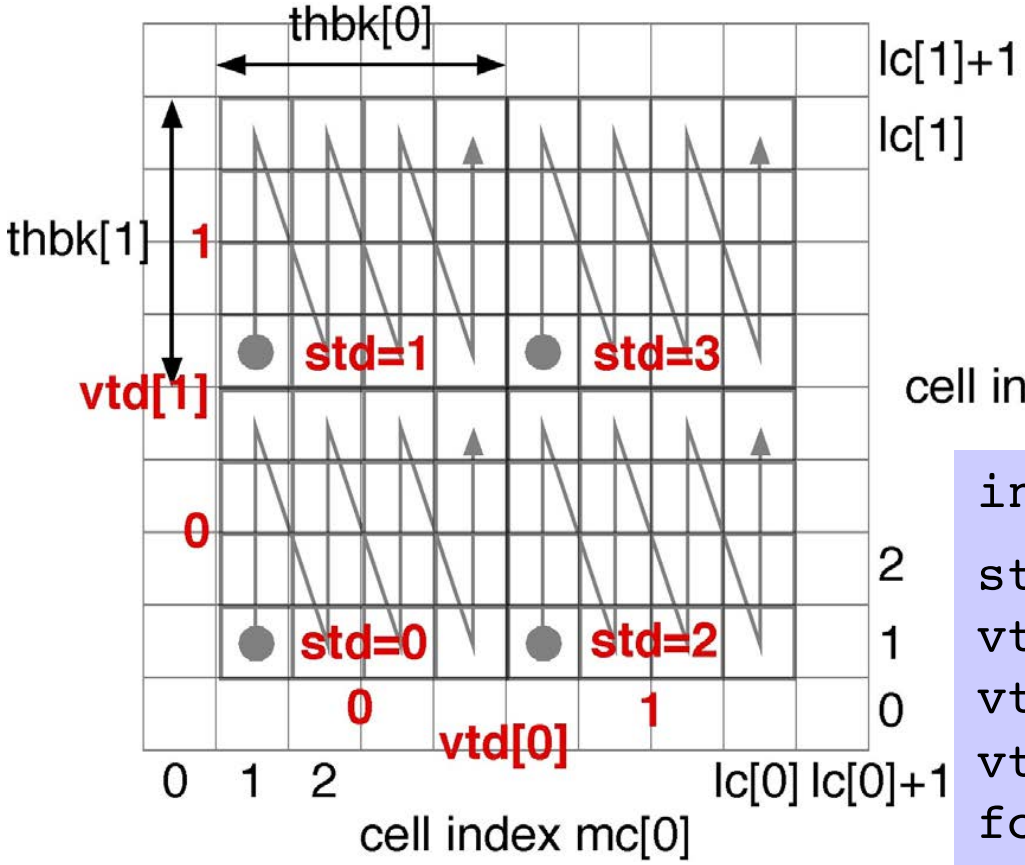
In init_params():

```
/* Compute the # of cells for linked-list cells */
for (a=0; a<3; a++) {
  lc[a] = al[a]/RCUT; /* Cell size ≥ potential cutoff */
  /* Size of cell block that each thread is assigned */
  thbk[a] = lc[a]/vthrd[a];
  /* # of cells = integer multiple of the # of threads */
  lc[a] = thbk[a]*vthrd[a]; /* Adjust # of cells/MPI process */
  rc[a] = al[a]/lc[a]; /* Linked-list cell length */
}
```

OpenMP Threads for Cell Blocks

Variables

- `std` = scalar thread index.
- `vtd[3]`: `vtd[0|1|2]` is the x|y|z element of vector thread index.
- `mofst[3]`: `mofst[0|1|2]` is the x|y|z offset cell index of cell-block.




```
int std,vtd[3],mofst[3];

std = omp_get_thread_num();
vtd[0] = std/(vthrd[1]*vthrd[2]);
vtd[1] = (std/vthrd[2])%vthrd[1];
vtd[2] = std%vthrd[2];
for (a=0; a<3; a++)
  mofst[a] = vtd[a]*thbk[a];
```

Call `omp_get_thread_num()` within an OpenMP parallel block.

Threads Processing of Cell Blocks

Start from your `pmd_irecv.c` instead

- Start with the MPI parallel MD program, `pmd.c`
- Within each MPI process, parallelize the outer loops over central linked-list cells, `mc[]`, in the force computation function, `compute_accel()`, using OpenMP threads
- If each thread needs separate copy of a variable (*e.g.*, loop index `mc[]`), declare it as `private` in the OpenMP parallel block

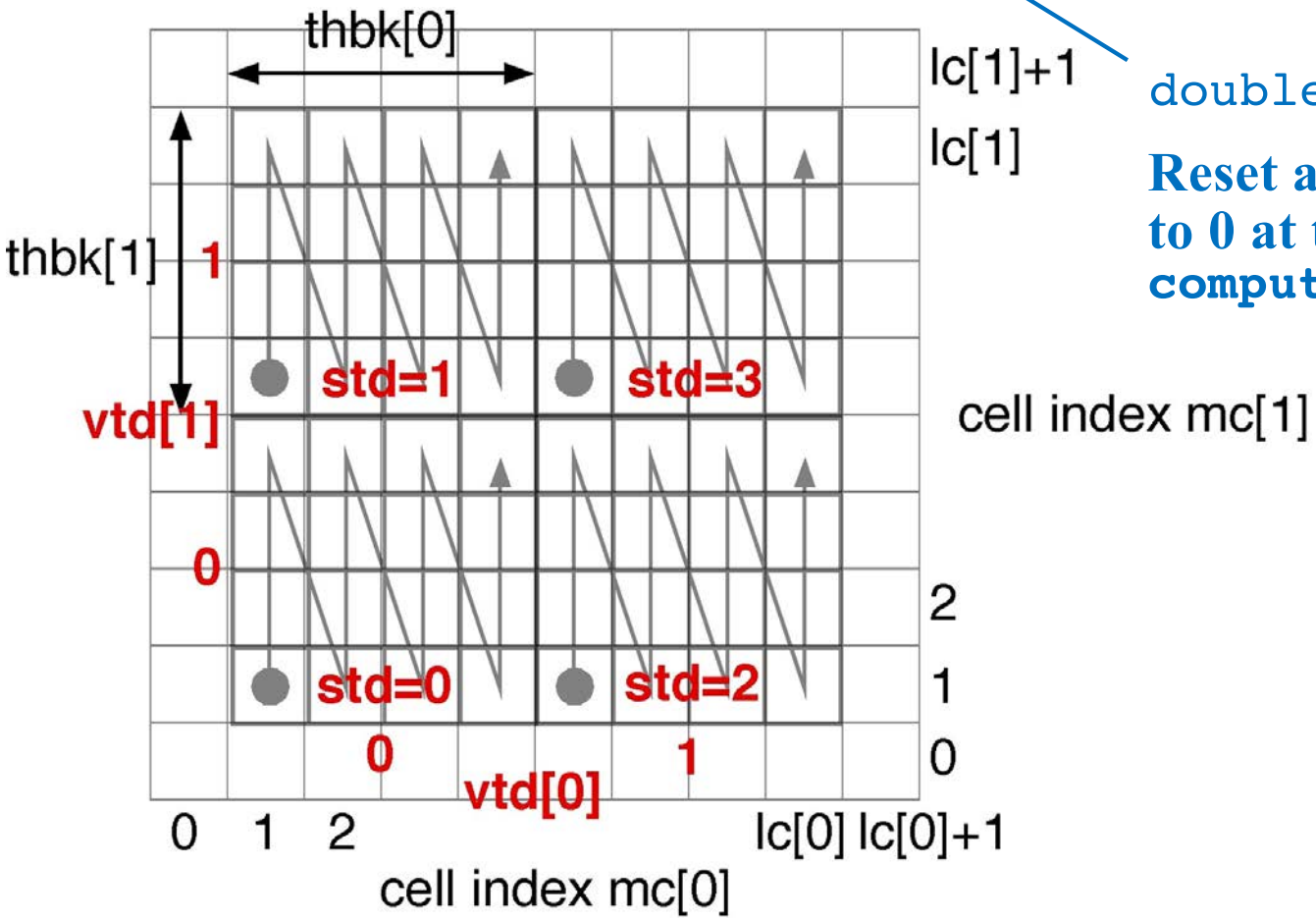
```
#pragma omp parallel private(mc,...)
{
  ...
  for (mc[0]=mofst[0]+1; mc[0]<=mofst[0]+thbk[0]; (mc[0])++)
  for (mc[1]=mofst[1]+1; mc[1]<=mofst[1]+thbk[1]; (mc[1])++)
  for (mc[2]=mofst[2]+1; mc[2]<=mofst[2]+thbk[2]; (mc[2])++) {
    Each thread handles thbk[0]×thbk[1]×thbk[2] cells independently
  }
  ...
}
```

Avoiding Critical Sections (1)

- **Remove the critical section**

```
if (bintra) lpe += vVal; else lpe += 0.5*vVal;
```

by defining an array, `lpe_td[nthrd]`, where each array element stores the partial sum of the potential energy by a thread




```
double lpe_td[nthrd];
```

