AI5002 - (Compulsory) Assignment

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Download code and LaTeX from below hyperlinks
1. Code/Binom_3_7.py
2. LaTeX

***Problem* 3.7**

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X? Find the pmf/cdf of the difference of two binomial random variables.

Solution

Let Z represent 'Difference between the number of heads and tails when a coin is tossed six times'.

# Heads	# Tails	$Z = \# Heads - \# Tails$
0	6	-6
1	5	-4
2	4	-2
3	3	0
4	2	2
5	1	4
6	0	6

TABLE 0: # of Heads, Tails and r.v. Z

Possible values of $Z = \{ -6, \ -4, \ -2, \ 0, \ 2, \ 4, \ 6 \}$

$$Pr \ (p = \text{" Head in a single toss "}) = \frac{1}{2} \tag{0.0.1}$$

$$Pr \ (q = \text{" Tail in a single toss "}) = \frac{1}{2} \tag{0.0.2}$$

1) Note: $\Pr(p) + \Pr(q) = 1$

Let us define two independent binomial r.v.s K and L.

$$X = \{\text{" Number of heads in six tosses"}\}$$
$$X \sim Bin(n = 6, p = 0.5) \tag{0.0.3}$$

$$Y = \{\text{" Number of tails in six tosses"}\}$$
$$Y \sim Bin(m = 6, q = 0.5) \tag{0.0.4}$$

The r.v Z can be represented as -

$$Z = X - Y \tag{0.0.5}$$

The binomial pmf is given by -

$$\Pr(Z = k) = f(k, n, p) = \binom{n}{k} p^k \ (1 - p)^{(n-k)} \tag{0.0.6}$$

The probability distribution of the difference of two or more independent random variables is the convolution of their individual distributions.

Let k = Y represent all values that Y can take and these implies X = Z + k.

$$P \ (Z = z) \ = \sum_{k=0}^{6} P \ (X = z + k) \ . \ P \ (Y = k) \tag{0.0.7}$$

$$P \ (Z = z) \ = \sum_{k=0}^{6} f(z+k, \ n, \ p) \ . \ f(k, \ m, \ q) \tag{0.0.8}$$

We define two cases for the positive and negative part of Z, and thus pmf is given as -

$$P \ (Z = z) \ = \begin{cases} \sum_{k=0}^{n} \ f(z + k, \ n, \ p) \ . \ f(k, \ m, \ q), & z \geq 0 \\ \sum_{k=0}^{m} \ f(k, \ n, \ p) \ . \ f(z + k, \ m, \ q), & z < 0 \end{cases}$$

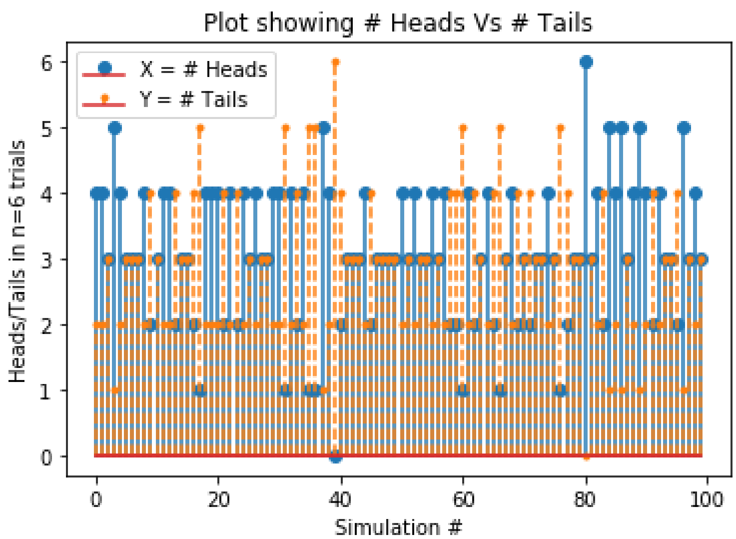

Fig 1.1: Simulation of X and Y

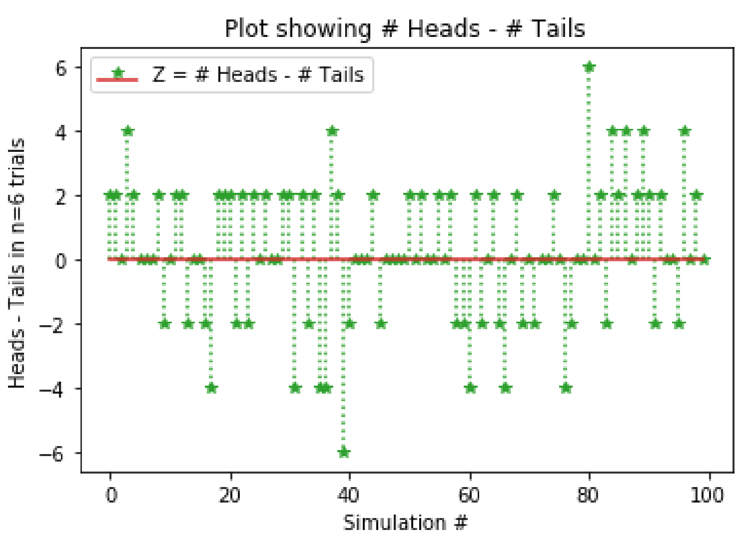

Fig 1.2: Plotting Z = X - Y