

AI5002: Challenging Problem: Mixture

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Download all Python codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/blob/main/Assignment_11/assignment11_code_drawing%20balls.py

and latex-tikz codes from

https://github.com/Debolena/AI5002-Probability-and-Random-Variables/tree/main/Challenging%20Problem_Mixture

Now,

$$P(X + Y \geq 5) = 1 - P(X + Y < 5) \quad (2.0.9)$$

$$= 1 - P(X + Y \leq 4) \quad (2.0.10)$$

$$P(X + Y \leq 4) = \sum_{k=0}^4 P(X = k, Y \leq 4 - k) \quad (2.0.11)$$

$$= \sum_{k=0}^4 P(X = k) P(Y \leq 4 - k) \quad (2.0.12)$$

$$= \sum_{k=0}^4 \binom{5}{k} \left(\frac{1}{2}\right)^5 P(Y \leq 4 - k) \quad (2.0.13)$$

$$= \left(\frac{1}{2}\right)^5 [1 \times 1 + 5 \times 1 + 10 \times 1 + 10 \times 1 + 0] \quad (2.0.14)$$

$$= \frac{26}{32} \quad (2.0.15)$$

Using (2.0.10) and (2.0.15),

$$P(X + Y \geq 5) = 1 - \frac{26}{32} \quad (2.0.16)$$

$$= \frac{6}{32} \quad (2.0.17)$$

From (2.0.8) and (2.0.17),

$$\frac{P(X + Y \leq 2)}{P(X + Y \geq 5)} = 1 \quad (2.0.18)$$

1 PROBLEM

Let $X \sim \text{Bin}\left(5, \frac{1}{2}\right)$ and $Y \sim U(0, 1)$. Then $\frac{P(X+Y) \leq 2}{P(X+Y) \geq 5}$ is equal to?

2 SOLUTION

As $X \sim \text{Bin}\left(5, \frac{1}{2}\right)$,

$$P(X = k) = \binom{5}{k} \left(\frac{1}{2}\right)^5, \text{ as } n=5 \text{ and } p = \frac{1}{2} \quad (2.0.1)$$

$Y \sim U(0, 1)$. So, the CDF of Y is:

$$F_Y(y) = P(Y \leq y) \quad (2.0.2)$$

$$= \begin{cases} 0, & \text{if } y \leq 0 \\ y, & \text{if } 0 < y < 1 \\ 1 & \text{if } y \geq 1 \end{cases} \quad (2.0.3)$$

$$P(X + Y \leq 2) = \sum_{k=0}^2 P(X = k, Y \leq 2 - k) \quad (2.0.4)$$

$$= \sum_{k=0}^2 P(X = k) \cdot P(Y \leq 2 - k) \quad (2.0.5)$$

$$= \binom{5}{0} \left(\frac{1}{2}\right)^5 \cdot 1 + \binom{5}{1} \left(\frac{1}{2}\right)^5 \cdot (2 - 1) + \binom{5}{2} \left(\frac{1}{2}\right)^5 \cdot (2 - 2) \quad (2.0.6)$$

$$= \left(\frac{1}{2}\right)^5 [1 + 5 \times 1 + 0] \quad (2.0.7)$$

$$= \frac{6}{32} \quad (2.0.8)$$