

AI5002 - Assignment 12

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The probability that P does not apply for the job given that Q does not apply for the job is given by -

Problem GATE12

P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

- (A) $\frac{4}{5}$ (B) $\frac{5}{6}$ (C) $\frac{7}{8}$ (D) $\frac{11}{12}$

Solution

Let us define two r.v. $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ representing P and Q respectively.

We further define the values taken by the r.v.s and their corresponding meaning,

$X = 0$ or $Y = 0$ represents that P or Q does not apply for a job.

$X = 1$ or $Y = 1$ represents that P or Q apply for a job.

Given,

$$\begin{aligned} \Pr(X = 0) &= \frac{3}{4}, \\ \Pr(X = 1) &= \frac{1}{4}, \\ \Pr(X = 1 \mid Y = 1) &= \frac{1}{2}, \quad \text{and} \\ \Pr(Y = 1 \mid X = 1) &= \frac{1}{3}. \end{aligned} \quad (1.0)$$

$$\Pr(X = 0 \mid Y = 0) = \frac{\Pr((X = 0) \cap (Y = 0))}{\Pr(Y = 0)} \quad (1.1)$$

Also, from (1.0) we can write,

$$\begin{aligned} \Pr(Y = 1 \mid X = 1) &= \frac{\Pr((X = 1) \cap (Y = 1))}{\Pr(X = 1)} \\ \Rightarrow \frac{1}{3} &= \frac{\Pr((X = 1) \cap (Y = 1))}{\frac{1}{4}} \\ \Rightarrow \frac{1}{12} &= \Pr((X = 1) \cap (Y = 1)) \end{aligned} \quad (1.2)$$

Similarly we can write,

$$\begin{aligned} \Pr(X = 1 \mid Y = 1) &= \frac{\Pr((X = 1) \cap (Y = 1))}{\Pr(Y = 1)} \\ \Rightarrow \frac{1}{2} &= \frac{\frac{1}{12}}{\Pr(Y = 1)} \\ \Rightarrow \frac{1}{6} &= \Pr(Y = 1) \end{aligned} \quad (1.3)$$

From (1.3), we can also find,

$$\begin{aligned} \Pr(Y = 0) &= 1 - \frac{1}{6} \\ \Pr(Y = 0) &= \frac{5}{6} \end{aligned} \quad (1.4)$$

To find (1.1) we use the below equation,

$$\begin{aligned}
 \Pr((X = 0) \cap (Y = 0)) &= 1 - \left[\Pr((X = 1) \cup (Y = 1)) \right] \\
 &= 1 - \left[\Pr(X = 1) + \Pr(Y = 1) \right. \\
 &\quad \left. - \Pr((X = 1) \cap (Y = 1)) \right] \\
 &= 1 - \left[\frac{1}{4} + \frac{1}{6} - \frac{1}{12} \right] \\
 &= 1 - \frac{1}{3} \\
 &= \frac{2}{3}
 \end{aligned} \tag{1.5}$$

Using (1.5) and (1.4), we solve equation (1.1),

$$\begin{aligned}
 \Pr(X = 0 \mid Y = 0) &= \frac{\Pr((X = 0) \cap (Y = 0))}{\Pr(Y = 0)} \\
 \Pr(X = 0 \mid Y = 0) &= \frac{\frac{2}{3}}{\frac{5}{6}} = \frac{4}{5}
 \end{aligned} \tag{1.6}$$