EAST WEST UNIVERSITY

Course Title : Co-ordinate Geometry and Vector Analysis.
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Assignment ①

① Given that,

$$3x^2+6y^2-12x+12y+18=0$$

$$\Rightarrow x^2+2y^2-4x+4y+6=0 \quad -①$$

putting $x=x+\alpha$, $y=y+\beta$ in equation ①

$$(x+\alpha)^2+2(y+\beta)^2-4(x+\alpha)+4(y+\beta)+6=0$$

$$\Rightarrow x^2+2x\alpha+\alpha^2+2(y^2+2y\beta+\beta^2)-4(x+\alpha)+4(y+\beta)+6=0$$

$$\Rightarrow x^2+2y^2+x(2\alpha-4)+y(4\beta+4)+(\alpha^2+4\beta-4\alpha+2\beta^2+6)=0 \quad -②$$

equation ② contains only terms second degree if, $2\alpha-4=0$ and $4\beta+4=0$

$2\alpha=4$ | $4\beta=-4$

$\alpha=2$ | $\beta=-2$

putting the value of α and β into equation ② $\rightarrow$ $x^2+2y^2=0$ [This is our required equation]

(2) Given,

$$2x^2 - 5xy + 2y^2 = 0$$

$$\therefore x = \frac{5y \pm \sqrt{(5y)^2 - 4.2.2y^2}}{2.2}$$

$$= \frac{5y \pm 3y}{4}$$

$$\therefore 4x = 5y + 3y$$

$$\Rightarrow 4x - 8y = 0 \quad \text{—(i)}$$

or, $4x = 5y - 3y$

$$4x - 2y = 0 \quad \text{—(ii)}$$

another diagonal is, $5x + 2y = 1$ —(iii)

By solving (i) and (ii) we get.

$$O = (x_1, y_1) = (0, 0)$$

By solving (ii) and (iii), and (iii) and (i) we get,

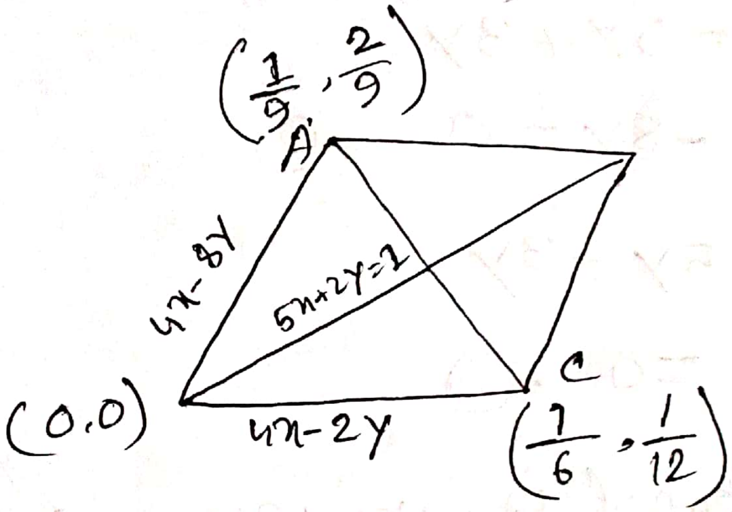
$$A = (x_2, y_2) = \left(\frac{1}{9}, \frac{2}{9}\right)$$

$$C = (x_3, y_3) = \left(\frac{1}{6}, \frac{1}{12}\right)$$

Middle point of A and C,

$$= \left(\frac{1/9 + \frac{1}{6}}{2}, \frac{2/9 + \frac{1}{12}}{2} \right)$$

$$= \left(\frac{5}{36}, \frac{11}{72} \right)$$



$$\therefore \text{ area} = \begin{bmatrix} 0 & \frac{1}{9} & \frac{1}{6} & 0 \\ 0 & \frac{2}{9} & \frac{1}{12} & 0 \end{bmatrix}$$

$$= \frac{1}{36} \text{ unit}^2$$

So, we can write,

$$\frac{y-0}{0-\frac{11}{72}} = \frac{x-0}{0-\frac{5}{36}}$$

$$\Rightarrow -\frac{5}{36}y = -\frac{11}{72}x$$

$$\Rightarrow \frac{11}{72}x - \frac{5}{36}y = 0$$ [divided by $\frac{1}{36}$]

$$\Rightarrow \frac{11}{2}x - 5y = 0$$ [multiply by 2]

$$\Rightarrow 11x - 10y = 0$$

This is our required equation

(3) Since the lines are parallel and perpendicular to the line,

$$5x - 3y + 1 = 0$$

So the lines will be.