Reset all array elements to 0 at the beginning of `compute_accel()`

Data privatization: *cf.* `omp_pi.c` & `hpi.c`

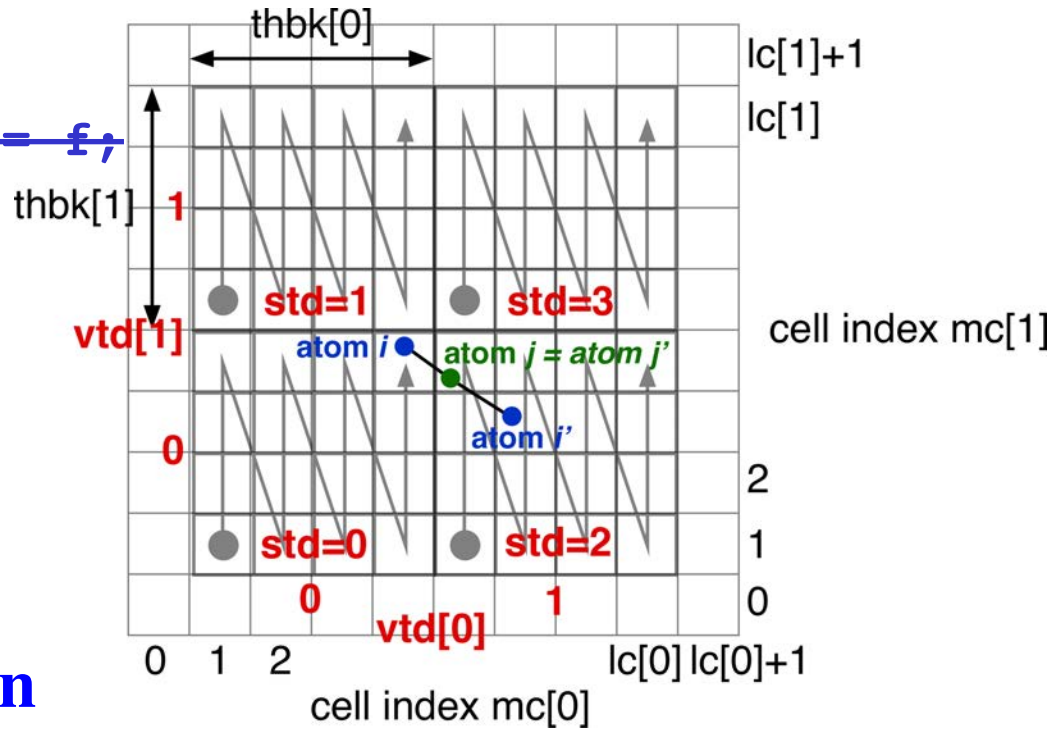
Avoiding Critical Sections (2)

- To avoid multiple threads to access an identical force array element, stop using the Newton's third law:

```
int bintra;
...
bintra = (j < n);
...
if (i<j && rr<rrCut) {
  ...
  if (bintra) lpe += vVal; else lpe_td[std] += 0.5*vVal;
  for (a=0; a<3; a++) {
      f = fcVal*dr[a];
      ra[i][a] += f;
      if (bintra) ra[j][a] -= f;
  }
}
```

Note the data privatization

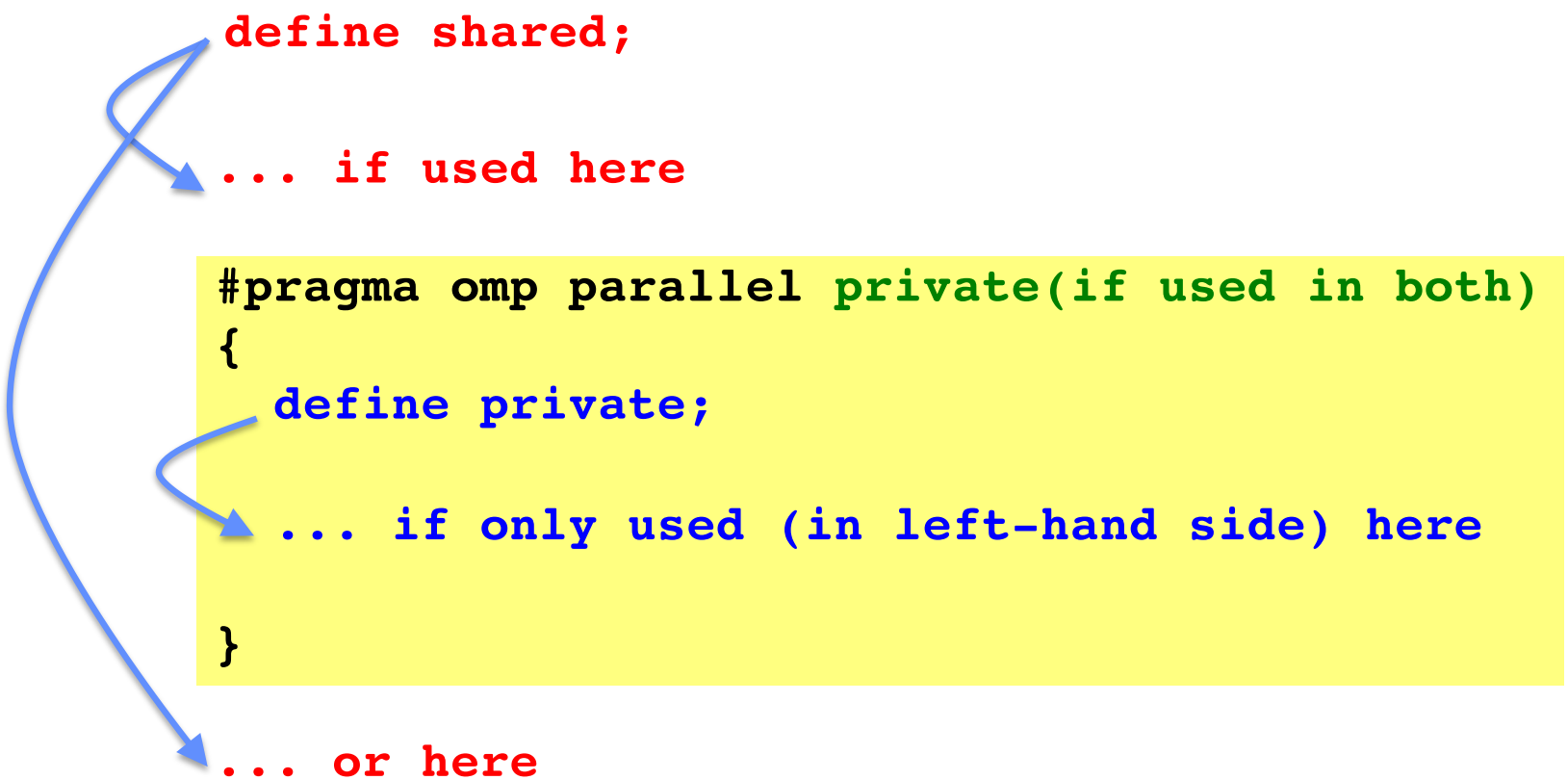
Mutually exclusive access to ra[][] for preventing race conditions



- Interthread reduction after join

```
for (i=0; i<nthrd; i++) lpe += lpe_td[i];
```

OpenMP Essential



Running HMD at CARC

- **Submit a batch job using the following Slurm script.**

```
#!/bin/bash
#SBATCH --nodes=2
#SBATCH --ntasks-per-node=1
#SBATCH --cpus-per-task=4
#SBATCH --time=00:01:59
#SBATCH --output=hmd.out
#SBATCH -A anakano_429

mpirun -bind-to none -n 2 ./hmd
```

To be explained later

- **Note that hmd.c must have been compiled in the same directory as you submit this Slurm script:**

```
mpicc -O -o hmd hmd.c -lm -fopenmp
```

Interactively Running HMD at CARC (1)

1. **Interactively submit a Slurm job & wait until you are allocated nodes. (Note that you will be automatically logged in to one of the allocated nodes.)**

```
$ salloc --nodes=2 --ntasks-per-node=1 --cpus-per-task=4 -t 29
salloc: Pending job allocation 6064886
salloc: job 6064886 queued and waiting for resources
salloc: job 6064886 has been allocated resources
salloc: Granted job allocation 6064886
[anakano@d05-35 ~]$
```

You are logged in to one of the allocated nodes

For CPU information, type `more /proc/cpuinfo`

https://carc.usc.edu/user-information/user-guides/hpc-basics/discovery-resources

Interactively Running HMD at CARC (2)

2. **Submit a two-process MPI program (named hmd); each of the MPI process will spawn 4 OpenMP threads.**

```
[anakano@d05-35 cs596]$ mpirun -bind-to none -n 2 ./hmd
```

