Solutions to the Homework Problems

1. Transform the equation $11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0$ to rectangular axes through the point $(2, -1)$ and inclined at an angle $\tan^{-1}\left(-\frac{4}{3}\right)$.

Solution:

Given,

$11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0$ ……..(1)

Putting $x = x + 2$ and $y = y - 1$ in equation (1),

$$11(x+2)^2 + 24(x+2)(y-1) + 4(y-1)^2 - 20(x+2) - 40(y-1) - 5 = 0$$

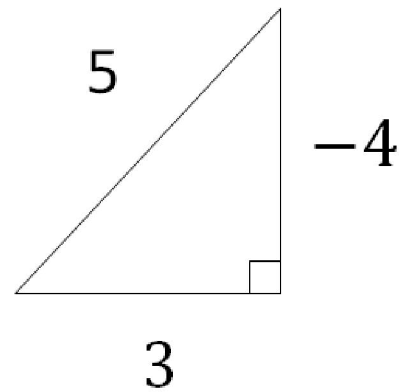
$$\Rightarrow 11(x^2 + 4x + 4) + 24(xy - x + 2y - 2) + 4(y^2 - 2y + 1) - 20(x+2) - 40(y-1) - 5 = 0$$

$$\Rightarrow 11x^2 + 24xy + 4y^2 + x(44 - 24 - 20) + y(48 - 8 - 40) + 44 - 48 + 4 - 40 + 40 - 5 = 0$$

$\Rightarrow 11x^2 + 24xy + 4y^2 - 5 = 0$ ……………(2)

Now,

$$\theta = \tan^{-1}\left(-\frac{4}{3}\right)$$



$$\Rightarrow \quad tan\theta = -\frac{4}{3}$$

$$\therefore sin\theta = -\frac{4}{5} \text{ and } cos\theta = \frac{3}{5}$$

We know,

$$x = xcos\theta - ysin\theta$$

$$\therefore x = \frac{3x}{5} + \frac{4y}{5}$$

And $y = xsin\theta + ycos\theta$

$$\therefore y = \frac{3y}{5} - \frac{4x}{5}$$

Putting $x = \frac{3x}{5} + \frac{4y}{5}$ and $y = \frac{3y}{5} - \frac{4x}{5}$ in equation (2),

$$11\left(\frac{3x}{5}+\frac{4y}{5}\right)^2+24\left(\frac{3x}{5}+\frac{4y}{5}\right)\left(\frac{3y}{5}-\frac{4x}{5}\right)+4\left(\frac{3y}{5}-\frac{4x}{5}\right)^2-5=0$$

$$\Rightarrow \quad 11\left(\frac{9x^2}{25}+\frac{16y^2}{25}+\frac{24xy}{25}\right)+24\left(\frac{9xy}{25}-\frac{12x^2}{25}+\frac{12y^2}{25}-\frac{16xy}{25}\right)+4\left(\frac{9y^2}{25}-\frac{24xy}{25}+\frac{16x^2}{25}\right)-5=0$$

$$\Rightarrow \quad x^2\left(\frac{99}{25}-\frac{288}{25}+\frac{64}{25}\right)+xy\left(\frac{264}{25}+\frac{216}{25}-\frac{384}{25}-\frac{96}{25}\right)+y^2\left(\frac{176}{25}+\frac{288}{25}+\frac{36}{25}\right)-5=0$$

$$\Rightarrow \quad -5x^2+20y^2-5=0$$

$$\Rightarrow \quad x^2-4y^2+1=0$$

This is our required transformed equation.

2. Through what angle must the axes be rotated to remove the term containing xy in $11x^2 + 4xy + 14y^2 = 5$? Find the transformed equation.

