

AI5002 - Assignment 1

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Download code and LaTeX from below hyper-links

1. [Codes/RayleighDist_CDF_PDF_Plot.py](#)
2. [LaTeX](#)

Problem 6.1.3

Plot the CDF and Pdf of

$$A = \sqrt{V} \quad (0.0.1)$$

Solution

We can write $A = \sqrt{V}$ as

$$A = \sqrt{X_1^2 + X_2^2} \quad (0.0.2)$$

since it is given $X_1 \sim N(0, 1)$, $X_2 \sim N(0, 1)$ and $V = X_1^2 + X_2^2$.

Since joint distribution is not mentioned so we assume X_1 and X_2 to be independent otherwise the distribution of A would be unknown.

By definition, the distribution of A is Chi with two degrees of freedom or Rayleigh.

The CDF and PDF plot is as shown in Fig 1.1 and Fig 1.2.

Problem 6.1.4

Find an expression for $F_A(x)$ using the definition. Plot this expression and compare with the result of problem 6.1.3

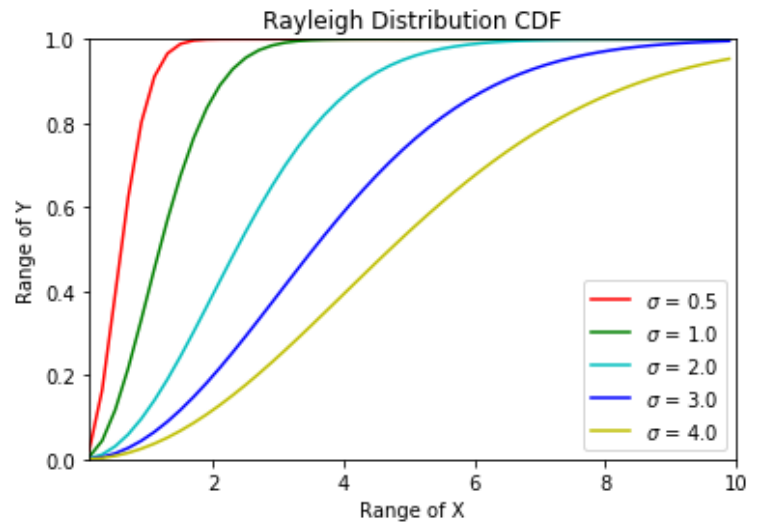


Fig 1.1: Cumulative distribution function

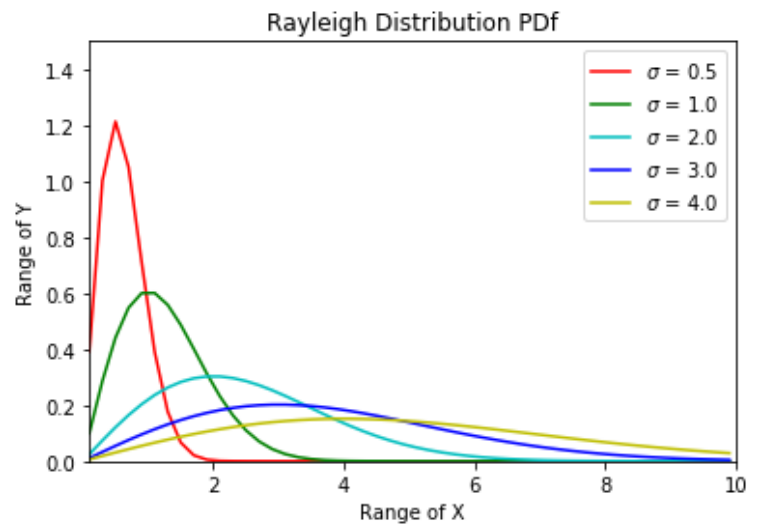


Fig 1.2: Probability density function

Solution

Given,

$$A = \sqrt{V} \quad (0.0.3)$$

$$F_A(x) = P (A \leq x) \quad (0.0.4)$$

$$F_A(x) = P (\sqrt{V} \leq x) \quad (0.0.5)$$

$$F_A(x) = P (V \leq x^2) \quad (0.0.6)$$

$$F_A(x) = F_V(x^2) \quad (0.0.7)$$

From (6.1.2.1) we get

$$F_V(x) = \begin{cases} 1 - e^{-\alpha x} & ; x \geq 0 \end{cases}$$

Now for x^2 , we substitute x^2 in place of x

$$F_V(x^2) = \begin{cases} 1 - e^{-\alpha x^2} & ; x^2 \geq 0 \end{cases} \quad (0.0.8)$$

