Dyson Equations in a Nutshell

Density response function

$$\chi(t-t') = \delta\ \overbrace{\rho(t)}^{\text{electron density}} \Big/ \delta\ \overbrace{\phi(t')}^{\text{external potential}} \xrightarrow{\text{Fourier transform}} \chi(\omega) \tag{1}$$

Equation of motion (EOM) in a mean-field approximation
[https://aiichironakano.github.io/phys760/DynCorr.pdf]

$$\chi = \chi_0 + \chi_0 v \chi. \tag{2}$$

where v is the Coulombic interaction and the free-electron response function arises from the excitation of electron-hole pairs, described by their energy ($\epsilon_{e|h}$) and occupation ($f_{e|h}$),

$$\chi_0 = \sum_{e,h} \frac{f_h(1-f_e)}{\omega-(\epsilon_e-\epsilon_h)}. \tag{3}$$

From Eq. (2),

$$(1 - v\chi_0)\chi = \chi_0, \tag{4}$$

$$\therefore \chi = \frac{\chi_0}{(1-v\chi_0)} = \chi_0 \left[\overbrace{1 + v\chi_0 + (v\chi_0)^2 + \cdots}^{\text{infinite sum}}\right], \tag{5}$$

Eigenmodes

$$1 - v\chi_0 = 0\ (\text{pole}) \rightarrow \text{eigenmodes} \tag{6}$$

$\because \delta\rho/\delta\phi = 0$, hence finite oscillation can exist without external potential (7)

Emergent collective mode: plasmon