<u>Solution:</u>

Given,

$$11x^2 + 4xy + 14y^2 = 5 \text{ ....(1)}$$

Putting $x = xcos\theta - ysin\theta$ and $y = xsin\theta + ycos\theta$ in equation (1),

$11(xcos\theta - ysin\theta)^2 + 4(xcos\theta - ysin\theta)(xsin\theta + ycos\theta) + 14(xsin\theta + ycos\theta)^2 - 5 = 0$

⇨ $11(x^2\cos^2\theta - 2xysin\theta cos\theta + y^2\sin^2\theta) + 4(x^2sin\theta cos\theta + xycos^2\theta - xysin^2\theta - y^2sin\theta cos\theta) + 14(x^2\sin^2\theta + 2xysin\theta cos\theta + y^2\cos^2\theta) - 5 = 0$

⇨ $x^2(11\cos^2\theta + 4sin\theta cos\theta + 14\sin^2\theta) + xy(-22sin\theta cos\theta + 4\cos^2\theta - 4\sin^2\theta + 28sin\theta cos\theta) + y^2(11\sin^2\theta - 4sin\theta cos\theta + 14\cos^2\theta) - 5 = 0$ ....(2)

To remove the xy term in equation (2), we can write,

$$-22sin\theta cos\theta + 4\cos^2\theta - 4\sin^2\theta + 28sin\theta cos\theta = 0$$

⇨ $$4\cos^2\theta + 6sin\theta cos\theta - 4\sin^2\theta = 0$$

⇨ $$4\cos^2\theta + 8sin\theta cos\theta - 2sin\theta cos\theta - 4\sin^2\theta = 0$$

⇨ $$4cos\theta(cos\theta + 2sin\theta) - 2sin\theta(cos\theta + 2sin\theta) = 0$$

⇨ $$(cos\theta + 2sin\theta)(4cos\theta - 2sin\theta) = 0$$

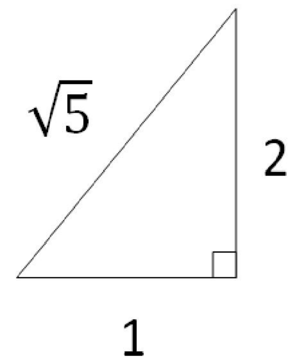
$$\therefore (4cos\theta - 2sin\theta) = 0$$

⇨ $$tan\theta = 2$$

Or, $(cos\theta + 2sin\theta) = 0$

⇨ $$tan\theta = -\frac{1}{2}$$

when $tan\theta = 2$



$\therefore\ sin\theta = \frac{2}{\sqrt{5}}$ and $cos\theta = \frac{1}{\sqrt{5}}$

Putting $sin\theta = \frac{2}{\sqrt{5}}$ and $cos\theta = \frac{1}{\sqrt{5}}$ in equation (2),

$$x^2\left(11.\frac{1}{5} + 4\,.\frac{2}{\sqrt{5}}\,.\frac{1}{\sqrt{5}} + 14\left(\frac{2}{\sqrt{5}}\right)^2\right) +$$
$$xy\left(-22.\frac{2}{\sqrt{5}}\,.\frac{1}{\sqrt{5}} + 4.\frac{1}{5} - 4.\frac{4}{5} + 28\,\frac{2}{\sqrt{5}}\,.\frac{1}{\sqrt{5}}\right) +$$
$$y^2\left(11.\frac{4}{5} - 4\frac{2}{\sqrt{5}}\,.\frac{1}{\sqrt{5}} + 14.\frac{1}{5}\right) - 5 = 0$$

⇨ $15x^2 + 10y^2 - 5 = 0$

⇨ $3x^2 + 2y^2 - 1 = 0$

⇨ $3x^2 + 2y^2 = 1$

$\therefore$ The transformed equation is, $3x^2 + 2y^2 = 1$ after rotating the coordinate axes through an angle, $\theta = \tan^{-1} 2$.

