# Confidence Intervals

For $\alpha \in (0,1)$, $100(1-\alpha)\%$ CI:

**1.** For a population mean, $\mu$,

$-$ large sample ($n > 30$) or normal underlying population and $\sigma$ known,

$$\mu \in \left(\overline{x} - \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}},\ \overline{x} - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}\right) = \left(\overline{x} - \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}},\ \overline{x} + \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}}\right),$$

where the quantiles refer to the $N(0,1)$ distribution;

$-$ large sample ($n > 30$) or normal underlying population

$$\mu \in \left(\overline{x} - \frac{s}{\sqrt{n}} t_{1-\frac{\alpha}{2}},\ \overline{x} - \frac{s}{\sqrt{n}} t_{\frac{\alpha}{2}}\right) = \left(\overline{x} - \frac{s}{\sqrt{n}} t_{1-\frac{\alpha}{2}},\ \overline{x} + \frac{s}{\sqrt{n}} t_{1-\frac{\alpha}{2}}\right),$$

where the quantiles refer to the $T(n-1)$ distribution.

**2.** For a population variance, $\sigma^2$, for a normal underlying population,

$$\sigma^2 \in \left(\frac{(n-1)\, s^2}{\chi^2_{1-\frac{\alpha}{2}}},\ \frac{(n-1)\, s^2}{\chi^2_{\frac{\alpha}{2}}}\right),$$

where the quantiles refer to the $\chi^2(n-1)$ distribution.

**3.** For the difference of two population means, $\mu_1 - \mu_2$, for large samples ($n_1 + n_2 > 40$) or normal underlying populations and independent samples,

$-$ $\sigma_1, \sigma_2$ known,

$$\mu_1 - \mu_2 \in \left(\overline{x}_1 - \overline{x}_2 - z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},\ \overline{x}_1 - \overline{x}_2 + z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right),$$

where the quantiles refer to the $N(0,1)$ distribution;

$-$ $\sigma_1 = \sigma_2$, unknown,

$$\mu_1 - \mu_2 \in \left(\overline{x}_1 - \overline{x}_2 - t_{1-\frac{\alpha}{2}} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}},\ \overline{x}_1 - \overline{x}_2 + t_{1-\frac{\alpha}{2}} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right),$$

where the quantiles refer to the $T(n_1 + n_2 - 2)$ distribution and $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$;

$-$ $\sigma_1 \neq \sigma_2$, unknown,

$$\mu_1 - \mu_2 \in \left(\overline{x}_1 - \overline{x}_2 - t_{1-\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},\ \overline{x}_1 - \overline{x}_2 + t_{1-\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right),$$

where the quantiles refer to the $T(n)$ distribution, with

$$\frac{1}{n} = \frac{c^2}{n_1 - 1} + \frac{(1-c)^2}{n_2 - 1} \quad \text{and} \quad c = \frac{\dfrac{s_1^2}{n_1}}{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}.$$

**4.** For the ratio of two population variances, $\dfrac{\sigma_1^2}{\sigma_2^2}$, for normal underlying populations and independent samples,

$$\frac{\sigma_1^2}{\sigma_2^2} \in \left(\frac{1}{f_{1-\frac{\alpha}{2}}} \cdot \frac{s_1^2}{s_2^2},\ \frac{1}{f_{\frac{\alpha}{2}}} \cdot \frac{s_1^2}{s_2^2}\right),$$

where the quantiles refer to the $F(n_1 - 1, n_2 - 1)$ distribution.