$$5x - 3y + a = 0 \quad -(1)$$

and $3x + 5y + b = 0 \quad -(2)$

Here (1) and (2) passing through the origin (0,0),

$a = 0$

$b = 0$

putting the value of a and b into equation (1) and (2)

$$5x - 3y = 0 \quad -(3)$$

and $3x + 5y = 0 \quad -(4)$

So the combined equation of (3) and (4)

$$(5x - 3y)(3x + 5y) = 0$$

(Answer)

④ Given that,

$$x^2 - y^2 - 3x + 3y = 0 \quad -①$$

We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, we can write,

$$x^2 - 3x + (-y^2 + 3y) = 0$$

$$\therefore x = \frac{3 \pm \sqrt{(-3)^2 - 4.1.(-y^2+3y)}}{2.1}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 - 4(-y^2+3y)}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 \cancel{+} 4y^2 - 12y}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{4y^2 - 12y + 9}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{(2y)^2 - 2.2y.3 + (3)^2}}{2}$$

$$\Rightarrow x = \frac{3 \pm 2y - 3}{2}$$

$$\Rightarrow 2x = 3 + 2y - 3$$

$$\Rightarrow 2x - 3 - 2y + 3 = 0$$

$$\Rightarrow 2x - 2y = 0 \quad -②$$

or,

$$2x = 3 - 2y + 3$$

$$2x + 2y - 6 = 0 \quad - (3)$$

After solving equation (2) and (3)

$$x = \frac{3}{2}, \quad y = \frac{3}{2}$$

The traingle form by the y axis.

so, $x = 0$

Now putting the value of $x = 0$ into equation

(2) $\rightarrow$ we get,

$$2.0 - 2y = 0$$

$$-2y = 0$$

$$y = 0$$

$$(x, y) = (0, 0)$$

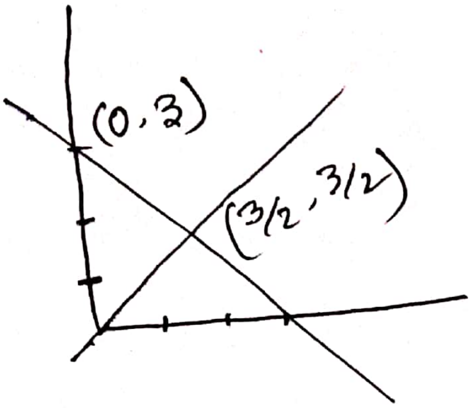
Now putting the value of $x = 0$ into equation (3)

$$2.0 + 2y - 6 = 0$$

$$2y = 6$$

$$y = 3$$

$$(x_1, y_1) = (0, 3)$$



Now the area of traingle,

$$\Delta = \frac{1}{2}\left\{-\frac{3}{2}(0-3)\right\}$$

$$= \frac{1}{2} \times \frac{9}{2}$$

$$= \frac{9}{4} \text{ unit}^2$$

$\therefore$ The point of intersection $(x, y) = \left(\frac{3}{2}, \frac{3}{2}\right)$

And area of traingle $\frac{9}{4}$ unit^2 (Answer)

