AI5002 - Assignment 1

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Download code and LaTeX from below hyperlinks
1. Codes/RayleighDist_CDF_PDf_Plot.py
2. LaTeX

***Problem* 6.1.3**

Plot the CDF and PDf of

$$A = \sqrt{V} \tag{0.0.1}$$

Solution

We can write A = $\sqrt{V}$ *as*

$$A = \sqrt{X_1^2 + X_2^2} \tag{0.0.2}$$

since it is given $X_1 \sim N(0,1)$, $X_2 \sim N(0,1)$ *and* V = $X_1^2 + X_2^2$.

Since joint distribution is not mentioned so we assume X_1 *and* X_2 to be independent otherwise the distribution of A would be unknown.

By definition, the distribution of A is Chi with two degrees of freedom or Rayleigh.

The CDF and PDF plot is as shown in Fig 1.1 and Fig 1.2.

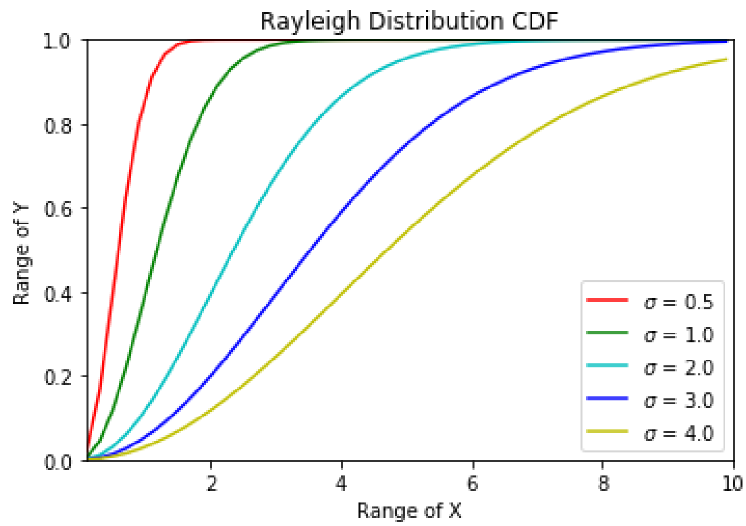

Fig 1.1: Cumulative distribution function

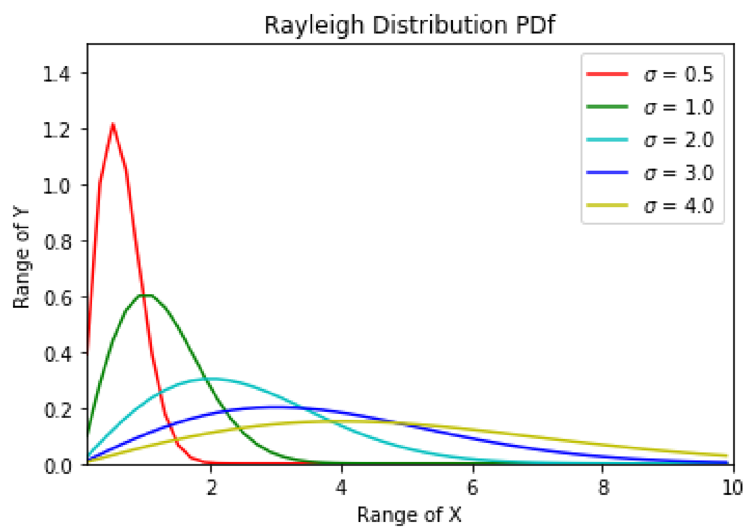

Fig 1.2: Probability density function

***Problem* 6.1.4**

Find an expression for $F_A(x)$ using the definition. Plot this expression and compare with the result of problem 6.1.3

Solution

Given,

$$A = \sqrt{V} \tag{0.0.3}$$

$$F_A(x) = P\ (\ A\ \leq\ x) \tag{0.0.4}$$

$$F_A(x) = P\ (\ \sqrt{V}\ \leq\ x) \tag{0.0.5}$$

$$F_A(x) = P\ (\ V\ \leq\ x^2) \tag{0.0.6}$$

$$F_A(x) = F_V(x^2) \tag{0.0.7}$$

From (6.1.2.1) we get

$$\mathrm{F}_V(x) \ = \ \begin{cases} 1\ -\ e^{-\alpha x} & ;\ x \geq 0 \end{cases}$$

Now for x^2, *we substitute* x^2 *in place of* x

$$F_V(x^2) = \begin{cases} 1\ -\ e^{-\alpha x^2} & ;\ x^2 \geq 0 \end{cases} \tag{0.0.8}$$

$$Putting\ \alpha\ =\ \frac{1}{2\sigma^2}\ =\ \frac{1}{2}\quad [\because\ \sigma^2\ is\ given\ as\ 1]$$

We get,

$$F_V(x^2)\ =\ 1\ -\ e^{\frac{-x^2}{2}}\quad ;\ x^2\ \geq\ 0 \tag{0.0.9}$$

Thus the CDF is derived as

$$F_A(x) = F_V(x^2)\ =\ 1\ -\ e^{\frac{-x^2}{2}}\quad ;\ x^2\ \geq\ 0$$

The plot of this equation is shown in Fig 1.3.

