AI5002 - Assignment 2

Tuhin Dutta ai21mtech02002

Download code and LaTeX from below hyperlinks

- 1. Code
- 2. LaTeX

Problem 2.5

A couple has two children,

- i. Find the probability that both children are males, if it is known that at least one of the children is male.
- ii. Find the probability that both children are females, if it is known that the elder child is a female.

Solution

Let a boy be denoted by 'b' and girl be denoted by 'g'.

As given a couple has two children then the possible set of combinations of children can be represented by this set

 $S = \{(b,b), (b,g), (g,b), (g,g)\}.$

i) Let 'E' be the event of both children to be males and 'F' be the event of at least one children to be male.

'E' represents the set $\{(b,b)\}$.

The probability of 'E' is given by $P(E) = \frac{1}{4}$.

'F' represents the set $\{(b,b), (b,g), (g,b)\}$.

The probability of 'F' is given by $P(F) = \frac{3}{4}$.

To find probability that both children are males, given at least one of the children is male can be represented by Bayes theorem as

$$P(E | F) = \frac{P(E \cap F)}{P(F)} \dots \dots \dots \dots (1)$$

From the set contents of 'E' and 'F', we can get $E \cap F = \{(b, b)\}.$

Thus,
$$P(E \cap F) = \frac{1}{4} \dots (2)$$

Using (2) into (1), we get,

$$P(E|F) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

ii) Let 'E' be the event of both children to be females and 'F' be the event of elder child being a female.

'E' represents the set $\{(g,g)\}$.

The probability of 'E' is given by $P(E) = \frac{1}{4}$.

'F' represents the set {(b,g), (g,g)}.

The probability of 'F' is given by $P(F) = \frac{2}{4} = \frac{1}{2}$.

To find probability that both children are females, given that the elder child is a female can be represented by Bayes theorem as

$$P(E | F) = \frac{P(E \cap F)}{P(F)} \dots (1)$$

From the set contents of 'E' and 'F', we get $E \cap F = \{(g, g)\}.$

Thus,
$$P(E \cap F) = \frac{1}{4} \dots (2)$$

Using (2) into (1), we get, P (E | F) =
$$\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$