$$\text{Putting } \alpha = \frac{1}{2\sigma^2} = \frac{1}{2} \quad [\because \sigma^2 \text{ is given as } 1]$$

We get,

$$F_V(x^2) = 1 - e^{-\frac{x^2}{2}} \quad ; x^2 \geq 0 \quad (0.0.9)$$

Thus the CDF is derived as

$$F_A(x) = F_V(x^2) = 1 - e^{-\frac{x^2}{2}} \quad ; x^2 \geq 0$$

The plot of this equation is shown in Fig 1.3.

Problem 6.1.5

Find an expression for $p_A(x)$ using the definition.

Solution

The PDF can be derived by differentiating the CDF expression from the previous problem 6.1.4

$$f_A(x) = f_V(x^2) \quad (0.0.10)$$

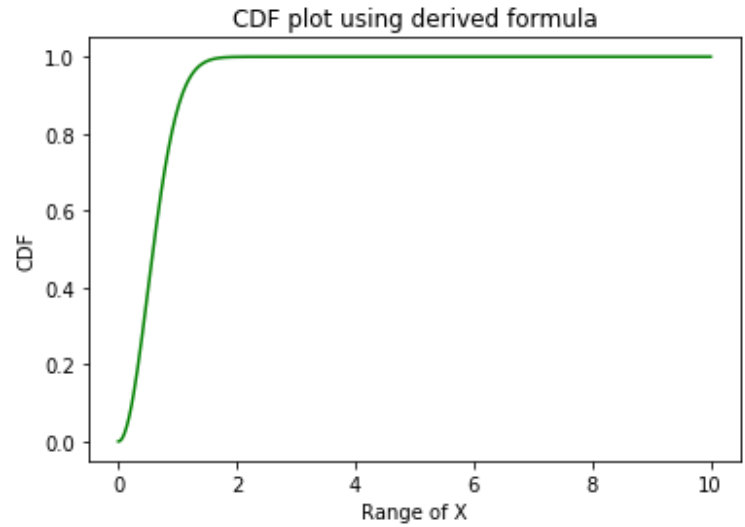


Fig 1.3: CDF plot from derived equation

$$f_V(x^2) = \frac{d}{dx}(F_V(x^2)) \quad (0.0.11)$$

$$f_V(x^2) = \frac{d}{dx}(1 - e^{-\frac{x^2}{2}}) \quad (0.0.12)$$

$$f_V(x^2) = e^{-\frac{x^2}{2}} . x \quad (0.0.13)$$

$$p_A(x) = f_A(x) = e^{-\frac{x^2}{2}} . x$$

The plot of this equation is shown in Fig 1.4.

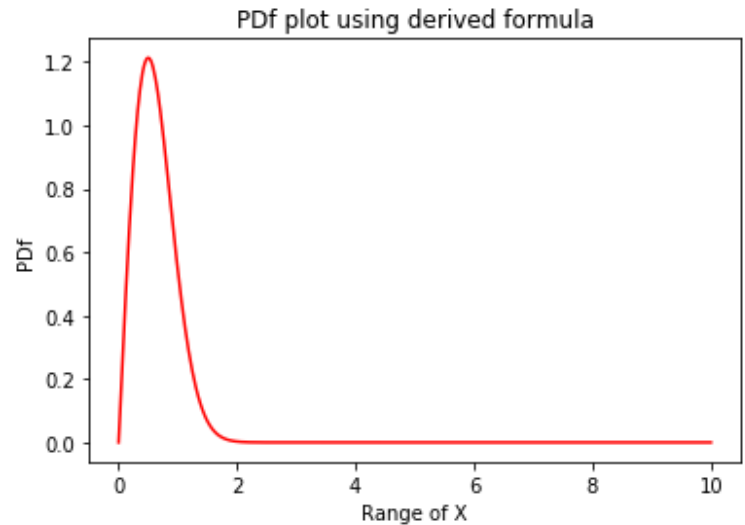


Fig 1.4: PDF plot from derived equation

The theoretical along with simulated PDF and CDF plot of 10,000 samples of Rayleigh random variables is shown from Fig 1.5 through Fig 1.8 -

