AI5002 - Assignment 4

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Download code and LaTeX from below hyper-links
1. Codes/Binomial_3_3.py
2. LaTeX

***Problem* 3.3**

Suppose X has binomial distribution.
Show that X = 3 is the most likely outcome.
(Hint: P (X=3) is the maximum among all P (x_i), $x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution

We can write,

$$X \sim B\left(n, p\right)$$

where n is the number of independent trials,
p is the probability of success for each trial and
q is the probability of failure = 1 - p for each trial.

Then,

$$P\left(X = x\right) = \left(\begin{array}{c} n \\ x \end{array}\right) . \ q^{\ n-x} \ . \ p^{\ x} \tag{0.0.1}$$

Putting value of x = 3 and assuming p = $\frac{1}{2}$ while q = 1 - p = $\frac{1}{2}$ in (0.0.1), we get

$$P\left(X = 3\right) = \left(\begin{array}{c} n \\ 3 \end{array}\right) . \left(\frac{1}{2}\right)^{\ n-3} . \left(\frac{1}{2}\right)^{\ 3} \tag{0.0.2}$$

$$P\left(X = 3\right) = \binom{n}{3} . \left(\frac{1}{2}\right)^{\ n} \tag{0.0.3}$$

From (0.0.3) we can see that for P (X = 3) to be maximum the term $\left(\begin{smallmatrix} n \\ 3 \end{smallmatrix}\right)$ should be maximum.

From combinatorics we know for the term $\left(\begin{smallmatrix} n \\ 3 \end{smallmatrix}\right)$ to be maximum, n value should be = 3 * 2 = 6.

For all given values of

$$x_i = \left(0, 1, 2, 3, 4, 5, 6\right)$$

we find the P (X = x_i).

For x_i = 0,

$$P\left(X = 0\right) = \binom{6}{0} . \left(\frac{1}{2}\right)^{\ 6} = \frac{1}{64} = P\left(X = 6\right) \tag{0.0.4}$$

For x_i = 1,

$$P\left(X = 1\right) = \binom{6}{1} . \left(\frac{1}{2}\right)^{\ 6} = \frac{6}{64} = P\left(X = 5\right) \tag{0.0.5}$$

For x_i = 2,

$$P\left(X = 2\right) = \binom{6}{2} . \left(\frac{1}{2}\right)^{\ 6} = \frac{15}{64} = P\left(X = 4\right) \tag{0.0.6}$$

For x_i = 3,

$$P\left(X = 3\right) = \binom{6}{3} . \left(\frac{1}{2}\right)^{\ 6} = \frac{20}{64} = 0.3125 \tag{0.0.7}$$

Comparing (0.0.4) through (0.0.7), we see X = 3 (0.0.7) to be the most likely event.

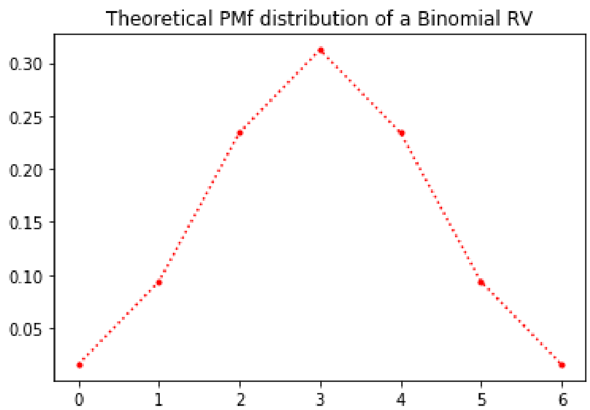

Fig 1.1: Theoretical PMf plot

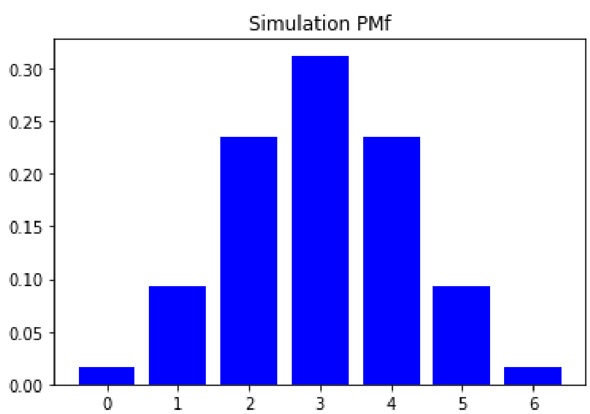

Fig 1.2: Simulation plot of PMf

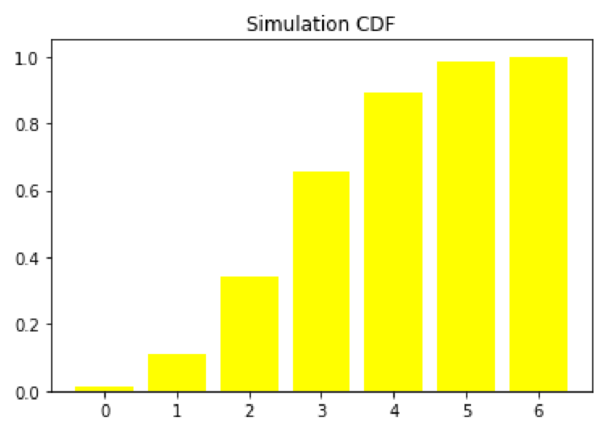

Fig 1.4: Simulation plot of CDF

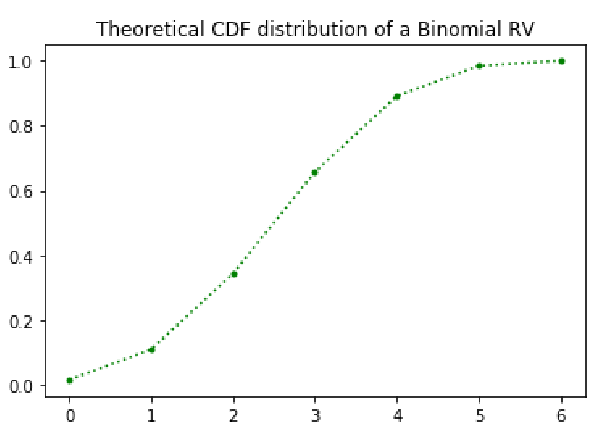

Fig 1.3: Theoretical CDF plot