AI5002 - Assignment 10

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Download code and LaTeX from below hyperlinks
1. Code/GATE_11.py
2. LaTeX

Problem GATE11

The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is.....

Solution

Number of positive integers lying between 1 and 100 (both inclusive) is given by the set S.
n(S) = 100

Let us define a random variable A as 'Integers between 1 and 100 (both inclusive) divisible by 2'. The sample space defined by A is given by -
A = {
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28,
30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54,
56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80,
82, 84, 86, 88, 90, 92, 94, 96, 98, 100}

$$Pr(A = x) = \begin{cases} \frac{1}{50} & x \in A \\ 0 & otherwise \end{cases} \tag{0.0.1}$$

Let us define a random variable B as 'Integers between 1 and 100 (both inclusive) divisible by 3'. The sample space defined by B is given by -
B = {
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42,
45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81,
84, 87, 90, 93, 96, 99}

$$Pr(B = x) = \begin{cases} \frac{1}{33} & x \in B \\ 0 & otherwise \end{cases} \tag{0.0.2}$$

Let us define a random variable C as 'Integers between 1 and 100 (both inclusive) divisible by 5'. The sample space defined by C is given by -
C = {
5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70,
75, 80, 85, 90, 95, 100}

$$Pr(C = x) = \begin{cases} \frac{1}{20} & x \in C \\ 0 & otherwise \end{cases} \tag{0.0.3}$$

Let us define a random variable AC as 'Integers between 1 and 100 (both inclusive) divisible by 2 and 5 or 10'. The sample space defined by AC is given by -
The sample space is given by -
AC = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}

$$Pr(AC = x) = \begin{cases} \frac{1}{10} & x \in AC \\ 0 & otherwise \end{cases} \tag{0.0.4}$$

Let us define a random variable AB as 'Integers between 1 and 100 (both inclusive) divisible by 2 and 3 or 6'. The sample space defined by AB is given by -
AB = {
6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78,
84, 90, 96}

$$Pr(AB = x) = \begin{cases} \frac{1}{16} & x \in AB \\ 0 & otherwise \end{cases} \tag{0.0.5}$$

Let us define a random variable BC as 'Integers between 1 and 100 (both inclusive) divisible by 3 and 5 or 15'. The sample space defined by BC is given by -
BC = {15, 30, 45, 60, 75, 90}

$$Pr(BC = x) = \begin{cases} \frac{1}{6} & x \in BC \\ 0 & otherwise \end{cases} \tag{0.0.6}$$

Let us define a random variable ABC as ‘Integers between 1 and 100 (both inclusive) divisible by 2, 3 and 5 or 30’. The sample space defined by ABC is given by -
$ABC = \{30, 60, 90\}$

$$Pr(ABC = x) = \begin{cases} \frac{1}{3} & x \in ABC \\ 0 & otherwise \end{cases} \tag{0.0.7}$$

From inclusion-exclusion principle we know,

$$\Pr(A + B + C) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(AB) - \Pr(AC) - \Pr(BC) + \Pr(ABC) \tag{0.0.8}$$

$$\Pr(A + B + C) = \frac{50}{100} + \frac{33}{100} + \frac{20}{100} - \frac{16}{100} - \frac{10}{100} - \frac{6}{100} + \frac{3}{100} = \frac{74}{100} = 0.74 \tag{0.0.9}$$

Let us define a random variable X as ‘Integers between 1 and 100 (both inclusive) NOT divisible by 2, 3, or 5’.

The sample space is given by -
= {
1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 97
}

The probability of X is given by -

$$\Pr(X) = 1 - \Pr(A + B + C) = 1 - 0.74 = 0.26 \tag{0.0.10}$$