AI5002 - Assignment 1

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Download code and LaTeX from below hyper-links
1. Codes/RayleighDist_CDF_PDf_Plot.py
2. LaTeX

Problem **6.1.3**
Plot the CDF and PDf of

$$A = \sqrt{V} \tag{0.0.1}$$

Solution
We can write A = $\sqrt{V}$ *as*

$$A = \sqrt{X_1^2 + X_2^2} \tag{0.0.2}$$

since it is given $X_1 \sim N(0,1)$, $X_2 \sim N(0,1)$ *and* $V = X_1^2 + X_2^2$.

Since joint distribution is not mentioned so we assume X_1 *and* X_2 to be independent otherwise the distribution of A would be unknown.

By definition, the distribution of A is Chi with two degrees of freedom or Rayleigh.

The CDF and PDF plot is as shown in Fig 1.1 and Fig 1.2.

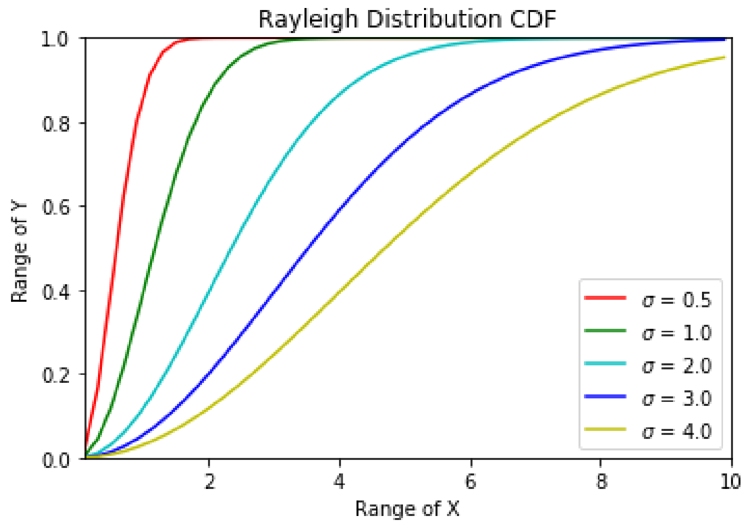

Fig 1.1: Cumulative distribution function

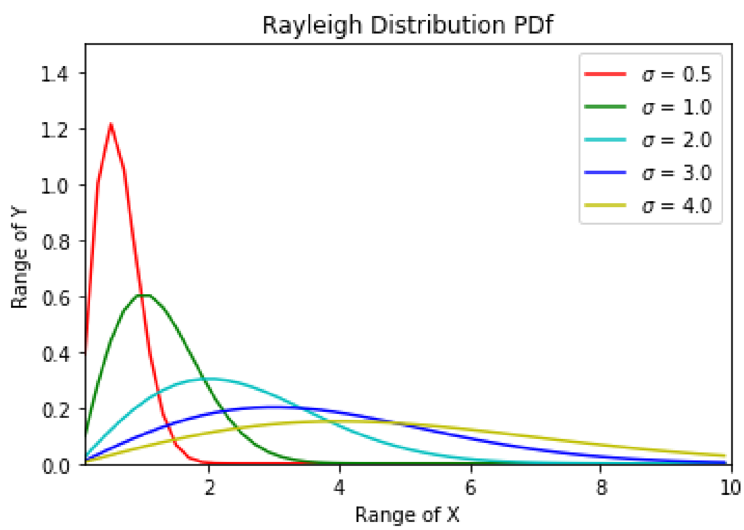

Fig 1.2: Probability density function

Problem **6.1.4**
Find an expression for $F_A(x)$ using the definition. Plot this expression and compare with the result of problem 6.1.3

Solution
Given,

$$A = \sqrt{V} \tag{0.0.3}$$

$$F_A(x) = P\ (\ A\ \le\ x) \tag{0.0.4}$$

$$F_A(x) = P\ (\ \sqrt{V}\ \le\ x) \tag{0.0.5}$$

$$F_A(x) = P\ (\ V\ \le\ x^2) \tag{0.0.6}$$

$$F_A(x) = F_V(x^2) \tag{0.0.7}$$

From (6.1.2.1) we get

$$F_V(x) = \begin{cases} 1 - e^{-\alpha x} & ;\ x \ge 0 \end{cases}$$

Now for x^2, *we substitute* x^2 *in place of* x

$$F_V(x^2) = \begin{cases} 1 - e^{-\alpha x^2} & ;\ x^2 \ge 0 \end{cases} \tag{0.0.8}$$

$$Putting\ \alpha = \frac{1}{2\sigma^2} = \frac{1}{2}\quad [\because\ \sigma^2\ is\ given\ as\ 1]$$

We get,

$$F_V(x^2) = 1 - e^{\frac{-x^2}{2}} \quad ;\ x^2 \ge 0 \tag{0.0.9}$$

Thus the CDF is derived as

$$F_A(x) = F_V(x^2) = 1 - e^{\frac{-x^2}{2}} \quad ;\ x^2 \ge 0$$

The plot of this equation is shown in Fig 1.3.