3. **While the job is running, you can open another window & log in to the node (or the other allocated node) to check that all processors are busy using top command. Type 'H' to show individual threads (type 'q' to stop).**

```
[anakano@discovery ~]$ ssh d05-35
[anakano@d05-35 ~]$ top (then type H)
...
  PID USER      PR  NI    VIRT    RES    SHR S %CPU %MEM     TIME+ COMMAND
29861 anakano   20   0  443776 102836   7976 R 99.9  0.1   0:09.12 hmd
29871 anakano   20   0  443776 102836   7976 R 99.9  0.1   0:09.06 hmd
29869 anakano   20   0  443776 102836   7976 R 99.7  0.1   0:09.02 hmd
29870 anakano   20   0  443776 102836   7976 R 99.7  0.1   0:09.04 hmd
29661 anakano   20   0  164504   2624   1628 R  0.3  0.0   0:02.34 top
1 root      20   0   43572   3944   2528 S  0.0  0.0   2:06.33 systemd
...
```

Interactively Running HMD at CARC (3)

4. Type '1' to show core-usage summary.

```
top - 12:36:48 up 48 days, 23:35,  1 user,  load average: 3.62, 3.75, 2.86
Threads: 378 total,   5 running, 373 sleeping,   0 stopped,   0 zombie
%Cpu0  :  0.3 us,  0.0 sy,  0.0 ni, 99.7 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
%Cpu1  :100.0 us,  0.0 sy,  0.0 ni,  0.0 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
%Cpu2  : 99.7 us,  0.3 sy,  0.0 ni,  0.0 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
%Cpu3  :100.0 us,  0.0 sy,  0.0 ni,  0.0 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
%Cpu4  :100.0 us,  0.0 sy,  0.0 ni,  0.0 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
%Cpu5  :  0.0 us,  0.0 sy,  0.0 ni,100.0 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
%Cpu6  :  0.0 us,  0.3 sy,  0.0 ni, 99.7 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
%Cpu7  :  0.0 us,  0.0 sy,  0.0 ni,100.0 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
...
%Cpu19 :  0.0 us,  0.0 sy,  0.0 ni,100.0 id,  0.0 wa,  0.0 hi,  0.0 si,  0.0 st
```

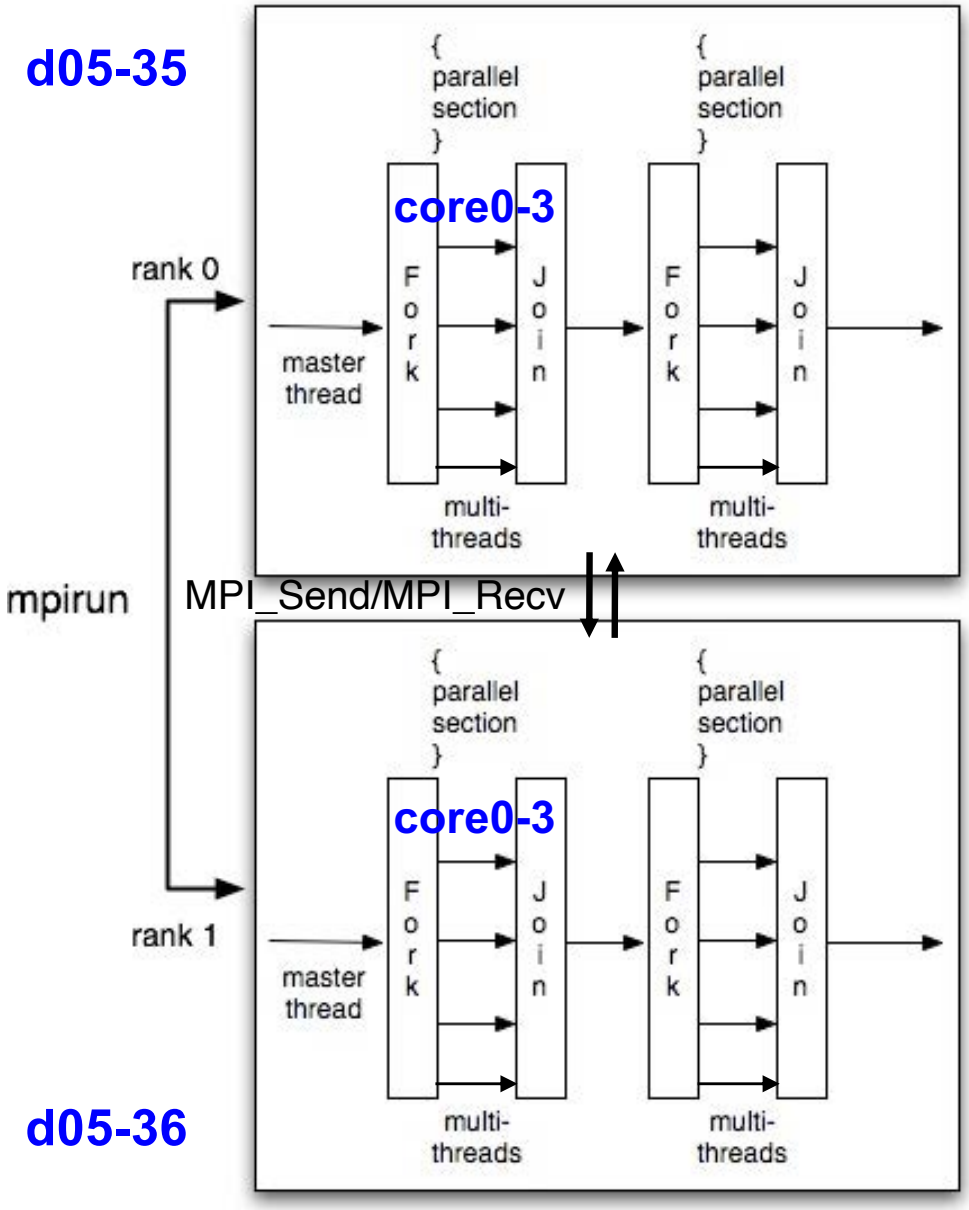
Interactively Running HMD at CARC (4)

5. **Without -bind-to none option, hmd process (and all spawned threads by it) is bound to one core.**

```
[anakano@d05-35 cs596]$ mpirun -n 2 ./hmd

[anakano@d05-36 ~]$ top
...
  PID USER      PR  NI    VIRT    RES    SHR S %CPU %MEM     TIME+ COMMAND
29363 anakano   20   0  443556 108340   7580 R 27.9  0.1   0:18.43 hmd
29373 anakano   20   0  443556 108340   7580 S 24.3  0.1   0:15.96 hmd
29371 anakano   20   0  443556 108340   7580 S 23.9  0.1   0:16.06 hmd
29372 anakano   20   0  443556 108340   7580 S 23.9  0.1   0:15.96 hmd
29341 anakano   20   0  164504   2476   1608 R  0.7  0.0   0:00.37 top
1 root      20   0   43572   3944   2528 S  0.0  0.0   2:06.30 systemd
...
```

How Hybrid MPI+OpenMP MD Runs



In hmd.h:

```
int vproc[3] = {1,1,2}, nproc = 2;
int vthrd[3] = {2,2,1}, nthrd = 4;
```

In hmd.c:

```
omp_set_num_threads(nthrd);
```

On discovery:

```
salloc --nodes=2 --ntasks-per-node=1
  --cpus-per-task=4 -t 30
```

On d05-35:

```
mpirun -bind-to none -n 2 ./hmd
```

On d05-35 & d05-36:

```
top (then type H and 1)
```

Try it yourself!

Validation of Hybrid MD

2 MPI process; 4 threads

In hmd.h:

```
vproc = {1,1,2}, nproc = 2;
vthrd = {2,2,1}, nthrd = 4;
```

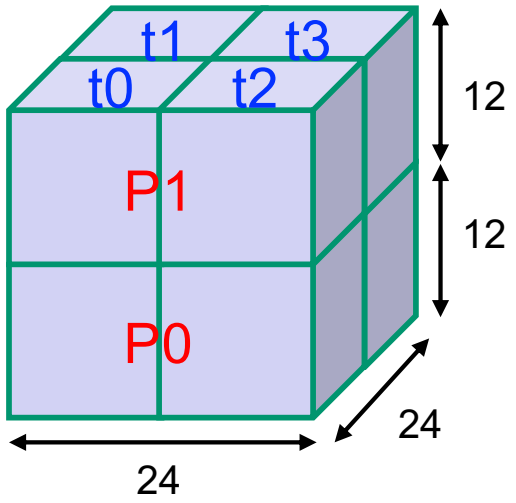
pmd.in

```
24 24 12  InitUcell[3]
0.8       Density
1.0       InitTemp
0.005     DeltaT
100       StepLimit
10        StepAvg
```

Make sure that the total energy is the same as that calculated by pmd.c using the same input parameters, at least for ~5-6 digits

```
0.050000  0.877345 -5.137153 -3.821136
0.100000  0.462056 -4.513097 -3.820013
0.150000  0.510836 -4.587287 -3.821033
0.200000  0.527457 -4.611958 -3.820772
0.250000  0.518668 -4.598798 -3.820796
0.300000  0.529023 -4.614343 -3.820808
0.350000  0.532890 -4.620133 -3.820798
0.400000  0.536070 -4.624899 -3.820794
0.450000  0.539725 -4.630387 -3.820799
0.500000  0.538481 -4.628514 -3.820792
```

Time — Temperature — Potential energy — Total energy



See the lecture on "order-invariant real-number summation"

