Lecture: Mathe: 400

Agendo

Permutation

Combination

Gcd basics

belete one

Out Cliven 3 distinct characters. In how many ways can we arrange them? $\delta = abc$ abc bac cab acb bca cba

$$\frac{3}{\uparrow} \quad \frac{2}{\uparrow} \quad \frac{1}{\uparrow} \quad = 3 * 2 * 1 = 3!$$

$$0 \quad b \quad c$$

$$0 \quad b \quad c$$

$$0 \quad c$$

In how many ways n distinct characters can be arranged?

nPr formulae

Que Given 5 distinct characters, in how many places can we arrange them in 2 places?

$$\frac{5}{1} \frac{4}{1} = 5*4 = 20$$

ab code.

ab ba ca do ea ac bc cb db eb ac bd cod dc ec ac be cc de ed

Que Given 5 distinct characters, in how many places can we arrange them in 3 places?

$$\frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} = 5 *4 *3 = 60$$

Ou Given 5 distinct characters, in how many places can we arrange them in 4 places? 5*4*3*2 = 120

$$\frac{n}{\uparrow} \frac{n-1}{\uparrow} \frac{n-2}{\uparrow} = n *(n-1) *(n-2)$$

Ou Given n distinct characters, in how many places can we arrange them in 4 places? n + n-1 + n-2 + n-3

Ou Given n distinct characters, in how many places can we arrange them in x places?

$$\frac{n}{\uparrow} \quad \frac{n-1}{\uparrow} \quad \frac{n-2}{\uparrow} \quad \frac{n-3}{\uparrow} \quad \frac{n-(r-1)}{\uparrow}$$

Ans = $n * n-1 * n-2 * n-3 - \dots - n-(r-1)$ Mutiply d divide by (n-r)!

 $n * n-1 * n-2 \cdots n-r+1 * n-r * n-r-1 * n-r-2 \cdots 1$ (n-r)!

 $\frac{n!}{(n-r)!} = \frac{n}{r}$ $\frac{n!}{(n-r)!}$ $\frac{n}{(n-r)!}$ $\frac{n}{(n$

Com bi nation

No of ways to select something

Order of selection does not matter

To simplify (i,j) = (f,i)

<u>Oul</u> Given 4 players, count no of ways of selecting 3 players.

Rohit Konli Namer Head

 $4_{C_3} = 4$

Rohit konli Warner

Ronit Konli head => 4

Konli Warner head

Ronit Warner head

<u>Ou</u> No of ways to arrange 4 players in 3 slots.

for every possible selection = 6 arrangements.

$$|8el = 6 \text{ arrangements}|$$

 $|8el = 6 * 4 = 24 \text{ arrangements}|$

 \underline{Ou} Given n distinct items, how many ways can we select r items. I such that $O \le r \le n$.

| selection =
$$x$$
 | arrangements.
 x | arrangements = 1 selection
| arrangement = $\frac{1}{x!}$ selection.

$$n_{p_r}$$
 arrangement = $\frac{n_{p_r}}{r!}$

$$u^{c\lambda} = \frac{xi(u-x)i}{ui}$$

Property of combination

Property! No of ways of selecting 0 items from n items, i.e.,

no of ways to not select anything, will always

be 1.

$$u^{c_0} = \frac{0!(u-0)!}{u!} = \frac{u!}{u!} = 1$$

Property 2 No. of ways of selecting n items from n items, i.e., no. of ways to not select anything, will always be 1.

$$u^{cu} = \frac{u^{i}(u-u)^{i}}{u^{i}} = \frac{u^{i} * 0^{i}}{u^{i}} = 1$$

$$n_{c_0} = n_{c_0}$$

Property 3 No of ways of selecting (n-x) items from n items, i.e., no of ways to not select anything, will always be 1.

$$= \frac{\lambda i (v-x)}{u i} = u^{C\lambda}$$

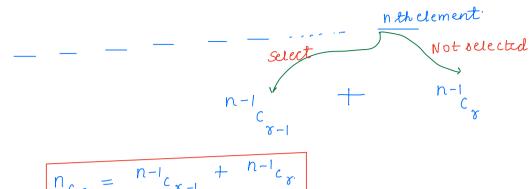
$$= \frac{(u-x)i (\psi-\lambda+x)i}{-u i}$$

$$= \frac{(u-x)i (\psi-\lambda+x)i}{-u i}$$

$$= \frac{(u-x)i (u-(u-x))i}{-u i}$$

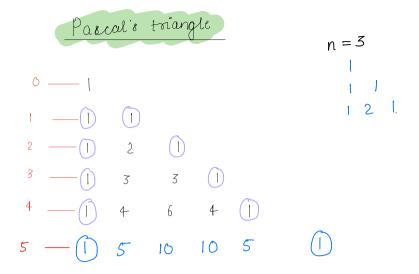
$$n_{c_{n-x}} = n_{c_x}$$

Property 4 Given n distinct elements. select r items. nex



$$n_{c_{\mathcal{X}}} = n_{-1}c_{\mathcal{X}-1} + n_{-1}c_{\mathcal{X}}.$$

And =
$$3c_2 + 3c_3$$
 $n-1c_{s-1} + n-1c_s$



0 ——	0 _{C0}
1 —(1) (1)	1 _{C0} 1 _{C1}
2 — 1 2 1	$2c_0$ $2c_1$ $2c_2$
3 — () 3 3 ()	3 _{C0} 3 _{C1} 3 _{C2} 3 _{C3}
4 — (1) 4 6 4 (1)	4 _{Co} 4 _{C1} 4 _{C2} 4 _{C3} 4 _{C4}
5 — ① 5 10 10 5	5c0 5c1 5c2 5c3 5c4 5c5

