AI5002 - Challenge Problem 9

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1. LaTeX

Problem IES_ISS_2015_stat1_Q3c

Two points are chosen on a line of unit length. Find the probability that each of the 3 line segments will have length greater than $\frac{1}{4}$?

Solution

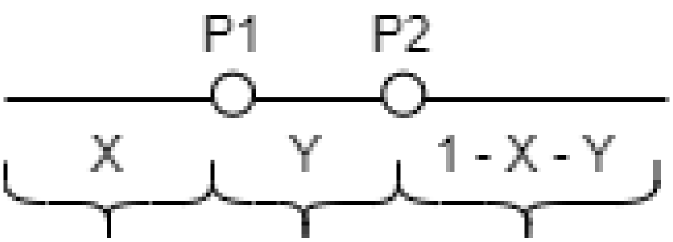

Fig 1: Line Segment

We can choose two points randomly say P_1, and P_2. Let the length of the first segment from 0 to P_1 be X, length of the second segment from P_1 to P_2 be Y, and length of the third segment from P_2 to 1 be $1 - X - Y$.
We want the length of each of the segments to be

$$X > \frac{1}{4} \tag{1}$$

$$Y > \frac{1}{4} \tag{2}$$

$$1 - X - Y > \frac{1}{4} \ or \ X + Y < \frac{3}{4} \tag{3}$$

Let us plot these three inequalities (1), (2), and (3) in the X-Y plane as shown in Fig 2. From figure 2 we see that all three inequalities are satisfied in the highlighted area marked by A, B and C.

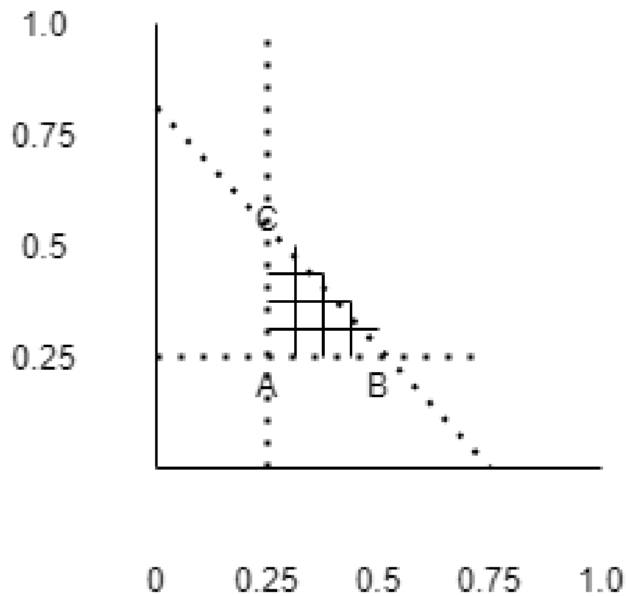

Fig 2: Favourable and Total Area

$\therefore$

$$\begin{aligned} Favourable\ Area &= \frac{1}{2} \cdot \text{BASE} \cdot \text{HEIGHT} \\ &= \frac{1}{2} \cdot (B - A) \cdot (C - A) \\ &= \frac{1}{2} \cdot (\frac{1}{2} - \frac{1}{4}) \cdot (\frac{1}{2} - \frac{1}{4}) \\ &= \frac{1}{32} \end{aligned} \tag{4}$$

Now to calculate the sample space or total area we have is the triangle with points at (0, 0), (0,1), and (1, 0).
$\therefore$

$$\begin{aligned} Total\ Area &= \frac{1}{2} \cdot \text{BASE} \cdot \text{HEIGHT} \\ &= \frac{1}{2} \cdot (1 - 0) \cdot (1 - 0) \\ &= \frac{1}{2} \end{aligned} \tag{5}$$

$\therefore$

$$\Pr\left(X > \frac{1}{4}\ and\ Y > \frac{1}{4} and\ X + Y < \frac{3}{4}\right) = \frac{\text{Favourable Area}}{\text{Total Area}} = \frac{1/32}{1/2} = \frac{1}{16} \tag{6}$$