EAST WEST UNIVERSITY

Mid-I Examination, Fall-2021
Department of Mathematics and Physical Sciences
Course Code: MAT 205 (Linear Algebra and Complex Variables)
Section: 8, Time: 90 minutes, Full Marks: 40
Course Instructor: Dr. Nepal Chandra Roy (DNCR)

N.B.: Answer all the questions. Figure in the right margin indicate full marks.

1. Determine the values of k such that the system in unknowns x, y and z has (i) a unique solution, (ii) no solution, (iii) more than one solution: [10]

$$x - 3z = -3$$
$$2x + ky - z = -2$$
$$x + 2y + kz = 1$$

2. (a) A diet research project includes adults and children of both sexes. The composition of the participants in the project is given by the matrix [5]

$$\begin{array}{cc} & \begin{array}{cc} \text{Adults} & \text{Children} \end{array} \\ A = & \begin{bmatrix} 80 & 120 \\ 100 & 200 \end{bmatrix} \begin{array}{l} \text{Male} \\ \text{Female} \end{array} \end{array}$$

The number of daily grams of protein, fat and carbohydrate consumed by each child and adult is given by the matrix

$$A = \begin{bmatrix} 20 & 20 & 20 \\ 10 & 20 & 30 \end{bmatrix}.$$

(i) How many grams of protein are consumed daily by the males in the project?

(ii) How many grams of carbo-hydrate are consumed daily by the females in the project?

(b) Consider the vectors $u_1 = (2,1,4), u_2 = (1,-1,3), u_3 = (3,2,5)$ in $\mathbb{R}^3$. Show that V=(5, 9, 5) is a linear combination of u_1, u_2 and u_3. [5]

3. (a) Show that the set of vectors $\{(3,0,1,-1),(2,-1,0,1),(1,1,1,-2)\}$ is linearly dependent. [5]

(b) Find the Hermitian matrix of [5]

$$A = \begin{bmatrix} 5i & 1+i & 2-5i \\ 1-4i & 4 & 3-2i \\ -1+3i & 1-8i & 2+3i \end{bmatrix}$$

4. (a) Show that $T = \{(x, y, z, t) \in \mathbb{R}^4 : x + y + z + t = 0\}$ is a subspace of $\mathbb{R}^4$. [5]

(b) Let S and T be the following subspaces of $\mathbb{R}^4$: [5]

$$S = \{(x, y, z, t) \in \mathbb{R}^4 : y - z + t = 0\}$$

$$T = \{(x, y, z, t) \in \mathbb{R}^4 : x - t = 0, y - 2z = 0\}.$$

Find a basis and the dimension of S and $S \cap T$.