Strong Scalability of Hybrid MD

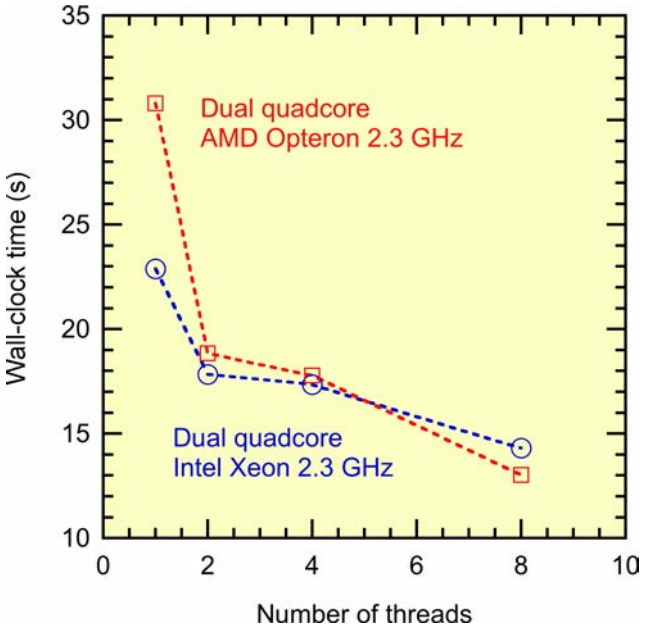
1 MPI process; 1-8 threads

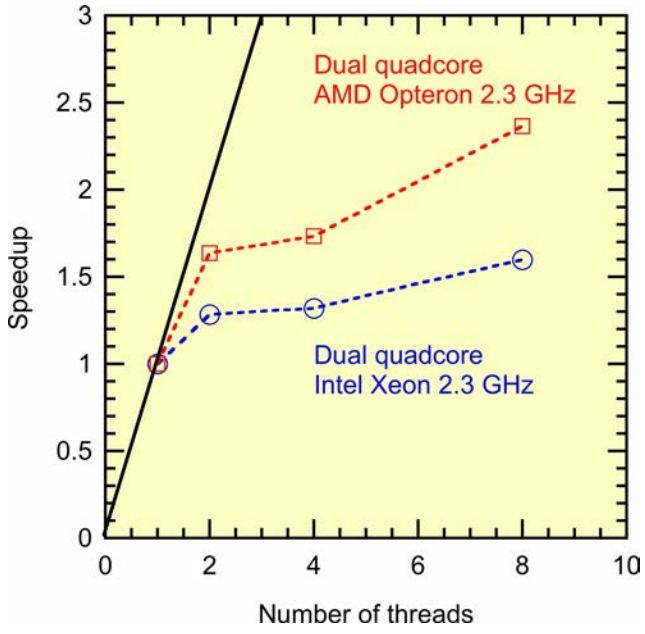
In hmd.h:

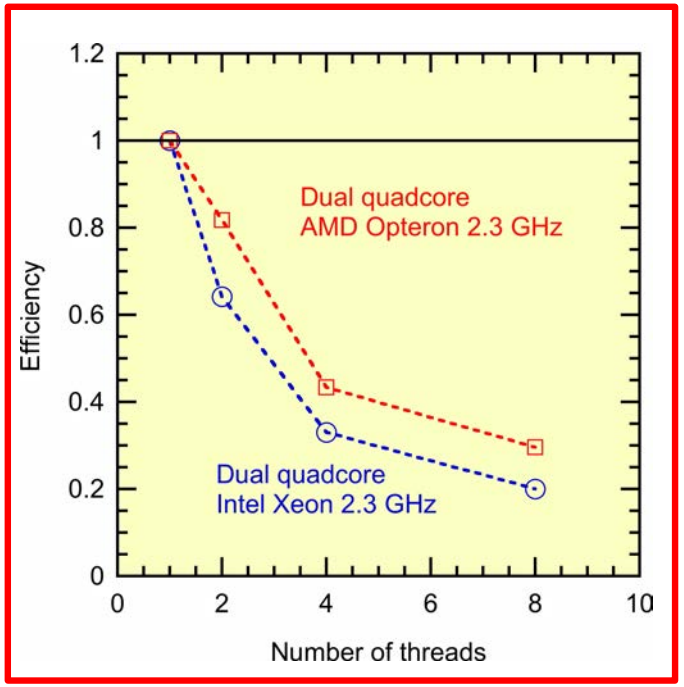
```
vproc = {1,1,1}, nproc = 1;
vthrd = {1,1,1}, nthrd = 1;
         2 1 1           2
         2 2 1           4
         2 2 2           8
```

pmd.in

```
24 24 24 InitUcell[3]
0.8      Density
1.0      InitTemp
0.005    DeltaT
100      StepLimit
101      StepAvg
```







`InitUcell[] = {24,24,24}`

$N = 4 \times 24^3$

$= 55296$ atoms

$$S_P = \frac{T(N,1)}{T(N,P)}$$

$$E_P = \frac{S_P}{P}$$

P: Number of cores

Improved Strong Scalability of Hybrid MD

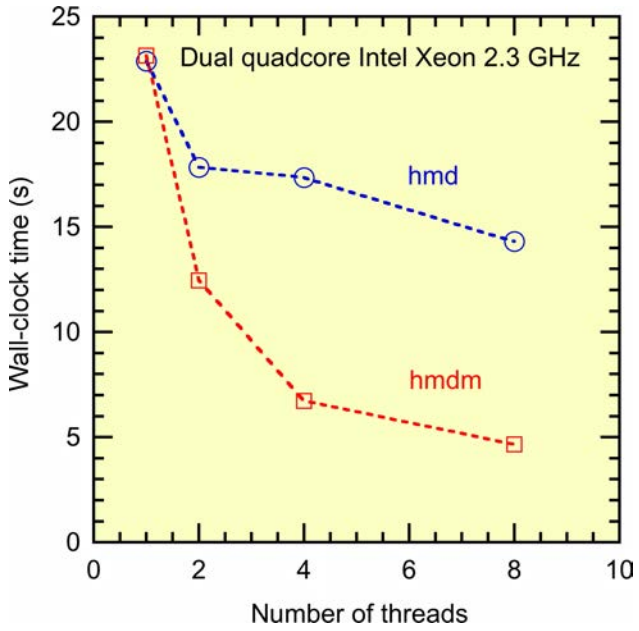
1 MPI process; 1-8 threads

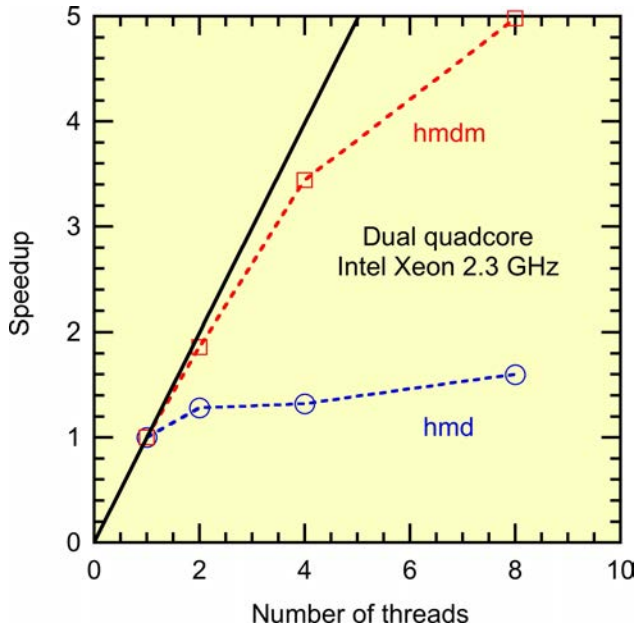
In hmd.h:

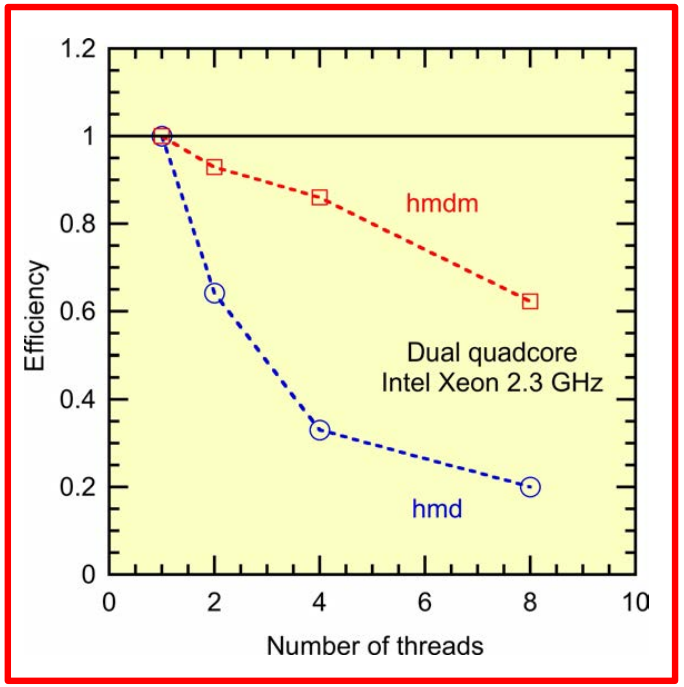
```
vproc = {1,1,1}, nproc = 1;
vthrd = {1,1,1}, nthrd = 1;
         2 1 1           2
         2 2 1           4
         2 2 2           8
```

```
#SBATCH --nodes=1
#SBATCH --ntasks-per-node=1
#SBATCH --cpus-per-task=8

mpirun ... –n 1 ... ./hmd1
                        2
                        4
                        8
```








```
InitUcell[] = {24,24,24}
```

