

$r = x^i \hat{i} \equiv x^1 \hat{i} + y^1 \hat{j}$ Casos donde las componentes de r transforman como verdaderas componentes de vectores.

1. $(x, y) \rightarrow (-y, x)$

3. $(x, y) \rightarrow (x-y, x+y)$

2. $(x, y) \rightarrow (x, -y)$

4. $(x, y) \rightarrow (x+y, x-y)$

Matriz A

1. $(x, y) \rightarrow (-y, x)$

$\tilde{x} = (0)(x) + (-1)(y)$

$\tilde{y} = (1)(x) + (0)(y)$

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Luego, matriz asociada $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ y $A^T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Multipliquemos y si da como resultado $A \cdot A^T = I$ si cumple! (criterio que aplica siempre).

$$AA^T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Se cumple!

2. $(x, y) \rightarrow (x, -y)$

$\tilde{x} = (1)(x) + (0)(y)$

$\tilde{y} = (0)(x) + (-1)(y)$

$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ y $A^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$AA^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Se cumple!

3. $(x, y) \rightarrow (x-y, x+y)$

$\tilde{x} = (1)(x) + (-1)(y)$

$\tilde{y} = (1)(x) + (1)(y)$

$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ y $A^T = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$$AA^T = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

No es la identidad, $2I$.

No cumple!

4. $(x, y) \rightarrow (x+y, x-y)$

$$x = v(x) + v(y)$$

$$y = v(x) + (-v(y))$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, A^T = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

No Comple