3. Determine the equation of the parabola $x^2 - 2xy + y^2 + 2x - 4y + 3 = 0$ after rotating of axes through $45°$.

Solution:

Given,

$x^2 - 2xy + y^2 + 2x - 4y + 3 = 0$ ....(1)

and , $\theta = 45°$

We have,

$$x = xcos\theta - ysin\theta$$

⇨ $x = xcos\, 45° - ysin\, 45°$

$$\therefore x = \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$$

And $y = xsin\theta + ycos\theta$

⇨ $y = xsin\, 45° + ycos\, 45°$

$$\therefore y = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$$

Putting $x = \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$ and $y = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$ in equation (1),

$$\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)^2 - 2\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)\left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right) + \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)^2 + 2\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right) - 4\left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right) + 3 = 0$$

$$\Rightarrow \left(\frac{x^2}{2} - xy + \frac{y^2}{2}\right) - 2\left(\frac{x^2}{2} + \frac{xy}{2} - \frac{xy}{2} - \frac{y^2}{2}\right) + \left(\frac{x^2}{2} + xy + \frac{y^2}{2}\right) + 2\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right) - 4\left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right) + 3 = 0$$

$$\Rightarrow x^2\left(\frac{1}{2} - 1 + \frac{1}{2}\right) + xy(-1 - 1 + 1 + 1) + y^2\left(\frac{1}{2} + 1 + \frac{1}{2}\right) + x\left(\sqrt{2} - 2\sqrt{2}\right) + y\left(-\sqrt{2} - 2\sqrt{2}\right) + 3 = 0$$

$$\Rightarrow 2y^2 - \sqrt{2}x - 3\sqrt{2}y + 3 = 0$$

This is our required transformed equation.

4. Verify that when the axes are turned through an angle $\frac{\pi}{4}$, the equation $5x^2 + 4xy + 5y^2 - 10 = 0$ transforms to one in which the term xy is absent.

Solution:

Given,

$5x^2 + 4xy + 5y^2 - 10 = 0$ ……………………………(1)

and ,

$$\theta = \pi/4$$

We have,

$$x = x\cos\theta - y\sin\theta$$

$$\Rightarrow \quad x = x\cos 45° - y\sin 45°$$

$$\therefore x = \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$$

and $\quad y = x\sin\theta + y\cos\theta$

$$\Rightarrow \quad y = x\sin 45° + y\cos 45°$$

$$\therefore y = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$$

Putting $x = \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$ and $y = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$ in equation (1),

$$5\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)^2 + 4\left(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\right)\left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right) + 5\left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)^2 - 10 = 0$$

⇨ $$5\left(\frac{x^2}{2} - xy + \frac{y^2}{2}\right) + 4\left(\frac{x^2}{2} + \frac{xy}{2} - \frac{xy}{2} - \frac{y^2}{2}\right) + 5\left(\frac{x^2}{2} + xy + \frac{y^2}{2}\right) - 10 = 0$$

⇨ $$x^2\left(\frac{5}{2} + 2 + \frac{5}{2}\right) + xy(-5 + 2 - 2 + 5) + y^2\left(\frac{5}{2} - 2 + \frac{5}{2}\right) - 10 = 0$$

⇨ $$7x^2 + 3y^2 - 10 = 0$$

Hence, the given statement is true.

5. Transform the equation $17x^2 + 18xy - 7y^2 - 16x - 32y - 18 = 0$ to one in which there is no term involving x, y and xy.

Solution:

Given,

$17x^2 + 18xy - 7y^2 - 16x - 32y - 18 = 0$ ……(1)

Putting $x = x + \alpha$ and $y = y + \beta$ in equation (1),

$$17(x+\alpha)^2 + 18(x+\alpha)(y+\beta) - 7(y+\beta)^2 - 16(x+\alpha) - 32(y+\beta) - 18 = 0$$

$\Rightarrow$ $17(x^2 + 2x\alpha + \alpha^2) + 18(xy + x\beta + y\alpha + \alpha\beta) - 7(y^2 + 2y\beta + \beta^2) - 16(x+\alpha) - 32\text{-}(y + \beta\ ) - 18 = 0$