***Problem* 6.1.5**

Find an expression for $p_A(x)$ using the definition.

Solution

The PDf can be derived by differentiating the CDF expression from the previous problem 6.1.4

$$f_A(x) = f_V(x^2) \tag{0.0.10}$$

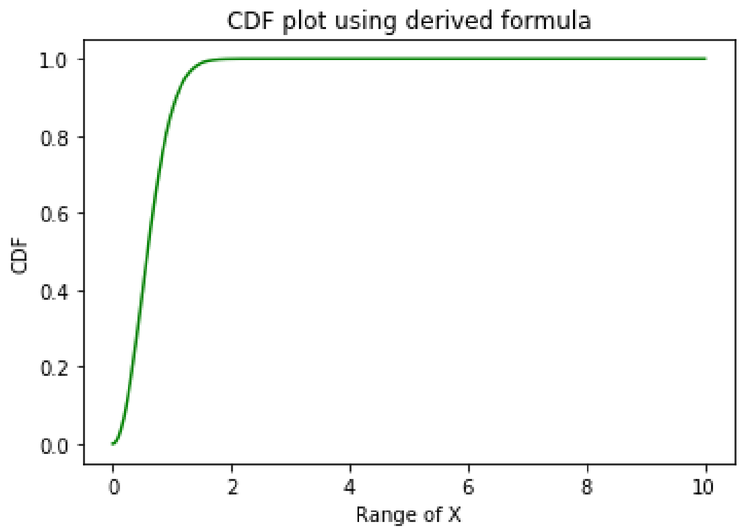

Fig 1.3: CDF plot from derived equation

$$f_V(x^2) = \frac{d}{dx}(F_V(x^2)) \tag{0.0.11}$$

$$f_V(x^2) = \frac{d}{dx}(1 - e^{\frac{-x^2}{2}}) \tag{0.0.12}$$

$$f_V(x^2) = e^{\frac{-x^2}{2}}.x \tag{0.0.13}$$

$$p_A(x) = f_A(x) = e^{\frac{-x^2}{2}}.x$$

The plot of this equation is shown in Fig 1.4.

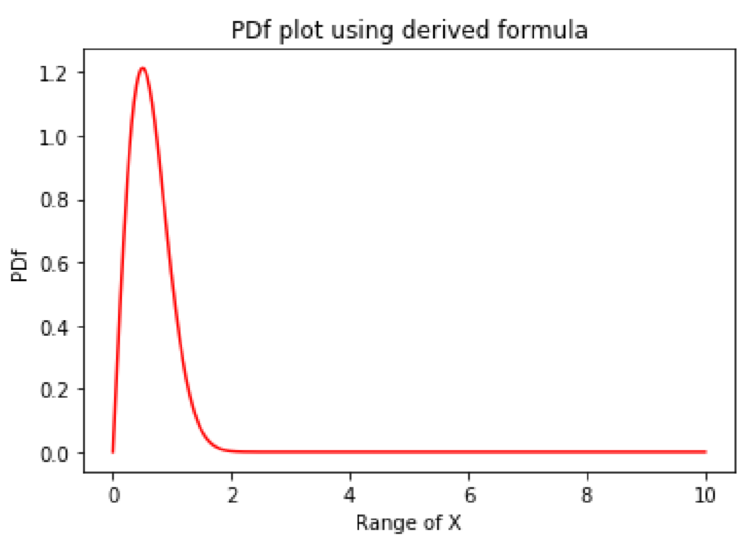

Fig 1.4: PDf plot from derived equation

The theoretical along with simulated PDf and CDF plot of 10,000 samples of Rayleigh random variables is shown in Fig 1.5 and Fig 1.6 -

