Math Background

1. Show that (assume $|x| < 1$)

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

2. $$\sum_{n=0}^{\infty} \frac{x^n}{n!} = ?$$

3. Write down the real and imaginary parts of z, where

$$z = \frac{2+i}{1-i}, \quad i = \sqrt{-1}$$

4. Expand cos(A+B) in terms of cosines and sines of A and B.

5. Write down partial derivatives of

$$f(x, y) = x^2 \sin(y) + y \cos(x^2)$$

with respect to x and y, *i.e.*, $\partial f/\partial x$ and $\partial f/\partial y$.

6. Given a probability density function,

$$p(x) = Ae^{-\alpha|x|}, \qquad -\infty < x < +\infty.$$

where α is a positive number. Express A in terms of α. Hint: Consider the normalization.

7. Integrate

$$\int_0^{\infty} \frac{dx}{1+x^2}$$

Hint: Consider a coordinate transformation, $x = \tan\theta$.