$N = 4 \times 24^3$

$= 55296$ atoms

$$S_P = \frac{T(N,1)}{T(N,P)}$$

$$E_P = \frac{S_P}{P}$$

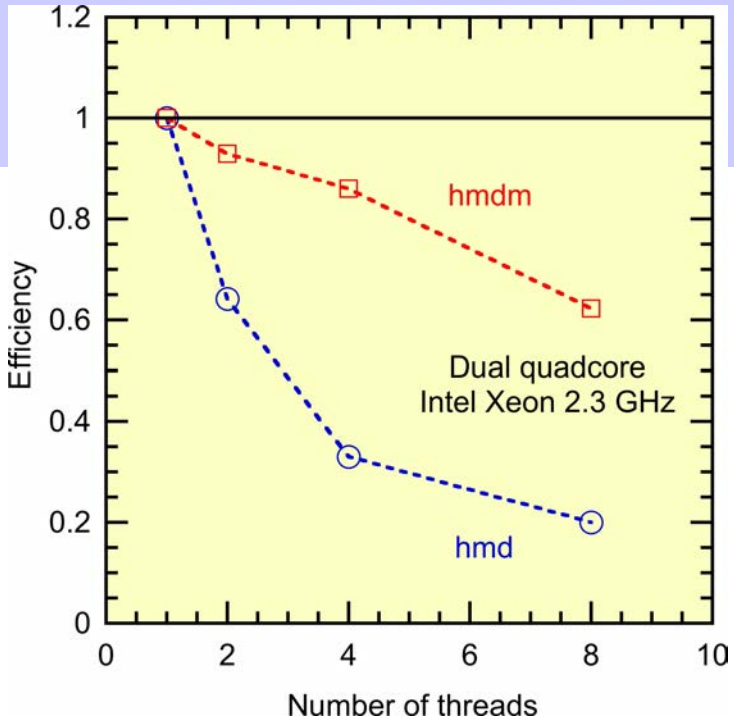
P: Number of cores

More on Multithreading MD

- **Large overhead is involved in opening an OpenMP parallel section**
  → Open it only once in the main function

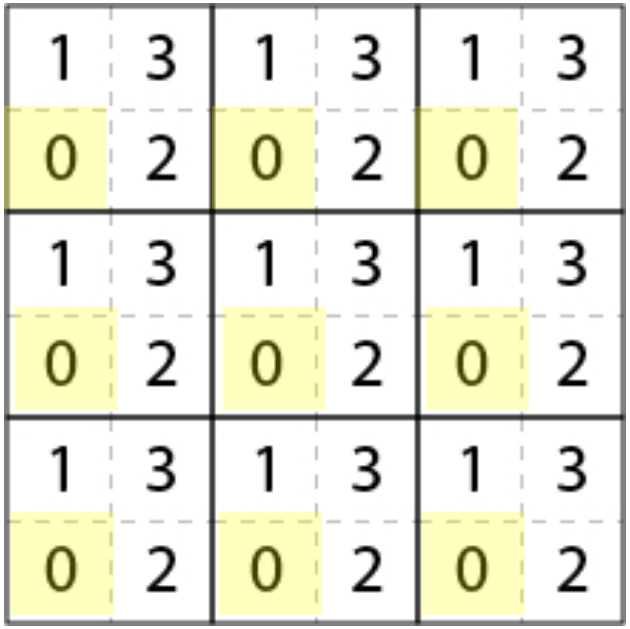
In hmdm.c:

```
int main() {
  ...
  omp_set_num_threads(nthrd);
  #pragma omp parallel
  {
    #pragma omp master
    {// Do serial computations here}
    ...
    #pragma omp barrier // When threads need be synchronized
    ...
  }
  ...
}
```



More on Avoiding Race Conditions

- **Program `hmd.c`: (1) used data privatization; (2) disabled the use of Newton's third law → this doubled computation**
- **Cell-coloring**
  - **> Race condition-free multithreading without duplicating pair computations**
  - **> Color cells such that no cells of the same color are adjacent to each other**
  - **> Threads process cells of the same color at a time in a color loop**



Four-color (eight colors in 3D) solution requires the cell size to be twice the cutoff radius r_c

H. S. Byun *et al.*, *Comput. Phys. Commun.* **219**, 246 ('17)

- **Use graph coloring in more general computations**

False Sharing

- **While eliminating race conditions by data privatization, the use of consecutive per-thread accumulators, `lpe_td[nthrd]`, degrades performance by causing excessive cache misses**

See false sharing Wiki page

- **Solution 1: Padding**

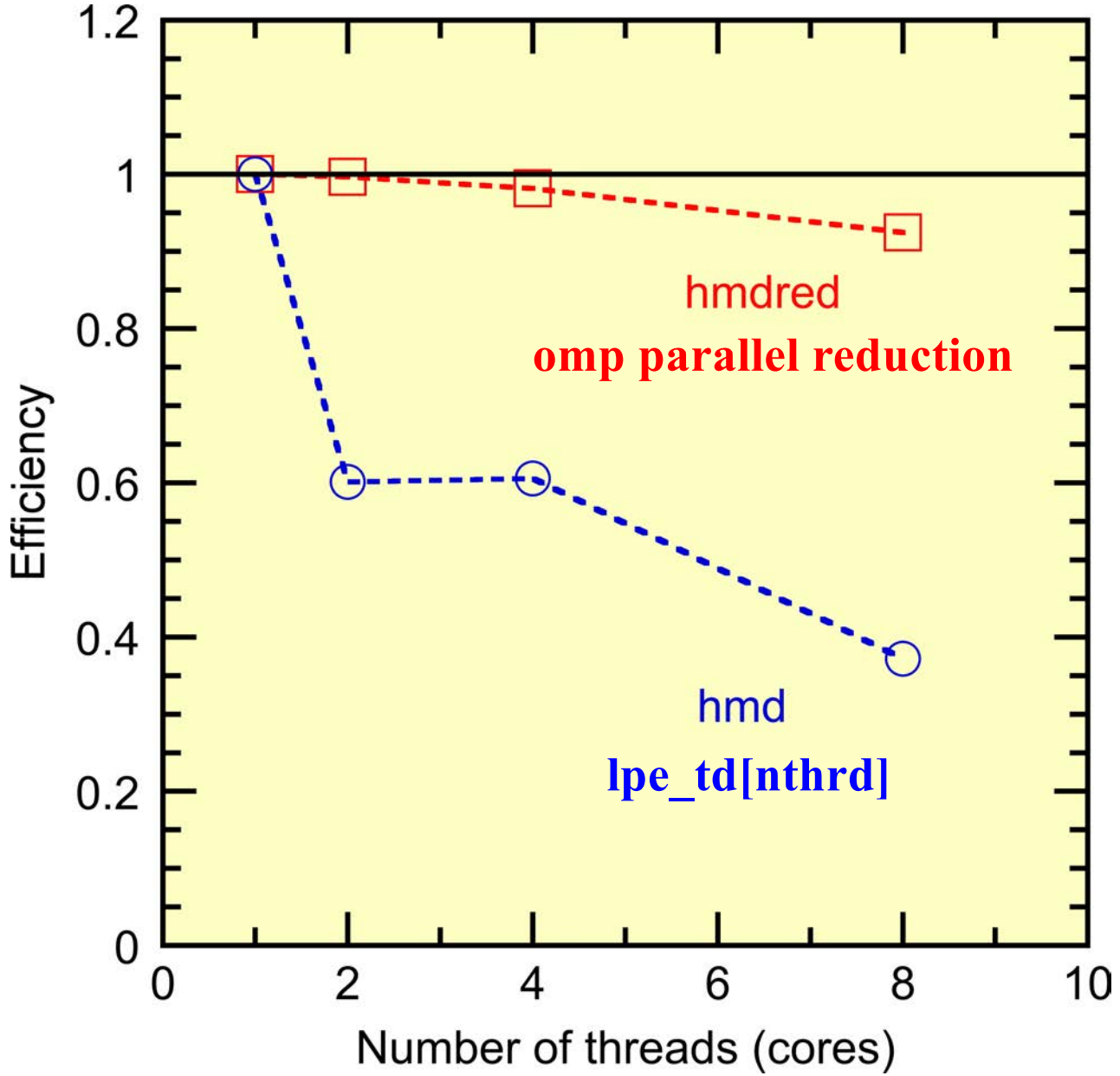
```
struct lpe_t {
  double lpe;
  double pads[7]; // assume intel CPU with 64 byte cache line
};
struct lpe_t lpe_td[nthrd];
```

- **Solution 2: System-supported data privatization**

```
#pragma omp parallel private (...) reduction(+:lpe)
{
  ...
  lpe += 0.5*vVal;
  ...
}
// No reduction over the threads is required here
```

