Chapter 4

4.1 - Kelper's Law

When modeling the Solar System, the only force to focus on is gravity which is represented by Newton's law of gravitation,

$$F_G = \frac{GM_SM_E}{r^2}$$

M_S and M_E are the masses of the Sun and Earth respectively, r is the distance between the two, and G is the gravitational constant. What we want to do is calculate the position of the Earth as a function of time. Newton's Second Law of Motion:

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_E}$$
$$\frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_E}$$

This represents the x and y components of the gravitational force

$$F_{G,x} = -\frac{GM_SM_E}{r^2}cos\theta = -\frac{GM_SM_Ex}{r^3}$$

$F_{G,y}$ *is in a similar form. The negative sign represents how the force is going toward the Sun.*

$$\frac{dv_x}{dt} = -\frac{GM_Sx}{r^3}$$
$$\frac{dx}{dt} = v_x$$
$$\frac{dv_y}{dt} = -\frac{GM_Sy}{r^3}$$
$$\frac{dy}{dt} = v_y$$

The units used for planetary models would be astronomical units($AU = 1.5e11$m) Unit of mass:

$$\frac{M_Ev^2}{r} = F_G = \frac{GM_SM_E}{r^2}$$

v *is the velocity of Earth*

$$GM_S = v^2r = 4\pi^2AU/yr^2$$

Converted equations of motion into difference equations:

$$v_{x,i+1} = v_{x,1} - \frac{4\pi^2x_i}{r_i^2}\Delta t$$
$$x_{i+1} = x_i + v_{x,i+1}\Delta t$$

A similar formula is used for the y components. Take note that the Euler Cromer method is used.

Kepler's Law for planetary motion states:

- All planets move in elliptical orbits, with the Sun at one focus.
- The line joining a planet to the Sun sweeps out equal areas in equal times.
- If T is the period and a is the semimajor axis of the orbit then $\frac{T^2}{a^3}$ is constant. Note that for a circular orbit, a is the radius of the orbit.

4.2 - Inverse Square Law and Stability of Planetary Orbits

In a two-body system all three of Kelper's Laws are consequences of the fact that the gravitational force follows an inverse-square law. If the interaction force only depends on the separation of the two bodies, the relative motion can be studied in the form of a one-body system. Similar to the Sun and Earth. The reduced mass of the moving body:

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

m_1 and m_2 are the masses of the two bodies. Orbital trajectory for a body of reduced mass μ

$$\frac{d^2}{d\theta^2}(\frac{1}{r}) + \frac{1}{r} = -\frac{\mu r^2}{L^2}F(r)$$

$L = \mu r^2 \theta$ is the angular momentum and $F(r) = -\frac{GM_S M_P}{r^2}$ is the force acting on the body.

4.3 - Precession of the Perihelion of Mercury

Oftentimes, there are deviations in Kepler's Law due to the presence of multiple planets acting on one another at all times. The planets that deviate the most from cicular orbit are Mercury and Pluto(Although, Pluto isn't regarded as a planet anymore), the orientation of Mercury's axes would rotate with time, being known as the Precision of the Perihelion of Mercury. The perihelion is the point in orbit where it's closest to the Sun. Mercury's perihelion is 566 arcseconds per century.

4.4 - Three Body-Problem and the effect of Jupiter on Earth

In a three-body problem, the presence of an analytical theory becomes less likely. Thus, the three-body problem is also considered the n-body problem, a central focus of celestial mechanics. In a three-body system like the Sun, Earth, and Jupiter, the force of Jupiter on Earth would be mostly negligible until the mass of Jupiter increases. Should the mass of Jupiter increase too much, Earth would be ejected from the Solar system due to the gravitational pull from both Jupiter and the Sun.

4.5 - Resonances in the Solar System: Kirkwood Gaps and Planetary Rings

As you move farther out from the Sun in the Solar System, the distance between each planet increase at regular intervals. That pattern is known as the Titus-Bode formula, where the distances of the planets to the Sun are related to the sequence;

$$0, 3, 6, 12, 24...$$

Each term, starting at 0, and 3 double the previous term. It was because of the Titus-Bode formula that asteroids were first discovered in 1800, leading to the discovery of Kirkwood Gaps. Looking at asteroids surrounding Jupiter, we can see examples of chaotic behaviour as they tend to orbit around Jupiter for long periods of time before sporadically changing. The physics behind Kirkwood gaps are also responsible for the creation of rings around certain planets, being massive collections of particles orbiting around the planet.

4.6 - Chaotic Tumbling of Hyperion

One of Saturn's moons, Hyperion, and its motion is often considered to be a chaotic case in our Solar System. Being considerably smaller than any other moon.

The reson for Hyperion's different behaviour is its unusual shape and highly elliptical orbit, being shaped more like an egg than a sphere. Because of its strange shape, Hyperion ends up having incredibly erratic orbital patterns. In order to simulate the position of Hyperion we'll need to xtend the original planetary motion program to include the rotation of the object around the perpendiculat axis to the plane of the figure.