AI5002 - Assignment 13

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Download code and LaTeX from below hyperlinks
1. Code/GATE_40.py
2. LaTeX

*Problem GATE*40

A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times (0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favour of a 0, and if the number of ones exceeds the number of zeros, the decoder decides in favour of a 1. Assuming a binary symmetric channel with crossover probability p = 0.1, the average probability of error is ....

Solution

Let the crossover probability be α. Since the channel is symmetric,

$$\Pr(1 \mid 0) = \Pr(0 \mid 1) = \alpha = 0.1 \tag{1.0}$$

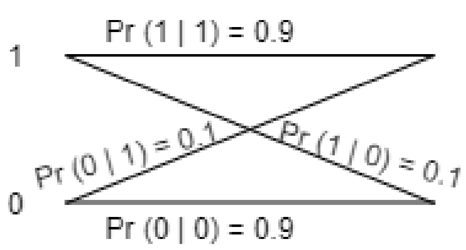

Fig 1.0: Direct and Crossover Probability

Let a binomial r.v be X $\in \{0, 1\}$ representing the number of bits transmitted incorrectly. The sample space is given as $\{0, 1, 2, 3\}$.

$$X \sim Bin(n = 3, p = 0.1) \tag{1.1}$$

The binomial p.m.f. is given by:

$$\Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{1.2}$$

Putting the values of X in (1.2) and sum up to get the probability of correct decoding,

$$\begin{aligned} \Pr(X = 0) &= \binom{3}{0} (0.1)^0 (1 - 0.1)^{3-0} \\ &= (0.9)^3 \\ \Pr(X = 1) &= \binom{3}{1} (0.1)^1 (1 - 0.1)^{3-1} \\ &= 3\ (0.1)\ (0.9)^2 \end{aligned} \tag{1.3}$$

The probability of correct decoding is given by,

$$\begin{aligned} P_c &= \Pr(no\ error) + \Pr(one\ bit\ error) \\ &= \Pr(X = 0) + \Pr(X = 1) \\ &= (0.9)^3 + 3\ (0.1)\ (0.9)^2 \\ &= 0.972 \end{aligned} \tag{1.4}$$

The average probability of error

$$\begin{aligned} P_e &= 1 - P_c \\ &= 1 - 0.972 \\ &= 0.028 \end{aligned} \tag{1.5}$$