1. **Create private copies of the variable (`lpe`) in the reduction clause for all the threads**
2. **Perform the specified reduction operation (+) on the variable at the end of the parallel section**

Scalability Test: False Sharing Matters



Some Like It as Arguments

- Use command line arguments for scaling tests without re-compiling multiple times
- `hmd.c` → `hmdarg.c` by adding the following lines in `main()`

```
int main(int argc, char **argv) {
  ...
  vthrd[0] = atoi(argv[1]);
  vthrd[1] = atoi(argv[2]);
  vthrd[2] = atoi(argv[3]);
  nthrd = vthrd[0]*vthrd[1]*vthrd[2];
  printf("Number of threads = %d\n", nthrd);
  ...
}
```

string-to-integer conversion

command-line argument

- Compiling

```
mpicc –o hmdarg hmdarg.c –fopenmp -lm
```

Strong-Scaling Test with `hmdarg.c`

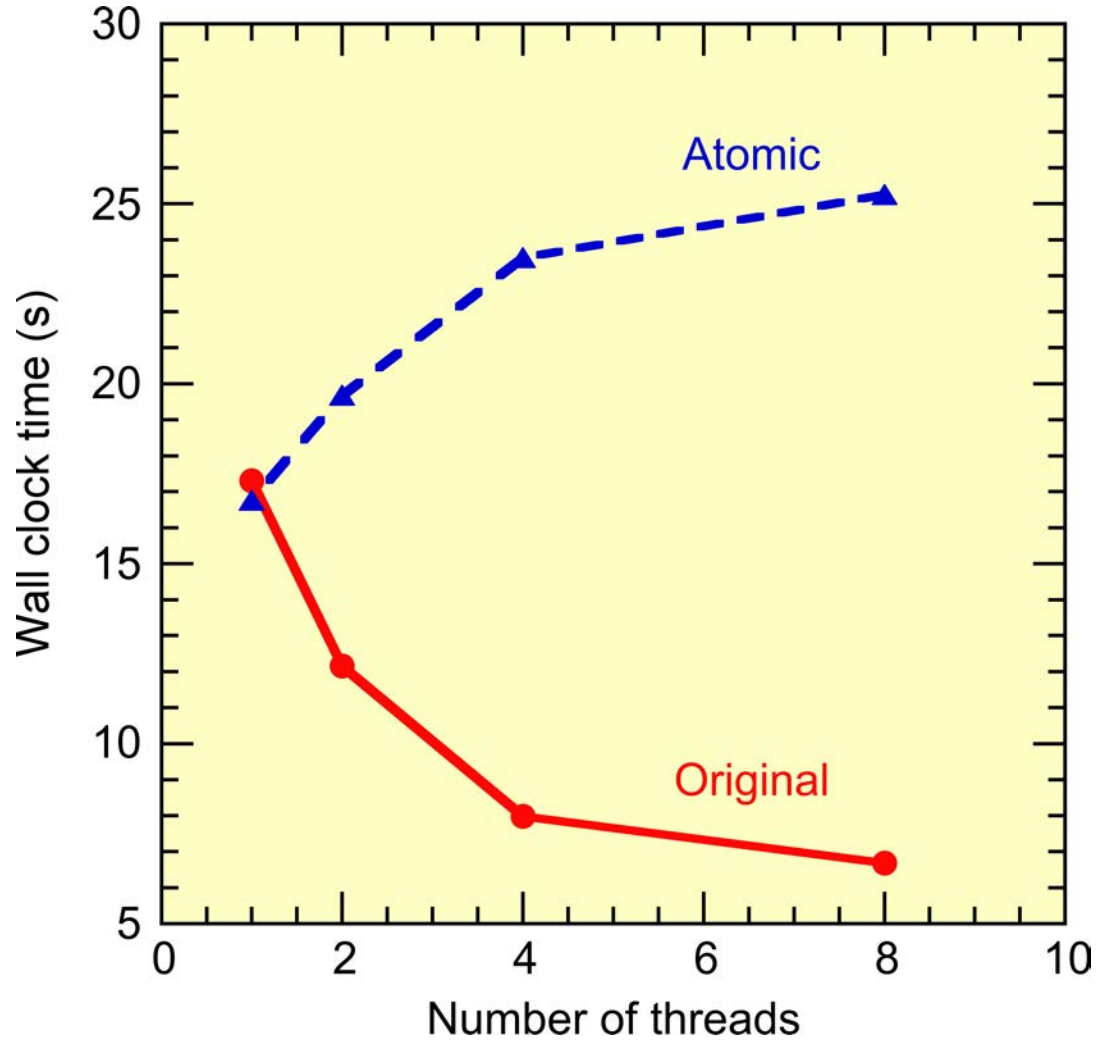
```
[anakano@discovery cs596]$ salloc --nodes=1 --ntasks-per-node=1 --cpus-per-
task=8 -t 59
...
[anakano@d05-29 cs596]$ mpirun -bind-to none -n 1 ./hmdarg 1 1 1
Number of threads = 1
al = 4.103942e+01 4.103942e+01 4.103942e+01
lc   = 16 16 16
rc   = 2.564964e+00 2.564964e+00 2.564964e+00
thbk = 16 16 16
nglob = 55296
CPU & COMT = 1.073547e+01 2.005649e-02
[anakano@d05-29 cs596]$ mpirun -bind-to none -n 1 ./hmdarg 2 1 1
Number of threads = 2
...
thbk = 8 16 16
nglob = 55296
CPU & COMT = 6.804797e+00 1.980424e-02
[anakano@d05-29 cs596]$ mpirun -bind-to none -n 1 ./hmdarg 2 2 1
Number of threads = 4
...
thbk = 8 8 16
nglob = 55296
CPU & COMT = 4.956142e+00 1.981378e-02
[anakano@d05-29 cs596]$ mpirun -bind-to none -n 1 ./hmdarg 2 2 2
Number of threads = 8
...
thbk = 8 8 8
nglob = 55296
CPU & COMT = 4.078273e+00 2.253795e-02
```

Atomic Operation

- **Restore Newton's third law & handle race conditions with the omp atomic directive**

```
int bintra;
...
if (i<j && rr<rrCut) {
  ...
  if (bintra)
    lpe_td[std] += vVal;
  else
    lpe_td[std] += 0.5*vVal;
  for (a=0; a<3; a++) {
    f = fcVal*dr[a];
    ra[i][a] += f;
    if (bintra) {
      #pragma omp atomic
      ra[j][a] -= f; // Different threads can access the same atom
    }
  }
}
```