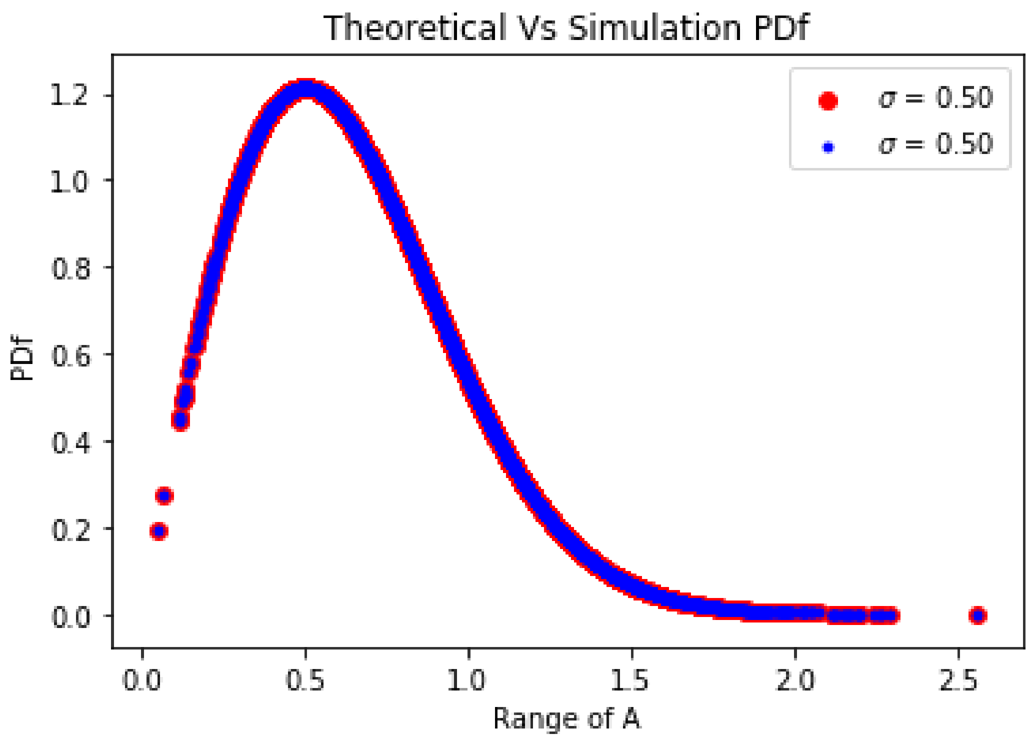

Fig 1.5: Theoretical Vs Simulation PDf

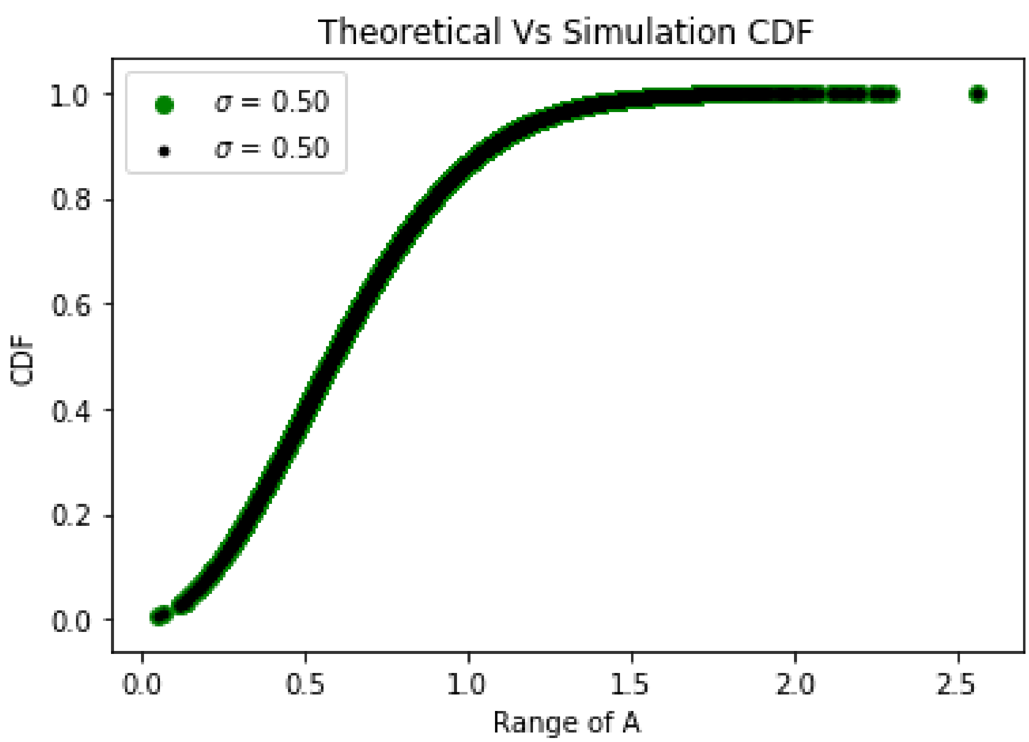

Fig 1.6: Theoretical Vs Simulation CDF