***Problem* 6.1.5**

Find an expression for $p_A(x)$ using the definition.

Solution

The PDf can be derived by differentiating the CDF expression from the previous problem 6.1.4

$$f_A(x) = f_V(x^2) \tag{0.0.10}$$

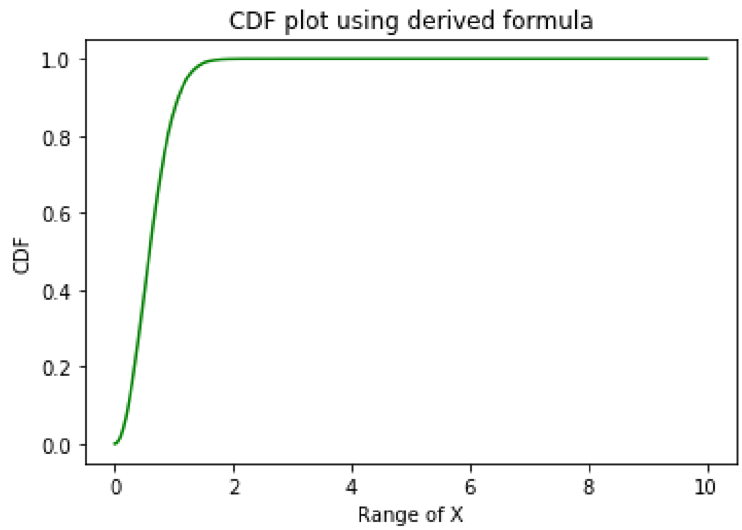

Fig 1.3: CDF plot from derived equation

$$f_V(x^2) = \frac{d}{dx}(F_V(x^2)) \tag{0.0.11}$$

$$f_V(x^2) = \frac{d}{dx}(1 - e^{\frac{-x^2}{2}}) \tag{0.0.12}$$

$$f_V(x^2) = e^{\frac{-x^2}{2}}.x \tag{0.0.13}$$

$$p_A(x) = f_A(x) = e^{\frac{-x^2}{2}}.x$$

The plot of this equation is shown in Fig 1.4.

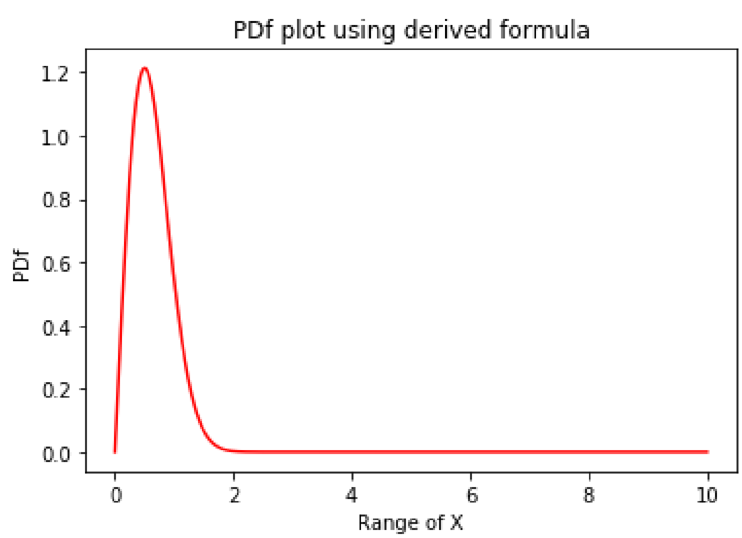

Fig 1.4: PDf plot from derived equation

The theoretical along with simulated PDf and CDF plot of 10,000 samples of two normal random variables which are further squarred, summed and taken square root of is shown in Fig 1.5 and Fig 1.6 -