⑤ Given that,

$$x^2+y^2-4x-7y+6=0 \quad -①$$

$$x^2+y^2+3x-14y-1=0 \quad -②$$

$$3x^2+3y^2+2x-35y+4=0 \quad -③$$

$$\Rightarrow x^2+y^2+\frac{2}{3}x-\frac{35}{3}y+\frac{4}{3}=0$$

Now from equation ① and ②, we get,

$$\cancel{-7x}+7y+7=0 \quad -\cancel{④}$$

$$=-7(x-y-1)=0 \quad \Rightarrow x-y-1=0 \quad -④$$

from equation ① and ③, we get

$$-\frac{14}{3}x+\frac{14}{3}y+\frac{14}{3}=0 \quad \cancel{⑤}$$

$$\Rightarrow -\frac{14}{3}(x-y-1)=0$$

$$\Rightarrow x-y-1=0 \quad -⑤$$

from equation ② and ③, we get

$$\Rightarrow \frac{7}{3}x-\frac{7}{3}y-\frac{7}{3}=0$$

$$\Rightarrow \frac{7}{3}(x-y-1)=0$$

$$\Rightarrow x-y-1=0 \quad -⑥$$

So, finally, we can say that equation ①, ② and 3 have a common radical axis.

[verified]

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(6) Given that,

$$x^2+y^2-6x+4y-3=0 \quad -(1)$$

here, $g=-3$, $f=2$ and $c=-3$

we know the general equation of circle is,

$$x^2+y^2+2gx+2fy+c=0 \quad -(ii)$$

Since equation (1) and (2) cuts orthogonally so,

$$2g_1g_2+2f_1f_2-c_1-c_2=0$$

$$\Rightarrow 2(-3)g+2.2f+3-c=0$$

$$\Rightarrow -6g+4f+3-c=0$$

$$\Rightarrow 6g-4f-3+c=0 \quad -(3)$$

Now equation 2 passes through the point $(3,0)$ we get,

$$3^2+0^2+2.3g+2.0.f+c=0$$

$$\Rightarrow 9+6g+c=0$$

$$\Rightarrow c=-6g-9 \quad -(4)$$

putting $c=-6g-9$ in the equation (3)

$6g - 4f - 3 - 6g - 9 = 0$

$\Rightarrow -4f - 12 = 0$

$\Rightarrow -4f = 12$

$f = -3$

As equation ② touches y-axis

so, we get,

$f^2 = c$

$c = (-3)^2 = 9$

putting the value of $f = -3$ and $c = 9$ and $g = -3$ in the equation ②

$x^2 + y^2 - 6x - 6y + 9 = 0$

So, our required equation is

$x^2 + y^2 - 6x - 6y + 9 = 0$

(Answer)

putting $c = 9$ into equation ④ $\rightarrow$

$9 + 6g + 9 = 0$

$g = -3$

Assignment-(11)

(1) Given that,

$$3x^2 - y + 5x - 2 = 0 \quad -(1)$$

Here,

$a = 3 \qquad h = 0$

$b = 0 \qquad f = -1/2$

$c = -2 \qquad g = 5/2$

$$\therefore \Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 3.0.(-2) + 2.(-\tfrac{1}{2}).\tfrac{5}{2} \times 0 - 3.(-\tfrac{1}{2})^2 - 0.(\tfrac{5}{2})^2 - (-2).0^2$$

$$= 0 + 0 - \frac{3}{4} - 0 - 0$$

$$= -\frac{3}{4}$$

Again,

$$h^2 - ab = 0^2 - 3.0 = 0$$

So the given equation (1) represents a parabola.

From equation (1) $\rightarrow$

$$3x^2 - y + 5x - 2 = 0$$

$$\Rightarrow x^2 - \frac{y}{3} + \frac{5x}{3} - \frac{2}{3} = 0$$

$$\Rightarrow x^2 + 2.\frac{5}{6}.x + \left(\frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 - \frac{y}{3} - \frac{2}{3} = 0$$

$$\Rightarrow \left(x + \frac{5}{6}\right)^2 - \frac{49}{36} = \frac{y}{3}$$

$$\Rightarrow \left(x + \frac{5}{6}\right)^2 = \frac{y}{3} + \frac{49}{36}$$

$$\Rightarrow \left(x + \frac{5}{6}\right)^2 = \frac{1}{3}\left(y + \frac{49}{36}\right)$$

$$\Rightarrow \left(x+\frac{5}{6}\right)^2 = 4\cdot\frac{1}{12}\cdot\left(y+\frac{49}{12}\right)$$