$\Rightarrow$ $17x^2 + 18xy - 7y^2 + x(34\alpha + 18\beta - 16) + y(18\alpha - 14\beta - 32) + 17\alpha^2 + 18\alpha\beta - 7\beta^2 - 16\alpha - 32\beta - 18 = 0$ ……………………(2)

The terms of x and y in equation (2) will be absent if

$$34\alpha + 18\beta - 16 = 0$$

And $\quad 18\alpha - 14\beta - 32 = 0$

Solving these two equations, we get

$$\therefore \alpha = 1 \text{ and } \beta = -1$$

Putting $\alpha = 1$ and $\beta = -1$ in equation (2),

$$17x^2 + 18xy - 7y^2 + 17 + 18(-1) - 7 - 16 + 32 - 18 = 0$$

⇨ $\quad 17x^2 + 18xy - 7y^2 - 10 = 0$ ....(3)

Now Putting $x = xcos\theta - ysin\theta$ and $y = xsin\theta + ycos\theta$ in equation (3),

$$17(xcos\theta - ysin\theta)^2 + 18(xcos\theta - ysin\theta)(xsin\theta + ycos\theta) - 7(xsin\theta + ycos\theta)^2 - 10 = 0$$

⇨ $\quad 17(x^2\cos^2\theta - 2xysin\theta cos\theta + y^2\sin^2\theta) + 18(x^2 sin\theta cos\theta + xycos^2\theta - xysin^2\theta - y^2 sin\theta cos\theta) - 7(x^2\sin^2\theta + 2xysin\theta cos\theta + y^2\cos^2\theta) - 10 = 0$

$\Rightarrow x^2(17\cos^2\theta + 18sin\theta cos\theta - 7\sin^2\theta) + xy(-34sin\theta cos\theta + 18\cos^2\theta - 18\sin^2\theta - 14sin\theta cos\theta) + y^2(17\sin^2\theta - 18sin\theta cos\theta - 7\cos^2\theta) - 10 = 0$ ....(4)

To remove the xy term in equation (4), we can write,

$$-34sin\theta cos\theta + 18\cos^2\theta - 18\sin^2\theta - 14sin\theta cos\theta = 0$$

$$\Rightarrow \quad 18\cos^2\theta - 48sin\theta cos\theta - 18\sin^2\theta = 0$$

$$\Rightarrow \quad 3\cos^2\theta - 8sin\theta cos\theta - 3\sin^2\theta = 0$$

$$\Rightarrow \quad 3\cos^2\theta - 9sin\theta cos\theta + sin\theta cos\theta - 3\sin^2\theta = 0$$

$$\Rightarrow \quad 3cos\theta(cos\theta - 3sin\theta) + sin\theta(cos\theta - 3sin\theta) = 0$$

$$\Rightarrow \quad (cos\theta - 3sin\theta)(3cos\theta + sin\theta) = 0$$

$$\therefore (3cos\theta + sin\theta) = 0$$

$$\Rightarrow \quad tan\theta = -3$$

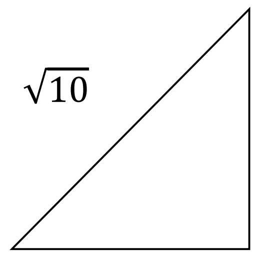
Or, $(cos\theta - 3sin\theta) = 0$

$\Rightarrow \quad tan\theta = \frac{1}{3}$

When, $tan\theta = -3$

$\therefore sin\theta = -\frac{3}{\sqrt{10}}$

and $cos\theta = \frac{1}{\sqrt{10}}$



Putting $sin\theta = -\frac{3}{\sqrt{10}}$ and $cos\theta = \frac{1}{\sqrt{10}}$ in equation (4),

$$x^2\left(17.\frac{1}{10} + 18\left(-\frac{3}{\sqrt{10}}\right)\frac{1}{\sqrt{10}} - 7.\frac{9}{10}\right) + xy\left(-34\left(-\frac{3}{\sqrt{10}}\right)\frac{1}{\sqrt{10}} + 18.\frac{1}{10} - 18.\frac{9}{10} - 14\left(-\frac{3}{\sqrt{10}}\right).\frac{1}{\sqrt{10}}\right) + y^2\left(17.\frac{9}{10} - 18\left(-\frac{3}{\sqrt{10}}\right).\frac{1}{\sqrt{10}} - 7.\frac{1}{10}\right) - 10 = 0$$