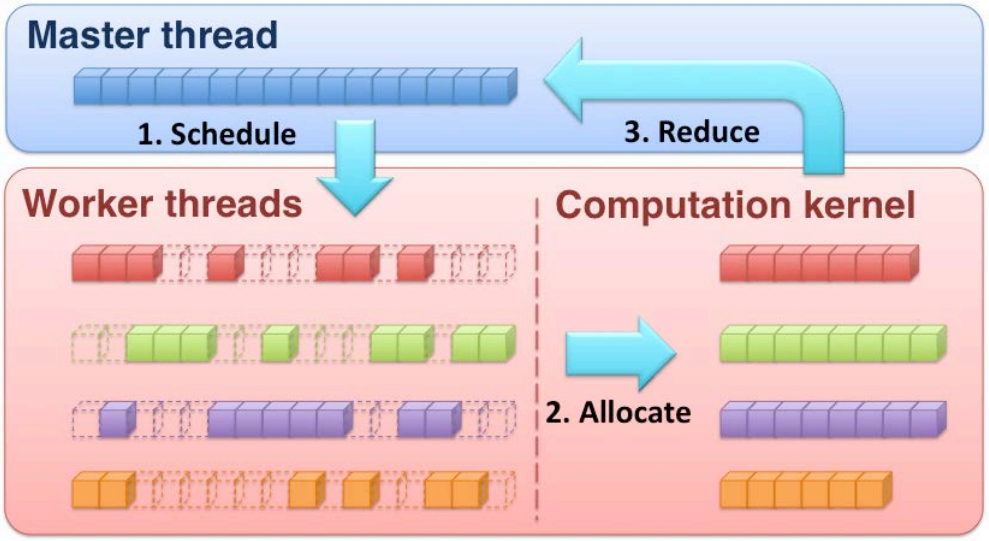
Atomic Operation Is Expensive

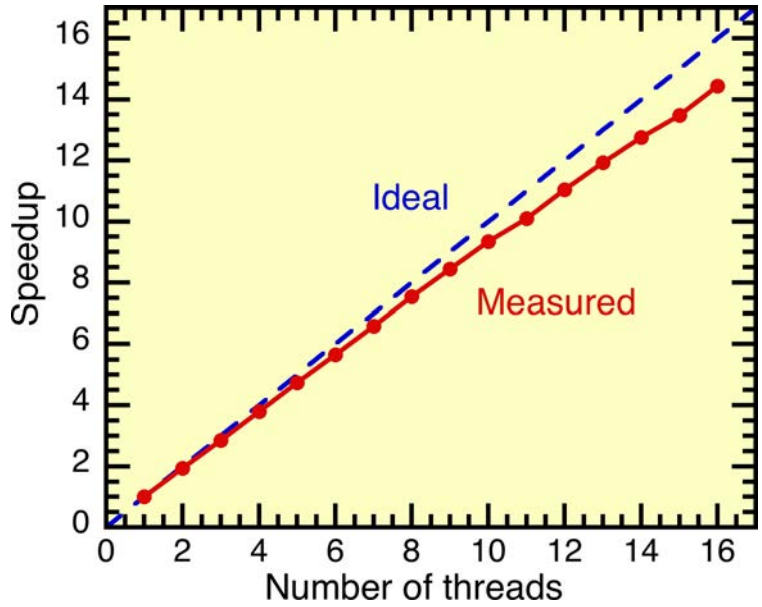


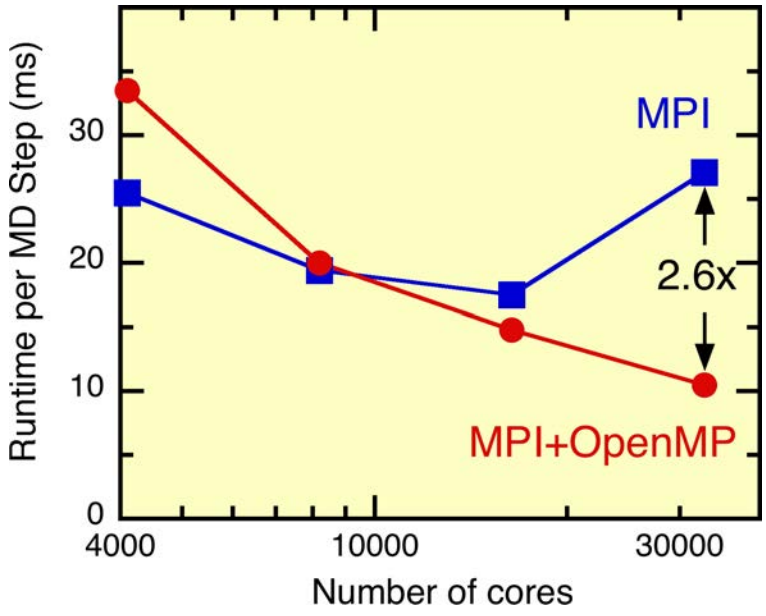
Spatially Compact Thread Scheduling

Concurrency-control mechanism:
Data privatization (duplicate the force array)

- **Reduced memory:** # of atoms, # of threads
  $\Theta(nq) \rightarrow \Theta(n+n^{2/3}q^{1/3})$
- **Strong scaling parallel efficiency 0.9 on quad quad-core AMD Opteron**
- **2.6× speedup over MPI by hybrid MPI+OpenMP on 32,768 IBM Blue Gene/P cores**







M. Kunaseth *et al.*, *PDPTA'11*; *J. Supercomput.* ('13)

Concurrency-Control Mechanisms

A number of concurrency-control mechanisms (CCMs) are provided by OpenMP to coordinate multiple threads:

- **Critical section: Serialization**
- **Atomic update: Expensive hardware instruction**
- **Data privatization: Requires large memory $\Theta(nq)$**
- **Hardware transactional memory: Rollbacks (on IBM Blue Gene/Q)**

# of atoms per node

# of threads

CCM performance varies:

- **Depending on computational characteristics of each program**
- **In many cases, CCM degrades performance significantly**

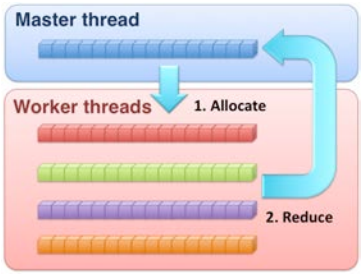
HTM/critical section

```
#pragma omp <critical|tm_atomic>
{
ra[i][0] += fa*dr[0];
ra[i][1] += fa*dr[1];
ra[i][2] += fa*dr[2];
}
```

Atomic update

```
#pragma omp atomic
ra[i][0] += fa*dr[0];
#pragma omp atomic
ra[i][1] += fa*dr[1];
#pragma omp atomic
ra[i][2] += fa*dr[2];
```

Data privatization



Goal: Provide a guideline to choose the "right" CCM

Hardware Transactional Memory

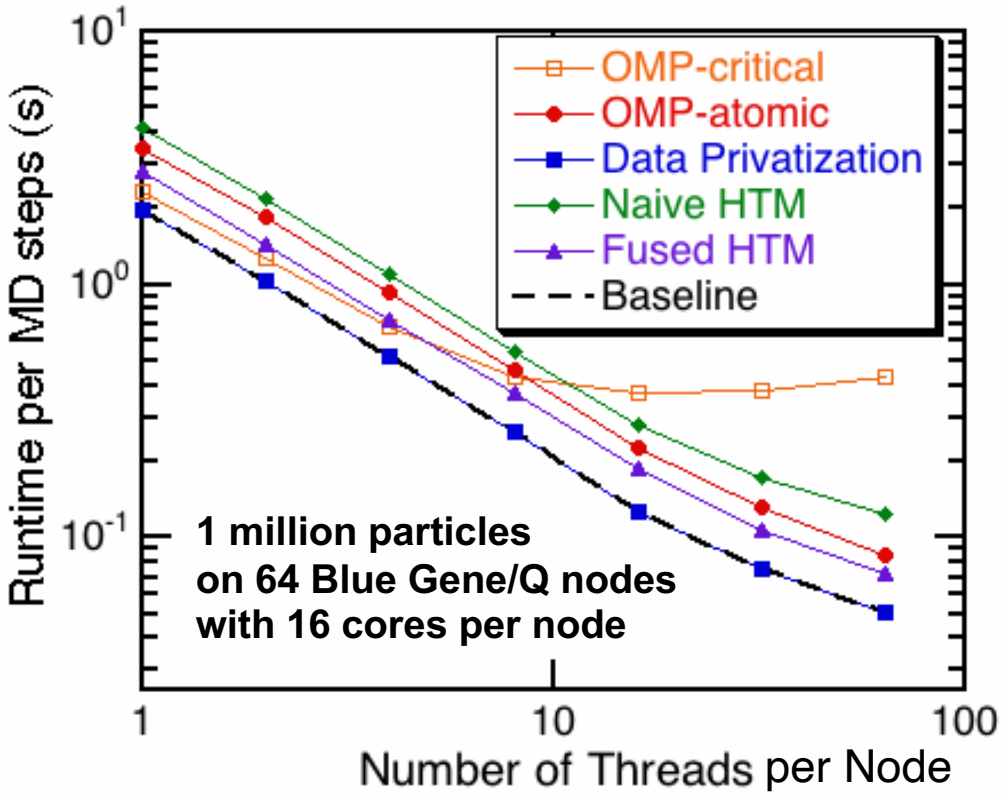
Transactional memory (TM): An opportunistic CCM

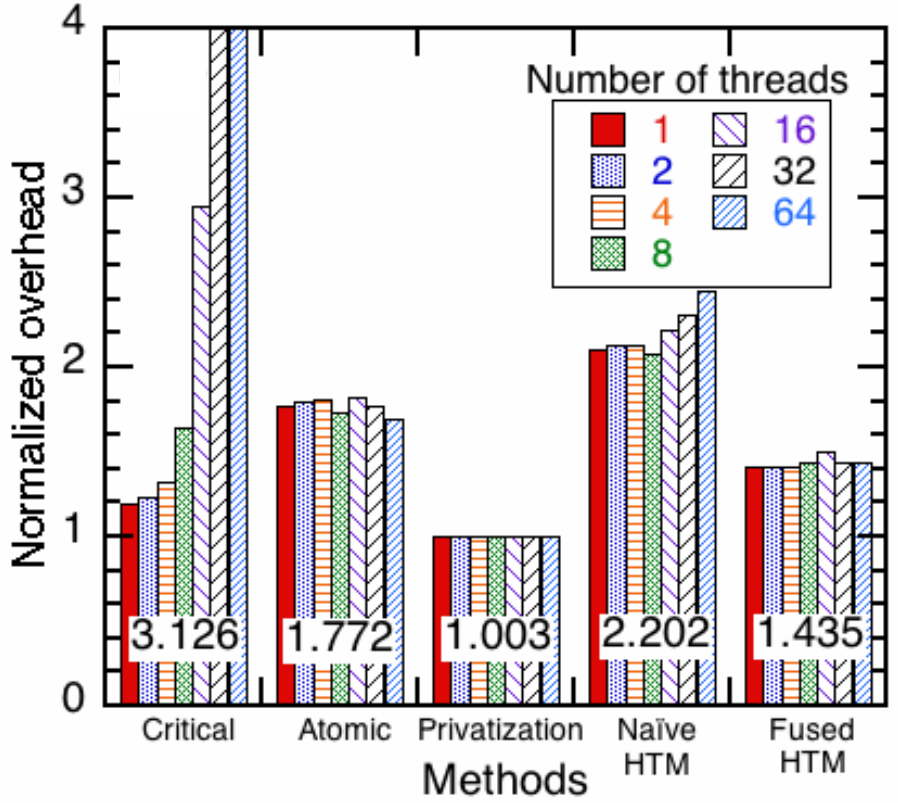
- **Avoids memory conflicts by monitoring a set of speculative operations (*i.e.* transaction)**
- **If two or more transactions write to the same memory address, transaction(s) will be restarted—a process called rollback**
- **If no conflict detected in the end of a transaction, operations within the transaction becomes permanent (*i.e.* committed)**
- **Software TM usually suffers from large overhead**