$$X^2 = 4aY$$

where,

$$X = x+\frac{5}{6}$$

$$Y = y+\frac{49}{12}$$

$$4a = \frac{1}{3}$$

$$a = \frac{1}{12}$$

vertex,

$$X = 0$$

$$x+\frac{5}{6}=0$$

$$x = -5/6$$

$$Y=0$$

$$y+\frac{49}{12}=0$$

$$\Rightarrow y = -\frac{49}{12}$$

focus,

$$X=0$$

$$\Rightarrow x+\frac{5}{6}=0$$

$$\Rightarrow x = -\frac{5}{6}$$

$$Y=a$$

$$y+\frac{49}{12} = \frac{1}{12}$$

$$y = -4$$

Equation of the axis,

$$X=0$$

$$x+5/6=0$$

Equation of tangent at the vertex,

$$Y=0$$

$$y+\frac{49}{12}=0$$

Equation of directrix.

$$Y = -a$$

$$\Rightarrow Y + \frac{49}{12} = -\frac{1}{12}$$

$$\Rightarrow Y + \frac{25}{6} = 0$$

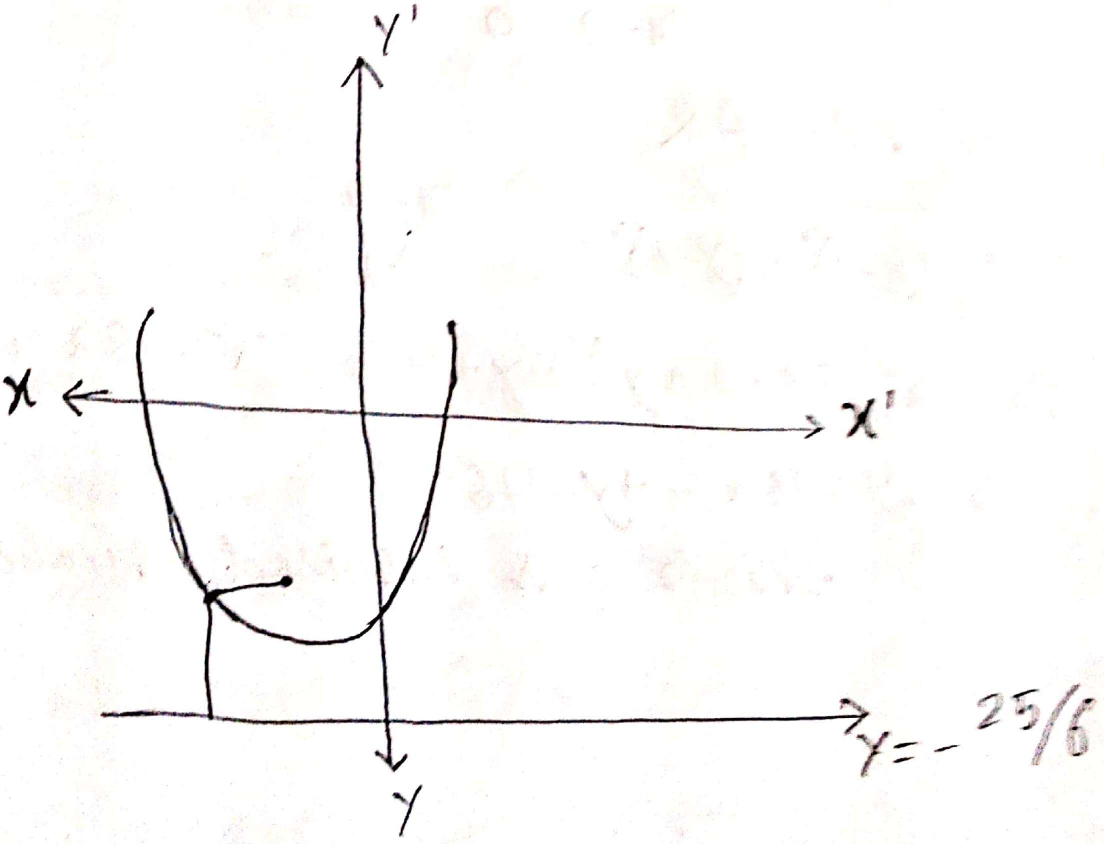
Length of latus rectum $= |4a| = \frac{1}{3}$ unit