$\Rightarrow -10x^2 + 20y^2 - 10 = 0$

$\Rightarrow \quad x^2 - 2y^2 + 1 = 0$

This is our required transformed equation.

Important notes for the Identification of Curves

The general equation of second degree can be written as-

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{----(1)}$$

where a, b, c, f, g, h are constants.

Consider, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Now

(1) If $\Delta = 0$, then equation (1) represents a pair of straight lines.

(2) If $\Delta = 0$ and $a + b = 0$, then equation (1) represents a pair of perpendicular straight lines.

(3) If $\Delta = 0$ and $h^2 - ab = 0$, then equation (1) represents a pair of parallel straight lines.

(4) If $\Delta \neq 0$ and $h^2 - ab = 0$, then equation (1) represents a parabola.

(5) If $\Delta \neq 0$ and $h^2 - ab < 0$, then equation (1) represents an Ellipse.

(6) If $\Delta \neq 0$ and $h^2 - ab > 0$, then equation (1) represents a Hyperbola.

(7) If $\Delta \neq 0$, $h = 0$ and $a = b$, then equation (1) represents a Circle.

Topic 2: Pair of Straight Lines

Important Things:

1. Verification
2. The Intersection between two straight lines
3. Angle
4. Parallelism
5. Perpendicularity
6. Equation of the bisectors of the angles
7. Distance

Important Formulas:

The general equation of second degree can be written as-

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{----(1)}$$

where a, b, c, f, g, h are constants.

1. The angle between the two straight lines represented by (1) is given by

$$tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

2. If equation (1) represents two straight lines and let (α, β) be the point of intersection of these straight lines, then the equation of the bisectors of the angles is given by

$$\frac{(x-\alpha)^2 - (y-\beta)^2}{a - b} = \frac{(x-\alpha)(y-\beta)}{h}$$

3. The distance between two straight lines represented by (1) is given by

$$d = 2\sqrt{\frac{g^2 - ac}{a(a + b)}}$$

Solved Problems:

1. Show that the equation $2x^2 + 7xy + 3y^2 + 8x + 14y + 8 = 0$ represents a pair of straight lines and find them. Find also their point of intersection.

Solution:

Given,

$2x^2 + 7xy + 3y^2 + 8x + 14y + 8 = 0$ …….(1)

Here,

$$a = 2$$

$$h = \frac{7}{2}$$

$$b = 3$$

$$g = 4$$

$$f = 7$$

$c = 8$

Now,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

⇨ $\Delta = 2.3.8 + 2.7.4.\left(\frac{7}{2}\right) - 2.49 - 3.16 - 8.\frac{49}{4}$

⇨ $\Delta = 0$

∴ The equation (1) represents a pair of straight lines.

