# *Solutions to the Homework Problems*

**1.** Find the direction cosines of the line which is equally inclined to the axes.

**Solution**:

Let the direction cosines of a line are $l, m, n$ where

$$\alpha = \beta = \gamma$$

That means, $l = m = n$

Let,

$$l = m = n = a$$

We know,

$$l^2 + m^2 + n^2 = 1$$

$\Rightarrow \quad 3a^2 = 1$

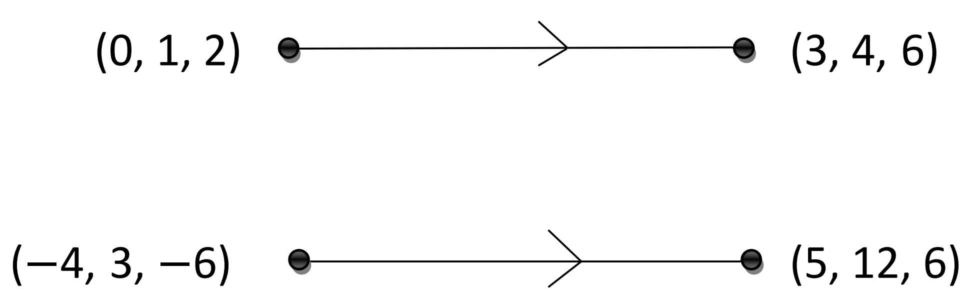
$\Rightarrow \quad a = \pm \frac{1}{\sqrt{3}}$

$$\therefore l = m = n = \pm \frac{1}{\sqrt{3}}$$

**2.** Show that the line joining the two points (0, 1, 2) and (3, 4, 6) is parallel to the line joining the two points (−4, 3, −6) and (5, 12, 6).

**Solution:**

We have,




$a_1 = 3$

$b_1 = 3$

$c_1 = 4$

$a_2 = 9$

$b_2 = 9$

$c_2 = 12$

For the first line,

$l_1 = \frac{3}{\sqrt{34}}$

$m_1 = \frac{3}{\sqrt{34}}$

$$n_1 = \frac{4}{\sqrt{34}}$$

For the second line,

$$l_2 = \frac{9}{\sqrt{306}}$$

$$m_2 = \frac{9}{\sqrt{306}}$$

$$n_2 = \frac{12}{\sqrt{306}}$$

Now we have,

$$l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$= \frac{3}{\sqrt{34}} \times \frac{9}{\sqrt{306}} + \frac{3}{\sqrt{34}} \times \frac{9}{\sqrt{306}} + \frac{4}{\sqrt{34}} \times \frac{12}{\sqrt{306}}$$

$$= 1$$

**3.** Show that the three lines whose direction cosines are proportional to $2, 1, 1$ ; $4, \sqrt{3}-1, -\sqrt{3}-1$ and $4, -\sqrt{3}-1, \sqrt{3}-1$ respectively are inclined to each other at an angle $\frac{\pi}{3}$ .

**Solution:**

Given,

$$a_1 = 2$$

$$b_1 = 1$$

$$c_1 = 1$$

$$a_2 = 4$$

$$b_2 = \sqrt{3} - 1$$

$$c_2 = -\sqrt{3} - 1$$

$$a_3 = 4$$

$$b_3 = -\sqrt{3} - 1$$

$$c_3 = \sqrt{3} - 1$$

For the first line,

$$l_1 = \frac{\sqrt{6}}{3}$$

$$m_1 = \frac{\sqrt{6}}{6}$$

$$n_1 = \frac{\sqrt{6}}{6}$$

For the second line,

$$l_2 = \frac{\sqrt{6}}{3}$$

$$m_2 = \frac{-\sqrt{6}+3\sqrt{2}}{12}$$

$$n_2 = -\frac{\sqrt{6}+3\sqrt{2}}{12}$$

For the third line,

$$l_3 = \frac{\sqrt{6}}{3}$$

$$m_3 = -\frac{\sqrt{6}+3\sqrt{2}}{12}$$

$$n_3 = \frac{-\sqrt{6}+3\sqrt{2}}{12}$$

Now, the angle between first line and second line is

$$cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\Rightarrow \quad cos\theta = \frac{\sqrt{6}}{3} \times \frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{6} \times \left(\frac{-\sqrt{6}+3\sqrt{2}}{12}\right) + \frac{\sqrt{6}}{6} \times \left(-\frac{\sqrt{6}+3\sqrt{2}}{12}\right)$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

The angle between first line and third line is,

$$cos\theta = l_1 l_3 + m_1 m_3 + n_1 n_3$$

$\Rightarrow \quad cos\theta = \frac{\sqrt{6}}{3} \times \frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{6} \times \left(-\frac{\sqrt{6}+3\sqrt{2}}{12}\right) + \frac{\sqrt{6}}{6} \times \left(\frac{-\sqrt{6}+3\sqrt{2}}{12}\right)$

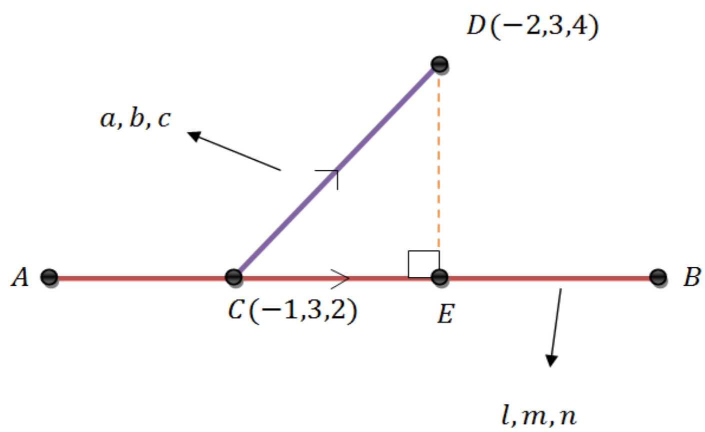
$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

$\therefore$ The three lines are inclined to each other at an angle $\frac{\pi}{3}$.

**4.** Find the distance of the point $(-2, 3, 4)$ from the line through the point $(-1, 3, 2)$ whose direction cosines are proportional to $12, 3, -4$.

**Solution:**




Now,

$$CD = \sqrt{(-2+1)^2 + (3-3)^2 + (4-2)^2}$$

$$\Rightarrow \quad CD = \sqrt{5}$$

Since the direction cosines are proportional to $12, 3, -4$ of the line $\overrightarrow{AB}$.

That is, the direction ratios of the line $\overrightarrow{AB}$ are

$$a = 12$$

$$b = 3$$

$$c = -4$$

Now the direction cosines of the line $\overrightarrow{AB}$ are

$$l = \frac{12}{13}$$

$$m = \frac{3}{13}$$

$$n = -\frac{4}{13}$$

Direction ratios of the line $\overrightarrow{CD}$ are

$$a = -1$$

$$b = 0$$

$$c = 2$$

Now,

The Projection of $\overrightarrow{CD} = CE = |al + bm + cn|$

$$= \left|(-1) \times \frac{12}{13} + 2 \times \left(-\frac{4}{13}\right)\right|$$

$$= \left|-\frac{20}{13}\right|$$

$$= \frac{20}{13}$$

From the right-angled triangle CED we can write

$$CD^2 = ED^2 + CE^2$$

$$\Rightarrow \quad ED^2 = CD^2 - CE^2$$

$\Rightarrow \quad ED = \sqrt{\left(\sqrt{5}\right)^2 - \left(\frac{20}{13}\right)^2}$

$\Rightarrow \quad ED = \frac{\sqrt{445}}{13}$ unit.