**CMPT 440 – Spring 2020: Quantum Finite Automata**

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# Theoretical Background

The quantum finite automata is define by the 15-tuple that is: $T = (\Sigma, Q, q_0, P, \alpha)$.

$$P = \frac{1}{R}\sum_{i-0}^{N} x - \delta(1,0)$$

QFAs are exponentially more space efficient when compared to a DFA while recognizing the same language. They are considered as a quantum counterpart of the usual DFAs. This means that they are typically more powerful than DFAs because a broader range of operations can be established due to the implementation of quantum mechanics. There are two types of QFAs, which are 1-way QFAs and 2-way QFAs. 1-way QFAs will recognize regular languages whereas 2-way QFAs will recognize non-regular languages. When comparing the two types, 2-way QFAs are more powerful because it allows superposition where the head of QFA can be at different locations.

# An Example

This is an example of a 1-way QFA from Ambainis and Freivalds. Alphabet $\sum = \{a\}$, state space is $Q = \{q_0, q_1, q_{acc}, q_{rej}\}$, accepting states $Q_{acc} = \{q_{acc}\}$ and rejecting states $Q_{rej} = \{q_{rej}\}$. The transition function will be defined by $V_x$ where $x \in \Gamma$. $V_x$ will be defined as:

$$V_a(|q_0\rangle) = \frac{1}{2}|q_0\rangle + \frac{1}{2}|q_1\rangle + \frac{1}{\sqrt{2}}|q_{rej}\rangle,$$

$$V_a(|q_1\rangle) = \frac{1}{2}|q_0\rangle + \frac{1}{2}|q_1\rangle - \frac{1}{\sqrt{2}}|q_{rej}\rangle,$$

$$V_\$(|q_0\rangle) = |q_{rej}\rangle, V_\$(|q_1\rangle) = |q_{acc}\rangle.$$

With the word aa, the automaton starts in $|q_0>$ then Va is applied, resulting in ½| $q_0>$ + ½$|q_1>$ + 1/√2$|q_{rej}>$. Ther2e are then two possible outcomes which are rejecting state where the superposition collapse to $|q_{rej}>$ then the word is rejected and a non-halting state where the superposition collapse to ½$|q_0>$ + ½$|q_1>$ then the computation continues. These two possibilities have a probability of ½. ½|q0> + ½|q1 is mapped to itself by Va, which a non-halting state is observed. This superposition does not have an accepting or rejecting states. After this, the word ends and the transformation $V_\$$ corresponding to the right endmarker $ is done, mapping the superposition to ½|qrej> + ½|qacc>. With the probability of ¼ for both, either the rejecting state $q_{rej}$ or the accepting state $q_{acc}$ is observed. This means that the total probability of accepting is ¼ whereas the probability of rejecting is ¾.

# References

Ambainis, Andris. (2011). “Quantum Finite Automata”. ResearchGate. January 2011.

www.researchgate.net/publication/221443789_Quantum_Finite_Automata

Ambainis, Andris, and Rusins Freivalds. “1-Way Quantum Finite Automata: Strengths, Weaknesses and Generalizations.” *Cds*, cds.cern.ch/record/346937/files/9802062.pdf.