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Given:

- $m\angle A = \frac{7\pi}{36}$ (in radians)
- $m\angle B = 120^\circ$

We need to find $m\angle C$ in both degrees and radians.

Step 1: Convert $m\angle A$ to Degrees

We use the conversion:

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

$$m\angle A = \frac{7\pi}{36} \times \frac{180}{\pi}$$

$$= \frac{7 \times 180}{36}$$

$$= \frac{1260}{36} = 35^\circ$$

Thus, $m\angle A = 35^\circ$.

Step 2: Find $m\angle C$ in Degrees

In a triangle, the sum of the angles is always 180°:

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$35^\circ + 120^\circ + m\angle C = 180^\circ$$

$$m\angle C = 180^\circ - 155^\circ = 25^\circ$$

Step 3: Convert $m\angle C$ to Radians

Using the conversion:

$$m\angle C = 25^\circ \times \frac{\pi}{180}$$

$$= \frac{25\pi}{180} = \frac{5\pi}{36} \text{ radians}$$

Final Answer:

- $m\angle C = 25^\circ$
- $m\angle C = \frac{5\pi}{36}$ radians

(1) Find the length of an arc of a circle which subtends an angle of 108° at the centre, if the radius of the circle is 15 cm.

Let's solve the problem step by step.

Given:

- Central angle $\theta = 108^\circ$
- Radius $r = 15$ cm

Formula for Arc Length:

$$\text{Arc length} = \frac{\theta}{360^\circ} \times 2\pi r$$

Substituting the values:

$$\text{Arc length} = \frac{108}{360} \times 2\pi \times 15$$

$$= \frac{3}{10} \times 30\pi$$

$$= 9\pi$$

$$\approx 9 \times 3.1416$$

$$\approx 28.27 \text{ cm}$$

Final Answer:

The arc length is **9π cm or approximately 28.27 cm.**

6) Find all trigonometric functions of angle in standard position whose terminal arm passes through point (3, −4).

Let's solve the problem step by step.

Given:

A point $(x, y) = (3, -4)$ lies on the terminal arm of an angle in standard position.

Step 1: Find the Hypotenuse r

The hypotenuse r is given by:

$$r = \sqrt{x^2 + y^2}$$

Substituting the given values:

$$r = \sqrt{3^2 + (-4)^2}$$

$$r = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 2: Calculate the Trigonometric Functions

Using the definitions of trigonometric ratios:

1. **Sine:**

$$\sin\theta = \frac{y}{r} = \frac{-4}{5}$$

2. **Cosine:**

$$\cos\theta = \frac{x}{r} = \frac{3}{5}$$

3. **Tangent:**

$$\tan\theta = \frac{y}{x} = \frac{-4}{3}$$

4. **Cosecant (Reciprocal of Sine):**

$$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$$

5. **Secant (Reciprocal of Cosine):**

$$\sec\theta = \frac{r}{x} = \frac{5}{3}$$

6. **Cotangent (Reciprocal of Tangent):**

$$\cot\theta = \frac{x}{y} = \frac{3}{-4} = -\frac{3}{4}$$

Final Answer:

$$\sin\theta = -\frac{4}{5}, \quad \cos\theta = \frac{3}{5}, \quad \tan\theta = -\frac{4}{3}$$

$$\csc\theta = -\frac{5}{4}, \quad \sec\theta = \frac{5}{3}, \quad \cot\theta = -\frac{3}{4}$$

If $\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{1}{5}$ and A, B are angles in the second quadrant then prove that $4\cos A + 3\cos B = -5$.

Let's solve the problem step by step.

Given:

$$\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{1}{5}$$

$$\Rightarrow \sin A = \frac{3}{5}, \quad \sin B = \frac{4}{5}$$

Also, A and B are in the second quadrant.

Step 1: Find $\cos A$ and $\cos B$

Using the identity:

$$\sin^2\theta + \cos^2\theta = 1$$

For A:

$$\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{3}{5}\right)^2$$

$$= 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos A = \pm\frac{4}{5}$$

Since A is in the second quadrant, $\cos A$ is negative:

$$\cos A = -\frac{4}{5}$$

For B:

$$\cos^2 B = 1 - \sin^2 B = 1 - \left(\frac{4}{5}\right)^2$$

$$= 1 - \frac{16}{25} = \frac{9}{25}$$

$$\cos B = \pm\frac{3}{5}$$

Since B is in the second quadrant, $\cos B$ is negative:

$$\cos B = -\frac{3}{5}$$

Step 2: Compute $4\cos A + 3\cos B$

$$4\cos A + 3\cos B = 4 \times \left(-\frac{4}{5}\right) + 3 \times \left(-\frac{3}{5}\right)$$

$$= -\frac{16}{5} - \frac{9}{5}$$

$$= -\frac{25}{5} = -5$$

Conclusion:

$$4\cos A + 3\cos B = -5$$

Memory Full ⓘ

4) If $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$, prove that $A+B = \frac{\pi}{4}$

We are given:

$$\tan A = \frac{5}{6}, \quad \tan B = \frac{1}{11}$$

We need to prove:

$$A + B = \frac{\pi}{4}$$

Step 1: Use the Tan Addition Formula

We use the identity:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Substituting the given values:

$$\tan(A+B) = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \left(\frac{5}{6} \times \frac{1}{11}\right)}$$

Step 2: Simplify the Numerator

$$\frac{5}{6} + \frac{1}{11} = \frac{5 \times 11 + 1 \times 6}{6 \times 11} = \frac{55 + 6}{66} = \frac{61}{66}$$

Step 3: Simplify the Denominator

$$1 - \left(\frac{5}{6} \times \frac{1}{11}\right) = 1 - \frac{5}{66} = \frac{66}{66} - \frac{5}{66} = \frac{61}{66}$$

Step 4: Compute $\tan(A+B)$

$$\tan(A+B) = \frac{\frac{61}{66}}{\frac{61}{66}} = 1$$

Since $\tan \frac{\pi}{4} = 1$, we conclude:

$$A + B = \frac{\pi}{4}$$

Conclusion:

Thus, we have proved that:

$$A + B = \frac{\pi}{4}$$

7i) $x+y+z=6,\ x-y+z=2,\ x+2y-z=2$

The given system of equations is:

1. $x+y+z=6$
2. $x-y+z=2$
3. $x+2y-z=2$

Step 1: Solve for x, y, z

We solve this system using elimination or substitution.

