AI5002 - Assignment 3

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Download code and LaTeX from below hyperlinks
1. Codes/Bayes_2_5.py
2. LaTeX

***Problem* 2.5**

A couple has two children,

i. Find the probability that both children are males, if it is known that at least one of the children is male.
ii. Find the probability that both children are females, if it is known that the elder child is a female.

Solution

Let a boy be denoted by 'b' and girl be denoted by 'g'.

As given a couple has two children then the possible set of combinations of children can be represented by this set
S = {(b,b), (b,g), (g,b), (g,g)}.

i) Let 'E' be the event of both children to be males and 'F' be the event of at least one children to be male.

'E' represents the set {(b,b)}.

The probability of 'E' is given by

$$P\ (E) = \frac{1}{4}. \tag{0.0.1}$$

'F' represents the set {(b,b), (b,g), (g,b)}.

The probability of 'F' is given by

$$P\ (F) = \frac{3}{4}. \tag{0.0.2}$$

To find probability that both children are males, given at least one of the children is male can be represented by Bayes theorem as

$$P(\ E \mid F\ ) = \frac{P(E \cap F)}{P(F)} \tag{0.0.3}$$

Also, we get E $\cap\ F = \{(b, b)\}$.

Thus,

$$P\ (E\ \cap\ F) = \frac{1}{4} \tag{0.0.4}$$

Using (0.0.4) and (0.0.2) into (0.0.3), we get,

$$P\ (\ E \mid F\ ) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \tag{0.0.5}$$

ii) Let 'E' be the event of both children to be females and 'F' be the event of elder child being a female.

'E' represents the set {(g,g)}.

The probability of 'E' is given by

$$P\ (E) = \frac{1}{4}. \tag{0.0.6}$$

'F' represents the set {(b,g), (g,g)}.

The probability of ’F’ is given by

$$P\ (F) = \frac{2}{4}. == \frac{1}{2}. \tag{0.0.7}$$

To find probability that both children are females, given that the elder child is a female can be represented by Bayes theorem as

$$P(\ E \mid F\ ) = \frac{P(E \cap F)}{P(F)} \tag{0.0.8}$$

From the set contents of ’E’ and ’F’,
we get E $\cap\ F = \{(g, g)\}$.

Thus,

$$P\ (E\ \ \cap\ \ F) = \frac{1}{4} \tag{0.0.9}$$

Using (0.0.9) and (0.0.7) into (0.0.8) we get,

$$P(\ E \mid F\ ) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} \tag{0.0.10}$$