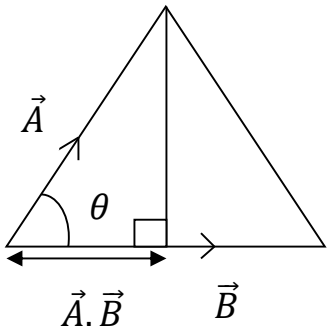
Topic 8: Vector Algebra

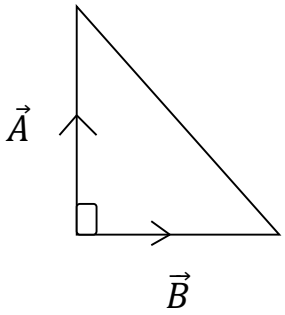
(a) Physical Significance of Dot product

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|cos\theta$$

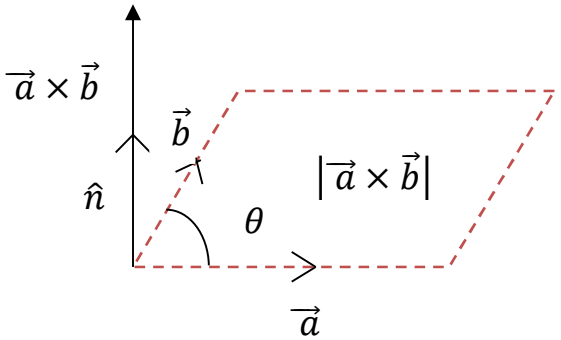
$\vec{A}.\vec{B} \rightarrow$ Projection of $\vec{A}$ on $\vec{B}$



Note: $\vec{A}.\vec{B} = 0$ represents that, $\vec{A}$ and $\vec{B}$ are perpendicular to each other.



(b) Physical Significance of Cross product

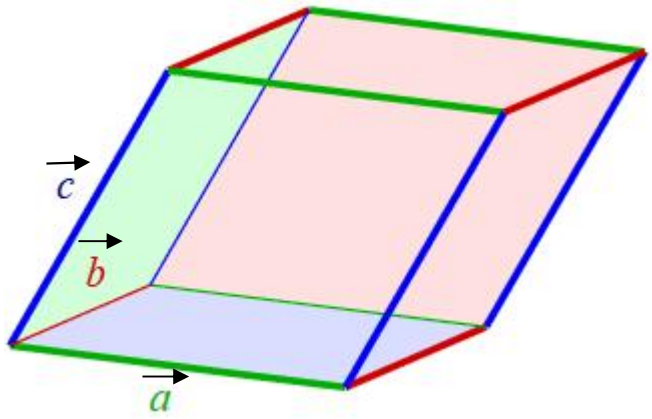


$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|sin\theta\ \hat{n}$$

$|\vec{a} \times \vec{b}|$ → area of a parallelogram

Note: $\vec{a} \times \vec{b} = \vec{0}$ represents that, $\vec{a}$ and $\vec{b}$ are parallel to each other.

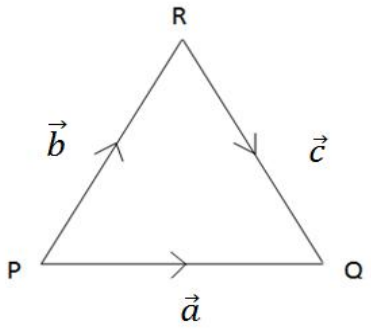
(c) Physical Significance of Box product



$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left|\vec{a} \cdot (\vec{b} \times \vec{c})\right|$$

$\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ → volume of a parallelepiped

(d) Triangle Law for Vectors



$$\vec{a} = \vec{b} + \vec{c}$$

Solved Problems

1. Show that the vectors $\vec{A} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{B} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$, $\vec{C} = 2\hat{\imath} + \hat{\jmath} - 4\hat{K}$ form a triangle. Determine whether ΔABC is right-angled or not.

Solution:

Given,

$$\vec{A} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$$

$$\vec{B} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$$

$$\vec{C} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$$

1st part:

$$\vec{A} = \vec{B} + \vec{C}$$

$$\Rightarrow \quad 3\hat{\imath} - 2\hat{\jmath} + \hat{k} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$$

$\therefore$ The vectors $\vec{A}$, $\vec{B}$ and $\overrightarrow{C}$ form a triangle .

2nd part:

$$\vec{A}.\vec{B} = 14$$

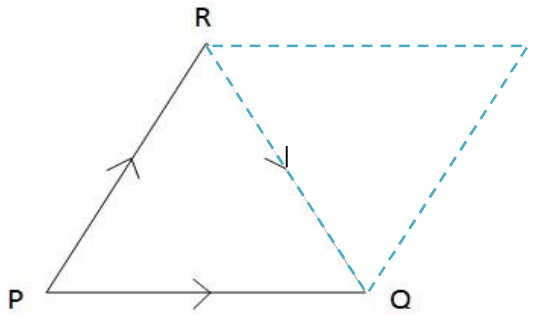
$$\vec{B}.\vec{C} = -21$$

$$\vec{C}.\vec{A} = 0$$

So, ΔABC is a right-angled triangle.

2. Find the area of the triangle having vertices at $P(1, 3, 2)$, $Q(2, -1, 1)$, $R(-1, 2\ 3)$.

Solution:



Now

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= \hat{\imath} - 4\hat{\jmath} - \hat{k}$$

$$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$$

$$= -2\hat{\imath} - \hat{\jmath} + \hat{k}$$

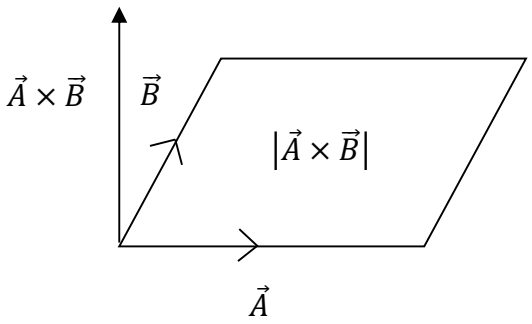
$$\therefore \overrightarrow{PQ} \times \overrightarrow{PR} = -5\hat{\imath} + \hat{\jmath} - 9\hat{k}$$

∴ Required area of the triangle

$$= \frac{1}{2} \times |\overrightarrow{PQ} \times \overrightarrow{PR}| = {}^{\sqrt{107}}/_{2}\ unit^2$$

3. Determine a unit vector perpendicular to the plane of $\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

Solution:



$$\vec{A} \times \vec{B} = 15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}$$

$$\left|\vec{A} \times \vec{B}\right| = 35$$

$$\text{Required unit vector} = \frac{\vec{A} \times \vec{B}}{\left|\vec{A} \times \vec{B}\right|}$$

$$= \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$$

Homework Problems

1. Find the volume of the parallelepiped whose edges are represented by $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, $\vec{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

2. The position vectors of A, B, C and D are $2\hat{\imath} + 4\hat{k}$, $5\hat{\imath} + 3\sqrt{3}\hat{\jmath} + 4\hat{k}$, $-2\sqrt{3}\hat{\jmath} + \hat{k}$ and $2\hat{\imath} + \hat{k}$ respectively. Show that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel and $CD = \frac{2}{3}AB$.

3. Find the angles α, β, γ, which the vector $\vec{A} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ makes with the coordinates axes and also show that $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$.

4. If the position vectors of the three points A, B and C are (2, 4, -1), (1, 2, -3) and (3, 1, 2) respectively. Find a vector perpendicular to the plane ABC.