

AI5002 - Assignment 9

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Download code and LaTeX from below hyperlinks

1. [Codes/AxiomProb_6_20.py](#)
2. [LaTeX](#)

Problem 6.20

An unbiased die is thrown twice. Let the event A be 'odd number on the first throw' and B be event 'odd number on the second throw'. Check the independence of the events A and B.

Solution

We know two events are said to be independent if $P(A \cap B) = P(A) \cdot P(B)$

Let us define the random variable S as 'Throwing a unbiased dice twice'. The sample space of random variable S is given by -

$S = \{$
 (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6),
 (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6),
 (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6),
 (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6),
 (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6),
 (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) $\}$

We define two random variables X and Y where X denotes 'Throwing an odd number on first throw' and Y denotes 'Throwing an odd number on second throw'.

The sample space of X is given by -

$X = \{$
 (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6),
 (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6),
 (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) $\}$

Probability of odd number on the first throw

$$P(X) = \frac{n(X)}{n(S)} = \frac{18}{36} = \frac{1}{2} \quad (0.0.1)$$

The sample space of random variable Y is given by -

$Y = \{$
 (1, 1) (1, 3) (1, 5),
 (2, 1) (2, 3) (2, 5),
 (3, 1) (3, 3) (3, 5),
 (4, 1) (4, 3) (4, 5),
 (5, 1) (5, 3) (5, 5),
 (6, 1) (6, 3) (6, 5) $\}$

Probability of odd number on the second throw

$$P(Y) = \frac{n(Y)}{n(S)} = \frac{18}{36} = \frac{1}{2} \quad (0.0.2)$$

We define one more random variable Z which is defined as 'Throwing odd numbers on both first and second throws' and the sample space is given by -

$Z = \{$
 (1, 1) (1, 3) (1, 5),
 (3, 1) (3, 3) (3, 5),
 (5, 1) (5, 3) (5, 5) $\}$

Probability of odd number on the first and second throw.

$$P(Z) = \frac{n(Z)}{n(S)} = \frac{9}{36} = \frac{1}{4} \quad (0.0.3)$$

$$\text{Also, } P(X) \cdot P(Y) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Since,

$$P(Z) = P(X) \cdot P(Y)$$

Therefore, X and Y are independent random variables.