

Lecture :- Binary search-1

Agenda

- Introduction
- first occurrence in sorted array
- unique element
- Max element in inc/dec array.
- Local minima in an array.

Introduction to searching

bro|sis \longrightarrow Police \longrightarrow target = bro|sis
search space = fair|mela.

Word \longrightarrow Dict ✓
Newspaper
Book

Target
Search space.
condition

Binary Search

Qu. Given $A[n]$ of sorted distinct elements. Return the index of an element k . If not found, return -1 .

0	1	2	3	4
3	8	11	19	28

$k = 19$ $ans = 3$

0	1	2	3	4
3	8	11	19	28

$k = 27$ $ans = -1$

Brute force:

$O(n)$ — traverse the array.

3	1	2	8	18
---	---	---	---	----

$k = 8$ — $O(n)$.

Approach 2

0	1	2	3	4	5	6	7	8	9
3	6	9	12	14	19	20	23	25	27

$$k = 12$$

$$\text{target} = 12$$
$$\text{mid value} = 14 \left[\frac{0+9}{2} \right]$$

$$\text{target} < 14$$

$$\text{search space} = (0-3) \text{ idx}$$

↑
mid-1

$$k = 25$$

$$\text{target} = 25$$
$$\text{mid value} = 14 \left[\frac{0+9}{2} \right]$$

$$\text{target} > 14$$

$$\text{search space} = (5-9) \text{ idx}$$

↑
mid+1

Dry run:

0	1	2	3	4	5	6	7	8	9
3	6	9	12	14	19	20	23	25	27

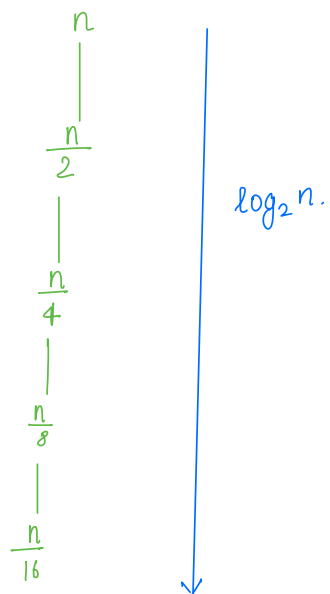
target = 12.

s	e	Action
0	9	mid = 14 target < mid s = 0, e = 3.
0	3	midIdx = 1 mid = 6 12 > 6 s = 2, e = 3
2	3	midIdx = $\frac{2+3}{2} = 2$ mid = 9 12 > 9 s = 3, e = 3
3	3	midIdx = 3 mid = 12 12 = 12 (return midIdx).

Binary Search pseudocode

```
int binarySearch(int[] A, int target) {  
    s = 0;  
    e = A.length - 1;  
    while( s <= e ) {  
        midIdx =  $\frac{s+e}{2}$ ;  
        mid = A[midIdx];  
        if( target == mid ) {  
            return midIdx;  
        } else if( target < mid ) {  
            e = mid - 1;  
        } else {  
            s = mid + 1;  
        }  
    }  
    return -1;  
}
```

TC: $\log_2 n$.
SC: $O(1)$



Q. Given $A[n]$, find first occurrence of target element.
Sorted

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
-5	-5	-3	0	0	1	1	5	5	5	5	5	5	5	10	10	15	15

target = 5 ans = 7

target = 15 ans = 16

target = -5 ans = 0

Brute force:

$O(n)$

Approach 2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
-5	-5	-3	0	0	1	1	5	5	5	5	5	5	5	10	10	15	15

target = 5.

ℓ	e	$(\ell + e) / 2$ midIdx	Action
0	17	8	mid = 5 5 = 5 ans = 8 $\ell = 0$ $e = 7$
0	7	3	mid = 0 5 > 0 $\ell = 4$ $e = 7$
4	7	5	mid = 1 5 > 1 $\ell = 6$ $e = 7$
6	7	6	mid = 1 5 > 1 $\ell = 7$ $e = 7$
7	7	7	mid = 5 5 = 5 ans = 7 $\ell = 7$ $e = 6$
7	6		stop

Pseudocode

```
int firstOccurrence(int[] A, int target) {  
    s = 0;  
    e = A.length - 1;  
    ans = -1;  
    while (s <= e) {  
        midIdx =  $\frac{s+e}{2}$ ;  
        mid = A[midIdx];  
        if (target == mid) {  
            ans = midIdx;  
            e = mid - 1;  
        } else if (target < mid) {  
            e = mid - 1;  
        } else {  
            s = mid + 1;  
        }  
    }  
    return ans;  
}
```

TC: $O(\log_2 n)$

SC: $O(1)$

Q. Given $A[n]$. Every element occurs twice except for one element.

find that unique element.