Then,

$$2x^2 + 7xy + 3y^2 + 8x + 14y + 8 = 0$$

⇨ $2x^2 + (7y + 8)x + (3y^2 + 14y + 8) = 0$

⇨ $x = \frac{-(7y+8)\pm\sqrt{(7y+8)^2-4.2(3y^2+14y+8)}}{2.2}$

⇨ $x = \frac{-(7y+8)\pm\sqrt{49y^2+112y+64-24y^2-112y-64}}{4}$

$$\Rightarrow \quad x = \frac{-(7y+8) \pm \sqrt{25y^2}}{4}$$

$$\Rightarrow \quad x = \frac{-(7y+8) \pm 5y}{4}$$

$$\therefore 4x = -7y - 8 + 5y$$

$$\Rightarrow \quad 4x + 2y + 8 = 0$$

$$\Rightarrow \quad 2x + y + 4 = 0 \quad \dots\dots\dots\dots\dots\dots(2)$$

Or,

$$4x = -7y - 8 - 5y$$

$$\Rightarrow \quad 4x + 12y + 8 = 0$$

$$\Rightarrow \quad x + 3y + 2 = 0 \quad \dots\dots\dots\dots\dots\dots(3)$$

Solving (2) and (3), we get

$\therefore x = -2$ and $y = 0$

So, the point of intersection is,

$$(x, y) = (-2,0)$$

2. Prove that the equation $2x^2 + xy - y^2 - x - 7y - 10 = 0$ represents a pair of straight lines. Then find their point of intersection and the equation of the bisectors of the angles between them.

<u>Solution:</u>

Given,

$$2x^2 + xy - y^2 - x - 7y - 10 = 0 \quad \text{..........(1)}$$

Here,

$$a = 2$$

$$h = \frac{1}{2}$$

$$b = -1$$

$$g = -\frac{1}{2}$$

$$f = -\frac{7}{2}$$

$$c = -10$$

Now,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$\Rightarrow \quad \Delta = 2.(-1).(-10) + 2.\left(-\frac{7}{2}\right).\left(-\frac{1}{2}\right).\frac{1}{2} - 2.\frac{49}{4} - (-1).\frac{1}{4} - (-10).\frac{1}{4}$

$\Rightarrow \quad \Delta = 0$

$\therefore$ The equation (1) represents a pair of straight lines.

Then,

$$2x^2 + xy - y^2 - x - 7y - 10 = 0$$

$\Rightarrow \quad 2x^2 + (y-1)x - (y^2 + 7y + 10) = 0$

$\Rightarrow \quad x = \dfrac{-(y-1) \pm \sqrt{(y-1)^2 + 4.2(y^2+7y+10)}}{2.2}$

$\Rightarrow \quad x = \dfrac{-(y-1) \pm \sqrt{y^2 - 2y + 1 + 8y^2 + 56y + 80}}{4}$

$$\Rightarrow \quad x = \frac{-(y-1) \pm \sqrt{9y^2+54y+81}}{4}$$

$$\Rightarrow \quad x = \frac{-(y-1) \pm \sqrt{9(y^2+6y+9)}}{4}$$

$$\Rightarrow \quad x = \frac{-(y-1) \pm 3\sqrt{(y+3)^2}}{4}$$

$$\Rightarrow \quad x = \frac{-(y-1) \pm 3\,(y+3)}{4}$$

$$\therefore 4x = -y + 1 + 3y + 9$$

$$\Rightarrow \quad 4x - 2y - 10 = 0$$

$$\Rightarrow \quad 2x - y - 5 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Or,

$$4x = -y + 1 - 3y - 9$$

$$\Rightarrow \quad 4x + 4y + 8 = 0$$

$$\Rightarrow \quad x + y + 2 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Solving (2) and (3), we get,

$$(x, y) = (1, -3)$$

∴ The point of intersection is,

$$(\alpha, \beta) = (1, -3)$$

Now the equation of the bisectors of the angles between two lines is given by,

$$\frac{(x-\alpha)^2-(y-\beta)^2}{a-b} = \frac{(x-\alpha)(y-\beta)}{h}$$

$$\Rightarrow \quad \frac{(x-1)^2-(y+3)^2}{2+1} = \frac{(x-1)(y+3)}{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{(x^2-2x+1)-(y^2+6y+9)}{3} = 2(xy + 3x - y - 3)$$

$$\Rightarrow \quad x^2 - 2x + 1 - y^2 - 6y - 9 = 6xy + 18x - 6y - 18$$

$$\Rightarrow \quad x^2 - y^2 - 2x - 6y - 8 - 6xy - 18x + 6y + 18 = 0$$

$\Rightarrow \quad x^2 - 6xy - y^2 - 20x + 10 = 0$

3. If the following second-degree equation $ax^2 + 6xy + 9y^2 + 2gx + 12y - 5 = 0$ represents two straight lines parallel to each other then find the values of a & g and also find the distance between them.