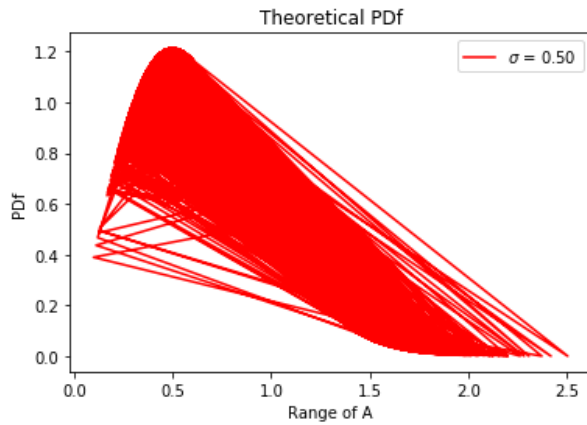


Fig 1.5: Theoretical PDF

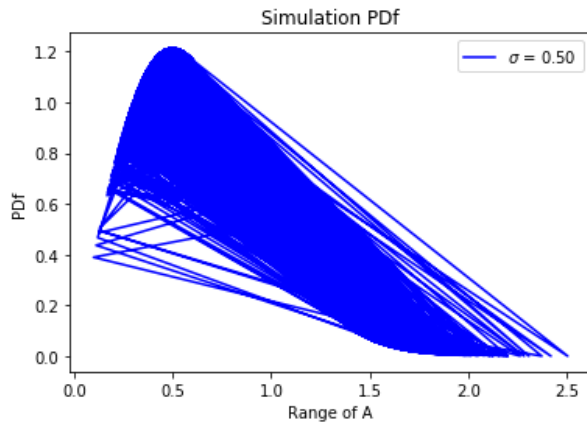


Fig 1.6: Simulation PDF

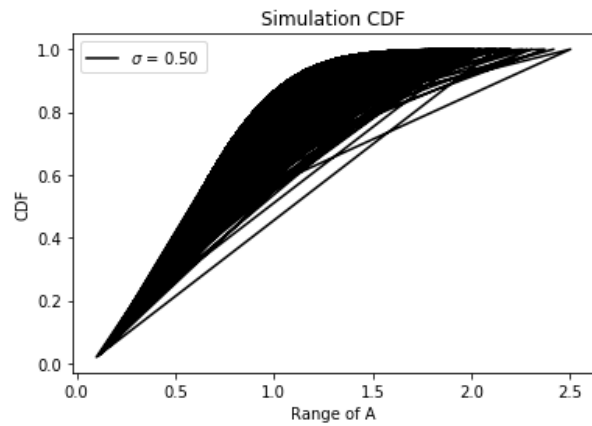


Fig 1.8: Simulation CDF

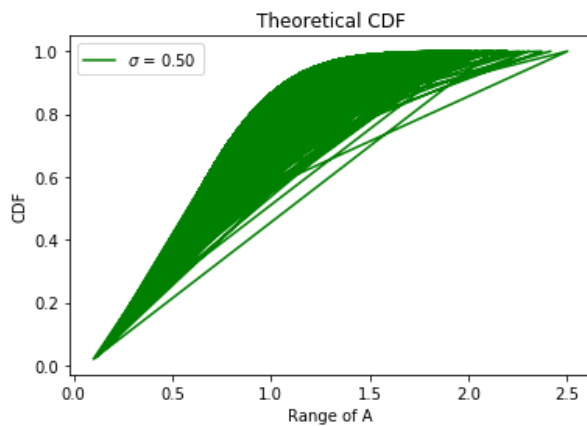


Fig 1.7: Theoretical CDF