Duplicate el are adjacent to each other but array is unsorted.

8	8	5	5	6	2	2
---	---	---	---	---	---	---

ans =

Brute force

xor — $O(n)$

Hashmap — $O(n), O(n)$

Approach 2

0	1	2	3	4	5	6	7	8	9	10
7	7	6	6	3	8	8	1	1	9	9

first occ of 7 = 0
" " " 6 = 2

↓
fir occ of any no is even
idx.

first occ of 8 = 5
" " " 1 = 7
" " " 9 = 9

fir occ of any no is odd idx

$A[mid] \neq A[mid-1]$ & $A[mid] \neq A[mid+1]$ } return A[mid]

Dry run:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	3	1	1	8	8	10	10	19	6	6	2	2	4	4

s	e	mididx	mid	Actions
0	14	7	10	$10 == A[mid-1]$ Not unique el. fir occ of 10 = 6 $arr[mid] == arr[mid-1] \rightarrow mid-1$ $f.o = 6$ $s = 8$ $e = 14$
8	14	11	2	Not unique. fir occ of 2 = 11 $arr[11] == arr[10] \rightarrow f.o = 10$ $!=$ $f.o = 11$ $s = 8$ $e = 10$
8	10			
...				

Pseudocode

```
if (A[mid] != A[mid-1]    &&  
    A[mid] != A[mid+1])  
    return A[mid]
```

```
f.o = if (A[mid] == A[mid-1])  
        mid-1 is f.o  
    else mid is f.o.
```

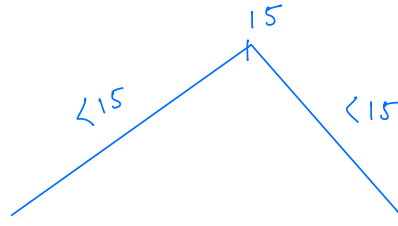
```
if (f.o is odd idx) {  
    left side
```

```
} else {  
    right side.  
}
```

Qn Given increasing-decreasing array with distinct elements.
find maximum element.

1	3	5	2
---	---	---	---

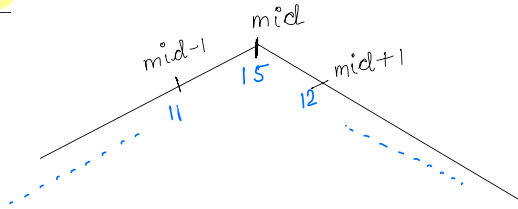
1	3	5	10	15	12	6
inc.					dec.	



Brute force: $O(n)$

Approach

Case 1

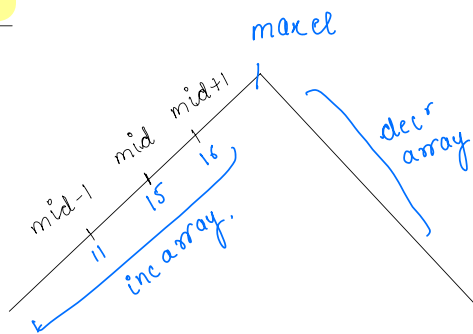


$$arr[mid] > arr[mid-1] \quad \&\&$$

$$arr[mid] > arr[mid+1]$$

ans \Rightarrow $arr[mid]$

Case 2

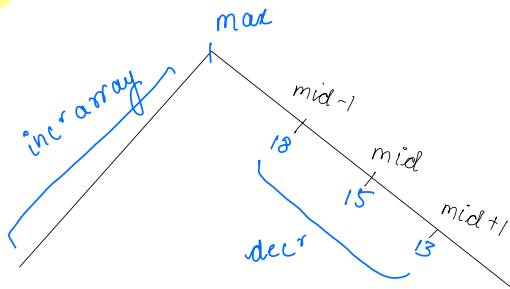


$$arr[mid] > arr[mid-1] \quad \&\&$$

$$arr[mid] < arr[mid+1]$$

ans \Rightarrow go to right

Case 3



$$arr[mid] < arr[mid-1] \quad \&\&$$

$$arr[mid] > arr[mid+1]$$

ans = go left

$$arr[mid] > arr[mid-1] \quad \&\&$$

\longrightarrow return $arr[mid]$.