Example:

$$5c_2 = 4c_1 + 4c_2$$

 $n_{cr} = n-1c_{r-1} + n-1c_r$
solved using DP.
 $c[i][j] = c[i-1][j-1] + c[i-1][j]$

Qui finding nth column tile (Amazon, 400 gle) Excel cheets 3 25 26 27 28 -- 52 53 54 A B C Y z aa ab az ba bb -- $n = 30 \longrightarrow ad$ 0 - 0 ax n = 50so **a**z 3 26 n = 7979, 25 - x 24 - y 25 - z. n = 78 bz. 50-1 = 49 n = 5026 1-1 = 0 26

$$n = 99$$
. $\frac{26}{26} = \frac{99 - 1}{3 - 1} = \frac{98}{20} = \frac{20}{0}$

```
Pseudwde
                                                        0- A
          String columnTile(int n) {
                                                         2 - \mathcal{L}
    n = 50
                string and = ";
10t its =
 rem = 49%26 = 23 while (n >0) {
  on=x
                      int rem = (n-1) 1/26;
  n = \frac{49}{26} = 1
                      are = getcode(rem) + are;
2nd its
                    n = \frac{n-1}{26};
 rem = 0 \%26 = 0
on = 20
            return reverse (ans);
                       TC: Log 26 n.
                       sc: 0(1)
```

Break: 8:40-8:50

Ged basics

Gcd
$$\rightarrow$$
 Greatest common divisor.

hcf \rightarrow highest common factor

gcd(a,b) \rightarrow greatest factor dividing a d b

If we have $gcd(a,b) = \kappa$.

a $\%$ $x = 0$

b $\%$ $x = 0$

x is highest factor of (a,b)

Example
$$g(d(15,25)) = 5$$
 $g(d(12,30)) = 6$
 $g(d(10,-25)) = 5$
 $g(d(0,-25)) = 5$
 $g(d(0,-10)) = 0$
 $g(d(0,-10)) = 0$
 $g(d(0,-10)) = 0$
 $g(d(0,0)) = 0$

Properties of 4CP

$$gcd(a,b) = gcd(b,a)$$

$$gcd(4,6) \qquad gcd(6,4)$$

$$1$$
2

$$g cd(0, A) = |A| ab 8(A)$$

 $g cd(0, 4)$
 $g cd(0, -4)$

$$gcd(a.b.c) = gcd(a, gcd(b,c))$$

 $gcd(gcd(a,b), c)$
 $gcd(gcd(a,c),b)$

$$gcd(4.8)18) = 2$$

$$gcd(8.4)18) = 2$$

$$gcd(8.18.4) = 2$$

Special property of 4cd

*
$$g(d(a,b)) = g(d(a-b,b))$$
 $a,b>0 & a>=b$

$$a,b>0$$
 { $a>=b$

$$g(d(23,5) = 1)$$
 $g(d(18,5))$
 $g(d(13,5))$
 $g(d(8,5))$
 $g(d(8,5))$
 $g(d(3,5) = 1)$

Ou write a function to find gcd(a.b).

int gcd(a.b) {

if (b==0) {

ycd(120, 30)

return a;

return gcd(b, a.b);

function a; a > breturn a; a > b a > breturn a; a > b

```
Qui given an array, calculate gid of entire array.

a\pi = \begin{bmatrix} 6 & 12 & 15 \end{bmatrix} \longrightarrow gcd(arr) = 3
int gcd Array (arr()) {
an = a\pi(0);
for(i=1; i'(a\pi(ength; i+1));
an = gcd(ans. arr(i));
}
return ans;
```

<u>Ou:</u> Given arr(n) elements. clelete one element such that 4cd of remaining elements becomes maximum.

	0	1	2	3	4	_
varr[] =	24	16	18	30	15	

Delete one element of array.	gcd of remaining el	on.
21 16 18 30 15	gcol (16,18,30,15)	1
0 1 2 3 4	gcd (24, 18, 30, 15)	3
0 1 2 3 4	g col (24.16, 30, 15)	1
0 1 2 3 4	gcd (24,16,18.15)	1
0 1 2 3 4	gcd(24.16,18.30)	2

Boute force: n² log mar (a.b)

Approach

$$arr(i) = 24 | 16 | 18 | 30 | 15$$
 $prefix = 24 | 8 | 2 | 2 | 1$
 $suffix = 1 | 1 | 3 | 15 | 15$

After deleting

Oth ide

 $gud(i) = 1 | 1 | 3 | 15 | 15$

After deleting

Oth ide

 $gud(i6.18.30.15)$
 $gud(i6.18.30.15)$
 $gud(24)$, $gud(i8.80)$, $gud(pf[i-1], cf[i+1])$
 $gud(24)$, $gud(i8.80)$, $gud(pf(0), cf(2))$
 $gud(24.16.30.15)$
 $gud(24.16.30.15)$
 $gud(pf(i), cf(3))$
 $gud(pf(i), cf(3))$

Code: H W

Thankyou (=)