Hardware TM on IBM Blue Gene/Q:

- **The first commercial platform implementing TM support at hardware level *via* multiversioned L2-cache**
- **Hardware support is expected to reduce TM overhead**
- **Performance of HTM on molecular dynamics has not been quantified**

Strong-Scaling Benchmark for MD





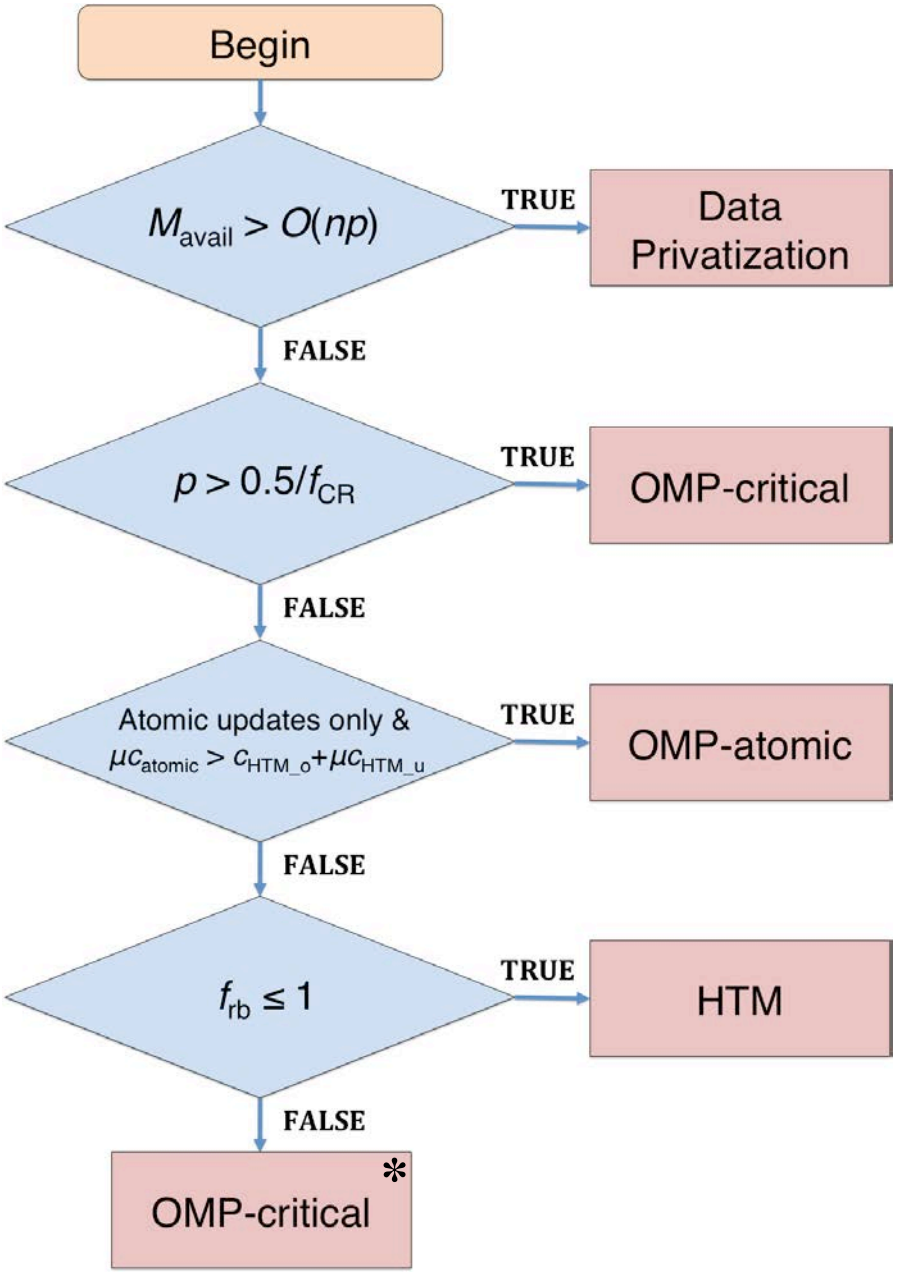
*Baseline: No CCM; the result is wrong

Developed a fundamental understanding of CCMs:

- **OMP-critical has limited scalability on larger number of threads ($q > 8$)**
- **Data privatization is the fastest, but it requires $\Theta(nq)$ memory**
- **Fused HTM performs the best among constant-memory CCMs**

M. Kunaseth *et al.*, *PDSEC'13* Best Paper

Threading Guideline for Scientific Programs



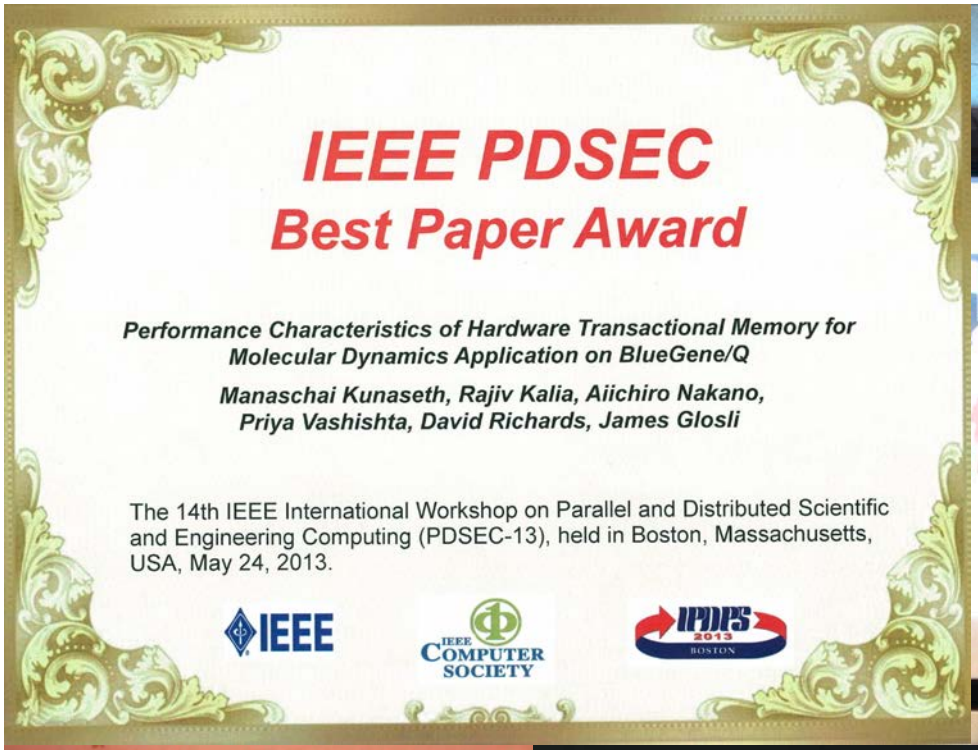
Focus on minimizing runtime (best performance):

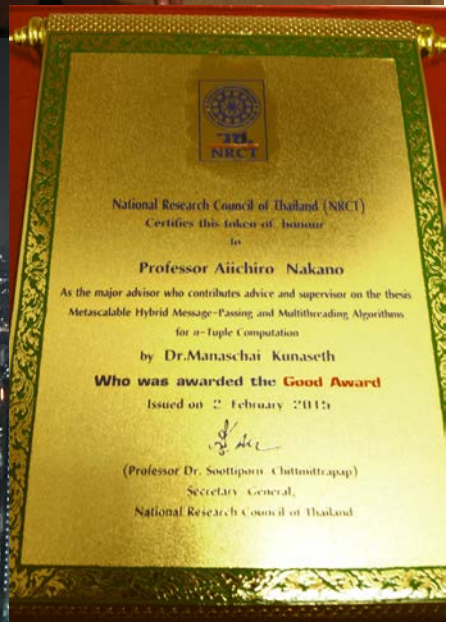
- **Have enough memory → data privatization**
- **Conflict region is small → OMP-critical**
- **Small amount of updates → OMP-atomic**
- **Conflict rate is low → HTM**
- **Other → OMP-critical* (poor performance)**

Concurrency control mechanism	Parallel efficiency
OMP-critical	$e = \min(\frac{1}{pf_{CR}}, 1)$
OMP-atomic	$e = \frac{t_{total}}{t_{total} + m\mu c_{atomic}}$
Data privatization	$e = \frac{t_{total}}{t_{total} + c_{reduction} n \log p}$
HTM	$e = \frac{t_{total}}{t_{total} + m(c_{HTM\_overhead} + \mu c_{HTM\_update})}$

M. Kunaseth *et al.*, *PDSEC'13* Best Paper

IEEE PDSEC Best Paper & Beyond





It All Started as a CSCI596 Final Project