$$arr[mid] > arr[mid+1]$$

$$arr[mid] > arr[mid-1] \quad \&\&$$

\longrightarrow right $e = mid+1$

$$arr[mid] < arr[mid+1]$$

$$arr[mid] < arr[mid-1] \quad \&\&$$

\longrightarrow left $e = mid-1$

$$arr[mid] > arr[mid+1]$$

Break: 8:26:8:36

Qn Given $A[n]$ of distinct elements, find any local minima in array.

Local minima: A no. smaller than its adjacent neighbours

3	6	1	0	9	15	8
---	---	---	---	---	----	---

21	20	19	17	15	9	7	
----	----	----	----	----	---	---	--

5	9	15	16	20	21
---	---	----	----	----	----

5	8	12	3
---	---	----	---

Brute force: $O(n)$

Approach

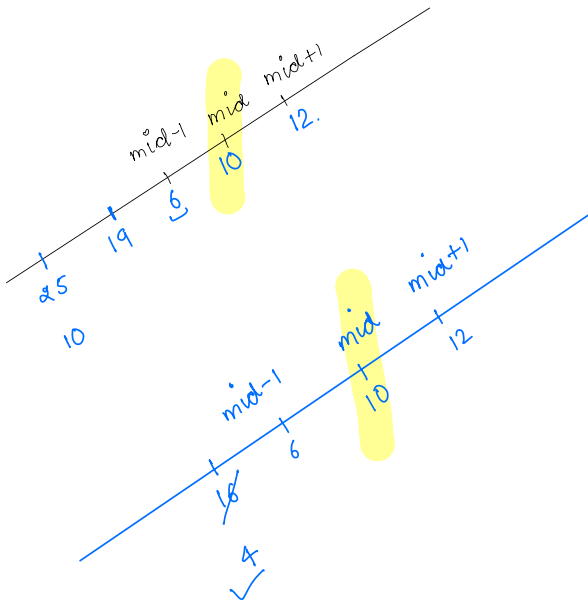
Case 1.



$$arr[mid] < arr[mid-1] \quad \&\&$$

$$arr[mid] < arr[mid+1] \quad ans = 10$$

Case 2:

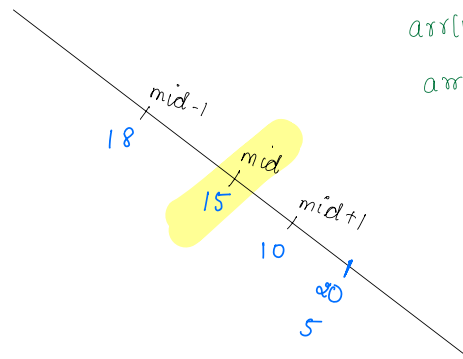


$$arr[mid] > arr[mid-1]$$

$$\&\& arr[mid] < arr[mid+1]$$

Left

Case 3

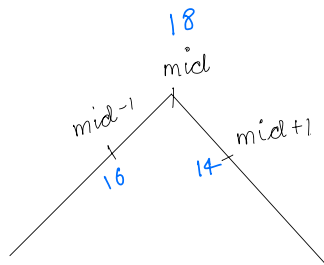


$arr[mid] < arr[mid-1]$ &&

$arr[mid] > arr[mid+1]$

right

Case 4



if ($arr[mid] > arr[mid-1]$) &&
 $arr[mid] > arr[mid+1]$)

any sides left
right

Dry run:-

0	1	2	3	4	5	6	7
9	8	2	7	6	4	1	5

s	e	midIdx	Action
0	7	3	mid = 7 7 > 2 && 7 > 6 left s = 0, e = 2
0	2	1	mid = 8 8 < 9 && 8 > 2 right s = 2, e = 2
2	2	2	mid = 2 2 < 8 && 2 < 7 return ans (2).

TC: $O(\log_2 n)$
SC: $O(1)$

Pseudocode

```
int localMinima(int[] A) {
```

h/w.

Thankyou 😊