AI5002 - Assignment 7

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***Problem* 5.17**

Suppose a girl throws a dice. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the dice?

Solution

Let A be the event of getting a 5 or 6 and A^c be the event of getting a 1, 2, 3, or 4 after rolling a dice.

$$P\ (\ A\ )\ = \frac{2}{6} = \frac{1}{3} \tag{0.0.1}$$

$$P\ (\ A^c\ )\ = 1 - \frac{1}{3} = \frac{2}{3} \tag{0.0.2}$$

Let B be the event of getting exactly one head when a coin is tossed any number of times.

Probability of exactly one head shows up when coin is tossed thrice is given by:

$$P\ (B \mid A) = P\ (\{HTT, THT, TTH\}) = 3/8 \tag{0.0.3}$$

Probability of exactly one head shows up when coin is tossed once is given by:

$$P\ (B \mid A^c)\ = P\ (\{H\}) = 1/2 \tag{0.0.4}$$

Given that she obtained exactly one head, then the probability that she threw 1, 2, 3 or 4 with the dice is given by:

$$P(A^c \mid B) = \frac{P(A^c) * P(B \mid A^c)}{P(A^c) * P(B \mid A^c) + P(A) * P(B \mid A)} \tag{0.0.5}$$

$$P(A^c \mid B) = \frac{\frac{2}{3} * \frac{1}{2}}{\frac{2}{3} * \frac{1}{2} + \frac{1}{3} * \frac{3}{8}} \tag{0.0.6}$$

$$P(A^c \mid B) = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{8}} \tag{0.0.7}$$

$$P(A^c \mid B) = \frac{8}{11} \simeq 0.73 \tag{0.0.8}$$