equation of the latus rectum:

$$Y = a$$

$$Y + \frac{49}{12} = \frac{1}{12}$$

$$Y + 4 = 0$$



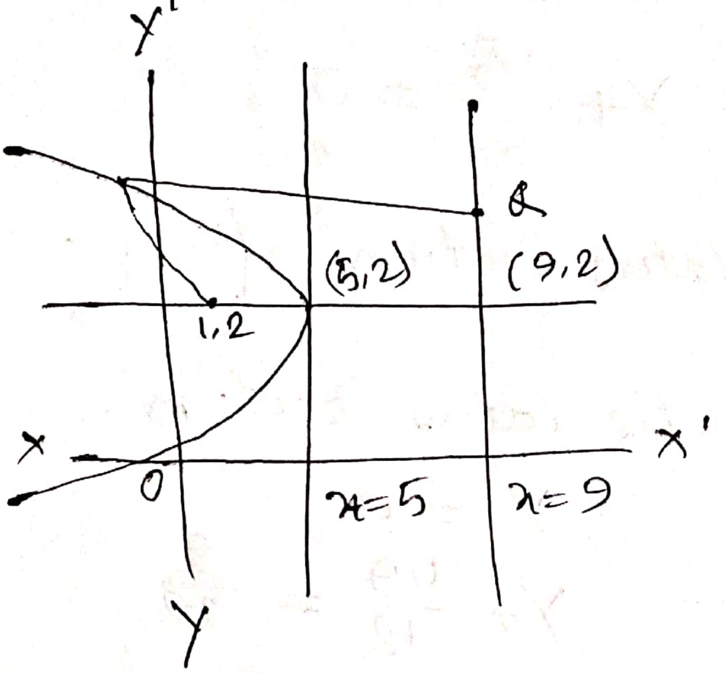
The parabola

(2) Given.

Focus, $(1, 2)$

tangent at the vertex $x-5=0$

$x = 5$



equation of the directrix,

$x-9=0$

Now, $FP = PQ$

$\Rightarrow \sqrt{(x-1)^2+(y-2)^2} = \dfrac{x-9}{\sqrt{1}}$

$\Rightarrow x^2-2x+1+y^2-4y+4 = x^2-18x+81$

$\Rightarrow y^2+16x-4y-76=0$

This is our required equation.

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(3) Given that,

$$x^2+2y^2+2x+4y-3=0 \quad -(1)$$

Here, $a=1$, $b=2$, $c=-3$ | $g=1$, $h=0$, $f=2$

We know,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 1.2.(-3) + 2.2.1.0 - 1.2^2 - 2.1^2 - (-3).0^2$$

$$= -6 + 0 - 4 - 2$$

$$= -12$$

and,

$$h^2 - ab = 0^2 - 1.2$$

$$= -2 < 0$$

Since $\Delta \neq 0$ and $h^2 - ab < 0$. So the given equation represents an ellipse.

Now putting $x = x+\alpha$ and $y = y+\beta$ into equation (1) $\rightarrow$

$$(x+\alpha)^2 + 2(y+\beta)^2 + 2(x+\alpha) + 4(y+\beta) - 3 = 0$$

$$\Rightarrow x^2 + 2x\alpha + \alpha^2 + 2y^2 + 4y\beta + 2\beta^2 + 2(x+\alpha) + 4(y+\beta) - 3 = 0$$

$$\Rightarrow x^2 + x(2\alpha+2) + y(4\beta+4) + 2y^2 + \alpha^2 + 2\beta^2 + 2\alpha + 4\beta - 3 = 0 \quad -(2)$$

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The term of x and y in equation ② will be absent if.

$$2\alpha+2=0$$

$$\alpha=-\frac{2}{2}=-1$$

and, $4\beta+4=0$

$$\beta=-1$$

Now, putting the value of $\alpha=-1$ and $\beta=-1$ into equation 2, we get,

$$x^2+2y^2-6=0$$

$$x^2+2y^2=6$$

$$\frac{x^2}{6}+\frac{2y^2}{6}=1$$

$$\frac{x^2}{6}+\frac{y^2}{3}=1$$

This is the standard equation of an ellipse.