Solution:

Given,

$$ax^2 + 6xy + 9y^2 + 2gx + 12y - 5 = 0$$

Here,

$$h = 3$$

$$b = 9$$

$$f = 6$$

$$c = -5$$

Now,

$\Delta = 0$

⇨ $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

⇨ $a.9.(-5) + 2.6.g.3 - a.36 - 9g^2 - (-5).9 = 0$

⇨ $-45a + 36g - 36a - 9g^2 + 45 = 0$

⇨ $-81a - 9g^2 + 36g + 45 = 0$ ……….(1)

Then,

$h^2 - ab = 0$

⇨ $9 - a.9 = 0$

⇨ $9a = 9$

⇨ $a = 1$

Putting $a = 1$ in equation (1),

$$-81 - 9g^2 + 36g + 45 = 0$$

$$\Rightarrow \quad -9g^2 + 36g - 36 = 0$$

$$\Rightarrow \quad g^2 - 4g + 4 = 0$$

$$\therefore g = 2,2$$

Now the distance between two parallel straight lines is given by,

$$d = 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$$

$$\Rightarrow \quad d = 2\sqrt{\frac{4+5}{10}}$$

$$\Rightarrow \quad d = \frac{3\sqrt{10}}{5}$$

4. Find the joint equation of the lines passing through the point $(-1, 2)$ and perpendicular to the lines $x + 2y + 37 = 0$ and $3x - 4y - 53 = 0$.

Solution:

Since lines are perpendicular to the lines $x + 2y + 37 = 0$ and $3x - 4y - 53 = 0$, so the lines are

$$2x - y + a = 0 \quad \ldots\ldots\ldots\ldots(1)$$

$$4x + 3y + b = 0 \ldots\ldots\ldots\ldots\ldots(2)$$

Since (1) and (2) passes through through the the point $(-1, 2)$, so we get

$$a = 4 \text{ and } b = -2$$

The lines are $2x - y + 4 = 0$ and $4x + 3y - 2 = 0$

Now the joint equation is

$$(2x - y + 4)(4x + 3y - 2) = 0$$

Special Information

If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, then the corresponding equation $ax^2 + 2hxy + by^2 = 0$ always represents two straight lines passing through the origin.

Special Properties of Some Special Geometrical Shapes:

Name of the Geometrical Shapes	**Special Properties**
Parallelogram	Opposite sides are parallel.
Rectangle	1. **Parallelogram**. 2. All angles are at right angles. Or, Diagonals are equal in length.

Rhombus	1. **Parallelogram**. 2. All sides are equal Or, Diagonals are at right angles.
Square	1. **Parallelogram**. 2. **Rectangle.** 3. **Rhombus.**

Homework Problems

1. Find the value(s) of k so that the equation

$$3x^2 + 10xy + 8y^2 - kx - 26y + 21 = 0$$

represents a pair of straight lines and then find the angle between the lines.

2. Find the values of p and q if the equation

$$\mathrm{px}^2 - 8\mathrm{xy} + 3\mathrm{y}^2 + 14\mathrm{x} + 2\mathrm{y} + \mathrm{q} = 0$$

represents a pair of perpendicular lines.

3. Find the equation to the pair of straight lines through the origin perpendicular to the pair given by

$$2x^2 + 5xy + 2y^2 + 10x + 5y = 0$$

4. Show that the equation

$$x^2 + 4xy - 2y^2 + 6x - 12y - 15 = 0$$

represents a pair of straight lines which together with $x^2 + 4xy - 2y^2 = 0$ form a rhombus.