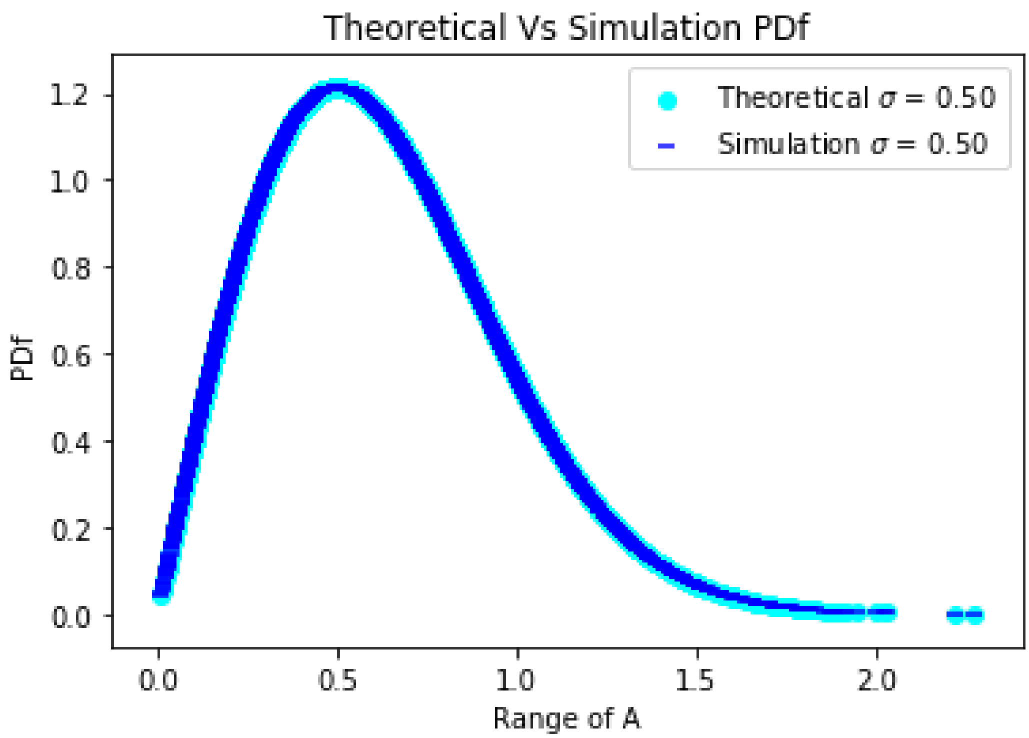

Fig 1.5: Theoretical Vs Simulation PDf

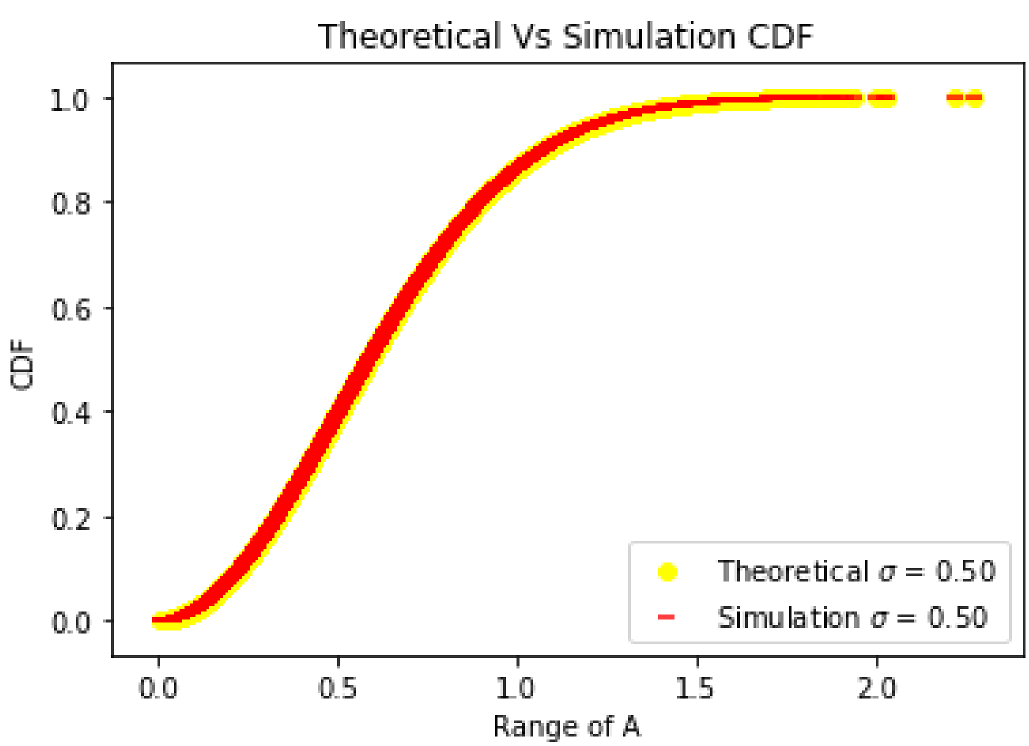

Fig 1.6: Theoretical Vs Simulation CDF