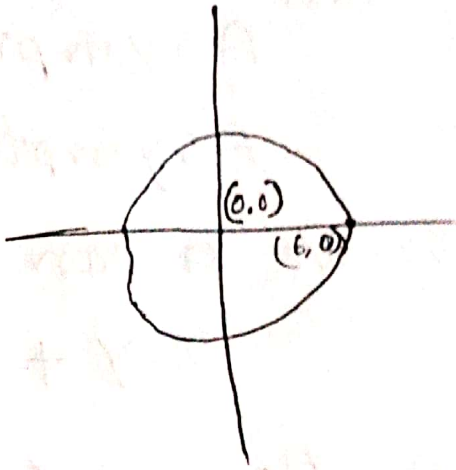
(4) Given that.

Foci are $(0,0)$, $(6,0)$

and length of the major axis is 8.

We know.

$$FF' = 2ae$$



$$\Rightarrow \sqrt{(6-0)^2} = 8e$$

$$\Rightarrow 8e = \sqrt{36}$$

$$\Rightarrow 8e = 6$$

$$e = \frac{6}{8}$$

$$2a = 8$$
$$a = 4$$
$$a^2 = 16$$

We know,

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \left(\frac{6}{8}\right)^2 = 1 - \frac{b^2}{16}$$

$$\Rightarrow \frac{36}{64} = 1 - \frac{b^2}{16}$$

$$\Rightarrow \frac{36}{64} - 1 = -\frac{b^2}{16}$$

$$\Rightarrow -\frac{7}{16} = -\frac{b^2}{16}$$

$$\Rightarrow b = \pm\sqrt{7}$$

Length of the minor axis,

$$2b = 2 \times \pm\sqrt{7}$$

$$= 2\sqrt{7}$$

or $-2\sqrt{7}$

eccentricity.

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \frac{3}{4} \text{ (Answer)}$$

(5) Here, given

$$(\text{Asymptote } 1) \times (\text{Asymptote } 2) + K = 0$$

Now,

Asymptote 1, $x = 0$

Asymptote 2, $y = 0$

So, we ~~is~~ can write the equation of hyperbola →

$$x \cdot y + K = 0$$

As the equation (1) passes through the point $\left(am, \frac{a}{m}\right)$

So, we can write,

$$am \times \frac{a}{m} + K = 0$$

$$\Rightarrow a^2 + K = 0$$

$$K = -a^2$$

putting the values of $K = -a^2$ in the equation (1)

We get,

$$xy - a^2 = 0$$

$$xy = a^2$$

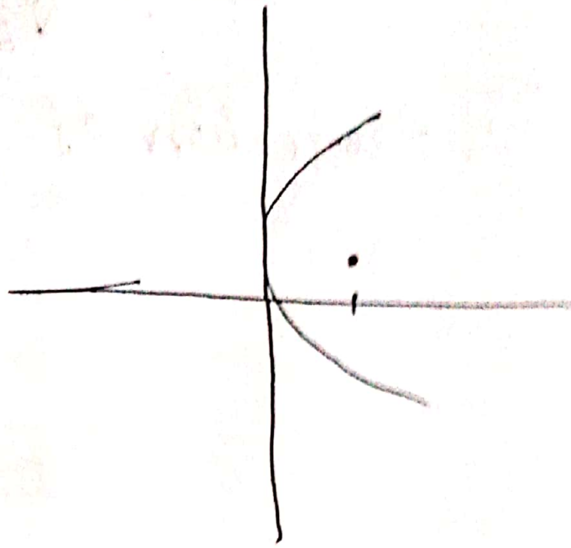
This is our required equation of hyperbola.

