

Solutions to the Homework Problems

1. Find the eccentricity and latus rectum of the ellipse
 $3x^2 + 4y^2 + 6x - 8y - 5 = 0$.

Solution:

Given,

$$3x^2 + 4y^2 + 6x - 8y - 5 = 0 \dots\dots\dots(1)$$

Putting $x = x + \alpha$ and $y = y + \beta$ in equation (1),

$$3(x + \alpha)^2 + 4(y + \beta)^2 + 6(x + \alpha) - 8(y + \beta) - 5 = 0$$

$$\Rightarrow 3(x^2 + 2x\alpha + \alpha^2) + 4(y^2 + 2y\beta + \beta^2) + 6(x + \alpha) - 8(y + \beta) - 5 = 0$$

$$\Rightarrow 3x^2 + 4y^2 + x(6\alpha + 6) + y(8\beta - 8) + 3\alpha^2 + 4\beta^2 + 6\alpha - 8\beta - 5 = 0 \dots\dots\dots(2)$$

The terms of x and y in equation (2) will be absent if

$$6\alpha + 6 = 0$$

And $8\beta - 8 = 0$

Solving these two equations we get,

$$\therefore \alpha = -1 \text{ and } \beta = 1$$

Putting $\alpha = -1$ and $\beta = 1$ in equation (2),

$$3x^2 + 4y^2 - 12 = 0$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1 \dots \dots \dots (3)$$

where $a = 2$ and $b = \sqrt{3}$

so the latus rectum is $2 \frac{b^2}{a} = 3$

and the eccentricity is $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}$

2. Find the equation of the ellipse whose latus rectum is 5 and eccentricity is $\frac{2}{3}$.

Solution:

Let the equation of the ellipse be of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots(1)$$

Now, the latus rectum is $2 \frac{b^2}{a} = 5$

$$\Rightarrow 2b^2 = 5a \dots\dots\dots(2)$$

and the eccentricity is $e = \sqrt{1 - \frac{b^2}{a^2}}$

$$\Rightarrow \frac{4}{9} = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \frac{5}{9} = \frac{b^2}{a^2}$$

$$\Rightarrow 9b^2 = 5a^2 \dots\dots\dots(3)$$

From (2) and (3) we get,

$$\frac{5a}{2} = \frac{5a^2}{9} \text{ [Equating the value of } b^2]$$

$$\Rightarrow \frac{1}{2} = \frac{a}{9}$$

$$\Rightarrow a = \frac{9}{2}$$

$$\text{So, } a^2 = \frac{81}{4}$$

Now putting the value of a^2 in equation (3), we get

$$b^2 = \frac{45}{4}$$

From (1),

$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

3. Find the equations of the tangent to the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ which is parallel to the line $y = 2x + 1$.

Solution:

Consider, the required equation of the tangent is

$$y = mx + c \dots\dots\dots(1)$$

Given, $y = 2x + 1$

So, $m = 2$

From (1),

$$y = 2x + c \dots\dots\dots(2)$$

Since equation (2) touches the ellipse,

$$\frac{x^2}{8} + \frac{y^2}{4} = 1$$

$$\therefore c = \pm \sqrt{a^2 m^2 + b^2}$$

$$\Rightarrow c = \pm \sqrt{32 + 4}$$

$$\Rightarrow c = \pm 6$$

From (2) ,

$$y = 2x + 6$$

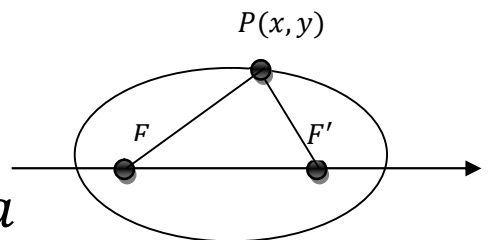
$$\text{and } y = 2x - 6$$

4. Find the equation of the ellipse whose foci are $(1, 0)$, $(0, 0)$ and length of the major axis is 2.

Solution:

We have

$$FP + F'P = 2a$$



$$\Rightarrow \sqrt{(x - 1)^2 + y^2} + \sqrt{x^2 + y^2} = 2$$

$$\Rightarrow \sqrt{x^2 - 2x + 1 + y^2} = 2 - \sqrt{x^2 + y^2}$$

$$\Rightarrow x^2 - 2x + 1 + y^2 = 4 - 4\sqrt{x^2 + y^2} + x^2 + y^2$$

$$\Rightarrow -2x - 3 = -4\sqrt{x^2 + y^2}$$

$$\Rightarrow 2x + 3 = 4\sqrt{x^2 + y^2}$$

$$\Rightarrow 4x^2 + 12x + 9 = 16x^2 + 16y^2$$

$$\Rightarrow 12x^2 + 16y^2 - 12x - 9 = 0$$