Step 1.1: Add Equation (1) and (2)

$$(x+y+z)+(x-y+z)=6+2$$

$$2x+2z=8$$

$$x+z=4 \quad \text{(Equation 4)}$$

Step 1.2: Solve for x in terms of z

From Equation (4):

$$x=4-z$$

Step 1.3: Substitute $x=4-z$ in Equation (3)

$$(4-z)+2y-z=2$$

$$4+2y-2z=2$$

$$2y-2z=-2$$

$$y-z=-1 \quad \text{(Equation 5)}$$

Step 1.4: Solve for y in terms of z

$$y=z-1$$

Step 1.5: Substitute $x=4-z$ and $y=z-1$ in Equation (1)

$$(4-z)+(z-1)+z=6$$

$$4-z+z-1+z=6$$

$$3+z=6$$

$$z=3$$

Step 1.6: Find x and y

$$x=4-z=4-3=1$$

$$y=z-1=3-1=2$$

Final Answer

$$\boxed{x=1, y=2, z=3}$$

If A(1,3) and B(2,1) are points, find the equation of the locus of point P such that PA = PB.

We are given two points:
$A(1,3)$ and $B(2,1)$, and we need to find the equation of the locus of point $P(x,y)$ such that $PA = PB$.

Step 1: Use the Distance Formula

The distance formula between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Since $PA = PB$, we equate the distances:

$$\sqrt{(x-1)^2 + (y-3)^2} = \sqrt{(x-2)^2 + (y-1)^2}$$

Step 2: Square Both Sides

$$(x-1)^2 + (y-3)^2 = (x-2)^2 + (y-1)^2$$

Expanding both sides:

$$(x^2 - 2x + 1) + (y^2 - 6y + 9) = (x^2 - 4x + 4) + (y^2 - 2y + 1)$$

Cancel x^2 and y^2 from both sides:

$$-2x + 1 - 6y + 9 = -4x + 4 - 2y + 1$$

Step 3: Simplify

$$-2x - 6y + 10 = -4x - 2y + 5$$

Rearrange:

$$-2x - 6y + 10 + 4x + 2y - 5 = 0$$

$$2x - 4y + 5 = 0$$

Final Answer

$$\boxed{2x - 4y + 5 = 0}$$

or

$$\boxed{x - 2y + \frac{5}{2} = 0}$$

Find equations of lines which contains the point $A(1,3)$ and the sum of whose intercepts on the co–ordinate axes is zero.

We need to find the equation of a line that:

1. Passes through the point $A(1,3)$.
2. Has the sum of its intercepts on the coordinate axes equal to zero.

Step 1: General Form of the Line with Given Condition

The equation of a line in intercept form is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the x-intercept and y-intercept, respectively. The given condition states:

$$a + b = 0 \quad \Rightarrow \quad b = -a$$

Thus, the equation of the line becomes:

$$\frac{x}{a} + \frac{y}{-a} = 1$$

Multiplying both sides by a, we get:

$$x - y = a$$

Step 2: Use the Given Point (1,3)

Since the line passes through $(1,3)$, substitute $x = 1$ and $y = 3$ into the equation:

$$1 - 3 = a$$

$$a = -2$$

Step 3: Find the Equation

Substituting $a = -2$ back into the equation:

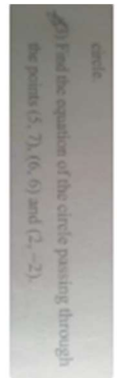
$$x - y = -2$$

or equivalently,

$$x - y + 2 = 0$$

Final Answer:

$$\boxed{x - y + 2 = 0}$$



To find the equation of a circle passing through three given points $(5, 7)$, $(6, 6)$, and $(2, -2)$, we use the general equation of a circle:

$$x^2 + y^2 + Dx + Ey + F = 0$$

Step 1: Set Up the System of Equations

Substituting each point into the equation:

1. **For (5,7):**

$$5^2 + 7^2 + 5D + 7E + F = 0$$

$$25 + 49 + 5D + 7E + F = 0$$

$$5D + 7E + F = -74$$

2. **For (6,6):**

$$6^2 + 6^2 + 6D + 6E + F = 0$$

$$36 + 36 + 6D + 6E + F = 0$$

$$6D + 6E + F = -72$$

3. **For (2,-2):**

$$2^2 + (-2)^2 + 2D - 2E + F = 0$$

$$4 + 4 + 2D - 2E + F = 0$$

$$2D - 2E + F = -8$$

Step 2: Solve for D, E, and F

We now have the system of equations:

1. $5D + 7E + F = -74$
2. $6D + 6E + F = -72$
3. $2D - 2E + F = -8$

Solving this system will give us D, E, and F. Let me compute the values.

The solution to the system of equations is:

$$D = -4, \quad E = -6, \quad F = -12$$

Step 3: Write the Equation of the Circle

Substituting these values into the general equation of a circle:

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

This is the required equation of the circle passing through the given points. [>-]