⑥ Given

focus $(2, 1)$

eq. of directrix. $x-2y+3=0$

ecentricity $=2$



$\therefore FP = e.PQ$

$$\sqrt{(x-2)^2+(y-1)^2} = 2.\frac{x-2y+3}{\sqrt{5}}$$

$$\Rightarrow \sqrt{x^2-4x+4+y^2-2y+1} = 2.\frac{x-2y+3}{\sqrt{5}}$$

$$\Rightarrow x^2-4x+4+y^2-2y+1 = 4.\frac{(x-2y+3)^2}{5}$$

$$\Rightarrow 5x^2-20x+20+5y^2-10y+5=4\{x^2-2.x.(2y-3)+(2y+3)^2\}$$

$$\Rightarrow 5x^2-20x+20+5y^2-10y+5=4\{x^2-4xy+6x+4y^2+12y+9\}$$

$$\Rightarrow 5x^2-20x+20+5y^2-10y+5=4x^2-16xy+24x+16y^2$$

$$\Rightarrow x^2-11y^2+44x+38y+16xy-11=0$$

(Answer)

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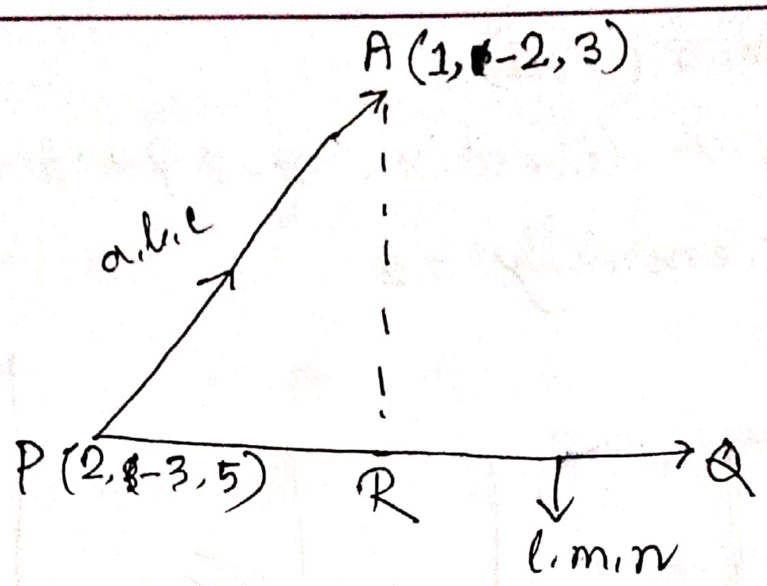

Here, $\overrightarrow{PA} = \sqrt{(1-2)^2 + (-2+3)^2 + (3-5)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + (-2)^2}$

$= \sqrt{6}$

Direction Ratio of the line $\overrightarrow{PA}$ are

$$\boxed{\begin{aligned} a &= -1 \\ b &= 1 \\ c &= -2 \end{aligned}}$$

Let, direction of cosine of $\overrightarrow{PQ}$ line are

$l, m, n \rightarrow$

Where, $\alpha = \beta = \gamma$ | ~~l=a, m=a~~

So, $l = m = n$

Let $[l = m = n = a]$

We know,

$$l^2+m^2+n^2=1$$

$$\Rightarrow 3a^2=1$$

$$a^2=\frac{1}{3}$$

$$a=\pm\frac{1}{\sqrt{3}}$$

$$l=m=n=\pm\frac{1}{\sqrt{3}}$$

$\therefore$ Projection of $\vec{PA}=PR$

$$=|al+bm+cn|$$

$$=\left|(-1)\times\frac{1}{\sqrt{3}}+1\times\frac{1}{\sqrt{3}}+(-2)\times\frac{1}{\sqrt{3}}\right|$$

$$=\frac{2}{\sqrt{3}}$$

from the right angle triangle PRA, we can write

$$PA^2=RA^2+PR^2$$

$$\Rightarrow RA^2=PA^2-PR^2$$

$$RA=\sqrt{(\sqrt{6})^2-\left(\frac{2}{\sqrt{3}}\right)^2}$$

$$=\sqrt{6-\frac{4}{3}}$$

$$=\frac{\sqrt{42}}{3}\text{ unit}$$

(Answer)