AI5002 - Assignment 11

Tuhin Dutta
ai21mtech02002

Download code and LaTeX from below hyperlinks
1. Code/GATE_13.py
2. LaTeX

Problem GATE13

Two players, A and B, alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is ...

(A) $\frac{5}{11}$ (B) $\frac{1}{2}$ (C) $\frac{7}{13}$ (D) $\frac{6}{11}$

Solution

Let us define a r.v. X.

$$\begin{aligned} X &= \{\text{'Odd \# trial to get six by A'}\}. \\ &= \{\ 1, 3, 5, 7, ...\ \} \end{aligned} \tag{1.01}$$

Probability of throwing a die to get 6.

$$\begin{aligned} Pr\left(p = \text{'Getting a 6'}\right) &= \frac{1}{6} \\ Pr\left((1-p) = \text{'Getting a non 6'}\right) &= \frac{5}{6} \end{aligned} \tag{1.02}$$

The probability distribution of X is geometric.

$$X \sim Geo\left(p\right) \tag{1.03}$$

Probability of first six thrown by A in the kth odd trial to win the game -

$$\Pr(X = k) = (1-p)^{k-1}.p \tag{1.04}$$

for k = 1, 2, 3, ...

Getting a six in the 1st trial thrown by A

$$\Pr(X = 1) = (1-p)^0.p = \left(\frac{5}{6}\right)^0.\frac{1}{6} \tag{1.05}$$

Getting a six in the 3rd trial thrown by A

$$\Pr(X = 3) = (1-p)^2.p = \left(\frac{5}{6}\right)^2.\frac{1}{6} \tag{1.06}$$

Getting a six in the 5th trial thrown by A

$$\Pr(X = 5) = (1-p)^4.p = \left(\frac{5}{6}\right)^4.\frac{1}{6} \tag{1.07}$$

Getting a six in the 7th trial thrown by A

$$\Pr(X = 7) = (1-p)^6.p = \left(\frac{5}{6}\right)^6.\frac{1}{6} \tag{1.08}$$

$$\vdots$$

Probability that A wins the game

$$\begin{aligned} \Pr(Y = \text{'A wins the game'}) = \Pr(X = 1)\ &+ \Pr(X = 3)\ + \\ \Pr(X = 5)\ &+ \Pr(X = 7)\ + \\ &\cdots \end{aligned} \tag{1.09}$$

$$\begin{aligned} \Pr(Y = \text{'A wins the game'}) = \frac{1}{6} + \left(\frac{5}{6}\right)^2.\frac{1}{6} + \left(\frac{5}{6}\right)^4.\frac{1}{6} + \\ \left(\frac{5}{6}\right)^6.\frac{1}{6} + \\ \cdots \end{aligned} \tag{1.10}$$

From (1.10), we compare it to a infinite geometric sequence as below,

$$s = ar^0 + ar^1 + ar^2 + ar^3 + ar^4 + \cdots \tag{1.11}$$

The first term and common ratio of (1.11) is given by -

$$\begin{aligned} a &= \frac{1}{6} \\ r &= \left(\frac{5}{6}\right)^2, \quad 0 \le r \le 1 \end{aligned} \tag{1.11}$$

The closed form summation of (1.11) is given by

$$s = \frac{a}{1-r} \tag{1.12}$$

From (1.12), on substituting values, we get

$$s = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}} \tag{1.13}$$
$$= \frac{6}{11}$$