

AI5002 - Challenge Problem 9

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Problem IES_ISS_2015_stat1_Q3c

Two points are chosen on a line of unit length. Find the probability that each of the 3 line segments will have length greater than $\frac{1}{4}$?

Solution

We can choose one point X , and then a second point Y . We assume that at random means here that the distributions of X and Y are uniform on $[0, 1]$ and that X and Y are independent.

We want the probability such that

$$\begin{aligned} 1/4 < X < 3/4, \\ 1/4 < Y < 3/4, \\ |X - Y| > 1/4. \end{aligned} \quad (1)$$

and the two lines $x - y = \pm 1/4$.

We want to find the probability that (X, Y) lands in the part of S that is not in between these two lines. That consists of two isosceles right-angled triangles marked by the solid lines inside square S in Fig 1.0.

Each of these triangles has legs $\frac{1}{4}$, so their combined area is

$$2 \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16} \quad (2)$$

Since the ordered pair (X, Y) has uniform distribution on the full square, so probability is $\frac{1}{16}$.

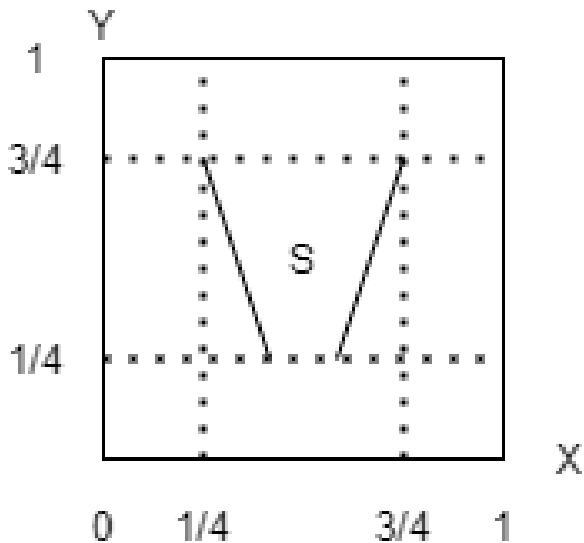


Fig 1.0: Unit Square

A unit square S is drawn in Fig. 1.0 along with the dotted lines $(x = 1/4, x = 3/4, y = 1/4, y = 3/4)$